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A MARKET-BASED APPROACH TO PRICING WITH-PROFITS GUARANTEES

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ABSTRACT

This paper addresses one of the fundamental issues faced by offices in the transacting of with-profits business, namely the provision of equity-related performance along with maturity guarantees that increase over the lifetime of the policy. The approach commonly followed of using capital to cover the office's investment mismatch risk is considered from an individual policy viewpoint, and, through stochastic modelling, the degree of security represented by different reserve levels identified. An alternative approach using derivatives to provide similar levels of security is then presented and the range of policy proceeds resulting from the two methods compared. While it is recognised that the latter approach may have limited application in practice, the ideas presented could form the basis of a new approach to pricing guarantees under with-profits business that would be consistent with the pricing of current derivative-backed guaranteed products.

KEYWORDS

With-Profits; Guarantees; Policyholders' Reasonable Expectations; Capital; Risk; Derivatives; Statutory Reserves; Stochastic Modelling; Low Inflation; Bonus Rates; Asset Shares

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The real voyage of discovery consists not in seeking new lands but seeing with new eyes. M. Proust

1. INTRODUCTION

1.1 *The 'With-Profits' Concept*

1.1.1 'With-profits' can mean many different things, with policyholders' reasonable expectations (PRE) varying from what is virtually a smoothed managed fund to sharing in all aspects of an office's value creation activities.

1.1.2 Central to all variations on the with-profits theme, however, is the granting of guarantees, whether they take the form of a (conventional) sum assured augmented by periodic reversionary bonus declarations or, in the case of unitised contracts, a guaranteed rate of growth (or, at the very least, a guarantee of non-reduction) in unit prices.

1.1.3 In addition, most (if not all) policyholders do not anticipate receiving just the guaranteed amounts at maturity. Despite warnings that terminal bonuses are not guaranteed, the with-profits concept tends to carry with it the hope that investment performance over the life of the policy will be such that the guaranteed return will not be all that is received.

1.1.4 Thus the strategies followed by offices transacting with-profits business have to reconcile a conflict between the need to meet the guarantees and the requirement for investment freedom in the interests of total policyholder returns — and that in the context of various practical complications.

1.2 *Some Practical Considerations*

1.2.1 The date on which a policy will terminate is not always known with certainty. For example, it may be surrendered or the maturity date may be at the policyholder's option. The latter is a particularly relevant complication, because guarantees will apply at more than one date.

1.2.2 The levels of initial guarantees as well as the size of subsequent reversionary bonus declarations (or unit price increases) have to be set, and the bonus philosophy followed can influence significantly an office's financial management.

1.2.3 Another key aspect of with-profits business is the smoothing of payouts to reduce the effect of market volatility on policy proceeds. The particular approach to smoothing affects, not just policyholder payouts, but can prove very significant to the financial health of the office, particularly in times of falling asset shares due to sustained poor investment returns.

1.2.4 Finally, while this section is not meant to be exhaustive, mention should certainly be made of the financial effects of the structure of the portfolio of business an office has, as well as the rate at which its new business is allowed to grow.

1.3 *Scope of the Paper*

1.3.1 In writing this paper, the authors have aimed to present some fundamental points as clearly as possible. As a consequence, important simplifications have been made.

1.3.2 The investigation has been restricted to guarantees vesting at maturity, and how, and at what price, they can be provided. In addition, the modelling has been carried out on individual policies rather than on portfolios.

1.3.3 The concentration on maturity guarantees is not meant to imply that practical considerations, such as those in Section 1.2, do not matter, which, of course, they do. However, extending the study to take account of these aspects could have so complicated the investigations that the fundamental points might have been lost or, at best, the issues clouded. These practical considerations will certainly throw up additional issues, but these will tend to be of a more particular nature than the generic points that this paper seeks to present. The situation with flexible maturity dates is particularly interesting, but is outwith the scope of this paper, being better dealt with in the context of portfolio studies.

1.3.4 Considering individual policies seemed the obvious way to highlight the risk/return trade-off implicit in all with-profits business, and demonstrates how this varies with duration, guarantee level and asset-backing strategy. The authors recognise that the presence of a portfolio (depending, of course, on its structure and, particularly, how widely spread anticipated outstanding terms are) could affect the degree of security aimed at for each policy in isolation. Such an investigation has been left to other papers (although some further comments are presented in Appendix A). Nevertheless, the results presented in this paper are a necessary first step in quantifying any benefit that may be gained from being part of a portfolio and the extent to which it could be passed on to policyholders.

1.3.5 In particular, it should be noted that, where an office has launched a tranche-based with-profits product (or ended up with something similar through its pattern of sales), it would be relatively straightforward to apply the concepts discussed in this paper. Where all the policies in a tranche share the same investment experience (through having similar start and end dates), there can be no diversification of investment risk over time. Thus, the risk to the office associated with the guarantee under any particular policy is the same whether that policy is viewed in isolation or as part of a portfolio.

1.4 *Outline of the Paper*

1.4.1 Further comments on the operation of with-profits business are presented in Section 2, highlighting, particularly, the relationship between investment strategy (i.e. risk profile) and capital requirements.

1.4.2 Section 3 introduces the investment model used for the stochastic simulations presented in later sections and highlights some of the issues to be taken into account when interpreting the results and drawing conclusions.

1.4.3 In order to compare the price of the capital-backed approach with that of the derivative-based alternative presented in Section 6, it is necessary to quantify the extent of capital support required under the former method. Section 4 presents the results of a stochastic investigation into this, with Section 5 discussing various approaches to charging for the capital levels derived.

1.4.4 Section 7 compares the approaches described in Sections 5 and 6, showing, in particular, the range of policyholder returns that the stochastic model projects.

1.4.5 Section 8 makes some concluding remarks.

2. The Capital-Backed Approach to With-Profits Business

2.1 *Two Familiar Alternatives*

2.1.1 As described above, the authors see the writing of with-profits business essentially as the provision of some form of increasing investment guarantee with ultimate payouts that reflect equity performance to some (ideally significant) extent. Two approaches to delivering such a combination of investment guarantees with equity upside are commonly presented.

2.1.2 In the first, an additional amount of money (i.e. over and above the sum of the individual policy asset shares) is used to 'support' the investment strategy being followed for the asset shares (in particular, the extent to which the funds are invested in equity-type assets). These additional assets may also be available to facilitate the smoothing of payouts over time.

2.1.3 In what is sometimes known as the 'revolving fund' approach, no additional assets are assumed to exist. Instead, each cohort of with-profits business is intended to be self-supporting in terms of both investment mix and payout smoothing.

2.2 *Variations on a Theme*

2.2.1 Whichever approach is followed obviously has consequences for the running of the office and, in particular, its capital management strategy. This encompasses such issues as policy design, guarantees granted, bonus declarations and investment mix.

2.2.2 From the standpoint of providing investment guarantees with additional equity upside, however, we see these two strategies as being variations on the same theme, rather than two dramatically different approaches.

2.2.3 Essentially, the office is choosing to bear the balance of the investment risk of backing guarantees, at least in part, with equity-type investments. It does this by having sufficient capital available to meet the guarantees (or whatever is considered to represent PRE), even under adverse investment experience.

2.2.4 When an individual policy is being considered, as it is in this paper, it is immaterial whether the capital support comes from an estate or not. For the purposes of this paper, therefore, we will not distinguish between the existence and non-existence of an estate, but refer to both of these strategies as the 'capital-backed' approach.

2.3 *The Need to Quantify Capital Requirements*

2.3.1 Obviously, the more capital there is available (either from an estate or from asset shares, or both), the greater the investment risk that can be run. However, quantifying, for a given amount of capital, just how far the investment mix of the asset shares can be biased towards equity-type investments, although very topical, is not straightforward.

2.3.2 Where an office is involved in capital-raising activities, whether through demutualisation, reinsurance financing or securitisation, to improve the financial position of a block of with-profits business, the extent to which the investment mix can be less constrained is often the key reason for the exercise.

2.3.3 At the opposite end of the spectrum, as it were, the recent report of the Closed Fund Alternative Working Party (Hairs *et al.*, 1999) highlighted similar issues. In this case, though, the problem is quantifying the extent to which the investment mix for a closed book of business should be constrained during its run-off, including the possibility that the investment constraints could be reduced initially.

2.3.4 A third example can be found in the identification of 'orphan surplus' within a with-profits fund. In order to demonstrate that a pool of assets is not needed to back PRE in respect of the affected policies, one has to quantify the assets that are required.

2.3.5 Finally, for an ongoing business where no restructuring or capital raising is envisaged, the possible approaches to charging asset shares for whatever capital support is given will be limited if no mechanism exists to quantify the capital involved. For the purposes of this paper too, it is necessary to quantify what capital support is needed to back a particular guarantee, in order that an appropriate charge for it can be compared with the price of the derivative-based approach which will be introduced later.

2.4 *Methods of Quantifying Capital Requirements*

2.4.1 Perhaps the most common approach to this problem, particularly since the publication of the Maturity Guarantees Working Party's report in 1980, is to carry out some form of stochastic simulation.

2.4.2 An alternative would be to perform a series of deterministic projections on specially chosen 'worst-case' scenarios.

2.4.3 The former requires the choice of an investment model, along with all the relevant assumptions and parameters. It also involves the identification of the probability of failure to meet guarantees which the office is prepared to tolerate.

2.4.4 The latter approach requires similar choices, although, in this case, volatility in investment returns and acceptable risks of failure are combined (perhaps implicitly) in the selection of the deterministic worst cases to be withstood.

2.4.5 In this paper stochastic projections have been used.

3. Some Comments on the Stochastic Modelling used

3.1 *A Word of Explanation*

The authors recognise that there is a wide range of experience and views on stochastic methods in the United Kingdom actuarial profession. In order to keep the paper accessible to a wide audience, the following chapter presents a brief overview of the model used and the key points to bear in mind when drawing conclusions from the results.

3.2 *Means and Tails*

3.2.1 The expected cost of a liability to an office and the reserve needed to be $x\%$ sure of being able to meet it are not necessarily the same, or even close, particularly where x is close to 100. While both involve assumptions about future experience, the former focuses on the mean (and how it might change over time), whereas the latter is concerned with both the mean and the variance (in particular the length of the lower tail) of the likely distribution of outcomes.

3.2.2 For risks such as mortality, a reserve based on a prudent assumption of future mean experience is usually sufficient, provided that due allowance is made for possible future changes in experience. (These may result from, say, changes in underwriting practices or, perhaps more significantly, general changes in human mortality.) Claims volatility is rarely an issue, for several reasons:

(a) The distribution of possible mortality rates at any age is closely grouped around the mean.
(b) Where lives are independent, volatility in mortality experience decreases with portfolio size.
(c) Reassurance can be used to cater for 'outlying risks', where the office does not have sufficient exposure for the mean to be relevant.

3.2.3 Where the distribution of possible outcomes is wider, however, reserves that give a high probability of being sufficient to meet the associated obligations can be considerably more than the mean. For example, in their study of Canadian maturity guarantees (for segregated fund business, akin to U.K. unit-linked business), Boyle & Hardy (1996) presented figures showing reserves up to fifteen times the expected cost of the corresponding guarantees.

3.2.4 Not only is the distribution of possible investment returns widely spread out, but the risk is such that the variance in possible outcomes does not decrease with an increase in the number of similar policies in the portfolio (where 'similar' particularly refers to the dates on which guarantees fall due). Appropriate diversification of assets may reduce the volatility of the portfolio, but the size of the portfolio does not. In other words, the more policies with a particular guarantee, the greater the cost to the office — in direct proportion. Given the current experience with guaranteed annuity rates granted when interest rates were higher (along with mean mortality rates), this is not a point which needs elaboration.

3.2.5 Thus, in modelling the consequences of an office providing with-profits guarantees through the capital-backed approach, we will be particularly concerned with the tails (principally the lower tail) of the distribution of possible investment performance.

3.2.6 This is akin to restricting attention to deterministic worst-case investment scenarios, but a stochastic approach to this problem has the advantage that the rest of the distribution of investment outcomes can then be used to give an indication of the likely spread of maturity values that a policyholder may receive.

3.2.7 Of course, this then leads into one of the main criticisms of stochastic modelling; that the results from using a model are just that — results from one particular model of the future — and are simply a function of the underlying assumptions one has made.

3.3 *The Choice of Model*

3.3.1 For the purpose of this paper, it is not necessary that the 'right' model of future investment returns be used, even if such a thing exists. Rather, 'something sensible' will suffice, so long as the particular characteristics of the model are borne in mind when interpreting the results.

3.3.2 There are many different models of investment performance from which to choose, although not all are fully in the public domain.

3.3.3 Perhaps the best known model, at least in actuarial circles, is that developed by Wilkie. It is well documented and used widely. Its particular characteristics are well understood, and it is easily adaptable to different economic outlooks.

3.3.4 It is this model, as presented in Wilkie (1995), but adjusted to reflect a low inflation environment, which the authors have used for this piece of research.

3.3.5 The authors recognise that the use of a different model would have given different numerical results, but are confident that the main messages from the paper would have remained unchanged. (The results of sensitivity tests are available from the authors.) In any case, it is not the absolute size of the numbers that matters. Rather, so long as the modelling is internally consistent (for example, in terms of asset share projections, statutory valuation bases, guarantees and bonus declarations), it is their shape and the relationships between them that are important.

3.3.6 Before concluding this section, it should be noted that Wilkie's model is what is known as 'autoregressive', or 'mean reverting'. In other words, the further away from the mean of its distribution one year's observation is, the more likely it is that next year's observation will be closer to the mean. The effect this has in practice will be considered further in various sections.

3.4 *Adjusting the Model for Low Inflation Conditions*

3.4.1 Wilkie developed his model from a study of U.K. investment returns over the period 1923-1994. By investigating possible relationships between certain key variables, he built up a complex structure of inter-relatedness. He then incorporated a random element in order that appropriate projections of future investment returns could be made.

3.4.2 Wilkie's model is not limited to the U.K. market and its historic performance. By using appropriate values for the 37 parameters within the model, other markets, or the U.K. in a variety of different economic conditions, can be simulated.

3.4.3 The parameters cover both mean returns expected (in terms of forces of interest) and the corresponding standard deviations, along with a correlation structure encompassing the principal economic measures. In his 1995 paper, Wilkie presented a selection of input parameter assumptions based on historical analyses for a variety of countries. Using the U.K. set as a basis for future projections gives a (geometric) mean equity return of 11% p.a., mean inflation of 4.8% p.a. and mean return on Consols of 8%. The authors felt that these means were too high for use in a study of guarantee pricing in current conditions, and that assumptions that reflected the expectation of continuing low inflation would be more appropriate. "An important feature of the way I believe this model should be used", writes Wilkie in the same paper, "is that those using it should form their own opinions about the choice of appropriate mean values".

3.4.4 The authors chose to alter just the parameters relating to mean returns, and to leave the corresponding standard deviations and correlation structure unchanged. While it may seem appropriate to have altered the latter in some way, it was not clear to the members of the group that returns were currently, or could be expected to be in the future, any less volatile than in the past. On the other hand, increasing the standard deviations from the historically derived figures would only have made the capital requirements presented below even greater. Rather than 'over-sell' the conclusions in the paper, the authors felt that it was better to leave different views of future variance to be incorporated when interpreting the results.

3.4.5 The parameters changed were the following:

	Value used (%)	Wilkie 1995 value (%)
Mean annual force of inflation (QMU)	2.40	4.70
Mean annual dividend yield (YMU)	2.20	3.75
Mean real dividend growth rate p.a. (DMU)	2.55	1.60
Mean long-term real interest rate p.a. (CMU)	2.50	3.05

It can be seen that, with a dividend yield similar to current levels, an increase had to be assumed in the mean annual rate of real growth in dividends in order to get a mean equity return that was felt to be reasonable.

3.4.6 The resulting projected mean inflation rate and mean nominal rates of return by asset class are the following:

	Annual rate (%)	Wilkie 1995 rate (%)
Inflation	2.5	4.8
Cash	4.2	6.6
Consols (i.e. undated gilts)	5.0	8.0
Equities	7.6	11.0

3.4.7 It is obvious that, for unchanged guarantee levels (and volatility of returns), higher assumed means would result in lower capital requirements in Section 4. Nevertheless, the authors feel that the above returns are not unreasonable long-term assumptions for the purposes of this paper. Those readers who would have preferred slightly higher assumptions are referred to sensitivity results available from the authors.

3.5 *Using the Model in Practice*

3.5.1 In the interests of simplicity, but without losing more generic applicability, this research has chosen to distinguish between only two asset classes, namely gilts and U.K. equities. While the authors recognise that offices usually include cash, debenture and property holdings in the assets backing their with-profits liabilities, it was felt that restricting attention to just two asset classes should be sufficient for the purpose of this paper. The proportion of a fund invested in equities will be referred to as the 'equity backing ratio' (EBR).

3.5.2 Equity returns have been modelled as described in the previous section. For fixed-interest returns, however, a simpler approach has been used.

3.5.3 Wilkie's model does not attempt to project a yield curve shape for fixed-interest returns. Instead, returns on cash and undated gilts are produced, and the derivation of dated gilt performance left to the user. A variety of approaches can be used for this, ranging from the assumption of a straight line to more complicated shapes, possibly even varying over time.

3.5.4 In considering the amount of capital required to back a particular guarantee, the key assumptions are the mean and variance of the assets in which the capital is invested. Gilts investment involves some uncertainty, since the market prices of future purchases are unknown. However, stochastic modelling of gilts returns would complicate the interpretation of the results without adding much to the conclusions.

3.5.5 Were it considered appropriate to model some degree of negative correlation between fixed-interest and equity returns, then the authors accept that a deterministic approach to gilts would result in modelled fund return volatility being too high. Given, however, that past U.K. experience has tended to show a positive correlation between equity and bond total returns, the approach adopted (which, in effect, is an assumption of exact matching of outstanding terms) should be sufficient for this paper. (In fact, the authors found that the model produced a slightly negative correlation between undated gilt returns and equity returns over a ten-year period. Over very short periods, however, slightly positive correlation was observed.)

3.5.6 Thus, for all the results shown in Section 4 onwards, any gilts holdings are projected deterministically with an annual rate of return of 5.0%.

3.5.7 Due to the assumption of a higher mean return for equities than gilts, the EBR of a fund can be expected to increase over time. In what follows, there will be times when this will be allowed to happen and times when it will not. In the latter case, the assumption of annual rebalancing of the investment mix will be referred to as 'rebasing'.

3.5.8 Rather than alter the underlying model to allow for, say, monthly time periods, this study retains a yearly time period. A consideration of day-to-day office solvency would, of course, require some more dynamic approach. For an investigation into guarantee pricing, however, annual simulations are sufficient. The only slight drawback is that, in modelling the maturity values of regular premium business, part of the effect of what is known as 'pound-cost averaging' is lost by the use of annual rather than of monthly premiums.

3.5.9 In what follows, when reference is made to the investment growth projected, this will be expressed in terms of an annualised growth rate which would have produced the same maturity value as the corresponding stochastic projection.

3.5.10 As already mentioned, of particular relevance to this study is the size of the lower tail of the distribution of modelled equity returns. In other words, attention will be given to the bottom 'so many' simulations from each set. Where only 1,000 simulations were run for each scenario, it was found that the sparseness of the tails could give rise to anomalous comparisons. As the '100% EBR' line in Figure 3.1 shows, even with 10,000 10-year simulations, that the results are not completely smooth.

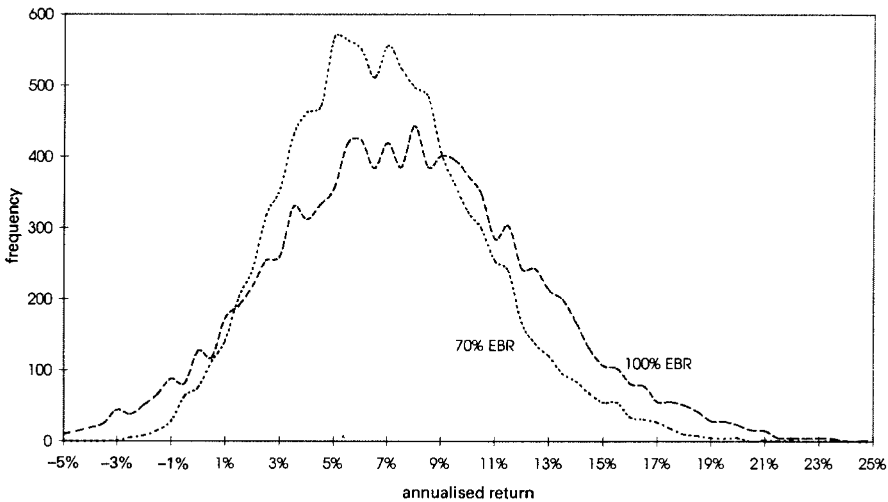

Figure 3.1. Distribution of annualised returns from 10,000 simulations

With 20,000 simulations smoothness improved, but not sufficiently to justify the considerable increase in calculation time. In order to keep the computing time manageable, all the results that follow are based on 10,000 simulations.

