

INVESTMENT POLICY AND INDEX NUMBERS

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WE have chosen *Investment Policy and Index Numbers* as the title of this paper, because the origin of the Actuaries' Investment Index is to be found in a paper, *The Statistical Groundwork of Investment Policy*, by the late C. M. Douglas (*T.F.A.* 12, 173).

The main policy consideration discussed by Douglas was the extent to which offices should invest in equities. Douglas maintained that funds should only be invested in equities if greater security was thereby obtained. To establish the suitability of equities, it was necessary to study the relative variations in equity prices, debenture prices and business activity. For this purpose, index numbers were required.

From a study of index numbers covering the period 1924-28 Douglas concluded that share prices were related to the level of business activity, i.e. to the progress of the 'trade cycle', equity prices following the cycle directly, and debenture prices following the cycle inversely, but with a different tempo and interval. If, therefore, an investor could judge the future trend of business activity, a sound investment policy could be pursued.

In this paper we propose to review some of the developments that have taken place in the principles and practice of investment policy over the last twenty-five years, and to study the contribution to these principles that the Actuaries' Investment Index and other index numbers can provide. In addition, we propose to review the methods used in the construction and maintenance of the Actuaries' Investment Index in the light of current conditions.

PART I. SHARE VALUES AND ECONOMIC CONDITIONS

(1) *General considerations*

In 1929, ordinary shares were only just beginning to be recognized as suitable investments for life assurance funds. Douglas was, of course, a pioneer, and he wanted to encourage actuaries to undertake research and experiment in this class of investment. With the limited information then available, Douglas had to be somewhat vague about general policy, and he did not, of course, specifically deal with the concepts of security and yield as Pegler (*J.I.A.* 74, 179) and Clarke (*J.I.A.* 80, 335) have done recently.

When we study later papers on investment policy, we find that very little has been done in the direction indicated by Douglas. Equities have become a recognized investment for life office funds, but there still exists some apparent disagreement about general principles of investment policy. Both Pegler and Clarke give as their first principle of investment the maximization of the expected yield (we shall discuss this principle in some detail in Part III). The principle includes that of capital security, since the 'expected yield' takes

into account the probability of receiving each item of income from the investment and also the probability of receiving back the capital. Pegler insists that a rough estimate of expected yield is in fact carried out, consciously or instinctively, every time an investment is considered. For this purpose, the investor requires a knowledge of the likely fluctuations in dividend returns and market prices for different classes of security under varying economic conditions; in addition, he has to make reasoned judgments as to present and future economic conditions. It is suggested that the Actuaries' Investment Index, with its extensive historical tables of prices and yields of a wide range of securities under the varied economic conditions of the last 25 years, should surely be a valuable part of the investor's equipment.

(2) *Changes since 1929*

Since 1929 there have been considerable and significant changes in economic theories and practice. One of the most noticeable changes is in the treatment of money, a change which is largely due to the work of Keynes. In the nineteen-twenties a typical text-book on the 'Principles of Economics' hardly mentioned 'money' except perhaps to dismiss it. 'Principles' dealt with the theory of the determination of relative prices, money being regarded as neutral and only fixing the general level of prices. The business cycle was dealt with in a separate volume, the causes being the psychology of business men, weaknesses in the monetary and banking system, the effects of economic progress, or some combination of these causes. At the present time good textbooks attempt to formulate an integrated theory, including the determination of prices, the determination of holdings of various kinds of assets, including money, and the causes of fluctuations in business activity. A considerable part of such a book is given to social accounting, to the clarification of such concepts as national income, savings, consumption, and investment, and to the relationships that exist between such aggregates. Statisticians have attempted to find empirical relationships between these aggregates, and the output of statistics required for these purposes has been increased and improved. No doubt many feel disappointed with the attempts that have been made by Governments in recent years at predicting economic events and with the economic policies that have resulted. At the time of writing the so-called new monetary policy would seem to be a revulsion from these policies. We feel that this may well be a temporary phase; economic model-building still goes on within Government departments, although few of the results are now published. Both the volume and quality of available statistics are improving; only recently, for example, the Board of Trade announced that statistics would be collected about the amount of investment in machinery and plant that industries *intended* to make in the short term. In the United States almost all investment advisers make short-term forecasts of the national product and make use of a wide variety of economic time series, so that the correct economic background is provided on which to base investment policy (see *Cases and Problems in Investments*, by F. J. Calkins, New York, 1955). Thus research work is not so much directed to a study of share charts by themselves, but towards more accurate methods of forecasting business conditions; on the basis of the past experience of the behaviour of share prices in varying business conditions, it is then possible to arrive at a rational judgment about their probable future behaviour.

There have also been, since 1929, changes in Company Law and Stock Exchange Rules. There was a general feeling after the abuses of the bubble flotations of 1928-29, so dramatically and painfully revealed after the Hatry crash, that such things should not happen again. The progressive tightening up of the permission-to-deal rules, and finally the relevant provisions of the Companies Act of 1948, have ensured that those who would raise money from the public through the channel of the Stock Exchange should provide the public with all the relevant information to enable them to judge for themselves the investment merits of the securities being issued. This has led to a great improvement in the standards of company practice, particularly in new issues procedure, in the form of accounts, and in the adequacy and accuracy of interim statements. The changes that have taken place in the sources of funds for investment have helped in bringing about these improvements. The fall in investment by private persons, and the great increase in institutional investment, have tended to concentrate investment in fewer and more expert hands, that is to say in the hands of investors who demand, and are able to judge, the relevant financial and trading information about the concern issuing the securities.

(3) *Share prices and business conditions*

How have share prices varied with business conditions since the war? A. G. Ellinger, in his recent book *The Art of Investment*, points out that the trend of share values no longer follows that of business activity. His main evidence is based on the data for the years 1946-50. Between 1947 and mid-1949 share values fell appreciably (the Actuaries' Investment Index fell from 168 to 126) although the trend of industrial production was rising throughout. This fact requires careful consideration since it is a very significant departure from previous experience. Also it should be noticed that during the recession of 1952 share values behaved in accordance with expectations. The Actuaries' Investment Index (1st series) commenced to fall in mid-1951, falling eventually from about 160 to about 127, and very slightly leading the fall in production.

Since the war it has been generally accepted that a trade recession in the United States would probably spread to the United Kingdom and other industrial countries. In the United States there was in fact a fairly extensive recession during 1949. The index of industrial production was about 195 in December 1948 and fell to 161 by July 1951. Standard and Poor's index of ordinary share values commenced to fall in June 1948, eventually falling from about 135 to 112 in June 1949. Thus, in accordance with expectations, the share index led the production index. These economic events in the United States were given much publicity in this country and the recession did have serious consequences for our external balance-of-payments position. It was also reasonable that many individuals would act on the assumption that the recession would spread to this country. Such expectations would account quite well for a substantial fall in share values. It would appear, then, that one basic difficulty at the present time is that in assessing future economic conditions in the United Kingdom one must also consider economic conditions in the United States, and we have not yet accumulated sufficient data to assess accurately the dependence of our economy on theirs. It is rather a superficial judgment to say that share values no longer 'forecast' business conditions. It would be wrong to regard a share chart by itself as a business barometer. One should rather look at the question the other way round—what are current

business conditions and what are the probable trends, and if this assessment turns out to be correct, what will be the effect on share values in general? There are also, of course, political and social conditions to take into account. What will be the likely attitude of the public and the Government to a rising trend in profits if this results in increasing dividends? Factors, in addition to those already discussed, which contributed to the uncertainty of 1949 and the consequent depression in share values were Government interference in the bond market, nationalization of certain industries, and dividend restraint. Clearly, if this country should get into economic difficulties there is always the possibility that the Government will take actions which cannot be to the advantage of holders of ordinary shares. In any consideration of general investment policy these factors must be given due weight.

In spite of the changes in economic knowledge and policy since 1929 we still appear to know very little about some of the factors affecting share values. There appears to have been any amount of guesswork about the psychology of 'bears' and 'bulls', about fluctuations in 'confidence', and the causes and spreading of market panics, and about the various systems of market forecasting, with very little data to support the various generalizations that have been made. Reliable data about the different types of investors, their motives and behaviour patterns do not exist. We know that a large proportion of investors are investors in the strict sense who buy securities because they expect to obtain a reasonable yield with security of capital, we know that others are motivated by a desire to get rich quickly, whilst yet others enjoy operating in the market. It would be very helpful to obtain some definite and extensive data on these subjects. Without this knowledge it is not possible to add the effects of these purely market factors to those due to the basic economic position.

(4) *Inflation*

Another important factor at the present time is inflation. When Douglas wrote his paper, many countries had experienced violent inflation as a result of the 1914-18 war, but it was not then expected that this factor would become important in peace-time. At present there is a widespread feeling that a continued policy of full employment makes continued inflation highly probable. If investors take inflation into account, then their estimates of 'expected yield' must be couched in 'real' values rather than in money values. The effect will be to reduce the value of the expected income from fixed interest securities relative to that from ordinary shares.

(5) *Fixed interest securities*

We have confined the discussion so far to the prices of ordinary shares. Douglas also considered that the prices of fixed interest securities had a trend which was related to the economic situation. During the nineteen-thirties and since the end of the war, however, Government attempts both here and in the United States to control the prices of Government securities have been the main factor in the bond market. Since 1951 interest rates have been more or less free to find their own level, but the data are insufficient for any reliable conclusions to be drawn. It is also possible that Governments in the future will again attempt to control interest rates and rely on methods other than monetary policy to control the economy.

Mr A. M. Khurso* has shown that for the period 1926-49 the yield of Consols, after allowing for income tax, can be 'explained' by the ratio of idle money to the value of securities (excluding preference and ordinary shares) and by an index of dividends on ordinary shares. The multiple correlation coefficient is very high, .97. In so far as the authorities attempt to control the level of interest rates, they would generally operate on the ratio of idle money to other assets. They have also used the device of restraining dividends, but the motive has not been to reduce interest rates.

PART II. PRACTICAL USES

Now that the Actuaries' Investment Index has been computed for some 25 years, it is appropriate to survey the various practical uses that have been developed for price index numbers and average yields. For some of these purposes, competing index numbers such as those produced by the *Financial Times* or the *Investors' Chronicle* are the most suitable; for others, the Actuaries' Investment Index provides special features unobtainable from other sources.

It should be noted that in the case of the Actuaries' Investment Index there are two series of index numbers, the first with 1928 as the base year (subsequently altered to 1938) and terminating in 1953, and the second with 1945 as the base year. For some purposes (see Part IV, Section (6)) it is legitimate to link the two series of index numbers for Industrials (All Classes Combined) so as to form a single series. The subdivision into industrial groups differs appreciably as between the two series and generally the sub-index numbers cannot be so linked.

(1) *Day-to-day investment management*

(a) *Approximate valuations*

If a detailed valuation be made at the end of each quarter, index figures may be employed to determine intermediate values as at the date of the Actuaries' Investment Index calculations (the last Tuesday in each month). For this purpose the detailed valuations of the main groups of securities would be adjusted by means of index ratios appropriate to the classes concerned. Debentures and preference shares would be valued by means of our Index figures; for ordinary shares, the portfolio would probably be divided into broad industrial groupings, to which the relevant indices are applied. British Government Securities require individual valuation because no satisfactory index exists for this class. Adjustments must be made for securities purchased since the last valuation, and for the book profits or losses involved in sales made during the same period.

When calculating the 'break-up' value of an investment trust share, the present market value of the investments must be estimated; for this purpose, the market value, given in the balance sheet at the end of the previous financial year, should be carried forward to the present time by means of index ratios.

(b) *Checking portfolio performance*

When a report is being prepared on the progress of an investment portfolio, it is useful to be able to compare the appreciation or depreciation experienced

* 'An Investigation of Liquidity Preference', *Yorkshire Bulletin of Economic and Social Research*, January 1952.

over the period with what would be expected from a standard portfolio, as represented by the Index constituents. This technique is sometimes used to check the performance of different investment trusts, studying the changes in the market value of the underlying investments from year to year and comparing these movements with the expected figures derived from index ratios.

In making such a comparison, it must be borne in mind that the Index, by virtue of its geometric construction, appreciates less and depreciates more than a static portfolio made up of the Index constituents. Consequently, the results achieved by a share portfolio can in practice always be expected to 'beat the Index'.

(c) Pricing unquoted shares

Sometimes it is necessary to determine a price of privately owned shares, when no stock exchange quotation is available. The price may be needed for a normal share transaction, for a deal consequent on take-over or amalgamation, or for official purposes such as a probate valuation. Naturally, it is necessary to allow for the absence of a market quotation, and for any other adverse features which may distinguish the share concerned from the high-class issues covered by the Index.

The Index figures provide a standard to compare the terms for dealing in a normally unquoted share at different times. Once a certain basis has been agreed between two parties, subsequent transactions can be arranged on consistent terms, adjusting the price by reference to changes in the Index over the intervening period.

It may be of interest to describe some of the applications of the Index that have been encountered in practice.

(i) In amalgamating several small brewery companies, the compensation for the various classes of shareholders was based on the Index figures.

(ii) The preference shares of a private company were valued for probate purposes. An excessive value, suggested by the Estate Duty Office, was reduced when the accountants showed that the price employed was almost as high as that of the much better class shares comprised in the Index.

(iii) An institution arranged to buy small lines of an unquoted preference share, whenever the family shareholders needed to sell. The first transaction was negotiated after lengthy discussion between the two parties. Subsequent purchases were made without delay or argument, by reference to movements in the Index figures.

(d) Pricing new issues

When a new issue is being made, the Index records may be of use, both to the Issuing House in fixing the price, and to the Institutions in considering the offer. Naturally, the Index figures are most valuable when recent market precedents are lacking. For example, when the first denationalized Steel issue was made in 1953 after five years of state control, the Index records of yields, etc., prior to 1948, were helpful in assessing the issue terms.

It must be admitted that our Index figures are of little use in connexion with debenture or preference issues, because the indices are composed of types of these stocks which are now to all intents and purposes obsolete. (Debenture

issues are invariably redeemable, whilst the index constituents are all irredeemable; preference issues are normally repayable on liquidation at an average market value, but the constituents of this index are, in theory anyhow, liable to repayment at par.)

(e) *Comparing gilt-edged holdings with alternative securities*

The Index includes separate categories for certain classes of investment where the income is particularly well secured, and prices and yields are mainly determined by the level of the gilt-edged market. The classes concerned are:

- (i) Corporation Stocks (irredeemable).
- (ii) Debentures (irredeemable).
- (iii) Preference shares.
- (iv) Bank shares.
- (v) Insurance shares.

By means of the monthly yield tables it is possible to see how these investment categories, as a whole, compare in yield with the gilt-edged basis (i.e. the yield on $2\frac{1}{2}\%$ Consols). Such comparisons facilitate decisions as to whether these investment categories should be increased or decreased; in addition, the regular monthly figures provide a standard of comparison helpful in the assessment of specific market offers.

Suppose, for example, that it has been decided to buy preference shares in exchange for $2\frac{1}{2}\%$ Consols; then from the Index figures for 24 November 1953 the yields are:

$2\frac{1}{2}\%$ Consols	.	.	.	.	.	.	.	3.88%
Investment Trust preference shares (all rates)	.	.	.	.	.	.	.	5.03%

The yield of 5.03% for the preference shares is calculated at the 'middle' price, ignoring purchase expenses. To obtain a practical yield, based on buying prices and appropriate expenses, the yield must be reduced by, say, 5% to 4.78%. This calculation suggests that Investment Trust preference shares of high quality can perhaps be bought to show a yield advantage of some 18s. % over Consols. If, therefore, a campaign is organized for buying on this basis, good class stocks should be available (subject to the limitations of the market). Should, however, a higher yield differential be required, only inferior stocks will normally be obtainable.

(f) *A yield criterion for the future*

Occasionally, an office may be interested in financing some project where the money will not be required for some years (i.e. the building of a large property or, perhaps, a ship such as an oil tanker). In such circumstances it may be necessary to guarantee a rate of interest to apply at some future date when the property or ship is complete. For this purpose it is convenient to provide in the legal documents for the mortgage to be granted at the Index Debenture rate, with, say, $1\frac{1}{2}\%$ addition for the special circumstances.

(2) *Forecasting*

As explained in the introductory section, the original purpose of the Index was the ambitious objective of assessing basic economic trends and studying

the present position in relation to the trade cycle. In this section of the paper we discuss certain methods which have been developed for this purpose. We are not concerned here with economic forecasting in general. This is a very large subject and a great deal has been written about it, particularly in the United States. In this section we are concerned only with methods which specifically use index numbers of security prices and stock exchange activity but may well take into account a wide background knowledge of the economic situation.

(a) The relation between the index components and stock exchange activity

The ordinary share index, by itself, is useless for studying the trade cycle. A chart of the index over a long period (see Fig. 1) shows merely historical information, i.e. whether the basic trend of share prices in the past has been upwards or downwards. Any interruption in the progress of an index may be a change in this basic trend, or merely a temporary reaction; there is no means of distinguishing by scientific methods, between these alternative possibilities.

Share prices are affected, among other factors, by

- (1) the dividends being paid, and
- (2) the rate of interest.

A relationship involving these factors has been used by certain investment advisers in the following form:

$$P_o = K \times P_c \times D \times R, *$$

where P_o is an index of ordinary share prices, P_c is an index of $2\frac{1}{2}\%$ Consols, D is an index of dividends being paid, K is a constant, and R is the residual factor.

