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MALE SOCIAL CLASS MORTALITY DIFFERENCES AROUND 1981: AN EXTENSION TO INCLUDE CHILDHOOD AGES

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ABSTRACT

The aim of this paper is to extend the analyses of our earlier work (Haberman & Bloomfield, 1988) by incorporating mortality rates of male children categorised by age and social class. Data recently made available in an annex to the Decennial Occupational Mortality Investigation are utilised to produce graduated mortality rates specific for age and social class and to extend the social class specific life tables down to cover the full age range.

KEYWORDS

Social Class Mortality Differences; Life Tables

1. INTRODUCTION

In an earlier paper (Haberman & Bloomfield, 1988), we considered social class mortality differentials in Great Britain in a period centred on the 1981 Census. There, adult mortality data from two sources, the Decennial Supplement and the Longitudinal Study, were described and used to construct life tables from ages 20 onwards for three composite social classes (I and II, IIIN and M, IV and V—see below). Detailed results were given with particular reference to mortality differentials between the social classes. A full discussion of the difficulties of interpreting these results was also provided.

The social classes, to which we referred in the earlier paper, and also in the present article, are based on a categorisation of occupation used by the Office of Population Censuses and Surveys (OPCS), and are as follows:

I	professional occupations,
II	employers and managers,
IIIN	skilled non-manual occupations,
IIIM	skilled manual occupations,
IV	semi-skilled occupations, and
V	unskilled manual occupations.

Our purpose in this paper is to take advantage of recently published mortality rates for infants and children (from an annex to the Decennial Supplement—Registrar General, 1989) in England and Wales, and to extend the earlier analyses to cover the full age range. Thus, social class specific life tables are produced here covering all ages.

2. DESCRIPTION OF THE DATA

For the Decennial Supplement investigation of occupational mortality centred on 1981, OPCS published for the first time a separate annex on infant and children's mortality—Registrar General's Decennial Supplement—Occupational Mortality Childhood Supplement (Registrar General, 1989). We refer to this publication in the rest of the paper as the Annex. This new emphasis on childhood mortality specific for social class arises from the capability to supplement the traditional census-based mortality data with stillbirth and infant mortality data taken from the system set up by OPCS to link directly infant death records with details collected at the registration of the infant's birth.

The Annex places more emphasis on analyses by social class than does the main Occupational Mortality volume (Registrar General, 1986), but much less emphasis on analyses by individual occupations. Two principal reasons are given. First, the small number of childhood deaths markedly affects the precision and confidence with which estimates can be made of mortality rates in relation to specific occupations. Hence, this effect makes uncertain the meaning of any occupational associations which might be found. The presentation of the data grouped into broadly homogeneous social classes overcomes some of these practical difficulties. Secondly, the Annex mentions the general belief that mortality differences among children do not directly reflect occupational hazards or health-related changes in parental occupations. The main determinants are likely to be the general socio-economic and environmental conditions in which the children were conceived and reared (Registrar General, 1989).

The data referred to relate to deaths among children aged under 15. For deaths at ages under 1, the linked files for 1979–83 (in respect of infant deaths and birth registrations) are used as the source of data. For deaths at ages 1–15 (last birthday), the data available refer to deaths in years 1979, 1980, 1982 and 1983 (as for adult mortality), 1981 being omitted because of an industrial dispute.

Occupational information of relevance here is collected at birth registration, death registration and the census. One of the main criticisms levelled at occupation and social class specific mortality rates which use census data to provide denominators is that of bias arising from the different ways in which information regarding occupation is collected at death registration and the census. This objection would not apply to stillbirth and infant mortality rates by social class, since data for both numerator and denominator are collected through similar registration processes. The objection would remain for the social class specific mortality rates derived for ages 1–15, although it would be less acute than for adult data. At the death registration of a child, the informant is often the same person as the head of household at the census; in many cases, the same parent describing their own occupation on both occasions. It would, therefore, appear reasonable to assume that the appreciable problems of bias between occupational and social class data collected at death registration and at census, and discussed in detail by Goldblatt (1986) and by Haberman & Bloomfield (1988), are much less acute for childhood mortality data than for adult data.

Infant mortality data have been analysed primarily by father's social class, so the emphasis in the tabulations used here is on legitimate births. During the period in question, 1979–83, illegitimate births formed about 13% of total live births in England and Wales (OPCS, 1986).

