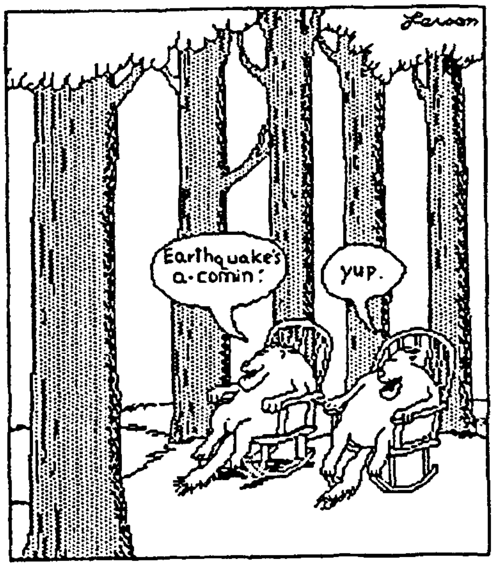
Insurance Futures



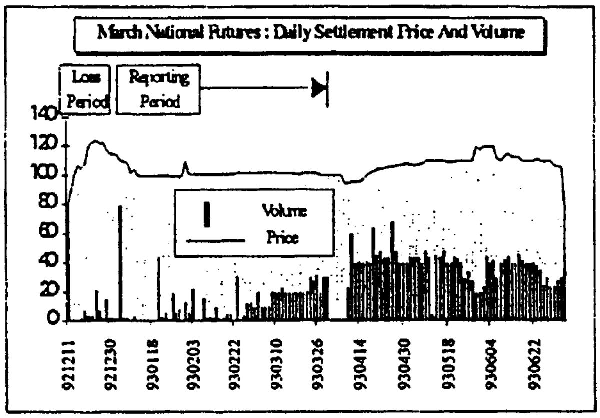
The Browns relaxed, knowing they had hedged their losses with insurance futures.

Members of the Working Party:

John Ryan (Chairman)
Helen Murphy
Peter Clark
Michael Lomax
Julian Lowe
Stephen Rix

1. Executive Summary

On 11 December 1992 trading commenced in Insurance futures at the Chicago Board Of Trade. The graph shows the trading experience of the March National futures contract in the first 6 months after launch. Two points stand out. Firstly, the futures price had doubled before the end of 1992 while the market found its feet. Secondly, the trading volumes were initially quite low but have picked up over the summer.



The launch of the CBOT contracts could not have been better timed. Global capacity in the reinsurance market was low after unprecedented losses in the late 1980's and early 90's. Hurricane Andrew had just created the largest insured natural catastrophe loss in USA history, at over $15bn.

As a proxy for conventional reinsurance, Insurance futures and options should have had plenty of natural buyers.

So why were the initial trading volumes so low? We offer the following reasons :

- regulatory obstacles
- steep learning curves
- lack of interested sellers
- imperfect risk transfer

There are barriers to the market on both sides: the finance specialists and insurers were one culture separated by two languages.

One objective of the Working Party has been to overcome the learning curve that investors face, be they speculator or hedger.

We consider the uses of Insurance and futures and options contracts as proxies for reinsurance - futures behave like proportional reinsurance whereas options behave like non-proportional reinsurance.

We have also looked at pricing, although much work remains to be done in this area (but will it be published?!).

The dynamics of an Insurance futures market have been a particular interest for us, since the survival of the CBOT contracts appears to lie in the balance at the time of going to press. There are valuable lessons to be learnt from other futures markets.

We hope that the outcome of CBOT's exciting initiative is positive. We are encouraged by the recent increase in trading volume and the fact that LIFFE is currently considering a UK / European contract, although few details are yet available.

In any event, it seems that the convergence of finance and insurance is unstoppable. We welcome this trend and believe it will offer exciting challenges as well as some threats for the actuarial profession.

Contents

2. The CBOT Contracts And Their Experience To Date

2.0 Overview

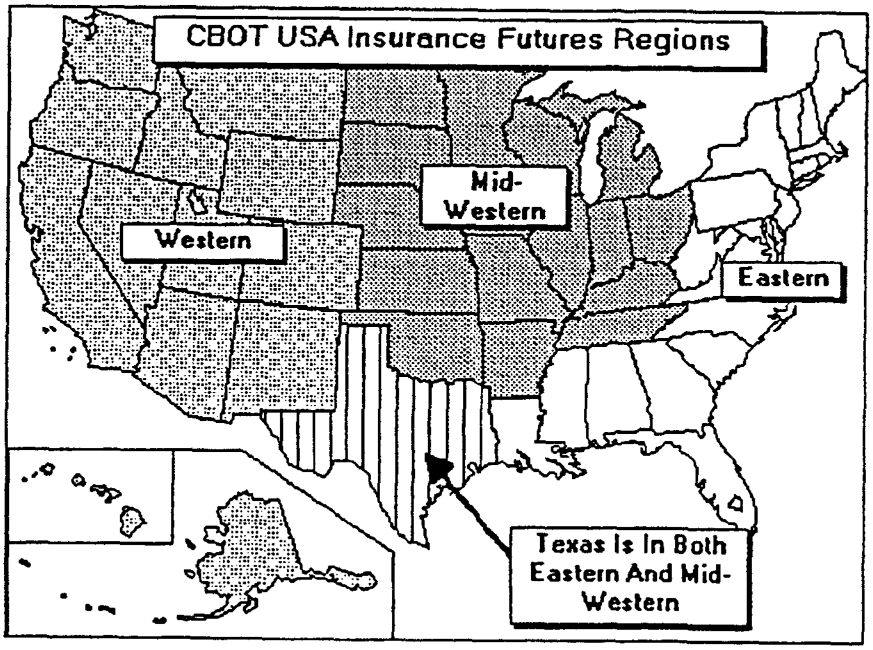
Section 2 summarises the Insurance futures and options contracts traded on the Chicago Board of Trade (CBOT) exchange. Details of the trading to date and further technical information regarding the contracts are provided in Appendices I and II.

2.1 The CBOT Insurance Futures And Options Contracts

Regions Covered

Insurance futures and options can be purchased covering three different regions of the United States: Eastern, Mid-Western and Western. There is also a National contract covering the whole of the United States.

The following graph shows the states covered by each contract:



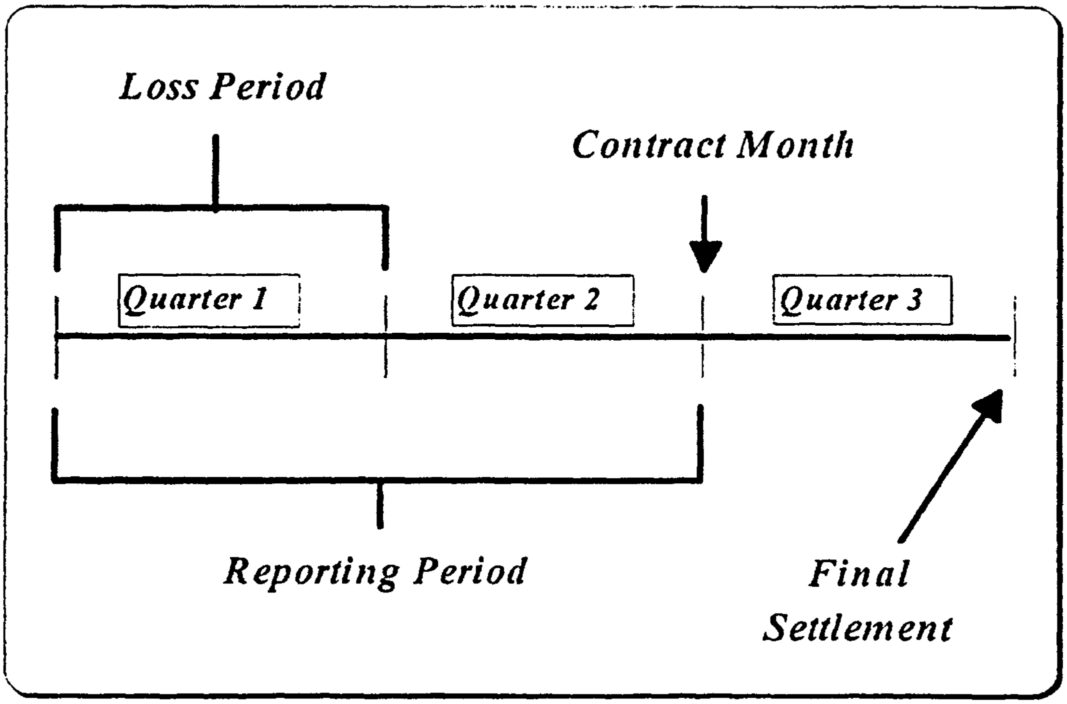
Period Of Loss

For each region, futures and options contracts are available in respect of losses incurred during one of a number of calendar quarters. Each calendar quarter is identified by the "contract month". The contract months being traded at the time of writing are as follows:

Contract Month	Loss Period
Sep 93	Apr - Jun 93
Dec 93	Jul - Sep 93
Mar 94	Oct - Dec 93

Trading And Loss Reporting Period

Trading commences at an ad hoc date, well in advance of the loss period, as decided by CBOT. Trading ceases six months and five days after the end of the loss period. Losses reported during the loss period and during the three months following the end of the loss period are included in the calculation of the settlement value. This information is summarised for the contracts currently trading in Appendix II.1. The following graph summarises the trading and loss reporting periods:



Contract Value

The futures contract value depends on an index reflecting the ratio of catastrophe losses to estimated premium:

$$\text{Index Value of Future} = \$25{,}000 \times \text{Reported Loss Ratio}$$

$$= \$25{,}000 \times \frac{(\text{Incurred Catastrophe Losses})}{(\text{Estimated Property Premium})}$$

where the estimated property premium is based on the most recent statutory annual statements filed by the ISO reporting companies.

Prices are quoted in tenths of a point (i.e.: 0.1% of $25,000 or $25). Thus a price of 5.2 points is equivalent to 0.052 x 25,000 = $1,300, and the minimum amount by which the price of a contract can move (the Unit of Trading) is $25.

The final Settlement Value will be determined at the close of the last trading day. An interim value will also be computed as at the end of the loss quarter.

Compilation Of Loss Data

ISO DATA Inc. a subsidiary of The Insurance Services Office Inc. (ISO) compiles loss data for certain causes of loss for certain lines of business. Essentially the losses arising from Wind, Hail, Earthquake, Riot, and Flood are recorded. Details of the loss causes for each line of business are shown in Appendix II.2.

We understand that losses included in the Settlement Value calculation are not restricted to events with aggregate losses in excess of a given threshold. It is assumed that the catastrophe element of the losses recorded will be equal to a pre-stated percentage of all losses arising from the specified causes.

The information is provided by a number of reporting companies. ISO DATA select the reporting insurers based on insurer size, diversity of property insurance business, and the receipt of complete and usable loss data on a quarterly or monthly basis in a form specified by the Insurance Services Office.

ISO DATA select at least 10 insurance company groups to be the reporting insurers. A list of the reporting companies to be used for any contract month will be announced by the exchange prior to the listing of that contract month.

Weighting Of Premium and Losses

The distribution of companies who report to ISO DATA is not entirely representative of the US as a whole. In order to achieve more representative national and regional results ISO DATA will weight both premium and losses by state and line. Weights are announced prior to the start of trading in any contract month, and will not be changed during the life of the contract.

Restrictions On Price Movements

To maintain an orderly market the CBOT impose restrictions on the maximum value of a futures contract and the daily price movement. Furthermore, the CBOT has a reporting limit of 100 contracts in one month, and limits the maximum net long or short position that any person may own, control, or carry to 10,000 contracts.

Further details of the maximum daily price movement and Future contract value are given in Appendix II.4

Security

At the end of each day's trading the Board of Trade Clearing Corporation (BOTCC) substitutes itself as the opposite party to every transaction. That is it interposes itself as the buyer to every clearing member seller, and the seller to every clearing member buyer. Consequently, a buyer does not need to locate the original seller.

The BOTCC operates a margin system whereby the clearing members are required to maintain a deposit with the Futures Commission Merchants in respect of their open positions. The margin system has two components; the initial margin, which is in effect a deposit, and a variation margin.

At the end of each day each clearing member's account is marked to market. In this process the BOTCC collects money from those that lost money and redistributes it to those that gained. The amount collected is known as the variation margin. Failure to pay the required variation margin in cash by 10 am the following day results in the investors position being closed out. The BOTCC guarantees the performance of every contract.

A result of the margining system and the BOTCC's guarantee is that the contracts should be highly secure. The CBOT emphasise that since the start of this system, in 1925, no customer has lost money due to default on a futures position.