3.6 *Interpreting some Initial Results*

3.6.1 As can be seen from Figure 3.1, when the EBR is reduced to 70% the results are intuitively reasonable. Despite the absence of rebasing the investment mix each year, the distribution of annualised return over 10 years is less spread out, but with a lower mean.

3.6.2 *Distribution percentiles*

3.6.2.1 An alternative approach to presentation is to consider the percentiles of the distribution of annualised returns. Figure 3.2 shows how some of the more significant percentiles vary with the EBR assumed for the same 10-year example (again without rebasing). Various observations can be made from this.

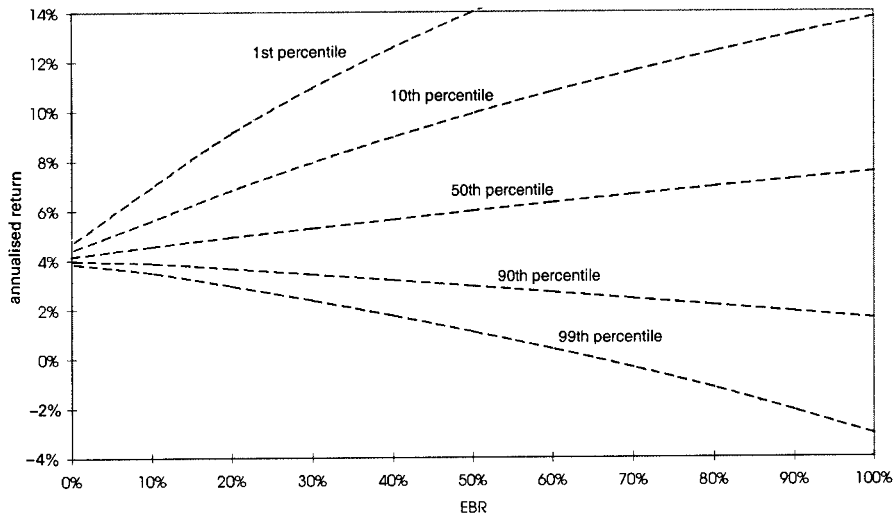

Figure 3.2. Distribution of annualised returns from 10,000 simulations

3.6.2.2 Not surprisingly, the median annualised rate of return increases with EBR, as, in fact, does the mean annualised return, reflecting the different mean returns assumed for gilts and equities.

3.6.2.3 The extent of the volatility in equity returns produced by the Wilkie model (and the parameter assumptions used for this paper) can be seen in the way that the percentiles become more and more spread out as EBR increases. (As a result, for EBRs of 70% or more, a negative return over 10 years was recorded in at least 1% of simulations.) Were the model not auto-regressive, the spread in modelled returns would be even greater.

3.6.2.4 Another influence on the spread of modelled returns may be the small degree of correlation between equity and gilt total returns in the model, since, for the purposes of this graph, the gilts portion was projected stochastically on the assumptions of perfect matching of outstanding term, a straight line yield curve and a cash return of 4.2%.

3.6.2.5 The median gilt return is a little over 4% (much less than the mean of 5% given in ¶3.4.6) as a result of these yield curve and term assumptions.

3.6.2.6 The relative importance of the volatility in returns from gilts compared with that for equity returns can be seen clearly by comparing the spread of the percentiles on the left hand side of the graph with that on the right. This confirms that the decision to model gilts deterministically throughout the modelling does not significantly detract from the conclusions drawn.

3.6.3 *Implications for guarantee levels*

3.6.3.1 Expressing the results shown in Figure 3.2 in terms of the ability to meet particular levels of guarantees makes interesting reading. In Figure 3.3 the percentage of runs that failed to produce an annualised rate of return over 10 years above each of 0%, 1%, 2%, ..., 5% is plotted for EBRs of 50% and above.

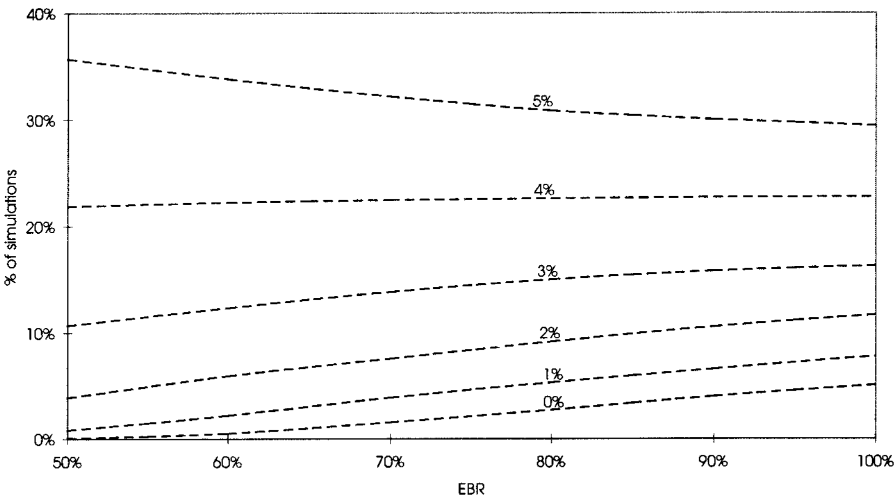

Figure 3.3. Proportion of annualised returns from 10,000 simulations which fail to meet a defined level

Several aspects of the results should be noted.

3.6.3.2 The extent of upward slope of each line reduces with increase in target rate of return. Perhaps a better way of expressing this is to say that increasing the target rate of return has a proportionately greater effect the smaller the EBR assumed.

3.6.3.3 As EBR increases, so does the mean rate of return (see ¶3.6.2.2), thereby reducing the proportion of simulations which fail to produce an annualised return of at least 5%.

3.6.3.4 The increased volatility that accompanies the increased EBR, however, results in a greater spread of annualised returns, thereby increasing the number of simulations that fail to make each of the lower target rates.

3.6.3.5 The number of simulations that fail to return, over 10 years, even the original amount invested is non-zero for almost all the EBRs featured above. Indeed, the proportion of simulations failing to make what many investors would consider modest rates of return is quite striking. While it must be stressed that these results are 'only' the results of the model and its assumed parameters, they, nevertheless, give some food for thought.

3.6.4 *Rebasing 'distortions'*

In all the stochastic projections that follow the asset mixes of the funds being projected are never rebased to the starting EBR. Doing so would result in artificially high returns because of the use of an autoregressive model. Each year the disinvestment would be from the better performing asset into the poorer performer (at least, as far as the immediately preceding year was concerned). Where the former had exceeded the mean of its distribution that year, it would be more likely to perform less well in the following year. Conversely, where the poorer performer had underperformed the mean of its distribution, its performance would be more likely to improve. While such a situation may represent an investor's dream, the authors felt that it was rather too optimistic to model!

4. Capital Backing and Probabilities of Shortfall

4.1 *Introduction*

4.1.1 This section presents the results of the stochastic modelling of capital requirements introduced in Sections 2.3 and 2.4. It starts by considering, in some detail, single premium conventional with-profits business (CWP), for a variety of EBRs and durations in force. A summary of the results for corresponding annual premium examples is then presented, with the main consideration placed in Appendix B. The section concludes by extending the analysis to single premium unitised with-profits cases (UWP).

4.1.2 The tables that follow show results for policies with 10-year original terms. The equivalent figures for policies with other original terms are available from the authors. They tell a similar story, and so would add little to the conclusions of the paper.

4.1.3 The key quantity to measure is the probability that there are insufficient assets at maturity to meet the guarantee — the probability of shortfall.

4.1.4 The analysis of the conventional examples starts by deriving the probabilities of shortfall in the case where the assets at the start of the projection period are equal to the sum of the minimum reserve (including resilience reserve) and the required minimum margin under current U.K. reserving regulations.

While the absolute numbers should be treated with caution, the shape of the results is well worth considering.

4.1.5 Amounts of starting assets required to meet a target probability of shortfall are then derived, and, in order to enable chargeable support to be identified, expressed as percentages of corresponding asset shares. Where the percentage is greater than 100, it is assumed that the additional assets (i.e. the capital support that is needed) will be invested with the same EBR as the associated asset share.

4.1.6 In order to produce meaningful results, all the various assumptions that need to be made must be consistent. How this has been addressed is the subject of Section 4.2.

4.2 *Ensuring Consistency and Transparency*

4.2.1 Results are given for every duration t, from outset to the anniversary one year before maturity (i.e. $0 \leq t \leq 9$). What is shown, for each t, is the result of the projection (basically, stochastic for equities, deterministic for gilts) from time t until time 10.

4.2.2 For some of the tables of projections, all that is required is to know the level that the maturity guarantee has reached by time t. For other tables of results, corresponding asset shares are also required.

4.2.3 Generating these values stochastically at each time t could have been done, but introducing such additional complexity would not have added anything to the points being illustrated, just made the results much more complicated to interpret! A simpler approach has, therefore, been used to generate the starting values.

4.2.4 For each current duration t, the asset share has been accumulated from outset deterministically using a 7.6% annual return on equities and 5% p.a. return on gilts, weighted by the appropriate EBR (thereby assuming rebasing of the asset mix each year). The projection from time t onwards does not incorporate rebasing of the EBR, thereby avoiding the artificial inflation of returns described in ¶3.6.4.

4.2.5 The levels of initial sums assured for the conventional policies (taken to be the accumulation of premiums to maturity at 1% p.a.) have been set reasonably low compared with the mean rates of return being assumed.

4.2.6 The same two-tier reversionary bonus rates have been assumed for both single and annual premium conventional cases for all EBRs. At 1% of sum assured and 2% of attaching bonus, the rates are obviously more onerous (compared with deterministic asset share growth) the lower the EBR. Nevertheless they seem reasonably plausible, as well as broadly consistent with the other assumptions being made. These rates of reversionary bonus are assumed to apply at every policy anniversary up to and including time t. During the stochastic projection no further bonuses are assumed to be declared. (The authors recognise that this may not sit well with PRE in all situations, particularly in the cases where the bonus rates are relatively high compared with modelled returns, but is sufficient for the purposes of the paper.)

4.2.7 As mentioned in ¶4.1.4, the initial sets of results are not based on the accumulated asset share at time t, but take as the asset value the sum of the statutory minimum reserve (including resilience reserve) and the required minimum margin. In calculating the reserve, the EBR of the asset share has been allowed for in deriving the appropriate yield to use. No attempt has been made to maximise the yield by efficient hypothecation of assets. Thus, while the reserves may be slightly higher in some cases than the minimum an office would be permitted to publish, they are consistent with the EBRs assumed at the start of their projection to time 10.

4.2.8 The authors would not disagree with readers who feel that these assumptions are somewhat simple and idealistic. They have been deliberately set that way, since the more easily what has been modelled can be grasped, the more straightforward it will be to think through the implications for more complicated scenarios.

4.3 *Modelling the Strength of Statutory Minimum Reserves for Conventional Business*

4.3.1 *Introduction*

As stated in ¶4.1.1, the results given in this section concentrate on single premium business. The equivalent annual premium results are summarised briefly with the various tables presented in Appendix B.

4.3.2 *Shortfall probabilities*

4.3.2.1 Table 4.1 presents the 'probabilities of shortfall', i.e. the probabilities that the accumulation, for $10-t$ years, of the sum of the statutory reserve (including resilience reserve) and required minimum margin at time t is less than the sum assured and bonuses attaching at time t (i.e. the amount at time t which is guaranteed to be paid at maturity). While the absolute values of these figures are probably not completely unrealistic, it is the relationship between them that is of particular interest.

4.3.2.2 For most durations, as EBR increases, so does the probability of shortfall. In other words, the statutory reserving basis is not uniformly strong across different investment strategies (for the same level of guarantees).

4.3.2.3 For the two latest durations, the shape of each row is not so straightforward, with the probabilities for the higher EBRs being less than those for the lower ones. The reason lies partly in the assumptions made and partly in the way that the reserving basis being used allows for equity investment.

4.3.2.4 Reserves are calculated using a discount rate which is the weighted average (in line with EBRs) of 4.9% and 2.4% (being 97.5% of 5% and 2.5%). The gilts proportion of the reserve is then projected forward at 5%, thereby creating a small surplus at maturity. The equity proportion, on the other hand, is stochastically projected using a distribution with a much higher mean. The greater

Table 4.1. Probability of shortfall at maturity (CWP, with resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	2.7%	3.2%	3.7%	4.2%	4.4%	4.9%
1	3.9%	4.6%	5.2%	5.7%	6.1%	6.5%
2	5.1%	5.8%	6.3%	6.9%	7.3%	7.7%
3	5.8%	6.3%	6.7%	7.2%	7.7%	8.0%
4	6.6%	7.0%	7.3%	7.7%	8.1%	8.5%
5	7.6%	7.9%	8.1%	8.5%	8.8%	9.1%
6	8.3%	8.4%	8.6%	8.8%	9.0%	9.2%
7	8.0%	8.0%	8.0%	8.1%	8.3%	8.4%
8	7.3%	7.0%	6.9%	6.9%	7.0%	7.1%

the EBR, the greater the volatility in maturity value, but also the greater the expected return and the lower the valuation yield. It therefore becomes more likely that growth at least equal to the valuation rate of interest will be projected as EBR increases. These different 'forces' are at work in all the rows, and, as the outstanding duration changes, so does their relative strength.

4.3.2.5 As might have been expected, for each EBR, the probabilities of shortfall increase (at least initially) as t increases and the duration over which equities are held to back a guarantee reduces. Intuitively this seems reasonable, particularly for results from an autoregressive model. Basically, the shorter the outstanding duration, the fewer the years over which, it is hoped, good performance from equities may be received to compensate for any poorer years.

4.3.2.6 That the probabilities do not continue to increase as outstanding duration decreases is testament to the security provided by the resilience reserve.

4.3.3 *Removing the resilience reserve*

4.3.3.1 Table 4.2 presents the equivalent probabilities of shortfall when the statutory minimum reserves are calculated without the resilience reserves.

4.3.3.2 As expected, the probabilities of shortfall increase with expired duration, thereby indicating the extent of strengthening in the previous table provided by the resilience test.

4.3.3.3 All the figures in Table 4.2 are greater than the corresponding ones in Table 4.1, but the extent of the increase reduces as EBR increases. In other words, the resilience test, when combined with the modelling assumptions, seems to be more onerous on gilt investment than on equity. The use of a weighted average valuation yield (which results in the post-resilience-scenario reserve not moving in line with either of the two asset types in the backing assets) will certainly be affecting the results. Nevertheless, it does not appear unreasonable, particularly for the earlier durations, that 300 basis points on top of an annual gilt yield of 5% should prove stronger than assuming a one off 25% drop in equity values (this being the resilience test that generally proved the most stringent).

Table 4.2. Probability of shortfall at maturity
(CWP, without resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	10.1%	10.8%	11.2%	11.5%	11.6%	11.9%
1	13.6%	14.1%	14.7%	15.0%	15.1%	15.4%
2	16.3%	16.8%	17.1%	17.3%	17.5%	17.6%
3	18.4%	18.9%	19.2%	19.4%	19.7%	19.8%
4	20.5%	20.9%	21.3%	21.4%	21.5%	21.7%
5	23.4%	23.5%	23.7%	23.8%	23.9%	24.1%
6	26.0%	26.1%	26.2%	26.3%	26.4%	26.4%
7	29.7%	29.5%	29.5%	29.4%	29.5%	29.5%
8	32.7%	32.2%	32.0%	31.8%	31.8%	31.8%
9	35.8%	34.8%	34.2%	33.9%	33.7%	33.7%

4.3.3.4 There is again a change in shape towards the later rows as the different discount rates and projection assumptions inter-relate to favour, slightly, the higher EBR cases.

4.3.3.5 The greater similarity between the different columns is the result of using only the dividend yield (rather than total return) for the equity-related contribution to the weighted-average valuation yield, thereby creating a form of resilience reserve within the 'basic' statutory minimum reserve.

4.3.4 *Asset share comparison*

4.3.4.1 It is worth considering the relationship between the statutory reserves calculated (including the resilience reserve) and the deterministic asset shares, shown in Table 4.3.

4.3.4.2 For each duration the proportions increase with EBR, although not quickly enough to avoid the probabilities of shortfall also rising.

4.3.4.3 With some of the proportions greater than 100%, the results suggest that with-profits business cannot be considered always to provide capital to an office, even on a published free assets basis.

4.3.4.4 The use of minimised reserves (see ¶4.2.7) would have made very little difference to the figures (the greater the proportion of gilts, the greater the effect, but, for the 50% EBR, the absolute reductions in Table 4.3 figures range from only 3.5% down to 1.8%) and certainly would not have changed the previous conclusion.

4.3.5 *Some tentative conclusions*

4.3.5.1 Even though relatively few results have been presented so far, certain tentative conclusions can be made.

4.3.5.2 First, that the statutory valuation basis, including the three resilience tests as well as the required minimum margin, is not uniformly strong over either policy duration or reserve investment mix. (While the probabilities of shortfall at

Table 4.3. Statutory reserve as a percentage of deterministically accumulated asset share (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	88.4	92.5	96.9	101.6	106.4	111.3
1	87.1	90.8	94.7	98.9	103.4	107.8
2	85.7	89.1	92.7	96.4	100.4	104.5
3	84.4	87.5	90.7	94.0	97.6	101.3
4	83.2	85.9	88.7	91.7	94.9	98.2
5	81.9	84.3	86.9	89.5	92.3	95.2
6	80.7	82.8	85.0	87.3	89.8	92.3
7	79.6	81.4	83.3	85.3	87.3	89.5
8	78.5	80.0	81.6	83.2	85.0	86.8
9	77.4	78.6	79.9	81.3	82.7	84.2

maturity presented here may seem rather high, the absolute numbers are dependent, to an extent, on the model and assumptions chosen. In addition, as mentioned in Appendix A, the risk to an office of a given policy, within the context of its portfolio, is not necessarily the same as when that policy is viewed in isolation.)

4.3.5.3 Second, that the notion sometimes presented, of with-profits business providing capital to an office need not always be the case.

4.3.5.4 Of course, in many other cases, there is capital available within the asset share that could (subject, in practice, to the smoothing policy of the office) be considered to be available to help meet guarantees, and hence reduce the probabilities of shortfall, as shown in Table 4.3.

4.3.6 *Annual premium business*

4.3.6.1 As can be seen from Appendix B, the results for corresponding annual premium cases are similar, and confirm the tentative conclusions above.

4.3.6.2 The general shape of the results is the same, although the annual premium case gives rise to a greater spread in the size of the shortfall probabilities.

4.3.6.3 Unlike the single premium case, though, the annual premium statutory reserves almost always exceed the corresponding modelled asset shares.

4.3.7 *Comparing the single and annual premium examples*

4.3.7.1 A comparison of the annual and single premium results must be made with care, because the levels of initial guarantee are different and, particularly, because, in the annual premium case, part of the guarantee relates to money not yet received.

4.3.7.2 There are also durational differences to consider, with the annual premium contract being of shorter discounted mean term than the corresponding single premium.

4.3.7.3 The greater range of shortfall probabilities for the annual premium case highlights an interesting consequence of using a net premium valuation. The sum assured modelled is based on a 1% interest rate, but the valuation rate (reflecting the different equity proportions) is higher, and so the net premiums used in the statutory reserves are smaller than the gross figures. Thus, effectively, the annual premium statutory reserves include an expense provision which, in the stochastic projections, is used to help offset investment volatility and hence reduce shortfall probabilities.

4.3.7.4 Thus, not only could the investment risks that an office runs between its single and annual premium portfolios be very different, but the published reserves need not give the whole picture. The implications for inter-office comparisons, on the basis of Form 9 free asset considerations, are clear.

4.4 *Capital-Backing for Conventional Business with a Standardised Shortfall Probability*

4.4.1 *The choice of standardised shortfall probability*

4.4.1.1 There is probably no 'right answer' to what probabilities of shortfall should be tolerated for with-profits business. For the purposes of the comparison between the capital-backed and derivative-based approaches to providing the guarantee, a consistent figure is required which, so long as it is 'sensible', should not influence the conclusions unduly. In keeping with the work done by the Maturity Guarantees Working Party, the authors have chosen to use a probability of 1%.

4.4.1.2 It could be argued that 99% security is too high to target for individual policies within a portfolio, since the resulting shortfall probability for the portfolio as a whole is likely to be less than 1%. The authors would not disagree, but stress that, as Appendix A develops, the extent of any portfolio benefit is a function of the structure of the portfolio and, particularly, the spread of guarantee vesting dates. It has already been noted, in ¶1.3.5, that some tranches of policies might give rise to no portfolio benefits.

4.4.1.3 The quantification of possible portfolio benefits is outwith the scope of this paper, as is a study of the issues surrounding their distribution.