For example, in May 1955, the Share Index (all industrials) was 156.9. The average yield was 5.29%, and the Dividend Index $156.9 \times \frac{5.29}{100} = 8.3$; the Consols Index, reflecting the level of the gilt-edged market, was 84.9. If the Share Index be divided by 'dividend' (8.3), and by 'Consols' (84.9), we obtain the residual factor $R = \frac{156.9}{8.3 \times 84.9} = .2226$ (taking K as unity). Alternatively, $R = \frac{\text{Constant} \times \text{Consols yield}}{\text{All industrials yield}}$.

This residual factor R reflects the 'confidence' that the investing public has in ordinary share prospects relative to those of gilt-edged stocks. A rising R is usually interpreted as an indication of a growing preference for ordinary shares. For example, the 'expected yield' on ordinary shares might be rising relative to the yield based on past dividends, so that in these circumstances we might well call R an index of expectations.

* More elaborate statistical techniques have been used in attempts to measure these factors; for example, 'A study of share prices, 1918-1947', by I. M. Sahni, *Yorkshire Bulletin of Economic and Social Research*, February 1951.

Table 1 below shows the components of the Actuaries' Investment Index at six-monthly intervals from 1945 to date. In this table, and in subsequent discussion, the term 'confidence' is used for the factor *R*.

Table 1

Date	Price Index	Yield	Dividend Index	Consols Index	Confidence
12/45	103.7	3.76	3.90	128.5	2069
6/46	117.1	3.52	4.12	136.6	2081
12/46	120.9	3.57	4.32	138.7	2018
6/47	121.1	3.95	4.78	128.5	1972
12/47	114.7	4.26	4.89	115.5	2031
6/48	103.3	4.81	4.97	107.7	1930
12/48	108.3	4.62	5.00	112.7	1922
6/49	90.2	5.63	5.08	106.3	1670
12/49	93.1	5.49	5.11	98.6	1848
6/50	96.7	5.36	5.18	97.2	1921
12/50	100.0	5.27	5.27	100.0	1898
6/51	115.9	4.97	5.76	91.5	2199
12/51	102.7	5.78	5.94	86.6	1997
6/52	86.2	7.05	6.08	78.2	1813
12/52	97.5	6.29	6.13	82.4	1930
6/53	98.4	6.42	6.32	85.2	1827
12/53	113.1	5.76	6.51	90.1	1928
6/54	133.1	5.56	7.40	93.7	1920
12/54	152.1	5.24	7.97	93.0	2052
6/55	165.9	5.10	8.46	83.8	2340

Confidence was high in 1945; dividends which had remained low during the war were then expected to rise. This hope was subsequently fulfilled, as the dividend index increased from 3.90 to 4.78 over the next 18 months. In June 1949, confidence was low, owing to the difficult economic situation, and the prospects of dividend limitation. A high value of confidence was recorded in June 1951, when the rapidly rising cost of living and consequent fear of inflation resulted in a marked preference for ordinary shares. A similar position obtained in June 1955, when confidence reached a particularly high level.

In addition to the Index and its components, another market indicator is available. This is the level of Stock Exchange activity. The Stock Exchange Official List shows the number of bargains marked during the day, in total, and for each subdivision of the list. The *Financial Times* calculates an Activity Index (the average number of markings during the second half of 1942 is used as a base for this purpose). Separate figures are shown for 'gilt-edged', 'industrial', 'speculative' and 'total', based in each case both on the number of markings for the day and the five-day mean.

Fig. 1 shows this Industrial Activity Index (five-day mean), plotted against the *Financial Times* Industrial Ordinary Share Index for the period 1946 to 1955. There is clearly a remarkable similarity between certain important aspects of the Activity Index and the Share Index. Peaks and troughs of the two charts correspond closely; the Activity Index, if roughly graduated, would become something very like an enlarged version of the Share Index.

The relationship between the Activity Index and the Share Index is perfectly logical, bearing in mind the way the jobbing system works on the London

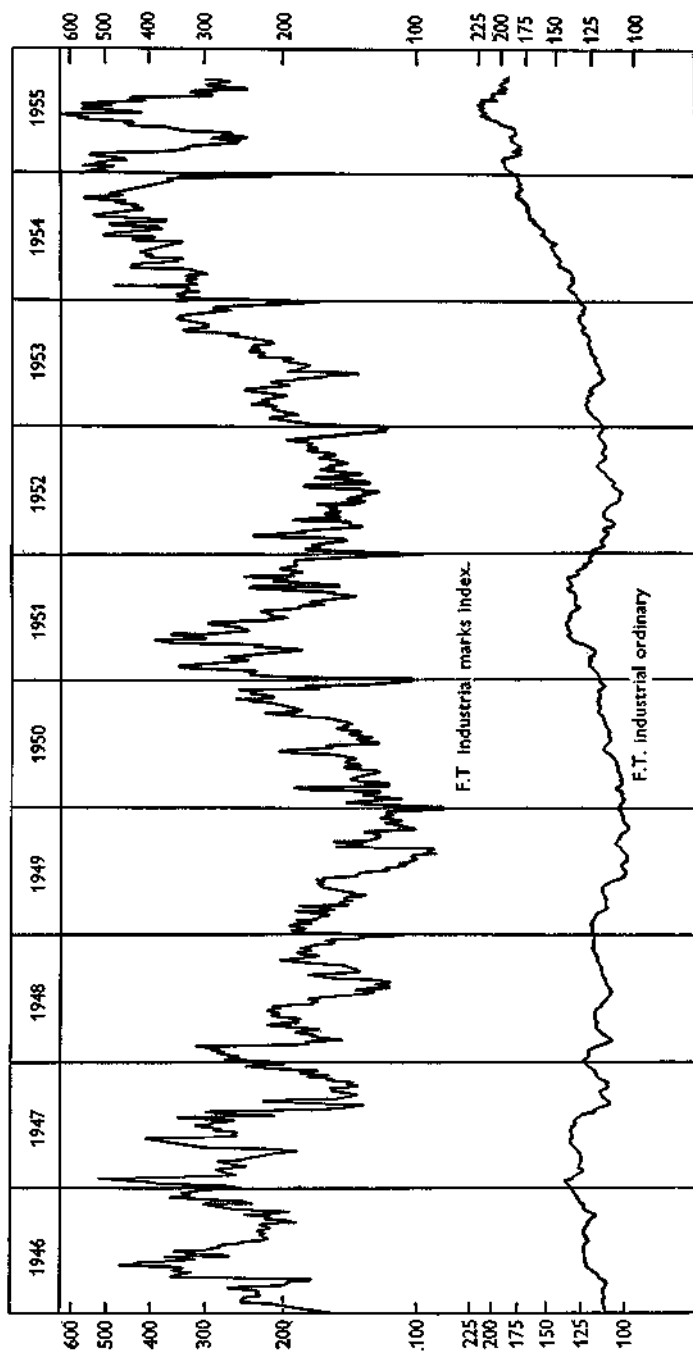


Fig. 1

Stock Exchange. When the industrial market is good, there is widespread interest in the Stock Exchange and many insistent buyers. Determined purchasers can almost always find a seller at a suitably enhanced price, and the activity rises to a high level. On the other hand, when the share market is weak there are few buyers and many sellers; in such conditions, however, the jobbers refuse to take stock except on severe terms, thereby discouraging sales and reducing activity.

A forecasting system has been developed, using the Index components and the Activity Index. The basis of this system is:

(i) that the components of an index sometimes show signs of a changing trend before the index itself; and

(ii) that such signs, if confirmed by corresponding moves in the Activity Index, often give early warning of an impending major move in the Share Index.

This technique is described by Mr A. G. Ellinger in his recent book, *The Art of Investment*. We have not investigated this forecasting system in any detail, and are therefore scarcely competent to express opinions on its validity. Its possibilities, however, appear to merit serious study and to provide an interesting field for research.

Index records show how this system has worked in practice over the post-war years. In Fig. 2, the *Financial Times* Industrial Ordinary Index is plotted against the three components over the period 1946-55. For this period, the trend of dividend is clearly upwards throughout. In the last quarter of 1946, the components of the Index all showed an upward trend, resulting in a steep rise in the Index to a peak of 135 in January 1947. During the first half of 1947, confidence and interest gave signs of a changing trend, since the 1947 peaks were in each case below the corresponding 1946 values. During the same period, the successive activity peaks were each lower than the one before. A month or so after these adverse symptoms appeared the Index fell some 25 points.

The next interesting stage in the progression of the Index came in the spring of 1950. Then the trend of confidence appeared favourable, and interest seemed to have ceased to decline. The rising Activity Index confirmed these hopeful indications. Over the next year the Index rose some 30 points.

Subsequently, the Index turned sharply down in October 1951 and upwards in 1953. In each case, a careful study of Figs. 1 and 2 will show that these changes were anticipated by corresponding moves in the Index components, associated with declining activity (1951) and rising activity (1953).

Analysis of these market movements in the past may not seem particularly convincing when subsequent history is well known. Records show, however, that users of the technique did in fact succeed in forecasting these changes of trend before any important move occurred in share prices.

(b) *The relation between profits and dividends and the share index*

Each month, the *Financial Times* prepares a summary of profits and balance sheet analyses, covering the results of all those companies publishing annual reports between 1 January and the end of the previous month. Specimen figures for 'Total Industrials' are given below from the *Financial Times*

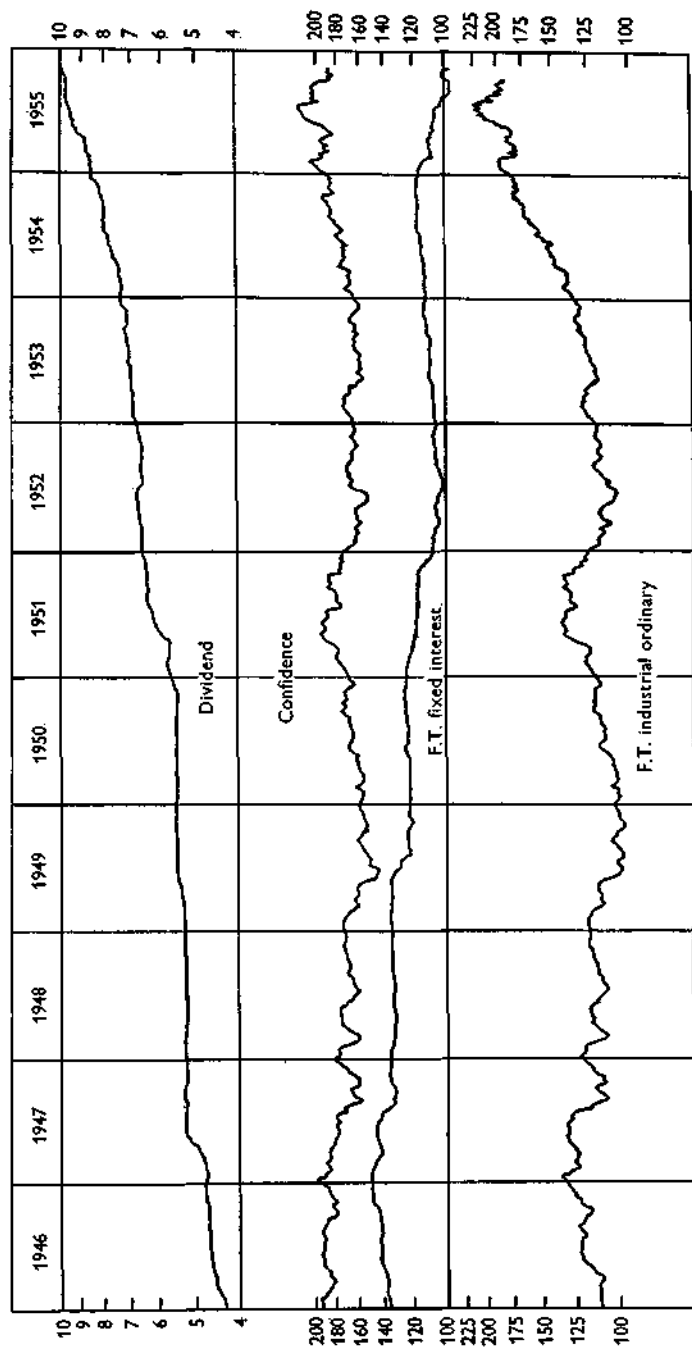


Fig. 2

Table of 10 September 1955, summarizing reports made during the first eight months of 1955.

No. of companies	(£,000's)			Earned ordinary dividends
	Profits	Depreciation	Tax	
2,054	1,475,464 (1,309,891)	219,011 (191,185)	566,086 (556,802)	448,877 (348,642)
Ordinary dividends	Reserve allocations	Issued ordinary capital	Capital and revenue reserve totals	Net working capital
162,997 (134,300)	321,878 (249,997)	1,831,452 (1,508,190)	2,685,281 (2,652,648)	3,093,491 (2,840,175)

Results of the same companies for their previous financial year are shown in brackets.

Separate figures are given for 25 industrial groupings and for 5 categories of raw material producers.

These extensive tables provide a means of studying the progress of industry and the prospects for ordinary shares, both in the aggregate and for the various industrial groups. The most significant factors from the point of view of share prospects are the 'earned ordinary dividends' and the 'ordinary dividends'. These factors are recorded after taxation and are thus influenced from year to year by changes in income tax, etc. Despite this practical difficulty, it is instructive to compare the trend of these two items over a period with the trend of a share index.

The *Financial Times* tables show monthly cumulative totals for the year to date. The trends of 'earnings' and 'dividends' are more clearly revealed if these totals are differenced to give results for each month. From these monthly figures, the ratio of 'latest years' results to 'previous years' results is calculated. Table 2, below, records these ratios each month from March 1950, when these tables first appeared, to September 1955. For each month in which reports are published the Actuaries' Investment Index (all industrials) is also recorded.

It is interesting to pick out from Table 2 the months in which the ratios appear to be at a maximum or minimum, and to compare these results with the corresponding 'Highs' and 'Lows' of the Share Index. Table 3 shows the results of this analysis.

Certain months (September, October, December and January) provide fewer reports than the average, so that the ratios for these months are subject to more than normal fluctuation. Consequently, 'high' and 'low' ratios for these months are only regarded as significant if supported by adjacent figures of similar dimensions.

The three indicators show distinct correlation, subject to a certain difference in timing. Table 3 suggests that the turning-point of the earnings ratio is after that of the share index, the delay being from two to eight months. On the other hand, the dividend ratio appears to turn shortly before the share index.

It is instructive to try to assess how, and when, the fortunes of industry can be expected to affect the share price, the earnings and the dividends of an individual company, whose financial year ends on 31 December. Normally,

Table 2. The share index compared with the trend of earnings and dividends

Month of publication of reports	Index	Earnings ratio	Dividend ratio	Month of publication of reports	Index	Earnings ratio	Dividend ratio
1950:				1953:			
Mar.	93	1.07	1.01	Jan.	100	.73	1.02
Apr.	93	1.08	1.01	Feb.	102	.78	1.00
May	95	1.00	1.03	Mar.	103	.88	1.03
June	97	1.08	.97	Apr.	101	.85	1.08
July	95	1.43	1.05	May	97	.83	1.13
Aug.	97	1.10	1.03	June	98	.75	1.11
Sept.	102	1.25	1.15	July	101	.81	1.10
Oct.	101	1.14	1.02	Aug.	104	.84	1.10
Nov.	102	1.25	1.09	Sept.	107	.89	1.12
Dec.	100	1.04	1.12	Oct.	110	1.13	1.08
1951:				Nov.	112	1.00	1.15
Jan.	103	1.03	1.11	Dec.	113	1.21	1.17
Feb.	107	1.06	1.10	1954:			
Mar.	102	1.23	1.14	Jan.	117	1.09	1.21
Apr.	111	1.11	1.09	Feb.	120	1.15	1.11
May	114	1.35	1.09	Mar.	121	1.24	1.46
June	114	1.11	1.14	Apr.	126	1.14	1.26
July	107	1.27	1.11	May	131	1.16	1.17
Aug.	110	1.54	1.09	June	133	1.13	1.21
Sept.	114	1.27	1.07	July	137	1.21	1.20
Oct.	114	1.44	1.03	Aug.	144	1.21	1.15
Nov.	105	1.23	1.02	Sept.	148	1.23	1.24
Dec.	102	1.52	.93	Oct.	155	1.19	1.19
1952:				Nov.	153	1.33	1.19
Jan.	96	1.19	1.01	Dec.	152	1.11	1.15
Feb.	95	1.09	.99	1955:			
Mar.	91	1.18	.98	Jan.	160	1.14	1.08
Apr.	95	1.11	1.03	Feb.	148	1.36	1.14
May	89	1.06	1.05	Mar.	146	1.26	1.20
June	86	1.15	1.02	Apr.	150	1.18	1.17
July	92	1.02	1.06	May	157	1.24	1.25
Aug.	96	1.09	1.02	June	166	1.48	1.28
Sept.	96	1.17	1.12	July	168	1.34	1.23
Oct.	96	.57	1.10	Aug.	156	1.23	1.25
Nov.	91	.79	1.12	Sept.	153	1.35	1.39
Dec.	97	.88	1.02				

Table 3. Turning-points of earnings, dividends, and share prices

	'High'	'Low'
Earnings	Aug. to Oct. 1951 (1.54-1.44)	Oct. 1952 to Jan. 1953 (.57-.73)
Dividends	Mar. to June 1951 (1.14)	Feb. 1952 to Mar. 1952 (.98)
Share Index	June to Sept. 1951 (114)	June 1952 (86)

the 1953 results would be announced on, say, 1 May 1954; shortly after this date, the share price can be presumed to have adjusted itself in accordance with these latest results, and with the expectations for the future. Subsequent

movements in price will be caused by investors' anticipations regarding the 1954 trading results and how they compare with the 1953 figures.

