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The ARC seeks to deliver research programmes that bridge academic rigour with practitioner needs by working collaboratively with academics, industry and other actuarial bodies.
The ARC supports actuarial researchers around the world in the delivery of cutting-edge research programmes that aim to address some of the significant challenges in actuarial science.

Effect of Size of Exposure on Parameter Estimates and Correlations

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Background and Motivation

- Most mortality models are designed for large population size, e.g. England and Wales
- Actuaries are interested in modelling relatively much smaller population, e.g. Pension scheme
- Investigate how population size affects the accuracy of parameter estimates
- Mortality model, e.g. M7

$$\operatorname{logit} q(t,x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \kappa_t^{(3)}\left((x-\bar{x})^2 - \hat{\sigma}_x^2\right) + \gamma_{t-x}^{(4)}$$

- Sampling variation in death number, $D(t,x)$, leads to noise in $\kappa_t^{(i)}$, hence in drift μ_κ, variance σ_κ^2 of random walk

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Method, $\boldsymbol{\theta} = (\boldsymbol{\kappa}^{(1)}, \boldsymbol{\kappa}^{(2)}, \boldsymbol{\kappa}^{(3)}, \boldsymbol{\gamma}^{(4)})$

$E^{EW}, D^{EW} \rightarrow \hat{\theta}^{EW}$

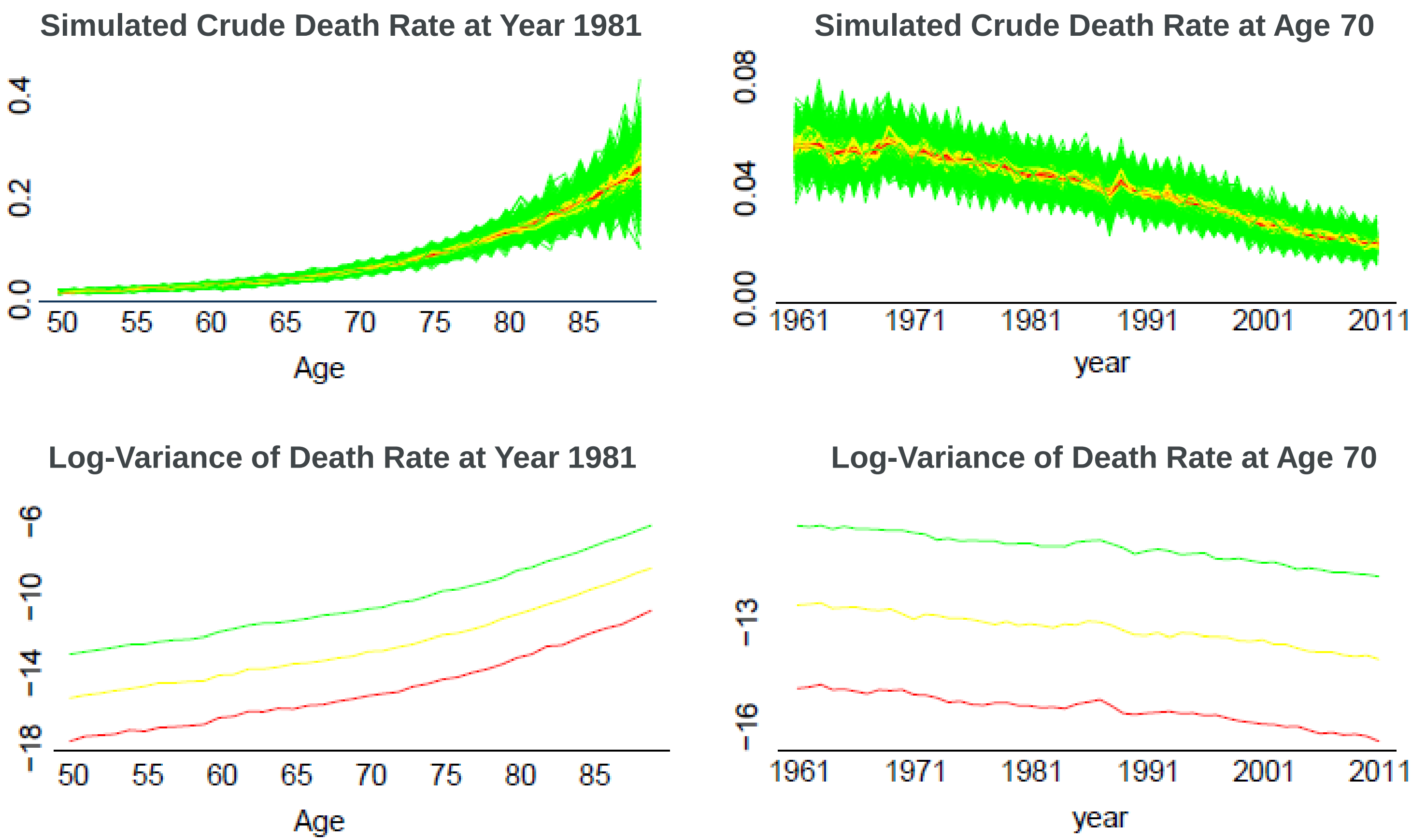
$\hat{m}^{EW} \rightarrow E^{w} = w \times E^{EW}$