2.2 Reasons For Launching Insurance Futures And Options Contracts

The launch was a result of marketing by CBOT, not demand from insurance companies. So why did CBOT decide to set up a market in insurance futures in the first place? The reason is quite simple: financial reward. Futures exchanges throughout the world are in a continual race to discover new products that will generate large trading volumes and, therefore, large profits for the exchange.

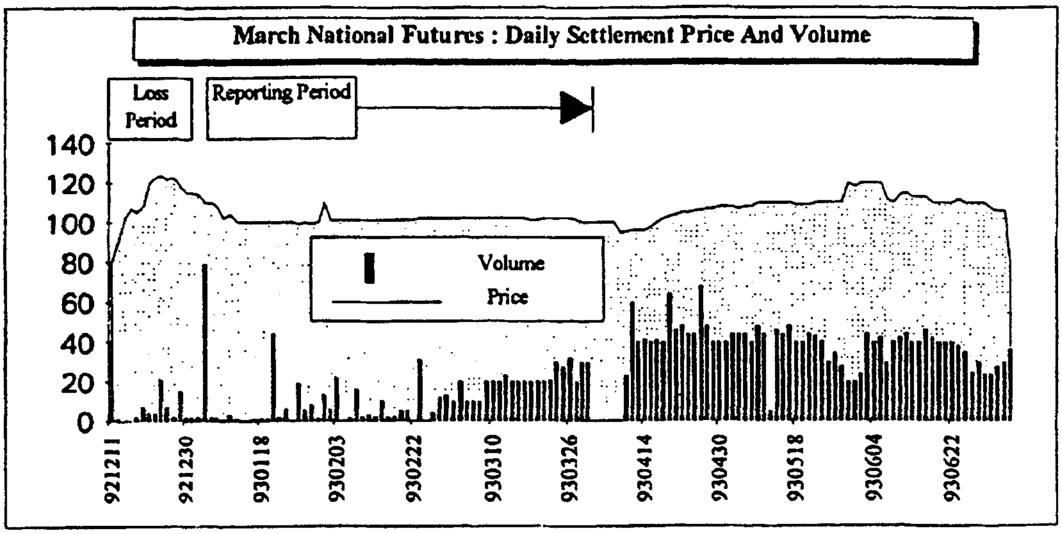
The stakes are very high. Growth in the world's derivative markets has been truly phenomenal in the last 20 years. According to the Bank for International Settlements, there were $4.5 trillion-worth of outstanding derivatives contracts (of all types) traded on exchanges (such as the CBOT and London's LIFFE) as at the end of 1992.

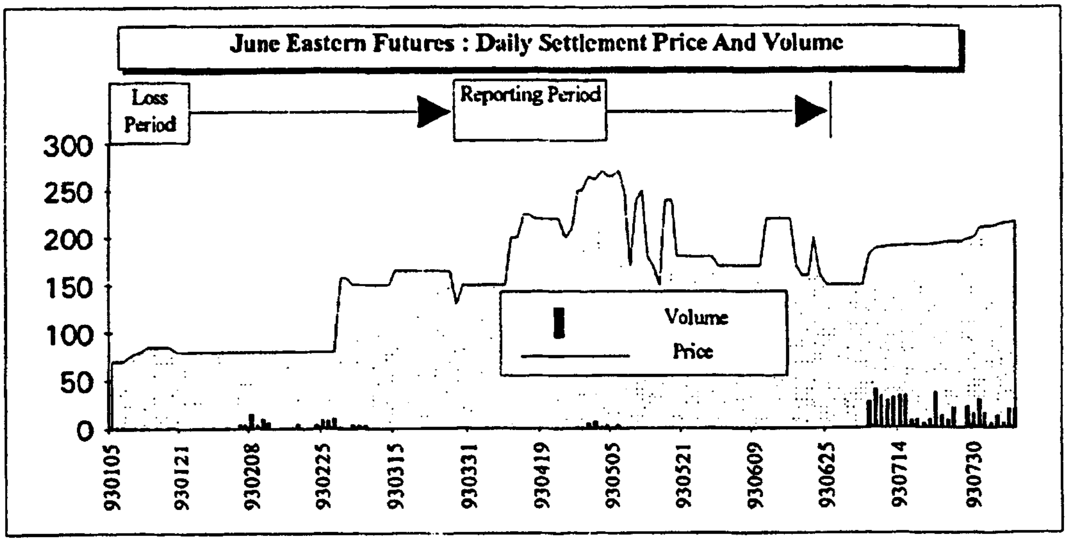
[What is total world-wide insurance premium volume and cat premium volume by way of comparison ?]

On the downside, the failure rate of new futures contracts is quite sobering (see Carlton (1984)). Products that fail to achieve adequate trading volumes are mercilessly shelved. The majority of futures contracts are withdrawn within 10 years of their introduction. Not surprisingly, the longevity of new contracts is quite skewed, since those products that do succeed last for many years. One third of all new contracts do not make it beyond 2 years however.

2.3 The Trading History Of CBOT Catastrophe Futures And Options

The following graphs show the futures prices and volumes for the March National and June Eastern contracts:





It can be seen that although the number of trades per day has been increasing, there is still very little trading.

2.4 The Benefits Of A Market In Insurance Futures And Options

We need to distinguish between the benefits to speculators and hedgers. For the natural hedgers - the insurers and reinsurers - a market in Insurance futures offers two potential benefits, namely risk transfer and price discovery.

The risk transfer benefit is very familiar, since conventional reinsurance achieves this already. However, conventional reinsurance may be unacceptably expensive or it may not be available at all. An open, liquid, competitive market in reinsurance based on standardised, guaranteed contracts could potentially attract new capital from outside the insurance industry and provide guaranteed capacity at a fair price.

Some have suggested that the attraction of capital from outside the insurance industry via an organised market could dampen the underwriting cycle. The agency relationships in traditional reinsurance could be broken. Traditional long term relationships between reinsurer and cedant, founded on mutual trust and retrospective pricing mechanisms, could give way to an era in which normalising pricing mechanisms cannot be relied upon because the barriers to the market have been torn down.

For the speculator, an Insurance futures and options market offers the opportunity of reaping spectacular rewards from insurance without making a long term commitment to the business and without incurring the up-front fixed expenses of overcoming regulatory obstacles and developing the necessary operational expertise. There are times when the risk-reward relationship in the traditional reinsurance market is enough to tempt even the most risk averse speculator.

2.5 LIFFE's Reaction To The CBOT initiative

The working party are not aware of any plans to issue a corresponding contract to the North American Insurance futures and options contracts. However there is a working party, including some of the major brokers and some ABI members, which is looking at the possibility of putting together a European index. This would allow a similar contract to be offered in Europe to cover European catastrophes. Most of the major London futures brokers would have no problems in actually organising trades in North American futures and options contracts. Consequently from a reinsurance point of view there is no particular need to have a special facility in the UK for US exposures.

2.6 A Comparison Of Insurance Futures With Other Futures Contracts

We can learn a lot about Insurance futures by comparing them to existing futures products and markets. Experience of other products and markets can point towards answers on questions such as :

What makes a good Insurance futures contract ?

How should we value Insurance futures and options ?

Commodity Futures And Financial Futures

Futures are traded on a very large number of commodities and financial assets on exchanges throughout the world, the largest of which are the Chicago Board Of Trade, the Chicago Mercantile Exchange and LIFFE.

The commodities include types of grain and oilseeds, livestock and meat, food and fibre, metals and petroleum and lumber. Financial "assets" include currencies, bonds and various securities indices (although in the case of an index there is no physical asset).

An Insurance future should strictly be labelled a financial future since it is based on an underlying financial "asset", namely the value of an index of insurance losses. However, the characteristics of an Insurance futures market are perhaps more akin to a market in commodity futures than a market in financial futures. A market in commodity futures responds to the supply and demand characteristics of a particular industry segment that meets its risk transfer needs by buying and selling in the market.

Hedgers And Speculators

All futures markets are made up of hedgers and speculators. Hedgers are meeting their risk transfer needs and speculators are in it for the profit. The characteristics of a futures market are determined by the existence or otherwise of active hedgers on the buy and / or sell side.

A market in Insurance futures and options is likely to be dominated by natural hedgers - in this case insurers and reinsurers - meeting their specific risk transfer needs. Compare this to other financial futures, where the buyers and sellers are very diverse and numerous and speculators play an active part in the market.

One could say that an insurance future is a future in the commodity "insurance capital".

Assets Held For Investment Versus Assets Held For Consumption

Many commodity futures are held for the purpose of consumption. Copper futures are an example. Financial futures on the other hand are based on underlying financial assets that are generally held for the purpose of investment by a significant number of investors. Bond futures are an example. Some commodity futures such as futures in gold and silver are also held by a significant number of investors for the purpose of investment, rather than consumption.

The distinction between futures on assets held for different purposes - investment or consumption - is an important one, much more important than the "commodity"/ "financial" label. Insurance futures are assets held for consumption - the consumption of "insurance capital". This distinction is discussed further in section 4.1.

3. Hedging And Market Players

3.0 Overview

Section 2 described what Insurance futures are, this section describes, politely, what you can do with them and who might want to do it, or not.

3.1 Who Might Do What And Why ?

The main potential doers are insurers, reinsurers, and speculators. As with any such contract, professional "market makers" are needed to provide the buffer between buyers and sellers.

There are two main things an insurer might want to do with them. First they might want to buy Insurance futures to "hedge" away, that is reduce, the insurance risk they are exposed to for some of their portfolio; this is akin to Quota Share reinsurance. Secondly, they might want to protect themselves against catastrophic losses beyond a certain amount by buying Insurance options; this is akin to Excess of Loss Reinsurance.

In a similar vein, reinsurers might want to sell Insurance futures to allow them to write profitable high level reinsurance covers whilst protecting their capital position. They may also want to buy Insurance options as a proxy for reinsuring their own portfolio.

Speculators might sell or buy Insurance futures or options to make money. On a slightly less blunt level, they can use them to diversify their portfolio and obtain an exposure to insurance risk. They can do this without going through the troublesome business of setting up an insurance company, or, throwing caution to the wind, buying shares in an insurance company.

3.2 How Do They Do It ?

Doing It With Futures - The Theory

Consider someone who has taken out an Insurance futures contract to buy at a price X. The holder will make a profit if the price goes up above X and a loss if the price goes below X. Now the price of an Insurance future increases as the amount of Catastrophe losses for a quarter increases. This means that the holder will make a profit if Catastrophe losses exceed a certain amount and a loss if they are less than a certain amount. This is basically good news for insurers ! If they can pitch the number of Insurance futures and the price at which they are taken out, so that the gains on the Insurance futures balance out the losses due to greater than expected catastrophes, they can immunise themselves against higher than expected Catastrophe losses - they have "hedged" away that part of their exposure to Catastrophe losses.

There are two downsides to the hedging process. The first is that they immunise the hedger against lower than expected Catastrophe losses - if Catastrophe losses are lower than expected they make a loss on their Insurance futures position, which cancels out the "gain" they would otherwise have made. The second is that the "If" in "If they can pitch the number of Insurance futures and the price...." is really more of an **"If"** More on this in section 3.3.

Doing It With Futures - The Practice

A simple example illustrates how a hedge might be achieved. Take the industry as a whole as having a premium base of $3,000m per quarter (for lines of business that are exposed to Catastrophes) and expecting ultimate Catastrophe losses of $400m per quarter for the third quarter. Consider an insurer whose portfolio represents precisely one percent of the market as a whole, i.e. premiums of $30m and expected ultimate Catastrophe losses of $4m. Assume that by the end of the reporting period for the Futures Contract, 75% of the ultimate losses have been reported.