4.4.1.4 Obviously the capital requirements presented below would have been different if another shortfall probability had been used, but the same is true for all the other modelling assumptions. The authors hope that the generic points developed will prove of interest, even if the size of some of the numbers is more than some readers would feel comfortable with.

4.4.2 *Single premium CWP business capital support*

4.4.2.1 Table 4.4 presents the results for the 10-year single premium case introduced in Section 4.3. What is shown, for each duration t, is the assets, expressed as a percentage of the deterministic asset share at time t, required to give a 99% chance of having sufficient assets at time 10 to meet the maturity guarantee applying at time t.

Table 4.4. Proportion of asset share needed for 1% shortfall probability (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	93.5%	101.2%	110.3%	121.1%	134.3%	150.7%
1	95.0%	103.1%	112.7%	124.5%	139.0%	157.4%
2	94.6%	102.4%	111.7%	122.9%	136.8%	154.4%
3	94.5%	102.0%	111.0%	121.9%	135.5%	152.6%
4	93.2%	100.0%	108.0%	117.7%	129.6%	144.5%
5	92.4%	98.7%	106.1%	115.1%	126.0%	139.5%
6	90.9%	96.5%	103.1%	110.9%	120.2%	131.7%
7	88.8%	93.4%	98.8%	105.1%	112.5%	121.4%
8	86.3%	89.9%	94.0%	98.7%	104.2%	110.7%
9	80.9%	82.7%	84.7%	86.9%	89.5%	92.3%

It is worth drawing attention to several points.

4.4.2.2 As might have been expected, for a given duration, the amount of capital required for '1% security' increases with EBR, reflecting the increased volatility in the assets while the guarantee level is left unchanged.

4.4.2.3 Most of the percentages are greater than 100, some considerably so. On these assumptions, it is only for lower EBRs that any surplus capital is available from asset shares to fund other aspects of the office's activity.

4.4.2.4 Apart from a small initial rise, the percentages decrease as duration increases, with the rate of decrease greater the larger the EBR. This is a consequence of the guarantee growing from outset to time t at a lower rate than the deterministic asset share. For each extra year of the pre-stochastic starting value accumulation, the asset share is modelled to grow at the appropriate weighted average of 5% and 7.6%, whereas the guarantee is modelled to grow at a little over 1%.

4.4.2.5 Table 4.4 will be used later in the paper when comparing the effects of different approaches to the charging for capital support with the costs of the derivative-backed alternative.

4.4.3 *An alternative presentation*

4.4.3.1 As an aside, when the amounts of capital-backing underlying Table 4.4 are re-expressed as percentages of the corresponding minimum statutory reserves (with resilience and required minimum margin), the following picture emerges. While this table is effectively a combination of Tables 4.3 and 4.4, it does give rise to some interesting points in its own right.

4.4.3.2 For a given duration, the larger the EBR, the greater the required uplift on the statutory minimum reserve (including resilience reserve and required minimum margin), which is consistent with the previous conclusions about the relative treatment of gilts and equities in the statutory approach.

4.4.3.3 For each EBR, as outstanding duration decreases, the statutory reserve does not grow as fast as the asset share, and, unlike the asset share (on the

Table 4.5. Ratio of 1% reserve to statutory minimum reserve (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	105.8	109.4	113.8	119.2	126.2	135.4
1	109.1	113.5	119.0	125.9	134.4	146.0
2	110.4	114.9	120.5	127.4	136.3	147.8
3	111.9	116.7	122.4	129.7	138.8	150.7
4	112.0	116.4	121.8	128.3	136.6	147.2
5	112.7	117.0	122.2	128.6	136.5	146.5
6	112.6	116.5	121.2	126.9	133.9	142.7
7	111.6	114.8	118.6	123.3	128.8	135.6
8	109.9	112.3	115.2	118.6	122.6	127.4
9	104.5	105.1	106.0	107.0	108.2	109.6

modelling assumptions being used), never reaches the level required to give 99% security of meeting the guarantee.

4.4.4 *Some comments on the equivalent annual premium results*

4.4.4.1 As was the case with the statutory reserving results, the annual premium results (which are given in Appendix B) tell a similar story to the single premium case.

4.4.4.2 One significant difference to note is that the capital support required over the asset share does not reduce so quickly with increasing duration in the annual premium case. This is a consequence of the guarantee being related to all the premiums payable, whereas the asset share, at any time, only reflects the premiums that have been paid to date (which has the effect of reducing the rate at which the asset share 'out-grows' the guarantee). This would tend to suggest that, all other things being equal, single premium conventional with-profits business should be less capital intensive than annual premium business (even without allowing for any difference in initial expense levels).

4.4.4.3 The additional strength within the net premium reserves already noted can also be seen.

4.5 *Extending the Standardised Shortfall Probability Capital Backing to Unitised With-Profits Business*

4.5.1 At the time of writing, discussions are continuing regarding what changes should be made to the statutory valuation regulations, particularly as far as unitised with-profits business (UWP) is concerned. Rather than produce results on a basis that could be overtaken by events, the extent of capital backing required, in order to restrict the probability of shortfall at maturity to 1%, is expressed, in this section, only in relation to the deterministically-modelled asset shares. This approach ties in with the comparative work presented later. An alternative presentation, expressing the capital support at time t as a percentage of the amount then guaranteed is included, by way of an interesting aside, in Section 4.6.

4.5.2 Only the results for single premium cases are presented in this section.

4.5.3 *Some comments on the approach to modelling used*

4.5.3.1 As in the conventional case, asset shares are assumed to accumulate to time t at the appropriate weighted average of the assumed returns on equities and gilts.

4.5.3.2 Unit values are assumed to grow at the gilt rate of return, namely, 5% p.a., irrespective of the EBR of the underlying assets. Allocation rate complexities are ignored, as are bid/offer spreads. Thus, premiums are assumed to be invested fully in units.

4.5.4 *Single premium UWP business capital support*

4.5.4.1 In Table 4.6 the maturity guarantee, which (for all single premium UWP projections) starts equal to the asset share at time 0, is assumed to grow at 5% (being a combination of the guaranteed rate plus discretionary bonus) until time t, and then at the guaranteed rate thereafter (i.e. no future discretionary bonus is assumed).

4.5.4.2 Table 4.6 shows the assets, expressed as a percentage of the deterministic asset share at time t, that are required to give a 99% chance of having sufficient assets at time 10 to meet the maturity guarantee applying at time t. While the shape is broadly similar to the corresponding conventional table (Table 4.4), various features are worth noting.

4.5.4.3 Compared with the conventional results, the percentages here start smaller, reflecting the smaller initial guarantee (the conventional case started by guaranteeing an accumulation rate throughout the term of 1% p.a.).

4.5.4.4 It takes three years for the quicker-growing UWP guarantee to 'overtake' the conventional guarantee, purely a consequence of the various assumptions made.

4.5.4.5 For the last few durations the percentages decrease, reflecting the larger number of past years in which the return on the asset share has exceeded the guarantee rate and the shorter future period of investment volatility.

4.5.4.6 Table 4.6 will be used later in the paper when comparing the effects of different approaches to the charging for capital support with the costs of the derivative-backed alternative.

4.6 *Alternative Presentation for UWP Business and More Onerous Guarantee Levels*

4.6.1 The capital support requirements in Section 4.5 are presented as percentages of the corresponding asset shares, as is appropriate for a study into possible policyholder charges. An alternative approach would be to express the same results as percentages of the corresponding unit values. While, in isolation, such a presentation would not naturally lead to an identification of capital support benefits, it would enable a comparison to be made with the reserving levels

Table 4.6. Proportion of asset share needed for 1% shortfall probability (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	84.7%	91.6%	99.8%	109.6%	121.6%	136.4%
1	89.4%	97.0%	106.1%	117.1%	130.8%	148.2%
2	92.6%	100.2%	109.2%	120.2%	133.8%	151.1%
3	96.1%	103.8%	112.9%	124.0%	137.7%	155.2%
4	98.5%	105.7%	114.2%	124.4%	137.0%	152.7%
5	101.4%	108.4%	116.6%	126.4%	138.3%	153.2%
6	103.8%	110.2%	117.6%	126.5%	137.2%	150.3%
7	105.3%	110.8%	117.1%	124.6%	133.4%	144.0%
8	106.3%	110.7%	115.7%	121.6%	128.3%	136.3%
9	103.5%	105.8%	108.4%	111.2%	114.5%	118.1%

currently being used by companies. In addition, it might also throw some light on aspects of the current discussions concerning statutory reserving requirements for UWP business.

4.6.2 Table 4.7 shows the required assets as a proportion of the unit value in the single premium case with a 0% guarantee.

Table 4.7. Proportion of unit value needed for 1% probability of shortfall (guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	84.7%	91.6%	99.8%	109.6%	121.6%	136.4%
1	90.5%	98.5%	108.0%	119.5%	133.7%	151.9%
2	94.9%	103.2%	113.1%	125.0%	139.9%	158.6%
3	99.7%	108.5%	118.9%	131.5%	147.2%	167.0%
4	103.4%	112.1%	122.3%	134.6%	149.6%	168.4%
5	107.9%	116.7%	127.0%	139.4%	154.5%	173.1%
6	111.7%	120.4%	130.4%	142.3%	156.6%	174.1%
7	114.8%	122.8%	132.1%	142.9%	155.7%	170.9%
8	117.3%	124.5%	132.8%	142.2%	153.1%	165.8%
9	115.7%	120.8%	126.5%	132.7%	139.6%	147.2%

As might have been expected, the figures increase with duration relative to those in Table 4.6, reflecting the slower growth in unit value compared with asset shares. (The underlying capital needs are, of course, the same in both tables.) The general shape of each column is still the same, but with a smaller decrease for later durations, since here no 'terminal bonus cushion' is being assumed.

4.6.3 Had the equivalent of Table 4.7 been shown for different guaranteed rates of future unit growth, the general shape would have been the same, but the percentages, particularly for longer outstanding durations, considerably larger.

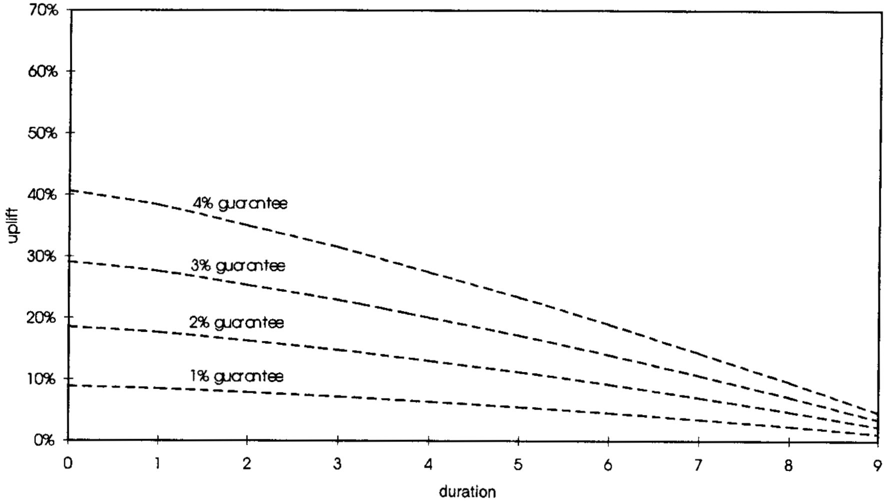

Figure 4.1. Additional percentage of unit value needed to maintain 1% probability of shortfall for non-zero guarantee rates (50% EBR)

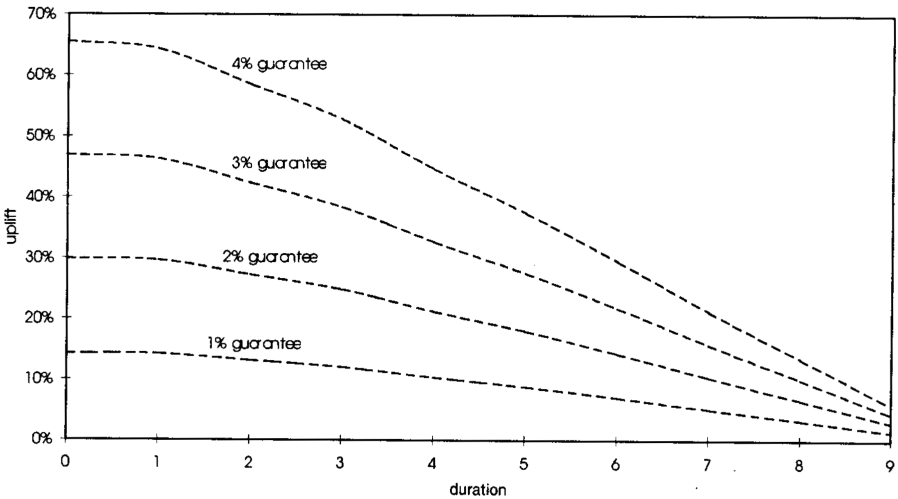

Figure 4.2. Additional percentage of unit value needed to maintain 1% probability of shortfall for non-zero guarantee rates (100% EBR)

Figures 4.1 and 4.2 show this for a variety of different guaranteed unit growth rates for, respectively, the 50% EBR and 100% EBR cases.

4.6.4 These results confirm what is intuitive. Guarantees are expensive in terms of capital requirements. This is particularly so at high EBRs, because of the greater volatility of investment returns. However, higher guarantees are relatively less onerous at longer durations, because of the shorter future periods of volatile returns.

4.6.5 While portfolio cross-subsidies may enable lower levels of capital support to be held without compromising shortfall security, the general message still remains — unit growth rates (and unit growth rate guarantees) must be kept affordable.

4.7 *Conclusion*

4.7.1 The main purpose of this section is to prepare for the comparison, in Section 7, by identifying the extent of capital support (in addition to the asset share) required by some reasonable levels of guarantee. The key tables are, therefore, Tables 4.4 and 4.6.

4.7.2 Clearly, many assumptions have been made in deriving these tables. While these may be rather artificial, the authors are confident that they are not unreasonable in current conditions.

4.7.3 Extrapolation to other situations should not be difficult — for example the case of past bonus declarations being higher than those projected in future. Another example would be where historic investment returns have enabled more substantial terminal bonus cushions to be built up than were modelled above.

4.7.4 Many of the results are for conventional business. This is appropriate, because the bulk of the in force with-profits business in the U.K. is conventional, and there may be particular relevance to demutualisation arrangements. While most new business is unitised, its statutory reserving basis is under review.

4.7.5 By covering all policy anniversaries, the results presented are applicable to both in force and new business.

4.7.6 In studies such as the one presented here, it is often not the actual size of the results which matters, but their relative shape. This is particularly so for the case for the statutory reserve experiments in Section 4.3. The reader should not conclude from those results that the authors are implying that published reserves are not appropriate, rather that a comparison of 'Form 9' free reserves may not give a complete picture of offices' relative positions.

5. Charging for Capital Backing

5.1 *Introduction*

5.1.1 The purpose of Section 4 was to quantify the extent to which capital support could be required by with-profits policies over their lifetime, in order to maintain a given level of exposure to equity investment, particularly in a low inflation environment. This section follows on from this, and considers different approaches that an office might use, should it be considered appropriate to charge the policyholder for such support. (Obviously, for the purpose of this paper, were the capital not to be charged for, then there would be little point in considering the cost of the derivative-backed approach to be presented later.)

5.1.2 To avoid numerous tables of results, those illustrated in Sections 5.4 and 5.5 have been limited to single premium 10-year policies for both the CWP case, introduced in Section 3, and the UWP case with 0% p.a. guarantee rate in the future.

5.1.3 Whenever the effects of charges on policyholder returns are shown, the capital support involved is that needed to target a 1% shortfall probability on an individual policy basis.

5.2 *Market Practice*

5.2.1 *Current approaches*

5.2.1.1 An indication of the variety of approaches currently followed by offices in the U.K. towards with-profits business and, in particular, the charges levied on asset shares can be found in the asset share survey carried out from time to time by Tillinghast-Towers Perrin (1997). The following points were drawn from these results.

5.2.1.2 Currently the majority of offices do not charge for capital backing of with-profits business, although there is an increasing trend in offices that do. No indication is given about the number of offices that measure the capital support required for different cohorts of business.

5.2.1.3 It is less common to charge for capital requirements of UWP business than CWP.

5.2.1.4 The charges are for, amongst other things, covering the cost of guarantees, covering the cost of smoothing, providing an extra return on capital or building up an estate.

5.2.1.5 Charges are deducted via reductions in investment return on asset shares or reductions in surrender or maturity values.

5.2.2 *Likely trends*

5.2.2.1 The results in Section 4 suggest that, in an environment of sustained low inflation and low expected investment returns, the amount of capital required to maintain an acceptable level of guarantee shortfall risk could be quite large (depending, of course, on the definition of 'acceptable' and the extent that any portfolio cross-subsidy was passed on to policyholders).

5.2.2.2 Added to this, the capital of life offices is coming under increasing pressure, due to such issues as reserving for guaranteed annuity options, lightening annuity mortality rates and the possible new UWP reserving requirements.

5.2.2.3 As capital becomes a more precious resource, the trend towards monitoring the use to which it is put, and the value created through it, is likely to continue.

5.2.2.4 Capital is required for a variety of purposes by a life office. Money used to back with-profits guarantees is not then available to support the writing of new business. Nor is it available to be invested without constraints, in order to maximise its expected return and hence build up the ongoing strength of the fund. In addition, capital put aside to back guarantees (or smoothing policy) may be called upon at times and so it is not just opportunity cost that is the issue. It seems natural, therefore, that more specific charging of asset shares for whatever capital support is allocated will receive more attention in the future.

5.2.2.5 Paragraph 5.2.2.4 looks at the situation where capital support is provided by the estate of the office. Where the capital is supplied by other asset shares, similar considerations apply. For one asset share to gain, another may be losing. Of course, views will differ as to the appropriateness of seeking to identify such cross-subsidy. Considering the effect of different approaches to charging for the access to another's capital need not imply that the charges must be levied in practice. Rather, the exercise could be regarded as a way of quantifying the benefits of membership of such an arrangement.

5.3 *Alternative Approaches to Charging for Capital Support*

5.3.1 *Opportunity cost*

5.3.1.1 In order to keep the research as widely applicable as possible, the authors chose to investigate two theoretical approaches to charging asset shares for the opportunity cost of their access to capital. In both cases it was anticipated that the charges would only be levied on asset shares which were less than the appropriate capital requirements for 1% shortfall probabilities derived in Section 4.

5.3.1.2 The first approach is to apply a charge equal to a proportion of the whole asset share (the 'asset share charging approach'). This is a broad-brush approach that does not specifically reflect the extent to which capital support is required at any given time. However, it is straightforward to apply, and is equivalent to the favoured method mentioned in the Tillinghast report. For the purposes of the comparative modelling work, the charge was set at (a rather arbitrary) 0.15% p.a.

5.3.1.3 The second method investigated is a more targeted approach (the 'capital support charging approach'). In this case, the charge is applied only to the capital support which is deemed to be allocated (i.e. the excess of the appropriate reserve derived in Section 4 over the asset share). It is assumed that capital support set aside is invested in the same equity proportion as the asset share, and that otherwise it would have been invested fully in equities. Capital is also presumed to be limited. The charge, therefore, has two parts:

(a) the loss of expected future equity out-performance over gilts; based on the amount of excess capital invested in gilts; plus

(b) a further charge (1% in the results shown below) as a percentage of the total excess capital to 'pay' for its use.

5.3.2 *Risk cost*

5.3.2.1 The above charges only reflect the opportunity cost of use of capital. They do not cover the cost of the guarantee itself.

5.3.2.2 In general, the extra capital support will not be required at maturity to meet the guarantee. Rather, it is held to allow the investment mismatch risk to be taken over the outstanding duration of the policy. Nevertheless, within the 99% of cases where (after allocating the appropriate supporting capital) there should be no shortfall, there will be some which require to draw on some of the support capital in order to meet the guarantee.

5.3.2.3 This was certainly the case in the modelling work carried out for this paper. For each guarantee level considered, the expected amount of capital support (i.e. in excess of asset shares) used per policy to meet guarantees was calculated, and then deducted as a 'one-off' charge at maturity.

5.3.2.4 As far as the bottom 1% of runs are concerned, the charge in ¶5.3.2.3 takes account of the cost of losing all the capital support allocated. Of course, further money will also be required in order to meet the guarantees in these worst cases. Depending on the source of this further capital, it could be argued that an additional charge should be levied. While the authors appreciate this, such additional charges have been ignored in order to maintain consistency with the derivative alternative presented in Section 6. Ignoring this across all charging methods will not affect the relative size of the results.

5.4 *The Impact of Charges on Capital Support Requirements for CWP Example*

5.4.1 The capital requirements developed in Section 4 were based simply on modelled investment returns (with no charges). Obviously, were the asset shares to grow slightly more slowly, as a result of annual charges, the capital support needs would be correspondingly greater.