$D^{w} \sim \text{Poi}(\hat{m}^{EW} E^{w}) \rightarrow D^{w,j}$

$\tilde{\boldsymbol{\theta}}^{w,j}$

- England and Wales(EW), Male, age 50-89, year 1961-2011
- $w = 1, 0.1, 0.01$
- $j = 1, \dots, 1000$, independent scenarios
- Fit M7 to $D^{w,j}$, E^{w} of each scenario

Simulated Crude Death Rate $m^w = \frac{D^w}{E^w}$



$w = 0.01$ $w = 0.1$ $w = 1$

Figure: The simulated death rate(upper) and its log-variance(lower), at year 1981(left), age 70(right)

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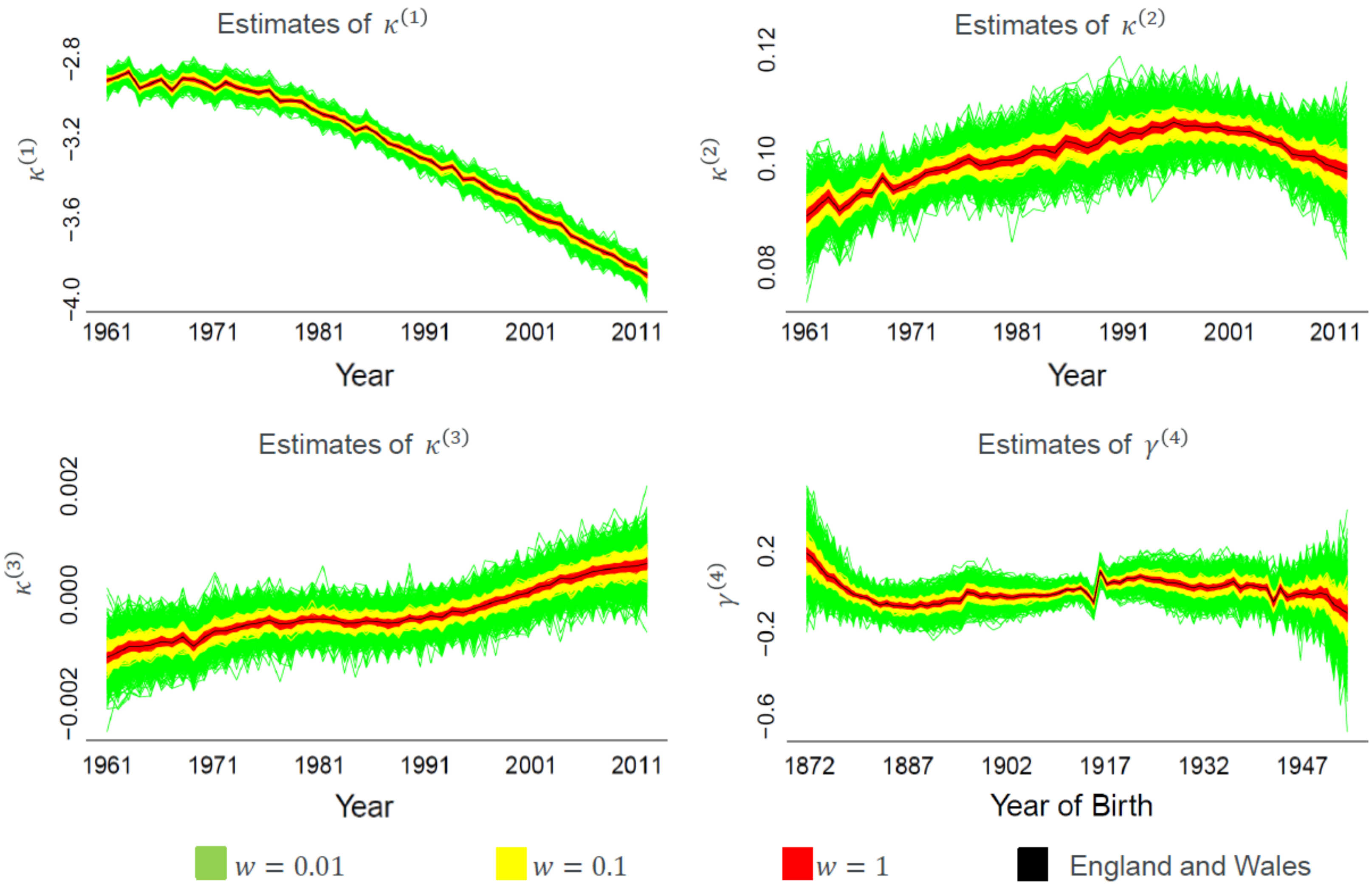
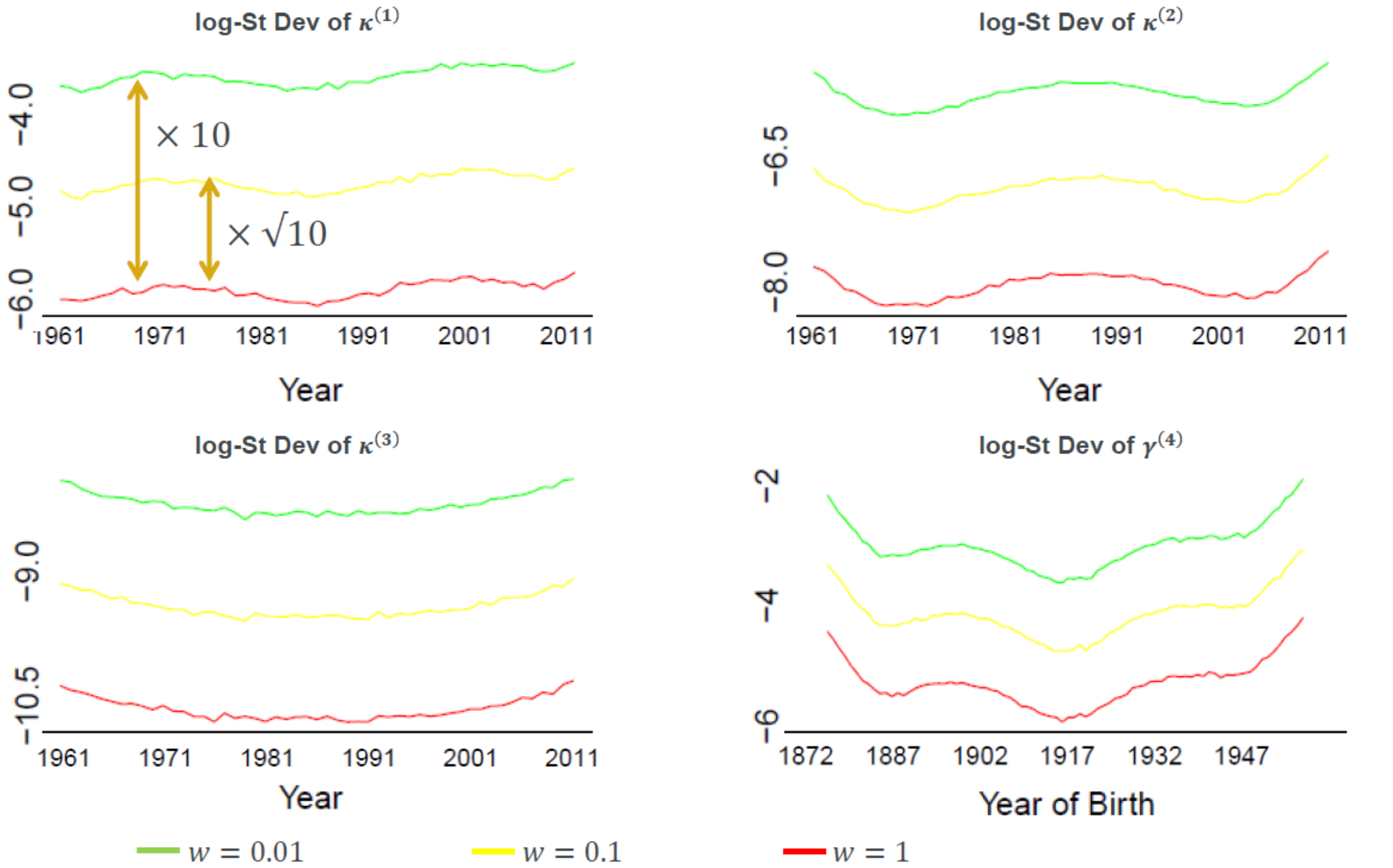
Parameter Estimates