At or before the start of the quarter, we expect that the Catastrophe loss ratio for the July-September quarter will be 4/30, and that only 75% of this will have been reported by the end of December. The price one puts on the Insurance future is therefore:

$25,000 x 4 / 30 x 0.75 = $2,500

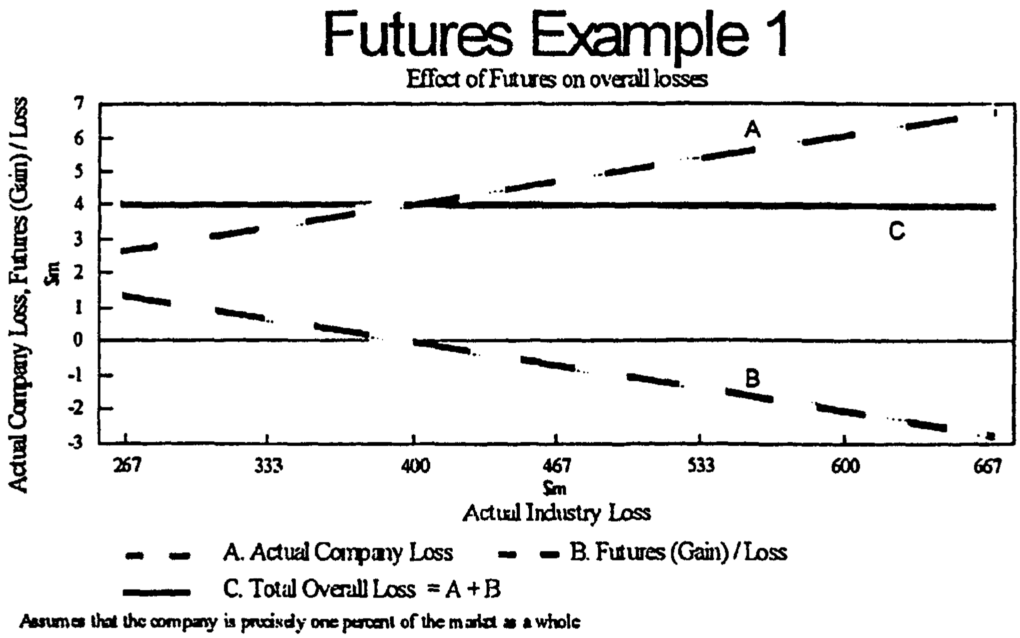
$25,000 is the base price of the contract, equivalent to a 100% loss-ratio. The number of contracts the company needs to take out is such that the futures gains / losses tie to the company's own premium base, that is:

$30,000,000 / $25,000 / 0.75 = 1,600 contracts.

If the Catastrophe losses are more than one assumed, the gain from the Insurance futures offsets those losses, and vice versa. For example, if the actual industry ultimate losses are $600m, we have the following situation:

Ultimate Industry losses:	$600m (and so Company losses of $6m)
Reported Industry losses by Dec.:	$450m
Futures Settlement Price:	$3,750 = 25,000 x 450 / 3,000
Gain per Futures Contract:	$1,250
Total Company Gain:	$2m = $1,250 x 1,600
Company losses - Futures Gain:	$4m = $6m - $2m

Thus the actual losses were $2m higher than expected, but this amount has been off-set by the futures gain. To help the reader see how this logic works for other loss sizes, Appendix III, Futures Example 1, shows the resultant gains and losses for a range of Industry losses. The results are summarised graphically as follows:



As the above graph shows, whatever the actual size of industry losses, assuming that the company has losses of exactly one per cent of the Industry, the overall losses (= Cat losses + Futures Gain / (Loss)) are now fixed at the $4m level the company expected. It has immunised itself against higher (or lower) than expected losses.

Companies that expect to have loss-ratios higher or lower than the Industry as a whole can also achieve a hedge by adjusting the number of contracts they purchase. Futures Examples 2 and 3 in Appendix III illustrate how this can be done. Companies may if they wish hedge more or less than 100% of their catastrophe risk by buying more or less contracts. If a company has a different reporting speed than the Industry as a whole, they will have to use the Industry reporting speeds when calculating the number of contracts, and their own reporting speeds when estimating their own losses, paying particular reference to the estimated reporting speeds of large Catastrophe losses, such as those arising from a hurricane.

The problem with hedging in this fashion arises because an individual company is not precisely matched to the Industry as a whole: the company may suffer large losses when as a whole lower than average losses are experienced and vice versa. These difficulties are discussed in section 3.3

Doing It With Options

Doing It With Options - The Theory

Consider someone who has bought an Insurance option at an Exercise price of X. The buyer will make a profit if the price goes up above X, and will make no money at all if the price goes below X. The price is in this case that of the Insurance future, and so will increase as the amount of Catastrophe losses for a quarter increases. This means that the holder will make a profit if Catastrophe losses exceed a certain amount, but make no money at all if Catastrophe losses are below a certain amount. Again, the trick is to pitch the number of Options and the price at which they are exercisable, so that the profits on the Insurance options balance out the losses due to Catastrophes beyond a certain size. If they can do this, insurers have effectively obtained a proxy for Excess of Loss reinsurance, covering losses above a certain size.

Again, there are practical difficulties, as it is not possible to exactly match the company's portfolio with that of the Industry as a whole. The difficulties this throws up are discussed in section 3.3.

Doing It With Options - The Practice

A simple example illustrates how buying an Insurance option protects a company against Catastrophe losses in excess of a certain amount. Take the same industry and company data considered in the Insurance futures example above. Say the company wishes to protect itself against Catastrophe losses in excess of 20% of its premium. The price at which one would which to have an Insurance Option exercising is:

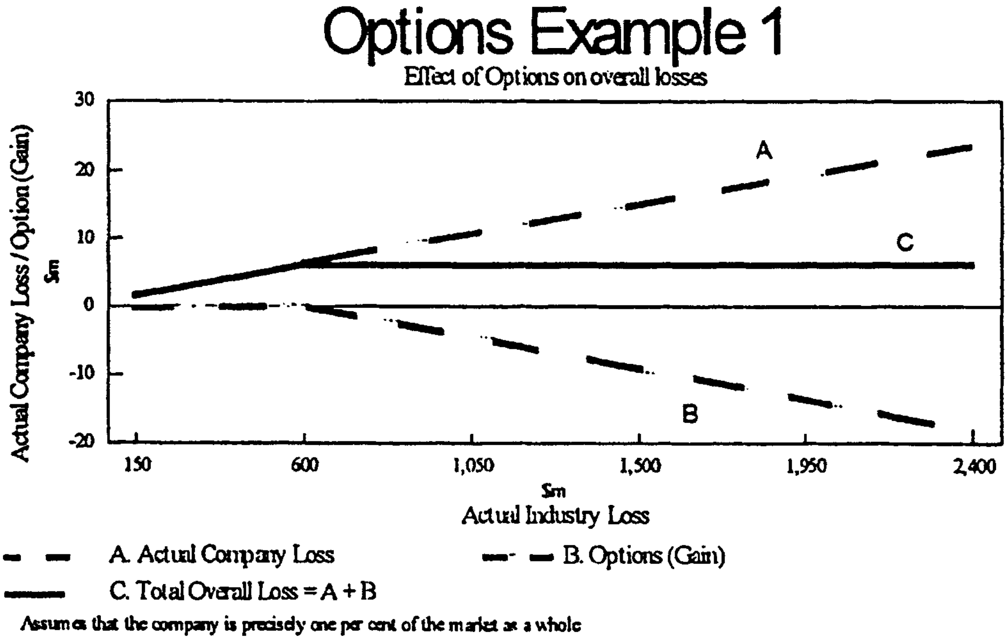
$2,500 x 0.20 / (4 / 30) = $3,750 ($2,500 is our current estimate of the futures price)

The number of contracts the company would wish to purchase is the same as for the futures example, namely 1,600

If the Catastrophe losses for the industry (and hence the company in this example) are greater than 20% of the premium, the company will make a gain on the Insurance option which offsets losses in excess of 20% ($6m in this case). For example, if the actual industry ultimate losses are $1,200m, we have the following situation:

Ultimate Industry losses:	$1,200m (and so Company losses of $12m)
Reported Industry losses by Dec.:	$900m
Futures Settlement Price:	$7,500 = 25,000 x 900 / 3,000
Gain per Options contract:	$3,750
Total Company Gain:	$6m = $3,750 x 1,600
Company Losses-Options Gain:	$6m = $12m - $6m

Thus the actual losses were $6m higher than $6m but the amount in excess of $6m has been off-set by the options gain. To help the reader see how this logic works for other loss sizes, Appendix III, Options Example 1, shows the resultant gains for a range of Industry losses. The results are summarised graphically as follows:

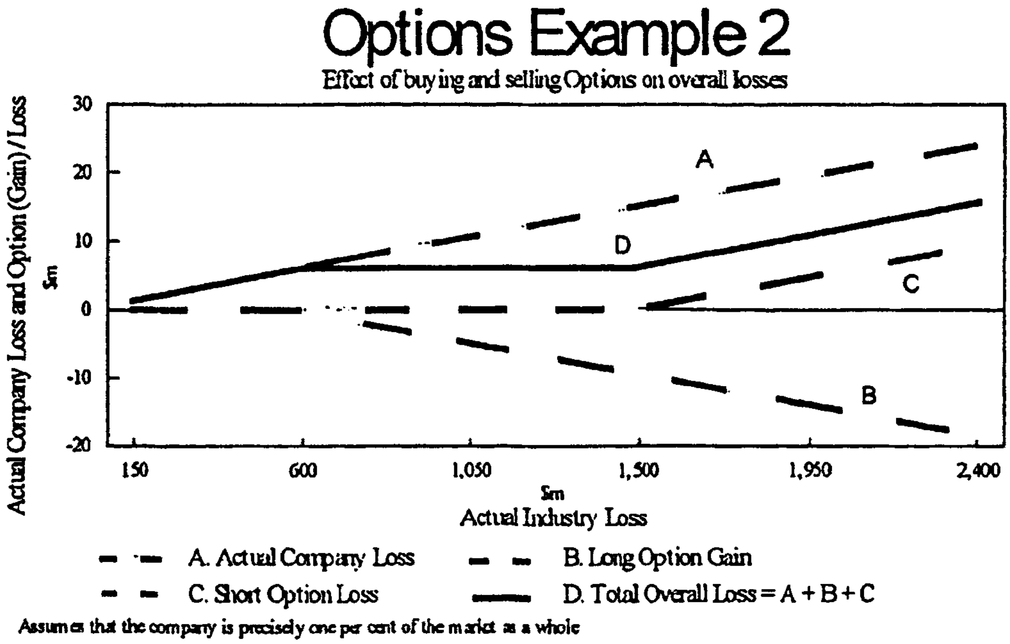


As the above graph shows, whatever the actual size of Industry Catastrophe losses, the company has now limited its' Catastrophe losses to a maximum of 20% of its' premium ($6m). It has of course had to pay for this privilege - the cost of buying the Insurance option.

Companies that expect to have different loss-ratios than the Industry as a whole can adjust the number of contracts of the Insurance option in a similar fashion to that illustrated in Futures Examples 2 and 3 in Appendix III. Similarly, if a company has a different reporting speed than the Industry as a whole, they will have to use the Industry reporting speeds when calculating the number of contracts, and their own reporting speeds when estimating their own losses, as before. In practice a company will not have losses that are a precise multiple of those of the Industry as a whole which will affect the efficiency of this method of protecting against losses, as discussed in section 3.3.

The protection purchased by such an Insurance option goes up to the 200% Loss-ratio level, at which losses are capped - in practice then, this is effectively an "open layer". Even Hurricane Andrew did not lead to the 200% level being breached for the Eastern Contract for the simulated futures prices calculated by CBOT for that quarter.

It is possible to obtain a proxy for a "layer" of reinsurance by selling an Insurance option at a price corresponding to a higher Loss-Ratio. The following graph shows a similar situation to the Options Example 1, but with the addition of a Short position at an Exercise price of $9,375. This corresponds to a layer of protection for the company in the range 20%-50%.



In this case, the protection in excess of $6m works as before but now for Industry losses in excess of $1,500m (company losses in excess of $15m), the company will suffer additional losses in excess of $6m. These losses are in excess of the layer of protection afforded by the combination of buying and selling Insurance options. This reduced level of protection has a reduced cost as the company now receives money for selling the option.

The calculations backing the numbers in the Options Example 2 graph are also given in Appendix III, Options Example 2.

Doing It Together !

A company can adopt a combination of Insurance futures and options positions to combine hedging and capping, in a similar fashion to an insurer who takes out both Quota Share and Excess of Loss reinsurance together.

3.3 A Comparison With Traditional Reinsurance

Tailor-Made Products

Conventional reinsurance has the advantage that it directly passes the risk to the reinsurer. It is a tailor made product and, subject to any coverage disputes and the security of the reinsurer, will indemnify the reinsured in respect of the losses that arise. It is therefore much easier for the reinsured to determine his requirements and to be certain that he will be reimbursed in the event of a covered loss.

Pricing Transparency

The price paid for, say, Insurance options as a proxy for excess-of-loss reinsurance will be more clearly visible than for reinsurance and comparisons more easily made between the cost of the options and the expected costs of the underlying risks. The current cost of the options appears, as one might expect, to lie broadly between the cost of conventional reinsurance and the expected pure risk costs of the underlying events.

Lack Of Correlation

Insurance futures have the obvious disadvantage that they only pay based on an industry index. To the extent that the reinsureds losses are not 100% correlated with that index the coverage may not pay when it is required. Of course, there may be a windfall gain for losses that the reinsured have not participated in to the extent of the market as a whole. Clearly this is an issue that an individual company needs to consider from its own point of view.