The share price. By mid-1954 a certain level of activity and profitability, relative to the year before, will have been attained by the company, and will have become known to a limited circle of directors, executives, agents, and other trade connexions. As this special knowledge spreads, the resulting share transactions gradually affect the price. The time-lag can only be guessed; perhaps we might reckon that a movement of the share price occurs, say, two months after a corresponding change in trade conditions.

The earnings. Assuming trade conditions change fairly steadily over the year, the situation at mid-year will be representative of the year's results as a whole. These results will not, however, be published until 1 May 1955. Consequently, any trade change developing in June-July will not be revealed by movements in published earnings for some ten months.

The dividend. This is announced on 1 May 1954, along with the results. When deciding on the dividend, the directors consider not only the 1953 trading results but also the company's progress in the first four months of 1954. Any important change in trading conditions in 1954 will be known to the directors with the minimum of delay and will affect their dividend decision with only a small time-lag—almost certainly a smaller time-lag than that between the trade change and the corresponding share movement.

The considerations set out above suggest that 'dividends' should turn just before the share index, and that 'earnings' should turn some 8 months after. This theory seems roughly consistent with the results recorded in Table 3. To demonstrate this point graphically, Fig. 3 shows the three indicators month by month since March 1950. For this purpose, the earnings ratio has been back-dated 8 months. When this time-lag is allowed for, the correlation between the three indicators becomes very close.

The earnings ratio is a sensitive indicator, and despite fluctuations the general trend is not difficult to discern. Its value as a forecaster, however, is limited by the severe time-lag. Unfortunately, the dividend ratio, which appears to give a more up-to-date assessment of business conditions, is more difficult to interpret, because its trend changes only slowly, and month-to-month fluctuations are considerable.

Eventually, as accounting methods improve, company results will be published sooner, and quarterly reports will become usual (as is already the practice in America). The earnings ratios, once they are available with less time-lag, will become considerably more valuable.

(3) *Assessing the growth prospects of industries*

For 19 industries, the Actuaries' Investment Index classifications and the *Financial Times* groupings correspond, so that the relative figures may be associated in an attempt to compare the outlook for the trades concerned.

From the *Financial Times* summaries for January-June 1955, figures have been extracted for earnings, dividends, and dividend ratios. For all industries, the dividends ratio is 1.21, the individual figures varying from 1.56 (paper) to 1.01 (wool textiles). In Table 4, below, the various industries have been graded in order of dividend ratio; in addition, the Actuaries' Investment

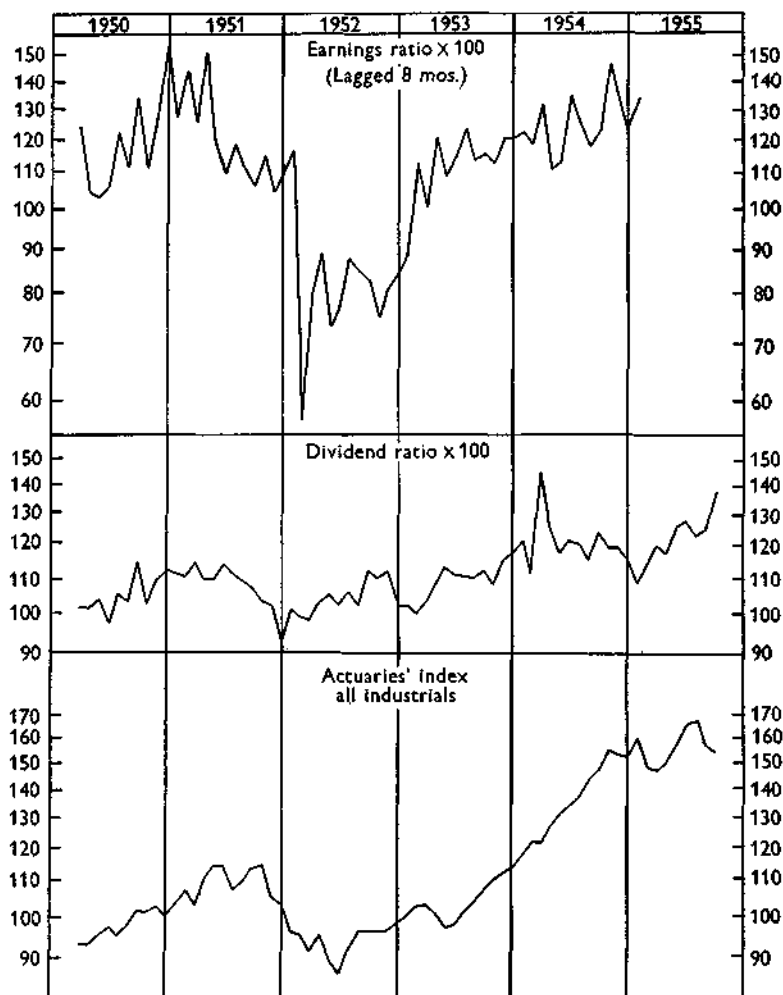


Fig. 3

Index dividend yield at 28 June 1955 is shown, together with the earnings yield (obtained by multiplying the dividend yield by the ratio of earnings to dividends from the *Financial Times* tables).

In calculating the earnings yield, the assumption is made that the dividend yield from the Actuaries' Investment Index constituents is appropriate to the much larger number of companies included in the *Financial Times* summaries. In practice, the Actuaries' Investment Index selections, being the largest and most reputable companies in each industry, almost certainly afford a lower dividend yield than the whole range of companies in the group concerned. This feature is, however, of little importance in the present context, because it applies with similar force to each industry.

Table 4

Industry	Dividend ratio	Dividend yield	Earnings yield
Paper	1.56	4.48	13.5
Oil	1.43	5.33	16.0
Chemicals	1.31	3.80	10.3
Household Goods	1.31	5.73	12.6
Stores	1.31	5.45	9.3
Electrical	1.26	4.36	13.9
Engineering	1.26	3.83	11.9
Building	1.24	5.17	15.0
Rayon	1.24	7.42	20.8
Motors	1.22	3.81	13.7
Boots and Shoes	1.21	6.44	10.9
Food	1.18	4.44	18.6
Newspapers	1.18	6.61	18.5
Shipping	1.15	5.06	14.2
Shipbuilding	1.12	5.74	23.0
Tobacco	1.11	6.73	17.5
Cotton	1.09	7.74	14.7
Breweries	1.06	6.40	10.9
Wool	1.01	5.92	14.8

A high dividend ratio, if maintained, may denote an industry with above-average growth prospects. For example, the 10 industries with a ratio higher than the average are: Paper, Oil, Chemicals, Household Goods, Stores, Electrical, Engineering, Building, Rayon and Motors.

These industries include most of the trades expected to provide good growth prospects; for most of these the dividend yields are comparatively low. Conversely, the four groups at the foot of the table (Tobacco, Cotton, Breweries and Wool) appear to offer fairly static dividends and record present yields of 6% to nearly 8%.

For a complete investigation, it is desirable to calculate the dividend ratios monthly for each industry and to estimate the current ratio from a careful inspection of the trend of these figures. These ratios, if continually watched and studied in conjunction with other economic data, such as production figures, employment statistics, wage rates, etc., might provide some hint as to changes in the fortunes of the industry concerned.

(4) *Share prospects in 'real' terms*

Almost all institutional funds carry some degree of liability in 'real' terms. Even a life fund, with liabilities apparently solely expressed in monetary values, is prejudiced by severe inflation, owing to rising administrative costs (see C. M. Gulland, 'Investment Policy and the Neutralisation of Future Expense Fluctuation', *Transactions of XIVth International Congress*). A pension fund is, of course, almost entirely concerned with 'real' values, whilst a fund covering the reserve liabilities of non-life business becomes inadequate when these liabilities are increased by persistent inflation. In these circumstances, it is most important to study the extent to which a portfolio of ordinary shares can be expected to maintain its value when the cost of living rises and money values deteriorate in 'real' terms.

The following figures show how the cost of living and wage rates have risen over the last twenty-five years:

Year	Index of weekly wage rates	Index of retail prices (all items)
1930	100	100
1935	97	91
1940	120	118
1945	164	147
1950	209	180
1955	282	233

Clearly, the exceptional war-time and post-war conditions have been responsible for the considerable decline in the 'real' value of the currency, which has occurred over the last fifteen years. Nevertheless, records show that the 'real' value of the currency has been declining slowly but steadily for hundreds of years. According to a recent article in *Economica*—'Seven Centuries of Building Wages', by Phelps Brown and Hopkins—the wages of a building craftsman, amounting to 3d. per day in the thirteenth century, increased by some 150 times to 445d. per day in 1955. This spectacular increase represents an annual rise of some $\frac{3}{4}\%$ per year. Over the last ten years the annual rate of increase of weekly wage rates has been $5\frac{1}{2}\%$ —almost 8 times the long-term average.

In general, ordinary shares, being backed by 'real' assets, can be expected to appreciate in monetary terms, both as regards capital and income, as the currency value declines. In Table 5 we set out figures for capital values from various share indices compared with wage and living cost figures for the last twenty-five years. The index numbers used are yearly averages of monthly figures for London and Cambridge, retail prices, and wages; and for the others mid-year figures have been taken.

Comparison of the three indices shows:

- (1) a remarkably close agreement between the two arithmetic indices, and
- (2) the consistently lower figures for the Actuaries' Investment Index due to the geometric averaging.

The comparative performance of arithmetic and geometric indices will be fully discussed in Part IV of this paper. Further comments on this point will therefore be reserved; at this stage we would point out that the arithmetic index indicates the appreciation or depreciation of a portfolio invested in the index constituents at the original base date and maintained thereafter without alteration. The geometric index gives undue weight to those securities which depreciate, and allows insufficiently for those which appreciate. Consequently, for a long-term study of the results expected from a typical portfolio kept unaltered over the years, the *Investors' Chronicle* Index is more appropriate than the Actuaries' Investment Index. According to the results shown in Table 5 for the *Investors' Chronicle* Index, such a portfolio has been more than maintained in 'real' value over the last 25 years. Furthermore, if the portfolio were managed with reasonable success, a better result than that shown by this Index might be expected.

Table 5. Share indices and the cost of living

Year	Actuaries	London and Cambridge	Investors' Chronicle	Index of retail prices	Index of wages
1930	100	100	100	100	100
1931	74	78	80	94	99
1932	60	75	65	91	98
1933	84	92	95	89	96
1934	94	112	110	89	96
1935	103	124	120	91	97
1936	111	144	140	93	99
1937	110	134	137	98	103
1938	88	110	110	99	106
1939	82	102	107	101	107
1940	52	85	65	118	120
1941	67	90	90	129	130
1942	81	101	105	138	139
1943	100	121	130	142	146
1944	113	132	150	145	155
1945	114	140	152	147	164
1946	124	152	170	149	177
1947	128	160	190	158	186
1948	109	153	150	171	200
1949	95	145	130	176	205
1950	102	147	140	180	209
1951	123	—	170	198	224
1952	92	—	132	216	241
1953	105	—	160	222	252
1954	142	—	204	227	263
(June) 1955	176	—	276	233	282

The indices shown above have been derived from the following sources:

- (a) Actuaries' Investment Index
1st series 1930-1945 and subsequently linked to 2nd series.
- (b) London and Cambridge Index
Based on prices of 93 shares, arithmetically averaged with appropriate weights.
This index was calculated from 1926 until 1950, when it was discontinued.
- (c) Investors' Chronicle Index
Derived from prices of 96 shares, arithmetically averaged. Base year—1923.
- (d) Index of retail prices
London and Cambridge Economic service 1930 to 1950: subsequently linked to Ministry of Labour figures.
- (e) Index of wages
Professor Bowley's index, calculated for the London and Cambridge Economic Service to 1950—then Ministry of Labour figures.
In each case the indices have been proportionately changed so as to show a figure of 100 for the year 1930.

From the Actuaries' Investment Index, a dividend indicator can be obtained by multiplying the price indices by the relevant yields. The yield index is an arithmetic average of the individual yields. At the same time, the price index represents the progress of a share portfolio in which equal amounts of money have been invested in each security, and thereafter the money values have been kept continuously constant by appropriate purchases and sales of the securities in the portfolio.* Consequently, the dividend index represents correctly the

* This statement applies strictly if the unweighted geometric mean is used. If weights are applied in obtaining the over-all index, then money values are equal within each group, but vary from group to group according to the weights.

results that would be obtained from such a share portfolio.* The figures below show values of this Index at quinquennial intervals:

Year	Actuaries' Dividend Index
1930	100
1935	59
1940	70
1945	73
1950	146

When the new series was introduced in December 1950, the 'all industrials' yield was distinctly lower than the corresponding figure for the original index. It is therefore impracticable to link up the two series and calculate the dividend index beyond 1950.

The dividend index obtained above is unduly conservative, as is the share index. In an attempt to study the dividend performance of a representative share portfolio over the last 25 years, a special index has been calculated. The constituents are the 48 shares selected by Moody's Economic Services in 1929 for a 'Blue Chip' share index. The dividends paid in each subsequent calendar year have been averaged arithmetically, allowance being made for bonus issues, etc. The results of this investigation are shown in Table 6.

Table 6. Dividend index (arithmetic average)

Year	Index	Year	Index	Year	Index
1930	100	1939	99	1948	114
1931	88	1940	93	1949	118
1932	78	1941	89	1950	122
1933	78	1942	88	1951	141
1934	84	1943	92	1952	146
1935	92	1944	94	1953	154
1936	98	1945	94	1954	180
1937	107	1946	108	1955	(198)
1938	106	1947	119		

The 1955 index is not yet available: we show in brackets an estimate, 10% above the 1954 result. Such an estimate appears conservative, since the net dividends (after income tax) announced during the first eight months of 1955 are some 22% above the corresponding 1954 figures (see *Financial Times* Profits Tables).

According to this table, the share dividends were cut by only 22% during the exceptionally severe slump conditions of 1932-33. By the end of the period, enhanced dividends had accrued, sufficient to compensate for some 75% of the higher living costs.

In contrast, if the fund had been invested at 31 December 1929 in 2½% Consols, rather than in this share portfolio, a fixed income would have been secured equal to some £80 per annum per £100 of share dividends. Con-

* The *Investors' Chronicle* publishes monthly a dividend index based on the Actuaries' Investment Index.

sequently, the share portfolio provided practically the same income in the two depression years, a higher return every subsequent year, and a final result some $2\frac{1}{2}$ times the Consols revenue. By the end of the 25 years, which included a most severe slump and a world war, dividends had increased by 98%, representing an over-all rate of growth of $2\frac{3}{4}\%$ per annum.

PART III. YIELDS

The Actuaries' Investment Index service includes a statement of the arithmetic average of the yield for all the groups for which price index numbers are given. For fixed interest securities a comparison of yields is vital information for those engaged in investment and a handy historical record must be of great value. In the case of ordinary shares the position is rather different. The yield is based on the most recent dividends and these had depended on the earnings during a period which has probably ended several months previous to the declaration, and up to 18 months before the date of the yield calculation. The market price, however, will reflect investors' expectations about future dividends. In so far as these expectations may be different from the recent past, the yield, based on past dividends and current price, may well have very little meaning. This point must be borne in mind when comparisons are made between fixed interest security and equity yields. It might be more helpful to investors to publish an index of dividends declared similar to the one used in Part II, Section 4.

(1) *Expected yield*

In Part I the concept of 'expected yield' as developed by Pegler and Clarke was mentioned. In the discussion on these two papers there was some criticism of the way in which the concept was defined and used. In this paragraph we have attempted to extend these ideas so as to allow for some of the criticism. In addition to the 'expected yield', Clarke considered also the 'degree of reliability' of the estimate. Securities have varying ranges of possible incomes. For example, the range of possibilities for the income from an ordinary share is generally greater than that from a fixed interest share. The narrower the range of possible incomes, the greater is the reliability that can be placed on the estimate of the expected yield. Thus, *other things being equal*, and since the estimate of expected yield involves the probabilities of receiving both income and capital, the income-cautious investor will prefer the investment providing the narrowest range of possible incomes. It should be mentioned here that we are confining the theory to institutional investors such as life assurance companies and pension funds, which, generally speaking, are income-cautious, i.e. their main aim is to assure themselves a steady or increasing income in the future.

In estimating the expected yield for a security, such an investor will have in mind some standard security, say a British Government security, for which he assumes that the expected yield is the actual yield i , and for practical purposes the degree of reliability is, say, unity. If he then finds that some other security X is equally attractive, that is to say if he is indifferent as between investing in the British Government security and some other security X , for which the expected yield is i' and the degree of reliability is k , then the difference in yield $i' - i$ just compensates for the difference in degree of reliability $1 - k$. We define i as the normalized expected yield of X for this investor. In

other words, given a security X with expected yield i' , and degree of reliability k (< 1), the normalizing process is simply the mental process of the investor in choosing a fixed interest security, actual or hypothetical, with degree of reliability unity, such that he would be indifferent as between investing in this security and in X . The expected yield of the fixed interest security is the normalized expected yield of X . We are reluctant to introduce this statistical terminology in a paper of this kind. It is, however, a fact that two securities might have the same expected yield and yet not be equally attractive to an investor because he does not have the same confidence in the two estimates of yield. The process of normalizing adjusts the yields to allow for the degree of reliability. Two securities having the same normalized expected yields are equally attractive to the investor. In practice, the procedure is to compare securities with some standard riskless security, that is to say riskless in the sense of security of income and security of capital at some redemption date. We are not concerned here with the 'liquidity' risk, i.e. the risk of having to sell the security at a loss because of unforeseen circumstances.