5.4.2 The extent of increase in capital requirements is an indication of the severity of the charges involved. Thus, in the tables that follow, it is this 'uplift percentage' which is shown.

5.4.3 There is a feature of the asset share charging approach which is worth noting at this point. In cases where capital support is required, this charging method involves making deductions in proportion to the size of the asset share, and not in proportion to the capital support. As a result, any increase in capital support does not get reflected in increased charges.

5.4.4 The results under the two charging approaches are presented separately.

5.4.5 *Asset share charging approach*

5.4.5.1 Table 5.1 presents the results for the 10-year single premium CWP case when the asset share charging approach (at a rate of 0.15% p.a.) is used. The figures shown are the ratios of the post-charging reserves required to deliver a

1% shortfall probability to the pre-charging ones. Where a pre-charging reserve (for 1% shortfall probability) is less than the corresponding asset share, no capital backing is needed and therefore no charges are imposed. Thus, that reserve is unchanged from Table 4.4 and 100% is shown. Certain features of Table 5.1 are worth noting and are covered in ¶¶5.4.5.2 to 5.4.5.6.

Table 5.1. Capital required if asset share charging employed as a proportion of reserves for 1% shortfall probability with no charges (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	101.4%	101.5%	101.5%	101.6%	101.7%
1	100.0%	101.2%	101.2%	101.2%	101.2%	101.3%
2	100.0%	101.3%	101.3%	101.4%	101.5%	101.6%
3	100.0%	101.0%	101.0%	101.0%	101.0%	101.1%
4	100.0%	100.0%	101.0%	101.0%	101.1%	101.1%
5	100.0%	100.0%	100.8%	100.9%	100.9%	101.0%
6	100.0%	100.0%	100.6%	100.6%	100.7%	100.7%
7	100.0%	100.0%	100.0%	100.5%	100.6%	100.6%
8	100.0%	100.0%	100.0%	100.0%	100.4%	100.4%
9	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

5.4.5.2 The ratios which are greater than 100% are simply a function of outstanding duration and the size of the 99th percentile (relative to the guarantee) from the corresponding projection to maturity. In particular, the amount of capital support needed to achieve a 1% shortfall probability is irrelevant.

5.4.5.3 In the main the percentages fall as duration increases reflecting the fewer number of years over which charges are applied.

5.4.5.4 The percentages increase gradually with EBR. This is because, the larger the EBR, the smaller is the 99th percentile of the projection from a given duration. The smaller the 99th percentile at maturity, the greater the proportionate effect of the accumulated charges and hence the slightly higher ratio above.

5.4.5.5 The relatively low increase in percentages with EBR is a consequence of what was mentioned in ¶5.4.5.2. Charging asset shares (in effect) only in proportion to the term over which capital support is required, and not in proportion to the amount of capital involved, is bound not to reflect well the relative benefits being gained for different EBRs. For example, at duration 0 the extra capital needed for an EBR of 60% is 1.2% of asset share, increasing to 50.7% of asset share for an EBR of 100% (see Table 4.4). The charges, as reflected by an increase of reserves ranging from 1.4% for 60% EBR to 1.7% for 100% equities, do not bear a similar relation to each other.

5.4.5.6 The same point applies for portfolios where all asset shares enjoy the same EBR, irrespective of their outstanding duration.

5.4.6 *Capital support charging approach*

5.4.6.1 Table 5.2 contains the same results for the capital support charging approach. This table differs from Table 5.1 in several ways.

Table 5.2. Capital required if capital support charging employed as a proportion of reserves for 1% shortfall probability with no charges (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	100.3%	102.2%	104.1%	105.8%	107.2%
1	100.0%	100.6%	102.2%	103.6%	104.9%	105.8%
2	100.0%	100.5%	102.3%	104.1%	105.9%	107.6%
3	100.0%	100.3%	101.5%	102.6%	103.6%	104.3%
4	100.0%	100.0%	101.1%	102.1%	103.1%	103.9%
5	100.0%	100.0%	100.7%	101.5%	102.3%	102.9%
6	100.0%	100.0%	100.2%	100.8%	101.2%	101.6%
7	100.0%	100.0%	100.0%	100.3%	100.6%	100.9%
8	100.0%	100.0%	100.0%	100.0%	100.1%	100.3%
9	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

5.4.6.2 The uplift over capital needs ignoring charges is greater for higher EBRs and less for lower ones. In fact, the increases in reserves look, in some way, proportional to the extra capital required over and above the asset share. There are several effects at work.

5.4.6.3 The charges imposed have two components (opportunity cost and risk cost) which are both based on the excess of the 1% reserve over the asset share. The corresponding effects on maturing asset shares will also be proportional to the pre-charging capital support needs. In other words, the smaller the maturing asset share before charges, the bigger the effect of the charges, and hence more capital support is needed to maintain the shortfall probability. This increase in capital support will, in turn, lead to a further increase in charges, leading to a further increase in reserves, and so on. In a sense, therefore, the impact of this charging approach is geared.

5.4.6.4 The opportunity cost component of the charge is proportional to the percentage invested in gilts. This portion of the charge, therefore, decreases as EBR increases. This goes some way to mitigating the effect of the much higher capital requirements at higher EBRs.

5.4.6.5 A third effect is a consequence of the risk cost component and the sheer size of the capital support amounts for high EBRs. Even without the opportunity cost component, a 1% p.a. charge applied to capital support of, say, 30% of an asset share is much more onerous than a 0.15% p.a. charge on the whole of the asset share.

5.5 *The Impact of Charges on Policyholder Returns for CWP Example*

5.5.1 From the discussion in the previous section, it will be obvious that the effect of the asset share charging approach on policyholder returns is more straightforward than that of the capital support alternative. For that reason, this section concentrates on the latter.

5.5.2 *Policyholder return percentiles*

5.5.2.1 Before quantifying the effect of charges on policyholder returns in specific cases, it is worth looking at the spread of possible returns. Figure 5.1 gives an indication of that spread for the 10-year single premium CWP case at duration 0. The picture at other durations would not be dissimilar.

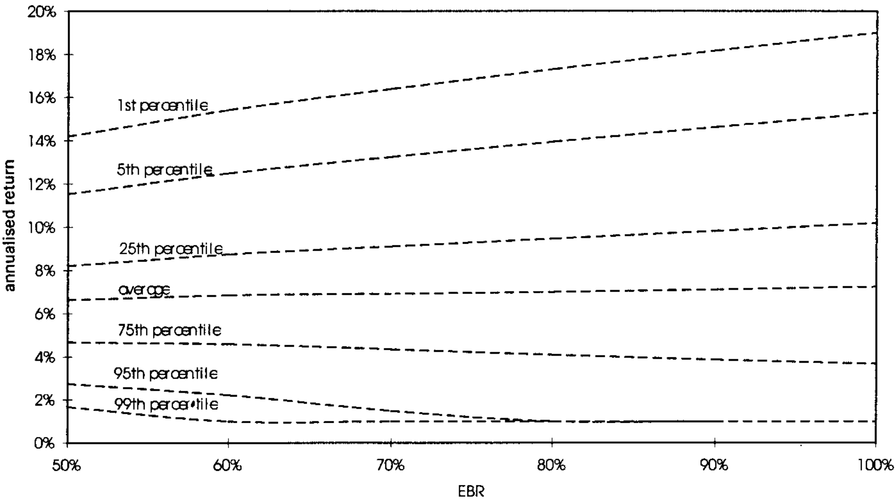

Figure 5.1. Spread of projected returns for different EBRs (CWP with capital support charging approach)

5.5.2.2 The following comments can be made:

(a) The volatility of potential return reduces as EBR reduces, as expected, since all the volatility of return is modelled within the equity component, as discussed in Section 3.

(b) The returns at the lower tails are better for low EBRs than for high EBRs. This is also as expected, since the lower tails of the distribution of equity returns exhibit annualised returns below that assumed for gilts. In addition charges are lower, as a consequence of lower capital support requirements.

(c) The guarantees bite (in more than 1% of cases) for EBRs of around 60% and above, as can be seen from the shape of the 99th percentile.

(d) Equity outperformance at higher percentiles of the distribution of returns is sufficient to outweigh charges, and mean returns are better at higher EBRs than at lower ones.

5.5.3 *Policyholder mean returns*

5.5.3.1 While the relative effect will vary for different percentiles, an indication of the impact of the different charging approaches can be got from looking at their effect on mean returns (over the 10-year period).

5.5.3.2 Table 5.3 presents the appropriate actual maturity accumulations from the modelling work carried out. As stated above, these projections start from duration 0. In other words, the stochastic projection (of equities) is over the whole term of the policy. Table 5.4 presents the same information, but expressed in terms of annualised returns (or reductions in annualised return or yield (RIY)) over 10 years.

Table 5.3. Comparison of mean accumulations on the different charging bases

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
No charging	18,993	19,439	19,879	20,319	20,758	21,197
Asset share charging	18,993	19,215	19,635	20,042	20,439	20,829
Capital support charging	18,993	19,392	19,521	19,671	19,864	20,126

Table 5.4. Mean returns and comparative reductions in yield on the different charging bases

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
Yield (no charging)	6.6%	6.9%	7.1%	7.3%	7.6%	7.8%
Asset share RIY	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%
Capital support RIY	0.0%	0.0%	0.2%	0.3%	0.5%	0.6%

5.5.3.3 These tables confirm several of our previous observations:

(a) Obviously, in the cases where no extra capital is required, no charges are taken and mean returns are the same.

(b) The shapes of the RIYs differ for the two approaches to charging, with the result that the mean payout on asset share charging improves relative to that on capital support charging as EBR increases (reflecting the increased capital needs as a result of the greater investment risk).

(c) The returns at high equity backing ratios under the capital support approach are substantially lower than under the asset share charging method. This probably says more about the 0.15% chosen as the level for the latter than about the deductions under the former.

(d) Despite the drain on policyholder returns from the charge for the capital needed, it is still worth the policyholders' while having a high EBR, at least in terms of average returns relative to lower EBRs.

5.6 *The Impact of Charges on Capital Support Requirements for UWP Example*

5.6.1 Tables 5.5 and 5.6 are the UWP equivalents of Tables 5.1 and 5.2, and are included for completeness. The percentage uplifts relate to the figures shown in Table 4.6, and so apply to the case where no future unit growth (from the duration in question) is allowed for.

5.6.2 There is little comment to add beyond what has been said for CWP business. The uplift in reserves again can be seen to reflect the charging method used, particularly the correspondence between the charges under the capital support approach and the amount of capital support required.

Table 5.5. Capital required if asset share charging employed, as a proportion of reserves for 1% shortfall probability with no charges (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	100.0%	100.0%	101.5%	101.6%	101.7%
1	100.0%	100.0%	101.2%	101.2%	101.2%	101.3%
2	100.0%	101.3%	101.3%	101.4%	101.5%	101.6%
3	100.0%	101.0%	101.0%	101.0%	101.0%	101.1%
4	100.0%	100.9%	101.0%	101.0%	101.1%	101.1%
5	100.8%	100.8%	100.8%	100.9%	100.9%	101.0%
6	100.6%	100.6%	100.6%	100.6%	100.7%	100.7%
7	100.5%	100.5%	100.5%	100.5%	100.6%	100.6%
8	100.3%	100.3%	100.3%	100.4%	100.4%	100.4%
9	100.2%	100.2%	100.2%	100.2%	100.2%	100.2%

Table 5.6. Capital required if capital support charging employed, as a proportion of reserves for 1% shortfall probability with no charges (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	100.0%	100.0%	101.8%	103.5%	104.9%
1	100.0%	100.0%	101.0%	102.5%	103.8%	104.8%
2	100.0%	100.0%	101.8%	103.6%	105.4%	107.1%
3	100.0%	100.6%	101.8%	102.9%	103.8%	104.6%
4	100.0%	100.8%	101.9%	103.0%	103.9%	104.7%
5	100.2%	101.0%	101.9%	102.7%	103.4%	104.0%
6	100.4%	100.9%	101.4%	101.9%	102.3%	102.5%
7	100.4%	100.8%	101.2%	101.5%	101.7%	101.9%
8	100.3%	100.5%	100.7%	100.8%	100.9%	101.0%
9	100.1%	100.1%	100.2%	100.2%	100.2%	100.2%

5.6.3 The relative overall shape of the two tables differs slightly compared with those in Section 5.4, but only because of the differing shapes of capital requirements derived in Sections 4.4 and 4.5.

5.6.4 Furthermore, had higher UWP guarantee rates been considered here, the difference in the two tables would have been even greater, on account of the higher capital requirements (and hence larger charges on the capital support method) involved.

5.7 *Concluding Comments*

5.7.1 Some of the above results will be considered further in Section 7, when the performance of the capital-backed approach to providing with-profits guarantees is compared with the derivative-based alternative introduced in Section 6.

5.7.2 *Comparison of approaches*

5.7.2.1 It is apparent from the above that to charge for access to capital via a simple annual percentage of asset share does not achieve a good fit between the incidence of the capital support needed and the charge for it. Nor would it necessarily produce equitable charges where asset share EBRs were not all set equal within the portfolio.

5.7.2.2 This could be problematic if, as is usually the case, there are other potential uses of capital or capital is limited. If different cohorts of business have substantially different capital needs, not reflecting this in different levels of charge could result in quite significant smoothing across them.

5.7.2.3 The capital support approach may be a more natural method to use for this purpose, in that it better targets the real opportunity costs. However, it is not so easy to explain to policyholders, particularly those whose asset shares need a fair amount of capital support, and are therefore subject to quite large charges as a result.

5.7.3 *Are higher EBRs worth having?*

5.7.3.1 At the charging levels used above for the capital support approach, the higher capital strain from high EBRs resulted in substantially higher charges that reduced the benefit of long-term equity outperformance over gilts, but not to the extent that mean overall returns did not increase with EBR. Of course, were a different opportunity cost to be used, then it is quite possible that the opposite could be the case.

5.7.3.2 It is not just mean returns which are relevant, however. The volatility in possible return also needs to be considered, since this will affect how much, if any, terminal bonus may be payable. Figure 5.1 showed something of the spread of possible policyholder returns and, in particular, the greater volatility resulting from higher EBRs. Thus, in deciding the extent to which with-profits asset shares should be invested in real assets, PRE considerations must encompass acceptable levels of volatility in terminal bonuses.

5.7.4 *Giving and receiving*

5.7.4.1 This section has concentrated on the situation where asset shares need capital support. There is also the 'other side of the coin', namely, where asset shares are greater than the amount needed to maintain a given shortfall probability, and so can be net providers of capital support.

5.7.4.2 Where the policies receiving support have to pay for it, it could be argued that any policy providing support should receive at least some of the benefit. Were this to be the case, the extent of cross-subsidy implicit in the asset share approach might require particular consideration.

5.7.4.3 The authors accept that it is possible to manage a with-profits fund at a much higher level than is set out above, without seeking to identify precisely the degree to which any particular policy is benefiting at any particular time. However, such an approach does not lend itself to a comparison with the market price of alternative investment structures, which may be viewed by policyholders as simpler means to the same end.

6. The Derivative-Based Alternative

6.1 *Introduction*

This section of the paper contains the development of the derivative-based alternative for providing with-profits guarantees. Derivatives can be used to provide as complete a portfolio hedge as practicalities permit. However, this is costly, and security for the office would be obtained at the expense of lower policyholder returns. In order to ensure a fair comparison of costs, the derivatives used must offer (only) the same level of financial security as the capital backing derived in Section 4. Once the option strategy is in place, no further capital should be needed (beyond the asset share less option premium) in order to meet the guarantees 99% of the time. Estimated market prices of such derivatives are presented later in this section. Discussion of the resulting policyholder payouts, particularly compared with those under the capital-backed approach, is kept until Section 7.

6.2 *The Put Spread Strategy*

6.2.1 As the gilts proportion of any asset share is assumed to be perfectly matched by outstanding duration, it is only the maturity value of the equity portion which is uncertain. The derivatives purchased, therefore, only relate to the equity component.

6.2.2 Purchasing a European put option with the strike price determined by the relationship between the amount of guarantee at maturity not covered by gilts and the current level of equities within the asset share (less the amount of equities sold to purchase the options) would give 100% protection, subject to:

(a) the assets on which the option is written being a perfect hedge for the equity portfolio in which the asset share is invested; and
(b) counterparty risk.

A study of the latter is outwith the scope of this paper. The former will be ignored for the time being, but discussed later in the paper.

6.2.3 Comparing the cost of this 100% protection approach with the capital-backed alternative would not be consistent, due to the, albeit small, differences in the risks being run. In order to be consistent with the 99% criterion used in setting the capital requirements in Sections 4 and 5, the degree of security provided by the derivatives must be reduced in some way.

6.2.4 Buying 99% of the required put option would, of course, not give what is required, but, rather, less than 100% security in more than 1% of cases. Selling a second European put option, on the other hand, could do what is needed. If the strike price of this option were determined by the relationship between the amount of the 99th percentile projected asset share at maturity not covered by gilts and the current level of equities within the asset share (adjusted for the amount of equities used to purchase the options), then its sale would remove the security against equity downside not offered by the capital-backed approach.

6.2.5 Thus, for each policy whose asset share is insufficient to offer the desired level of security, two options need to be considered. The one to be sold will have a lower strike price than the one bought, and so only offset part of the cost of the latter rather than make any new money. In all other respects (i.e. in terms of underlying asset, term to expiry, and so on) the options will be identical. This type of arrangement is often referred to as a 'put spread'.

6.2.6 Figure 6.1 shows the maturity profile for a put spread strategy with a high strike price of 80% of the initial value and a low strike of 60%. The payoff is zero for values above 80%, where (it is assumed for this example) no capital is required to support the guarantee. As the value of the underlying equity drops below 80% of the initial value, there is an increasing return from the options to cover the shortfall versus the guarantee. This return reaches a maximum at a strike of 60%, which, in this example, represents the 1% lower tail of likely equity performance.

6.2.7 Another possible derivative strategy would be to hold cash and sell a 'call spread'. In the example above this would involve holding sufficient cash to provide 20% payoff at maturity, selling a call option with a strike price of 60% and buying a call option with a strike price of 80%. This arrangement would have the same economic effect as the put spread and, in theory (assuming 'put/call parity'), the same cost. For the remainder of the paper, only the put spread will be considered.

6.3 *Modelling the Cost of the Put Spread Strategy*

6.3.1 The cost of the put spread has been calculated using the standard Black-Scholes equation for European options on current market conditions at the time of writing (August 1999). Whether or not the market prices are consistent with the approach taken to the stochastic modelling of the long-term future is not important for the purposes of this paper; it is relevant to use current market

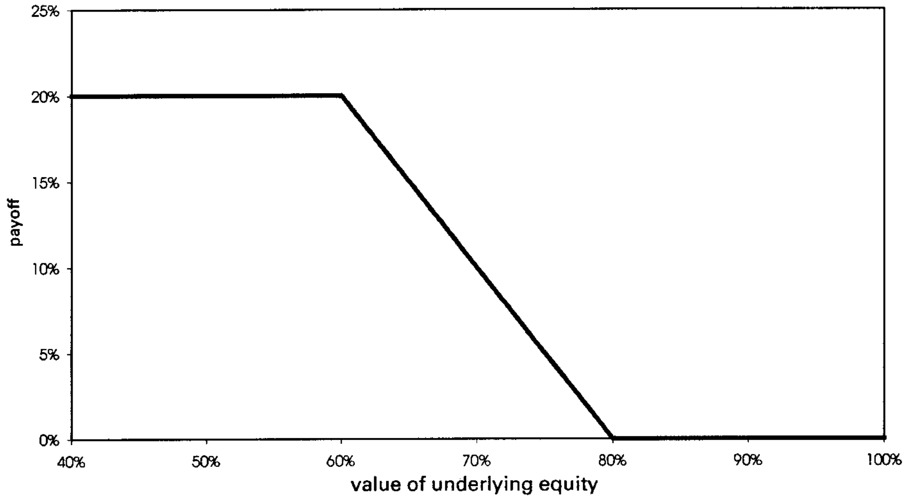

Figure 6.1. Profile of payoff from a put spread at expiry

conditions for the options, as they would be purchased 'up front', thereby locking in the conditions at the time.

6.3.2 *Black-Scholes input parameters*

6.3.2.1 There are several input parameters required in order to calculate the price of each put option. The underlying equities have been assumed to be FTSE 100 equities. This could introduce a mismatch, in practice, between the options and the underlying with-profits fund, and is discussed in Section 6.5. The strike prices have been determined, as described above, as proportions of the spot price of the equities available within the asset share. The term to expiry of each option has been taken to be the same as the outstanding duration of the policy being considered. The dividend yield assumption is consistent with the economic assumptions given in Section 3.4. The appropriate discount rates have been derived from the current (August 1999) term structure of sterling swap rates in the market.