The lack of correlation refers not only to events but also to being much more heavily exposed to a specific loss. This seriously hampers the use of Insurance futures to hedge away insurance risk. Consider a company that feels it has taken on too much exposure in a given region, and wishes to reduce this by buying Insurance futures in a similar fashion to Quota-Share reinsurance. It could be the case that it may make a loss on its futures position, while still having higher than expected losses itself. Purchasing Insurance futures would in this case <u>increase</u> the <u>maximum</u> amount the company was exposed to through Catastrophe losses, rather than the intended effect of decreasing its exposure to Catastrophe losses. Conversely however, it could make a profit on a futures position at the same time as it made smaller than expected losses on its' own book of insurance business.

To estimate the extent to which an insurer's catastrophe losses are correlated with those of the industry as a whole can be a complex process. In the extreme it may involve assessing an insurer's exposure at, say, post-code level relative to some total insured exposure and comparing these figures to storm-tracks of hurricanes.

It is possible that markets may arise providing coverage for the difference between the index and a particular company's account. This would have the advantage of being able to have the flexibility of a futures or options and without the disadvantages of the lack of correlation. If Insurance futures become widely used, it is not difficult to imagine how these books of business could be put together. Essentially a major player who provided a lot of this cover would have a correlated index. The parallel is exact with diversifying an investment portfolio.

Flexibility Of Futures Contracts

Assuming the market develops from its present state, then a futures contract becomes much easier to organise during the year. There is also a greater degree of flexibility of cover. There is no need to worry so much about continuity or indeed about the volumes of business that one is likely to write in the current year. Against this of course the reinsured needs to evaluate the exposure he needs. It may be more difficult to layer unless the option market develops.

Security

The security issues are different in that it is unlikely that a major catastrophe will create any security problems in respect of a futures contract. It is unlikely, at least for the major exchanges, that these contracts will ever become a major part of their business. Given the importance of financial derivatives and their use by the banking system, it is almost inconceivable that the financial authorities could allow a major securities futures exchange default and the security of the contracts is a function of the exchange as a whole. Consequently security is less likely to be a problem. Clearly some of the major reinsurers are very soundly based, but obviously in the event of a major catastrophe, their security is going to be tested to its limits.

Underwriting Cycle

The cycle of the futures will be different. The price on the futures market will be purely market driven and indeed will provide some secondary check on company pricing. There will not be any element of pay-back as clearly players can either enter or leave the market according to its perceived attraction without any difficulty. This is not the case with a reinsurance company. A futures contract will not provide access to the other services that a reinsurer might provide such as claims handling advice and so on.

Evaluation Of Exposure

A futures contract puts a greater onus on the ceding company to determine their actual exposure to any particular loss in determining the amount of cover required, whereas the reinsurer will need to evaluate that more when undertaking pricing.

Transaction Costs

Transaction costs are also likely to be less in terms of futures driven market than with a conventional market. In part this will be due to the fact that futures contracts are not tailor made to the reinsured, whereas reinsurance contracts are. It is also likely that introductory or brokerage costs would be less.

3.4 The Players In An Insurance Futures Market

A market is formed by buyers and sellers coming together to trade. A "natural" market is one with a lot of automatic buyers and sellers. For example, a market in grain is a natural market, because producers and users are on different sides of the market. Farmers want to fix the price at which they sell their produce and the users want to fix the price at which they buy their raw materials.

If there are only "natural" market participants on one side of a market - buyers or sellers - then trading volumes may suffer, and an illiquid market is a bad market. Basic economics suggests that the forces of supply and demand will generate a market if someone is willing to trade. For example, if hedgers desperately want to hedge then they will be willing to pay a price that is attractive enough to attract speculators. This argument assumes well-informed investors.

We suggest that a market in Insurance fiutures is not a "natural" market. It is likely to be a one-sided market. There are natural hedgers - the insurers and reinsurers - but one only expects them to be on one side of the market at any point in time, depending on the current position in the underwriting cycle. When traditional reinsurance prices are high and capacity low, the insurers and reinsurers will be buyers of insurance futures, each meeting their risk transfer needs. In theory, reinsurers could be also be sellers, but why should they write insurance via an organised futures or options market when there is the conventional route ? Speculators are likely to be the sellers, although the high possibility of a small profit alongside the small probability of a large loss does not fit the normal perception of a speculator's utility curve. This risk profile could however be diversified away.

4. Pricing Insurance Futures And Options

4.0 Overview

Both the modelling of insurance losses and the pricing of futures and futures options are huge and complicated subjects, so in a general paper such as this, an outline of the approaches that can be adopted is all we can realistically hope to achieve. To put our thoughts on valuation of Insurance futures and options into context, it is necessary to give a brief resume of the techniques for valuing conventional futures and options. The reader's attention is drawn to the Glossary in section 9 for a description of technical finance terms.

4.1 Valuation Of Traditional Futures Contracts

Price Versus Value Of A Futures Contract

It is important to appreciate the distinction between a futures ***price*** and the ***value*** of a futures contract.

The futures price at any given time during the trading period of a particular contract is the delivery price of a contract entered into at that time. When the contract is initiated, the delivery price is agreed between buyer and seller, bearing in mind that there is no initial cash outlay; therefore in theory, the initial value of the contract should be zero. This provides the basis for determining the futures price at all times.

Forward Agreements Are Easier To Value....

Forward agreements are easier to value than futures contracts because they only involve a single payment at maturity. In contrast, futures contracts are "marked to market", that is, they are settled daily during the life of the contract.

Forward and futures prices are generally very close to one another when the maturities of the two contracts are the same. It is therefore fairly safe to make life a little easier and to consider forward prices as a proxy for futures prices.

In fact, when the risk-free interest rate is constant and the same for all maturities, forward prices and futures prices can be shown to be exactly the same, using a dynamic arbitrage argument (see Hull (1993)). This argument can be extended to cover situations where the interest rate is a known function of time.

When interest rates vary unpredictably, forward and futures prices are no longer the same and the relationship between them is complex (see Cox, Ingersoll and Ross (1981)) and depends on the correlation of the underlying asset with interest rates.

We find that when the underlying asset is strongly positively correlated with interest rates, futures prices will tend to be higher than forward prices. This is because when the price of the underlying asset increases, an investor with a long futures position makes an immediate gain that is invested at a higher than average rate of interest, and vice versa. An investor holding a forward is not affected in this way by interest rate movements.

Assets Held For Investment Versus Assets Held For Consumption

Futures and forward prices on assets held significantly for investment are fairly straightforward to calculate, whereas this is not the case for contracts on assets that are held for consumption. This is because if a market is strongly influenced by hedgers, then levels of inventory and storage costs and supply and demand characteristics will tend to influence pricing to a potentially great degree.

Valuing Forwards On Assets Held For Investment

The general approach to valuing forwards on assets held for investment by a significant number of investors is to use a simple arbitrage argument. A portfolio in the underlying security plus cash is constructed that replicates the pay-off of the forward at maturity. In the absence of arbitrage opportunities, the value of the forward must be the same as the value of the replicating portfolio.

This is a common approach used in modern financial economics to value many types of derivative securities, including option contracts.

Forwards On Non-income Paying Securities

Appendix IV.2 considers a forward on a simple financial asset that provides the holder with no income, such as a discount bond. A forward price at time t, F(t), is derived as:

$$F(t) = S(t)*e^{r(t)*(T-t)}$$

where S(t) is the price at time t of the asset underlying the forward contract and r(t) is the risk-free rate of interest per annum at time t for an investment maturing at time T.

Forwards On Other Types Of Assets Held For Investment

The valuation approach is the same as for non-income producing assets held for investment. Relationships between the forward price, delivery price, risk free interest rate, time to maturity and the value of the contract can be derived to produce similar results to that quoted above. These are outlined in Appendix IV.2.

Valuing Forwards On Assets Held For Consumption

The pricing of forward on assets held for consumption is complicated by the fact that storage of the asset (commodity) may be costly and also spot markets may be non-existent or too thin for arbitrage.

For commodities that are not significantly held for investment purposes, arbitrage valuation arguments need to be treated with extreme care. Individuals and firms require the commodity for its consumption value - it is utilised as part of their business rather than held as an investment. Holding the commodity may be preferred to holding a futures contract because futures contracts cannot be consumed. Benefits of ownership might include the ability to meet unexpected demand for the commodity or keeping a production process going.

These benefits are sometimes referred to as the convenience yield provided by the underlying asset. Convenience yield is therefore like a liquidity premium. When inventories of a commodity are plentiful we expect low convenience yields and vice versa.

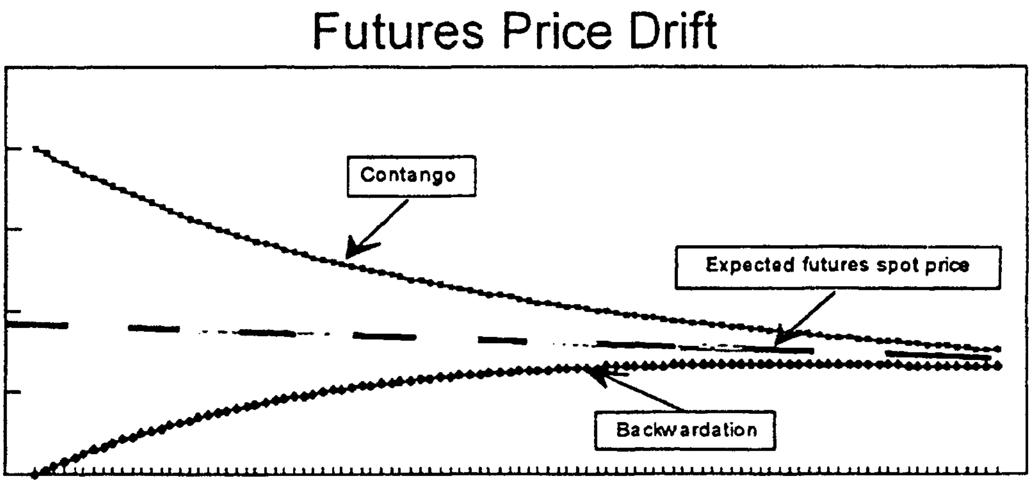
The usual approach to valuing forward contracts on assets held for consumption is based on "convenience yields" and "storage costs".

Extending The Arguments To Indices

We have shown that futures can be valued by looking at the forward price for one security / commodity. The arguments can be extended to an index; the security is just substituted by a portfolio of stocks comprising the index, and the dividend stream, where appropriate, replaced by the dividends on the stocks in the portfolio.

How Do Futures Prices Compare To Expected Future Spot Prices?

The futures price at a point in time is the delivery price of a contract entered into at that time. This may be different from the expected future Spot Price. If the buyers are hedgers, and the sellers are speculators, one can argue that the futures price will be below the expected future Spot Price. This is because the hedgers are prepared to accept slightly reduced returns, as they are happy to have the benefit of the hedge, whereas the speculators require compensation for the extra risk they are taking on. The position where the futures price is below the expected futures spot price is known as Normal Backwardation. The converse situation is known as Contango. These two cases are illustrated below:



Now the more risky an investment is, the more an investor might expect to be rewarded by higher returns. In a similar fashion to the argument developed above, by equating a security with a Long Forward position plus cash, one can show that:

$$F(t) = E(S(T)) * e^{(r(t)-k(t))*(T-t)}$$

E(S(T)) is our expectation of the Spot price at time T, r(t) is our risk-free rate of return, as before, k(t) is now a measure of the systematic risk of the investment over the period (T-t), i.e. the risk that cannot be diversified away. One can then go on to equate forwards with futures to develop the result for futures.

Clearly, as $k<r$, $k=r$, $k>r$, the value of F(t) is greater than, equal to, or less than E(S(T)). What does this for Futures prices in mean in practice? Well, if we knew that $F(t) < E(S(T))$, then one could take a long position which, over a period of time, would lead to a profit. Conversely, if $F(t) > E(S(T))$, a short position over a period of time would lead to a profit. This leads one to take the view that we should have $F(t) = E(S(T))$, or risk-free profits could be made. Various studies have been done examining whether futures prices are unbiased predictors of expected future Spot prices - these studies have come to various conclusions!! Two studies (see Houthakker(1975) and Chang(1985)) concluded that broadly $F(t) < E(S(T))$, two other studies (see Telser(1958) and Dusak(1973)) concluded that $F(t) = E(S(T))$ The reader can make up their own mind, and potentially fortune !

4.2 How Can We Extend These Arguments To Insurance Futures?

We have already seen that Insurance futures are somewhat different in nature to other financial futures. We have also made the important distinction between futures contracts on assets held for consumption and assets held for investment.

If we think of the asset underlying an Insurance future contract as being "insurance capital", we can see that this is akin to an asset held for the purpose of consumption (i.e. it is employed, or utilised, in the business of insurance). This puts Insurance futures in the "tricky" valuation category. Just as levels of inventory will affect commodity futures prices, so levels of industry capital will affect insurance futures prices, unless a significant number of speculators can be attracted to the market.