The investor, however, is not concerned only with the selection of a single security. His main concern is the income from and the security of the whole fund. Generally the degree of reliability of the average yield from the fund is greater than that for the yield from a single security. A principle followed by all large investors is that of spreading the funds over a wide field as regards type of security, and location of the firm and industry. For example, each of the normalized expected yields of n securities may be less than that from some, say, British Government security, yet it may be the best policy to spread one's funds over these securities, rather than invest the whole fund in the British Government security, because the *average* expected yield (*not* the average *normalized* expected yield) may be substantially greater than the expected yield of the British Government security and the degree of reliability of the *average* expected yield may well be little different from that of the yield from the British Government security. This, of course, is one of the reasons why it is good policy to invest a substantial proportion of a fund in equities.

An investor, therefore, does not simply maximize the normalized expected yield of a particular security. He has in mind a short range of normalized expected yields, depending on the yields obtainable, which he regards as a standard. He selects securities on the basis of this standard, and spreads his funds over these securities so as to maximize the degree of reliability of the average yield. This is probably what Douglas had in mind when he said that 'the only reason for investing in ordinary shares can be that greater security is to be found there'. The security depends on both the selection and the spread of investments. These principles are theoretical, they attempt to state precisely some of the operations that an investor performs more or less intuitively when he has gained a wide and varied experience in investing funds. The concepts are subjective in the sense that investors will make different estimates of the normalized expected yields both because their relevant knowledge and their risk attitudes differ. For example, an investor may prefer not to invest any funds in securities having an estimated value for k below a fixed limit. Again, some investors will not consider securities in respect of which their knowledge is so vague that they feel that it is impossible to estimate either i or k or both.

(2) Estimating the 'expected yield'

So far, we have discussed the expected yield solely in theoretical terms. It is, of course, impossible to place any definitive value on this function; most of the relative factors are no more than educated guesses, based on forecasts of future trends and conditions. The most that can be done in practice, when alternative investments are being considered, is to classify them roughly into expected yield and reliability groupings, assessed from whatever statistics and knowledge are available.

When classifying an ordinary share in this way, the investor will examine the current dividend yield and the earnings yield, and will try to assess the trend of future earnings. A proportion of the earnings not distributed as dividends is ploughed back into the business and eventually should provide further profits and added dividends. If profits are expected to grow steadily and surplus earnings are considerable, the expected yield should be somewhere above the dividend yield and below the earnings yield. The reliability will depend on the history of companies in the industry concerned or in similar industries. It will also depend on one's knowledge of the management.

A vital factor in this connexion is the return likely to be obtained from new money invested in the business. High earnings are usually made from the original assets, installed years ago; new plant bought at present-day prices is less remunerative. In exceptional circumstances the return on new money, the earnings yield and the expected yield might be approximately equal. For example, consider a new company which has just raised £1,000,000 of ordinary share capital. Suppose that this money invested in suitable plant can be expected to earn 8% after depreciation, etc. If a dividend of 5% is paid, the balance of 3% is available (after tax) to purchase further productive assets, which can also earn 8%. In these circumstances and assuming that the directors follow a liberal distribution policy, the expected yield to an investor, subject to tax, can be regarded as 8%, i.e. the earnings yield. In practice, an earnings yield based on heavily written down assets, and which considerably exceeds the return that new money will provide, should be heavily discounted.

(3) Comparative yields

In this section we set out the results of an investigation into data taken from the working sheets of the Actuaries' Investment Index. It compares the results of investments in low-, medium-, and high-yielding ordinary shares. We do not wish to draw any firm conclusions from the results; they are suggestive and the investigation may be considered mainly as an illustration of the kind of information that can be obtained from the Actuaries' Investment Index data. In order to obtain firmer conclusions, the investigation should be made over a much longer period.

For the purpose of this investigation, the statistics of the Industrial groups have been used; within each group, the constituents at 29 December 1950 have been arranged in order of the dividend yield at that date, and then subdivided into three categories, according to whether the yield is low, medium, or high. In this way it is possible to obtain three sample portfolios, evenly distributed over the various industrial groupings, but differing as regards the initial dividend yield. It would, of course, be a relatively simple matter to

calculate periodical index figures for the three samples and to compare the price ratios and the yields with each other, and with the over-all index. For the present purposes, however, it is adequate to extract the relative prices and yields in June 1955, so as to compare the growth of capital and income, recorded by these alternative portfolios, over a period of $4\frac{1}{2}$ years.

The 'Industrials' classification of the Index comprises 22 classes and 202 securities. It has been necessary to leave out of the investigation the following groups:

Chain Stores, Tobacco, and Chemicals. These classes, containing 2, 3, and 6 constituents respectively, are not sufficiently comprehensive to allow of subdivision according to yield.

Breweries, Newspapers, Departmental Stores. For these categories the respective average dividend yields are 6.17%, 6.92% and 6.71%, compared with an over-all yield of 5.27% for all industrials combined. Consequently, the low- and medium-yielding securities are insufficiently represented.

Miscellaneous Group. This class naturally covers a range of industrial trades, differing in scope and prospects, and is therefore best excluded.

Oil. Results for this group are unduly affected by political factors.

After removing these categories, 14 groups covering 134 securities remain.

Table 7, below, shows how these groups are represented in the three sample portfolios:

Table 7. Number of securities in the sample portfolios

Group	Yield category		
	Low	Medium	High
Building	3	4	3
Electrical Engineering	3	4	3
Engineering	4	4	2
Motors and Aircraft	4	2	4
Shipbuilding	2	3	2
Boots and Shoes	2	3	4
Chemists	4	3	3
Cotton	4	3	3
Food	4	3	3
Household	4	4	2
Paper and Publishing	4	4	2
Rayon	3	3	3
Wool	3	4	2
Shipping	2	4	4
	46	48	40

It is assumed that £100 is invested in each of the 134 securities. From the working sheets the following factors were recorded:

- (1) Price at 29 Dec. 1950.
- (2) Yield at 29 Dec. 1950.
- (3) Price at 28 June 1955.
- (4) Yield at 28 June 1955.

(5) Price-relative $\times 100$, i.e. value at 28 June 1955 of £100 invested in the security at 29 Dec. 1950.

(6) Income in June 1955 per £100 invested in 1950 ($100 \times$ price relative $\times$ yield in June 1955).

For each sample portfolio, arithmetic averages were calculated for the 1950 yields, the price relatives, and the 1955 incomes. Where necessary, allowance was made for 'casualties' among the index constituents. Where a 'casualty' was replaced by a new security, a corresponding change was reckoned to be made in the sample portfolio, selling the casualty at a depreciated price, and reinvesting in the new share. Sometimes a suitable replacement was not available; in that case, the proceeds of the sale were assumed to be reinvested equally in all the remaining shares of the industrial group concerned. Results of this investigation are given in the following table:

Table 8. Arithmetic average of prices, yields, and incomes

	Yield category		
	Low	Medium	High
100 $\times$ price-relative	210	172	170
Yields (1950)	4.21	5.00	6.53
Yields (1955)	4.13	4.90	5.68
Income in 1955 per £100 originally invested	8.67	8.42	9.69
Income in 1955 if low- and high-yielding securities be reinvested at 28 June 1955 in 'medium' group	10.29	8.42	8.34
Number of securities	46	48	40

Clearly, the low-yielding group has proved the most profitable. Admittedly, the period 1950 to 1955 was favourable to this group, which derived the maximum benefit from increased dividends, following the end of the 'dividend freeze'. In this connexion, however, we would point out that a somewhat similar investigation was made for private circulation by Miss M. Rix for the period 1948-51. At 31 Dec. 1948, four groups of 25 securities were selected (low, low-medium, high-medium, and high-yielding groups). Results over the next three years were as follows:

Group	Price at 31 Dec 1951 (1948=100)
Low	112.7
Low-medium	106.3
High-medium	94.9
High	102.1

Again the best performance was provided by the low-yielding shares.

In practice, institutional investors, seeking immediate income, often prefer the medium-yielding category to the low-yielding shares. The latter group, however, usually provides high earnings yields and/or considerable growth

prospects; consequently, 'expected yields' are high, and by this criterion the shares should merit serious attention for investors interested in long-term income.

PART IV. THE CONSTRUCTION OF INDEX NUMBERS OF SECURITY PRICES

Having surveyed the various uses of index numbers, we now proceed to discuss the methods of their construction and maintenance, bearing in mind the more important of the uses which are likely to be made in practical investment work.

(1) *Earlier papers on index numbers*

In 1930 A. C. Murray read a paper entitled *The Compilation of Price Index Numbers and Yield Statistics Relative to Stock Exchange Securities* to the Faculty of Actuaries (*T.F.A.* 13, 97). Mr Murray was a member of the Joint Committee responsible for the Actuaries' Investment Index, and this paper gives many of the reasons which guided the Joint Committee in deciding the methods to be used in calculating the index numbers.

Since this paper, very little has been written about index numbers of security prices. In 1930, however, at the same time as Murray's paper, the London and Cambridge Economic Service commenced to publish a new Index of Security Prices, and a Memorandum* was issued explaining the methods employed and the reasons for choosing these methods. It will be interesting to consider both Murray's paper and the London and Cambridge Memorandum. For convenience in this part of the paper we shall refer to the former as A.C.M. and to the latter as L. & C., while the Actuaries' Investment Index will be referred to as A.I.I.

Both papers consider the possible uses of the index numbers although not to the extent discussed in this paper. They both mention that the index numbers will be a guide to investment policy on the one hand and a guide to the general economic situation on the other. A.C.M. is more concerned with the effect of economic and financial and political events on the level of security prices, whilst L. & C. state that it is primarily concerned with the diagnosis or prognosis of the general industrial situation. That is to say that L. & C. regarded the share index as a barometer of business conditions. The Memorandum stated:

For this purpose it is not necessary to have exact measurements of any defined quantity; we only require that the turning-points of our index shall be rightly dated for our purpose, that major fluctuations shall appear of more importance than minor, and that in quite short periods we shall be able to distinguish between fortuitous variations and those which are tendentious.

In addition to the two purposes already mentioned, A.C.M. required primarily that the index should be suitable for the use of large investors, such as insurance companies, financial trust companies, etc., and also that the index numbers might be used to measure approximate changes in the value of a portfolio of securities.

* *A new Index of Price of Securities*, by A. L. Bowley, G. L. Schwartz and K. C. Smith.

Once the purpose has been decided, the following problems of compilation must be considered:

- The selection of the securities.
- The weights to be used.
- The method of averaging.
- The base year.
- Grouping.
- The removal and addition of securities and allowance for bonus issues.
- Periodical revision.

(2) *The rules for selecting securities*

In both cases the purposes aimed at required that the securities should be restricted to those of companies of importance and good standing. The A.I.I. (1st series) limited the selection to companies whose equity stock or shares were quoted in the London Stock Exchange Daily Official List and had a market capitalization on 31 December 1928 exceeding £2,000,000, and which operated at least in part in Great Britain. L. & C. did not give any precise rules. The A.I.I. Ordinary Share index was based on 124 securities whilst the L. & C. Ordinary Share index was based on 93 securities. In both cases the securities were subdivided into several industrial groups, for each of which separate index numbers were calculated.

For a reliable index, that can be easily understood by its users, rules are required to ensure that the sample always consists of the securities of companies of the specified standard. The L. & C. Memorandum does not give a precise statement of any rules. In the case of the A.I.I. (2nd series) the rules take account of the size of the equity of the company, the dividend record, and the type of industry. A security is removed if the company passes under the control of another concern or if the company fails to pay a dividend for two consecutive years in respect of that security. There is also a casualty rule under which a security is removed if its price-relative falls below a level which depends on the index of the sub-group to which the security belongs. This rule will be considered in more detail at a later stage. Securities not in the sample may qualify for inclusion because of new flotations of previously private companies or because of amalgamations. Finally, it has been decided to make a major review of the list of securities, of methods of grouping and of methods of calculation, every five years.

(3) *Weighting*

On the question of weighting, A.C.M. pointed out that the weights would depend on the purpose of the index. Since there were several desirable purposes to consider, a compromise would have to be made. The suggestion was made that in the case of securities with large capitalization, insurance companies' purchases would tend *within each class* to approximate more nearly to equal amounts of money in each security than to any other distribution, and it was therefore considered that an unweighted price index should be constructed for each class. In the case of the A.I.I. (1st series), the index for all classes combined was also unweighted. It should, however, be noted that the number of securities in each class varied so that in this sense the index for

all classes combined incorporated weights. In the case of the A.I.I. (2nd series), the index numbers for each class are again unweighted, but for combinations of classes of industrial ordinary shares weights have been used mainly for the reason discussed on p. 368. The method of choosing the weights was similar to the 'ideal method' suggested by the L. & C. and which is mentioned below.

The L. & C. Memorandum was emphatic that a rational system of weights was required for the purpose that the index was to serve. The ideal would be to ascertain the value, at the average price in the base year, of the aggregate of the ordinary shares of every company whose shares were currently purchasable and, by grouping them under the chief industrial headings, obtain an indication of the relative importance of each group. Adequate material for such an estimate was not, however, readily available and the labour involved was thought to be prohibitive. It was decided, therefore, to assign weights within each group according to the market value of the aggregate of each company's ordinary shares in the base year, but to obtain the over-all index by assigning weights to the group index numbers according to the net output of each group as given by the Census of Production. It will be seen from Fig. 4 that the two index numbers (the A.I.I. 1st series and the L. & C.) follow one another very closely and that either will serve the purpose stated in the L. & C. Memorandum. The diagram suggests that, provided the sample is large and representative, an elaborate weighting system is not required for this particular purpose.

(4) *The method of averaging and the base year*

A.C.M. discusses the method of averaging at considerable length. Again, much depends on the purpose of the index. For example, if one desired to measure the change in a portfolio consisting of the investment of equal amounts of money in the securities selected, then one would use the arithmetic average of the price-relatives. If P_{it} is the price of the i th security at time t and t_0 is the base time, then the index at time t would be

$$\frac{1}{n} \left[\frac{P_{1t}}{P_{1t_0}} + \frac{P_{2t}}{P_{2t_0}} + \dots + \frac{P_{it}}{P_{it_0}} + \dots + \frac{P_{nt}}{P_{nt_0}} \right].$$

However, investors do not generally behave in such a static fashion, and in order to be realistic it is necessary to revise the list of securities from time to time. Further, the amount of change shown by the index will depend upon the base year, that is to say, the index will measure only a portfolio consisting of equal money investments at the base date; it will not measure the change for equal money investments at some other date.

Although such averages as the median and the mode were mentioned, the only other average that was seriously considered by A.C.M. and L. & C. was the geometric mean. Both papers give calculations demonstrating how widely the arithmetic mean and the geometric mean can differ if the price relatives move erratically. The geometric mean, however, has the advantage that it is independent of the base year; on the other hand, it gives undue weight to a security whose price falls considerably—indeed, in the extreme case the index becomes zero if any price falls to zero. L. & C. considered that the grounds for using the geometric mean could be ones of convenience and accuracy only. When it becomes necessary to remove or add a security or replace one security by another the procedure is very simple if the geometric mean is used.

A.C.M. considered that his illustrations showed that on the whole the geometric mean measured the change in the investment position between any two dates. It is clear that the arguments for either average were not very convincing one way or the other. In practice the A.I.I. was based on the geometric mean and the L. & C. Index was based on the arithmetic mean. Fig. 4 again shows that the choice was immaterial if the main purpose of the index was to indicate the trend of the industrial position. The diagram, however, refers only to a short period of time and in the next few paragraphs we shall examine the problem in some more detail.

We are concerned with some measure that is representative of the price-relatives of a sample of securities. The sample is not a random one; it has been selected as typical of the securities in which certain investors are willing to keep their funds. At the base date all the price-relatives are equal to unity, and as the time from the base date increases the values of the relatives spread out in both directions, being limited in one direction by zero and being unlimited in the other direction. This means that in time the distribution becomes very skew. To illustrate this the following figures have been taken from the A.I.I. (2nd series). Taking 1945 as the base year and using the securities comprising the Consumption Goods group, Table 9 (i) shows the distribution of the price-relatives at December 1950 and Table 9 (ii) the distribution at February 1955.

Table 9

Value of price relative	(i)	(ii)
0- $\frac{1}{2}$	—	3
$\frac{1}{2}$ - $\frac{3}{4}$	14	13
$\frac{3}{4}$ -1	44	12
1- $1\frac{1}{4}$	34	18
$1\frac{1}{4}$ - $1\frac{1}{2}$	8	12
$1\frac{1}{2}$ - $1\frac{3}{4}$	5	13
$1\frac{3}{4}$ -2	1	8
2- $2\frac{1}{4}$	1	3
$2\frac{1}{4}$ - $2\frac{1}{2}$		6
$2\frac{1}{2}$ - $2\frac{3}{4}$		5
$2\frac{3}{4}$ -3		1
3- $3\frac{1}{4}$		2
$3\frac{1}{4}$ - $3\frac{1}{2}$		1
$3\frac{1}{2}$ - $3\frac{3}{4}$		2
$3\frac{3}{4}$ -4		1
4-		4

The unweighted arithmetic mean of distribution (ii) is 1.75 and the unweighted geometric mean is 1.41. If the weights adopted in the A.I.I. (2nd series) are used, the weighted arithmetic mean is 1.57 and the weighted geometric mean is 1.24. These figures show, first, that for skew distributions the geometric mean is considerably lower than the arithmetic mean and, secondly, that in this particular index the weights have had an important effect. The reason for the weight effect is that the Breweries and Tobacco groups have been given considerable weight, but the index numbers for these two groups have fallen appreciably since 1945, whilst those for all other groups except one have risen, many substantially.

