BOOK REVIEWS

Title: AN INVESTIGATION INTO POSSIBLE APPLICATIONS OF
FUZZY SET METHODS IN ACTUARIAL SCIENCE
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Fuzzy Set Theory is a branch of algebra which originated in 1965 and has attracted considerable interest since then. In very simple terms, it is an attempt to introduce an element of uncertainty or vagueness into conventional algebra. For example, let U be some set of objects and let E be a (conventional) subset of U. Then each element of U is either a member of E or not a member of E; there is no room for uncertainty here. However, a *fuzzy subset E* of U is defined by a *membership function* μ_E whose domain is U and whose range is the interval [0, 1]. For each element x belonging to U, $\mu_E(x)$ specifies the *degree to which* x *belongs to* U. These ideas can be extended to real numbers as in the next, more specific, example. Suppose we are interested in the rate of return next year on an investment portfolio. This will be some number whose value is at present unknown. The "fuzzy approach" to dealing with this uncertainty is to specify an interval [a, b] and a function μ: [a, b] → [0, 1]. The interval [a, b] contains all possible values for next year's rate of return and, for any x ∈ [a, b], $\mu(x)$ represents the *degree of certainty* with which x belongs to this set of values. At first sight, this may seem very much like modelling next year's rate of return as a random variable with a specified distribution. That the two approaches are not the same can be seen by noting that a continuous distribution on [a, b] would assign zero probability to any individual point, x.

In the Introduction to this book, the author proposes Fuzzy Set Theory as an alternative approach to stochastic methods for modelling uncertainty. He claims that the former can model types of uncertainty/vagueness/ambiguity that the latter cannot. The basic mathematical concepts of Fuzzy Set Theory are set out in Chapter 1. This chapter covers a great deal of material in a mere 14 pages. The reader who has not previously studied Fuzzy Set Theory should be warned that this chapter is uncompromisingly mathematical and is not a particularly easy read. Actuarial applications of this theory are outlined in the remaining six chapters (48 pages), some of them very briefly.

In my opinion, the two most interesting applications are the calculation of the net single premium for a term assurance using a fuzzy model for future rates of return in Chapter 3 and the, all too brief, outline of an expert system for life underwriting in Chapter 6. It is not clear to me that fuzzy methods have much to offer in the modelling of uncertain future rates of return compared with stochastic methods. Stochastic methods have the advantage that all actuaries, to a greater or lesser extent, have an intuitive understanding of probabilities. The application to life underwriting is more

convincing. It is no surprise to read on page 62 that "Fuzzy expert systems . . . is precisely the most successful application area for fuzzy sets . . .".

An objective of this book is to convince actuaries that Fuzzy Set Theory has a place in actuarial science. Although it is in many ways an interesting book, it might have gone further towards achieving its objective if it had covered less theory and described some of the applications in more detail. Before opening this book, any actuary interested in actuarial applications of Fuzzy Set Theory would be well advised to read the more accessible introduction to this subject written by Jean Lemaire and published in ASTIN, Vol. 20, 1990.

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