As a start to valuing Insurance futures, we can look at how the price may vary with the expected losses during a quarter. There are three separate periods to consider for arriving at an Insurance Futures price - before, during and after the Loss quarter. Before the Loss quarter, the buyer will simply have an expectation of what the losses will be at the end of the quarter to work on. During and after the Loss quarter, one will have further information about losses that have happened during the quarter, and then a view on how much of these losses have been reported three months after the quarter has finished.

Before the Loss quarter then, one could expect the price to be a function of the form:

$$F(t) = E(S(T))*e^{(r-k)*(T-t)}$$

During and after the quarter, on would expect the price to be of the similar form:

$$F(t) = E(S(T) \mid \text{perceived Incurred losses to date })*e^{(r-k)*(T-t)}$$

The r and k making allowance for the time value of money and the non-diversifiable riskiness of the investment. Following the arguments outlined previously, as the buyers are expected to be hedgers and the sellers are likely to be speculators, we would expect this approach to lead to the futures price exhibiting "Normal Backwardation", that is the futures price would tend to be less than the expected Settlement Price at the end of the period.

The expected value of the Settlement Price at the end of the period can be arrived at by making assumptions either about the Catastrophe Loss-ratio as a whole, or by building up a model for the frequency and severity of the Catastrophe losses one expects during a given quarter for a given futures contract (Eastern, National etc.) In Appendix IV.1 we have outlined an approach to deriving these expected prices at the end of the trading period, i.e. the expected Catastrophe losses at the end of the period, by making broad assumptions about the frequency of different types of Catastrophe and the severity of different types of Catastrophe. For illustration this is done for the National Contract, with no attempt to modify the results for seasonal effects by making the model specific to a particular Loss quarter. This is in no way meant to be a Working Party "view" on the likely level of Futures prices, and is intended just to illustrate the type of technique that could be applied.

As mentioned at the start of this section and in section 4.1, care needs to be taken in applying an arbitrage type of valuation to investments held for other than pure investment purposes. In the case of Insurance futures, one would want to try and make some allowance reflecting the added value or otherwise of being exposed directly to insurance risk compared to obtaining that exposure through Insurance futures.

4.3 Valuation Of Traditional Option Contracts

Brief Overview

The valuation of traditional option contracts is more complex than valuing Futures. In order to examine the applicability of some of the current models, we will give a quick description of how some of the current models work and the assumptions they make about the underlying stochastic process. This will enable us to give an indication as to why some of these models cannot be applied to value an option on an Insurance future and make suggestions as to which existing models seem to have the most desirable features for being adapted to cope with Options on Insurance Futures. We have indicated in the list of references where the interested reader may obtain details of the models mentioned here if they wish to investigate them further.

Black-Scholes Model And Variations

This is the classic option-pricing model (see Black and Scholes(1973)) around which many other models are based. In its basic form it is a model for pricing European options on non-dividend paying securities. The key starting assumption is that the stock price follows a Wiener process. This is defined in the Glossary, section 9. Broadly this means that the changes in stock price follow a Brownian motion type of process. The process assumes that returns over a period are normally distributed and it can be shown that this implies that given a starting stock price now, the distribution of the stock price at some future time is Log-normally distributed.

Black and Scholes showed that, making the assumption, amongst others, about the Stochastic process above, the option price could be arrived at in a closed form by solving a differential equation ($df/dt + r \times df/dS + 1/2 \times \sigma^2 \times S^2 \times d^2f/d^2S = r \times f$; r, f, S following the notation adopted previously, σ^2 being the volatility of the stock) given certain boundary conditions. This depends only on the risk-free interest rate, so the expected return over the period does <u>not</u> affect the results, and it is the volatility of the stock, amongst other factors, which affects the price of the option. Other assumptions have to be made, such as continuous trading, no transaction costs, interest rates and constant volatility for all maturities and so on.

This model can be extended to commodities and dividend paying stocks. It can also, with some persuasion, be adapted to provide information about American options. One can also assume that an <u>index</u> follows the geometric Brownian motion, and apply the method to various indices. This is not quite consistent, as if individual stocks follow this Brownian motion process, a weighted average of those stocks would not. Never-the-less, it is just a model for the process and works well in practice.

Black also showed that, assuming the futures price was of the form $F = Se^{\alpha\,(T-t)}$, one could liken futures to a security paying dividends and, with certain assumptions about α one could produce a pricing model for options on futures.

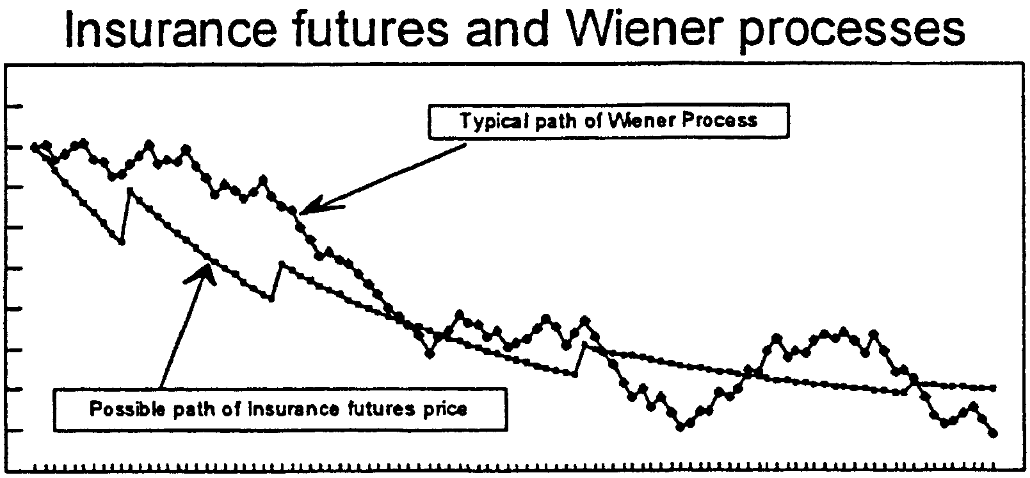
Variations include a model where the interest rate follows a stochastic process (see Merton(1973)) and one where the volatility follows a stochastic process (see Hull and White(1990)).

Can, Say, The Black Model Be Applied To Insurance Options (On Futures) ?

No.

Why Not ?

The underlying process in all the models above is the Brownian motion type of process. In English, this means that changes in the price of the derivative under consideration vary up and down in some random way. This does not seem to be consistent with the behaviour we might expect of Insurance futures, that is to say Catastrophe losses. At the start of a period we might have an expectation of the final Settlement Price at the end of the period. We might expect this to gradually decline over the Loss quarter if losses do not occur, as shown below:



As losses do occur, the futures price might be expected to jump. This will not be a Markov process, which is part of the assumptions of the Wiener process. Once the price has jumped to a given level, the price in the future will depend on the past. The stochastic variation will not be the symmetric variation underlying the movement in stock prices - there will be a much greater tendency to increase, as losses occur, than to decrease, as losses do not occur. The linch-pin of the assumptions that lead to much of the current option-pricing theory does not, it seems to us, apply in the case of Insurance options (on futures).

That is not to say that current models do not add some value. In reality stocks and indices do not precisely follow the Wiener process. If one has a feel for the bias that is introduced by applying a model to a process for which the assumptions are not quite correct, use can still be made of that model.

Say, for example, one thought that in reality the "tail" of the distribution of the instrument under consideration was fatter than the Log-normal tail implied in the Black-Scholes model and its' variations. This would mean that the model would tend to under-price the option - this is still useful information as it provides information about the minimum price of the option. It may also be that the Insurance futures market takes on a life of its own, and the "noise" from general futures trading becomes such that the insurance nature of the spikes in price are no longer a significant feature and the model can be more readily applied.

Are There Other Models One Could Try And Apply ?

Yes.

What Are They ?

There are models that assume jumps in the stochastic process, rather than just the drift up or down. One such model is the Pure Jump Model (see Cox, Ross & Rubinstein (1979)). The gist of this model is that in time dt, say, the current stock price, S, can jump from:

S to Su, with probability λdt

S to $Se^{-\omega\,dt}$ with probability $1 - \lambda dt$

This has a lot more appeal than the Wiener process, as an explanation of movements in Insurance futures prices. We expect the futures price to drift down as losses do not happen, but to "jump" up as a Catastrophe loss occurs. In this basic case the jumps are of fixed sizes; extensions of the model can allow for jumps of varying sizes. In the context of stock prices, this model has the disadvantage that it only allows jumps up - in an Insurance futures context, this becomes a desirable feature.

There is also a Jump Diffusion Model (see Merton (1976)), which superimposes the type of Jump structure above onto an underlying diffusion process.

Where Do We Go From Here ?

The pricing of options, especially the more exotic type of option we are considering, is a highly specialised area. All we have tried to do here is to indicate why we think that most standard option-pricing models cannot be validly applied in an Insurance Futures context and to indicate other types of model that could perhaps be a more fruitful source in trying to price Insurance options.

The details given in Appendix IV.1 describe an approach to simulating the price of Insurance futures at the end of the period, by simulating numbers and amounts of losses. This type of approach can lead one to arrive at the E(S(T)) indicated in our suggestion of valuing Insurance futures above, and could be the starting point for trying to value Insurance options by some sort of Jump process. The λ indicated above in the Pure Jump model, is akin to the Poisson probability of a Catastrophe occurring; the "u" in the Pure Jump model is akin to the amount of loss this gives rise to; a model which forms a proxy for Insurance Options would have to have the "u" itself a stochastic variable. Alternatively one could make a simpler assumption about the distribution of the overall Loss-ratio, and try to build a model around this.

5. Development Of New Futures Contracts

5.0 Overview

Efforts are currently underway to develop Insurance futures and options contracts based on UK and European insurance losses, for example windstorm losses. The main effort has been in trying to construct a reliable index of European windstorm losses. Can the Europeans learn anything from the CBOT experience to date ? Could CBOT have marketed the contracts better ? This section looks at the likely ingredients of a successful Insurance futures contract and points out a few unpalatable ones along the way.

5.1 What Makes A Good Futures Contract ?

Quite simply, a good futures contract is one that has a large, liquid market. The experience of the CBOT contracts to date has not so far been encouraging in this respect, although CBOT have professed themselves happy with the progress to date and pointed out that other futures markets have taken several years of market familiarization before the trading volume took off. Never-the-less, Insurance futures have been traded very little and there is, as yet, no significant market in the futures option. Is this because the Insurance futures as currently designed are inherently unsuitable for this type of market or because we are seeing initial reluctance to trade a new type of derivative?

Features Of The Underlying Asset

There is a lot of empirical evidence on what makes a successful futures contract (see Carlton (1984)). Carlton cites five important features that a commodity traded on a futures exchange should possess to be successful :

(a) uncertainty
(b) price correlations across slightly different products
(c) a large potential number of interested participants and industrial structure
(d) large value of transactions
(e) price freely determined and absence of regulation

Insurance scores on all of these counts, with the possible exception of the price correlation requirement. Since insurance risk can be very specific, a hedging instrument such as a future or an option based upon a standardised, industry exposure may only be weakly correlated with an insurers risk transfer needs.

The CBOT catastrophe futures are based on broad geographical zones - Western, Midwestern, Eastern and National - and average industry experience in each zone. These zones are not well matched to most insurers inwards risks (although natural-hazard-prone Texas is included in both the Eastern and Midwestern experience so a suitable trading strategy can isolate Texan experience)

Futures on an insurance index also present a number of particular problems that we can add to Carlton's list :

(i) no spot market
(ii) complex nature of the underlying asset
(iii) concerns about "insider" information
(iv) high margin requirements
(v) existence of a natural substitute - conventional reinsurance
(vi) natural hedgers only on one side of the market

These problems are dealt with in turn below.

No Spot Market

In common with certain other commodity futures, there is no established spot market in which insurance can be traded. For example, there is no spot market in the underlying asset for frozen orange juice concentrate futures when the oranges are still on the trees!

An underlying insurance "price" index must first be identified or constructed before a market in insurance derivatives can be established. CBOT's solution was to commission the ISO to create a loss ratio index based on catastrophe losses and premiums. A UK or European initiative would need to go down a similar route.

Investors' lack of familiarity with a new index is a real barrier. It steepens the learning curve for a new product and also makes arbitrage opportunities difficult to identify.

Complexity Of The Underlying Asset

The complexity of the underlying "asset" in the case of an insurance derivative is itself a barrier to market liquidity. A large contributory factor to futures market liquidity is the presence of "locals" - local exchange specialists that make their living out of trading narrow arbitrage anomalies. Locals tend to be attracted to simple contracts that are easily understood, with well established spot markets.