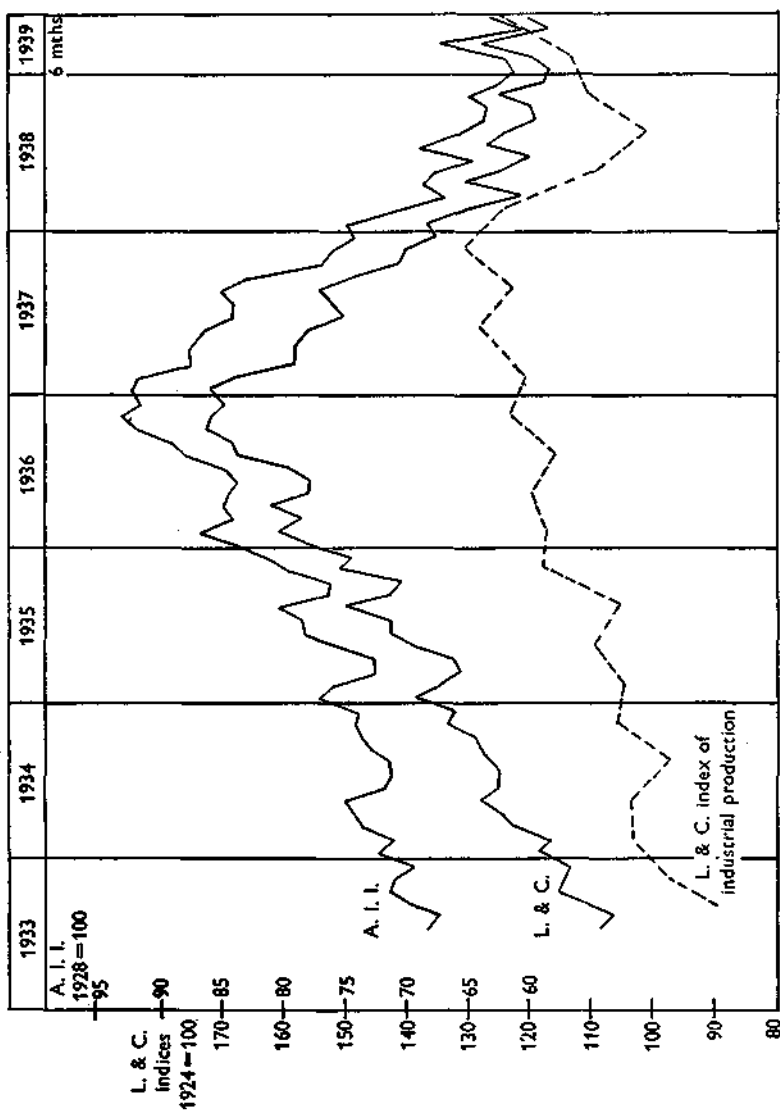


Fig. 4

The figures for the Capital Goods group are:

	Arithmetic mean	Geometric mean
Unweighted	1.96	1.70
Weighted	2.02	1.77

In this group the index numbers of the sub-groups comprising it have all moved appreciably upwards and the effect of using weights has not been very material. The distribution of the relatives, however, is significantly skew, as shown by the difference between the arithmetic and geometric means.

In the example on p. 361 we have a situation in which about three-quarters of the price-relatives have risen, some of them substantially. It could happen, if some other base year was chosen, that this would not be the case. In order to see what the distribution would be if the majority of price-relatives had fallen, we can take the above data and convert them to February 1955 as the base date, obtaining the distribution of the price-relatives at December 1945. The distribution is shown in Table 10 below.

Table 10

Price relative	Frequency
0- $\frac{1}{2}$	4
$\frac{1}{2}$ - $\frac{1}{4}$	22
$\frac{1}{4}$ - $\frac{1}{2}$	30
$\frac{1}{2}$ - $\frac{3}{4}$	20
$\frac{3}{4}$ -1	9
1- $1\frac{1}{4}$	6
$1\frac{1}{4}$ - $1\frac{1}{2}$	7
$1\frac{1}{2}$ -2	3
2- $2\frac{1}{2}$	2
$2\frac{1}{2}$ -3	1

The distribution is still positively skew, but not so skew as the distribution with 1945 as the base date. The averages of this distribution are:

	Arithmetic mean	Geometric mean
Unweighted	.838	.711
Weighted	.987	.809

Again the arithmetic mean is appreciably greater than the geometric mean and the weights make a significant difference. The figures also bring out the importance of the base date for the interpretation of the arithmetic mean. For example:

Index (unweighted arithmetic mean)

Base year	1945	1955	% change
1945	100	175	75
1955	83.8	100	19

Index (weighted arithmetic mean)

Base year	1945	1955	% change
1945	100	157	57
1955	98.7	100	1.3

The above table is an extreme illustration of the effect of changing the base date. Over the 10-year period many prices increased considerably and relatively few declined [see Table 9 (ii)]. The rise from 100 in 1945 to 175 in 1955 represents the appreciation of an investment of equal amounts of money in each security in 1945, whilst the rise from 83.8 to 100 represents the appreciation of an investment of amounts of money inversely proportionate to the price changes between 1945 and 1955. That is to say, in the latter case the least amount of money is invested in that security whose price rises the most, and the largest amount in that whose price rises the least.

The geometric means, of course, are independent of the base year and give changes of 41% and 24% respectively, figures not far from the mean of the respective changes given by the arithmetic means.

In judging which mean to use the purpose of the index is the main consideration. If we are concerned with the economic situation, we require a measure which typifies the distribution. The geometric mean is closer, often much closer, to the median and mode than to the arithmetic mean and, in so far as the range of values covering the median and mode can be said to typify the distribution, we have grounds for choosing the geometric mean.

If, however, we wish to consider the index as measuring the value of the portfolio of some typical investor, the arithmetic mean would seem to be the best measure; this point is demonstrated in Section (4) of Part II, where a comparison has been made between arithmetic and geometric indices over 25 years. However, it has been pointed out by C. D. Rich (*J.I.A.* 74, 338) that the unweighted geometric mean* can be regarded as the measure for a certain type of investor. It represents the result of investing equal amounts of money in each security and then of keeping the market value of each of the investments equal in time by following the rule that, on a change in prices which makes the values unequal, purchases and sales are carried out in order to restore the equality. For example, when the market in general is rising, there will be sales of securities which have risen in price the most and purchases of those which have risen in price the least. In a falling market there will be sales of those securities which have fallen in price the least and purchases of those which have fallen in price the most. In the extreme case, if the price of one security is tending to zero, all the remainder are sold and the proceeds are used to purchase this one, and in the limit the value of the index becomes zero. This type of investor is not quite so absurd as he might appear to be. In the first place one can put a limit on the amount of fall in price that is permitted before the security is removed from the sample, a procedure which has been adopted in the A.I.I. (2nd series). In the second place the rule is very similar to the rule of the so-called formula plan that has been fairly widely used in the United States.

The usual type of formula plan divides the fund, say, equally between equities and money or equities and bonds, and the money values of the two

* See note on page 351 for qualifications if weights are used.

categories are kept constant. For example, if the two categories are equities and bonds, then when the prices of equities are rising relatively to those of bonds, equities would be sold and bonds purchased. The rule then would require the equalization of money values at discrete intervals of time. This rule is the only practical rule corresponding to the theoretical rule of Rich's investor. The latter must equalize the money values of the investments continuously, an impossible procedure in practice. The shorter the period between the equalizations of the portfolio in the formula plan, the closer the rule is to that illustrated by the geometric mean. In practice, the period between equalizations would have to be reasonably long in order to avoid heavy costs of investment. If the period of equalization becomes infinite, the index at any point in time becomes the arithmetic mean of the price-relatives.

Formula planning, to be successful, assumes that prices will rise and fall and that the share prices that rise the most will on the average fall the most. As between equities and bonds this hypothesis is reasonable, generally the amplitude of the movement of share prices is greater than that of bond prices. If the market as a whole rises and subsequently falls to the original level and if on the average share prices rise more and consequently fall more than those of bonds (or vice versa), then it is easy to show that the investor will make a profit. The simplest proof follows from the fact that the arithmetic mean is greater than the geometric mean. Formula planning could be applied to a portfolio of equities, although in this case the hypothesis that the prices of those shares which rise the most will subsequently fall the most on the average is open to doubt. However, for our purpose, it is sufficient that the use of formula planning for a group of equities is a possible policy for investment. In this paper we are not concerned with the merits of this policy.

The index number that would measure the value of a portfolio subject to the formula plan rule would be one based on the method of 'link relatives'. Let the interval of time be denoted by 0, 1, ..., t , $t+1$, ..., and let the price of the i th security at time t be $P_{i,t}$ ($i=1, 2, \dots, n$). Then the link relative index at time t is given by either of the formulae

$$(i) \quad I_t = I_{t-1} \times \frac{1}{n} \left[\sum \frac{P_{i,t}}{P_{i,t-1}} \right],$$

or

$$(ii) \quad I_t = I_{t-1} \times \left(\prod \frac{P_{i,t}}{P_{i,t-1}} \right)^{1/n}$$

depending on whether one uses the arithmetic or the geometric average. The former is, of course, the indicator of the results of formula planning.

The L. & C. Memorandum discusses these two index numbers considering the following illustrations.

Suppose the prices of three securities a, b, c are as follows, at dates A, B, C :

	Percentages of prices at A		
	A	B	C
a	100	150	100
b	100	50	100
c	100	75	100
Arithmetic mean	100	91.7	100
Geometric mean	100	82.6	100

Now start with the price of each at 100 at *B*.

	<i>B</i>	<i>C</i>
<i>a</i>	100	66 $\frac{2}{3}$
<i>b</i>	100	200
<i>c</i>	100	133 $\frac{1}{3}$
Arithmetic mean	100	133 $\frac{1}{3}$
Geometric mean	100	121

The link method using the arithmetic mean would give from *A* to *C*

$$100 \times \frac{91.7}{100} \times \frac{133\frac{1}{3}}{100} = 122.2.$$

The Memorandum states that this result is absurd as showing the movement from *A* to *C*. This is true if the purpose of the index is simply to indicate the trend of security prices, but it would be correct as an indicator of the use of the formula plan. In other words, by using the formula plan the investor makes a profit provided the prices of the securities return to their original level. If he had left the portfolio unchanged he would have made neither a profit nor loss. If the geometric mean is used the result from *A* to *C* would be

$$100 \times \frac{82.6}{100} \times \frac{121}{100} = 100$$

and again there is neither profit nor loss, a result which the L. & C. Memorandum states is in accordance with the facts.

So far, the alternative methods of averaging have been considered in relation to price index numbers only; the question of average yields must not, however, be overlooked. The A.I.I. gives the arithmetic average of the individual yields; where weights are used for a price index, the same weights are used for the corresponding yield. These yield figures are usually regarded as merely giving the average current yield, they are not index numbers of yield. However, if the geometric average is regarded as an indicator of the results of the policy followed by Rich's investor, then the yield on such a portfolio is the arithmetic average of the individual yields, and as explained in Part II, Section (4), the A.I.I. average yield may be multiplied by the price index to obtain a dividend indicator. This argument would not apply if the price index was based on the arithmetic average.

In this part we have set out certain properties of the geometric average which have not been considered previously and which give a little additional support to the use of this method of averaging. However, the choice of method still remains arbitrary to a large extent, particularly if the index is required for several purposes.

(5) Grouping

This question of grouping has already been mentioned in connexion with weighting. In the new A.I.I. the choice of grouping for the various sub-index numbers varies with the type of share. For ordinary shares the grouping is by industry. Twenty-seven index numbers are published covering twenty-six

industries and a miscellaneous group. In addition, three more index numbers are published, the first for a combination of five industries producing mainly 'capital' goods, the second for a combination of thirteen industries producing mainly 'consumption' goods, and the third for a combination of all industries excluding the financial groups.

In the case of debentures there are three industrial groups: Investment Trusts, Breweries and Miscellaneous. In addition there is an index number for Breweries and Miscellaneous combined.

For preference shares there are two main groups: Investment Trusts and Industrial. Each of these is subdivided according to dividend rate, it being thought that such a subdivision might show some significant differences between the average yields. Investment Trusts are subdivided into two groups: dividend $4\frac{1}{2}\%$ and under, and dividend $4\frac{3}{4}\%$ to 6% . Industrials are subdivided into three groups: dividend $4\frac{1}{2}\%$ and under, dividend $4\frac{3}{4}\%$ to 6% , and dividend over 6% . In making a comparison on this basis a difficulty arises because of the varying quality of the shares. For example, the average yield for one group may be lower than the average yield for another because the average quality of the former is higher than that of the latter, rather than because the dividend rates differ.

This factor of quality is perhaps not important for Investment Trusts. The range of the yields of the individual shares is narrow. From 1945 until the end of 1954 the higher dividend group has had the higher average yield, the maximum difference being about $\frac{1}{2}\%$ during 1950. Since March 1955 the difference has been negligible, and at times the average yield of the lower dividend rate group has been very slightly the higher.

The problem is more difficult for the industrial preference shares. In these groups quality varies considerably. The range of yields within a group is often of the order of 1% , whilst the difference between the average yields has a maximum of about $\frac{1}{2}\%$. There are three groups, $4\frac{1}{2}\%$ and under, $4\frac{3}{4}\%$ to 6% and over 6% . Let us call them *A*, *B*, and *C* respectively. The average yield of group *C* has always been the highest. The maximum difference between *C* and *B* has been about $\frac{1}{2}\%$ and the minimum about $\cdot 02\%$ during 1946-47, when interest rates were very low. The difference between the average yields of *B* and *A* has always been small, $\frac{1}{4}\%$ at the most, but generally less than $\frac{1}{10}\%$ and it has frequently been negative. These results suggest that the present classification for the industrial shares now in the A.I.I. is of doubtful value. In order to eliminate the quality factor it might be better to confine such an investigation to a few very high-class shares. The authors have not had the time to experiment in this way.

(6) *The removal and addition of securities: periodic revision*

Our attention will be confined to the geometric average as used for the A.I.I. The general rule is that the continuity of the index should be maintained. If the portfolio is changed at time t , then the index I_t before the change should be the same as I_t after the change. In the case of the old series, all the index numbers were unweighted geometric averages of the price-relatives with adjustments when securities were added or removed, to preserve the continuity of all the index numbers. The procedure led to some apparent inconsistencies between the numbers. If one security is replaced by another no difficulty arises. The new security is given a notional base price such that its price-

relative at the date of change is equal to the price-relative of the security removed. If, however, a security is added or if a security is removed without replacement a complication arises when there are sub-group and group index numbers. Suppose, for example, there are three sub-groups indicated by A , B , and C and a group index I for A , B , and C combined. Now suppose a security is added to group B . Continuity in the index B_t can be preserved by giving the security a notional base price equal to its price at the date of change multiplied by $\frac{I}{B_t}$, that is to say it is assumed that the price-relative at time t is equal to the index B_t . Continuity in I_t , however, can be maintained only if the price-relative at time t is equal to I_t . It is thus necessary to give the security different base prices for the different index series. In this event it is clearly impossible for I_t after the date of change to be the average of the price-relatives entering into A_t , B_t , and C_t . After a large number of changes, this fact becomes apparent by a mere inspection of the index numbers. For example, at 30 June 1942 the price index numbers for the Distributive classes were:

Home Rails	45.4
Shipping	21.6
Stores and Catering	48.0
Combined	48.9

Thus there is the apparent anomaly that the combined index is higher than any of the sub-group index numbers. A similar difficulty arises if securities are removed without replacement. In the old series whole sub-groups had to be removed because of nationalization and the effect on the internal consistency of the series was very great. In the new series the difficulty has been partly overcome by making the combined index number the weighted geometric average of the sub-group index numbers, the weights being kept fixed whatever changes are made in the sample of securities. The weights are proportionate to the aggregate market value of all securities eligible for each sub-group. This method does not overcome the difficulty if a whole sub-group is removed.

In the new series a special casualty rule has been introduced. It will be remembered that it was pointed out that if the price of a security became zero, any index number in which that security was involved would become zero also. But even if the price of a security falls to a very low level relative to the prices of other securities in the group, the downward effect on the index is undesirable. Investors would not be likely to keep such a security, nor is the trend of such a security typical. In the old series the securities of the Royal Mail Lines had this undesirable effect on the index numbers. To avoid this eventuality in the future the rules of the new series provide for the removal of a security, subject to the discretion of the Committee, if the ratio of its price-relative to its price-relative at a date three years previously is less than one-half the corresponding ratio of the index numbers of the sub-group containing the security. The rule is arbitrary and it is worth noting that a security could be removed even if its price over the three-year period had not fallen. If the price of the security was unchanged, but the index number of the sub-group containing it had increased by more than 100%, then the security would be due for removal.

The adjustment for bonus issues and rights issues calls for no special comment. The guiding principle is that the base price should be so adjusted that the price-relative after the issue as far as possible correctly measures what has happened to a holding between the base date and the date of issue, remembering that in some cases the base price is notional.

The reasons for adding and removing securities to and from the sample were briefly set out on p. 359. It is clear that over a long period the changes might accumulate to a relatively large number, and this will add to the difficulty of interpreting movements over long periods. One way of regarding a series is to divide it into several short series of unequal length, each point of division being at a date when a change was made in the sample of securities. Then each short series measures the value of a fixed sample of securities. The short series are then 'chained' together to form a single continuous series. It now follows that if an investor is thinking in terms of the appreciation or depreciation of a representative portfolio, he should use the index over only short periods, say periods of not more than one or two years. In these circumstances the use of the geometric mean rather than the arithmetic mean will not seriously affect the calculation.

If, however, the investigator is comparing changes in the price level of shares with changes in business conditions, and hence requires only an index of the general level of prices, then comparisons based on the geometric mean may be made over long periods.

PART V. CONCLUSIONS

(1) *The Actuaries' Investment Index*

Finally, we have to consider the extent to which the Actuaries' Investment Index service meets current needs, bearing in mind the facilities provided by competing indices.

