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INTERNATIONAL STUDIES IN GENERATION MORTALITY

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THE two studies described below were made as part of a large-scale programme of official research into a series of topics relating to fertility, mortality and migration. The outcome of all this research (including that recorded below) will be published by the Government in due course but the manner and place of publication have not yet been decided. Although the present paper is announced under joint authorship, it is not a fully integrated effort: Part 1 was devised and written by the first-named contributor and Part 2 by the second. Nevertheless, their preparation took place over the same period of time and during that period steady contact was maintained between the authors by correspondence and meetings, and thus the work proceeded in parallel.

PART 1: SECULAR AND GENERATION MORTALITY

1.1. It is well over a hundred years since censuses and vital registration began in a number of Western countries; so, for people born during several decades, it is now possible to trace their mortality experience throughout the whole of their lives. For their successors, a similar but more restricted analysis can be made, but this can be extended with the aid of alternative mortality projections into the future. Such possibilities have not gone unnoticed, and papers have been published on the basis of data for various countries, notably England and Wales,[1] France,[2] Holland,[3] Sweden[4] and the USA.[5] The authors' aims are diverse and there is no common theme; but exposition rather than analysis has been the main feature. The patterns pursued by the rates as the age advances, and whether they are similar to or different from the corresponding patterns in the experience of secular periods, do not seem to have been much studied. The purpose of the present analysis is to examine this question of the relationship of q_x or m_x with x. It will be shown that the relationship is indeed different for generations to what it is for secular periods, and how the difference has varied in the past and may change in the future. Prediction will probably not be much helped by this, as the generation approach to forecasting seems neither essential[6] nor necessarily even advantageous[7] except for some specific groups of causes of death[8]; see, however, part 2. It does, however, seem reasonable to suggest that, in the future, attempts to frame 'laws' of mortality linking q_x with x should always take generation histories into account as well as the secular experiences on which, in the main, they have hitherto been founded even though the search may well be rendered more baffling thereby.

1.2. It is not difficult to see why, in economically well-developed countries today, this difference of shape of q_x between generations and secular periods has arisen. The basic cause is changes in mortality over a period of time in association with increasing general prosperity and with medical and social advances. How it happens can be shown by means of a simple model. Suppose first a situation in which death rates are and have for long been decreasing at the steady rate of $S\%$ per annum at all ages. In these circumstances, for any secular period, whatever the general level of mortality, the ratio of q_x to q_y for given values of x and y will be constant in time. As between all generations, too, whatever the year of birth the ratio of q_x to q_y will be unvarying. Nevertheless, this ratio will not be the same for generations on the one hand as it is for secular periods on the other. If $q_x = f(x)$ for secular periods, we shall have $q_x = f(x)\ (1-{\cdot}01S)^x$ for generations. So q_x/q_y (generations) will be equal to q_x/q_y (secular) multiplied by $(1-{\cdot}01S)^{x-y}$. Thus the pace of advance in the mortality rate as age increases will always be slower for generations than for secular periods. This model is not too unrealistic so far as the first half of the human life-span is concerned, but in the second half S has tended to diminish with age towards zero for centenarians. Let us now postulate, therefore, the substitution for S of a rate of improvement S_x which varies with x, diminishing as x advances. In these conditions both secular and generation experiences will be changing shape in time, and the outcome of a comparison between them must depend not only on the ages involved but also the size of the time-difference. If we postulate for example that

$$S_x = S.\left(\frac{100-x}{100}\right), (x<100),$$

then if q_x was equal to $g(x)$ in earlier days of stable q's, for secular period N years after the improvements began $(N<x)$:

$$q_x = g(x)\left(1-{\cdot}01\, S\frac{(100-x)}{100}\right)^N$$

whereas for generations born n years before this period $N(n<N)$:

$$q_x = g(x)\left(1-{\cdot}01\, S\frac{(100-x)}{100}\right)^{N-n+x}$$

which is a different pattern from the secular one, the degree of difference being dependent on n as well as x. The ratio of generation q's to secular q's is thus:

$$\left(1-{\cdot}01\, S\frac{(100-x)}{100}\right)^{x-n}.$$

1.3. On this formula, as x advances from zero upwards, the ratio falls at first. But at later ages it rises again, and near the end of life it is close to unity. This is illustrated by the following specimen figures for an S of 2% per annum and for n equal to 0, 15 and 30 years:

Age x	Value of $\left[1-\cdot 02\left(\frac{100-x}{100}\right)\right]^{x-n}$ where n = 0	15	30
0	1·00	1·36	1·85
15	·77	1·00	1·29
30	·65	·81	1·00
45	·61	·72	·87
60	·62	·70	·79
75	·68	·74	·80
90	·83	·86	·88
100	1·00	1·00	1·00

These figures show that, as n increases:

(a) the sag in the curve below unity in middle life is reduced; but
(b) the amount of fall in the values from age to age early in life is sharpened; as a net result
(c) the downward slope in the curve up to middle life is accentuated.

1.4. This pattern is a consequence of the kind of progress that has been made in practice in many countries in recent decades. The mechanism is further illustrated in the following two diagrams. Diagram 1 illustrates (exaggeratedly)

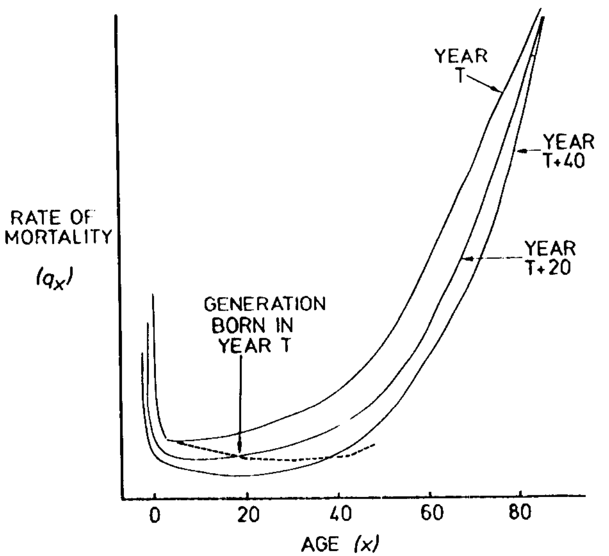

Diagram 1. Schematic representation of mortality rates by age at successive periods (———) showing relationships with generation mortality (- - - - - - -).

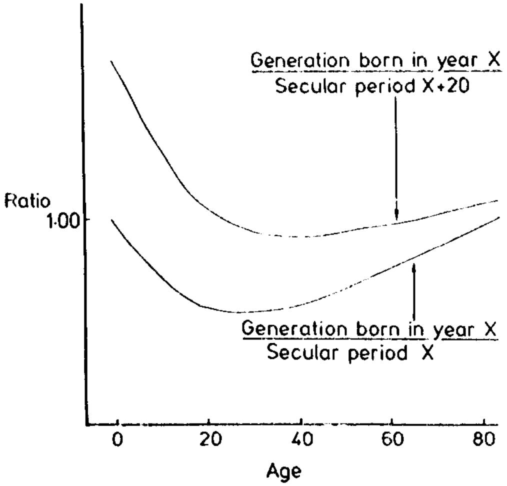

Diagram 2. Illustrative ratios of generation mortality rates to secular mortality rates.

how the generations straddle across the successive secular experiences—and it also brings out the practical difficulty of measuring the disparities from diagrams, owing to the smallness of the mortality rates at many ages. Diagram 2 shows the generation/secular ratios, which stand out much more clearly and exhibit the early fall and late rise with advancing age which are typical.

In practice, of course, both the size of S and the relationship of S_x with x (which clearly must be more complex than in the simple formula given above) have changed in time and also differ from one country to another. Mathematical models can only give a broad notion of what may be found when the actual data are analysed. Nevertheless, the concave pattern illustrated above seems to be of basic importance and indeed it is fairly typical of what has happened in practice. It is of interest to note that:

(a) for negative values of n, the ratios follow a similar general pattern to that illustrated in §1.3 but begin below unity at age 0 and show a narrower range of variation;
(b) for smaller values of S than 2% per annum, the pattern persists but the numerical values depart less far from unity;
(c) for larger values of S, again the pattern persists but this time with wider departures from unity.

Diagram 3 illustrates, for two specimen values of n, how the ratios compare for a lower value of S (1% per annum) and a high one (3% per annum). The 'sag' in the curves in middle life is greatly enhanced for the high value of S. It is also clear that the shape of the curves depends a lot on the value of n adopted.

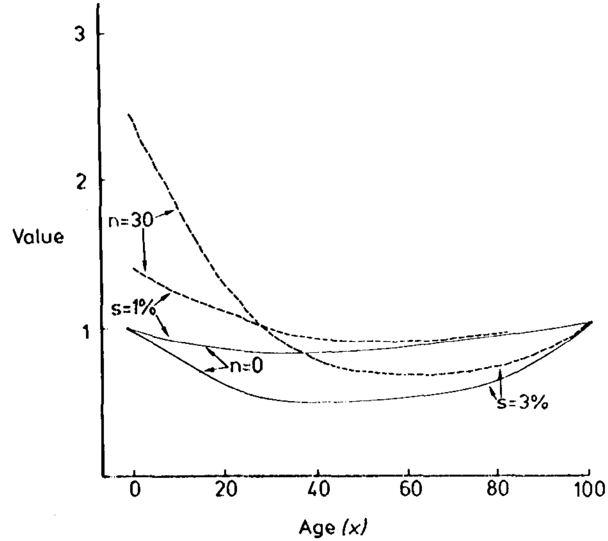

Diagram 3. Values of the theoretical ratio of generation to secular mortality

$$\left[1-S\left(\frac{100-x}{100}\right)\right]^{x-n}$$

But whereas the development of S is an inescapable matter of history, n can be chosen (within limits) at the will of the investigator; n need not be zero, and indeed should not be restricted to this value as it occasionally has in mortality studies.

DATA AND PROSPECTS FOR ENGLAND AND WALES

1.5. In order to give an idea of the magnitude of S, some approximations to the values actually experienced in England and Wales at times over the period 1841–1965 are set out for specimen ages, for men, in the following statement:

Age	*1841–45 to 1861–65*	*1861–65 to 1881–85*	*1881–85 to 1901–05*	*1901–05 to 1921–25*	*1921–25 to 1941–45**	*1941–45* to 1961–65*	*Unweighted averages*
0	·1	·2	–·2	3·1	2·3	5·0	1·8
20–24	·2	1·9	1·5	1·1	2·5	3·3	1·7
40–44	–·5	–·1	1·1	2·5	2·1	2·2	1·2
60–64	–·5	–·4	·3	1·2	·3	·1	·2
80–84	–·1	–·2	·4	–·3	·3	·1	·0
Unweighted averages	–·16	·28	·62	1·52	1·50	2·14	·98

* Excluding military and civilian casualties; in order to minimize further the effect of the Second World War, the experience of the average of the three periods 1936–40, 1941–45 and 1946–50 has been substituted for that of the period 1941–45 in compiling the table, though this does not cause a big difference in the results.

The corresponding values for women are similar but recent improvements at ages 60 and over have been better than for men. S has clearly increased very materially, reaching an average value of over 2% per annum for the specimen age-groups illustrated. It tends to diminish with advancing age, as assumed in the model mentioned above, but the manner of the decline is less straightforward than a simple linear relationship with x.

1.6. Various projections of British mortality rates for the 40 years starting at the present time suggest that S could be at a lower level in the near future than it has been in the past. The values of S_x in four such projections[8,9,10] lie within the following ranges:

Age x	*Men*	*Women*
0	·4 to 1·7	·4 to 1·7
20–24	·3 to 1·2	·4 to 1·2
40–44	−·2 to ·6	0 to 1·7
60–64	·3 to 1·1	·1 to 1·7
80–84	·1 to 1·1	·3 to 1·5

These figures are generally lower than on recent experience in infancy, in childhood and in early adult life. At older ages, however, they range from the same as recently to a notable improvement on the past. If such expectations are right, age-patterns of mortality, and particularly the relationships between them, must be expected to alter and develop in new ways. The generation/secular disparity in the shape of q_x should mostly diminish but, on the more extreme projection, could actually be enhanced in later life.