"Insider" Problems

CBOT is not using an index determined by an outside body but the published returns of about two dozen insurance companies. This inevitably raises the question of moral hazard. This is especially the case for the final three months of trading of a contract after the loss period. Outside investors may consider themselves to be at a disadvantage in trading these contracts, however this perceived risk should be largely eliminated due to the inclusion of a considerable number of insurers, no one of which constitutes a substantial part of the index.

High Margin Requirements

The volatile nature of catastrophe insurance means that margin requirements must be very high. This might discourage potential speculators.

Conventional Reinsurance As A Substitute

A hurdle for insurance futures as a hedging instrument is the existence of a well developed and understood substitute market for hedging risk - the conventional reinsurance market. Why use a standardised risk transfer tool when a "bespoke" solution can be negotiated with another party ?

Natural Hedgers Will Dominate One Side Of The Market

We have already commented in section 3 on the likely characteristics of a market in Insurance futures. One problem is that there do not seem to be natural hedgers on both sides of the market, buying and selling. While we might expect insurers and reinsurers to be on different sides of the market, the existence of the conventional reinsurance market casts some doubt on this.

Summary

Perhaps the most important attribute of a market in Insurance futures would be its liquidity. The one advantage that a futures market can offer over the conventional reinsurance market is guaranteed capacity and a fair market price, by accessing capital outside the insurance industry.

Unfortunately, we seem to have a Catch 22 (see Heller (1962)) that Yossarian would be proud of. To attract investors the market must be liquid, to be liquid the market must attract investors! The challenge for CBOT and, perhaps at a future date, LIFFE, is how to overcome this negative feedback.

6. Wider Repercussions Of Trading In Insurance Futures And Options

6.0 Overview

This section looks at some of the potential implications of an Insurance futures market.

6.1 Smoothing Of The Underwriting Cycle

If Insurance futures were to be widely traded, it is likely to smooth out the underwriting cycle. This would be to the benefit of all concerned. The reason for smoothing out the underwriting cycle is that it would be very much easier to enter or leave the futures market than the reinsurance market.

6.2 Implications For The Reinsurance Market

Transaction Costs

For reasons mentioned above, it is likely that the transactional cost would be less on the futures, which would tend to favour the use of futures especially if a market develops between the differences between a particular company and an index. This would not be good news for reinsurance brokers, unless they took some intermediate role in this process.

Cost Of Cover

If the use of Insurance futures contracts became wide-spread, it may make conventional reinsurance more expensive and not less. This is because the contracts would take a percentage of the bread and butter business out of the market, leaving reinsurers with a less well diversified portfolio. It could also mean that the reinsurance market may reduce for more esoteric risks. A further consequence of a reduction in the call on traditional reinsurance may be that reinsurers actually become sellers of Insurance futures, to obtain a diversified portfolio.

Conversely one could argue that in times of low capacity, if Insurance futures lead to an influx of new capital into the insurance industry, this can only serve to reduce the price of reinsurance.

Selling The Market Share

The issues of insurance companies manipulating the market might need to be considered.

6.3 Insurance Futures At Lloyd's

It has been suggested, though not widely, that one way of solving the Lloyd's open year problem would be to use futures. Essentially one could derive a contract whereby shares of pollution and asbestos risks were simply traded. This is an idea that has some interesting concepts within this area. However at this stage this working party has not made any attempt to analyse it further.

7. Regulatory Aspects

7.0 Overview

This section briefly describes some of the areas where clarification of the regulations regarding Insurance futures and options is required and some of the work being done in this area.

7.1 Insurance Futures And Options Are Not Insurance Contracts

Insurance futures can achieve similar risk transfer objectives as proportional reinsurance and options on Insurance futures can behave like non-proportional reinsurance (see section 3 for more details) However Insurance futures and options are not contracts of insurance. This has implications for :

(i) accounting treatment
(ii) statutory solvency
(iii) taxation treatment

7.2 Institute And Faculty Working Party

The Institute and Faculty of Actuaries have a working party, chaired by Bill Abbott, that is discussing with the DTI valuation regulations (from an insurance companies point of view) for derivative instruments including futures and options. However this is largely investment driven and the issues involved are not the same for Insurance futures, although initial regulations are likely to cover them in the same way as any other contract. It is likely that information will be released by the Institute and Faculty working party to a wider audience in the not too distant future.

North American regulators have not in general approved the use of futures contracts. At the time of writing Illinois is the only state to have authorised the use of Insurance futures and options, whilst New York and California are in the process of passing legislation to allow their use. American regulators have more draconian powers than UK authorisation in terms of actually banning the use of such instruments, so the authorization of the use of Insurance futures as hedging instruments by several states is a positive move. Now that several states have given them the green light, it is likely that other states will follow.

It should be recognised that even if the various regulatory authorities do not allow credit to be taken for statutory purposes for these instruments, they could be used to offset the accounting mechanisms. We are not aware of any further accounting standards taken on this area. Ideally profit / loss on reinsurance futures and options would be added to the underwriting result.

7.3 Other Considerations

As far as the Stock Exchange is concerned, it would be open to a company to point out that although it may be likely to show a large gross underwriting loss, this has no material net effect on its overall financial position because of the futures contracts it holds which will provide corresponding investment profits. On the assumption that the Stock Market behaves sensibly, one would expect the share prices to recognise this benefit.

8. Data Sources

Due to the low volumes of contracts traded none of the major financial journals are listing the Insurance futures contracts at present.

The CBOT maintain extremely good information services. The CBOT Education and Marketing Departments have compiled an explanatory folder and provide a query answering service, which are described below:

London

Explanatory folders can be requested from and questions can be directed to:

Suzanne Matus
52-54 Gracechurch Street
London
EC3V 0EH

Tel: 071-929-0021

Chicago

Explanatory folder can be obtained from the Literature Services Department:

Tel: 0101-312-435-3558

Questions can be directed to:

Dena Karras
The Chicago Board of Trade
141 West Jackson
Suite 2280
Chicago
Illinois 60604

Tel: 0101-312-435-3674

Historical statistics for the contracts can be obtained on diskette in various formats for a fee from:

Robert Pfannkuche
The Chicago Board of Trade
141 West Jackson
Information Systems Dept. Suite 940-A
Chicago
Illinois 60604

Tel: 0101-800-288-CBOT (2268)

The type of statistical information available is illustrated in Appendix I.

Information regarding the contracts can also be obtained from the usual Information agencies, such as Reuters.

9. Glossary

American Option Contract

These are options that may be exercised at any time up to the expiration date (maturity).

Most of the options traded on exchanges are American, but they are more difficult to value than European Options.

Arbitrage

An arbitrage strategy is a trading strategy in which the arbitrageur makes a profit while taking no risk at all. Forces of supply and demand mean that arbitrage opportunities will not exist for long in an open, liquid market.

Call Option

A Call Option gives the holder the right to buy the underlying security by a certain date for a certain price.

Contango

The futures price at a point in time is the delivery price of a contract entered into at that time. This may be different from the expected future Spot Price. If the buyers are speculators, and the sellers are hedgers, one can argue that the futures price will be above the expected future Spot Price. This is because the hedgers are prepared to accept slightly reduced returns, as they are happy to have the benefit of the hedge, whereas the speculators require compensation for the extra risk they are taking on. The position where the futures price is above the expected futures spot price is known as Contango.

Delivery Date (Futures/ Forward Contracts)

The date on which the holder of the short position delivers the underlying asset to the holder of the long position, or settles the cash difference between Spot Price and Delivery Price.

Delivery Price (Futures/ Forward Contracts)

This is the price at which the holder of a short position in a contract agrees to sell the underlying asset to the holder of the long position at maturity of the contract.

It is fixed at the commencement of the contract and is determined on the basis that there is no cost to entering into the transaction at the outset.

European Option Contract

These are options that may only be exercised at maturity of the contract.

Forward Agreement

This is a bilateral agreement to buy or sell an asset at a certain future Delivery Date for a certain Delivery Price.

The agreement is usually between two financial institutions or between a financial institution and one of its corporate clients. It is not normally traded on an exchange.

No cash changes hands initially. Therefore at the time the contract is entered into, the Delivery Price negotiated should in theory be such that the value of the forward contract to both parties is zero.

A Forward Agreement is settled entirely at maturity, when the holder of the short position delivers the amount of the underlying asset specified in the contract to the holder of the long position (if possible) in return for a cash amount equal to the Delivery Price.

Forward Price

This is the Delivery Price for a contract negotiated on the same day. In practice it is negotiated by buyer and seller. In theory this is the Delivery Price that would make a contract have zero value.

As time passes, the Forward Price is liable to change, while the Delivery Price remains fixed during the life of each contract (although it will be different for different contracts).

Futures Contract

This is the same as a Forward Agreement, except that :

- futures contracts are normally traded on an exchange;

- to make trading possible, the exchange specifies certain standardised features of the contract;

- since the two parties to the contract do not necessarily know one another, the exchange provides a mechanism which gives the two parties a guarantee that the contract will be honoured;

- an exact Delivery Date is not always specified;

- there is marking to market, or daily settlement.

Futures Option Contract

These are Options on Futures Contracts.

Futures Price

See Forward Price, reading "Futures" for "Forward".

The Futures Price is the price at which trades occur on a futures exchange. It is determined by the forces of supply and demand.

Hedger

An investor who buys or sells a contract for the primary purpose of risk transfer.

Initial Margin (Futures Contract)

This is the amount that an investor must deposit in his Margin Account at the time the contract is first entered into.

The Initial Margin will depend on the volatility of the underlying asset.

Maintenance Margin (Futures Contract)

To ensure that the balance in the Margin Account never becomes negative, a Maintenance Margin (a.k.a. Variation Margin) is set that is somewhat lower than the Initial Margin. If the balance in the Margin Account falls below the Maintenance Margin, the investor receives a "margin call" and is required to top up the Margin Account to the Initial Margin level immediately.

The Maintenance Margin is usually 70-80% of the Initial Margin for most types of futures contracts.

Margin Account (Futures Contract)

A broker will require an investor to deposit funds in a margin account to guarantee the investor's performance.

Normal Backwardation

This is the opposite to Contango. If the buyers are hedgers, and the sellers are speculators, one can argue that the futures price will be below the expected future Spot Price. This is because, following the same argument as for Contango, the hedgers are prepared to accept slightly reduced returns, as they are happy to have the benefit of the hedge, whereas the speculators require compensation for the extra risk they are taking on. The position where the futures price is below the expected futures spot price is known as Normal Backwardation.

Option Contract

There are two basic types : Call Option and Put Option (see definitions).

An option gives the holder the right to do something. The distinguishing feature of an option contract is that the holder does not have to exercise this right.

An investor must pay something to enter into an option contract, the "option price", unlike Futures and Forward Contracts.

Open Interest

This is the total number of outstanding contracts in a security at a certain time

Put Option

A Call Option gives the holder the right to sell the underlying security by a certain date for a certain price.

Settlement Price (Futures)

This price is set by the exchange at the end of each day. It is the official price to be used for determining daily gains or losses and margin requirements.

Speculator

An investor who is buying or selling a contract for investment purposes, rather than as a hedge against unwanted risk.

Spot Price

The is the price of an asset for immediate delivery.

Strike Price (Options - a.k.a. Exercise Price)

The price at which the long call/put option holder can exercise his right to buy/ sell the underlying security.

Terminal Value

This is the value of a long position in a contract at maturity.

Wiener Process

This is the process stock prices are assumed to follow in most option-pricing models. It is a stochastic Markov process; in English this means it is a combination of fixed and random variables, for which only the present value of the variable is needed to determine the future values (it doesn't matter what levels the variable has been before, just where it is at now).

The behaviour of a variable following a Wiener process has two key properties depending on how small changes in the variable are related to small changes in time. Firstly that a small change in the variable is proportional to (the small change in time)^(1/2) x N(0,1), where N(0,1) is a standardised Normal Random Variable. Secondly that changes in the variable over two different time intervals are independent.

In more common parlance, the model for stock price behaviour is known as Brownian Motion, that familiar term from O' level physics days! In a similar fashion to smoke particles, stock prices are assumed to wobble up and down, with an equally likely chance of going up or down, but can have an overall drift imposed on them one way or the other. It can be shown that stocks whose change in price follows a Wiener process have a future value whose distribution is Log-normal.