There is no doubt that, when economic trends are studied (see Part II, Section (2) (a)—Forecasting), a daily index is needed. For this purpose the *Financial Times* Index is invariably used, and the monthly Actuaries' Investment Index figures are of little, if any, value. Again, if we are concerned with a long-term investigation into the results of an ordinary share portfolio, an arithmetic index such as that produced by the *Investors' Chronicle* is the more suitable.

Whilst there are several other indices of share prices, analysed according to industry, there is no competing monthly index which provides average yields.

One of the more important applications of the Actuaries' Investment Index is the comparison between the monthly yield, and price tables and the corresponding data from the *Financial Times* profits analyses (see Part II, Section (2) (b) and (3)). In effect, such an investigation provides the basic data from which an investor can attempt a classification of the shares in the industry concerned according to 'expected yield'. Clearly, such comparisons would be facilitated if identical industrial groupings were adopted by the Actuaries' Investment Index and the *Financial Times* Index. As many categories already correspond closely, this result could be achieved with relatively minor changes in the Actuaries' Investment Index classifications.

From time to time, suggestions have been made for a weekly compilation

of the Actuaries' Investment Index. In practice, however, most of the applications of indices fall into two categories, either

- (a) those involving comparison with other factors available monthly,
- or
- (b) those requiring a daily index.

Consequently, there would appear to be no advantage to be gained from weekly compilation.

As a supplement to the normal geometrically averaged price and yield tables, further indices might be calculated, at six-monthly intervals, for share prices and dividends, averaged arithmetically. Such a supplement would render the index suitable for long-term comparison with subscribers' portfolios.

The working sheets used by the Actuarial Tuition Service Staff for computing the index each month contain valuable material for research purposes. Apart from the investigation into the results of different yield categories, described in Part III, Section (3), other comparisons might, with advantage, be made; for example, shares might be analysed according to the size of company or according to the gearing of the capital, and the results traced over a long period. Naturally, the working sheets, being the basic statistics of the Index, must be retained in the Institute offices. However, the Investment Research Committee of the Institute and Faculty would be glad to give every reasonable facility to any members wishing to undertake such research.

(2) Investment policy

In determining the distribution of a fund's assets, regard must be paid to the nature and duration of the liabilities. Usually estimates are made of the future progress of the fund, allowing for possible payments for claims, surrenders, withdrawals, pensions, etc. For most progressive life, annuity and pension funds, such an investigation suggests that to cover possible capital commitments only a portion of the fund needs to be invested in redeemable fixed interest securities. Once these commitments have been satisfied, however, the remaining liabilities are normally of very long duration, and thus require to be covered by corresponding long-term or irredeemable investments. The remarks in this section apply to this particular portion of the portfolio.

In the past, the major proportion of this part of the fund has frequently been invested in fixed-interest securities. Such procedure offends seriously against the principle of diversification. In addition, all past experience suggests that in the long run a well-spread and well-managed equity portfolio should provide a much higher return than these fixed-interest investments. Consequently, current practice requires that an important part of the 'irredeemable' fund be invested in equities, thereby securing a higher 'expected yield', particularly if the fund concerned (e.g. a pension fund) carries liabilities in 'real' terms.

The equities should be selected so as to provide the maximum 'expected yield' using the methods outlined in Part III. In practice, this criterion can only be interpreted in general terms. It does, however, mean that the long-term prospects of a share are regarded as of more importance than the immediate dividend yield; consequently, the portfolio would include a proportion of securities carrying a low current yield, provided that the earnings yields and/or the growth prospects were adequate.

In building up this equity portfolio, either by direct purchase or by exchange from gilt-edged stocks, successful timing of the transactions may be facilitated by use of investment indices examined in conjunction with profits tables, activity indices, production data and other economic factors.

To maintain the share portfolio to the best advantage, an active policy should be pursued, keeping the 'expected yield' as high as possible. This procedure would be particularly concerned with the distribution of the shares according to industry. It is suggested that study of the Actuaries' Investment Index industrial group tables, interpreted in conjunction with other economic data, is of considerable value in keeping the maximum weight in the more progressive industries and in reducing the stake in the declining trades.

ABSTRACT OF THE DISCUSSION
AT THE INSTITUTE*

Mr H. W. Haycocks, in introducing the paper, said the authors' objective had been to promote a discussion on the Actuaries' Investment Index, and that had required a description of the methods of construction, the consideration of possible uses and an introduction explaining the origin of the Index. The investigations in Part II were little more than illustrations of what could be done with index numbers. In order to obtain firm conclusions some of those investigations would have to be made over a longer period so as to see what happened under more varied economic conditions. He felt that in that section of the paper they should have referred to the two papers written by Mr H. E. Raynes in 1927 and 1937 dealing with investment in ordinary shares (*J.I.A.* 59, 21 and 68, 483).

In discussing yields, the authors had been tempted into saying something about the so-called expected yield. They had not attempted to set up a complete theory of investment; what they had in mind was a situation in which an investor had a list of securities and it was assumed that he could assess the expected yield and order the securities according to the reliability of that assessment. They should therefore have given the factor k an ordinal scale, so that $k = 1, 2, 3$, etc., according to the ranking of the security; the higher the value of k the less the reliability. With that information, together with general investment experience, it was thought possible to make an isolated choice of an investment in a rational way.

That was not the whole problem by any means; there was also the problem of the best distribution of a fund over the available assets. Naturally such a distribution would depend upon the objectives of the fund and the relations of the manager to the fund—for instance, was it his own or did it belong to somebody else? In any event, ultimately there was the problem of choosing a distribution which balanced expected yield on the fund against the risk of loss. The authors had not solved that problem. They would probably want more than a simple order scale for k . Whether a theory of investment was possible might well be a subject for another paper and further discussion.

When writing the paper the authors had overlooked a paper by A. D. Roy, published in 1952 in *Econometrica*, called *Safety First and the Holding of Assets*. He had postulated expected yields, standard errors and coefficients of correlation between the yields. It was possible in that way to obtain a neat mathematical solution; but he (the speaker) felt that as a theory of investment it was getting too far away from the actual methods adopted by investors to be of any value. Mr Roy's paper had, however, set out the problem clearly and stimulated interest in possible solutions.

In criticizing such theories, they must remember the part played by theories in the social sciences. In physics theories were explanatory and at the same time predictive, but in the social sciences they were more often only explanatory. They threw light on certain factors operating in the situation and worked out the implications of those factors, but in any actual situation there were so many other factors to consider that the element of judgment in making a decision was

* The discussion at the Faculty of Actuaries on 19 March 1956 is reported in *T.F.A.* 23, 430.

decisive. Practical men often smiled at theories, particularly when amateurs like himself suggested them. They were not always aware that they themselves theorized on matters which could be brought to the surface by suitable questions and discussion. It was well known in science that the more precise and certain the statements made about the world, the less could be said about it. It followed that social theories had to leave a large part of the phenomena they described out of the picture.

He did not wish in the introduction to direct too much attention to what was, after all, only a small part of the paper. The Research Committee was interested in the construction of suitable index numbers and it hoped to obtain some help from the meeting. Parts II and IV of the paper together showed that there was no one ideal method of construction. If it was decided to publish a single, general-purpose index, the problem was that of choosing a suitable method from several which had been proposed and used from time to time.

Mr H. C. Cottrell, in opening the discussion, said that the question of expected yield had started with an actuary who said that security of capital was always the primary objective. That view had prevailed for many years, but Pegler attacked it just after the war and produced a new definition of what they really sought in their investment approach; they sought to maximize the yield, defined as that rate of interest which equated the present value of all future returns (allowing for their respective probabilities) with cost.

So far he thought the approach which Pegler had substituted for that of his predecessor was quite sound, but from there Pegler had led them to the point at which they attempted to estimate in advance what that yield would be in the future on various hypotheses, or implicit hypotheses, regarding the future rates of payment on the securities they were considering buying. Next, there had been a paper by H. G. Clarke, who developed the idea and suggested that they should consider the standard error of the variables in their calculations. It seemed to the speaker that those writers had jumped too far for their theories to be used as a guide to future action or even as a rationalization of what was being done.

Pegler said, in effect, that if in looking at an ordinary share they said that the priority percentages, the asset backing, the economic prospects of the industry and so on were relevant factors, then they implicitly turned those factors into probable dividends over a future period of years and calculated the expected yield on the security. The speaker believed that the expected yield approach was the ideal approach if they had anything on which to work, but that they were deluding themselves if they imagined that that was what they were doing implicitly when using their cruder tools.

It seemed to him that when they produced an investment index they had two purposes in mind. The first was to investigate the correlation between movements in share prices and economic conditions and, if such a relation were established, to make such use of it as they might. Two approaches were mentioned in the paper. On page 333 there was the reference to Douglas, who concluded that share prices were related to the level of business activity—the progress of the trade cycle—equity prices following the cycle directly and debenture prices following the cycle inversely and with different tempo and intervals. If, therefore, an investor could judge the future trend of business activity, a sound investment policy could be pursued.

On page 358 they were told that 'L. and C.' stated that it was primarily concerned with the diagnosis or prognosis of the general industrial situation; that

was to say, they regarded the share index as a barometer of business conditions. There were two different approaches. They could either use share prices as a guide to what business conditions were going to be or use business conditions as a guide to how share prices would move.

There was, too, the difficulty which Douglas mentioned—the different tempo and interval. They could allow for the different tempo and interval if they were satisfied that they were following some regular pattern, but in fact the leads and lags seemed to vary widely as between business conditions and share prices, and they could be completely distorted by Government intervention. Any attempt to draw useful conclusions from movements of share prices and economic conditions since the war was hopelessly at sea because of Government interventions which affected movements of share prices—dividend limitation, differential rates of profits tax and other controls. All of those factors had made it difficult to interpret data relating to that period.

The other purpose of an index was to assist in the solution of day-to-day technical problems. The sections dealing with approximate valuations and checking portfolio performance, on page 337, could be accepted straight away; those techniques were well known and seemed sound. The authors then suggested using the Index to price new issues or to value unquoted shares. He thought that in considering such problems it was desirable to take as many comparable shares as possible which were already quoted in the market and try to get a rather finer approach to the question.

Then, on page 339 there was some discussion of preference shares in investment trusts compared with $2\frac{1}{2}\%$ Consols. He (the speaker) thought that indices would not be used much for that purpose, because the investor would be more concerned with daily variations and would look directly at the current yield of $2\frac{1}{2}\%$ Consols and at the margin which he had secured in the past. The index might, however, support his conclusions.

On page 347 the authors referred to the value of the Index in assessing the growth prospects of industries. In the main, they were concerned to argue that a high dividend ratio might denote an industry with above-average growth prospects. Emphasizing the word 'might', the speaker agreed with the authors, but thought that they were straining the point a little. The industry which was distributing a comparatively small proportion of its profits was providing itself with capital on attractive terms. Surely if they were looking to an assessment of the growth prospects of industries, it was more important to look at the particular industry, what it did, and what its markets were likely to be in, say, 20 years' time. On the list there were mixed together such industries as tobacco, cotton, and chemicals; but he was certain that the authors did not mean that they regarded a cotton share which distributed only a quarter of its earnings as more attractive than a chemical share which distributed half. The paper gave that impression; it said that the prime requirement in looking for growth prospects was that the earnings should exceed the dividends several times. He submitted that that was only one point and might not be the most important point.

On page 349 the paper discussed share prospects in 'real' terms. It would be expected that share prices and the cost of living would move in the same direction, but there again the authors seemed to be straining in trying to prove the degree of the correlation. It must be borne in mind that politicians influenced events. If the authors had stopped the comparison short about 1950, it would have looked very unhappy for share prices, but since then the shareholders had been allowed to receive a little more of what belonged to them. It seemed that

any attempt to derive a basis for future policy from that sort of comparison led to difficulties. He did not dispute that the ordinary share might well reflect the cost of living more than did gilt-edged stock; but he thought that the degree of the reflection was over-stated in the paper.

He was not satisfied with regard to the confidence index approach on page 340, because, the effects of a change in gilt-edged rates and of an increase in dividends having been eliminated, what was left was surely a measure of the extent to which there were more buyers than sellers. If that residual was called a confidence index, the inference was that it reflected the demand, whereas it in fact reflected the relationship between demand and supply. Considerable changes had been seen since the war in the flow of new issues of ordinary shares in the market. If there were a substantial change in the volume of new shares coming on the market, then the index did not reflect the confidence of investors in any way or even their enthusiasm for ordinary shares.

Turning, finally, to the construction of the Index itself, he agreed with the statement on page 369 that comparisons would be facilitated if identical industrial groupings were adopted by the Actuaries' Investment Index and the *Financial Times* Index. Whilst holding no particular brief for the *Financial Times* Index as compared with others which might be available, he would go further and try to co-operate as far as possible. The *Financial Times* had given the lead on the subject of earnings but plenty of scope remained for further work. He would have thought that in due course there would also be scope for further research on a question mentioned briefly—earnings on capital employed.

He was aware that the capital employed at written-down values was certainly not a homogeneous item throughout all balance sheets. Nevertheless, he thought that if people constructing the Index wanted a guide to the future it might well be for the meeting to say, 'Go ahead and look at some of the other aspects rather than devote too much time to a consideration of the best way to construct a price index of ordinary shares. If it is possible to sub-divide into groups, the more the better; and the further they can be linked with the standard industrial classification of the Central Statistical Office so that figures of employment and production in those trades may be examined, the better.' If future development moved in that direction, the Index, he thought, would become a more valuable aid to investors.

Mr C. M. Gulland, F.F.A., speaking as Chairman of the Faculty Section of the Investment Research Committee responsible for the Actuaries' Investment Index, said how greatly they valued the co-operation between the two sections of that Committee. They might from time to time hold different opinions, but in due course the differences were always resolved to their mutual satisfaction. He would also like to put on record how much the Committee valued the suggestions which had been made by the authors in their paper and by speakers in the discussion in Edinburgh for the improvement or extension of the Index. They would also welcome suggestions made in that evening's discussion, all of which would be given the fullest consideration when the construction and presentation of the Index next came up for review.

Mr L. Ginsburg, F.F.A., was particularly impressed by the introductory remarks regarding the impact of business conditions on share prices. It had been said that the essence of investment lay in successful timing, which was easy enough to expound but difficult to achieve in practice.

In Part I the authors had enumerated the phenomena which might have a profound effect on the timing of Stock Exchange purchases, but they had not developed that theme. Policy might determine the basic pattern of the investments—the proportions to be held in British Government securities, in debentures, in preference and ordinary shares, in mortgages and in property, and so on. If policy were to be implemented to maximum advantage, however, available resources should be concentrated on the most attractive investments for any particular moment of time. That led them to ask what economic indicators would help in deciding at what stage ordinary shares, for instance, might be regarded as dear and fixed interest investments as relatively cheap. When should they invest short and when long? It was easier to propound those questions than it was to answer them, and it was no criticism of the paper that, having brought the reader so far, the authors pursued that aspect no further.

Regarding the practical uses of the Index, a little experience showed that it had serious limitations in its current form when used for approximate valuation. The debenture index, being based on irredeemable stocks, was of no practical value for that purpose, the reference in that context apparently having slipped in through an oversight.

The preference index should give reasonable results, although, owing to time lags in altering the Stock Exchange Official List in the less active stocks, the securities constituting the Index were not fully representative, and their number could with advantage be increased. Possibly the grouping into low, medium and high dividend rates could be abandoned. The distinction between the investment rating of preference shares repayable at average market price and those repayable at par was very fine in practice and appeared to have little influence on prices.

Approximate valuation of quoted debentures and preference shares might be effected most readily by computing the unweighted mean price of the securities in the portfolio and comparing with the mean price at the last date of accurate valuation. Very small holdings might be disregarded. Fixed interest security values tended largely to move together and so good results were obtainable. Pricing might be facilitated by keeping the valuation schedules in the same order as the Stock Exchange Official List.

For ordinary shares the group indices produced a reasonable approximation when market conditions were relatively quiet. When the range of price movements was extensive, however, the implied assumption that an equal number of shares was held invested in each security was too wide, and the results should be treated with caution. For a similar reason the average yields given in the Index must to some extent be regarded as unrealistic. Those yields took no account of market or even nominal capitalization, and the assumption that equal amounts were invested in each security might in some cases produce results which were quite unreliable. In the oil group, for instance, the Index average yield for June 1955 was 5.33 %, to achieve which the investor would require to invest equal sums in companies ranging from Shell Transport, with a market capitalization of £405,000,000, to Apex (Trinidad) Oilfields, with a capitalization of £1,500,000. The weighted average yield was 3.6 %.

The approximate valuation of a portfolio of ordinary shares presented formidable difficulties, but at the cost of some effort good results might be obtained by computing $\Sigma n(a-b)$, where n was the number of shares in each security, a the current price and b the price at the last full valuation. Price differencing was easy, and since results were not required exactly, the calculation of $n(a-b)$

could be effected at a glance. The tiny holdings which crept into every portfolio might be disregarded. With experience, surprising rapidity was achieved and, of course, a high degree of accuracy was possible. The process need not be carried out more often than quarterly, but intermediate valuations could be estimated by use of the Index unless price movement had been extreme.

They were indebted to the authors for their Table 4, which showed the various industries graded in order of dividend ratio. Like the opener, however, the speaker thought that the statement that a high dividend ratio might denote an industry with above-average growth required some qualification. A high dividend ratio might represent only the effect of past expansion, on the cessation of which dividends were distributed on a more liberal scale. It might signify the end of a period of expansion rather than its beginning, and its manifestation was at once obvious to the whole body of investors. Growth required finance, and finance might call for conservatism in distribution of dividends. Thus, the substantial ploughing back of profits might be more significant of growth potentialities than their liberal distribution. If shares were then available on a high earnings yield basis, they might offer favourable opportunities for investment despite a low dividend ratio and a low current dividend yield. Increased demands for particular goods and services, if left freely to work their way through the economy, would generate expanded resources to meet those demands. Thus the economic data mentioned by the authors, such as production figures, employment statistics and wage rates, taken in conjunction with the annual statements made by Company Chairmen, were probably more indicative of growth than a high dividend ratio.