6.3.2.2 The implied volatility is a function of term to expiry and strike price. A mid-market volatility surface has been used to derive the volatility for each put option. One of the features of this surface is the large skew in volatility between different strike prices. At low strike prices the volatility is much higher than at high ones. This feature of the current U.K. market has the benefit of making the sold put option more valuable than if the volatility was flat, cheapening the overall structure.

6.3.3 *A minor practical point*

The net cost of the derivative spread is deducted from the asset share at the start of each stochastic projection in proportion to its EBR at the time. The EBR of the initial asset share therefore remains unchanged. There is a possible second order effect on the statutory reserves in this situation due to the potential impact on allowable yield from switching into a derivative. This small effect has been ignored since the paper does not specifically consider the cost of 'statutory capital' (i.e. the situation where an asset share is less than the corresponding statutory reserve).

6.4 *Cost of the Put Spread Strategy*

6.4.1 The results which follow are based on the same 10,000 simulations (and, in particular, their 99th percentiles) as were used for Sections 4 and 5. Each figure is obtained by comparing the 99th percentile projected return with the corresponding maturity guarantee. Where the former is smaller, the two are used to set the lower and upper strike prices of the put options and the overall put spread cost calculated.

6.4.2 Table 6.1 presents the one-off costs of the appropriate put spreads for the 10-year CWP single premium example. For ease of comparison, the costs are expressed as percentages of the underlying asset share (before the options are purchased). Table 6.2 presents the equivalent information for the corresponding UWP example (where no future increase in unit prices is assumed).

6.4.3 While the main discussion of these results (and their effect on policyholder returns) is reserved until Section 7, the following observations are made at this point:

(a) The shape of the costs is broadly in line with the extra capital required to support the guarantee. This is to be expected, since the relationship between capital required (guarantee at maturity) and asset share (return at maturity) is the basis of the assessment of the strike prices of the put options.

(b) There is not a precise correlation — the put spread becomes relatively cheaper as term to expiry decreases. This is due, partly, to falling time value within the options, and, partly, to the put options moving from at or slightly in the money (for early durations) to an increasingly out of the money position (for later durations). The latter is the result of the deterministic accumulation of starting asset shares at a rate in excess of the build up of guarantees.

(c) The pattern of costs for both CWP and UWP examples generally reflects the level of underlying guarantees. The effect of a higher UWP guarantee assumption can be seen in Table 6.3.

Table 6.1. Cost of put spread as a proportion of asset share (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	0.0%	0.2%	1.6%	3.2%	4.8%	6.5%
1	0.0%	0.5%	1.9%	3.4%	5.1%	6.7%
2	0.0%	0.4%	1.7%	3.1%	4.6%	6.1%
3	0.0%	0.3%	1.5%	2.8%	4.1%	5.5%
4	0.0%	0.0%	1.0%	2.1%	3.3%	4.5%
5	0.0%	0.0%	0.7%	1.7%	2.7%	3.7%
6	0.0%	0.0%	0.3%	1.1%	1.9%	2.8%
7	0.0%	0.0%	0.0%	0.5%	1.1%	1.8%
8	0.0%	0.0%	0.0%	0.0%	0.3%	0.8%
9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Table 6.2. Cost of put spread as a proportion of asset share (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	0.0%	0.0%	0.0%	1.4%	2.9%	4.5%
1	0.0%	0.0%	0.9%	2.3%	3.8%	5.4%
2	0.0%	0.0%	1.3%	2.7%	4.1%	5.7%
3	0.0%	0.5%	1.7%	3.1%	4.4%	5.9%
4	0.0%	0.8%	1.9%	3.1%	4.3%	5.5%
5	0.2%	1.1%	2.1%	3.2%	4.3%	5.4%
6	0.5%	1.3%	2.2%	3.1%	4.1%	5.0%
7	0.7%	1.3%	2.0%	2.8%	3.6%	4.3%
8	0.8%	1.3%	1.8%	2.4%	2.9%	3.5%
9	0.4%	0.7%	0.9%	1.2%	1.5%	1.8%

6.4.4 The cost of the put spread can be quite high, in excess of 5% of asset share at 100% EBR. This is not as bad as it seems, though, since this is a one off cost, and not incurred each year.

6.4.5 The costs are not dissimilar to the results for capital support charging in Section 5. While the different impact of the two methods will be analysed in Section 7, it is interesting to note that the cost of the options spread over the remaining years of the term appears to be of the same order of magnitude as the RIY under capital support charging.

6.5 *Practical Considerations*

6.5.1 The option strategy suggested above has been deliberately kept simple to aid transparency and comparability. At the same time, however, it is important to be aware of the additional complexity that would be involved in practice.

Table 6.3. Cost of put spread as a proportion of asset share (UWP, guarantee rate = 2% p.a.)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	0.6%	2.1%	3.8%	5.5%	7.3%	9.0%
1	1.2%	2.8%	4.5%	6.2%	8.0%	9.8%
2	1.5%	3.0%	4.6%	6.3%	7.9%	9.6%
3	1.8%	3.2%	4.7%	6.2%	7.8%	9.3%
4	1.8%	3.1%	4.4%	5.7%	7.1%	8.4%
5	1.9%	3.1%	4.2%	5.4%	6.5%	7.7%
6	1.9%	2.9%	3.9%	4.8%	5.8%	6.8%
7	1.7%	2.5%	3.3%	4.0%	4.8%	5.6%
8	1.5%	2.0%	2.6%	3.1%	3.7%	4.3%
9	0.7%	1.0%	1.2%	1.5%	1.8%	2.1%

6.5.2 In reality, with-profits funds are invested in a range of asset classes, the returns on which are volatile. Full hedging of downside risk with put options would require tailored options for each asset class.

6.5.3 Among the broad categories of asset class are U.K. equities, overseas equities, U.K. fixed interest and overseas fixed interest. These can be hedged in turn by FTSE 100 options, various overseas index options (e.g. S & P 500 for the United States of America, Eurostoxx 50 or individual European index options for Europe and so on), gilt options and overseas bond options (on, for example, Bunds and U.S. Treasuries). Nevertheless, there is a potential mismatch or basis risk between the coverage of put options and the underlying investments. Options tend to relate to blue chip indices, whereas with-profits investment performance reflects the actual basket of assets held.

6.5.4 For example, an office's U.K. equity portfolio may be closer to the FTSE All-Share Index than the FTSE 100. While this mismatch can be reasonably large, the largest companies are becoming a bigger proportion of the full market. For example, the ratio of market capitalisation between the FTSE 100 and FTSE All Share companies is around 76% at the time of writing (August 1999). Companies already use futures and options on blue chip indices for tactical asset allocation purposes, albeit over short terms. The performance of these two indices can easily diverge over the short term, but over longer terms they will tend to match each other reasonably well.

6.5.5 Long-term options, like those modelled in this paper, are only available as over the counter (OTC) derivatives from investment banks and the like. Current admissibility limits restrict the level of concentration of investment in OTCs. Liquidity and diversification of counterparty risk could be issues, due to the limited number of providers and the fact that the value of the spread can become an increasingly significant part of the portfolio. One method of mitigating counterparty exposure is to receive cash collateral from the counterparty, to be kept on default. This has the added benefit of also reducing the exposure for admissibility purposes.

6.5.6 In with-profits funds the levels of guarantees at various outstanding terms change all the time due to, amongst other things, new money, bonus declarations and policy exits. An office hedging its guarantees in the future will soon find the level of guarantee to be hedged has changed. The overall hedge could be reviewed on a regular basis and updated for the change in term structure and notional amount of guarantees written.

6.6 *Comment and Conclusions*

6.6.1 The main conclusions will be developed in Section 7, when the use of derivatives is compared with the capital support approach.

6.6.2 While the simple strategy presented above is somewhat theoretical, it is possible to devise more complex strategies which, while not a perfect hedge, would give some comfort that the guarantees would be met in the future and therefore free up some of the capital which would otherwise be required.

6.6.3 The derivative strategy involves spending money as insurance against reasonably unlikely events, whereas the reserving strategy locks up capital that is ultimately expected, in most circumstances, to be released to the office. The cost of the put spread is broadly proportional to the amount of reserves that would be required if excess capital were being used to back the guarantees. Thus, if the capital locked up is charged for, the two approaches are not as different as they seem.

7. Comparison of the Derivative and Capital-Backed Strategies

7.1 *Introduction*

7.1.1 Sections 4, 5 and 6 have presented two broad approaches which a life office could use, in conjunction with appropriate charges, to provide guarantees under with-profits policies. Overall there are four separate situations to compare, the first three of which are variants of the capital-backed strategy:

(a) capital-backed with no charges;
(b) capital-backed with charges proportional to asset share;
(c) capital-backed with charges proportional to the level of support; and
(d) the derivative-backed approach.

7.1.2 The comparison deals first with mean policyholder returns and then goes on to consider the possible range of payouts under each approach. For simplicity, attention is restricted to the 'duration 0' case.

7.1.3 When option (b) was first introduced in Section 5, the charging level illustrated was 0.15% of asset share p.a. In order to illustrate the sensitivity of results to this choice, an alternative, 0.25% p.a., is also considered below. Similarly, for option (c), the Section 5 choice of 1% over equity return for opportunity cost p.a. is augmented by a second possibility, 2% over equity return p.a. By interpolation or extrapolation, the level of charges equivalent to the put spread can therefore be derived.

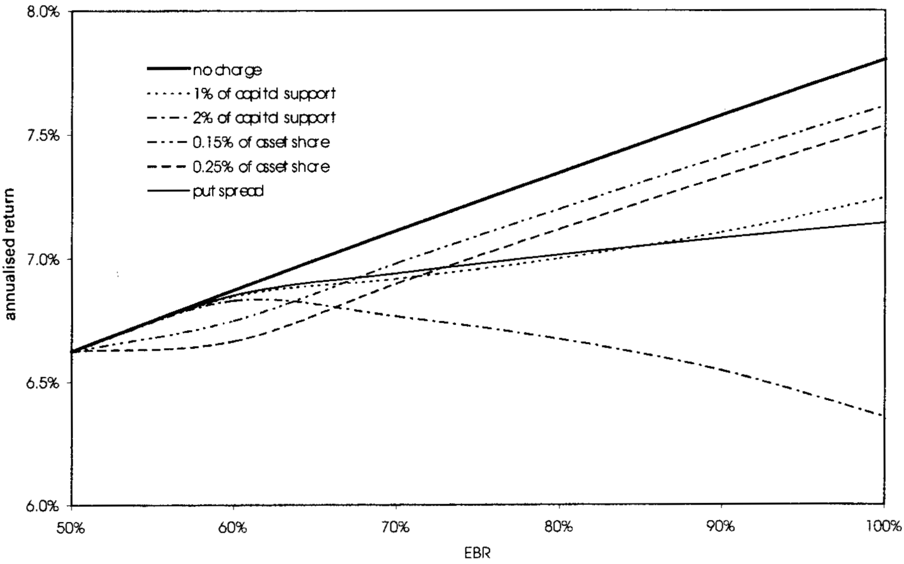

Figure 7.1. Mean returns on different charging methods, 10-year, single premium, CWP, duration zero

7.2 Comparison of Mean Returns for Conventional Business

7.2.1 Figure 7.1 illustrates, for a range of EBRs, the mean annualised returns under each of the charging approaches for the 10-year single premium CWP example at duration 0.

7.2.2 Not surprisingly, all the plots for methods allowing for charges lie on or below that for the no charge comparator.

7.2.3 The lines plotting returns for the two levels of option (b) (i.e. charging in proportion to the asset share) show the limitations of this method of charging for use of capital. At the lower EBRs, where capital backing is modest, the impact of charges is the highest of any method. For higher EBRs, the charges seem light compared with the cost of the put spread approach. In fact, as EBRs increase, the charges do not reflect the fact that the capital support is increasing exponentially.

7.2.4 As may be obvious, the problem highlighted in ¶7.2.3 is actually a consequence of keeping constant the level of asset share being charged, regardless of EBR and the consequential capital needs. This point will be picked up later.

7.2.5 On the other hand, the mean returns under the capital support charging approach follow a more intuitive pattern. There is little or no charge at low EBRs, due to little or no need for extra capital. In the case of the 1% charge (in

Table 7.1. Reductions in yield due to charges on the range of methods with reference to mean returns (CWP, duration zero)

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
Yield with no charges	6.6%	6.9%	7.1%	7.3%	7.6%	7.8%
Reductions in yield due to:						
Put spread cost	0.0%	0.0%	0.2%	0.3%	0.5%	0.7%
Asset share charge 0.15% p.a.	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%
Asset share charge 0.25% p.a.	0.0%	0.2%	0.2%	0.2%	0.2%	0.3%
Capital support charge 1% p.a.	0.0%	0.0%	0.2%	0.3%	0.5%	0.6%
Capital support charge 2% p.a.	0.0%	0.0%	0.3%	0.7%	1.0%	1.4%

addition to the opportunity cost of holding gilts) for use of capital, the mean returns increase with EBR, as does the extent of divergence from the 'no charges' line. Interestingly, however, the resulting line seems to track quite closely that for the put spread alternative.

7.2.6 The results in ¶7.2.5 seem reasonable. As capital requirements increase with EBR, so does the corresponding charge. Nevertheless, expected payouts still increase with EBR, justifying the preference for equity investment.

7.2.7 This is not the situation when the level of charge is increased to 2% (plus the opportunity cost of holding bonds). The high capital requirements at high EBRs, combined with this higher level of opportunity cost charge, result in deductions which cause the mean policyholder return to fall as EBR increases. While the 2% charge may be a little 'over-cooked', its inclusion does illustrate the fact that, if the cost of capital is high, it makes little sense for an office to aim for a high EBR, as policyholders cannot be expected to benefit from equity outperformance.

7.2.8 It is, perhaps, not surprising that the 1% of capital support line should approximately track that for the put spread approach. As explained in Section 6, under each approach, the deductions from asset share (whether through annual charges for capital or the one-off cost of the derivative) are, in some sense, proportional to the amount of capital support required. The fact that they are so close together suggests that the 1% charge on this capital is roughly equivalent to the estimated market costs of opting for the derivative-backed alternative, at least on the set of assumptions used.

7.2.9 An alternative way of presenting the financial effects of each of the charging alternatives is to express them as reductions in annualised policyholder return (RIY). These are shown in Table 7.1, from which can be seen again the similar impact on policyholder returns of the put spread and the capital support (with an opportunity cost of 1% over equity return) approaches.

7.3 *Comparison of Spread of Returns for Conventional Business*

7.3.1 Figure 7.2 illustrates, for an EBR of 100%, the spread of modelled

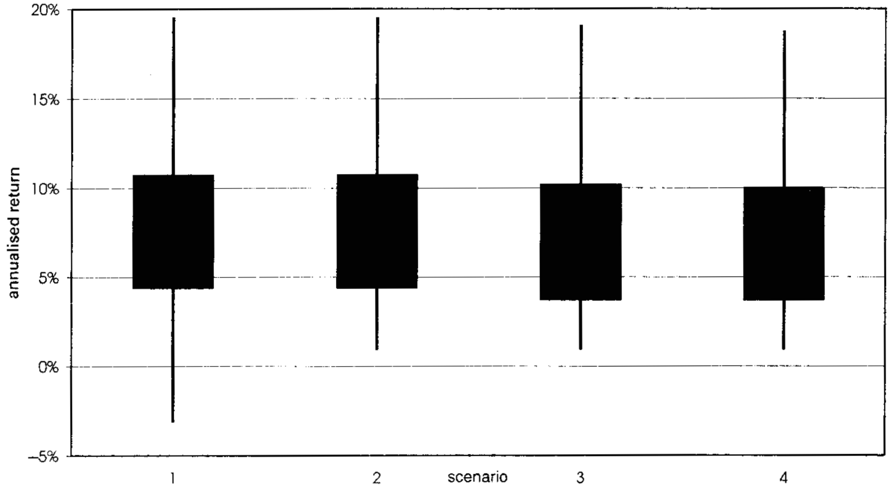

Figure 7.2. Spread of returns to policyholder and impact of guarantees and charges (CWP, duration zero)

policyholder returns under various scenarios. Again, the situation modelled is the 10-year CWP single premium at duration 0.

7.3.2 The spread of payouts is illustrated using the 25th and 75th percentile payouts (the 'boxes') to cover 50% of the expected range of returns and the 1st and 99th percentiles (the 'whiskers') to display the wider range of outcomes.

7.3.3 The scenarios modelled are the following:

(1) no charges and no guarantees – this simply represents the range of projected asset shares without any allowance for guarantees or their costs;
(2) no charges, but allowing for the with-profits guarantee;
(3) with-profits guarantees with charging for use of capital (capital support charging approach); and
(4) with-profits guarantees hedged using a put spread.

7.3.4 Figure 7.2 illustrates several important points:

(a) Offering a guarantee reduces the volatility of returns by putting a floor on the policyholder payout. This is illustrated in the difference between scenarios 1 and 2.
(b) Charging for the guarantee (through capital support charges) acts as a ceiling for returns. The 1st, 25th and 75th percentiles all fall while the 99th percentile remains at the guarantee floor, i.e. the spread of returns reduces.
(c) Buying put spread protection at the outset has much the same effect on policyholder payouts as charging for the guarantee at the levels modelled.

(d) An EBR of 100% was chosen for the figure in order to emphasise the points being made. Other EBRs would have exhibited similar features, but with less variability due to lower investment volatility and hence lower guarantee charges.

7.4 *Comparison of Mean Returns for Unitised With-Profits Business*

7.4.1 Figure 7.3, the UWP equivalent of Figure 7.1, shows the mean returns under the same six charging alternatives for the 10-year, single premium case at outset, where no unit growth is assumed at all. The patterns reflect the features identified for CWP, but with a few differences.

7.4.2 UWP guarantees tend to build up at different rates from those for CWP business. Their initial values are typically less than the CWP equivalents, being based on premiums paid rather than premiums payable. Of course, where a generous growth rate has been guaranteed (whether just on units bought or on all future purchases too), the situation can be very different. In the example modelled in Figure 7.3, the guarantee is set at the premium amount, less than the CWP example (which guaranteed 1% p.a. return), and this is reflected in the way the charges start to apply further to the right, around an EBR of 70%.

7.4.3 For EBRs above 70%, the plotted lines follow similar patterns to the CWP example, but with not such pronounced differences between them. This is due to the generally lower capital requirements, which are a function of the 0% guarantee rate used.

7.4.4 Again the mean returns on the put spread and the 1% capital support charge are very similar, as can be seen from Table 7.2, the UWP equivalent of Table 7.1.

7.4.5 An analysis of the spread of payouts reveals a similar pattern to that shown for CWP business in Figure 7.2.

7.5 *Conclusions*

7.5.1 In this section it has been demonstrated that, if an office wishes to charge for the capital support required to back a particular level of guarantees or a specific EBR, then a charge as a fixed proportion of asset share (or via a fixed reduction in asset share investment return) does not match the actual incidence of capital support particularly effectively.

7.5.2 A better approach appears to be applying the charge only in proportion to the actual support amount, and basing the level of the charge on what it is actually costing to tie capital up in this way (i.e. the loss of some equity return and whatever other opportunities are considered relevant).

7.5.3 It is, however, recognised that the asset share charging method is easy to apply in practice. Given a particular EBR for a block of business, an appropriate level of asset share charge could be derived from Figure 7.1. In other words, the charge implied by the put spread for that EBR could be converted into a charge as a proportion of asset share. This charge would need to be reassessed if the office wished to alter the block's EBR. The capital support charging

Table 7.2. Reductions in yield due to charges on the range of methods with reference to mean returns (UWP, guarantee rate = 0%, duration zero)

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
Yield with no charges	6.6%	6.9%	7.1%	7.3%	7.5%	7.7%
Reductions in yield due to:						
Put spread cost	0.0%	0.0%	0.0%	0.1%	0.3%	0.5%
Asset share charge 0.15% p.a.	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%
Asset share charge 0.25% p.a.	0.0%	0.0%	0.0%	0.2%	0.2%	0.2%
Capital support charge 1% p.a.	0.0%	0.0%	0.0%	0.2%	0.3%	0.4%
Capital support charge 2% p.a.	0.0%	0.0%	0.0%	0.3%	0.6%	0.9%

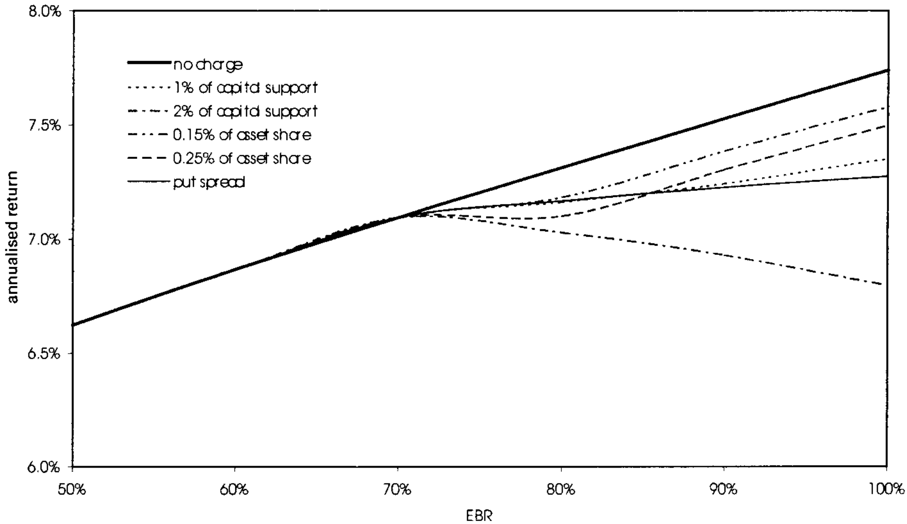

Figure 7.3. Mean returns on different charging method (UWP, guarantee rate = 0%, duration zero)

method is more robust, in that the charging automatically copes with a change in EBR (so long as the appropriate capital requirements are known). While the figures shown relate only to a 10-year policy at outset, similar considerations apply for other terms and durations.