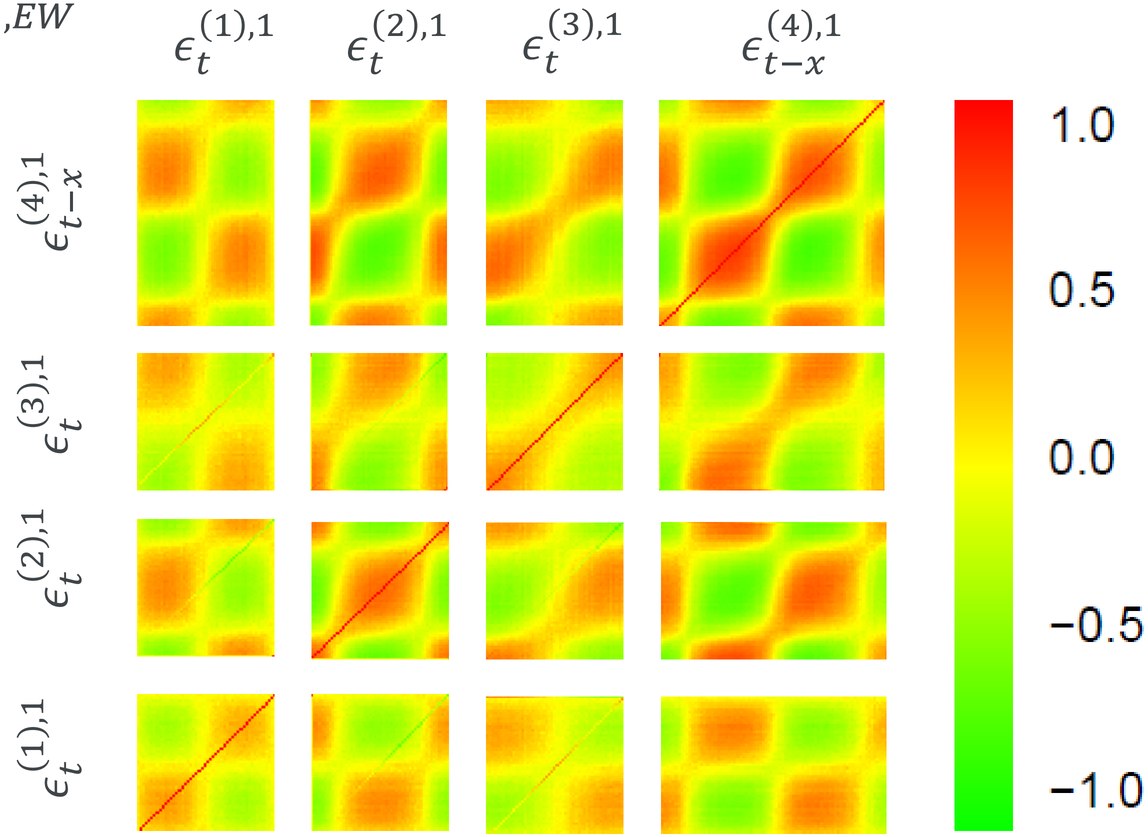


Uncertainty of Parameter Estimations

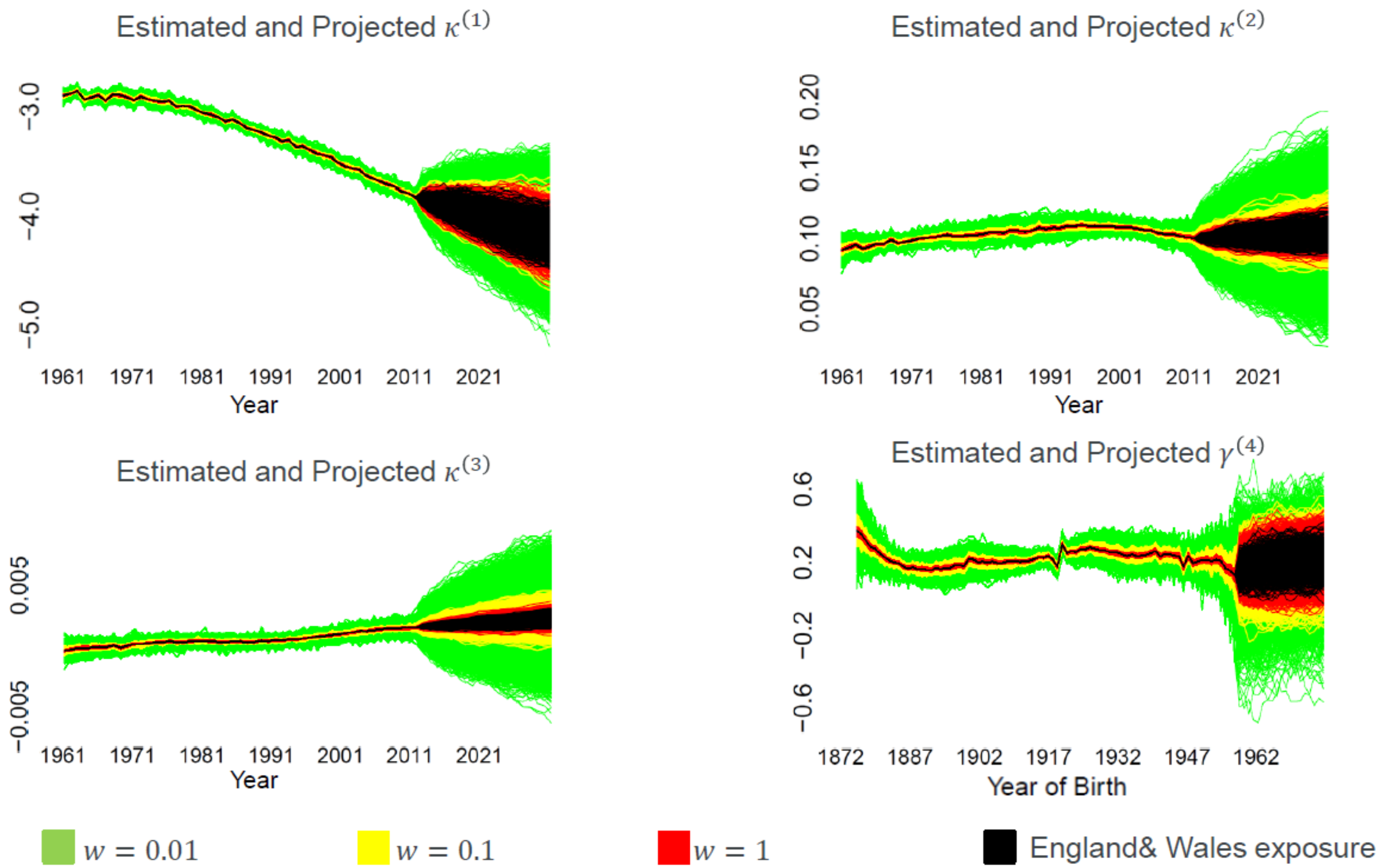


Correlation of $\widetilde{\theta}^{w,j} - \widehat{\theta}^{EW}$

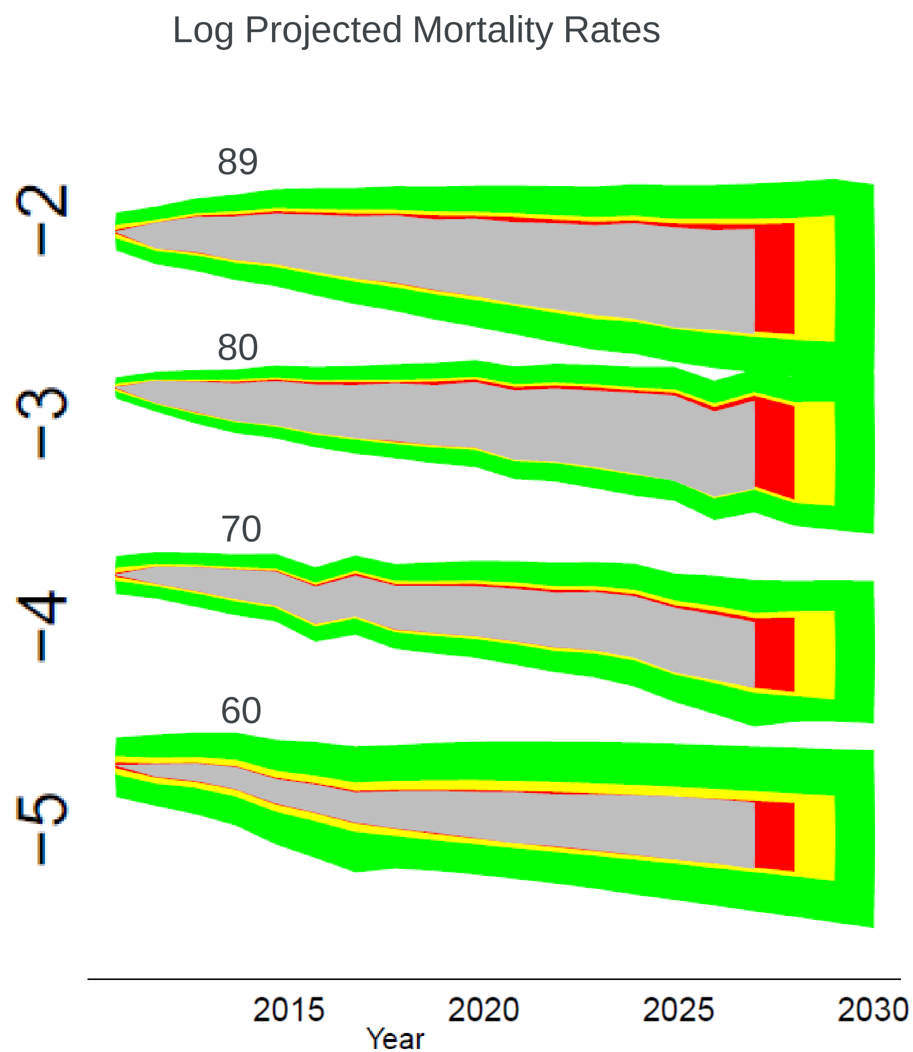
- $X^w = \left(\epsilon_1^{(1),w} \dots \epsilon_{n_y}^{(1),w}, \epsilon_1^{(2),w} \dots \epsilon_{n_y}^{(2),w}, \epsilon_1^{(3),w} \dots \epsilon_{n_y}^{(3),w}, \epsilon_1^{(4),w} \dots \epsilon_{n_y+n_a-1}^{(4),w}\right)$
- $\epsilon_t^{(1,2,3),w} = \tilde{\kappa}_t^{(1,2,3),w} - \hat{\kappa}_t^{(1,2,3),EW}$
- $\epsilon_{t-x}^{(4),w} = \tilde{\gamma}_{t-x}^{(4),w} - \hat{\gamma}_{t-x}^{(4),EW}$