Table 5 placed the share and cost-of-living indices side by side and was of major interest. Its significance might be enhanced, however, by examining the performance of a portfolio of ordinary shares which increased every year through the purchase of an equal number of each of the shares constituting the Index. Based on the *Investors' Chronicle* index, a fund started in 1930 would by 1939 represent an average cost of approximately 106 per share against an average value of 107, and by 1952 an average cost of approximately 123 against a value of 132. By 1952 the Index of Retail Prices had risen to 216, so that as far as capital was concerned the fund had fallen far behind the cost of living and, indeed, after allowing for expenses of purchase, had just maintained its value. Of course, 1952 might be regarded as an exceptional year, possibly merely an incident in the long-term upward trend. By July 1955 share values had leapt ahead of prices, and the value of the portfolio of ordinary shares had been more than maintained in real terms, besides participating in the increased dividends shown in Table 6. Nevertheless, the experience was of relatively short duration and for a variety of reasons, until 1953, it might have been unsafe, despite the evidence of an expanding economy, to draw firm conclusions. Since July 1955 there had been a reversal of the general upward movement in share prices, but he would not like to prophesy what would happen next week, far less next year. It would, therefore, be all the more interesting to study the interplay of the political, economic and psychological forces which determined, however imperfectly, the relationship between share prices and business conditions to which the authors had referred in Part I.

Mr H. G. Clarke said that one of the first things he was taught in connexion with investment policy was to be very chary of jobbing backwards. He had always taken that to imply that the statistician and the stockbroker should be

kept at a respectable distance from each other. It would be understood, therefore, that he was doubly apprehensive when he read first the title of the paper and then the names of the two authors who had combined to produce it. Nevertheless, he had found it an interesting paper which was well worth the study of those who had to do with investment matters.

In introducing the paper Mr Haycocks had implied that the authors might have some reservations about Part III. They had, however, introduced a modified form of expected yield which they termed normalized expected yield. Apparently, normalized expected yield was intended to allow for the fact that varying degrees of reliability attached to the expected yields of various securities. In his own paper in 1954 he had suggested as a first investment objective that the aim should be to secure maximum expected yield with minimum error. That might be alternatively expressed as maximum expected yield with maximum reliability, and it would appear that the authors' normalized expected yield represented an attempt to combine into one conception the two conflicting components in that objective.

At the end of the first paragraph of the section headed *Expected Yield*, the authors stated that they confined their theory to institutional investors; yet in the next paragraph they seemed to define normalized expected yield in such a way that it must apply to an investor faced with the selection of one investment only. That must be so, because the authors claimed that two securities having the same normalized expected yield were equally attractive to the investor, whereas in the third paragraph of the section they proceeded to argue—in his view rightly—that that might not be the case, and indeed was unlikely to be the case, when the investor was not concerned only with the selection of a single security. Finally the authors concluded that an investor does not simply maximize the normalized expected yield. It appeared, therefore, that no simple investment objective could be derived from the conception of normalized expected yield, and it was questionable whether it had any value. A fuller explanation of what the authors had in their minds might show that he was being unfair, but it seemed to him of some significance that throughout the remainder of Part III of the paper, although expected yield was referred to quite frequently, there was no further mention of normalized expected yield. For his part, he still held the view that in the field of investment, as they knew it, there was an unavoidable conflict between yield and security, which it was important to recognize but which could not be resolved, and any attempt to resolve it might prove misleading rather than helpful.

In the remainder of Part III there was an interesting illustration of the way in which recorded statistics, such as those available in the Actuaries' Investment Index, might be utilized to give some idea of the relative yields which had been experienced in the past on different classes of investment. Those yields might, of course, be no reliable guide to future expected yields, but at least they added to the background and experience on which the investor must rely to some extent in the selection of his investments.

It seemed to him that in the second paragraph on page 355 the expressions 'dividend yield' and 'earnings yield' had not been adequately defined. There was a statement half-way through the paragraph that, if profits were expected to grow steadily and surplus earnings were considerable, the expected yield should be somewhere above the dividend yield and below the earnings yield. It appeared to him that that was valid only if the dividend yield was based on current dividend alone and the earnings yield was based on current and future

estimated earnings. If the earnings yield were based on the current earnings alone, which was the normally accepted basis, it was quite possible for the expected yield to exceed the earnings yield.

In Section (2) of Part II, entitled *Forecasting*, the authors set out a technique, on pages 343, 345 and 347, which they described as a forecasting system. It was dealt with so fully that the authors seemed to be giving it their blessing, yet they were at some pains to avoid expressing an opinion on its validity. He, however, did not accept it as a valid forecasting system. He would perhaps accept it as an interesting and useful method of recording the past experience of ordinary shares, but even so it would, in his view, be of considerably greater value if it included a record set out in chronological order alongside Fig. 1 and Fig. 2 showing the major economic and political events which occurred during the period covered by the investigation. If that were done, it would show clearly that, taking any particular point of time during the period, the course of the charts following that point of time was not really determined by their course immediately before but by the economic and political events which occurred on and around that point of time.

There was one aspect of framing investment policy on charts to which he wished to refer. If a sufficiently large number of investors were to follow a system such as that which the authors had described, that fact in itself would tend to make the system appear to work, at least in the short run, merely for the reason that if investors followed a rising or falling market they might themselves make the market rise or fall further. In such circumstances, if a forecast indicated by the charts turned out to be correct, the movement of prices might tend to be overdone, and if the forecast turned out to be wrong the subsequent reversal of prices might be all the greater. It could therefore be undesirable from the point of view of market stability to encourage investors merely to follow each other like sheep.

Nevertheless, he did not regard charts as having no value in the context of making investment forecasts. There were, he thought, two ways in which they could be useful. An investment forecast should first of all be based on a consideration of economic and political factors. When a forecast had been made in that way, it might then be useful to see whether or not the recent trend of the charts was such as to confirm the view, which could well be the case if other investors had already been thinking on similar lines. A forecast, of course, was not necessarily wrong if it was not confirmed by the charts, but at least an investor would have greater confidence in his forecast if it was so confirmed.

The other value which he thought might be derived from a system of charts, provided it was studied in conjunction with the additional information he had suggested, was that it could give the investor a broad view of the experience of the past and thereby build up his investment background. In the selection of investments the investor relied to a considerable extent on his past experience. So, also, when faced with the problem of investment forecasting, an investor with a wide experience of the past would be the better equipped. Investment statistics such as those described in the paper could be of service to the investor, provided that their limitations were fully recognized and that they were employed only for appropriate purposes.

Mr L. G. Hall said that the authors had suggested that the fall in investment by private persons and the great increase in institutional investment had tended to concentrate investment in fewer and more expert hands—that was to say, in

the hands of investors who demanded and were able to judge the relevant financial and trading information about the concern issuing the securities. All that was true, but he was not sure that it had led, as it should have done, to increased stability in the equity market. Stocks were passing more and more into firm institutional hands, so that the market was narrower than it used to be. A temporary loss of confidence, bringing a few small sellers into the market, caused equity prices to fall heavily. Jobbers, often through shortage of capital resources, were unwilling to take stock on their books, and the private investor—or the private bargain-hunter, if they liked to call him that—was no longer strongly in evidence. Short-term uncertainties caused sharp falls in share prices and provided outstanding buying opportunities for the long-term investor of large and growing institutional funds, who was in a supremely favourable position to take advantage. Yet it was his impression that most institutional investment managers—restrained no doubt at times by the general lines of policy to which they were directed to conform—were not willing to take advantage of the long-term chances offered by short-term uncertainties; instead, they waited until the market had made a strong recovery, short-term confidence was high again, and lines of stock were difficult to buy.

He had had access to some interesting figures which, although they related to the past, appeared to demonstrate in no uncertain fashion the attractions of equities for the long-term investor. Notional funds of £1 million each were invested on 1 January 1919 and each succeeding 1 January in the 30 stocks constituting the *Financial Times* Industrial Ordinary Share Index. Two calculations were made. In one, the income was not accumulated. In the other, the gross income was re-invested each 1 January. By the end of 1955, when a total of £37 million new money had been invested, the capital value of the first fund—with no reinvestment of income—was £90 million and the gross annual income was £4.3 million, or 11.6% on total cost. The capital value of the second fund—with reinvestment of gross income—was no less than £299 million, and the income was £13.7 million. If the second fund had been invested in 2½% Consols instead of equities, it would have accumulated—again with reinvestment of gross income—to a capital value of £70 million instead of £299 million, and the income would have been £3 million instead of £13.7 million. As the authors had emphasized, all past experience suggested that in the long run a well-spread and well-managed equity portfolio should provide a much higher return than a portfolio of fixed interest investments.

Mr F. W. Bacon said an unkind critic might say that the paper was notable for its advocacy of two things—the Actuaries' Investment Index and investment in ordinary shares. He was not an unkind critic, and he recognized that many other valuable points were made in the paper. He therefore hoped that he would be forgiven if he concentrated on one point where he found that he could not go all the way with the authors—namely the question of investment in ordinary shares. He was not opposed to equity investment as such; it was rather that the arguments put forward for such investment were arguments which he did not find entirely convincing.

There seemed to be two main reasons advanced in the paper. The first was inflation and the need for the preservation of real values. Everyone would agree that if inflation were to continue indefinitely, then ordinary shares were an obvious choice, and no index numbers were necessary to tell them that. But the real problem, to his mind, was whether it would indeed continue or whether

the current period was exceptional. From that angle it was worth while taking a quick glance back at history. Looking at the last 150 years, it was found that wholesale prices fell more or less continuously throughout the nineteenth century; in fact, in 1900 they were about half what they were in 1800. Even after the first world war, in the 1930's, it was seen that prices were lower than they were in the 1830's. The periods when prices had risen rapidly were periods of war or preparation for war, and it was to be hoped that that might be regarded as exceptional. He therefore wondered whether they were unduly influenced by the fact that they were still living in a period of post-war inflation and 'cold war'; it was worth remembering that those circumstances might change in the future.

He agreed that even without inflation, provided there was full employment, the national income should grow and equities should benefit. Again, another doubt arose. He recalled what Lord Keynes had told them before the war about a falling rate of profit on new investment if there was full employment. At the present time, in what was described as a 'capital-hungry' age, in which the demands for capital seemed to be insatiable, such circumstances might seem a long way off, but he felt that they could not be ignored entirely. He put those points forward not to discourage investment in equities but in order that the risks involved should not be overlooked.

The other argument which the authors gave for investment in equities was that by spreading funds over a wide field it was possible to increase the average expected yield as compared with gilt-edged securities, with little loss in the degree of reliability. He was not sure that he altogether followed them there. He would have thought that the more risky the investment, the higher the yield, and the less the degree of reliability. On the authors' argument better results would therefore be expected from spreading funds over high-yielding than over low-yielding investments, because of the greater scope in the former for increasing the degree of reliability by averaging, but in a subsequent section the authors produced evidence to show that over a limited period that had not been the case.

He wondered whether spreading would necessarily increase the degree of reliability of a group subject to fluctuations affecting all the members of the group in the same way, such as economic factors affecting the whole economy. If that were the case, it was an argument for investing part of a fund in fixed interest securities on the ground that those securities were likely to be affected in the opposite direction from ordinary shares by such factors as trade depression. His conclusion was that it was desirable that an important part of the fund should be invested in ordinary shares, but such investment should not be indiscriminate, and timing and selection were all-important. On that point he wanted to make a reference to Table 6. That table should be supplemented by another table showing the movement of share prices, and he also wondered whether the table was affected by the fact that 1930 had been chosen as the starting point.

It was perhaps interesting to note that during the period 1928-38, equities fell from 115 to 100 while the fixed-interest index rose from 78 to 100, which again emphasized that timing was important. The essence lay in knowing when to buy and when to sell. The best time to buy was probably when everybody else was selling, and the best time to sell was when everybody else was buying.

Mr J. B. H. Pegler wished to refer to two points which interested him particularly. One was the discussion of yields in Part III. As was mentioned

in the paper, he had tried, in his own paper in 1948, not to produce anything new but to summarize what he thought investors of life office funds—or at least the more intelligent of them—were trying to do. It had been clear to him that they were not trying to do what most of the standard papers at that time defined as the aims of investment policy. His own attempt had been very loosely expressed, largely because, as he would honestly confess, he had been by no means clear exactly what the answer was. He did not think that either Mr Clarke in his paper of 1954, or the authors with their 'normalized expected yield', had quite got the answer, although each paper had made some progress.

He supposed that it was rather reprehensible of him to criticize the authors if he had no constructive comment to make, but he had to admit that so far inspiration had eluded him. The nearest he could get was a criticism on similar lines to that of Mr Clarke. He thought they were all agreed that life office investors, like many other investors, were aiming to maximize expected yields, although the precise assumptions made in determining the probabilities entering into the estimation were not the same for all. They were agreed, too, that the reliability of their estimates had a bearing on the choice of the security, or on its attractiveness to the investor who was thinking of buying it.

His difficulty over the authors' approach arose from the feeling that the importance of the reliability factor depended not on the security concerned alone, and not on the investor's needs alone, but also on the make-up of his portfolio. If an investor were contemplating putting all his money into one security, the reliability factor would be very much more important than if he proposed to spread his money over a fairly wide range, although he (the speaker) was not sure that however wide the spread it was altogether possible to eliminate the influence of reliability.

The authors defined the normalized expected yield of a given security by reference to a degree of reliability k such as to make the investor indifferent to investing in the security or in one with a degree of reliability of unity. That definition, he thought, needed tightening up, although he was afraid that he could not tell the authors exactly how to do it.

The opener had questioned whether any form of assessment of expected yield, whether normalized or otherwise, was made, even implicitly. He (the speaker) suggested that whether or not some sort of calculation of that kind was made in practice, the conception had some value. How otherwise was it possible to describe the kind of process which an investor went through in comparing the yield on, say, $2\frac{1}{2}\%$ Consols with the yield on a fixed interest share which had very poor cover and was issued by a company in an unpopular industry?

In the section dealing with the performance of shares with different dividend yields, it was interesting to see the better performance of the low-yielding shares, although an examination of Table 8 suggested that the evidence was by no means overwhelming. The test which the authors applied was perfectly fair, he thought, but it could nevertheless be argued against them that after the period of five years covered by the test the low-yielding shares were still giving the lowest income, and if it was income they were after that was a serious disadvantage. It was interesting to see that the least successful class appeared to be the medium yielders, and the authors' hint at the reason for that was probably right. It was because the securities in that class were much more 'board-worthy' than others. Boards of directors were inclined to fight shy of both high- and low-yielding shares, and the greater popularity of the middle classes of share inevitably depressed their yield.

Whenever there was a discussion on investment matters, something was usually said about investment being not a science but an art. What was meant was, he thought, that not enough was known about the subject of investment and, in particular, the requirements of various types of investor and the real value of various types of assets in which they might invest, for it to be possible to apply scientific principles alone. So far he agreed with the critics, but if they went on to criticize the authors, and for that matter Clarke and Pegler before them, for trying, not always successfully, to apply scientific methods to what was essentially an art, and if they said the whole attempt was misguided, he must disagree most emphatically.

Actuaries had long been used to applying scientific principles in dealing with the liabilities of a life office, and the fact that their estimates would not always be exactly realized in practice was not a sound reason for not making estimates at all. The fact that data were often indifferent or unreliable was not a reason for refusing to make the best use of the data available. They did their best and drew what conclusions they could—and, what was more, acted upon those conclusions, always bearing in mind the limitations of their conclusions arising out of the limitations of the data. He therefore found it difficult to understand why it should be suggested that any attempt to use scientific principles in dealing with assets was misconceived. He thought (unlike Mr Clarke) that it was possible to examine statistics and draw conclusions from them without jobbing backwards. He therefore applauded whole-heartedly what the authors had tried to do and hoped that others would pursue the same line in the future.

Mr A. G. Ellinger (a visitor) had been stimulated by Mr Pegler's remarks, although he did not think that the position was quite as he saw it. It must be remembered that there was still much investment, possibly very marginal, done by people who had never heard of the Institute of Actuaries, and sometimes those people exercised a considerable influence on the market. Further, their approach was purely 'artistic', although it might be that they were bad artists. On the other side, there were the actuaries, whose approach, when they got away from their boards of directors, was presumably purely scientific, which was as it should be. But a peculiar position was arising, in which there was the prospect of the artists pulling one way and the scientists pulling another; and, while the scientists were pulling hard the other way, another group of scientists from the same stable ran up and said 'You must drop that rope; we have a better rope on the other side of the room.' He strongly suspected that the attitude of life offices and pension funds to equities changed rapidly when the supply of fixed-interest securities was constantly enlarged at a rate of interest of which most of those present had never dreamed. It had a most unscientific effect on the whole structure of the Stock Exchange.

Eventually, of course, if modern trends continued, investment would become purely scientific and there would presumably be no artists from whom the actuaries could buy investments. What they would do then he did not know, but he thought that that situation was a long way off and he suspected that while one sort of artist was disappearing another sort might be growing up. There seemed to have been a considerable change in the pattern of holdings since 1950 and the gloom which used to be spread by Mr Cuthbertson, who said there would soon be no equity investors left, had become unfashionable. The degree of interest in the industrial share market, which was extremely low in 1949, had risen to a much higher level. He suspected that that was partly due to the fact

that some people were saving money and, instead of buying life assurance, were investing it in ordinary shares. There might be an increasing number of new artists able to pull on the other end of the rope.