Jointly registered illegitimate births formed 58·5% of all illegitimate births in the period 1979–83. This proportion rose from 55·2% in 1979 to 61·3% in 1983. Thus, an increasing proportion of the illegitimate infant deaths can be analysed by father's social class, because information on the occupation of the father was collected at birth and death registration. Unpublished analyses by OPCS have shown that mortality patterns for legitimate and jointly registered illegitimate births have similar social class distributions. The graduations discussed in this paper have not been able to utilise mortality rates for jointly registered illegitimate babies (which are unpublished). However, the Annex did analyse infant mortality by social class using combined data for legitimate and jointly registered illegitimate babies, and these results are referred to in a subsequent paragraph. It has not been possible to comment on the mortality of non-jointly registered illegitimate babies.

Table 1 shows, for males and females separately, infant mortality rates (for legitimate births only) and mortality rates through childhood, for deaths occurring in England and Wales in the years 1979–83 (excluding 1981), related to the population at risk in the census year 1981. The rates given are specific for age and social class.

Table 1. *Mortality of children aged under 15: rates per 100,000 by age, sex and social class; England and Wales, 1979–80, 1982–83*

Sex and Social Class	Age at Death (Last Birthday)			
	Infant* (0–1)	1–4	5–9	10–14
Males				
I	866·68	33·04	24·19	20·52
II	964·48	34·20	19·04	21·88
IIIN	1008·46	41·28	22·93	20·26
IIIM	1152·65	52·70	25·88	26·30
IV	1510·23	63·84	31·65	30·47
V	1814·67	111·54	50·14	36·23
I–V ratio	2·09	3·38	2·07	1·77
Females				
I	683·94	33·12	16·53	15·11
II	762·28	31·22	15·12	15·44
IIIN	816·64	35·84	17·54	14·02
IIIM	901·57	41·96	18·34	19·05
IV	1171·43	52·18	22·56	17·35
V	1326·28	85·55	30·55	23·94
I–V ratio	1·94	2·58	1·85	1·58

* For legitimate births only—rate per 100,000 live births.

Mortality rates tend to fall sharply throughout the first year of life; they continue to fall to a minimum at about age 10, thereafter rising. The grouping of ages used conceals the exact turning point.

For all ages and across all social classes, shown in Table 1, mortality rates for males are higher than those for females. For both sexes, death rates tend to rise consistently from social class I to social class V, although there are occasional small reversals of pattern. The slope of the rise through the social classes is similar for males and females, and is more marked at ages 1–4.

Using the standard statistical methodology set out in the Appendix, it is possible to test whether the differences between the rates in Table 1 are significant or whether they can be attributed to random, statistical fluctuations. For example, for males, for social classes I and II combined, it can be shown that the mortality rates decrease significantly from age groups 1–4 to 5–9, but the differences between the rates for age groups 5–9 and 10–14 are not significant. The same features hold for social classes IIIN and IIIM combined. However, for social classes IV and V combined, the rates decrease significantly from age groups 1–4 to 5–9 *and* from 5–9 to 10–14. A comparison of mortality rates across social class groups indicates that, for each of the age groups 1–4, 5–9 and 10–14, the gradient from social classes I and II to IIIN and IIIM and then to IV and V is significant. The excess of male mortality rates over female mortality rates can also be examined for significance: a comparison of the rates in Table 1, specific for age groups (1–4, 5–9 and 10–14) and social class, indicate that the excess is significant in all cases except at ages 1–4 for social classes I and II. (In each of these cases, a 5% significance level has been used in the test of the appropriate null hypothesis.)

The Registrar General (1989) has investigated the variation in infant death rates (i.e. at ages less than 1) between the social classes. Since these data were in the form of counts, it was assumed that the observations followed a Poisson distribution, and so a log-linear model was fitted. The technique adopted for fitting such a model was the same as that given by Renshaw & Haberman (1986), and depended on the computer package GLIM being applied to the numbers of deaths (after a logarithmic transformation was taken). The package provides estimates for factors measuring a social class effect on mortality and the corresponding standard errors. Testing the null hypothesis that there was no difference between the multiplying factors for social classes I and V revealed that this null hypothesis should be rejected at the 1% level:

(a) for deaths of legitimate births during 1975–82, when the analysis separately considered the mortality rates for each of:

all infant deaths,
stillbirths,
perinatal deaths (including still births and deaths in the first week of life),
neonatal deaths (deaths at ages under 28 days), and
postneonatal deaths (deaths at ages 28 days and over, but under 1 year); and

(b) for deaths of legitimate and jointly registered illegitimate births during 1975–82, when the analysis considered the mortality rates for each of the same 5 categories as above.