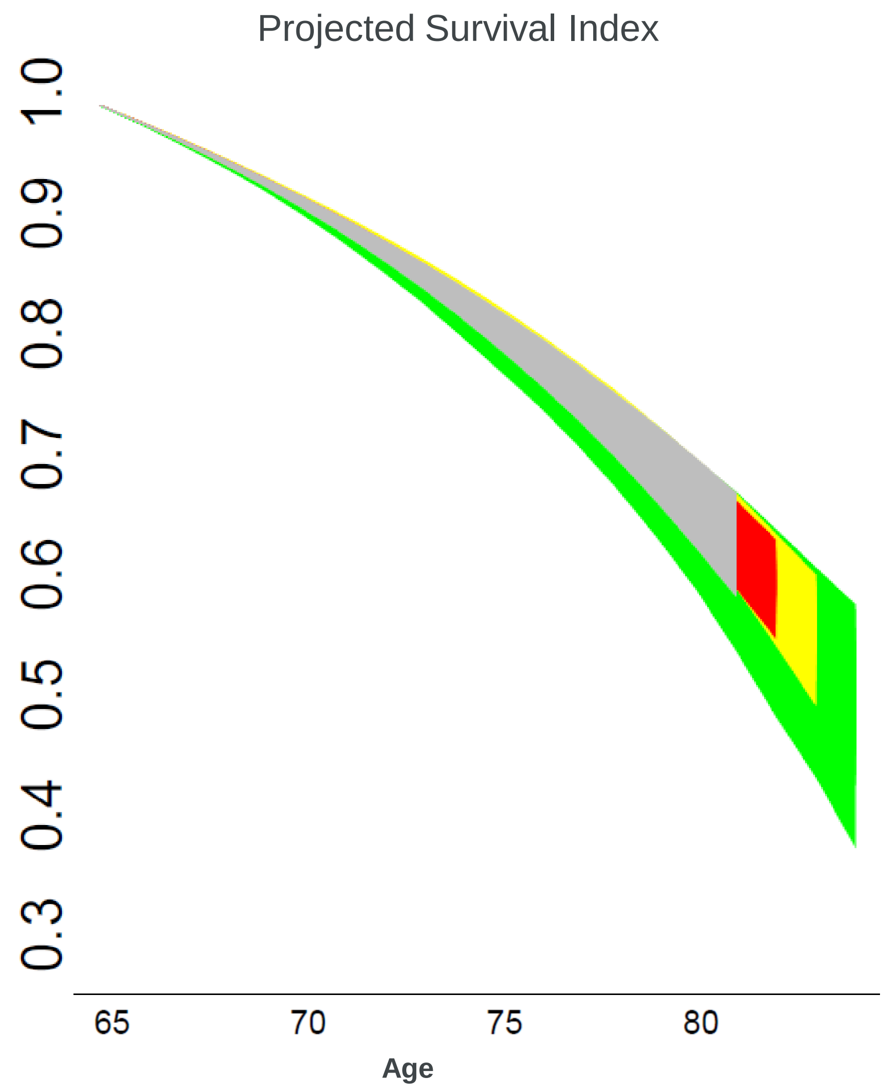


Parameter Projection



Projected Mortality Rate and Survival Index





$w = 0.01$ $w = 0.1$ $w = 1$ EW

Drift of Multivariate Random Walk $\mu_w^{(i)} = \frac{1}{n_y}\sum_{t=1}^{n_y} \Delta\kappa_t^{(i),w}$