1.7. Ratios of generation to secular mortality rates in various age-groups for England and Wales are set out in Tables 1, 2 and 3. The calendar years indicated

Table 1. *Ratios of generation to secular mortality rates: 1871–1921, England and Wales*

	n = −20			*n* = 0			*n* = 20			*n* = 40		
Age-group	1871–1881	1891–1901	1911–1921	1871–1881	1891–1901	1911–1921	1871–1881	1891–1901	1911–1921	1871–1881	1891–1901	1911–1921
Men												
0–4	·88	·83	·61	·98	·98	·93	1·15	1·50	1·91	1·77	3·09	5·58
10–14	·57	·55	·65	·76	·84	·77	1·17	1·00	1·19	1·39	1·54	3·12
20–24	·55	·59	·53	·67	·86	·62	·96	1·02	·94	1·14	1·53	2·14
30–34	·58	·38	·26	·60	·51	·35	·80	·67	·62	1·06	1·21	1·36
40–44	·60	·37	·24	·57	·44	·35	·67	·63	·52	·96	·94	·93
50–54	·63	·48	·43	·59	·50	·56	·62	·66	·72	·80	·84	·92
Women												
0–4	·86	·83	·60	·97	·99	·93	1·16	1·54	2·00	1·80	3·30	5·50
10–14	·58	·56	·60	·79	·84	·73	1·18	1·01	1·27	1·41	1·76	4·38
20–24	·48	·53	·54	·62	·82	·68	·95	1·02	·94	1·18	1·42	1·95
30–34	·47	·37	·26	·54	·51	·38	·75	·74	·62	1·07	1·19	1·45
40–44	·49	·33	·22	·52	·40	·33	·62	·61	·52	·93	·96	·93
50–54	·54	·37	·32	·54	·40	·41	·58	·53	·60	·77	·75	·89

are the average years of birth of the generations; n is the amount to be added to the mean of these years of birth in order to arrive at the mean of the twenty years of secular experience on which the ratios are founded. Table 1 relates to the first half of life and to data which derive wholly from the past. Tables 2 and 3 include projected mortality experience for the future: ratios based on projections are italicized. Table 3 is concerned mainly with the older ages. (*Note*: The ratios in these three tables were calculated from other ratios and may differ a little from those assessed directly from the death rates.)

1.8. It may be seen from Table 1 that, for both men and women:

(a) the age-pattern of the cohort–secular ratios is similar to that in the model developed in §1.2 and illustrated in §1.4;
(b) as time has progressed, the degree of dispersion of the ratios, between ages, has increased, for all values of n;
(c) the manner in which the figures vary for different values of n broadly matches the theoretical expectations;
(d) the figures for the earlier periods appear consistent with low values of S whereas those for the later periods seem to fit in with higher values of S; this is in accord with the developments in S recorded above.

Though there are some differences in the figures as between men and women, these do not stand out prominently.

Table 2. *Ratios of generation to secular mortality rates: 1931–61, England and Wales (based on actual experience and official projections)*

Age-group	*n = −20* 1931–41	1951–61	*n = 0* 1931–41	1951–61	*n = 20* 1931–41	1951–61	*n = 40* 1931–41	1951–61
Men								
0–4	·43	·31	·90	·84	2·51	*1·52*	*4·60*	*1·78*
10–14	·36	·30	·56	·81	1·49	*1.03*	*1·90*	*1·19*
20–24	·27	*·31*	·41	*·70*	·91	*·97*	*1·25*	*1·08*
30–34	·20	*·31*	·35	*·67*	*·76*	*·95*	*1·08*	*1·07*
40–44	*·32*	*·44*	*·48*	*·79*	*·86*	*·92*	*1·00*	*1·00*
50–54	*·51*	*·56*	*·66*	*·73*	*·85*	*·81*	*·95*	*·91*
Women								
0–4	·42	·30	·89	·85	2·51	*1·56*	*4·61*	*1·92*
10–14	·27	·20	·48	·69	1·64	*1·10*	*2·63*	*1·29*
20–24	·16	*·17*	·22	*·54*	·71	*·99*	*1·30*	*1·15*
30–34	·16	*·23*	·26	*·53*	*·62*	*·92*	*1·07*	*1·04*
40–44	*·29*	*·39*	*·45*	*·72*	*·82*	*·87*	*·99*	*·94*
50–54	*·37*	*·47*	*·53*	*·72*	*·80*	*·83*	*·91*	*·93*

1.9. The projected mortality rates underlying the *italicized* ratios in Table 2 are those adopted for the official population projections for Britain published in 1974.[10] Obviously, any conclusions to be drawn from this Table must be less strongly based than their counterparts from Table 1; nevertheless it seems worth while to find whether a continuation of the past relationships between secular and generation experiences, and of the trends in these relationships, is to be expected on the basis of forecasts made on principles unconnected with these relationships. It would appear that:

(a) in general, the decline with advancing age in the cohort–secular ratios continues for both sexes and for all values of n; but
(b) the pace of the decline is less steep for the generations born in 1951–61 than for earlier generations and the tendency to change to an increase in middle life is enhanced, especially for men;
(c) the degree of dispersion of the age pattern is no longer increasing in time but on the contrary is much reduced;
(d) the variations with n are still generally in line with theoretical expectation;
(e) the Table is broadly consistent with the values of S shown in § 1.6 above (for the official projection these are in the middle of the ranges there shown for ages 20–24 and all over 30, but at the lowest end of the range at ages under 15 and 25–34 and at the highest end at ages 15–19).

1.10. Tables 1 and 2 taken together can be looked at in a different way: for each age-group and for each value of n one may study the progress of the ratios in time, i.e. for successive groups of generations. The picture thus presented is as follows:

$n = -20$: For ages 0–4 and 10–14 the ratios decline steadily from left to right; at higher ages an earlier fall has been reversed.
$n = 0$: A downward trend among the earlier generations has now been reversed except at age 0.
$n = 20$: Whereas earlier the tendencies appear to have been upward at young ages and downwards at older ages, both these are now reversed.
$n = 40$: An upward tendency at young ages has been reversed; there seems little trend at all at older ages.

Some diminution in cohort–secular differences is to be expected as a result of the projected decline in S's. Indeed if S's became zero for several decades the cohort–secular differences would disappear.

1.11. The analysis thus far can be extended in two ways:

(a) by examining the variations which flow from the adoption of alternative mortality projections, and
(b) by looking at older age-groups—though the scope for this is limited.

If Dr Scott's principal projection[8] replaced the official 1976 figures, the picture presented by Table 2 would be little changed. In general only the ratios for ages 40–44 and 50–54 would be altered, in the central part of the table; they would be increased, but not very greatly. On a more 'optimistic' alternative mortality projection by Dr Scott,[9] the cohort–secular ratios are in general slightly reduced in middle life but increased at ages under 15. The conclusions

Table 3. *Ratios of generation to secular mortality rates in older life: England and Wales*

Age-group	$n = -20$				$n = 0$				$n = 20$			
	1871–1881	1891–1901	1911–1921	1931–1941	1871–1881	1891–1901	1911–1921	1931–1941	1871–1881	1891–1901	1911–1921	1931–1941
Men												
30–34	·58	·38	·26	·20	·60	·51	·35	·35	·80	·67	·62	·76
40–44	·60	·37	·24	*·32*	·57	·44	·35	*·48*	·67	·63	·52	*·86*
50–54	·63	·48	·43	*·51*	·59	·50	·56	*·66*	·62	·66	·72	*·85*
60–64	·76	·69	*·57*	*·58*	·71	·69	*·67*	*·69*	·71	·81	*·80*	*·72*
70–74	·80	·82	*·76*	*·76*	·79	·81	*·80*	*·84*	·79	·86	*·89*	*·89*
80–84	·92	*·81*	*·83*		·89	*·85*	*·84*		·93	*·85*	*·84*	
Women												
30–34	·47	·37	·26	·16	·54	·51	·38	·26	·75	·74	·62	·62
40–44	·49	·33	·22	*·29*	·52	·40	·33	*·45*	·62	·61	·52	*·82*
50–54	·54	·37	·32	*·37*	·54	·40	·41	*·53*	·58	·53	·60	*·80*
60–64	·57	·41	*·36*	*·39*	·57	·42	*·47*	*·51*	·58	·54	*·60*	*·72*
70–74	·61	·50	*·43*	*·43*	·61	·51	*·49*	*·51*	·62	·57	*·58*	*·65*
80–84	·74	*·63*	*·62*		·73	*·67*	*·66*		·77	*·70*	*·70*	

drawn in § 1.9 above would not be materially affected by the adoption of either of these alternatives.

1.12. Table 3 gives, for three values of n, the generation–secular ratios for ages 30–54 (repeated from Tables 1 and 2) with the addition of figures for ages 60–64, 70–74 and 80–84. Most of the data derive from actual experience but some (*italicized*) are based on the official projections. It is not practicable to include the generation born in the years 1951–61 (or the figures at 80–84 for those born in 1931–41) because these people will not reach advanced years until well into the 21st century, beyond the range even of the current 40-year projections. The Table shows clearly the tendency of the ratios to rise in old age, reversing the falling trend experienced up to middle age. The turn-round is much more delayed for women than for men, especially for $n = 20$, and particularly for the later generations. This is because women's mortality has improved so much in early old age recently. Projections involving slighter improvements make the turn-up come earlier in life, and those assuming weightier advances delay it.

1.13. The experience in England and Wales as it has been recorded, and as it is expected to develop, thus shows that the q_x curve for generations dips more in early and middle life than that for secular periods. After an apparently negligible start, the difference increased to a peak but now seems to be diminishing—unless current mortality projections are much too pessimistic (for a more radical forecast, see reference[7]); there is scope for difference of opinion as to the likely pace of diminution, particularly in old age.

EXPERIENCE IN SWEDEN

1.14. From the foregoing, there can be little doubt that a general pattern of divergent generation-secular mortality relationships similar to that in England and Wales must be present in the data of countries with a comparable socio-economic history. Nevertheless there could be significant differences in detail. For this reason, the statistics of a number of other Western countries have been examined. Sweden is especially worthy of study because its vital records are of the greatest antiquity, going back as they do to the middle of the eighteenth century, and they may thus provide useful evidence about the beginning of the divergence. The National Central Bureau of Statistics in Stockholm have sent extracts from a report[11] setting out all the recorded death rates, *inter alia* in quinary age-groups for quinary periods, up to the end of the nineteenth century. A paper[4] is also available showing secular and generation experiences for more recent years. Mainly from the latter of these, Table 4 has been compiled in a form parallel to those given above; but it does not include any ratios based on projected future mortality.

Table 4. *Ratios of generation to secular mortality rates: 1861–1921, Sweden*

Age-group	*n* = 0 1861	1881	1901	1921	*n* = 20 1861	1881	1901	1921	*n* = 40 1861	1881	1901	1921
Men												
0–4	1·14	·98	·92	·88	1·31	1·27	1·67	1·68	1·71	2·32	3·19	4·42
10–14	·62	·63	·69	·68	·67	1·06	1·14	1·50	1·11	1·73	2·50	3·75
20–24	·87	1·02	·74	·50	1·00	·96	·88	·91	·94	1·14	1·59	2·90
30–34	·89	·87	·53	·31	1·01	·98	·63	·59	1·15	1·17	1·22	1·14
40–44	·69	·57	·46	·40	·83	·68	·62	·61	·99	·90	·95	1·09
50–54	·61	·61	·50		·83	·69	·66		·93	·90	·77	
60–64	·61	·75	·75		·82	·80	·83		·90	·89	·90	
70–74	·67	·71			·77	·86			·93	·95		
Women												
0–4	1·14	·98	·90	·81	1·32	1·30	1·77	1·54	1·75	2·55	3·30	4·24
10–14	·63	·67	·76	·52	·72	·93	1·35	1·50	1·00	1·65	3·88	4·00
20–24	·96	1·08	·75	·75	1·02	·95	1·00	·95	·90	1·27	2·10	4·00
30–34	·90	·84	·49	·23	·97	·82	·69	·46	·96	1·17	1·38	1·57
40–44	·76	·67	·46	·27	·87	·73	·61	·46	·95	·97	1·03	1·00
50–54	·66	·81	·51		·90	·83	·59		·91	·96	·71	
60–64	·63	·74	·54		·91	·82	·60		1·02	·91	·62	
70–74	·59	·61			·70	·81			·96	·91		

The main features of Table 4 are much the same as those of Table 1; however:

(a) for men, the ratios tend to be a little higher than for England and Wales; but for women they are sometimes lower;
(b) the decrease in the ratios with advancing age is perhaps, on the whole, a little less steep than in England and Wales.