Direct comparison of these findings (at ages under 1) with those of the 1970–72 Decennial Supplement (Registrar General, 1978) is not possible, since the infant mortality statistics presented in that report included both legitimate and illegitimate births. Illegitimate births have higher death rates than legitimate births, and the occupation details collected are not always sufficient to assign a meaningful social class, particularly when the illegitimate birth is registered by the mother alone.

Table 2, taken from Registrar General (1978), gives the mortality rates at ages 1–14 for males and females separately, and for the social classes based on the 1970–72 decennial investigation into occupational mortality. A comparison of Tables 1 and 2 indicates that, for males, the spread of mortality rates across the social classes at ages 1–4 has increased over the period since 1970–72, while for age groups 5–9 and 10–14 the spread has decreased. For females, the spread of mortality rates has increased for age groups 1–4 and 5–9 and remained approximately the same at ages 10–14.

A detailed comparison of the results in Table 1 with the findings of the Decennial Supplements for 1970–72 and 1959–63 must be undertaken with caution, because of the likelihood of different degrees of numerator/denominator bias. It is worth noting, however, that in 1970–72 the social class gradient

Table 2. *Mortality of children aged over 1 and under 15: rates per 100,000 by age, sex and social class; England and Wales 1970–72*

Sex and Social Class	Age at Death (Last Birthday)		
	1–4	5–9	10–14
Males			
I	60·58	27·51	28·32
II	60·02	31·41	31·07
IIIN	74·53	38·93	34·81
IIIM	75·53	42·40	34·64
IV	92·89	44·06	40·06
V	128·72	69·10	56·47
I–V ratio	2·12	2·51	1·99
Females			
I	57·48	26·62	20·83
II	54·02	23·81	21·33
IIIN	62·16	27·46	19·89
IIIM	61·85	27·30	21·29
IV	84·21	32·53	26·44
V	109·09	43·42	32·76
I–V ratio	1·90	1·63	1·57

Table 3. *Children aged under 16 by social class of head of household, Great Britain, 1981*

Social Class	Percentage of Children in each Social Class
I	6·0
II	23·3
IIIN	10·3
IIIM	36·2
IV	15·3
V	5·2
Armed Forces and Inadequately Described	3·6

for males was largest for 5–9 year olds (Table 2), whereas in 1959–63 it was greatest for 1–4 year olds, as seen here also for the 1979–83 investigation.

Table 3 gives a summary of the social class distribution of the childhood population of Great Britain at the 1981 Census, as categorised by the social class of the head of household. This distribution largely matches the social class distribution of households (figures available in Census tabulations, but not given here), although there are relatively more children from social classes IIIM and V households and correspondingly fewer from most of the other classes.

3. GRADUATING THE RATES FOR MALE CHILDREN AND CONSTRUCTION OF THE LIFE TABLES

As in Haberman & Bloomfield (1988), the social classes were combined in pairs to produce three composite classes of approximately equal size: classes I and II; IIIN and M; IV and V.

From the grouped central mortality rates ${}_n\mathring{m}_x$, central mortality rates were calculated for single years of age for particular pivotal ages using the approximation:

$${}_n\mathring{m}_x = m_{x+\frac{1}{2}(n-1)}.$$

The five values for each composite class for male children were then plotted on a semi-logarithmic scale together with the graduated rates from ages 20 onwards, as produced in Haberman & Bloomfield (1988). The central mortality rates were then graduated graphically, using as a template the graduated mortality rates from English Life Table No. 14 for ages under 19 (Registrar General, 1987). For males, the ELT No. 14 graduation is based on cubic splines using an optimal set of 10 knots, of which 4 are at ages under 20. The crude male mortality rates underlying ELT No. 14 exhibit a rapid increase between ages 15 and 19. Reflecting this feature, the graduated curve of mortality rates has a minimum at age 10 and a local maximum at age 18.

For reasons of convenience and simplicity, the graduation was carried out using graphical means rather than using the more sophisticated methods of parametric curve fitting or spline graduation. The level of accuracy and degree of smoothing afforded by a parametric approach were not considered necessary in the likely applications of these results. Such an approach might be justified if full, precise, social class-specific versions of ELT No. 14 were to be produced. Graduation by reference to a standard table was not adopted, as it was felt to be too restricted a technique, in that it imposes a fixed relationship with the standard table across all ages.