Std Dev($\mu_w^{(i)}$)	$i = 1$	$i = 2$	$i = 3$
$w = 1$	0.0000842	0.0000086	0.00000113
$w = 0.01$	0.0008289	0.0000844	0.00001095

$\times 10$

- The higher the w, the lower the standard deviation of drift.
- w has no significant effect on the mean of the drifts
- No significant linear correlation between drifts for all w
- The correlation generally decays as w decreases

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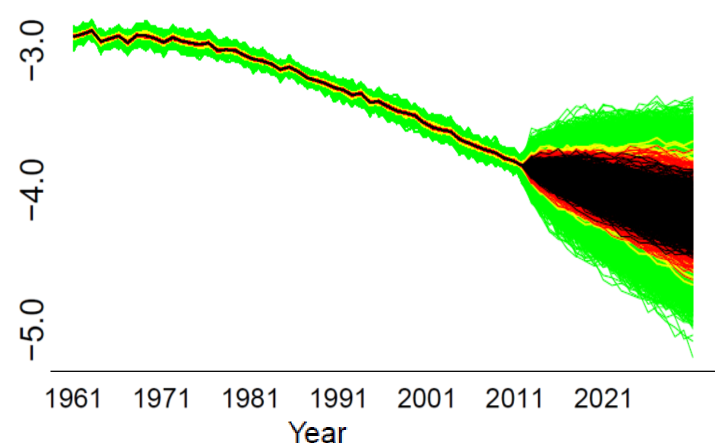
Co-Variance Matrix of Multivariate Random Walk, V^w

- $V_{i,j}^{w} = E\left[\left(\Delta\kappa_t^{(i),w} - \mu_w^{(i)}\right)\left(\Delta\kappa_t^{(j),w} - \mu_w^{(j)}\right)\right]$, where $i,j = 1,2,3$
- $V_{i,j}^{EW} = \begin{pmatrix} 6.80\times10^{-4} & 2.19\times10^{-5} & 4.95\times10^{-7} \\ 2.19\times10^{-5} & 1.31\times10^{-6} & 3.22\times10^{-8} \\ 4.95\times10^{-7} & 3.22\times10^{-8} & 3.33\times10^{-9} \end{pmatrix}$
- $\mathrm{E}[V_{i,j}^{1}] = \begin{pmatrix} 6.93\times10^{-4} & 2.15\times10^{-5} & 5.43\times10^{-7} \\ 2.15\times10^{-5} & 1.43\times10^{-6} & 2.99\times10^{-8} \\ 5.43\times10^{-7} & 2.99\times10^{-8} & 4.33\times10^{-9} \end{pmatrix}$
- $\mathrm{E}[V_{i,j}^{0.01}] = \begin{pmatrix} 19.1\times10^{-4} & -1.69\times10^{-5} & 4.84\times10^{-6} \\ -1.69\times10^{-5} & 1.28\times10^{-5} & -1.98\times10^{-7} \\ 4.84\times10^{-6} & -1.98\times10^{-7} & 1.01\times10^{-7} \end{pmatrix}$

Estimated and Projected $\kappa^{(1)}$



- The mean of $V_{i,j}^{w}$ shifts up from the original $V_{i,j}^{EW}$ as w decrease
- Lower w also results in higher standard deviation to the co-variance matrix

