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Longevity: A “simple” stochastic modelling of mortality

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Agenda

Context and basic idea

The model

- Stochastic process
- Parameter estimation
- Validation
- Determination of the volatility

Application

- Sample paths
- Bias on cash flows

Selected results

Conclusion

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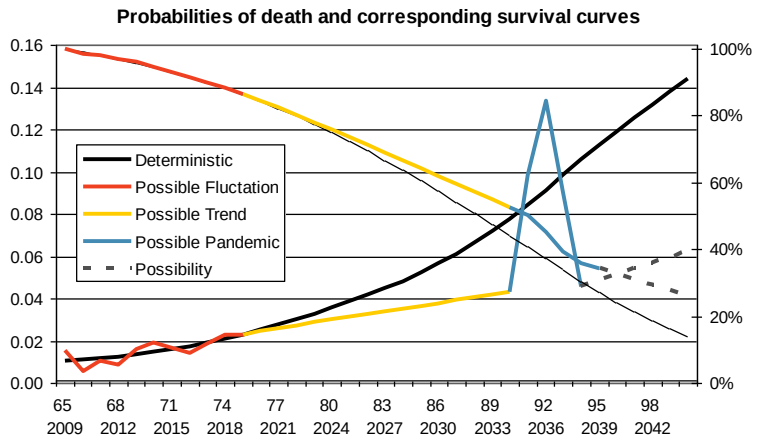
Basic idea



Male aged 65 in 2009 based on 100% PCMA00 in 2000 and 100% Medium Cohort from 2001 onwards. Source: CMI R-23 & WP-37

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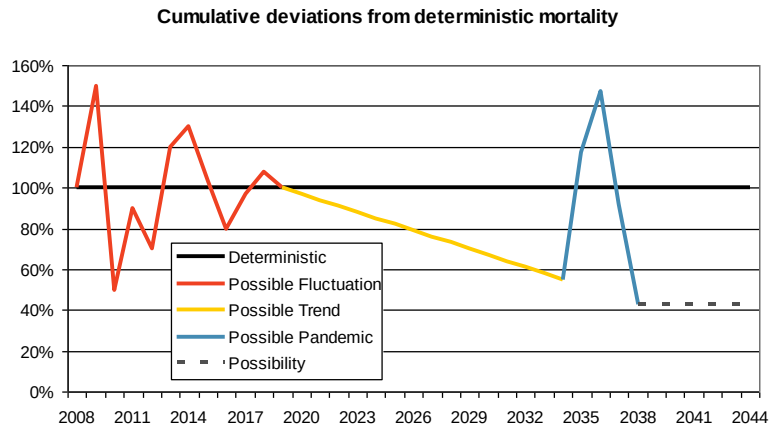
Basic idea



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The Model

Describes how mortality rates behave around any projected expected values

Focus on systemic risk

→ Multiplicative model

$$\hat{q}_{x,t} = q_{x,t} \times C_t$$

Where for age *x* at time *t:*

$q_{x,t}$ expected mortality

$\hat{q}_{x,t}$ observed mortality

C_t stochastic process

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The Model

$$\hat{q}_{x,t} = q_{x,t} \times C_t + \varepsilon_{x,t}$$ Restricted to [0,1]

Stochastic process:

$$\begin{cases} C_t = \exp(X_t) \times C_{t-1} & \forall t > 0 \\ C_0 = 1 \\ (X_i)_{i \in N} \sim \mathrm{N}(\mu, \sigma) \text{ iid} \\ \mathrm{E}[\exp(X_t)] = 1 \\ (\varepsilon_{x,i})_{i \in N} \text{ iid}, \quad \mathrm{E}[\varepsilon_{x,i}] = 0 \ \forall i \\ C, \varepsilon \text{ independent} \end{cases}$$

The Model

"E[exp (X_t)] = 1"

$$\mathrm{E}[\exp(X_t)] = 1 \Rightarrow \mathrm{E}[C_t] = 1 \Rightarrow \mathrm{E}[\hat{q}_{x,t}] = q_{x,t} \quad \text{as } \mathrm{E}[\varepsilon_{x,i}] = 0$$

The results of the C_t process are centred around the projected deterministic values.