At age $x=0$, the crude rates given in Table 1 are estimators of q_0. These were converted into crude central mortality rates m_0, according to the formula:

$$m_0 = \frac{q_0}{1-(1-\phi)q_0}$$

where ϕ is an estimate of the average age at death of those dying in the first year of life. As in Registrar General (1987), $\phi = 0{\cdot}15$ was employed here. These values of m_0 were then included in the graduation process.

As part of the graduation process, a comparison of actual and expected deaths was made at ages under 15. Detailed statistical tests of goodness of fit are not possible, because the crude data are presented in the broad age groups shown in Table 1. However, broad comparisons of observed deaths and expected deaths, on the basis of the graduated rates, give satisfactory results. At adult ages, formal comparisons of actual and expected deaths have not been carried out, because these graduations result from the 'blending' together of two separate sets of crude mortality rates (see Haberman & Bloomfield (1988) for further details). Second and third order differences of log m_x were examined and monitored in order to ensure a smooth progression in the graduated age-specific mortality rates.

There is an inconsistency in sources of data, in that the mortality rates for the adult ages have been derived from two data sources, one of which relates to Great Britain and one of which relates to England and Wales only (as discussed by Haberman & Bloomfield (1988)). The mortality data for children also relate to England and Wales only. Because our aim is to construct approximate social class specific life tables by blending together mortality rates from disparate data sources (rather than produce definitive life tables along the lines of ELT No. 14), we have not made any adjustments explicitly to deal with this point. However, some downward adjustments to the mortality rates at ages over 60 (as published in Haberman & Bloomfield (1988)), have been made in order to produce an improved progression of the graduated rates. To some extent, these adjustments deal with the requirement to reduce any rates for Great Britain so that they are more applicable to England and Wales.

The graduated central mortality rates were read off the respective graphs and were converted into mortality rates, q_x, at exact ages using the recursive formulae:

$$q_0 = \frac{m_0}{1+(1-\phi)m_0}$$

$$q_1 = \frac{m_1(1 + \frac{1}{2}m_2)}{1 + \frac{7}{12}m_1 + \frac{5}{12}m_2 + \frac{1}{3}m_1m_2}$$

$$q_2 = \frac{m_2(1 - \frac{13}{12}q_1)}{(1 - q_1)(1 + \frac{5}{12}m_2)}.$$

These last two equations are exact, if l_t is quadratic over the age interval [1,3]; and for $x=3,4\ldots$

$$q_x = m_x \cdot \frac{1 - \frac{1}{2}m_{x-1}}{1 + \frac{5}{12}(m_x - m_{x-1}) - \frac{1}{6}m_{x-1}m_x}.$$

This formula is exact if l_t is quadratic over the age interval $[x-1, x+1]$. This algorithm was used in the construction of ELT No. 14, and has been adopted here for consistency.

Table 4 provides a complete listing of the graduated male mortality rates for the three composite social classes.

Table 4. *Graduated male mortality rates,* q_x, *for Social Class Groupings. (England and Wales 1979–80, 1982–83)*

	Social Class Grouping				Social Class Grouping		
Age x	I and II	IIIN and IIIM	IV and V	Age x	I and II	IIIN and IIIM	IV and V
0	0·00941	0·01112	0·01637	25	0·00055	0·00074	0·00117
1	0·00079	0·00085	0·00110	26	0·00054	0·00076	0·00120
2	0·00040	0·00054	0·00087	27	0·00054	0·00078	0·00123
3	0·00030	0·00042	0·00069	28	0·00055	0·00079	0·00126
4	0·00027	0·00036	0·00059	29	0·00058	0·00081	0·00132
5	0·00025	0·00032	0·00050	30	0·00062	0·00085	0·00141
6	0·00022	0·00029	0·00043	31	0·00066	0·00091	0·00151
7	0·00020	0·00025	0·00036	32	0·00071	0·00098	0·00162
8	0·00019	0·00024	0·00033	33	0·00076	0·00105	0·00174
9	0·00019	0·00023	0·00032	34	0·00081	0·00115	0·00186
10	0·00019	0·00023	0·00032	35	0·00087	0·00126	0·00199
11	0·00020	0·00024	0·00031	36	0·00093	0·00138	0·00214
12	0·00021	0·00025	0·00031	37	0·00100	0·00151	0·00229
13	0·00024	0·00028	0·00032	38	0·00110	0·00166	0·00251
14	0·00028	0·00032	0·00036	39	0·00120	0·00182	0·00275
15	0·00034	0·00040	0·00043	40	0·00132	0·00204	0·00302
16	0·00044	0·00052	0·00053	41	0·00148	0·00229	0·00331
17	0·00081	0·00100	0·00126	42	0·00166	0·00257	0·00362
18	0·00087	0·00107	0·00149	43	0·00186	0·00288	0·00397
19	0·00079	0·00098	0·00138	44	0·00209	0·00323	0·00436
20	0·00074	0·00089	0·00129	45	0·00240	0·00362	0·00489
21	0·00069	0·00083	0·00123	46	0·00275	0·00407	0·00548
22	0·00065	0·00079	0·00117	47	0·00316	0·00456	0·00615
23	0·00060	0·00076	0·00115	48	0·00362	0·00512	0·00689
24	0·00058	0·00074	0·00115	49	0·00416	0·00574	0·00773