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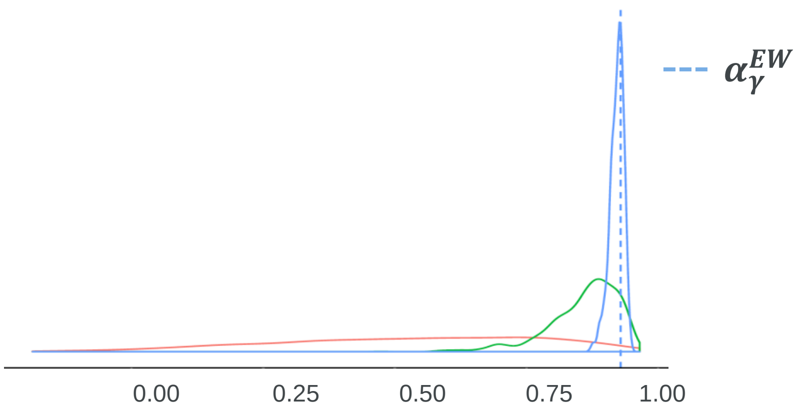
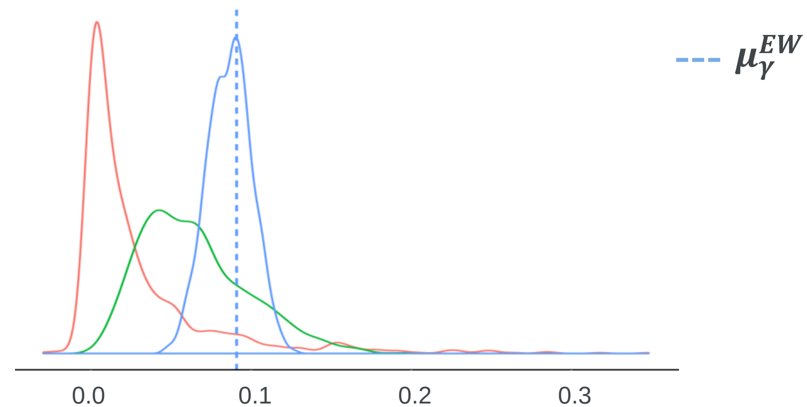
$AR(1)$ model for the Cohort Effect $\gamma_c^{(4),w}$

- $\gamma_c^{(4),w} = \mu_\gamma^w + \delta_1^w c + \alpha_\gamma^w \gamma_{c-1}^{(4),w} + \xi_c^w$

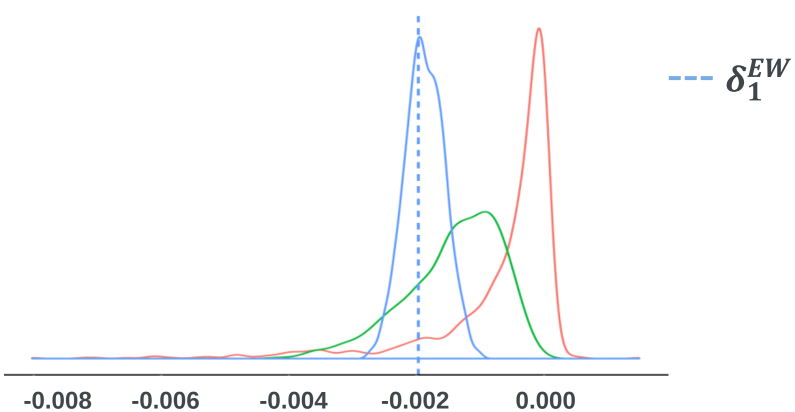
Density of α_γ^w



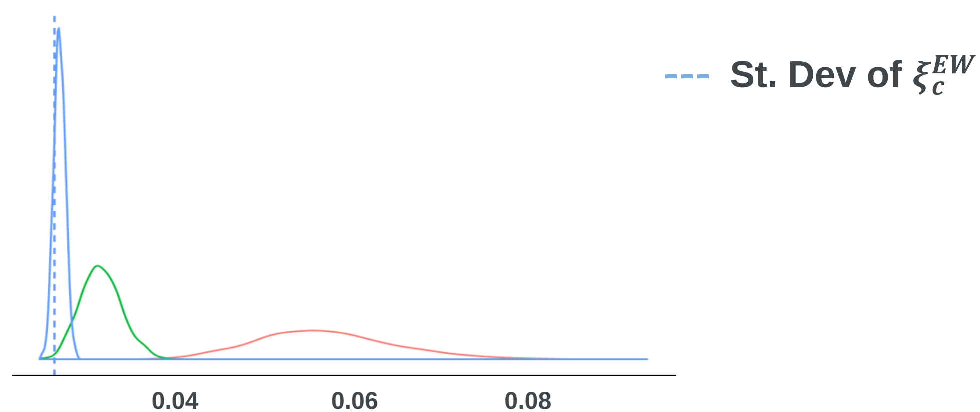
Density of μ_γ^w



Density of δ_1^w



Density of St. Dev of ξ_c^w



$w = 0.01$ $w = 0.1$ $w = 1$

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Conclusions

- The accuracy of the parameter estimates depends significantly on the population size

- Hence smaller population results in greater uncertainty in projections

- Forecasting needs to allow for small population bias in parameter estimates

Thanks

Questions?