1.15. The data available for Sweden for the period before 1861 are generally very reliable, according to advice received from the Swedish National Central

Table 5. *Ratios of generation to secular mortality rates: 1761–1841, Sweden*

Age-group	*n = 2*					*n = 22*					*n = 42*				
	1761–1776	1781–1796	1801–1816	1821	1841	1761–1776	1781–1796	1801–1826	1821	1841	1761–1776	1781–1796	1801–1816	1821	1841
Men															
0–4	1·00	1·00	1·00	1·00	1·00	1·08	1·04	1·25	1·08	·98	1·12	1·30	1·32	1·06	1·21
10–14	1·14	1·06	·83	1·07	1·15	1·35	1·06	1·10	1·08	1·14	1·46	1·34	1·04	1·07	1·22
20–24	·95	1·10	·83	·95	·92	1·00	1·00	1·00	1·00	1·00	·93	1·23	1·09	1·09	·99
30–34	·91	1·02	·97	1·05	·89	·98	1·03	·94	1·06	1·18	·98	·99	1·12	1·41	1·24
40–44	1·03	1·11	·89	·70	·64	1·06	1·04	·86	·72	·90	1·00	1·00	1·00	1·00	1·00
50–54	1·08	1·08	·78	·76	·57	1·13	·94	·81	·74	·73	·98	·97	·91	·95	·91
60–64	1·11	·94	·70	·65	·59	1·05	·83	·74	·68	·67	·93	·89	·84	·77	·86
70–74	1·06	·89	·66	·69	·64	·98	·84	·71	·70	·73	·91	·90	·76	·80	·85
80–84	1·13	·88	·74	·89	·81	·94	·82	·75	·83	·91	·91	·83	·81	·94	·99
Women															
0–4	1·00	1·00	1·00	1·00	1·00	1·10	1·07	1·26	1·11	·94	1·17	1·34	1·34	1·04	1·17
10–14	1·12	1·04	·85	·99	1·14	1·27	1·04	1·11	1·07	1·12	1·35	1·32	1·06	1·05	1·06
20–24	·93	1·13	·82	·94	·86	1·00	1·00	1·00	1·00	1·00	·91	1·23	1·11	1·16	·94
30–34	·88	·90	·84	1·06	·97	1·00	·93	·95	1·10	1·11	1·02	1·05	1·07	1·27	1·11
40–44	·94	·90	·79	·80	·73	1·03	·90	·88	·83	·88	1·00	1·00	1·00	1·00	1·00
50–54	1·00	·99	·73	·83	·67	1·06	·85	·83	·83	·78	·91	·98	·93	·97	·94
60–64	1·00	·88	·66	·67	·61	·98	·79	·75	·72	·67	·88	·90	·83	·79	·84
70–74	·96	·83	·63	·74	·62	·92	·81	·70	·70	·72	·90	·90	·74	·81	·86
80–84	·99	·84	·67	·94	·68	·94	·76	·74	·87	1·00	·85	·84	·78	·99	1·13

Bureau of Statistics, except in infancy in the earliest years. Their form makes it more suitable that the generation–secular ratios should be calculated for values of n ending, not in zero, but in 2 (strictly speaking, $2\frac{1}{2}$). Results for the generations born in 1821 and 1841 are shown in the right-hand portions of the three sections of Table 5; they exhibit the same falling (and at the end of life rising) tendencies with advancing age as do those in Table 4, though less markedly so. Similar ratios for the generations born in 1761, 1781 and 1801 were calculated; they fluctuate a good deal, and do not disclose any consistent pattern; but this is partly attributable to some very wide swings in secular mortality between 1750 and 1810, as the following crude death-rates (for the combined sexes) illustrate:

1766–70	26·4
1771–75	33·1
1776–80	24·9
.	.
.	.
1801–05	24·4
1806–10	31·5
1811–15	27·2

1.16. The effect of such variations can be much reduced by combining the generations (as indeed has been done in Tables 1–3) and accordingly in Table 5 they have been grouped for the periods 1761–76, 1781–96 and 1801–16. It can be seen that the pattern characteristic of later generations, including the upturn in the ratios (which does not, however, emerge until the age-group 80–84) was established by 1801–16. Moreover, the data for 1781–96 too show some diminishing trend with advancing age (only slight for $n = 2$), though without a discernible up-turn in old age. For the generations born in 1761–76, it seems doubtful whether any trend with age was yet established.

1.17. It thus appears that the difference in the shape of q_x between generations and secular periods first became a phenomenon in Sweden towards the end of the eighteenth century—in fact shortly before Malthus was to begin his personal investigations of the population situation in Scandinavia. This development in mortality was no doubt associated with the 'industrial revolution' which started in England about 1750 and soon emerged in other European countries. So, although reliable figures are not available for England and Wales for any period before 1840, very probably mortality did improve there at that time, and a generation–secular disparity then began to emerge.

FRANCE, HOLLAND, ITALY AND THE UNITED STATES OF AMERICA

1.18. Data are available for France for much the same period as for England and Wales and they show the same features, though less markedly so. The probable future experience for the generations born in 1931 and 1951 can also be studied with the aid of certain French official mortality projections, but these projections reach only as far ahead as 1985 and there is less evidence as to a recent reduction in the generation–secular disparity.

1.19. Adequate data have been traced for Holland, Italy and the U.S.A. (including projected mortality rates for the third of these countries),[12, 13] and they exhibit without exception the same kinds of features as do those for England and Wales, Sweden and France. But as the statistics at present available are, for a variety of reasons; incomplete, a close investigation of international disparities in shape, or in timing, is hardly practicable. In due course, it should be possible to get hold of fuller information; and a study of such disparities, seen against a background of differing histories of social and economic development, could well provide a good basis for new research.

1.20. Some attempt has, however, been made to make a valid contrast here between England and Wales, Sweden, France and the U.S.A. It seemed desirable to devise a simple arithmetical index by means of which the chief features of the countries' experiences could be succinctly summed up and then contrasted. The ideal would be to bring out for each generation, and for each value of n:

(a) the relative rapidity of the falls in the generation–secular ratios up to middle life;
(b) the ages at which the falls cease and upturns begin; and
(c) the timing of any sharpening or blunting in the intensity of generation–secular differences.

1.21. As a simple measure of (a), it was decided to use the average percentage fall in the ratio between each age-group and the next for the four pairs of age-groups 0–4:10–14; 10–14:20–24; 20–24:30–34; 30–34:40–44. Table 6 shows the values for England and Wales derived from Tables 1 and 2. For the purposes of international comparison these can be further condensed by averaging all the data for men and women and for values of n of 0, 20 and 40. On this basis, England and Wales, Sweden and the U.S.A. are compared in Diagram 4. The curves pass through the points calculated except for Sweden up to 1861, where graduation has been adopted. Figures cannot be produced on an identical basis

Table 6. *Indexes of generation–secular differences in the shape of the curve of q_x: England and Wales*

		$n = -20$	$n = 0$	$n = 20$	$n = 40$
Men	1871–81	7	12	12	14
	1891–1901	16	16	18	23
	1911–21	18	20	26	36
	1931–41	3	10	21	28
	1951–61	−10	1	11	13
Women	1871–81	12	14	14	15
	1891–1901	20	19	20	26
	1911–21	19	21	28	35
	1931–41	−1	2	18	30
	1951–61	−14	1	13	16

for France, but an approximation is included in the diagram. The diagram shows considerable international similarity in the rise and fall of the generation–secular shape difference, and so there seems no need to pursue further analysis under (c) in § 1.20. But the difference of shape appears to have been a little sharper in England and Wales than in the other countries, for the generations of 1871–1931.

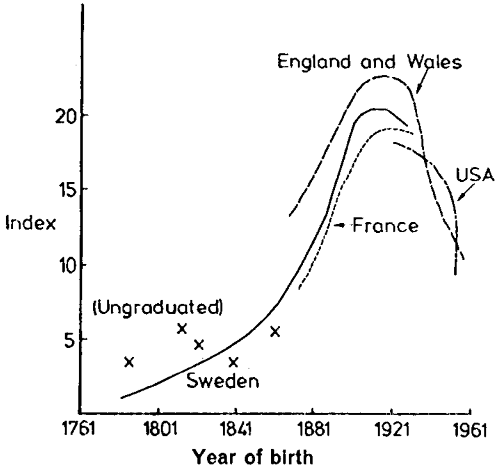

Diagram 4. Indexes of generation–secular shape disparities.

1.22. As to (b), Table 7 gives an impression of the turning-points for selected years of birth and for values of *n* of 0 and 20. Results are shown to the nearer 5. The upturn has been arriving earlier in life for the later generations than for the earlier ones and the data are rather similar for all countries.

Table 7. *Turning-points*

Year of birth	*England & Wales*		*Sweden*		*France*		*U.S.A.*	
	$n = 0$	$n = 20$	$n = 0$	$n = 20$	$n = 0$	$n = 20$	$n = 0$	$n = 20$
1861			65					
1871					55	50		
1876	45	55						
1881			50	45				
1891					50	50		
1896	45	50						
1901			45	45				
1911					50	40		
1916	40	45						
1921			35	35			35	45
1931					35	35	35	35
1936	35	35						
1951							35	35

CONCLUSION TO PART 1

1.23. Attempts to find a 'law' of mortality, which would show mathematically a relationship between the age and the chance of dying at that age, have been made by actuaries and statisticians from time to time over a period of nearly 200 years. The earlier formulae were simple but powerful, e.g. that the inability to withstand destruction increased in geometric progression (Gompertz). As time has passed, changes in experience and developments in the collection of statistics have caused the formulae to become more complex and less easily interpreted, as for example in relation to the graduation of the English Life Tables based on the deaths of the years around 1951 and around 1961. All or nearly all these attempts to find a law, or to graduate, have been founded on the mortality rates observed in some secular period; but it is now possible to see very clearly that for generations there is a pattern appreciably different from that for secular periods. This disparity has developed during a period of rapid economic and social change; currently it shows signs of levelling off, and mortality projections suggest that it may decline substantially, though not disappear.

1.24. In such circumstances it is only natural to ask the question: is the generation mortality pattern the genuine foundation for the 'law'? If it is, then secular patterns represent no more than ephemeral phenomena. If, however, the secular patterns do represent the reality, then the generation curves stem merely from the experiencing of the secular rates by people who happened to be of a certain age at a certain time, and such curves are not any more meaningful than this. It does not seem possible to resolve this dilemma by stating with confidence that the one alternative or the other is clearly the right one; no absolute standard of judgment is available. It might be possible to reach a decision on the basis of statistical considerations—for example, if there seemed to be a greater consistency of pattern for generations than for secular periods, or *vice versa*, in the relationship of q_x with x. Tests suggest, however, that neither side is markedly superior in consistency to the other.

PART 2: A STUDY OF GENERATION MORTALITY USING A LOCAL FORM OF GOMPERTZ' LAW

2.1. The principal aim of this part is to discuss generation mortality, with particular reference to a 'local' form of Gompertz' celebrated law

$$\mu_x = Bc^x.$$

The theory is applied to study the trends in mortality of various male and female cohorts in England and Wales, and Sweden, and to make mortality forecasts at the older ages, where the practical effect in population projections is greatest.

2.2. The famous law of mortality propounded by Benjamin Gompertz in 1825 may be stated, in modern notation, as follows:

$$\frac{d\mu_x}{dx} = k\mu_x \tag{2.1}$$

where k is a constant. A full description of Gompertz' paper and the processes leading up to the formulation of this law are given in a biographical article[14] where it is stated that the paper 'marked the beginning of a new era, not merely because his formula was, for several reasons, an enormous improvement on others which had been suggested previously . . . but because it opened up a new approach to the life table'.