Table 4 (*cont.*)

	Social Class Grouping				Social Class Grouping		
Age x	I and II	IIIN and IIIM	IV and V	Age x	I and II	IIIN and IIIM	IV and V
50	0·00478	0·00644	0·00867	78	0·09522	0·09950	0·11336
51	0·00548	0·00722	0·00973	79	0·10167	0·10856	0·12096
52	0·00615	0·00810	0·01091	80	0·10854	0·11840	0·12903
53	0·00689	0·00908	0·01223	81	0·11584	0·12629	0·13760
54	0·00773	0·01018	0·01371	82	0·12360	0·13469	0·14669
55	0·00867	0·01142	0·01537	83	0·13183	0·14674	0·15633
56	0·00973	0·01280	0·01684	84	0·14058	0·15972	0·16653
57	0·01091	0·01435	0·01845	85	0·15311	0·17373	0·17733
58	0·01223	0·01609	0·02021	86	0·16660	0·18883	0·19278
59	0·01371	0·01804	0·02214	87	0·18114	0·20507	0·20931
60	0·01537	0·02021	0·02425	88	0·19680	0·21791	0·22238
61	0·01723	0·02214	0·02656	89	0·21363	0·23154	0·23624
62	0·01931	0·02425	0·02909	90	0·23168	0·24087	0·25082
63	0·02164	0·02656	0·03185	91	0·25099	0·25065	0·26613
64	0·02425	0·02909	0·03487	92	0·26613	0·26076	0·28218
65	0·02717	0·03185	0·03817	93	0·28218	0·27119	0·29897
66	0·03044	0·03487	0·04177	94	0·29897	0·28194	0·31651
67	0·03409	0·03817	0·04571	95	0·31651	0·29303	0·33479
68	0·03817	0·04177	0·05001	96	0·33479	0·30445	0·34696
69	0·04273	0·04571	0·05471	97	0·35379	0·32252	0·35982
70	0·04782	0·05001	0·05983	98	0·37350	0·34104	0·37299
71	0·05231	0·05471	0·06541	99	0·39387	0·36028	0·38865
72	0·05721	0·05983	0·07149	100	0·40714	0·38022	0·40142
73	0·06256	0·06541	0·07812	101	0·42125	0·40081	0·42199
74	0·06838	0·07149	0·08533	102	0·44371	0·42199	0·44371
75	0·07473	0·07639	0·09317	103	0·46586	0·44371	0·46586
76	0·08165	0·08347	0·09948	104	0·48833	0·46586	0·49713
77	0·08917	0·09115	0·10620				

Three social class specific life tables were then constructed in the conventional manner using a radix of $l_0 = 10{,}000$. L_x and T_x were calculated from:

$$L_x = \frac{l_x - l_{x+1}}{m_x}$$

$$T_x = \sum_{t=0}^{\infty} L_{x+t}$$

and estimates of the complete expectation of life were calculated according to:

$$\mathring{e}_x = \frac{T_x}{l_x}$$

Figures 1–3 present the resulting sets of social class and age specific mortality rates for males, for ages 0–104 on a semi-logarithmic scale. Table 5 provides a

Table 5. *Complete expectation of life $\mathring{e}_x$ for males by Social Class Groupings at selected ages (England and Wales 1979–80, 1982–83)*