7.5.4 In particular, the structure and level of charge (i.e. 100 basis points over expected equity return) used in the capital support charging method do not look unreasonable. When a larger opportunity cost was incorporated (of 200 basis points), the impact on policyholder returns at high EBRs was extreme. This does not necessarily mean that this level of charge is too high. Rather, where capital is expensive (or in short supply), the results suggest that it is not always in the policyholders' interests to draw on it to support investment risk.

7.5.5 It has also been demonstrated in this section that, where an office chooses to hedge the investment mismatch risk with a suitable put spread, then the cost of doing so seems, irrespective of EBR, to track that of the capital-backed approach, with a required opportunity cost of 100 basis points above expected equity return. Of course, the profit margin of derivative providers could alter this relationship.

7.5.6 Were these costs to be translated into policyholder charges, then, while their incidences would be very different, their effects would be very similar (for policies held to maturity). A consideration of the situation for lapses and, indeed, for whole portfolios, is outwith the scope of this paper.

7.5.7 It may appear rather a coincidence that the put spread and capital charging method costs look so similar. The pricing of derivatives is based on risk-free discount rates derived from market swap rates. These, typically, are based on gilt rates plus an appropriate credit spread, and so are likely to lie somewhere between the long-term expected returns on gilts and equities. However, as previously discussed, the strike prices in the put spread (and hence the cost of the spread) are closely related to the amount of capital support which would otherwise be needed. It seems reasonable, therefore, that the costs of the two approaches could move in line as EBRs (and hence capital support needs) change, provided that the level of charge on support capital is consistent with market prices.

8. Conclusion

8.1 With-profits guarantees do not come free. Either an asset share's EBR is tightly constrained or some additional source of security must be found. This can come from additional capital, whether internal (e.g. other asset shares or an office's estate) or external.

8.2 This paper has presented another alternative: purchasing appropriate derivatives (with policyholder monies) which give the required degree of protection. While the current derivatives market may not provide a perfect match to the investment exposure of every U.K. with-profits fund, the approach presented could be used to derive a market-consistent charging level for capital backing.

8.3 It would appear, from a recent survey, that the majority of U.K. offices do not charge for capital backing of with-profits business. It may be that this will change, as capital becomes more of an issue in the industry. The authors hope that this paper will help offices to prepare.

8.4 The authors accept that some with-profits offices may continue to consider it inappropriate to charge each policy for the specific support it receives.

Nevertheless, it is hoped that this paper will still prove useful to such offices in indicating the extent of cross-subsidy which could be taking place.

ACKNOWLEDGEMENTS

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The views expressed are, of course, those of the authors and not necessarily those of their employers. However, the authors would like to thank their employers for accommodating time demands, particularly of writing the paper.

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The following list includes, not only the works referred to in the paper, but other publications that might prove helpful as a starting point for further research.

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APPENDIX A

SOME COMMENTS ON PORTFOLIOS

A.1 This paper has considered the investment risk and capital requirements at an individual policy level. As noted in Section 1.3, this was in order to highlight various aspects of the risk/return trade-off implicit in with-profits business. Results derived from hypothetical portfolios can be limited in their application and even disguise some of the underlying forces at work — hence the single policy approach. The authors believe the methods outlined in the paper can be applied in the context of a portfolio, but certain aspects need careful consideration.

A.2 In the comparison of the cost of capital-backed and derivative-backed security presented, the amount of capital support required has been calculated on the basis of a 'stand-alone' shortfall probability. One of the key considerations when portfolios are considered is the extent to which the capital needed to secure a given level of security for the portfolio as a whole might differ from the sum of the corresponding individual policy requirements.

A.3 In most cases, (particularly where there is a good spread of outstanding durations within the portfolio) it could be expected that the portfolio-based requirement will be less than the corresponding sum of individual policy requirements, through the diversification of investment risk over time. As it is most unlikely that every maturing asset share will lie at or below the bottom percentile of possible outcomes, in many cases it can be expected that capital will be released by some maturities which could then be used to offset the risk of future shortfalls.

A.4 Translating this portfolio cross-subsidy into a per-policy effect is not straightforward, however. While a given policy might still enjoy 99% security against shortfall (as well as lower capital support to be charged for) through being part of a portfolio, the nature of the risk being run is now changed. Rather than focusing on just an investment risk, the security afforded is now dependent on what happens in the rest of the portfolio. Thus, the situation is no longer equivalent to the derivative-backed approach presented in Section 6, and hence cost comparisons are not wholly valid. (Were an alternative, portfolio-based, derivative arrangement to be developed, it is not obvious how the cost could be translated into appropriate individual policy charges.)

A.5 It is even possible to construct a portfolio of policies with differing durations, where the investment risk is such that the portfolio requires a greater level of additional capital than the sum of the capital required at an individual policy level. While the authors did not investigate such portfolios in any depth, their existence emphasises the importance, for any office, of modelling its own particular portfolio, and it is hoped that the generic 'per-policy' results presented in this paper may help to shed light on some of the underlying issues.

A.6 Where a policy's asset share is less than the appropriate statutory minimum reserve, the derivation of a suitable charge (should the office consider it appropriate to apply it) for the capital support given is quite straightforward. In this case, however, the presence of a portfolio may provide a large number of asset shares capable of 'lending' the capital in question, but it will not influence the cost/risk in the way described above — statutory reserves must be calculated policy by policy, with future valuation strain eliminated. Where the capital support deemed to be required to ensure statutory solvency can be maintained in a range of adverse investment conditions more onerous than those represented by the three prescribed resilience scenarios, the presence of a portfolio may change things, but it depends on its nature and, in particular, what 'post-scenario' cross-subsidies exist.

APPENDIX B

RESULTS FOR ANNUAL PREMIUM BUSINESS

B.1 As for the single premium example, the sum assured for the 10-year annual premium case investigated below has been set to guarantee initially an accumulation rate of 1% p.a. (i.e. for an annual premium of £1,000, the sum assured is £10,567). The modelling of annual premium business involves dealing with a few additional issues. How they have been treated in this work is as follows:

(a) Premiums have been assumed to be payable annually in advance.
(b) The reserves at time t have been calculated before the payment of the premium then due (and so the reserve at time 0 is simply the required minimum margin of 0.3% of the sum assured).
(c) Premiums are always invested in line with the EBR at the top of the appropriate column, even though, during the stochastic part of the projection, the underlying asset share EBR could be more or less than at time t.

B.2 Again, as for the single premium example, the probabilities of shortfall for the reserves, including the resilience allowance, are presented first. While there are some obvious similarities to Table 4.1, there are also significant differences.

Table B.1. Probability of shortfall at maturity (with resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	1.9%	4.2%	6.5%	8.5%	10.4%	12.1%
1	1.8%	3.9%	5.9%	7.7%	9.6%	11.0%
2	1.8%	3.8%	5.8%	7.5%	9.3%	10.7%
3	2.1%	3.9%	5.8%	7.3%	9.1%	10.2%
4	2.2%	3.9%	5.8%	7.5%	9.1%	10.3%
5	2.6%	4.6%	6.3%	7.8%	9.3%	10.6%
6	3.2%	5.1%	6.6%	8.0%	9.4%	10.5%
7	3.7%	5.1%	6.4%	7.8%	8.8%	9.6%
8	3.2%	4.5%	5.6%	6.5%	7.3%	8.0%
9	1.6%	2.1%	2.5%	2.9%	3.3%	3.7%

B.3 The probabilities of shortfall again rise with EBR, indicating a 'softening' of reserve strength as EBR increases. The reason for this lies in the size of the net premiums, which increase with the decreasing valuation yield.

B.4 The probabilities also rise and fall with term expired, demonstrating the effect of the resilience test.

B.5 This time, however, the probabilities cover a much greater range than in the single premium case, showing a considerable increase towards the top right hand corner of the table. This is a consequence of the use of a net premium valuation with a relatively low rate of interest. The lower the valuation rate (i.e. the higher the EBR), the higher the net premium being valued, and hence the lower the reserves relative to the single premium situation.

B.6 When the resilience reserve is excluded, the resulting probabilities of shortfall are as shown in Table B.2. In comparing with Table 4.2, some of the same observations can be made as in the single premium case. It can also be noted that at short durations the resilience test has relatively little effect because of the much more gradual build up of the reserve than in a single premium policy.

Table B.2. Probability of shortfall at maturity (without resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	1.9%	4.2%	6.5%	8.5%	10.4%	12.1%
1	2.7%	5.3%	7.4%	9.7%	11.4%	13.2%
2	4.1%	6.6%	9.2%	11.4%	13.1%	14.7%
3	5.6%	8.4%	10.9%	13.0%	15.0%	16.5%
4	7.8%	10.9%	13.5%	15.5%	17.1%	18.4%
5	10.8%	14.2%	16.7%	18.5%	20.0%	21.4%
6	14.2%	17.2%	19.6%	21.4%	22.8%	24.1%
7	17.8%	21.0%	23.4%	25.2%	26.8%	27.9%
8	21.3%	24.3%	26.4%	28.3%	29.5%	30.6%
9	23.6%	26.4%	28.6%	30.5%	31.9%	33.1%

B.7 Expressing the statutory reserves as a percentage of modelled asset shares highlights a very different situation from that in the single premium example, as Table B.3 demonstrates.

Table B.3. Statutory reserve as a percentage of the deterministically accumulated asset share (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	–	–	–	–	–	–
1	115.9	118.6	121.4	124.3	127.2	130.2
2	112.6	115.1	117.8	120.5	123.4	126.3
3	110.2	112.7	115.2	117.8	120.6	123.4
4	108.2	110.5	112.9	115.4	118.0	120.8
5	106.3	108.5	110.8	113.1	115.6	118.2
6	104.5	106.5	108.7	110.9	113.3	115.8
7	102.6	104.7	106.7	108.8	111.0	113.3
8	100.7	102.7	104.7	106.7	108.7	110.9
9	98.9	100.8	102.7	104.6	106.6	108.6

B.8 As for the single premiums, for each duration the proportions increase with EBR, although (and to an even greater extent) not quickly enough to avoid the probability of shortfall also rising.

B.9 Almost all the proportions are greater than 100%, giving further confirmation that with-profits business cannot always be considered to provide capital to an office, even on a published free assets basis.

B.10 The extent to which the use of 'optimised' reserves (see ¶4.2.7) would have reduced these proportions is dependent upon the investment mix assumed for the additional assets required, over and above the asset shares, to cover the reserves. It has not been investigated.

B.11 Now that some results for annual premium business have been considered, the tentative conclusions presented in Section 4.3.5 have received further support, namely that:

(a) The statutory valuation basis, including the effects of the three resilience tests as well as the required minimum margin, is not uniformly strong over either policy duration or reserve investment mix.
(b) The notion sometimes presented, of with-profits business providing capital to an office, need not always be the case, particularly for annual premium business.

B.12 The relationship between the asset share and the reserve required for a 1% probability of shortfall at maturity in the annual premium case is shown in Table B.4. This is analogous to Table 4.4 for the single premium case.

Table B.4. Proportion of asset share needed for 1% shortfall probability (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	–	–	–	–	–	–
1	104.5%	111.0%	118.5%	126.9%	136.7%	148.2%
2	105.3%	111.8%	119.2%	127.7%	137.4%	148.9%
3	106.7%	113.5%	121.1%	130.0%	140.3%	152.4%
4	107.7%	114.5%	122.3%	131.3%	141.8%	154.3%
5	108.3%	115.2%	123.0%	132.0%	142.6%	155.2%
6	108.3%	114.8%	122.3%	130.9%	141.1%	153.0%
7	107.3%	113.3%	120.1%	127.8%	136.8%	147.4%
8	105.9%	111.1%	116.9%	123.6%	131.3%	140.2%
9	100.3%	103.5%	107.1%	111.0%	115.3%	120.1%

B.13 A similar picture emerges to that seen for the single premium examples, namely, probabilities increasing with EBR for a given t and initially rising, but mainly falling, with duration for a given EBR.

B.14 The only significant difference to note is the much slower rate of decrease in the proportions as duration increases. This is a consequence of the guarantee being related to all the premiums payable, whereas the asset share at any time only reflects the premiums that have been paid to date. This would tend to suggest that, all other things being equal, single premium conventional with-profits business should be less capital-intensive than annual (even without allowing for any difference in initial expense levels).

B.15 No proportions are shown for duration 0, since it is assumed that no premiums have yet been received and the asset share has been set to zero.

B.16 The annual premium table analogous to Table 4.5 is Table B.5. From this can be drawn conclusions similar to the single premium case, but with some important differences.

Table B.5. Ratio of 1% reserve to statutory minimum reserve (with resilience reserve and required minimum margin (CWP))

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	–	–	–	–	–	–
1	90.1%	93.6%	97.6%	102.1%	107.5%	113.8%
2	93.5%	97.1%	101.2%	105.9%	111.4%	117.9%
3	96.8%	100.7%	105.1%	110.3%	116.4%	123.5%
4	99.5%	103.6%	108.3%	113.8%	120.2%	127.7%
5	101.9%	106.2%	111.0%	116.7%	123.3%	131.2%
6	103.6%	107.8%	112.5%	118.0%	124.5%	132.2%
7	104.7%	108.2%	112.6%	117.5%	123.3%	130.0%
8	105.2%	108.1%	111.7%	115.9%	120.7%	126.3%
9	101.4%	102.8%	104.3%	106.1%	108.2%	110.6%

B.17 The effect of the more stringent treatment of gilts can again be seen in percentages which rise with EBR.

B.18 The percentages for the earlier durations are considerably smaller than in Table 4.5, reflecting the additional strength within the net premium reserves, first noted in Section B.4, which applies for all EBRs since, in all cases, the valuation rate of interest is greater than the 1% used to accumulate the premiums to arrive at the sum assured (and therefore the valuation net premiums will always be less than the office premium).

ABSTRACT OF THE DISCUSSION

HELD BY THE FACULTY OF ACTUARIES

Dr D. J. P. Hare, F.F.A. (introducing the paper)**:** It is now more than six years since J. E. O'Neill and H. W. Froggatt presented their paper, 'Unitised With Profits — Gamaliel's Advice', to a sessional meeting of the Faculty (*T.F.A.* **44**, 243-294 and *J.I.A.* **120**, 415-469). Times have moved on since then, and much has changed. Sadly, many of the issues surrounding with-profits business highlighted at that discussion still remain. In particular, comments regarding the need for clear and adequate disclosure of exactly what investors are 'buying' when they purchase a with-profits product seem just as appropriate now as then, if not more so, particularly in the light of the wide variation in products which form the 'with-profits bond' market of today.

In the paper we suggest that a central feature of with-profits business is some form of investment guarantee. Perhaps it is just as well that some companies seek to limit the extent to which certain guarantees apply, as relatively high reversionary bonus rates, or, more accurately, annual unit growth rates, eat into what would otherwise have been terminal bonus cushions, and hence valuable sources of capital backing for guarantees.

We did not set out to write a paper which criticised certain practices in the market. Nor, I hope, have we done so. Rather, we sought to investigate some 'generic' aspects of the transacting of with-profits business, leaving more particular conclusions to be drawn by the reader.

The direction of our work has been influenced by various factors. One of the more memorable was the battle, nearly three years ago, for the acquisition of a certain mutual life office. An interesting feature of the eventual transaction was the commitment to distribute the estate, replacing it with capital, to the extent that it was needed, from the acquiring office. This demutualisation touched on issues which we felt were of relevance to all with-profits funds. The investment risks involved in writing with-profits business had been considered many times in the literature, but little had been published on the more specific questions that we were now asking. In particular:

— How much capital support was needed to raise the equity backing ratio (EBR) of an asset share or a with-profits fund by a given amount?
— If such support had to be paid for, to what extent was the additional capital worth having?
— Furthermore, if, as a result of other estate distributions, self-supporting with-profits funds were to diminish in availability, and derivative-backed contracts completely take over as the means to secure investment guarantees, would that be a bad thing for investors?

Such questions strike right at the heart of with-profits business, and how the value that such policies offer compares with equivalent derivative-backed benefits.

Equivalence is sometimes a hard thing to pin down, particularly between different investment vehicles. We chose to address this through probabilities of failure to meet guarantees; hence the need for stochastic modelling and also the introduction of 'put spreads' to ensure that we were comparing 'like with like'. Mention of the word 'stochastic' can cause some actuarial eyes to glaze over, and the inclusion of equations like the Black-Scholes pricing formulae can prove major deterrents to a wide readership of a paper. By keeping the modelling work relatively simple, we hope that we managed to keep the paper accessible, while not invalidating our results. Obviously, the numbers that we show are a function of the modelling assumptions used, but, as we stress in the paper, it is the general shape of the answers which is relevant for our purposes.

This is particularly the case with Section 4. Doubtless, some readers will wish to discuss the appropriateness of using 99% as the degree of security to be considered for each policy in isolation. What is especially of interest to us, however, is how the capital needs vary with both term and EBR, and this, to a large extent, is independent of the level of security considered. We are particularly keen to hear the reaction of readers to the variability that we show, particularly in the light of how certain ratios are used in the marketplace.

Of course Section 4 is not the core of the paper, but, rather, a necessary stepping stone to the comparison of costs in Section 7. In making the link between derivative prices and possible with-profits charging levels, we were careful not to imply that an office would be wrong if it did not implement our ideas. There could be good reasons why a with-profits office may choose to offer 'cheaper' guarantees — some philosophical and some to do with the nature of particular portfolios. Nevertheless, we hope that the paper has helped to demonstrate the potential size of some of the cross-subsidies involved.

The with-profits concept has served many policyholders well over many years, but, if competition continues to increase and the terminal bonus cushions of the past disappear, offices transacting with-profits business must ensure that all the risks being run are well understood and also tolerable.

In a paper produced by a previous incarnation of this Research Group (Forfar *et al.*, 1989) a case study was included of a life office which, in the mid-1980s, was, through a combination of premium basis assumptions and reversionary bonus rates, granting single premium pension with-profits business guarantees which were greater than the level of long-term interest rates then available. The investment policy for the backing assets involved holding about 60% in equity and property, and the resulting writing up problems were becoming increasingly evident in the published returns. In the period from February 1986 to November 1987, steps were taken to lower the premium basis guaranteed rate of interest. However, the events of 17 October 1987 took their toll, and, to cut an interesting story short, by the time of the next Annual General Meeting, in May 1988, discussions were well advanced with potential merger partners.

It was the German philosopher Hegel who said: "We learn from history that we do not learn from history". If anyone should learn from history, particularly to make financial sense of the future, it should surely be actuaries.

Before I sit down I would like to make a personal reference to one of the authors and, with the encouragement of the other members of the Research Group, pay tribute to the courage and determination shown by Mr Priestley since his cycling accident last year. There cannot be many authors who schedule their meetings round hospital timetables, but we were glad to do so and are thrilled that he is now back at work and able to be with us this evening.

Dr A. J. G. Cairns, F.F.A. (opening the discussion)**:** It is my intention to focus much of my discussion on certain aspects of the modelling. In doing so, I will leave discussion of the more general implications for the life industry to those better qualified than myself.

Sections 1 and 2 give some background and motivation for the present paper. In particular, they draw attention to the work of the Maturity Guarantees Working Party (1980), and aim to apply some of the reserving principles in that work.

Section 4 is where the meat of the paper really starts. This is where the authors introduce the maturity guarantees concept of a stochastic reserve: that is, what reserve is required today to ensure that we can cover the guaranteed maturity value with 99% certainty. In banking circles this is often referred to as 'value at risk'. In this context we might describe the 1% value at risk as being the difference between the 99% stochastic reserve and the statutory reserve, or the asset share, or some other reserve held in respect of each policy. It is the amount of money that the office might lose if things go badly wrong.