2.3. The solutions of (2.1) are of course of the form

$$\mu_x = Bc^x$$

where $c = e^k$ is referred to as the Gompertz constant. In 1860 Makeham suggested his (first) modification of Gompertz' law by adding a constant force of mortality, to obtain

$$\mu_x = A + Bc^x.$$

In a recent study Redington[15] said of the third constant A of Makeham's law: 'her voice has been quiet this century—during peace time, that is—and in English Life Table No. 11 it has been reduced to a whisper'. This constant was therefore ignored in Redington's paper, and in the present study, which is most concerned with mortality at older ages where its relative weight is small.

2.4. Gompertz' law represents the exponentially increasing 'wearing-out' of the body's resistance to fatal illnesses, and Makeham's law adds a constant, independent of age, corresponding to 'accidental' deaths; thus both these classical laws are more than arbitrary mathematical curves, for they are derived from theories about the incidence of human mortality. This is perhaps more than can be said for many more recent formulae, such as the seven-parameter functions m_x of English Life Tables Nos. 11 and 12.[16] But there are of course important differences between twentieth-century conditions and those ruling around the time of Gompertz' discovery: one of these is that age-specific mortality rates were more or less constant with time in the early and middle nineteenth century.[17] It is therefore to be expected that period (or secular) life tables such as the English Life Tables will show patterns different from cohort (or generation) tables, and these differences are discussed in Part 1 above.

2.5. The fact that it has proved impossible to fit a simple law of mortality to the more recent English Life Tables does not mean that such laws cannot be fitted to generation data: it may reasonably be conjectured, on general grounds, that generation mortality may be 'smoother' than secular. It was found in practice that, although Gompertz' law does not hold over the entire spread of adult ages of recent cohorts, a 'local' form of this law could be developed as an aid to the study of generation mortality trends.

2.6. Let us consider central rate of mortality at ages x to $(x+t)$ and $(x+t)$ to $(x+2t)$, either for the same generation or during the same time period (in our applications $t = 5$ and we are concerned only with generations). As in Benjamin and Haycocks,[18] let $\mathring{l}_y$ be the number of lives attaining age y during the course of the investigation.

The observed central death rates at ages x and $(x+t)$ shall be denoted by ${}_t\hat{m}_x$ and ${}_t\hat{m}_{x+t}$. It is shown by Scott[19] that, under certain conditions which may be assumed to hold (at least at ages under about 85) in the applications considered below, the number of deaths at ages y_1 to y_2 in a population subject to the force of mortality μ_y at age y has, approximately, a Poisson distribution with mean (and hence variance)

$$\int_{y_1}^{y_2} \mathring{l}_y \, \mu_y \, dy.$$

It follows that the ratio

$$\frac{\text{standard deviation of the number of deaths}}{\text{mean number of deaths}}$$

is approximately equal to

$$\left\{\int_{y_1}^{y_2} \mathring{l}_y \, \mu_y \, dy\right\}^{-\frac{1}{2}}$$

The quantity in brackets is of the order of the observed number of deaths, and in our national investigations this is very large. We may therefore ignore random variations to obtain the approximations

$${}_t\hat{m}_y \doteqdot \frac{\int_0^t \mathring{l}_{y+s}\mu_{y+s}\,ds}{\int_0^t \mathring{l}_{y+s}\,ds}\ ; \ y = x, x+t.$$

2.7. We now make the further assumptions that: (a) mortality follows Gompertz' law, for some B,c, between ages x and $x+2t$, and (b) for $y = x,\ x+t$, $0 \leqslant s \leqslant t$,

$$\frac{\dot{l}_{y+s}}{\dot{l}_y} \doteqdot {}_sp_y$$

where ${}_sp_y$ is the life table function corresponding to the given force of mortality. This approximation is not unreasonable in large national investigations, if the age-group is not too wide and the period of observation is reasonably long.

The accuracy of the data must of course be considered, the largest source of error being misstatements of age in census returns. The effect of these errors is minimized by the Registrar General's choice of quinary age-groups, and the same groups are employed in Sweden. The accuracy of census age-statements at advanced ages is often extremely doubtful, and mortality at ages over 85 is considered separately in §§ 2.10–11.

2.8. If the above conditions hold, the observed central death rates ${}_t\hat{m}_x$, ${}_t\hat{m}_{x+t}$ are approximately equal to the 'life table' functions ${}_tm_x$, ${}_tm_{x+t}$ respectively, according to a table which follows Gompertz' law or mortality for some values of B and c. We may therefore attempt to solve the equations:

$${}_t\hat{m}_x = \frac{1 - {}_tp_x}{\int_0^t {}_sp_x\, ds} \tag{2.2}$$

$${}_t\hat{m}_{x+t} = \frac{1 - {}_tp_{x+t}}{\int_0^t {}_sp_{x+t}\, ds} \tag{2.3}$$

where, for $y = x,\ x+t$ and $0 \leqslant s \leqslant t$,

$${}_sp_y = g^{c^y(c^t - 1)},\ (g = e^{-B/\log c}),$$

for B and c. It was found in practice that, at the ages considered below, equations (2.2) and (2.3) could be solved uniquely for B and c, although there may be no guarantee that such a unique solution exists. The method of finding B and c is described in the Appendix.

2.9. The principal applications of the above theory are as follows:

(a) To obtain the force of mortality, and hence all the other life table functions, from a set of crude or graduated central death rates, by 'blending' the results for the age ranges $(x-t)$ to $(x+t)$ and x to $(x+2t)$, if necessary. This approach could be used, for example, to construct the other life table functions of the English Life Tables from a graduated set of values of m_x, or to construct an abridged life table from crude death rates in quinary age-groups. But since there are simpler techniques already available, and the above method might not result in any great improvement, this line has not been pursued at present.

(b) To 'monitor' the constants B and c, especially the latter. It was found in practice that the value of c is much less variable than B; since $\log c$ measures the proportionate rate of increase of mortality, rather than its absolute level, the values of c experienced at widely different ages and times may well be comparable, whereas B plays a minor supporting role in relation to c.

Reasons for movements in the value of the Gompertz constant c may be advanced in terms of particular causes of death or general factors, and, if a reasonably stable pattern emerges, future values of c may be extrapolated. The resulting mortality projections are restricted to older ages, but at younger ages the effect of changes in mortality rates is relatively unimportant in population projections.

For the reasons given in § 2.5, the work was arranged on a generation basis but studies could also be made on a secular plan.

2.10. Before proceeding to this study let us first consider mortality at ages over 85. It is convenient to treat this subject separately because there is a tendency for errors due to incorrect census age-statements to become quite pronounced at these ages in national investigations. Humphrey[20] investigated mortality at these ages by means of data based on death certificates alone, and came to the conclusion that recent mortality in England and Wales follows Gompertz' law at ages 86 and above. He found evidence that this is true not only in England and Wales but also in France and Sweden, the latter country being of particular interest since the registration of births began there in 1748.

The values of the Gompertz constant c obtained by Humphrey (his Table 12), partly from the data of Vincent[21] were as follows:

	Males	*Females*
England and Wales, 1910–1941	1·065	1·071
England and Wales, 1942–1957	1·074	1·080
Sweden, 1901–1945	1·086	1·089
France, 1920–1929	1·080	1·075
France, 1929–1938	1·067	1·078

2.11. Humphrey noted that the values for Sweden for the three fifteen-year periods 1901–15, 1916–30, 1931–45 are virtually identical, and considered it possible that this, and the relatively high values of c in Sweden, are due to higher standards of accuracy in that country. In other words, there is some evidence that c is not changing much with time at advanced ages, and the relatively high values in Sweden are perhaps more accurate than those of the other countries.

A study of mortality rates at advanced ages in Scotland during the years 1945–60 was conducted using the published data[24] and Humphrey's methods: the values of c in the Gompertz graduations were 1·076 (males) and 1·0845 (females).

These results are given in 'period' form, and if there is a constant annual improvement factor Q at all advanced ages, generation tables would follow Gompertz' law with constant $c' = cQ$. Humphrey found wide fluctuations in mortality from year to year, but detected a definite improvement in mortality between 1910–41 and 1942–57, the improvement being larger for females than for males. He also considered it probable that the rate of improvement was constant for all ages over 85. On the basis of past trends a value of $Q \doteqdot 0{\cdot}99$ to $1{\cdot}00$ may be anticipated but this will reduce the value of c only very slightly. There is therefore good evidence that, in the forseeable future, generation mortality of both sexes in England and Wales at ages over 85 will follow Gompertz' law with values of c of the order of 1·07 to 1·08.

2.12. Mortality rates on a generation plan have been collected by Case *et al.*[1] in England and Wales and Bolander[4] in Sweden. As explained by Bolander (her Diagram 1), most generation studies of mortality are based on a 'mixed cohort' plan rather than the more exact 'true cohort' arrangement: the former is employed by Case and by Bolander for rates recorded before 1895 but after 1895 the Swedish values are arranged in a pure cohort design.

In order to try to extend the values for England and Wales to a more recent date, the published data for the four-year period 1971–74, and some data released by the Office of Population Censuses and Surveys was used to obtain central death rates for that period. These rates are intended as approximations to those for the quinquennium 1971–75.

2.13. The cohorts relate to those born in periods of five years, and are arranged by 'central' year of birth; in England and Wales this is the 'central' year of birth in the mixed cohort design, and in Sweden it is the central year of five: the mortality rates used are the averages of those for five cohorts, each relating to an individual year of birth. The quinary age-groups are the same in both countries. Tables 8 and 9 show, for each listed cohort, the value of c obtained by solving equations (2.2) and (2.3) in respect of the given age-group and the next older age-group. Thus c (or, more precisely, $\log c$) measures the average proportionate rate of increase in the force of mortality over the ten years beginning at the lower age on the left, of the cohort born around the year shown at the top.

2.14. The first point of note is the relative stability of c with age and time, especially at ages over 50. An examination of the male cohorts shows a tendency for a 'peaked' pattern to emerge in the more recent generations: the value of c has become larger at middle ages and then declines with advancing age, and the peak has been reached at steadily younger ages. This trend is not nearly so pronounced in the female cohorts, where there is a tendency for the value of c to increase with age to about 1·105 from ages 60 to 75.

Reasons for these trends may be sought in an examination by separate causes of death, in so far as these statistics are reliable. The increasing value of c among middle-aged males is largely due to increasing mortality from heart diseases (compare reference [8]). c may be regarded as an index of the 'pressure' imposed (or self-imposed) on the generation, and there are many aspects of modern society which have tended to improve conditions for the old and the young more than for the working population, especially the male members aged about 40–50.

2.15. The pattern of values of c can be extrapolated and the resulting constants

Table 8. *Values of* c: *England and Wales*

age-group	*Male cohort* 1881	1886	1891	1896	1901	1906	1911	1916	1921
30–34	1·069	·958	1·040	1·037	1·040	1·024	1·000	1·008	1·046
35–39	·987	1·045	1·035	1·047	1·035	1·013	1·046	1·077	1·098
40–44	1·062	1·053	1·067	1·057	1·059	1·080	1·101	1·117	1·117
45–49	1·060	1·079	1·066	1·069	1·093	1·108	1·122	1·116	1·118*
50–54	1·089	1·065	1·080	1·102	1·106	1·117	1·113	1·110*	
55–59	1·070	1·087	1·103	1·099	1·103	1·103	1·097*	*1·100*	
60–64	1·082	1·098	1·089	1·098	1·095	1·093*	*1·093*	*1·093*	
65–69	1·104	1·087	1·094	1·091	1·088*	*1·088*	*1·088*	*1·088*	
70–74	1·088	1·092	1·086	1·086*	*1·086*	*1·086*	*1·086*	*1·086*	
75–79	1·091	1·081	1·081*	*1·081*	*1·081*	*1·081*	*1·081*	*1·081*	

age-group	*Female cohort* 1881	1886	1891	1896	1901	1906	1911	1916	1921
30–34	1·060	·962	1·021	1·024	1·010	1·006	·988	·983	1·033
35–39	·984	1·028	1·028	1·021	1·019	1·006	1·027	1·060	1·089
40–44	1·049	1·049	1·050	1·043	1·038	1·057	1·072	1·091	1·094
45–49	1·060	1·060	1·044	1·045	1·060	1·069	1·084	1·087	1·091*
50–54	1·066	1·045	1·056	1·066	1·070	1·085	1·086	1·089*	
55–59	1·060	1·076	1·083	1·086	1·098	1·095	1·093*	*1·091*	
60–64	1·086	1·095	1·092	1·101	1·098	1·097*	*1·097*	*1·097*	
65–69	1·106	1·099	1·109	1·103	1·103*	*1·103*	*1·103*	*1·103*	
70–74	1·101	1·108	1·099	1·111*	*1·110*	*1·110*	*1·110*	*1·110*	
75–79	1·107	1·096	1·110*	*1·107*	*1·107*	*1·107*	*1.107*	*1·107*	
80–84	1·106								

Note: the *italicized* figures are projections; these are discussed in § 2.7. The *asterisked* values use central death rates based on the experience of the four years 1971–74 in place of 1971–75.