"$\varepsilon_{x,i}$"

- "Noise" component
- Corresponds to the diversifiable risk

The Model

"E[exp (X_t)] = 1"

As X_t follows a Normal distribution by assumption, this is equivalent to:

$$\mu + \frac{\sigma^2}{2} = 0$$

Indeed, $E\left[\exp\left(X_t\right)\right] = \exp\left(\mu + \frac{\sigma^2}{2}\right) = 1$

→ The model calibration depends only on one parameter,

i.e. the volatility σ.

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Parameter estimation

The model can be calibrated using:

$$X_t = \ln\left(\frac{C_t}{C_{t-1}}\right) \qquad where \quad C_t = \frac{\hat{q}_{x,t}}{q_{x,t}}$$

Where $\hat{q}_{x,t}$ "**real**" observed mortality

$q_{x,t}$ "**expected**" mortality

→ Expected mortality could be **derived** from observed mortality.

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Parameter estimation

From a methodological point of view we should

- Split the observation period into 2 equal intervals
- Use the first interval to forecast the expected mortality
- Determine the volatility during the second interval based on observed and expected mortality
- Use more years per interval than you aim to project

→ **Difficult in practice…**

But "smoothing" is possible

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Parameter estimation

"Smoothing" methods

- M1: Lee-Carter
- M3: Currie "Age-Period-Cohort"
- M4: PB-Splines
- M7: Cairns-Blake-Dowd "Quadratic"

Example: England and Wales population

- Source: Human Mortality Database
- Age range 60-89, calendar years 1955-2005

→ **M1 "LC"** does not capture the „cohort effect" observed in the data
→ **M4 "PB-Splines"** results depend heavily on chosen parameters
→ **M3 "Currie APC"** and **M7 "CBD/Q"** have shown reasonably robust back-testing results

Source: A quantitive comparison of stochastic mortality models using data from England & Wales and the United States, Pensions Institute DP PI-0701

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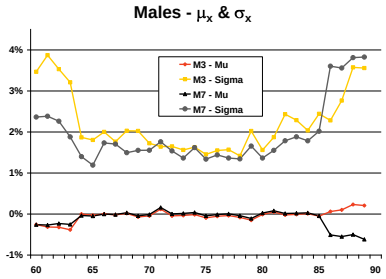
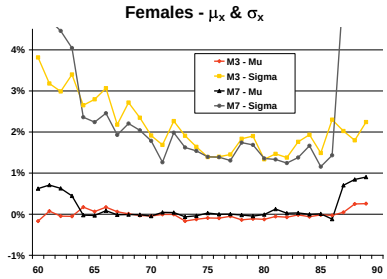
Validation

"Parameters μ and σ are independent of age"





- Reasonable for most ages
- Some effects at the extremes, partially due to smoothing

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Validation

"The X_t follow a Normal distribution"
We have used the Jarque-Bera test:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

Where ***S*** is the skewness, ***K*** the kurtosis of the tested distribution and ***n*** the number of observations

→ If the tested distribution is Normal, then

$$JB \rightarrow \chi^2(2)$$

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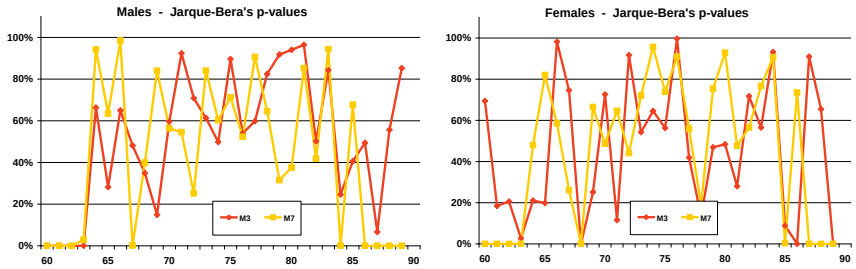
Validation

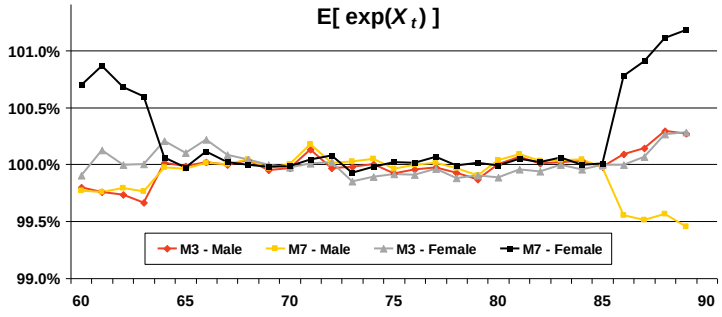
"The X_t follow a Normal distribution"