		Social Classes		
Age x	ELT14	I and II	IIIN and IIIM	IV and V
0	71·04	72·53	70·59	67·74
15	57·27	58·48	56·70	54·27
25	47·71	48·83	47·12	44·81
35	30·09	39·11	37·49	35·38
45	28·70	29·56	28·15	26·29
55	20·14	20·70	19·63	18·14
65	13·04	13·18	12·66	11·60
75	7·70	7·85	7·50	6·92
85	4·35	4·44	4·14	4·02

Table 6. *Social Class differences in expectation of life*

Age	I and II minus IIIN and M	IIIN and M minus IV and V
0	1·94	2·85
15	1·78	2·43
25	1·71	2·32
35	1·62	2·11
45	1·41	1·86
55	1·07	1·49
65	0·52	1·06
75	0·35	0·58
85	0·30	0·12

tabulation of expectation of life indices at selected ages from the three life tables, together with the corresponding values from ELT No. 14 for all males for comparison purposes.

Figures 1–3 and Table 3 indicate marked differences in mortality rates for males by social class, summarised through the comparison of $\mathring{e}_x$ values shown in Table 6.

4. SUMMARY

Using the data from two separate demographic sources, namely the latest Decennial Supplement on Occupational Mortality (including infant and children's mortality) and the OPCS Longitudinal Study, we have constructed approximate life tables from age zero onwards for three composite social class groupings. A brief discussion of the resulting mortality differentials has been given.

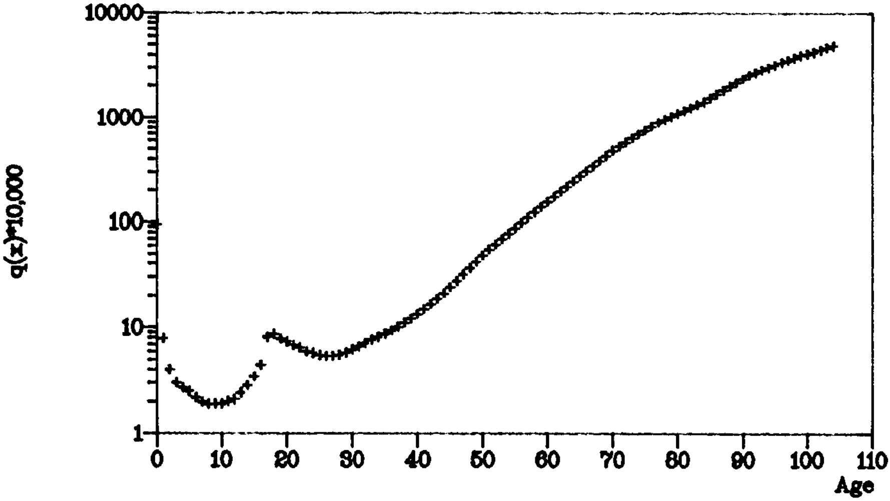

Figure 1. Age specific death rates for social classes I and II.

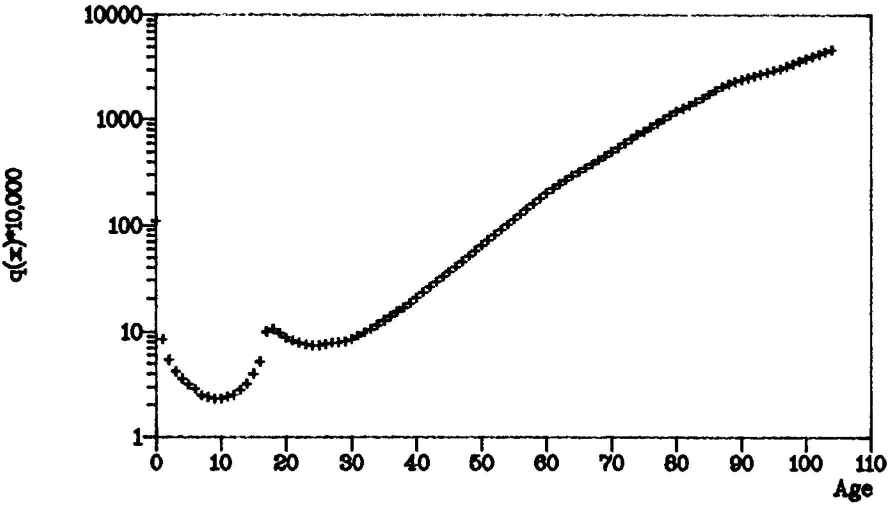

Figure 2. Age specific death rates for social classes IIIN and M.