Table 9. *Values of* c: *Sweden*

age-group	*Male cohort* 1880	1885	1890	1895	1900	*Female cohort* 1880	1885	1890	1895	1900
40–44	1·054	1·046	1·047	1·036	1·047	1·026	1·034	1·041	1·025	1·040
45–49	1·054	1·065	1·053	1·061	1·072	1·050	1·052	1·035	1·056	1·056
50–54	1·075	1·059	1·076	1·080	1·095	1·064	1·044	1·066	1·062	1·060
55–59	1·070	1·083	1·087	1·097	1·103	1·064	1·084	1·078	1·075	1·081
60–64	1·093	1·093	1·097	1·103		1·102	1·092	1·092	1·096	
65–69	1·096	1·103	1·106			1·102	1·106	1·105		
70–74	1·105	1·109				1·104	1·107			
75–79	1·109					1·109				

may be employed to project future mortality rates for the cohorts shown; these projections were carried out for England and Wales only.

The projected values of c were obtained after a consideration of the horizontal and vertical patterns; the results of § 2.5 show that c may be expected to tend towards a value of the order of 1·07 to 1·08 as age advances past 85, and this adds weight to the assumption that the pattern of values of c is relatively stable.

The value of B at ages x to $(x+10)$ may be deduced, by methods given in the Appendix, from the known central death rate at age x to $(x+5)$ and the estimate of c. This enables us to calculate the projected central death rate at ages $(x+5)$ to $(x+10)$. The process may be continued with the projected central death rate and the next value of c. Whether or not this method of projection is more or less reliable than other methods is an open question. Although the present approach is inapplicable to the younger ages, the practical effect of variation in mortality at these ages is not of great importance in population projections; mortality variations at the older ages, however, have a moderate effect on the total population and a considerable effect on its age-distribution, as discussed below.

2.16. The projected central death rates show a secular improvement in age-specific mortality relative to the previous cohort at nearly all ages considered (60 to 85) and in both sexes, the exceptions being the 1896 male cohort and the 1916 female cohort. In both these cases a small increase in mortality has recently been observed, and this is at least partly related to smoking (see reference[8] for statistics by separate causes of death); it is well known that cigarette smoking became prevalent among men during the 1914–18 war and among women in the 1939–45 war. From the projections, male mortality at ages 60 to 80 is expected to fall at a moderate rate in the foreseeable future, and at ages 80–84 mortality should begin to fall again after about 1980. For females the projections suggest a continuing moderate decline at ages over 65, but as the generations born from about 1916 onwards reach these ages, mortality rates will decline more slowly.

2.17. This method could not be used to forecast mortality directly for so long into the future as forty years, but the following rough estimates of forty-year improvement factors for comparison with other projections were obtained by the extrapolation of trend lines:

Age	*Males*	*Females*
62	·80	little change
67	·75	·85
72	·75	·75
77	·75	·75
82	·80	·80

These values are comparable with those used at present in the United Kingdom official projections,[10] although the improvement in the official projections decreases with advancing age. The above factors are also comparable with, if rather more optimistic than, the level of improvement suggested by Scott,[8] but they are much less optimistic than the factors projected by Brass[7] using a logit transformation.

2.18. The recently published variant official population projections for Great Britain[22] consider varying fertility and migration only, but population projections for England and Wales under different assumptions about future mortality are discussed by Cox and Scott.[23] An examination of the results of these projections shows that, if the improvement factors suggested by the present method were employed, the size and age-distribution of the projected population of England and Wales would be fairly similar to the values obtained from the current official ('central') projections. Marked differences occur only when the assumed rate of improvement at the older ages is very pessimistic (no improvement) or extremely optimistic (corresponding to the elimination of mortality from all forms of cancer, with considerable reductions in mortality from other causes, over the next forty years).

APPENDIX

THE CALCULATION OF THE CONSTANTS B AND c

A.1. The first step was to evaluate the integral

$$\int_0^t \exp\left\{-\frac{Bc^x}{\log c}(c^s-1)\right\} ds$$

by numerical methods. The integrand may be expanded in an exponential series and the terms $(c^s-1)^n$ may be found by the binomial theorem and integrated. The exponential series may then be integrated term by term, and an application of Leibniz' test for alternating series shows that the remainder after N terms is not greater than the first neglected term, which may be made arbitrarily small. (In practice $N = 10$ was sufficient.) There are of course other numerical techniques for the evaluation of this integral.

A.2. We then suppose that c is known and solve equations (2.2) and (2.3) for B separately, giving solutions B_1 and B_2. To find these values, $k = B\,c^y/\log c$ is regarded as the variable, and the equations

$$f(k) = \frac{1-e^{-k(c^t-1)}}{\int_0^t e^{-k(c^s-1)}ds} - {}_t\hat{m}_y = 0, \; (y = x, x+t),$$

are solved by Newton–Raphson techniques, the initial value of k being found by choosing $B = ({}_tm_y)/(c^{y+\frac{1}{2}t})$, or some similar value. This technique requires $f'(k)$ and hence $\int_0^t (c^s-1)e^{-k(c^s-1)}ds$ to be evaluated; this integral may be evaluated by methods similar to those given above. The resulting solutions B_1 and B_2 may be computed to any prescribed level of accuracy; let $U = B_1 - B_2$.

This procedure is followed for a pair of trial values of c and the resulting values of U are noted. By linear interpolation a third value of c may be found such that, if U were a linear function of c in the relevant range, the new value of U would be 0. This value of c may be made the centre of a new pair of trial values of c, rather closer together than the original pair, and by iteration a value of c may be found such that $U \doteqdot 0$ to a prescribed level of accuracy. The resulting values of c and B_1 (or B_2) are the required solutions.

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ABSTRACT OF THE DISCUSSION

Mr S. Haberman: Actuarial interest in generation mortality virtually disappeared after Derrick's paper of 50 years ago (*J.I.A.* **58**, 117), probably because it has not been considered a significant factor in the practical side of actuarial work.

The mortality rate $q(x, t)$ at age x, time t may be analysed in at least three ways: first, by holding t constant, which is the secular approach; secondly, by holding x constant, which was Derrick's approach and is not a true generation analysis; and thirdly, by associating $q(x,t)$ with $q(x + n,t + n)$ for all n—the cohort generation approach. If the second approach revealed that changes in the environment had no effect on q, then it is a valid description of mortality pattern and coincides with the cohort approach, that is. $s = 0$ in Mr Cox's model. Only in such conditions is secular analysis strictly justified, otherwise the mixing of cohorts would clearly lead to interpretation difficulties. In current conditions where mortality rates have been decreasing with time at most ages, secular analysis is not justified in isolation. Male mortality rates for cancer of the lung provide an example of the misleading conclusions that may be thrown up by secular analysis. The rates for any generation curve are exponential in shape. They increase with time but produce a peaked secular curve.

If the case for cohort analysis is so clear cut, why has there been the concentration on secular analysis for mortality description and projection? The reasons have been threefold: first, the well known statistical disadvantages of the cohort approach; second the problems presented by migration and third, most significantly, the need to describe the effect of current environmental influences. Concerning migration, it would be dangerous to use cohort analysis in countries such as Australia or Israel, or for sub-units of the population, e.g., counties or boroughs in the United Kingdom. At the time of Derrick's paper, cohort analysis was considered valid on the grounds that somehow a generation carried its mortality with it, determined in some mysterious fashion by the calendar year of birth. I believe that this is only one part of the story. Cohort rates may be thought of as the product of several factors; for instance, the effects of early life, the exogeneous effects caused by the environment in early adulthood, and the endogeneous effects caused by ageing and deterioration in the health of the generation. By omitting the latter two, early thinkers glossed over the potential importance of cohort analysis, except for Sir William Elderton in the discussion on Derrick's paper. This error is not similarly committed by the present authors who concentrate on these items.

Mr Cox compares secular and generation mortality rates by calculating their ratio and analysing that in detail. Dr Scott concentrates on ages over 30 and attempts a novel local fitting of a Gompertz curve.

Mr Cox's approach is strikingly simple and, as with all elegant innovations, the question arises why it has not been done before. The analysis is traditionally actuarial in that a hypothesis is proposed, 'actual' is compared with 'expected', and deductions are made about the applicability of the hypothesis. The variation between the secular and cohort descriptions of mortality is principally explained by the rate of improvement in mortality rates at the younger ages over the past century or so. Thus the model incorporates a rate of improvement which reduces linearly with age. The results arising from its application have been compared with past data, and that expected from the current projections. Mr Cox discusses the main features of the theoretical ratio of generation q : secular q. If we call this ratio R, it is a function of x, n and S, and the features illustrated may be established quite simply by a mathematical analysis. If we consider R as a function of x only, we find that a minimum occurs at age $\bar{x}$ which is given approximately by $(100 + n)/2$. This is almost independent of S: S only affects the third significant figure in the calculation.

The results of Tables 1 and 2 showing the ratios of generation to secular mortality rates for England and Wales conform with the features expected from Mr Cox's model in the variation with x and n, and suggest that the model is a reasonable representation. Table 1 suggests that the rate of improvement in mortality has risen over time. Table 2, which relates to more recent generations, displays features that may be explained by a fall in the rate of improvement.

Although the age differences are the most striking features of Tables 1 and 2, there are sex differences as well. These imply that the female rates may have enjoyed a higher rate of improvement in the past.

Table 3 extends the analysis to age groups over 60. The author comments that $\bar{x}$ is higher for women than for men, especially for $n = 20$, and for the later generations, and he blames this on the fact that female mortality has reduced more between ages 60 and 69. This feature is certainly not reproduced by the model, as I have indicated that $\bar{x}$ is virtually independent of S. For $n = 20$, $\bar{x}$ varies by ·3 between $S = 1\%$ and 5%.

We can summarize the results of Tables 1–3 by the statement that the generation–secular differences have increased in the past and are expected to narrow in the future without disappearing completely.

The author then examines Swedish data. He chooses Sweden because it is a country with a similar socio-economic history to England and Wales, and has vital records stretching back into the eighteenth century. The pattern of ratios in Tables 4 and 5 is similar to that displayed by both the model and the earlier tables relating to England and Wales. The values of the ratios for men are slightly higher and for women slightly lower than England and Wales. This would imply that men have enjoyed a higher rate of improvement and women a lower rate in Sweden compared with England and Wales. The examination of Swedish data provides an insight into the timing of the first appearance of the generation–secular disparity. Mr Cox dates this emergence at the end of the eighteenth century, that is, at the start of the industrial revolution, and postulates that a similar date may be suitable for England and Wales.

The analysis is extended to other countries where migration cannot be ignored as a confounding factor. Its importance may be illustrated by considering net immigration as a percentage of total population change for the period, say, 1960–1970. Immigration has caused 39%, 38% and 23% of the total population change for France, Sweden and the United States over that period. For Italy net emigration represents 16% of the natural increase over the same period. It is not clear whether Mr Cox's data for the United States, refers solely to whites or to the total population. In view of the large immigrant flows of whites into the United States during this century, it may be that generation analysis would be more fruitful if applied to negroes only.

Having analysed the ratio of what are essentially two curves, why did Mr Cox not go further and compare the gradients of the two curves, or, given the emphasis placed on the curve of deaths, why did he not compare the two curves of deaths? No attempt is made to fit a curve to the generation death rates, and this certainly displays interesting shapes with an extended low plain at the early adult ages, followed by a sharply rising slope. There is not a mortality rate to be seen in the paper, and that is not necessarily a bad thing.

Other extensions might include the use of the ratios for the cause of death studies. For cancer of the lung for males, an S_x of the quadratic form might be suitable, for instance, $bx(1 - cx)$.