Apart from the effects at the extremes, the p-values are ≥ 5% almost everywhere
→ We can accept the "Normal" hypothesis

Validation

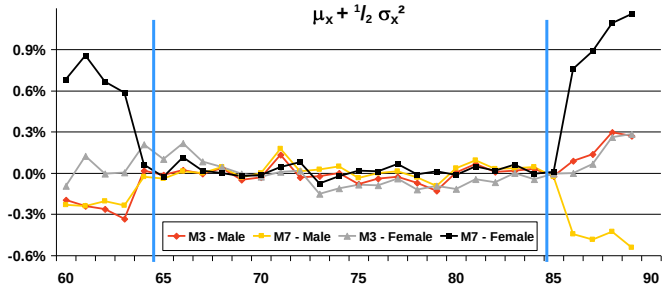
"E[exp (X_t)] = 1, i.e. around the expected mortality"



To avoid effects at the extremes, we consider only the age range 64 to 84

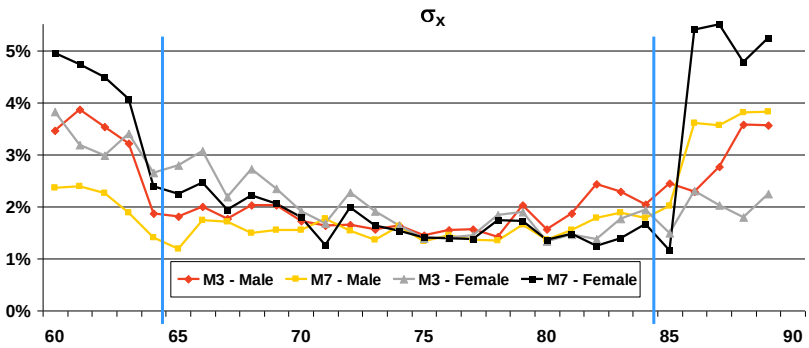
Validation

"E[exp (X_t)] = 1, i.e. around the expected mortality"



Almost 0 within the considered range, exceptions due to the effects at the extremes

Determination of the Volatility



- Volatility based on 1955-2005 data
- What do we use for the future?

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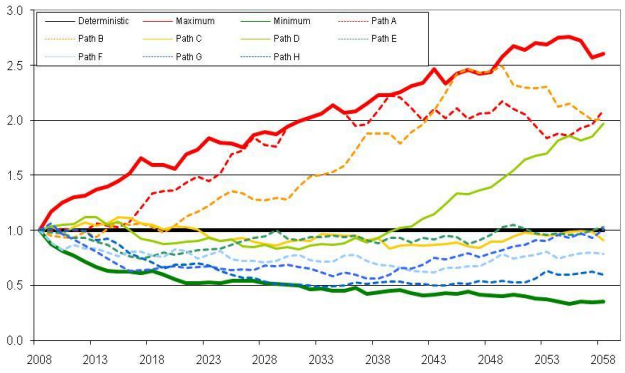
Sample paths for C(t)



- Arbitrary σ = 4%
- C(t) log-normally distributed by year

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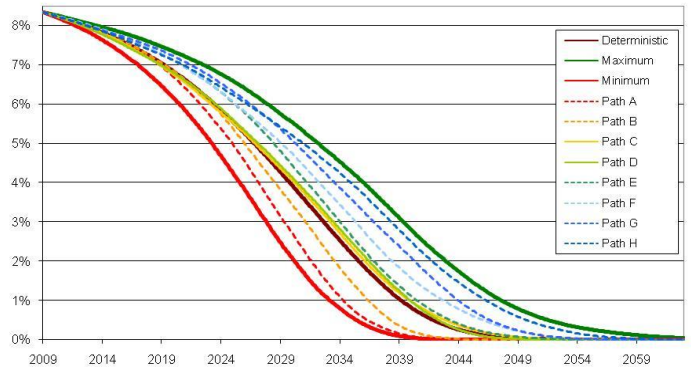
Corresponding annuity cash flows