He turned, next, to the question of confidence; apparently he should not have called it confidence. He claimed to have invented the word because he got hold of something and wanted to name it! It had been invented, in fact, because Mr Dalton was manipulating the whole Stock Exchange. He (the speaker) had wanted to see what was going on when Mr Dalton's influence was removed from the Stock Exchange, and he had gone through some thoroughly unmathematical calculations to produce what he wanted. He had drawn a chart which went up and down nicely. In his view, it had something to do with confidence, and he had called it a confidence indicator. Whether it indicated confidence, or lack of confidence, or anything else, he did not know, but it was an empirical index of the hypothetical investor; it was the way he felt inside himself. If he felt good inside himself and if a company declared that it would offer new shares to shareholders below the market price, he rushed in and bought the shares, and, because of the rights issue, the price rose. That had happened recently with Birmid, an excellent company, no doubt, which did not seem in need of new capital. On the other hand, when Associated Electrical Industries announced that it would make a rights issue, everybody was extremely glum and rushed to sell shares, which went down and down. That was because confidence was going down at the time of the A.E.I. issue and going up at the time of the Birmid issue.

He thought that the behaviour of the market when rights issues were being made did nothing to invalidate what had been said about the confidence indicator. What the confidence indicator tried to examine was what sort of change was going on between the investor's likes and his dislikes, between ordinary shares and fixed-interest stocks. Sometimes he felt more inclined to one and less inclined to the other. It might be a matter of what he disliked less rather than what he liked more. That was something which was always changing, and it was reflected in the yield ratio—the ratio of fixed-interest yields to ordinary share yields. There appeared no reason why the ratio should not move a long way in either direction, but for a long time it had not moved far out of a fairly narrow range. It had been suggested in 1954–55 that the ratio was going to move into an entirely new region which had never been approached before, but something went wrong in the Summer of 1955 and the region was not reached. The gap between the two was still historically rather narrow.

The question of rights was surely elementary and simple; companies which were issuing rights were selling shares and increasing the supply. When the supply was increasing, unless there was a corresponding increase of demand, confidence was less. In the case of Birmid, the increase in supply had produced an even greater increase in demand, and confidence had risen. But the theory of confidence could certainly take on board any burdens imposed on it by the issue of rights and could not be invalidated on that ground, although many other excellent things could no doubt be said against it.

Mr J. R. Hemsted said he was interested in the idea of expected yield. Although the paper was long, he would have liked to see a little more research on that subject. It seemed that they were in a position similar to a meeting 100 years earlier, when actuaries were asked, 'What is the expectation of life?', and one after another they had replied, 'We do not know what mortality will be

in the future and cannot work it out'. They had seen in the paper that investment should be done on the basis of maximum expected yield and, a little earlier, the statement that it was impossible to say what that was. He thought that they should work out what it had been in the past.

They had been able to work out mortality in the past and had used it to estimate mortality in the future, and they ought to get down to finding what the 'mortality' element in security yields had been in the past and use it for the future, guided, of course, by current economic conditions. For example, 30 years earlier there had been a preference index, and presumably it gave a yield higher than could be obtained on $2\frac{1}{2}\%$ Consols. It might be said that the market was offering the extra yield to compensate for the increased risk. It should be a fairly simple matter to work with the securities in the preference index over the past years and see whether the market valuation of that extra risk had been justified. An expected yield for each year in the past could be produced, with possibly a continuous 'mortality investigation' on security yields. The same thing could be done with ordinary shares. It would take into account the inflationary element.

A small pointer to what could be used immediately emerged from the fact that dividends had been increasing by roughly $2\frac{1}{2}\%$ per annum. When looking at an ordinary yield, they should automatically add another $2\frac{1}{2}\%$ to allow for the over-all inflationary trend, producing a yield of 7 to 8% to compare with the fixed-interest yield.

He had been pleased to hear Mr Pegler suggest that something might be done on those lines and he was convinced that there was much to be gained from the experience of the past. All they needed was somebody with time to do the research and present the results.

Mr L. Brown, in closing the discussion, acknowledged Mr Gulland's kind remarks about the co-operation between the two sections of the Committee and assured him that they had found that co-operation very helpful.

When a review was made of a major index, it was necessary for the Committee to ask themselves a number of questions. First, was the Index serving a useful purpose? Unfortunately, not many visitors had spoken in the discussion, but the interest shown in the paper and the fact that there were enough subscribers to pay for the cost of producing it suggested that the Index was probably worth while.

Secondly, was the construction of the Index satisfactory? He would not discuss the merits of the various methods of construction dealt with in the paper and the two discussions, for it was clear that the merits of each method would vary according to the purpose for which the Index was to be used. Their aim was as far as possible to produce an index for general use, and, since he thought he could claim that there had been an absence of serious criticism of the method of construction, that supported them in their belief that they were working on lines generally acceptable to members.

Should there be any changes in the method of presentation or was any further service desirable? That was a matter for the most careful consideration, and the meeting could rest assured that the Committee would study as fully as possible the suggestions which had been made. It was their desire to give the service for which there was a demand, provided it was statistically sound and practicable in operation, bearing in mind the over-riding need of continuity in the Index.

The opener had requested further information and investigation on earnings yield and earnings on capital employed. He therefore confirmed the invitation in the paper to any members who were prepared to enter into further research on those subjects to make use of the index figures and records available in Mr Haycocks's office.

The authors had a long section dealing with the uses of indices generally. They had recorded various examples of how they could be used and had developed numerous suggestions as to their possible use, not merely in respect of the Actuaries' Index. They had demonstrated, he thought, that considerable help could be obtained from the study of index figures and charts in ordinary investment work, but whether it was possible to forecast the movement of prices in the market by such study he was somewhat doubtful. Clearly, there were certain special uses for which an index was appropriate, such as approximate valuations and so on. The study of index figures could throw up anomalies in the market or indicate trends and so draw attention to points which might repay examination and investigation.

He supported Mr Clarke's general remarks on the value of index figures. To him, generally, they were extremely helpful and assisted in keeping the subject in perspective, in correcting and confirming general impressions of what had been happening in the markets and in comparing current conditions with past history, so as to help produce that balanced judgment which was essential in investment matters.

He referred, next, to the section on expected yields. The authors had carried a stage further the general discussion on that subject by developing the idea of the normalized expected yield, and he thought he agreed with that section of the paper. He would say that if the normalized expected yield of a security was equal to the gilt-edged yield on a comparable security at that time, he would not buy the other security; he would want a bigger yield. It did not matter whether the reason he wanted a bigger yield was because of the difficulty in assessing the reliability factor or because he wanted to be sure that he was on the right side in his estimates of the risks, or simply because he wanted a higher yield if he took an extra risk; they came to the same thing in the end.

Some of their City friends have been known to smile when the subject of expected yield was mentioned, and occasionally actuaries had their legs pulled on the subject; he sympathized with that view because he had never seen an expected yield in practice. Possibly one day they would discuss a paper on the methods of calculation of expected yield, or perhaps it called for a mathematical note in the *Journal*—but he doubted it.

Referring to Mr Pegler's remarks, he (the speaker) did not know whether investment was a science or an art or just plain business, but he supported Mr Pegler in saying that the development of the conception of the expected yield was worthwhile. Frequently in investment matters it was necessary to tackle a problem by going back to first principles, even if sometimes they had to invent new first principles.

The analysis involved in the conception of the earnings yield in focussing their minds on what were the essentials of an investment policy was extremely valuable, and it helped them to put the risks involved in different types of investment in proper perspective. Mr Ellinger had referred to the confidence indicator, and he thought there was something quite useful in that; it was simply a relationship between the yield on ordinary shares and the yield on Consols, and the movement of that ratio could be indicative and helpful in judging the

position. He was not sure, however, whether he agreed with Mr Ellinger's comment on the evidence from the market of confidence when he referred to rights issues by Birimid and A.E.I. He thought the important practical point was that the A.E.I. issue was one of £24,000,000 whereas the Birimid issue was quite small.

Finally, he spoke for the Committee in expressing their gratitude to the authors for their work and for the discussion which had arisen from the paper, both of which would be extremely helpful to the Committee.

The President (Mr J. F. Bunford), in moving a vote of thanks to the authors, said that the Joint Investigation Research Committee of the Institute and Faculty had a quinquennial review ahead of it and the paper had been deliberately laid before both the Faculty and the Institute in order that the help and guidance of the members of those bodies might be available to the Committee in reaching decisions arising out of the review. Both the authors had had a considerable part to play in the research connected with the Index.

He thanked Mr Cottrell, who had opened the discussion, and all who had taken part in it, particularly Mr Ellinger for his entertaining and provoking contribution. They had had a lively discussion. During the earlier discussion at the Faculty he had noted considerable discussion about the rival merits of arithmetical and geometrical indices. At the Institute there had been no mention of that point.

He thanked the authors for having instigated a most interesting discussion.

Mr J. Plymen, in reply, said that the authors had been criticized because of various detailed investigations outlined in the paper concerning the growth of industries. The authors had fully expected criticisms of that nature, as had been explained in the paper and in the introductory remarks. Those investigations had been deliberately provocative and had been designed as 'kite-flyers' to stimulate the interest of the profession. None of the investigations outlined in the paper was intended in any way to be a full-scale study. The authors had just made some explorations.

One criticism had been made about the various statistical investigations; members had said that they would much prefer to use their own economic judgment. He did not disagree with that, but he suggested that they should use statistical studies to fortify that judgment.

There had been some comment about the question of the correlation between the Index and the cost of living. Obviously the share index fluctuated widely and, according to the period they chose, they got a different result; but the authors contended that over a long period the correlation between the two was considerable. Possibly they might have put a slightly different emphasis on that part of the paper. Figures for wage rates had been shown against the Index and there were reasonable economic grounds for suggesting that share prices and wage rates might move together. It was certainly more or less common current political thought that increased productivity should be shared between capital and labour. Possibly over a very long term the association between wage rates and the share index might be closer than between the cost of living and the share index. That was a subject which called for further research.

Mr Bacon had commented on the cost of living having fallen between 1830 and 1930. Certainly, if the cost of living fell over that period, wages rose. Although he had no figures with him, he thought it would be found that wages

in 1930 were about four times wages in 1830 as a result of increased productivity, and he suggested that, if share prices could have been studied over the same 100 years, a somewhat closer association would have been found with the increasing wage rates than with the decreasing cost of living. He had some figures available for a share index on an arithmetical basis, compared with wage rates from 1870 onwards, and the correlation between the two over the 80 years was remarkably close; the two charts started off together and finished together, with the usual fluctuations in between.

He had been delighted by Mr Pegler's new adjective for medium-yielding stocks—'board-worthy'. That was particularly worthy of note and might prove a valuable result of the discussion!

Mr Clarke had been rather critical of what he called jobbing backwards, but he (the speaker) found it difficult to see how, when they were thinking of investment matters, they could overcome that difficulty. If they studied what happened in the past, they were told that they were jobbing backwards. But what had they done when they produced a mortality table and had to underwrite life assurance? Were they to be accused of jobbing backwards in that case?

Mr C. D. Rich has written:

I am sure that, in referring on page 366 of the paper to 'Rich's investor', the authors do not wish to imply that I recommend the method of investment followed by this individual.

The purpose of my note in Vol. 74 of the *Journal* was to draw attention to the fact, which so far as I know had not previously been grasped, that the geometric mean of a number of security prices does actually represent the result of a theoretical method of managing an investment portfolio. Its uses as an index cannot therefore be criticized as purely arbitrary. It is interesting also that the use of the geometric mean of prices for a price index is consistent with the use of the arithmetical mean of yields for a yield index.

As will be seen from the paper, the method of investment which underlies the 'geometric mean' index will not produce very satisfactory results for the investor. It involves 're-equalizing' the amounts invested in the various securities whenever there is a relative change in prices. Actually the investment manager who adopts an entirely inactive policy will in general achieve better results than this.

Unfortunately, however, the person who just 'sits on' his investments will lose his advantage if, for example, the price of one of the securities, having risen, falls back again to its original value. This is where the 'formula plan' referred to by the authors will, if applied to a portfolio of securities, help the investment manager. Under this plan the money values of the securities are equalized periodically. The old problem of whether to 'take one's profit' or to 'let it run' is really a problem of the institutional investor just as much as of the speculator. By suitable choice of formula the investment manager can hope to perpetuate the benefit of a rise in price of one security relative to the others.

The authors suggest that the equalizations of the portfolio should be carried out at specified intervals of time. I would, however, suggest that it would be better to carry out equalizations whenever the relative changes in prices of the securities pass a certain point. Thus, in times of Stock Exchange activity, alterations in the investments would be made more frequently than in times when prices are idle. A workable rule might be that, whenever the price of one of the securities in the portfolio rises more than, say, 10% relative to the mean then

the extra 10% should be realized, and the proceeds reinvested elsewhere. Such a rule would also, of course, be a useful automatic method of limiting the proportion of the fund invested in any one security, which many people will probably consider desirable.

Finally, the question arises, what relation the use of the geometric mean as an investment index has to all this. The answer, to my mind, is that the geometric mean index is very useful if regarded as a 'pessimum'. The investment manager whose investment portfolio has fared worse than the geometric mean can be fairly certain that his policy has been at fault.

The authors subsequently wrote:

In his remarks introducing the paper Mr Haycocks stated that the authors were not satisfied with the section on 'expected yield'. The concept of 'expected yield' is only one of several that would be required for a theory of investment and to deal with this subject fully would require another paper. The Actuaries' Investment Index does include figures of average yields and it was felt that something should be said to connect this information with the ideas about expected yield which had been put forward by Pegler and Clarke. It is clear from the discussion that some investors do not find any value in this concept. The authors believe, however, that such investors must have in mind some kind of expectation which is very closely related to it. There is also some disagreement among those who use the concept of degree of reliability or standard error. It is almost as if an investor made a hypothesis about expected yield and acted on it. Like the scientist he finds it difficult to explain how he formulated the hypothesis. Unfortunately investors do not appear to make systematic tests of their hypotheses. They do not keep records of their decisions and the grounds for them.

In the introductory remarks it was mentioned that Mr Roy had used coefficients of correlation as well as expected yields and their standard errors in estimating a 'best' distribution of assets. Since the date of the discussion on our paper, an article has appeared in *J.S.S.* 13, 260, by S. P. L. Kennedy and R. C. Howroyd, dealing with this problem. Their method is very similar to that used by Mr Roy. It is clear from Mr Bacon's remarks that this approach is realistic. The expected yields are not independent since there are individual risks special to individual securities and group risks applicable to groups of securities. In so far as there is a general risk applicable to equities as a whole, Mr Bacon is correct in saying that this is an argument for including some fixed-interest securities in a portfolio. The spreading of a fund over equities only does not reduce this very general risk.

In response to Mr Bacon's remarks on the history of living costs, etc., over the last 150 years, we give below figures for cost of living, wages, and share prices at decennial intervals from 1810 to date. Naturally, indices covering such a long period must be interpreted with considerable reserve, because a number of different series have to be employed and several 'link ups' have to be made. In particular, the share index before 1850 is based on limited data, owing to the scarcity of quoted shares in those days. Nevertheless, in selecting the indices and linking them up, we have paid careful regard to the scope of the different series, and we feel that the over-all result is a reasonable indicator of the general trends.

At the beginning of the period, living costs were high, consequent on the inflation caused by the Napoleonic wars. Subsequently, the trend of living costs was downwards until 1930; despite this long-term deflationary movement, wage

rates increased almost continually, being some $3\frac{1}{2}$ times higher in 1930 than 100 years before. Over the same period, the share index increased about four times, demonstrating the long-term growth of ordinary share prices, consequent on increased productivity and industrial development.

Year	Price Index	Wage rates	Share Index
1810	171	27	23 (1811)
20	124	28	22
30	98	26	26
40	105	25	27
50	77	26	24
60	95	30	25
70	92	35	36
80	84	38	38
90	69	43	37
1900	72	47	55
10	75	47	51
20	242	144	89
30	93	95	96
40	117	112	81
50	185	186	141
55	242	254	250

Lord Keynes's remarks about the falling rate of profit on new investment in conditions of full employment were made during the period of mass unemployment in the 1930's. Since then, technological advances and rising living standards have enormously increased the need for capital, whilst social changes have reduced the propensity to save. Consequently, we would suggest that even if there were a considerable cut in defence spending, it would be a very long time before capital requirements were satisfied and the rate of profit on new investment began to decline.

We agree with those speakers who emphasized the timing of the purchases and sales of investments. Rules, such as those of formula planning, are designed so that the investor can avoid having to make these decisions and yet obtain a little of the benefit of correct timing. Timing is a matter of judgment based on experience and all the relevant evidence. The past movements of index numbers form only some of this evidence.

Mr Hemsted suggested that the results of preference share investments made in the past should be studied, calculating the losses, etc. due to defaults, and obtaining the realized yield. We cannot, of course, reproduce the 'expected' yield that investors required from their securities years ago, so that it is necessary to assume that the yield on the market value represented the average of investors' 'expected yields'. Clearly, the technique used in Table 7, comparing the performance of high-, medium- and low-yielding ordinary shares over a period, might well be applied to preference shares. Probably a similar result might appear, in that the medium class might be more 'board-worthy' and hence stand at relatively higher prices.

In practical investment work, careful consideration is usually given to the proportions of ordinary shares, preference shares, debentures, etc. Deciding on these broad groupings is, however, only a part of the programme. The next stage is to select the securities in each class, deciding whether they should be high, medium or low yielders, or an assortment of each class. This is a subject on which little guidance other than individual prejudice is available.