Mention of causes of death leads to the work of Professor Beard who, in a series of papers published over a decade ago, analysed mortality data by cause of death and quantified the generation and environmental factors in mortality. He considered μ to be the product of three components: a function of the year of birth, representing genetic factors; a function of the year of experience, representing environmental factors; and a function of the age, representing the conditional resistance of the individual. By a simple combination of four μs at different ages and times, he was able to isolate the effect of each function, and determine them numerically. He then successfully related the genetic and environmental functions to cigarette consumption for both sexes.

Mr Cox's paper does not, regrettably, transcend beyond an alternative method of description of generation–secular disparities. His ratios are interpreted in terms of rates of improvement which are either known exactly or have already been estimated from projections. The ratios do provide a deeper insight into generation mortality, but no new techniques are advanced for actually using the ratios in the projection process.

Having assisted Mr Cox with the exhumation of one skeleton, Dr Scott produces another, namely, Gompertz' description of mortality, and in this connection he is following in the footsteps of Redington (*J.I.A.* **95**, 243). Like Mr Redington, Dr Scott proposes to omit A which corresponds to accidental deaths from the more general Makeham formula for the force of mortality, on the grounds that its relative weight is small at the old ages on which Dr Scott concentrates. This is certainly borne out on examining the numbers of deaths attributed to accidental causes at ages over 75. From the *Registrar General's Annual Review for 1973*, 2% of male and female deaths are assigned to these rubrics, but how many deaths at these advanced ages involve an accident as a contributing cause? If an elderly person has a slight stroke, falls and breaks a leg and gets pneumonia, he or she may subsequently die; this is not an uncommon train of events. Would such a death be classified under pneumonia, accident or cerebrovascular disease as the underlying cause? Would it contribute to the A term or to the Bc^x term of the Makeham formula? Assuming A is small, it is not justified when treating ages as young as 30 as Dr Scott does; thus accidental deaths form 24% of male, and 15% of female deaths at ages 30–45. The author's approach is to fit a Gompertz model locally to generation data. The trouble with local fits is that a very simple curve will do if the neighbourhood is small enough. Why not a straight line? Is not that how the use of a uniform distribution of deaths is justified to life contingencies' students?

In § 2.9 the author proposes to monitor only c from the fitting of his model. He states that "c is much less variable that B" between age groups, and that "B plays a minor supporting role". Redington, however, reported the importance of B in his 1969 paper. In particular he expressed it as c^{-z}, so that if B is constant, a higher c implies a higher μ, but if z is constant, a higher c implies a lower μ. Dr Scott is concentrating on the rate of increase of the generation mortality rate, and he hopes to answer questions in this field by examining his theories of local cs. By not publishing B he has left us in a dilemma. Do we regard B as constant so that c is a rate of interest, or do we regard z as constant so that c is a rate of discount? If we regard neither as constant, how are we to interpret Table 8 showing the variations in c between sexes, ages and cohorts? In fact B (or z) determine the main features of the curve of deaths; the peak, the point of inflection and the angle as described by Redington. These are the points where the first, second and third differential coefficients are zero.

Reference is made by Dr Scott to Humphrey's work in graduating mortality rates from registration data only at ages over 86 for England and Wales using a Gompertz formula. At all ages the female rates were lower, yet the value of c for females was higher than that for males. Why was this? The reason is that the two z's were approximately equal while the values of B were markedly different. The point made by the author in § 2.14 concerning the stability of c is true. If B is constant, reducing c from 1·08 to 1·07, a ·9% change, results in a 50% change in μ_{75}. The author is hardly being controversial when he states that he expects generation mortality to follow Gompertz' law with values of c lying between 1·07 and 1·08.

In Tables 8 and 9 Dr Scott presents the values of c arising from his fitting process for England and Wales and Sweden by date of birth of cohort, age and sex. Apart from the stability of c, he comments on the peaked progression of c for male cohorts born in 1911 and 1916 and associates this pattern with the increasing mortality from heart disease, thereby assuming that variations in B may be ignored between successive age groups in these cohorts. It is possible that this feature may have arisen because of Dr Scott's use of a Gompertz formula rather than a Makeham formula at the ages 30–45.

Unlike Mr Cox, Dr Scott puts forward an application of this method to the extrapolation of rates into the future on a generation basis. Over a 40-year period the results are comparable to those of the Government Actuary's official projections, but intermediate between the pessimism of Dr Scott's cause of death projection and the optimism of Professor Brass's logit projections.

Mr H. A. R. Barnett: On first reading, this is a very exciting paper, but on reflection, it appears to be merely a theoretical exercise in techniques which will certainly have no place outside population studies. Whether it will have a place inside population studies is not for me to say. Bearing this in mind, it is a pity that certain expressions are used. In § 1.1 it is stated that attempts to frame laws of mortality linking q_x with x should always take generation histories into account. Admittedly the year of birth as well as the year of experience and all the years between have a bearing on mortality, but the most important of these are the year of experience, the few years before and, of course, age. I know that it follows that if year of experience and age affect mortality, the two combined bring in the year of birth, but in my view, the environmental element is by far the more important. Furthermore, the authors' approach is unlikely to be appropriate for forecasting either assured lives' mortality or annuitants' mortality. The majority of life assurance policies do not remain in force for more than 30 years, if that; this applies also to annuity policies which are rarely purchased before 60 years of age. So the assured lives and annuitant populations are analogous to national populations with very high rates of emigration and immigration.

The summary in § 1.24 is somewhat sweeping in its polarization of the problem into whether the reality is the generation pattern or the secular pattern, without apparently admitting the possibility of a mixture of the two.

Coming to Part 2, I doubt whether anyone ever really believed that the accidental death rate was the same at all ages. We have had the constant A bandied about as being related to accidental deaths. I believe the Makeham formula was used, was useful and was convenient, because for a long time it happened to fit over a long range of ages, but any attempt to separate mortality by cause into the A section and the Bc^x section failed. In the same way, the seven-parameter formula used for E.L.T. 11 and E.L.T. 12, which bore a close, though never acknowledged, affinity with one which I produced in a paper discussed some years before E.L.T. 11 was published (*J.I.A.* **81**, 105), merely happened to fit over a wide range. Furthermore, Gompertz' law will never fit the childhood ages nor those young adult ages where the accidental death rate decreases with age. I do not accept that mortality at these ages is unimportant in making population forecasts. Incidentally, I believe there will always be a discontinuity at the age where driving first becomes legal!

How Mr Haberman's example of pneumonia, due to accident, due to cerebrovascular disease would be coded would depend on the way in which the certificate was made out. If it was simply pneumonia in 1(a) due to accident in 1(b) due to cerebrovascular disease in 1(c), the code would stop at whatever the accident was. If pneumonia was the only cause in part 1, and accident and cerebrovascular disease were both in part 2, the coder would stop at 'accident'. If it were pneumonia in 1(a) due to cerebrovascular disease in 1(b) and the accident was in 2, it would then stop at cerebrovascular disease. The number of permutations can be increased, but it is very difficult to devise a means of setting out a certificate in such a way that the coder would stop at 'pneumonia.'

Table 8 is the most exciting part of the paper, yet even this on reflection is disappointing. Extrapolation can equally well be achieved diagonally downwards from right to left, that is to say, on years of experience. We are not told the precise equations used for extrapolating c, or whether it was done graphically, but the equations certainly are not straight lines. If, for example, they are third difference extrapolations, is this not the same as saying that instead of two parameters there are five?

Another formula (*J.I.A.* **101**, 133) has four parameters for assured lives mortality from age 17 to the upper limit. Would it fit national data and, if so, might the extrapolation not be simpler even though there were more parameters? The authors may say that nothing is simpler than the projections they have used. I would agree, but in that case would not similar projections of the actual populations graphically between suitable upper and lower extremes give just as good an estimate for the future?

Mr R. H. Daw: At the end of Part 1 of the paper, Mr Cox discusses briefly whether generation mortality or that of secular periods is the more appropriate as the foundation of a 'law of mortality'. In 1970 I submitted a paper to the Institute (*J.I.A.* **97**, 17) comparing mortality and sickness in this country in relation to geographical and socio-economic factors. For the period around 1961 the Standardized Mortality Ratio for deaths from all causes in men aged 15–64 was 80% higher in Social Class V than in Social Classes I and II. For regions of the country the highest figure was 50% greater than the lowest. For certain causes of death the variations was even greater. I concluded that "In general it may be said that the more unfavourable the environment, whether in terms of social conditions like overcrowding, education, income, etc., or of diet, weather, or air pollution, the higher is the mortality and sickness." Thus at the present time, improvements in living conditions and air pollution are examples of factors which may lead to lower mortality; deterioration in the National Health Service is one which may perhaps tend to increase mortality.

In earlier times, the factors affecting mortality were of a very different nature. Picking out a few of the views expressed in the 1946 discussion of Pedoe's paper on mortality trends (*J.I.A.* **73**, 213) we have: that the provision for the masses of drinking water free from sewage had been the greatest factor in the reduction of disease in the nineteenth century, Sir John Orr's opinion that at the end of the nineteenth century the working class diet was less satisfactory than at the beginning, that improvements in mortality before 1900 were due to public health measures, but the improvements after 1900 were due to advances of medical science in dealing with specific diseases.

The effect of the various factors which have operated at different times need not produce a smooth or consistent time pattern in mortality rates, whether generation or secular. That there has been some sort of consistency of pattern is probably because any change in conditions takes effect only slowly and the full impact does not occur for some time. All this leads me to think that the secular patterns are more likely to show stability because they are at least the result of certain factors acting at the same time on all ages. Generation mortality is however mainly the result of different factors acting at succeeding ages so that any resulting patterns would be harder to interpret. I am not implying that earlier conditions, say at the time of birth, do not affect subsequent mortality. This may well be the case for tuberculosis (*J.I.A.* **76**, 143) but the effect is probably small.

Previous work on generation mortality has considered the ratios by age of one generation to another. This applies to Derrick's graphs of 1927 (*J.I.A.* **58**, 117) and Rhodes' mathematics (*J.R.S.S.* **104**, 15). If we take $n = 0$, Mr Cox's ratios in Tables 1–5 are broadly a comparison of the mortality at age x of a generation born x years ago, with the same secular mortality at age x, x years earlier. Thus the difference in time between the numerator and the denominator for a particular generation is not constant but increases with age. Looked at biologically, I doubt whether there is any real reason to expect a consistent pattern in such ratios. Each ratio in a column of the tables is a measure of the effect of factors tending to change the mortality at that age, operating over a time period of different length. Mr Cox shows that numerically there has been a fairly consistent and slowly changing pattern, but I feel that this may be of little real significance.

My comments have been made from the biological point of view of trying to relate improvements in mortality to the cause so far as we know, or can know, it. The authors have, however, been looking at numerical patterns without consideration of how and why they have arisen, or what reasons there are for expecting them to continue. Such an approach is of interest, and can lead to greater understanding of mortality rates, but there is a danger of drawing unwarranted conclusions unless there is some consideration of the causes which led to the pattern.

Dr A. M. Adelstein (a visitor): My interest in this subject is in trying to interpret vital statistics; so that it is more a causal than an actuarial one. I have found the paper very interesting from the point of view of the techniques it produces.

Since my interest is in interpreting data from the point of view of trying to alleviate conditions and in trying to improve the health of people, I would have to judge a generation mortality pattern from the point of view of what it tells us. Does it help us to interpret the data? Taking the tuberculosis mortality rates as an example, it is very interesting because the generation mortality rates do not rise with age; they fall with age, and do so quite sharply from about the age of 20 onwards. The pattern across the whole generation is a part of a *W*, that is to say, it is high in infancy, low in childhood, rises to a high peak in about the twenties, and then falls consistently. Generation after generation shows exactly the same pattern, but at successively lower levels. When you look at the cross-section of the data on current mortality, you get the illusion that mortality from tuberculosis rises with age, because you are comparing the children of today with the adults of yesterday. Tuberculosis is the only cause I know which gives this very distinct pattern; each generation seems to go on a course which is quite unique—high mortality, dropping, rising again and then falling. This does indeed help to interpret the current trends, and to interpret the way tuberculosis affects the population.