The tables in this section give the first set of important results in the paper. First, they give a good indication of the risks being run by the life office. Second, Table 4.5 considers the relationship between the statutory reserve and the stochastic reserve. The statutory reserve may well be cautious. However, it is clear that it is quite inadequate in each of the scenarios considered in the table. Furthermore, the extent of this inadequacy varies considerably with the term to maturity and with the EBR. In other words, the statutory reserve tells us nothing, not surprisingly, about the risks being carried by a company which holds precisely these reserves.

Section 5 deals with the charges which might be levied on a policyholder for the use of additional capital to protect the policy and its guarantees. I will not dwell on this, except to say that the capital support charging approach, described in Section 5.4.6, seems much fairer. In my mind, the earlier asset share charging approach is quite wrong. I cannot agree with any argument which suggests that it might be acceptable because it is easier to calculate. The authors, of course, come to the same conclusion later on, but the point needs to be reiterated.

Section 6 describes the method which, in my mind, is the most clear or fair method of charging for the maturity guarantee. The reason for this is that the use of put options can remove entirely the risks (or the risks up to the 99% level) of having insufficient assets at maturity. If the company was to charge more for the guarantee, the policyholder could make alternative arrangements rather than take out a new policy. If the company charges less, the policyholder is on to a good thing.

Section 7 pulls the results together. If we consider Figure 7.1, we see that the 1% capital support looks similar in cost to the derivative-backed strategy; but how do we know that 1% is the right level? This looks like luck. I agree with the conclusions in the paper that the capital support charging approach is the more appropriate method of this type, and Figure 7.1 backs this up. However, who could say in advance that the chosen charging level of 1% would be the right level?

There is one very significant aspect of this paper which the authors state repeatedly. The modelling described in the paper is intended to be qualitative in nature rather than quantitative. It is, therefore, quite acceptable to simplify the models used, in order to help us to focus on what is really important. If we use a model which is too complex, then it becomes difficult to see the wood for the trees. This is a very important part of the modelling process. Any actuary who rushes at a complex problem with a complex model runs the risk of making disastrous decisions, because he or she has not carried out the proper groundwork.

Quite recently I had a conversation with a derivatives trader with a London investment bank. Whenever his team introduces a new and innovative product they use a mixture of different models. Initially they will not know to what extent the price of the derivative will move in response to changes in the market or in the economy. The first thing that they do is to construct a simple model. This allows them to analyse carefully the sensitivity of the derivative price to changes in various input factors. This is exactly what we are seeing in the paper.

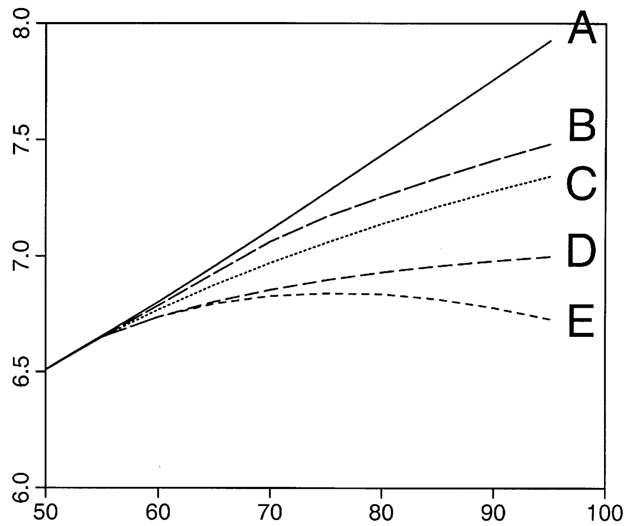
Nevertheless, I call into question the extent of the simplifications introduced in the paper. For example, is it really necessary to assume that gilt returns are constant? Indeed, this assumption introduces inconsistencies in the use of the Wilkie model, which then brings in an element of doubt over the reliability of the later conclusions. Then, in Section 6.3.2, there are further inconsistencies in the use of detailed market data to allow for the pricing of the put options, and this left me feeling rather uncomfortable.

As a consequence, I took another look at Figure 7.1, and attempted to reproduce some of the features of the no charge and the put-spread lines. In my calculations I used the most simple model possible for equity prices — the random walk model with no dividends. For cash and bonds I took a similar approach to the authors, and assumed the same constant rate of return. Although this model is very simple, it has the advantage over that used by the authors of internal consistency. Parameters were chosen to give consistency with the results in the paper.

Would I be able to reproduce the shape of the put-spread curve? The answer is ‘yes’, but it required several hours of investigation to achieve, despite the simple structure of the model. The EBR was used to purchase equal numbers of equity units and put-spreads. The strike prices on the put spread were chosen to ensure that the guarantee was met approximately 99% of the time or 99.5% of the time. The results are shown in Figure D.1.

The top line, curve A, shows the average return if there is no charge for the guarantee. Curves B and C give the picture if put prices are calculated using the same parameters as the asset model. With curve B we aim to match the guarantee 99% of the time. With curve C this is 99.5%. This was achieved by varying the amount of the put spread. Curves D and E use a lower market rate of interest and a higher assumed level of volatility, respectively, than in the asset model.

These curves are the closest to that in Figure 7.1. Furthermore, I see this as quite reasonable, because banks will charge more for their derivatives than the so-called fair price because of the



A: no charge
B: fair charge for puts, 99% certainty
C: fair charge for puts, 99.5%
D: low interest rate in puts, 99.5%
E: high volatility in puts, 99.5%

Figure D.1. Average return to policyholder

long-term nature of the contract. In particular, the average level of volatility in equity prices over the next ten years will be regarded as quite uncertain. The bank may take a cautious view, and assume a higher level of volatility than would be regarded as the best estimate. What this figure also shows us is that the cost to the policyholder of the guarantee is quite sensitive to each of these inputs.

We can also take some comfort that a similar result to that in the paper can be produced using quite a different model. This, again, is a qualitative statement; that is that our conclusions appear to be fairly robust relative to the choice of asset model. This is an important conclusion. All models are approximations to reality.

We should always check that our conclusions are not overly dependent on the choice of model. If they are, then we must exercise extreme caution.

Professor A. D. Wilkie, C.B.E., F.F.A., F.I.A.: Table 4.1 shows exactly the pattern one would expect if the underlying process were a random walk with drift. Consider Figure D.2, which shows the mean and selected symmetric quantiles of such a process, with time t on the horizontal axis, and the value of the process, say $X(t)$, on the vertical axis. For any constant level of X, other than the starting value (zero in this case), the probability of $X(t)$ falling below that level starts at zero when $t = 0$, then increases as t increases, then falls again. This is a result that was shown by Bachelier as long ago as 1900.

The authors' capital backing method, which is sometimes also called 'quantile reserving', suffers from the same deficiency as value at risk (VaR), in that it can be mathematically

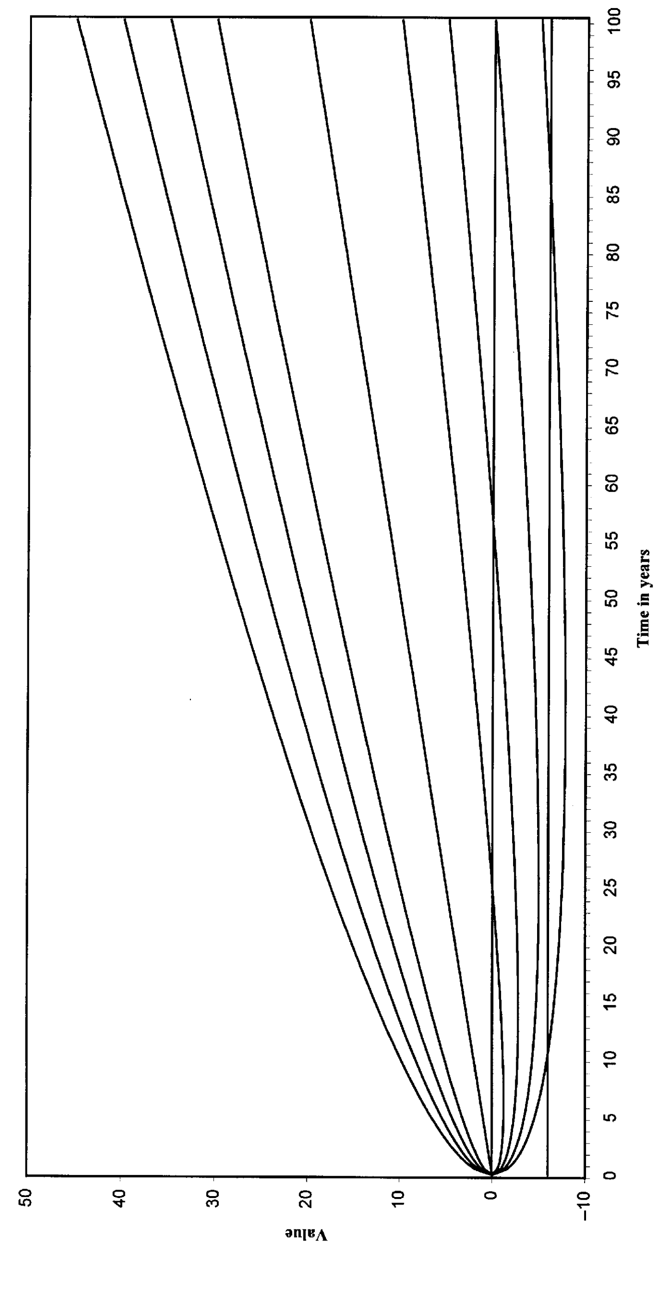


Figure D.2. Median and quantile lines for a biased random walk, mean 0.2, standard deviation 1; lines for 0 and −6 are also marked, showing how the probability of falling below any line, except that for 0, rises and then falls as time increases inconsistent, as the authors point out in ¶A.5. Professor Delbaen of ETH, Zurich, in recent talks, has shown that a more consistent reserve is the quantile reserve plus the expected value of the loss in excess of the quantile, what one might call the conditional tail reserve. He has shown that this is suitably convex, i.e. when you combine two portfolios the reserve for the combined portfolio can be no greater than the sum of the reserves for the separate portfolios, and may well be less.

I would have suggested a 2% charge on the capital put up for the reserve, rather than 1%. What I would compare it with are commercial bonds. Often a good quality commercial loan stock yields 2% more than comparable government stock, and my feeling is that the sort of risk that the shareholders, or whoever provides the contingency reserves, are running is rather similar.

We should also note that options are, quite correctly, priced using risk-neutral probabilities, i.e. assuming that the mean return on shares is the same as that on risk-free bonds. However, we normally expect shares to have a higher mean return than bonds, otherwise why should we invest in them? That means that call options are 'cheap' relative to our expectations for them, and put options are 'dear'. I wonder whether the apparent accident that the put option strategy appears to cost the same as the quantile reserve with a 1% charge is related to this fact, and with the other parameters used.

Reference

Bachelier, L. (1900). Theory of speculation. Doctoral thesis translated and reprinted in *The random character of stock market prices*, edited by Paul Cootner. M.I.T. Press, Cambridge, Mass., 1964.

Mr T. J. Sheldon, F.I.A.: I feel that the paper is an attempt to justify the ways of actuaries to financial economists, and the comparison between the actuarial approach to charging for use of capital and the price of equivalent options is illuminating. It certainly suggests that, perhaps, we are not making adequate charges for the guarantees being underwritten in with-profits funds, though the degree of give and take between different generations and classes of policyholders would need investigating before reaching a firmer conclusion.

The paper illustrates and discusses alternative approaches to charging for capital support. I agree with the authors that the asset share charging approach is not particularly scientific, and that the capital support charging approach seems preferable. The charge under the capital support approach has two components. The first reflects the loss of the expected higher return from equities over that earned on gilts; and the second is a flat charge on the excess capital required. While the second component seems reasonable, I have some doubts about the first. It seems to ignore the fact that equities are riskier than gilts, and that this extra risk is reflected in a higher expected rate of return. If we do make a charge for this component, we seem to be arguing that £100 invested in equities is more valuable than £100 invested in gilts.

Admittedly Figure 7.1 shows that this method of charging for capital support is close to the cost of the put spread strategy, but I suspect that a reasonable fit can also be obtained by simply charging a slightly higher flat rate on the extra capital required of, say, between 1% p.a. and 1.25% p.a. The comparisons between the cost of capital approach and the put spread will, of course, depend on the stochastic asset model used to derive the former, and the parameters, such as the equity risk premium, assumed in the model.

Section 5.2 points out that most companies do not make charges for capital backing. This is endorsed by the findings of the Asset Share Working Party, which confirm that the number of companies making such charges has actually reduced, and that only a small number of companies who currently make no charge intend to start making a charge in the future.

Section 5.7.3 poses the question of whether higher EBRs are worth having. It has become accepted wisdom that they are, and, indeed, the corporate strategy of many with-profits offices has, to a large extent, been dictated by this acceptance. The results of the paper support this view, but the comparison between the various EBRs becomes considerably more marginal when a reasonable charge for the cost of capital is introduced. It is of interest to note that many financial advisers seem to regard safety as the most important feature of with-profits, and the comparison is often with deposit accounts rather than with other forms of riskier investments. Maybe the time has come for a reappraisal of equity backing ratios in with-profits funds.

In ¶7.5.7 it is mentioned that the pricing of derivatives is based on risk-free discount rates with an appropriate allowance for credit risk. As Professor Wilkie has already mentioned, this is only half the story. The probability distribution of the pay-offs from derivatives is adjusted from their real-world probabilities before their expected values are discounted at these risk-free rates. The adjusted probabilities are commonly referred to as the risk neutral distribution because of this.

The investigations described in the paper are based on an individual policy or a tranche of identical policies. Appendix A comments on the additional complexities of modelling a portfolio of contracts with a spread of maturity dates. In order to apply a market-based approach to a portfolio, allowing for cross-subsidies within the portfolio, a particular type of stochastic asset model is required. The model needs to be capable of pricing the various options, which are being written both by the fund and between different cohorts of business, in a market consistent manner. This can be achieved by projecting probability distributions of maturity values, and then discounting the means of those distributions at an appropriate discount rate. Stochastic investment models used by actuaries carry out the first task, that of producing the distribution of future outcomes, but, in general, cannot place a value on the outcomes to derive market consistent values. Fortunately, models are now available which can carry out both tasks. They work by adjusting the probability distribution of outcomes before discounting at a risk free rate of interest. This is precisely the approach used by traders of derivative instruments. The parameters in these models are calibrated against known market prices of tradeable securities.

One advantage of this new type of stochastic model is that the ownership of a with-profits fund can be analysed on a market-consistent basis. The value of each stakeholder's ownership rights can be derived, and the total will equal the market value of the assets of the fund. The effect of management action on the values of the various stakes — for example, those of different generations or classes of policyholders, shareholders, company agents and the tax authorities can then be assessed. Use of this type of model overcomes the criticism mentioned in ¶3.2.7, since the results of the model can be shown to be consistent with market prices. We do, therefore, have the tools to carry out the type of analysis suggested by the authors for portfolios of contracts.

Mr D. O. Forfar, F.F.A., A.I.A.: The subject matter of the paper is really about the importance of charging for guarantees on with-profits business. You cannot get back both the undiluted asset share at maturity and have the additional advantage that, if the asset share happens to have fallen below a certain level, then the asset share will be made up to that level. As the authors point out, if you want that additional advantage you need to pay for it, and one way of doing this, in the case of with-profits, is through the asset share being slightly diluted, that is by a charge being levied to pay for the cost of the guarantee. In the language of derivatives, you have to give away some of the upside for the downside to be protected. The key question is: "How much of a charge needs to be levied to pay for this?"

You not only have to decide how much to make for such a charge, but you also need to decide how to invest the charge premium. It is interesting to characterise a with-profits policy in the language of financial economics. You start with a straight unit-linked contract where you get back your undiluted asset share. You then add the first special feature — if the asset share falls below a minimum level, which is increased every year, you will get a top up to this minimum level. This could be described as a ratcheted minimum level. You then add the second special feature — that your payout will be determined, not by the undiluted asset share, subject to the minimum level, but by the asset share after smoothing, also taking into account the minimum level. You then need to pay for these additional features.

This gives you something close to the with-profits endowment policy. The ratcheted minimum level, in insurance terminology, is called the sum assured and reversionary bonus.

Indeed, the time has, perhaps, now come, given that the traditional endowment appears to be receiving a certain amount of adverse comment, to market it as a type of equity derivative contract with these special features. It might give it a new lease of life!

The paper looks at the cost of adding the ratcheted minimum feature, and how the charge for this should be levied. The possibilities are:

(a) a charge on asset shares;
(b) a charge on capital support; or
(c) a charge determined by Black-Scholes principles.

There is a fundamentally different philosophy behind (a) and (b), on the one hand, and (c) on the other. In cases (a) and (b) the charge works because of the law of large numbers. In any individual realisation of the future you either make a profit or a loss, but, on the average, the profits and losses cancel out. I will call this type of charge a 'law of large numbers charge'.

In the case of (c) the charge works because of the no-arbitrage principle. That means that any individual realisation of the future will give rise to neither profit nor loss, because the charge premium is cleverly invested in a way described as dynamic hedging — for actuaries it would be better described as dynamic immunisation — so as to exactly meet the cost of the downside protection required at maturity without profit or loss. I shall call this the 'immunised charge'.

It is important, not only to pay attention to the cost of the guarantee, but also to the investment strategy to be associated with the charge. You cannot answer the question as to the charge for the downside protection without saying what you are going to do with the charge premium that you levy in order to give this protection. If you are going to invest the premium in the same assets as the asset share, then you have to accept that you will make profits and losses, you will have swings and roundabouts, but, on average, you will be all right. If, however, you immunise dynamically, as per Black-Scholes, you will not encounter swings and roundabouts, but have exactly the right amount at maturity to cover the downside protection.

The charge for downside protection is heavily dependent on the degree of smoothing adopted. If there is very heavy smoothing, then the charge for the downside protection reduces, both under the 'law of large numbers charge' and the 'immunised charge'.

The authors have not considered a portfolio of policies, but, if they did so, I think that you would find that the cohorts of policies in a portfolio are not independent. If, for one cohort maturing in a given year, the cost of the downside protection, in actuality, in that year is very hefty, then the cost in the next year should not be as hefty, as the stock market is likely to have recovered. This would seem to reduce the 'law of large numbers charge' for the guarantees, but it would not seem to reduce the 'immunised charge', which works cohort by cohort, treating them as independent. However, it is interesting that the authors found that, if you charge 1% of capital support, then you get practically the same charge overall as the Black-Scholes charge, based on the immunised approach. However, it is important to recognise that the results can be very different on these two approaches in any individual case, because there will be only one future, one simulation, that will actually be realised.

Mr A. C. Smith, F.F.A.: The sections in the paper follow in a logical order. There is even some logic which some might say is typically actuarial. In ¶5.5 the authors note that one approach to charging for capital support is more straightforward than the other, and therefore they conclude that they should concentrate on the more complex approach! Nevertheless, despite the logical order of the paper and the lucid presentation of results, the paper does prompt further questions, such as: "Do derivative-backed products make a with-profits fund redundant, or can a with-profits fund still add value? Can synergies be created in a with-profits fund by the pooling of risks, and thereby create value effectively out of nothing?" These questions prompted the following more specific comments:

(1) Section 7 shows that policyholder returns under the 1% capital support charge and the derivative-backed approaches are very similar. I wonder, however, to what extent this is

dependent on the 1% capital support charge formula. As described in ¶5.3.1.3, there are two charges for the opportunity cost of use of capital:

— the loss of expected investment return from the support capital being invested in the same EBR as the asset share instead of 100% in equities; and
— a further 1% to pay for the availability of capital.

The first of these charges could be eliminated by investing the capital support 100% in equities rather than in the same EBR as the asset share. This would, however, increase the amount of capital support required, and therefore increase the second charge. Whether the combined charge increased or decreased as the EBR in which the capital support is assumed to be invested is varied would give an indication as to the appropriateness of the 1% level. Indeed, it would be interesting to solve for the rate of the second charge, which leaves the combined charge unchanged, as the EBR in which the capital support is assumed to be invested is varied, and, using this rate, to see how policyholder returns under the two approaches compare.

(2) I wonder at what extent similar policyholder returns produced under the two approaches are partly a result of:
   (1) the Wilkie model used; and
   (2) the lack of any dynamic investment strategy.

   In ¶3.6.4 the authors state that, in the stochastic projections, the asset mix of the fund is not rebased to the starting EBR each year, because, to do so in conjunction with an autoregressive model, would produce artificially high returns. Professor Wilkie made his model autoregressive presumably because he believed that that reflected investment markets, and, therefore presumably, believed that an investment strategy which switched each year out of the better performing asset class and into the poorer performer would not only be a good strategy in terms of his model, but also in reality. If one did rebase the EBR each year, then one would tend to reduce the amount of the support capital required. The position with the derivative-backed approach would be more complex, because, in theory, one would have to buy and sell derivatives consistent with the rebasing of the EBR. This inflexibility with the derivative-backed approach could give the capital support approach an advantage. If the authors believe that the combination of an autoregressive model and a dynamic investment strategy is too optimistic, it would be interesting to re-run the results on an economic model, for example Andrew Smith's model, which is not mean reverting, but assumes that the investment return on an asset class each year is independent of the return in the previous year.