10. Bibliography

Black, F. and Scholes, M. (1974), 'The pricing of Options and Corporate Liabilities', Journal of Political Economy, Vol.81 (May-June) (pp.637-659)

Carlton, D.W. (1984), 'Futures Markets: Their Purpose, Their History, Their Growth, Their Successes And Failures', The Journal Of Futures Markets Vol.4 No.3 (pp. 237-271)

Chang, E.C (1985), 'Returns to Speculators and the theory of Normal Backwardation', Journal of Finance, Vol.40 (Mar.) (pp.193-208)

The Chicago Board Of Trade (1992), 'Catastrophe Insurance Futures And Options : A Reference Guide'

Cox, J.C., Ingersoll J.E. and Ross S.A (1981), 'The relation between forward and futures prices", Journal Of Financial Economics, Vol.9 (Dec.) (pp. 312-346)

Cox, J.C., Ross, S.A., Rubinstein, M. (1979), 'Option pricing: a simplified approach', Journal of Financial Economics, Vol.7 (Sep.) (pp.229-263)

Cox, S.H. and Schwebach, R.G. (1992), 'Insurance Futures And Hedging Insurance Price Risk', Symposium On Insurance Futures, The Journal Of Risk And Insurance, Vol.LIX, No.4 (Dec.)

D'Arcy, S.P. and France, V.G., (1990), 'Will Insurance Futures Fly ?', Journal Of Commerce (Oct.)

D'Arcy, S.P. and France, V.G., (1992), 'Catastrophe Futures : A Better Hedge For Insurers', Symposium On Insurance Futures, The Journal Of Risk And Insurance, Vol.LIX, No.4 (Dec.)

D'Arcy S.P and France, V.G. (1993), 'Catastrophe Insurance Futures', Faculty Working Paper 93-0132, Bureau of Economic and Business Research, College of Commerce and Business Administration, University of Illinois at Urbana-Champaign

Duffie, D. (1989), 'Futures Markets', Prentice-Hall Inc.

Dusak, K (1973), 'Futures Trading and Investor Returns: an Investigation of Commodity Risk Premiums', Journal of Political Economy, Vol.69 (Dec.) (pp. 250-260)

Hogg and Klugman (1984), 'Loss Distributions', J.Wiley & Sons Inc.

Houthakker, H.S (1957), 'Can Speculators Forecast Prices?' Review of Economics and Statistics, Vol 39 (pp. 143-151)

Hull, J.C. (1993), 'Options, Futures And Other Derivative Securities', Prentice-Hall Inc., Second Edition

Hull, J. and White, A. (1990), 'Pricing Interest Rate Derivative Securities', Review of Financial Studies, 3,4 (pp.573-592)

Kidder, Peabody & Co. Inc. (1992), 'Introducing Catastrophic Insurance Futures'

Levi C. and Partrat C. (1989), 'Statistical Analysis of Natural Events in the United States', presented at ASTIN Colloquium, New York (Nov. 1989)

Merton, R.C (1973), 'Theory of Rational Option Pricing', Bell Journal of Economics and Management Science, 4 (pp.141-183)

Merton, R.C (1976), 'Option Pricing when Underlying Stock Returns are Discontinuous', Journal of Financial Economics, Vol.3 (Mar.) (pp.125-144)

Niehaus, G. and Mann, S.V.(1992), 'The Trading Of Underwriting Risk : An Analysis Of Insurance Futures Contracts And Reinsurance', Symposium On Insurance Futures, The Journal Of Risk And Insurance, Vol.LIX, No.4 (Dec.)

Renshaw, A.E (1991), 'The Interpretation and Implementation of Generalized Linear Models (with applications)', City University London

Robertson, J.P. (1991), 'Insurance Futures', The Actuarial Review (March)

Roll, R. (1984), 'Orange Juice And Weather', American Economic Review, Vol.74

Sherman, R.E. (1991), 'Actuaries And Insurance Futures', The Actuarial Review (Feb.)

Smith, C.W. and Stulz, R.M. (1985), 'The Determinants Of Firms' Hedging Policies', Journal Of Financial And Quantitative Analysis, Vol.20 No.4 (Dec.)

Telser, L.G (1958) 'Futures Trading and the Storage of Cotton and Wheat', Journal of Political Economy, Vol.66 (Jun.) (pp. 233-255)

Example Of The Statistical Information Available

CBOT Eastern Insurance Futures Pricing Information
5, 6, 7, 8 and 11 Jan 1993

Eastern Contract

Trade Date

Settlement Price

Daily Trading Volume

COMMOD	FUTURE	T-DATE	OPEN1	HIGH	LOW	CLOSE1	SETTLE	VOL	OPEN-INT
UE	93H	930105	158	160	158	158	158	14	18
UE	93H	930106	160	160	158	160	160	4	18
UE	93H	930107	160	169	160	168	168	68	24
UE	93H	930108	165	167	165	167	167	10	22
UE	93H	930111	166	167	166	167	167	14	20
[illegible]	93U	930105	85	88	85	87	87	2	0
[illegible]	93U	930106	87	87	87	87	87	3	0
[illegible]	93U	930107	87	87	87	87	87	2	0
[illegible]	93U	[illegible]	[illegible]	0	0	85	85	0	0
[illegible]	93U	[illegible]	[illegible]	85	85	85	85	1	1
UE	93Z	[illegible]	[illegible]	140	138	138	138	1	0
UE	93Z	930106	138	138	131	131	131	3	0
UE	93Z	930107	130	130	130	130	130	3	0
UE	93Z	930108	0	0	0	135	135	0	0
[illegible]	93Z	930111	0	0	0	152	152	0	0
[illegible]	94H	930105	60	61	60	61	61	0	0
[illegible]	94H	930106	0	0	0	61	61	0	0
[illegible]	94H	930107	80	80	80	80	80	2	0
[illegible]	94H	930108	0	0	0	80	80	0	0
UE	94H	930111	0	0	0	80	80	0	0

1993 March Contract (Oct-Dec Loss Period)

June Contracts (Jan-Mar Loss Period)

Sept Contracts (Apr To Jun Loss Period)

Dec Contracts (Jul-Sep Loss Period)

Open Interest (note that there is greatest interest in contracts during the loss period)

Summary Of Trading Periods And Reporting Periods For Contacts Currently Traded

Contract Month	Loss Period	Reporting Period	Trading Commences	Last Day of Trading
Sep 93	Apr - Jun 93	Apr 93 - Sep 93	4 Jan 93	5 Jan 94
Dec 93	Jul - Sep 93	Jul 93 - Dec 93	4 Jan 93	5 Apr 94
Mar 94	Oct - Dec 93	Oct 93 - Mar 94	4 Jan 93	5 Jul 94

Lines of Business Included In The Contract By Cause of Loss

Lines of Business	Cause of Loss: Wind	Hail	Quake	Riot	Flood
Homeowners	X	X			
Commercial Multi Peril	X	X		X	
Earthquake			X		
Fire	X	X		X[1]	
Allied	X	X		X[1]	
Auto, Physical Damage (Private & Commercial)	X	X	X	X	X
Farmowners	X	X		X	
Commercial Inland Marine	X	X	X	X	X

[1] commercial element only

Companies Contributing To ISO DATA

American Financial Group
AMICA Mutual Insurance Company
CIGNA Group
CNA Insurance Companies
CU Insurance Companies
Continental Insurance Companies
Employers Mutual Companies
Fireman's Fund Companies
General Accident Group
Hanover Insurance Companies
ITT Hartford Group
Kemper Corporation Group / Kemper National Insurance Companies
Liberty Mutual
Lincoln National Group
Royal Insurance Group
Safeco Insurance Group
St Paul Group
Transamerica Corporation Group
United States F&G Group
USAA Group
Westfield Companies
Zurich Insurance Group - US

CBOT Limitations Of Price Movement And Holdings

The maximum value of an Insurance futures contract is $50,000 (because the catastrophe losses would usually be expected to comprise only part of the total loss ratio, the limitation of $50,000 represents considerably more than a doubling of expected catastrophe losses, as witnessed by Hurricane Andrew not causing this level to be breached).

Should a contract have a daily settlement price greater than 90% of its maximum final settlement price, a new contract for delivery in the same calender month may be listed, with a maximum final settlement amount to be determined by the CBOT Board.

The daily price should not move by more than ten points (i.e.: 10% of $25,000 or $2,500), though a variable limits of fifteen points may be applied (i.e.: 15% of $25,000 or $3,750)

The maximum net long or short position that any person may own, control, or carry is 10,000 contracts. There will be no additional limits imposed in the spot months.

There is a reportable limit of 100 contracts in any month.

Futures Example 1

Forecasts in July:	$m
(1) Company Incurred Loss	4
(2) Company Premium	30
(3) Company Reported Loss	75% by Dec.
(4) Industry Incurred Loss	400
(5) Industry Premium	3,000
(6) Industry Reported Loss	75% by Dec.

(7) Futures Price in July $2,500
(= 25,000 x (4) x (6) / (5))

(8) To hedge 100% of premium1,600 contracts need to be bought
(= (2) / 25,000 x 100% / (6))

	$m	$m	$m	$m	$m	$m	$m
Industry Reported Loss	200	250	300	350	400	450	500
Industry Ultimate Loss	267	333	400	467	533	600	667
(9) Futures Price ($)	1,667	2,083	2,500	2,917	3,333	3,750	4,167
Company Expected Loss	4.00	4.00	4.00	4.00	4.00	4.00	4.00
(10) Company Actual Loss	2.67	3..33	4.00	4.67	5.33	6.00	6.67
(11) Futures Gain / (Loss)	(1.33)	(0.67)	0.00	0.67	1.33	2.00	2.67
(12)Overall losses ((10)-(11))	4.00	4.00	4.00	4.00	4.00	4.00	4.00

Futures Example 2

Forecasts in July:	$m
(1) Company Incurred Loss	2

(2) to (7) as previously

(8) To hedge 50% of premium: 800 contracts need to be bought
(= (2) / 25,000 x 50% / (6))

	$m	$m	$m	$m	$m	$m	$m
Industry Reported Loss	200	250	300	350	400	450	500
Industry Ultimate Loss	267	333	400	467	533	600	667
(9) Futures Price ($)	1,667	2,083	2,500	2,917	3,333	3,750	4,167
Company Expected Loss	2.00	2.00	2.00	2.00	2.00	2.00	2.00
(10) Company Actual Loss	1.33	1.67	2.00	2.33	2.67	3.00	3.33
(11) Futures Gain / (Loss)	(0.67)	(0.33)	0.00	0.33	0.67	1.00	1.33
(12) Overall losses ((10)-(11))	2.00	2.00	2.00	2.00	2.00	2.00	2.00

Futures Example 3

Forecasts in July: $m
(1) Company Incurred Loss 6
(2) to (7) as previously
(8) To hedge 150% of premium: 2,400 contracts need to be bought
(= (2) / 25,000 x 150% / (6))

		$m	$m	$m	$m	$m	$m	$m
	Industry Reported Loss	200	250	300	350	400	450	500
	Industry Ultimate Loss	267	333	400	467	533	600	667
(9)	Futures Price ($)	1,667	2,083	2,500	2,917	3,333	3,750	4,167

	Company Expected Loss	6.00	6.00	6.00	6.00	6.00	6.00	6.00
(10)	Company Actual Loss	4.00	5.00	6.00	7.00	8.00	9.00	10.00
(11)	Futures Gain / (Loss)	(2.00)	(1.00)	0.00	1.00	2.00	3.00	4.00
(12)	Overall losses ((10)-(11))	6.00	6.00	6.00	6.00	6.00	6.00	6.00

Options Examples 1 And 2

Forecasts in July: $m
(1) Company Incurred Loss 4
(2) to (7) as previously

(8) To hedge Loss-ratio in range of 20-50% 1,600 (= (2) / 25,000 x 100% / (6)) contracts need to be bought at a price of $3,750 (= (7) x 0.2 / ((1) / (2))) and sold at $9,375 (= (7) x 0.5 / ((1) / (2))

		$m	$m	$m	$m	$m	$m
	Industry Reported Loss	113	450	788	1,125	1,463	1,800
	Industry Ultimate Loss	150	600	1,050	1,500	1,950	2,400
(9)	Futures Price ($)	938	3,750	6,563	9,375	12,188	15,000

	Company Expected Loss	4.00	4.00	4.00	4.00	4.00	4.00
(10)	Company Actual Loss	1.50	6.00	10.50	15.00	19.50	24.00
(11)	Long Option Gain	0.00	0.00	4.50	9.00	13.50	18.00
(12)	Overall Effect ((10)-(11))*	1.50	6.00	6.00	6.00	6.00	6.00
(13)	Short Option Loss	0.00	0.00	0.00	0.00	(4.50)	(9.00)
(14)	Overall Effect ((12)-(13))+	1.50	6.00	6.00	6.00	10.50	15.00

* Figures for Options Example 1

\+ Figures for Options Example 2

Pricing Mathematics - Simulating The Expected Settlement Value

The following indicates a simple approach that can be taken in building a model for the expected final Settlement Value of an Insurance future, based on assumptions about the frequency and severity of losses. This expected value can then be used as a base point in arriving at the value of an Insurance future, as indicated in section 4.2.

The Model

The following crude model is for the National contract and takes no account of seasonality of the losses. In practice one would want to adjust the model for different quarters and extend it to the three different regions.