There are three causes with which I am familiar in terms of generations, that is, tuberculosis, cancer of the lung and cancer of the cervix of the uterus. The last displays an interesting pattern, because if the age over time curves are drawn and put over date of birth rather than date of death, that is, pushing each of the higher ages back one step, a very distinctive pattern appears. The women who were born around two points have high mortality which then falls again: the first point is at about the turn of the century, and the second point is about the early twenties. All these curves rise to a peak, come down, and then rise again, and the simplest and perhaps the most obvious interpretation is that the two world wars caught these women when they were about 20. The curve of venereal disease in young women, is exactly the same. So it appears that the cancer is associated in some way with sexual behaviour, and possible increase in promiscuity in those times. The generation curve is essential in this case for interpreting and helping us to understand how this occurs, and in helping us to explain whether the preventive measures thought of are now actually useful. If the generation as well as the current curve is not studied, we are unable to say whether there is really a falling rate or not, because it depends on which generation of women is being looked at.

For lung cancer, the pattern is that mentioned by Mr Daw: each generation carries a certain propensity with it, but it is not just a generation, it is a generation *and* a time effect. The cause in this case turns out to be largely smoking. During the First World War, smoking increased considerably, and in each of the generations after that mortality rates have risen. The generation curves are needed to interpret the picture in which the mortality rates continue to rise in older people because of the effects of their earlier habits, probably exposure to other factors as well as cigarettes.

Dr F. Rueff (a visitor): The authors' observations are important for annuity policies in Germany. A system of generation tables has been prescribed by our National Supervisory Body for at least 15 years. They have served very well, and the insurance companies have not made losses as a result of mortality in respect of annuity policies. The generation tables in Germany have also been of assistance on the basis of the so-called *Vergleichsmethoden*. There is a description of the method in the report of the Tokyo Congress (*Trans. 20 Int. Cong. Actuaries*, **2**, 371). I have used this method on the German mortality period tables and from 1870 till the present there is an almost precise correspondence between prediction and fact. The same conclusions apply in the case of England and Wales, Norway, the USA, Switzerland and France. The results of the German method are comparable with those of Mr Cox and Dr Scott, and the formulae used are the same.

Professor W. Brass (a visitor): I am never surprised at the irregularities of mortality, and perhaps there has been too little consideration of the really astonishing thing that there are regularities. It does not surprise me that it is difficult to forecast mortality, but that it can be done at all.

Some years ago I delivered a lecture to the Faculty of Actuaries in Edinburgh (*T.F.A.* **33**, 123). Partly because of its nature, the technical basis was not very well spelled out. I attempted, through practical examination, to ask the old question: does a generation carry its mortality with it? I devised a method by which I thought an answer could be found. The method was to determine whether, in childhood, a generation did particularly well relative to the generations before or after, that mortality advantage continued in a relative sense to what happened to the generations on each side, that is, trends over time were allowed for. I applied this method to

Sweden and the results were enormously impressive. They suggested that if a generation started off with an advantage, then that advantage continued. I applied it to England and Wales, and the results were more ambiguous. I then tried the USA and the results came out the other way round, that is the generations that started off well did badly at later ages, and *vice versa*. This is typical of what is found when attempting to deal empirically with generation mortality effects. There are great problems of disturbances. In the USA, e.g., the influence of migration is enormous, and therefore no comparability over generations can be assumed.

In these studies I was not interested in the use of generation mortality for prediction purposes. It is not a very important question. In practice you obtain much the same results according to whether you use the secular or generation method. Mortality rates are affected by secular jumps, and this raises the issue of how far a sudden increase or a sudden decrease in mortality for a particular age or for a particular time extends. I think the answer is a little but not all that far. How far depends on the causes of death, which leads to the need for closer investigation.

There were a number of references to the mortality projections that I presented in a paper to the Royal Statistical Society (*J.R.S.S.*, **137,** 532). I made those projections with my tongue largely in my cheek as an illustration of methodology, and I hope that they are not quoted as a genuine expression of what I think will happen. I want to illustrate the reasons why I made them.

How you project mortality depends upon the time scale, both in the past and in the future. Changes in mortality patterns, whether generation or secular, can be described by four parameters. One of these describes level and the others pattern effects related to old-age mortality, early-age mortality, and the balance between the two. Over time there is a fairly regular change in level, but the pattern parameters fluctuate substantially. The period of fluctuation varies, but it is usually quite short, so that when trying to make mortality projections quite different conclusions are reached, according to whether the next 5 or 10 years, or the next 50 years, are being considered. Over the next 5 or 10 years the assumption that the shape of parameters will continue the current trend is a reasonable one. Over 50 years, it is thoroughly unreasonable, and it is necessary to introduce fluctuations over time, or neglect the pattern changes completely.

So, according to the aims, different irregularities and regularities enter into the study. This is closely related to cause of death issues. I am trying to produce models of mortality by cause of death in order to look at the relationship between different mixes of causes and age patterns. As the mix changes, and this can happen quite erratically in particular countries, then there are different relatively short-term pattern effects. Over a substantial period fluctuations would be expected to even out, because mortality will change in ways which are dictated by improvements in medical technology. Although the irregularities will be there in the short run, over long periods the regularities will return.

Mr J. Hamilton-Jones: I should like to widen the word 'International' in the title, and refer you to a paper submitted to the Tokyo Congress (*Trans. 20 Int. Cong. Actuaries*, **2,** 735). In South America in general, and in Colombia in particular, there exist so far no reliable statistics of population mortality. Illiteracy, mainly in rural areas, makes it impossible to collect complete census data. I think it is relevant to examine the thorny problem of mortality projection with an eye to the needs of demographers working in widely different conditions. Türler's Congress paper just mentioned quotes as his earliest available material population mortality in 1938.

A comparison of *S* from the data in Türler's paper with the last column headed "1941–45 to 1961–65" of the table in § 1.5 gives, perhaps, surprising results. I have no comparable result for age 0, because Turler combined it with age 1, but here are the answers: at age 20–24, 3·3 in the paper, 3·2 for Colombia; 40–44, 2·2 in the paper, 2·7 in Colombia; 60–64, ·1 in the paper, ·8 for Colombia; and 80–84, ·1 in the paper and ·9 for Colombia.

Of course, we are looking at two populations which are vastly different: many Colombians pass their lives at an altitude which would try our own constitution; others in stifling heat, yet they turn up 'improvement' factors comparable with those in Europe. Now the basic q's of course are much higher, and I think it is fair to expect that Colombian development will be on a different pattern. Cause of death in specific cases and other specific problems yield to analysis on the generation approach, but on turning to more recently developed countries, I cannot see how the technique can yet be of much help. Generation mortality, whatever we think of it, is a synthesis of observed data as is also secular mortality. This means that you cannot really fiddle about with them. It is when you start to blend and to select that the process of judgment and skill comes in. The secular mortality table wins every time when you think of the use actually made of pure and unanalysed mortality tables, that is, mortality tables not referring to a particular cause of death, nor mortality tables where, e.g., accidental deaths have been eliminated, but where all deaths are incorporated. The temptation to fiddle about with a generation mortality table is irresistible, and if you use a blend of the two, you are fiddling about.

Mr C. D. Daykin: In § 1.1 of the paper a sentence to which other speakers have already referred comments that attempts to frame laws of mortality should always take generation histories into account, as well as secular experiences. A previous speaker indicated that this was perhaps taking it a bit too far. But to take something into account does not necessarily mean that you have to use it as an overriding consideration, and generation mortality is something that needs to be taken into account more than is usually done. The existence or otherwise of a simple law, as is discussed in various parts of this paper, does not deflect from the usefulness of generation or cohort considerations in studying mortality as in many other fields. It is common practice, e.g., to speak of expectation of life as a measure of mortality, and although actuaries may be alert to their true significance, many other users are probably not. For example, in George King's E.L.T. 7 based on deaths in the period 1901–10, there is an expectation of life at birth for females of 52·4 years. The actual expectation of life of women born around 1900 is likely to be around 57 years, almost 5 years more, if the experience so far is taken into consideration together with projected future mortality to complete the generation. Differing views on mortality improvement from now on would not greatly affect this result. For males born in 1900 the generation expectation of life might be 51 years compared with 48·5 in E.L.T. 7. The effect for later generations is inevitably more dependent on projected future mortality, but even with what can be regarded as reasonably conservative levels of *assumed* future improvement, the true expectations of life for most recent generations will be 3–4 years higher than expectations based on mortality in the year of birth.

Expectations of life, as such, do not play an important part in actuarial work, but other functions, such as annuity values, are similarly sensitive to the effects of improving mortality, and it is frequently the mortality of this generation about which we are in fact implicitly making assumptions. The allowance made for future improvements in mortality when calculating annuity rates is often on a rather *ad hoc* basis, e.g. by taking a year from the age. It could be worthwhile for greater attention to be given to the effect of likely future improvements, and the precise way in which these might impinge on annuity values.

Mr A. D. Wilkie: I cannot see that there is any particular point in arguing whether secular or generation mortality is the more important. If you have a three-dimensional figure with mortality rates plotted against two dimensions of time in one direction and age in the other, the result is a steeply sloping surface. I do not see why it is more important that the hill slopes more in a northerly direction than in a north-easterly. The important point is the shape of the hill. You have to look at both dimensions.

Mr Cox says that it is very difficult to draw diagrams because the q_x's are small. But it is not so difficult if you transform the scale of q_x on μ_x. You can draw quite reasonable diagrams on four-cycle log graph paper. For example, in the paper which introduced the A67/70 tables, (*J.I.A.* **101**, 135), the q_x's were drawn on that scale. If μ_x's are plotted on the log scale, Gompertz' law gives a straight line which is the simplest possible function. It does not matter whether you log q_x or μ_x until you get to the very highest stages. Dr Scott refers to this, suggesting in § 2.11 that "if there is a constant annual improvement factor Q at all advanced ages, generation tables would follow Gompertz' law with constant $c' = cQ$." This is like treating the whole μ_x surface as a plane which not only slopes upwards with age, but slopes sideways and downwards as time goes on. The purpose of Dr Scott's part of the paper is to show that this assumption is more or less right which does not seem altogether implausible.

Mr Cox shows the difference between going diagonally across the surface and perpendicularly in the direction of age. If you follow Redington's suggestion, also put forward by Buus (*Trans. 16 Int. Cong. Actuaries*, **1**, 364), that if mortality at some high age such as 90 or 100 is constant, and the secular mortality produces an increasing c and a declining B, then the μ_x line is simply getting steeper but going through the same point. In order to demonstrate what is happening, imagine two rods of bamboo held parallel, one high up representing age 100 and the other low down representing age 30, with a cloth stretched taut between them. The surface is a plane, because there is no change with time. If mortality rates at all ages have the same improvement factor, both holders of the bamboo rods should tilt them at the same time, and the surface is still a plane. However, if mortality at age 100 is fixed, then the upper rod is kept horizontal, and the lower rod is tilted resulting in a much funnier shaped surface which is not nearly so easy to deal with. If it is straight in one direction, so that calendar-year mortality follows a reasonable sort of Gompertz' law, then it is not straight in the other direction so generation mortality does not follow Gompertz' law, and I am not sure what you do about that.

Giles and I (*T.F.A.* **33**, 375) produced a way to plot these slopes, but turning them flatter, by relating the levels of mortality to Coale and Demeny's *Regional Model Life Tables* and then looking to see how they were changing. To a great extent the changes for many countries, Netherlands, males; Sweden, females; Norway, females; and England and Wales, females, show very steady improvements with time, rather similar to those that Mr Cox has described, but quite a number of countries in our investigation did not behave in that way. Over the last 20 years the mortality in a great many countries increased at quite a number of ages. Mr Cox has been either careful or lucky in happening to choose England and Wales and Sweden for his examples, because they were two of the countries where mortality had decreased generally rather than increased. The typical pattern over 1950–70 was for mortality in the 30s to decrease in the way we would expect, and for mortality in the 80s to do the same as it has done in Britain. Mortality in the 60s for males rose, sometimes by quite a lot: in Norway and Germany, the rise was about 35% over that period. That gives a much more humped shape of surface, as if someone was sticking his fist underneath and pushing it up somewhere about the 60–70 age group. There was also quite a sharp rise in the 20s in many countries, which presumably resulted from deaths by violence. For females there is a small indication of a possible world-wide rise peaked at about age 50.