Annuity of 1 for a male aged 65, paid monthly from 2009 onwards based on 100% PCMA00 in 2000 and 100% Medium Cohort from 2001 onwards . Source: CMI R-23 & WP-37

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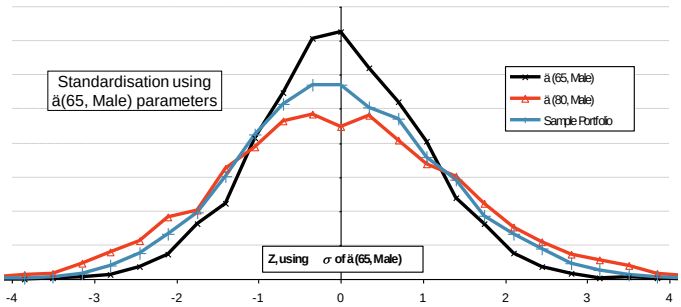
"Standardised" distributions of Present Values



- Same C_t paths and discount rate of 5%
- Depends on the portfolio composition and characteristics

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Adjusting cash flows

E[PV of CF^{Stochastic}] ≠ PV of CF^{Deterministic}

E.g. deviation of PV is 0.3% at outset in the case of ä(65, Male)

Reason:

$$ {}_t p_{x,y} = \Pi_{0 \le l \le t-1} \left(1 - q_{x+l,y+l}\right) $$

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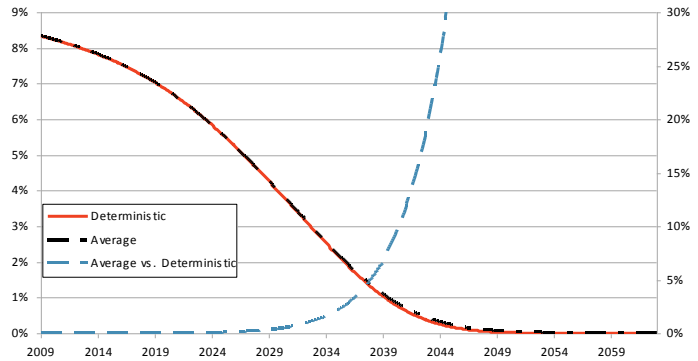
Adjusting cash flows



The bias should be taken into account when comparing deterministic scenarios to the stochastic distribution.

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Capital requirements

The UK regulator requests an Individual Capital Assessment (ICA) assessing capital requirements at a 99.5% confidence level for all types of risks over 1 year timeframe.

This is consistent with Solvency II requirements.

Examples for ä(65, Male)

- Capital ~ VaR_α $Z_{99.5\%} = 2.70$ (+8.4%) $Z_{97.5\%} = 1.99$ (+6.2%)
- Capital ~ $TVaR_\alpha$ $Z_{99.5\%} = 3.10$ (+9.6%) $Z_{97.5\%} = 2.42$ (+7.5%)

Sources: MAP/GN46 V2.0 Amendment 1, BAS Recommended Practice
CEIOPS-CP-49/09

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Probability of a specific scenario

A stochastic model allows you to estimate the probability of occurrence of any deterministic scenario.

Examples for ä(65, Male)

- Solvency II QIS 4 extreme scenario for longevity risk is a permanent drop of -25% on the base mortality level

  Z = 2.35 (+7.3%), i.e. the 98.9% percentile

- Combination of a drop of -15% with additional improvements of "+1%"

  Z = 2.68 (+8.3%), i.e. the 99.5% percentile

Source: CEIOPS-CP-49/09

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Application

Pricing of reinsurance solutions for longevity

- Swap of annuity payments with limits
- Capital allocated

Enable the inclusion of shock scenarios

- Pandemics

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Conclusion

Method to transform a deterministic scenario into a stochastic mortality model

Advantages

- Apply to any future mortality basis
- Depending on 1 parameter only
- Link with extreme scenarios
- Can include shock
- Consistent by market or even by product

Disadvantages

- Calibration
- Bias needs to be compensated for
- Computing time, as with any stochastic model

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Longevity: A “simple” stochastic modelling of mortality

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