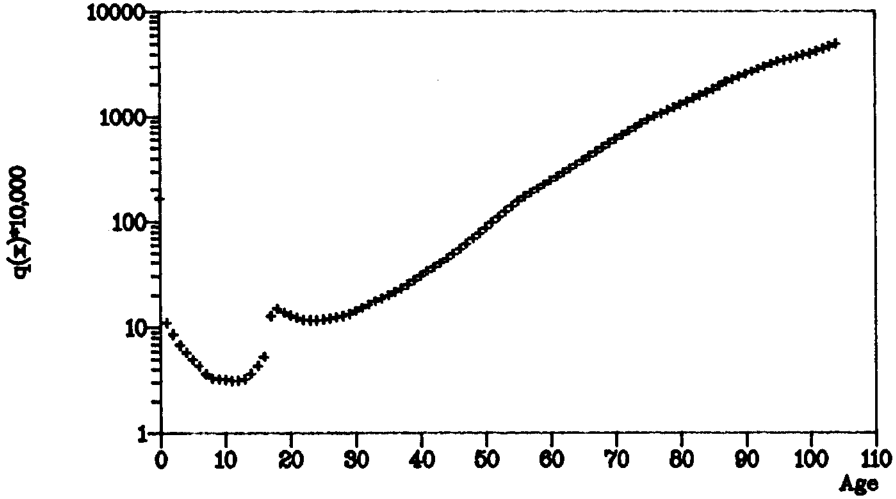

Figure 3. Age specific death rates for social classes IV and V.

As mentioned in Haberman & Bloomfield (1988), these mortality differentials are of potential importance in the pricing of, and reserving for, life insurance policies (individual and group), in particular those contracts with a significant protection component. Consulting actuaries may also find these life tables useful in setting up valuation bases for staff and works occupational pension schemes.

The mortality differentials at the youngest ages between the social classes are of wider importance and interest. These differences have been seen to be of central significance in discussions of the extent and persistence of social inequalities in health, and the reasons for these (Registrar General, 1989).

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APPENDIX

COMPARISON OF CENTRAL MORTALITY RATES

Suppose that there are two populations in which the true central mortality rates at a particular age (or age group) are M_1 and M_2, respectively.

From a mortality investigation we have, for population 1, the observed number of deaths A_1 and central exposed to risk R_1, and for population 2, A_2 and R_2.

Then, following the approach of Forfar *et al.* (1988), we have that, on the assumption that the number of deaths occurring in population 1 during the period of observation has a Poisson distribution, the maximum likelihood estimator of M_1 is:

$$m_1 = \frac{A_1}{R_1}$$

and $$E(m_1) = M_1$$

and $$\text{Var}\,(m_1) = \text{Var}\left(\frac{A_1}{R_1}\right) = \frac{1}{R_1^2}\text{Var}\,(A_1) = \frac{1}{R_1^2}R_1M_1 = \frac{M_1}{R_1}.$$

Since we do not know the true value of M_1, we substitute the maximum likelihood estimator to give an estimate of the variance of m_1, i.e.:

$$\text{Var}\,(m_1) \simeq \frac{m_1}{R_1}.$$

Similarly, for population 2 and with the corresponding assumption, we have that:

$$m_2 = \frac{A_2}{R_2}, \qquad E(m_2) = M_2 \qquad \text{and} \qquad \text{Var}\,(m_2) \simeq \frac{m_2}{R_2}.$$

Then $E(m_1 - m_2) = M_1 - M_2$.

Assuming that the two populations are independent:

$$\text{Var}\,(m_1 - m_2) \simeq \frac{m_1}{R_1} + \frac{m_2}{R_2}.$$

Suppose we wish to test the null hypothesis that $M_1 = M_2 = M$ say. Then m_1 and m_2 are both estimators of m. If the null hypothesis is true, both populations are samples from effectively the same population, and the 'best' estimator of m will be obtained by pooling the two sample populations to give:

$$m = \frac{A_1 + A_2}{R_1 + R_2}.$$

This pooled estimator is now substituted for both M_1 and M_2 to give:

$$\text{Var}\,(m_1 - m_2) \simeq m\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

under the assumption that the null hypothesis is true. The null hypothesis is then tested approximately by taking:

$$z = \frac{m_1 - m_2}{\sqrt{m\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)}}$$

as a standardised normal deviate.