I am not at all sure what this does to Mr Cox's curves. I think it pushes some of the figures above 1·0. We cannot make any assumptions about steadily declining mortality even at a decreasing rate. It is quite possible that we shall get increasing mortality in the U.K. That would mean that the expectations of life that we are quoting nowadays based on secular mortality are possibly too optimistic. They are higher than those experienced by generations in the past and they are higher than will be experienced in the future.

Professor B. Benjamin: I was going to begin by saying that I thought that Mr Cox had missed the point of his paper, by which I mean that it is inevitable that if you take a small part of the product of a continuing programme of research, which this is, then this part may not itself be clearly related to any particular aim. This is bound to some extent to inhibit discussion. It is rather tantalizing that the major part of the research is in two papers, which are still in process of publication.

The first attack of any reviewer or discusser is on the contrast, if any, between the stated aim of the paper and its achievement. If there is no clearly stated aim then this becomes difficult, and I think that Professor Brass had something of the same difficulty. Mr Cox says at the outset that prediction is not much helped by his analysis. Since I have spent most of my working life professing that statistics are for the making of predictions and not for making fun, I am bound to ask whether the journey is really necessary at all. Indeed, the more so because the author also says that generation analysis is essential for framing laws of mortality. I am one of those people who do not find much satisfaction in attempting to frame laws of mortality at all. The only law I can recognize at this stage in man's history is that propounded by Professor McKeown in his book *The Modern Rise in Population*, which is that we start by being well and we strive by our way of life to make ourselves ill! We do not need to have laws for anything except for making predictions. So what is the point of the paper? Not, I would suggest, the actual ratios of q_x (secular) to q_x (generation) which are the arithmetical accidents of the passage of particular generations through particular historical events affecting mortality. The main point of the paper is to try to elucidate why the shape q_x is different for mixed as distinct from single generations. As Mr Haberman pointed out, the main source of differences are those environmental influences which occur, not steadily over a long period of time, but relatively suddenly and sharply over a short period of time, so that $q_{x,t}$ is related to t in respect of durations subsequent to these relatively abrupt environmental changes. Mr Daw and Dr Adelstein both made this point.

There are many examples of these kinds of changes, some of them beneficial in lowering q_x and others operating to increase q_x. Of the beneficial changes, we think immediately of infectious diseases, which in the middle of the nineteenth century accounted for more than half of all deaths in the population. Of the changes that have helped to combat these around the early nineteenth century, largely as a result of the great work of William Farr, there occurred public acceptance that many of these diseases were either food- or water-borne. As a result of this acceptance and the actions that followed, the death rate from food- or water-borne diseases was almost halved in the last half of the century. Vaccination against smallpox became available free in 1840, and was compulsory for infants in 1854, but only actually enforced in 1871. There is a very steep fall in mortality from smallpox after about 1850.

Towards the end of the nineteenth century there was the development of bacteriology, and the isolation of bacteria. This had an effect on the attack against airborne infections. In the same period there was an increase in health education activity; the appearance of health visitors; the appearance of the milk depots which became the maternity child welfare centres, and there was a very sharp fall in infant mortality as a result. In the 1930s, there were the sulpha drugs which took away much of the threat of the secondary pneumonias which contributed to death from infectious diseases, and a little later the antibiotics, particularly streptomycin which brought down tuberculosis mortality very much more rapidly. I would accept what Dr Adelstein said about the consistent pattern of generation mortality with age for tuberculosis, but I would say that if it was an inverted *W*, it is a much smaller *W* now.

Then there is the role of nutrition. In 1874 in an epidemic of scarlet fever, there were 35,000 deaths in England and Wales. After World War II scarlet fever ceased to be a killing disease, and there was much discussion among those of us working in the public health field as to whether there had not been some mutation of the organism which reduced its virulence. However, even during the war, the priorities given to the nutrition of infants and nursing mothers, and the continuing efforts after the war, had raised the general nutritional level of children, and this had had a profound effect. It is now recognized that a well-nourished host is much more able to deal with invading organisms than a malnourished host.

When we come to cardiovascular disease, we have seen in the last 20 or 30 years a gradual recognition of the various environmental factors most of which came into play relatively suddenly. These factors include obesity, lack of exercise and the smoking of cigarettes. We have seen more recently the result of educational measures to combat these environmental factors. We have seen the development of drugs to aid damaged and arrhythmic hearts, and to reduce blood pressure, and we have seen replacement surgery. All these changes have been relatively sudden, and there are other examples of fairly great importance, although perhaps of small moment, in terms of overall mortality improvement, for example, the discovery of insulin. We also need to appreciate the very great effect of the introduction of compulsory education round about 1870, because those mothers who were so receptive at the end of the century to health education, were the same mothers who were the first generation products of compulsory education, and they were ripe for the acceptance of health education.

We can think conversely of deleterious changes. At the beginning of the nineteenth century there were the effects of rapid urbanization before there were any real improvements in hygiene, and the increase in child diseases. We think of the effects of child labour; the effect of poverty; the fact that people drank gin because they could not afford proper food, and so on, and more recently the effect of smoking manufactured cigarettes which affected men from the 1920s and women a little later.

Finally, there is the question of migration which, again, is a very sharp change in its effect on mortality. I recently made a world review of cancer mortality for the World Health Organization and it was noticeable that in Israel, which is not normally regarded as a country of advanced economic development, the mortality rates for lung cancer were characteristic of the more industrialized countries, and this was entirely because of the influx of migrants from industrialized countries.

So it would appear that all the generation-focused changes are related to specific diseases or groups of diseases, and that generation analysis does not really make much sense when applied to all-cause mortality. This raises the difficulty that death certification and disease classification practices have changed decade by decade with advancing medical knowledge, so that long-term analyses, either secular or generation, are difficult except for broad disease groups. Nevertheless, the author has drawn attention to the fact that only by taking more notice of cause of death analysis will progress be made in understanding mortality changes.

Turning to Dr Scott's contribution, I am bound to remark that however irregular a curve, the shorter the segments, the lower the order of curve that each can be forced into. Since you can fit almost anything to a Gompertz curve as long as it is increasing, it does not surprise me that localized curves fit very well. I doubt whether you can make any inverse act of prediction from such a fit. I do not think that a Gompertz curve tells you anything about the incidence of fatal disease in the individual. It tells you something about the frequency of disease incidents in a group of lives, but does not tell you much about mortality itself. This kind of exercise has been taken as far as it can be taken, and certainly as far as it needs to be taken, for the purposes of group mortality prediction.

The general momentum of mortality reduction, despite man's stubborness in finding new ways of killing himself, seems likely to take us to an expectation of life at birth of 80 at the end of the century, but if we are to do any more in the way of prediction, we have to begin with disease and not with death. That is the point which I tried to make to the Institute as long ago as 1957 (*J.I.A.* **83**, 266), and is something which we are trying to do in our research at the City University. It means abandoning reliance on curve-fitting and getting together with the medical profession to understand more about the point at which, or the conditions under which, deviation from normality in the individual ceases to be adapted to, and gives way to an inevitable procession to death, slow or fast but inevitable and relentless. For this reason I have always very much applauded the efforts made by a minority, led I think by Mr Barnett, to get more done in the way of cause of death analyses.

The President (Mr C. M. O'Brien): Mortality as a subject and as a problem has been the concern of actuaries since their earliest days. It is true that in times of great economic instability such as at present, our attention tends to be concentrated more on the economic factors than on mortality which, *pace* Professor Brown, does seem to have a comforting stability. Nevertheless, it is of fundamental importance to us as actuaries and in our calculations, and also of course of great importance to demographers. In these circumstances while it is perhaps true to say that the study of mortality is currently a specialist occupation, it is still a very necessary study, and those who work in it do a service to this profession in particular.

It is in that sense that I think we should be very grateful to Mr Cox and Dr Scott for having put this paper before us. It has provoked a very valuable and expert discussion tonight, and I am sure that you would all like to thank Mr Cox and Dr Scott.

Mr Cox: For 98% of my working career I did not see myself reading or joining in a paper on mortality to the Institute of Actuaries. Nevertheless, the Ross Panel Report did suggest that generation mortality deserved further investigation, and this idea was developed in O.P.C.S. papers to the thought that relative patterns of secular and generation mortality should be examined. The question has been raised whether this ties in with the other research that is being done. I am not at all sure that it has any close links. It would be fairer to consider this particular item by itself.

A further question was asked about the relative patterns of secular and generation mortality. I was interested to find that there were features common to a number of countries. This fact I thought was worth reporting. I would not reject this, as Mr Barnett seems to, because it cannot be used for forecasting. Scientifically it is surely interesting in itself.

Mr Daw seems to imply that numerical patterns are of little use unless they have biological significance. But surely if there are patterns there must also be a significance. I agree with those who say that for all practical purposes secular mortality is more important than generation mortality. In my experience actuaries nearly always take this point of view, but academicians usually look at it the other way round. They cannot think why actuaries are so obsessed with secular mortality.

Another point which was brought up was whether the right questions had been asked. This of course is extremely important. As far as I was concerned, the question had been asked, and I was asked to look into it. So I did not really bother whether it was the right question or not.

My approach was first of all to look at the ratios: the algebra came afterwards. It looks better the other way round, as presented in the paper. However, I will confess that that is not the way in which I approached it.

I was a little surprised when Mr Haberman said that the 'other' countries were plagued with migration problems which would confuse the analysis more than for Britain. I would have thought that taking the period we have covered—after all it is 200 years—all the countries in question have had sizeable migration flows, in and out, now and then, during this time. The fact that the patterns found are so similar for the United States, England and Wales, Sweden and France, surely means that they override the very varied experience of these countries in regard to migration.

Dr W. F. Scott wrote as follows: I think Mr Barnett is incorrect to say that mortality at childhood and young adult ages is important in population projections; this can easily be demonstrated by carrying out projections such as those of Cox and Scott (reference (23) on p. 320). We were not, of course, unaware of the four-parameter formula for assured lives but one cannot easily fit mathematical laws to generation data by the usual statistical methods because of differing environmental conditions, for example, influenza epidemics. Hence one may have recourse to the 'local' Gompertz' law advocated here, or other methods.

I think Professor Brass takes a long-term view of mortality, considering that such features as the recent upturn at middle ages are ephemeral in historical terms; he may well be right in the long term but the medium term—the next few decades—may show different patterns; mortality rates are increasing at many ages in many countries, as shown by Giles and Wilkie (*T.F.A.* **33**, 375), and several articles in *Proc. 20 Int. Cong. Actuaries.*

Dr Rueff has written to explain more fully some of his remarks during the discussion. When comparing the present paper's findings with this of his monograph [*Ableitung von Sterbetafeln für die Rentenversicherung und sonstige Versicherungen mit Erlebensfallcharakter* (Konrad Triltsch Verlag, Würzburg, 1955)]. Dr Rueff was referring mainly to the similarity between Mr Cox's formula

$$q_x^{t+m} = q_x^t(1 - \cdot 01\, S_x)^m$$

and Dr Rueff's equation (§ 11.2)

$$q_x^{t+m} = q_x^t\, e^{-m(\log_e 10)\, F(x)},$$

in which $F(x)$ is fitted at each age x by least squares approximation.

Since

$$(1 - \cdot 01\, S_x)^n \doteqdot e^{-m(\cdot 01\, S_x)}$$

the formulae of Mr Cox and Dr Rueff are approximately equivalent. In this paper Mr Cox suggested a straight line fit for S_x; Dr Rueff's method permits a more general function, but as is shown in his graphical results from German data (his graphs 4 and 5), $F(x)$ was roughly linear for males aged 40–100 and females aged 25–100.

The '$\Delta\tau$' method is also explained in Dr Rueff's monograph; this interesting approach deserves to be better known in the U.K. According to the $\Delta\tau$ method, for $t - x = \tau$, $x \geqslant 25$ and $t \geqslant T$ (the year of a basic period table),

$$q_x^t \doteqdot q_{x-\Delta\tau}^T$$

where $\Delta\tau$ depends only on τ, the year of birth.

Further, $\Delta\tau$ is given by the formula

$$\Delta\tau = e^{\alpha\tau - \gamma} - \beta.$$