The major one-off events that causes Catastrophe losses are Hurricanes. These are therefore modelled separately. There is a considerable body of work on the modelling of Catastrophe losses, particularly for Hurricane losses (see Levi & Partrat(1989), Hogg & Klugman(1984) and Renshaw(1991)).

Other losses are mainly due to tornadoes/windstorms; in addition there are losses due to Earthquakes, Hail and Riots. The study and modelling of any one of these Catastrophes would be a paper in itself, so we have here adopted a fairly broad-brush approach to their modelling.

The Parameters

The model we have constructed translates into an expectation of slightly over one Hurricane per year. The other sources of Catastrophe are expected to occur between four and twelve times per quarter, with an overall expectation of thirty-two losses per year.

The Hurricanes have an expected loss of about $1 billion. The "other" Catastrophes have losses ranging between roughly $200m to $1,800m per quarter, overall about $3.2 billion per year.

We have used a Poisson model for the frequency of these events. The severity of Hurricanes is modelled by the Log-Normal distribution, and the severity of the "other" Catastrophes is modelled by the Normal distribution. The parameters used are as follows:

Hurricane Losses

Quarterly frequency Poisson: mean 0.2803
Severity Log-Normal: mean 5.4912, Standard Deviation 1.74297

"Other" Losses

Quarterly frequency Poisson: mean 8
Severity Normal: mean 100, Standard Deviation 10

Other models were considered for frequency and severity. For example, Pareto or Weibull distributions could be used to model the Hurricane severity. The fit of the model was considered at two levels. First, that the frequency and severity were a good model for the known Hurricane data, which was drawn from several sources, including the references mentioned above. Secondly, that the overall losses, and hence simulated Insurance Futures prices, were a satisfactory model for CBOT's simulated Futures settlement prices over the last fourteen quarters. We will not dwell on the modelling process, as this paper is not meant to be a learned treatise on modelling Catastrophe losses. The model is simply meant to be an illustration of the approach that could be adopted.

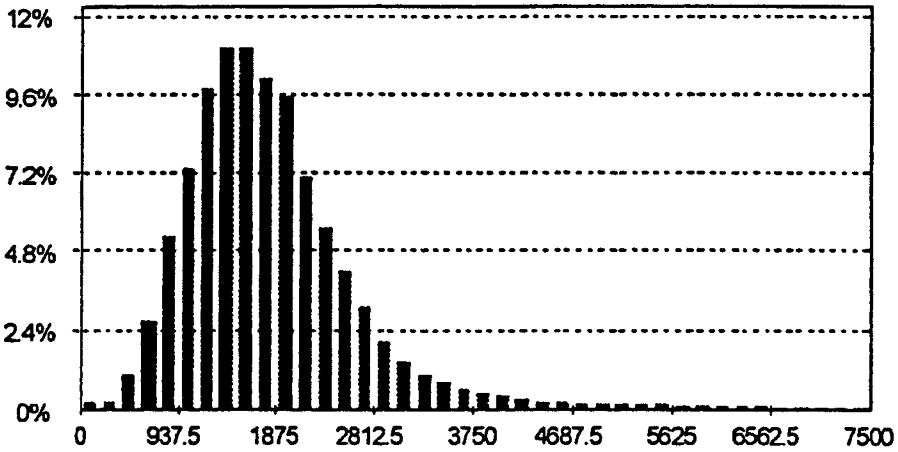
The Results

Simulations based on the above model produce a mean and a standard deviation of the Insurance futures price at the end of the period. These are compared to the CBOT simulation of Insurance futures prices for the fourteen quarters prior to the launch of the contract below:

	Model	CBOT Data (with Hugo and Andrew)	CBOT Data (without Hugo and Andrew)
Mean	\$2,200	\$4,000	\$2,100
SD	\$2,900	\$5,900	\$1,100

The Model has hurricanes as 26% of total losses, compared to 27% in the CBOT simulated data. The simulation allows one to examine the distribution of the expected final Settlement Value at the end of the period, as illustrated below:

Simulation of Insurance Futures Settlement Price : Expected Value $2,210

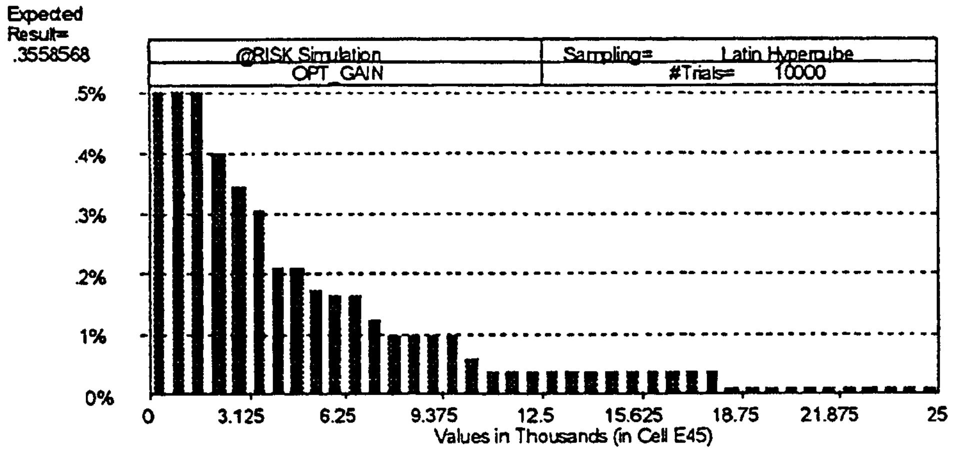


The model described above can be used to examine the effect on a hedging strategy of a company's losses not being perfectly correlated with the Industry as a whole. One can simulate Company losses at the same time as Industry losses, either by having company losses as a variable percentage of Industry losses (say in the range 0.5% - 1.5%, rather than precisely 1%), or one could simulate completely separate losses for the Company, but have them correlated to varying degrees with the Industry as a whole.

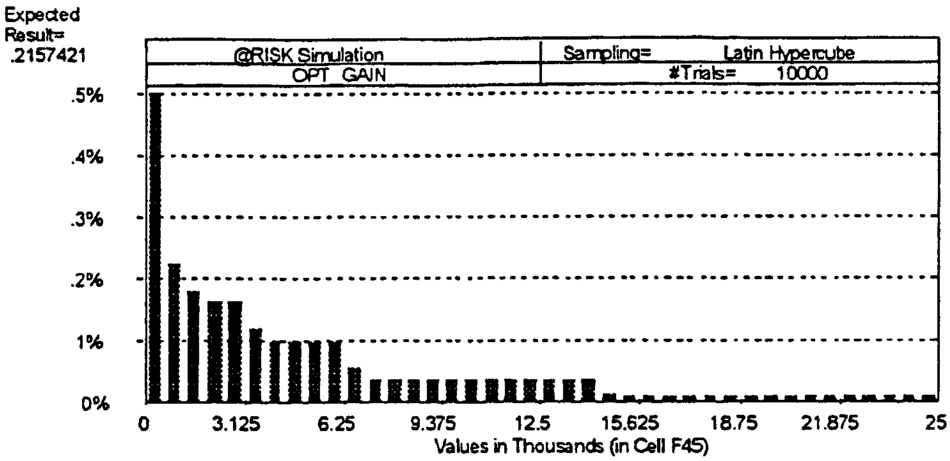
Using The Model To Make Comparisons With Reinsurance Costs

Section 4.3 described how the pricing of options, particularly exotic options of the kind represented by Insurance options (on futures) is bedevilled with problems and uncertainty. In order to make a rough comparison with the price of conventional reinsurance, we have examined the simulated expected pay-off of an Insurance Option, using the simple model above, in a risk-neutral world. The results of these broad-brush comparisons are described in section 3.3. The method involves simulating a series of Catastrophe losses, and examining the expected pay-offs on Insurance options with varying Exercise levels. This is really just akin to comparing typical current reinsurance costs with the underlying expected values of the losses. Just as reinsurance costs will reflect more than the pure expected losses underlying them, so Insurance options would reflect the riskiness of this type of investment, as perceived by different types of investor. Examples of the distribution of these sample Option pay-offs are reproduced below:

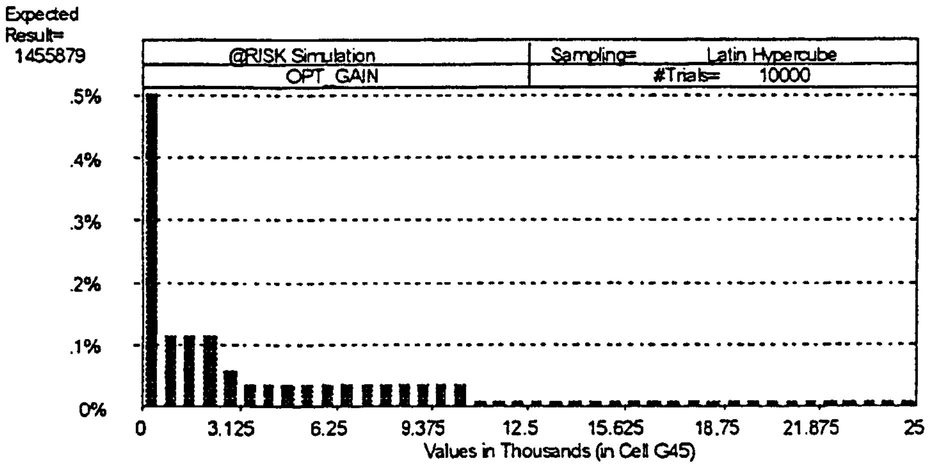
Simulation of Option Pay-off XS 20%: Expected Value $360



Simulation of Option Pay-off XS 40% : Expected Value $220



Simulation of Option Pay-off XS 60% : Expected Value $150



Pricing Mathematics - Developing Formulae For Pricing Forwards

Forwards On Non-income Paying Securities

Define the following variables:

T time when the forward contract matures
t current time
t_0 time when contract was first initiated
$S(t)$ price at time t of the asset underlying the forward contract
K delivery price in the forward contract
$F(t)$ forward price at time t
$f(t)$ value of a long forward contract at time t
$r(t)$ risk free rate of interest per annum at time t, with continuous compounding, for an investment maturing at time T

Consider 2 portfolios as follows :

Portfolio I

* Long forward contract on the underlying security plus cash equal to $Ke^{-r(t)*(T-t)}$.

* Value is $f(t) + Ke^{-r(t)*(T-t)}$.

* Pay-off at time T is $(S(T) - K) + K = S(T)$.

Portfolio II

* One security.

* Value is $S(t)$.

* Pay-off at time $S(T)$.

From a simple no-arbitrage argument, portfolios I and II have the same value, that is:

$$S(t) = f(t) + Ke^{-r(t)*(T-t)}$$

When the contract was first initiated, K should have been negotiated so that $f(t_0) = 0$, that is

$$K = S(t0)*e^{r(t)*(T-t)}$$

By definition, this is also the forward price at time t_0, so F(t0) = K. More generally, the forward price at time t, F(t) should be given by :

$$F(t) = S(t)*e^{r(t)*(T-t)}$$

At maturity, F(T) = S(T), that is, the spot price of the underlying security at time T.

Of course in practice F(t) is determined by market forces in a futures market and it may not equal the theoretical value $S(t)*e^{r(t)*(T-t)}$. If this is the case then arbitrage opportunities will exist and a speculator can take a riskless profit by adopting a suitable trading strategy.

Forwards On Other Types Of Assets Held For Investment

The value of the contract can be derived using arbitrage arguments in many situations, including :

(a) underlying securities that provide a known cash income

(b) underlying securities that provide a known dividend yield

(c) forward contracts on currencies

(d) forwards on gold and silver

Using the same notation as previously, it can be shown that a general formula linking F, K, r and f in these situations is as follows (see Hull (1993)):

$$f(t) = S(t) - I(t) - Ke^{-(r(t)-q)*(T-t)}$$

where : I(t) is the present value at t of the known cash income produced by the underlying asset from t to T; and
q is the annual dividend rate on the underlying asset, paid continuously

Returning to the four examples just mentioned, we can interpret the equation as follows :

(a) underlying securities that provide a known cash income

Put q=0 in the equation.

(b) underlying securities that provide a known dividend yield

Put I(t)=0 in the equation.

(c) forward contracts on currencies

Put q=0 and I(t) = 0

S(t) is the current price in Sterling of one unit of the foreign currency at maturity, namely $S'(t)*e^{-p(t)*(T-t)}$, where S'(t) is the current price in Sterling of one unit of the foreign currency and p(t) is the risk-free rate prevailing in the foreign currency.

(d) forwards on gold and silver

If we want to allow for storage costs, we can use appropriate negative values for either q or I(t) at the same time as zeroing the other, depending on the nature of these costs.