(3) The projections concentrate on a single policy. In practice, a life office will have a portfolio of business with a wide range of maturity dates. It would be interesting to re-run the results on a portfolio of business, with new business being written each year to replace that year's maturities, to investigate the extent to which a portfolio of business produces synergies enabling the traditional smoothing approach to outperform the derivative-backed approach. It could also be useful to set up a bonus smoothing account to which capital support charges would be transferred, and out of which maturity shortfalls would be paid, and to track this account as a percentage of the fund, noting the percentage of runs where the bonus smoothing account tends to plus infinity and the percentage where it tends to minus infinity.

Mr D. M. Pike, F.F.A.: The way in which the different approaches to charging are presented in the paper is that, first the authors investigate the system of capital support for policies, using a seemingly arbitrary 1% charge, and then they investigate, as an alternative, the derivative-backed approach. Figure 7.1 shows that these give remarkably similar answers. If I did not know the authors better, I would almost be tempted to suggest that, having first calculated the results of the derivative-backed approach, which is, perhaps, more soundly based in theory, they calibrated the capital support method accordingly. I am not being entirely facetious, because I wonder if it would be a more realistic approach to state that the derivative-backed approach does give us a more scientific method of getting a result, but, perhaps, the costs make it a less practical approach. The capital support method is more practical, but has to be calibrated by some means. Figure 7.1 shows, quite dramatically, that, once we get to 60% EBR and the 1% risk of ruin kicks in, if the cost of the downside is covered for free by the estate, then the higher the EBR, the better for the policyholder, whereas if the policyholder has to meet the cost of the downside risk, then, as Mr Sheldon and others have pointed out, the outperformance of equities over gilts is not so dramatic.

The answers do depend, crucially, on the initial assumptions, which are shown in ¶3.4.5, where the authors set the dividend yield to current levels, and, consequently, have had to increase the dividend growth rate to give an overall mean return of equities 'that was felt to be reasonable', presumably by the authors.

I am reassured by the comfort that Dr Cairns was able to give us that the results do not depend on the use of the Wilkie model compared to other models. This is probably because the main results are really comparative rather than absolute. The authors have highlighted the autoregressive feature of the Wilkie model. Another feature of it is that it is based on a lognormal distribution, so that the tails, as shown in Figure 3.1, are not so fat as other models, using other distributions, might produce. For instance, in Figure 3.3 the relationship between different yields may be consistent from one model to another. However, other models might, perhaps, give different absolute levels for the 0% and 1% yields.

Mr D. A. Smith, F.I.A.: The paper sets out a market-based approach to pricing with-profits guarantees, and draws into question the common method of costing for guarantees as a 'return' on the capital provided (the opportunity cost of Section 5.3.1). Section 4 considers the amount of capital expected to be sufficient to fulfil the guarantees 99% of the time. Section 5 assumes that this level of capital is available, and provides two possible methods of charging for the use of capital, but does not quantify what level of charge would be appropriate. Section 6 describes a derivative-based strategy that provides the same level of security, which avoids the need for internal capital and which puts a price on the guarantees as the cost of a put spread.

Table 7.2 is key, because, if one believes in an arbitrage-free world and also in the market value pricing basis of the paper, then there will be a maximum level of charge that a life office can make if it is using its own capital to underwrite the guarantees. The reductions in yield from the put spread cost is reasonably close to the 1% p.a. capital support line. It follows that 1% p.a. is the maximum charge that a life office could make for the guarantees illustrated. With a higher charge, policyholders would, in theory, choose another office, one using the derivative strategy, and thereby get better value. Further, the 1% charge would include the risk cost (as given in Section 5.3.2) as well as the opportunity cost on capital locked in (as given in Section 5.3.1). A 1% p.a. opportunity cost, as suggested in the paper, may, therefore, be too high. It does not seem logical to expect a fixed percentage on whatever level of capital is provided, as the level chosen is subjective. The opportunity cost concept is, therefore, unhelpful. A better approach, using the financial economics methods of the paper, is that the fair return on capital is the market return on the assets provided plus the market price of the risk. This does assume that a matching strategy is in place.

Tables 6.1, 6.2 and 6.3 give example prices for the guarantees, and are illuminating. We see that, for a 10-year single premium conventional with-profits contract with an 80% equity backing ratio, the price of the guarantee on the assumptions used is 3.2% of the single premium. For a unitised with-profits contract with no guaranteed bonus the price is 1.4%. These costs seem low, but, as is shown in Table 6.3, the results are sensitive to the level of guarantees. The cost increases to 5.5% for unitised with-profits with a guaranteed minimum bonus of 2%. Allowing for further discretionary bonus, it is clear that the market price of the guarantees on with-profits business can be considerable. The paper, therefore, not only provides a convenient pricing method, but it also serves as a warning that the levels of guarantees and bonus additions need to be set with care!

In ¶5.3.2.3 it is stated that the expected amounts of capital used, averaged over the simulations, have been calculated. I could not find them in the paper, but would be interested to learn from the authors how they compared with the costs of the derivative strategy. The former will have been based on the Wilkie model and the parameters of ¶3.4.5, and the latter on the Black-Scholes formula and the parameters of Section 6.3.2. Given that the underlying distributions will be different, and that the option costs depend on the tails of the distributions, it seems likely that the two could differ to a large degree. This needs comment.

Mr A. J. M. Chamberlain, F.I.A.: I was intrigued to read the first paragraph, which talked about what the words 'with-profits' can mean. I have great difficulty in considering a smoothed managed fund to be a with-profits contract at all, but what are policyholders actually looking for in terms of the level of guarantees? We have heard the suggestion that with-profits is primarily compared with deposit-type contracts, whereas others will tell you that it is primarily compared with heavily equity-backed products like ISAs. I do not pretend to know the answer to that.

Applying a fairly lay approach to the adoption of a deterministic return on gilts, I compared this decision with the decision in Section 3.4 to reflect the current low interest, low inflation environment within the model. How many people can assure me that this is going to be the condition indefinitely? Supposing that we revert to a higher inflation, higher interest environment within the period concerned? Do we not then see the sort of circumstances that were seen in the early 1970s? I remind you that, at one point, Consols were yielding 17%. What will that do to the capital value of the highly secure gilts in this model? It is the correlated risk of a fall in equities and a fall in gilts that may be a significant exposure for with-profits business.

To defend the approach that the Government Actuary takes on resilience testing, is it surprising, on these modelling assumptions, that the resilience test is more onerous on gilt investment than on equity, when gilts remain at a constant 5% return?

In many ways, although I am only about halfway through my career, I am beginning to feel extremely old-fashioned. I am quite a fan of the net premium method of valuation, because of the way in which it has withstood the test of time. To me the thinking in ¶4.3.7.3 is illustrative of a particular problem. If you write business on a 1% return to give the sum assured, then you must be including a bonus loading if you are going to value at a higher rate, unless you are abandoning that age-old actuarial principle of net premium valuations being based, at least loosely, on the premium basis, which was part of the theory behind them. It is not really reasonable to argue that all the difference in the premium is an expense provision. Some of it, surely, is intended to be future return.

If there is one message that I did get from this paper, it related to the importance of the required return on capital of the person providing the moneys to back these guarantees, whether that be an outside shareholder, whether it be an estate, or whether it be an investment bank, and, indeed, their appetite for risk. It might be interesting, hearing the debate on the merits or otherwise of the formula that the authors have used for the capital method, to consider the other decisions that life offices are making, at least every year, in relation to how they price non-profit products, whether to buy a new computer system, whether to purchase another life office or whatever they may be deciding at the time. Every time that they enter into a project they are looking for a return on capital. Is the investment of the estate in providing a level of guarantee to a new tranche of with-profits policyholders fundamentally different to any other investment decision for the capital of an office? For a mutual, I would suggest that the answer is not that different. So, maybe, that provides an alternative approach to the value, or otherwise, of the method that the authors have proposed, and, indeed more significantly, the quantum of the 1% parameter.

Dr A. S. Macdonald, F.F.A.: It is useful to look at the paper in terms of pricing principles (a term much more familiar in general insurance than in life insurance). There are at least four pricing principles used in the paper. The first is the most familiar one, the principle of equivalence, which we see in the methods that are used to price the policies used in the simulations. The second, which by now is quite well known, is the quantile pricing principle, or the equivalent reserving principle, avoiding a shortfall with a given probability. Both of these depend, in slightly different ways, on the laws of large numbers, and, therefore, on averaging over large numbers of policies or, more contentiously, over different time periods. The third is introduced in Section 5, and is known as the no-arbitrage principle. I think that there is a subtle difference between the use of the no-arbitrage principle in the authors' model and the no-arbitrage principle as it is used in financial economics. It depends upon the supposition that, if you can reproduce a given payoff more or less perfectly from instruments that are available, that can be bought and sold and are traded on a market, then that enforces a price for the payoff, because, if you either try to buy or to sell at a different price, somebody can reproduce that payoff using these traded instruments and make infinite amounts of money without risk. The no-arbitrage principle eliminates that possibility.

This is a slightly different situation, because it is not clear how someone might take advantage, in the example that Dr Cairns gave, of the life office charging more for the guarantee than was suggested by the put option strategy, by then making alternative arrangements. We are not dealing with a market in which all the instruments are traded both ways. You cannot decide arbitrarily to buy or sell a life insurance policy. Also, the very notion of doing so only considers the pure investment part of the risk, and ignores all other aspects of a with-profits policy, such as the pooling across generations and the smoothing. This is, therefore, a fourth pricing principle — no-arbitrage used as a benchmark rather than no-arbitrage put into action by explicit trading and construction of matching portfolios. In the same way as it was immediately acknowledged in 1952 that immunisation was, perhaps, more practically useful as a benchmark than as a strategy to be followed by the entire industry, so this use of no-arbitrage as a benchmark is, perhaps, a more useful idea than contemplating the use of no-arbitrage pricing on the scale of an entire industry. There are several reasons why it might not be possible to implement no-arbitrage pricing on such large scale. One is the construction of the alternative arrangements in a market where some of the primary instruments are not traded, such as life insurance policies. The other, and larger, problem is the scale of trading that might ensue if the life insurance industry, or the pensions industry, attempted to use derivatives on a large scale to back the extremely large guarantees that are given. That would result in a one-way demand in the market for put spreads if the institutions tried to implement the strategies by buying traded options, assuming that they were available, or, if they tried to carry out dynamic hedging, it would result in very large one-way movements in the underlying market, because everybody would be trying to do the same thing at the same time.

The extension of the work from single policies to portfolios is necessary before we can compare the various pricing principles in the paper. The following is only a suggestion that might offer an interesting way to dilute the impact of the whole industry trying to do the same thing in the traded markets at the same time. The authors mention, in ¶A.6, the now well known idea of the recycling of capital by with-profits business, so that, if asset shares exceed statutory reserves, capital can be recycled to support the new business strain of the younger generations of policies, providing the engine which has fuelled the expansion of mutual insurance. A similar idea might apply to the recycling of put-spread support for guarantees from one generation of policies to another, so that the older generations of policies approaching maturity are notional holders of put spreads which are underwritten or sold to them by the younger generations of policyholders, who hold these in their portfolios as an asset. Since that might involve nothing but internal and notional transfers or trading within the life company and not in the market, then the effect of the trading would not be apparent in the markets as a whole.

Mr M. D. Ross, F.F.A.: This paper is a very timely one, because the whole future of a with-profits contract is under much scrutiny and, indeed, some attack from within government circles and elsewhere.

The paper raises a number of issues like: "How do you charge for with-profits guarantees, and what is a suitable level?'' I am intrigued, in a way, because it does boil down to the risk appetite of the life office, whether it be free assets, or the with-profits members themselves, or the shareholders, or, in the case of the derivatives option, the investment bankers who make these options available. It depends on what the risk appetite is and the risk-reward relationship that the different parties might require. The inter-generational aspect, or the cross-generational aspect, has been a feature of with-profits business since the start, certainly for most life offices, although not for all is it an integral part. It also moves across from pure life business into the pensions fund industry and final salary schemes, where there is another inter-generational cross-guarantee going on. How is that charged for?

A life office which was set up, particularly, so far as a with-profits fund is concerned, largely to meet such guarantees and to give such guarantees might well take a different approach to the risk-reward relationship than an investment banker. There is freedom within a with-profits life office to alter the asset policy. It is going to vary over time. A life office is surely better placed to provide a fairer charge and to manage that over time. The trouble is, in the modern world, that it is not quite so transparent as some people would wish.

There is very much a scale issue in terms of trying to buy derivatives in the market. One only needs to look at some of the manoeuvring to try to hedge against interest rate risk in recent years, and, indeed, the fact that, even if you do not want to hedge, there are not enough long-dated assets around to buy to protect that liability. Some of us have had to find other ways to do it. In the marketplace one can do it, but it is not an easy thing, even for one life office. If you did it for an entire industry, then there would be major challenges. So, I question whether, in the real world, it is right to compare the cost in that way against what would actually be achievable in practice. After all, life offices have been doing this for over 100 years.

There is an issue in the with-profits concept of what terminal bonus target is being used for a portfolio. I would like to have seen rather more work done in terms of changing that target. It certainly does confirm the fact that with-profits bonus rates, certainly unitised ones, continue to be alarmingly high, and it will be worth us all taking note of that. Whilst I have much sympathy with Mr Chamberlain's comment about interest rates, I remember one of my predecessors telling me that he spoke at a Faculty meeting in the 1960s and said that it is not impossible for gilt yields to be in double figures. The others laughed at him. He told me this when gilt yields were 18%. We need to remember that environments can change, and life offices are — and certainly have been in the past — able to adapt to that. I would be very concerned if the world moved us to a position where that sort of flexibility was lost. It has served our customers well over very many years.

Mr D. G. Robinson, F.F.A.: The history books tell us that the origin of with-profits involved members joining together to share the risk of uncertain or adverse mortality experience. The world today is very different. The mortality risk is much more predictable. Instead of mortality, the investment return is now, arguably, the main element for which there is significant uncertainty. It seems to me that with-profits has evolved in the last 200 years from being true with-profits into something (and we are not really sure what) that involves smoothing, but I question the extent to which our new with-profits policyholders and those who advise them understand precisely what it is that they have bought. Do they really understand what smoothing means, how it will operate and over what period? Do we really understand what smoothing means, or do we make it up as we go along? I wonder how many Appointed Actuaries would be able to sit down in front of a typical client and explain how smoothing would work for the client's policy.

It seems to me that, not for the first time, with-profits is under some threat. It is perceived as being over-complicated, jargon-ridden and without a clear definition of the customer proposition. At present it is also being caught in the backwash of problems with a few mortgage endowments failing to repay the mortgages. So, with-profits is in jeopardy, but it would be a great shame if it were to fall into disrepair. It has served the public very well for over 200 years, but, if it is to continue to have a role, it is in need of some fundamental redefinition. The customer proposition needs to change. The basic design probably needs to change. Smoothing methods and guarantee levels need to be much more open in future, and the trade-off between the EBR and the guarantee levels needs be much clearer. We also need to improve the communication of the benefits of with-profits to customers.

If we are able to do this — and I believe that we can — then with-profits will have a healthy future. The profession has a key role to play in this debate. The paper is an important building block in the reconstruction of with-profits.

Mr J. Goford, F.I.A.: I tend to take the position of the customer. The rules of policyholders' reasonable expectations seem to be that you are not allowed to ask customers themselves, you are only allowed to ask informed observers, most of whom seem to be, or are recommended to be, IFAs. If you look at what IFAs say to their customers, then you get statements like: "It is like a deposit account with a bit better return and a kicker at the end", and: "smoothed participation in equities".

Another observation is that we reserve for what we are not expecting to pay, and we do not reserve for what we are expecting to pay. That is to say, we have a resilience reserve which we are not expecting to pay out, and, although we expect to pay out terminal bonus, we have no reserve for it.

Reading with-profits guides, I read about asset shares. If I understand anything about asset shares, I see the 90:10 mechanism working at policy level rather than at fund level. So I ask myself: "What have I got here; is it really with-profits?" I do not think that it is, and, at some point, the operation of the 90:10 mechanism has changed from fund level to policy level, and we have just about noticed that it has changed.

Would it not be a real shame if with-profits was not a valid part of the stakeholder pension arrangements? If the key to that is some demonstration that the customer is not being ripped off through deductions in the mechanics of asset shares, then we really must redesign with-profits to demonstrate that.

My conclusions are, as ever when one starts with the customer, something quite simple. I would reserve on asset shares and smoothing reserve, smoothing reserve defined, very simply, as the difference between asset shares and values paid out on maturity, death and surrender. The difference between claims payments and the asset shares on those policies would be published on a year-by-year and on a cumulative basis. The Actuary's table of the correlation of free asset ratios and EBRs would also be published. The reserve is then asset shares and smoothing reserve (and the glidepath, if you are intending to overpay), and the rest of the assets support the true capital, whether owned by the propriety shareholders or by the owners of mutuals. Most of that capital will be needed to support guarantees, the amount needed being assessed by a stochastic reserve calculation. The key question then is: "What risk of ruin do we use?" To my mind this is where the real judgement of the Appointed Actuary is called into play, with his or her experience of the specific company, and with evaluation of the company's preparedness to take risk. The paper supports the Appointed Actuary in this.

Mr A. J. Robertson, F.F.A. (closing the discussion)**:** Offices are under pressure on capital as a result of guaranteed annuity options, and, with the forthcoming introduction of stakeholder pensions, there is pressure on the basic concept of with-profits. Against that challenging background there is increasing pressure on offices to ensure that their investment and bonus policies are appropriate to current economic conditions and consistent with the office's available capital. The techniques described in the paper are a useful basis for carrying out some of that analysis.

The authors decided to focus on individual policies, and I welcome that. A clear understanding of the fundamentals is useful before developing the more complex models. However, a number of speakers highlighted some of the key features associated with the different generations of policies and the possibility of one generation subsidising or supporting subsequent generations.

The Wilkie model was chosen by the authors, and it provides a reasonable basis for illustrating the key principles. Dr Cairns had some questions on the concept of using a constant gilt return. However, his own analysis supported the general conclusions of the paper. Mr A. C. Smith queried the extent to which a dynamic investment strategy would have reduced the amount of capital required.

For the sake of clarity, the authors adopted a number of simplified assumptions in relation to the rates of reversionary bonus in calculations for the statutory minimum reserve. In practice, one would expect that an office with a high equity backing ratio would have set its bonuses at a more conservative level, in view of the additional volatility of the equity investments. It would have been interesting to have seen the results of alternative bonus strategies. This is, perhaps, a topic for a subsequent paper. The probabilities of shortfalls quoted in Section 4 look high, particularly given the assumption of no future reversionary bonus in the stochastic projections. However, it perhaps illustrates the importance of carrying out this type of analysis so that offices are comfortable with the relative balance between asset mix, bonus strategy and available capital.

There was a considerable amount of discussion on what constituted an appropriate basis for charging for the use of capital. Given increased pressure on the available capital, it is likely that more offices will incorporate a charge in their asset share calculations. Mr Forfar indicated that this was, perhaps, the crux of the paper, and that the question was really how much the charge levied should be. Mr Chamberlain highlighted the importance of determining the rate of return on capital required by whoever it was who was providing that capital. The paper provided an insight into the alternative methods of charging for the access to capital, either through a percentage of the asset share or an explicit charge based on the level of capital support. The authors concluded — and there was a certain amount of support for the view — that the simple approach of a charge based on a percentage of the asset share was somewhat inequitable, and an approach based on the amount of capital actually required was preferable.

Dr Macdonald made the valid point that, for practical reasons, the use of derivative-based pricing was, perhaps, more appropriate as a benchmark rather than as a practical strategy. However, it is useful to have a basis that at least allows you to compare the charge with a theoretical justification. Dr Macdonald's suggestion that this could be operated internally between successive generations was an interesting development of the concept.

It remains to be seen whether with-profits will be permissible in the stakeholder regime that will shortly be unveiled. There are obviously clear benefits to policyholders in giving access to the benefits of the equity returns while protecting them against the volatility of the underlying asset values. There is, however, considerable criticism of with-profits in terms of the lack of transparency. The techniques in the paper provide a useful insight into how with-profits can be managed, and are a useful step forward in terms of greater transparency of the operation of the with-profits concept.

Mr J. A. Dickson, F.F.A. (replying)**:** There have been comments on the choice of the model and assumptions, and, in particular, the perceived inconsistency of using Black-Scholes pricing of derivatives alongside a Wilkie model projection of the future. There is obviously no correct model for projecting returns in the future. The Wilkie model is merely a method of generating a distribution of future equity returns to enable us to look at the nature of equity risk. The parameters chosen are consistent with our current view of future returns. The model used for option prices is largely irrelevant. The option strategy that we adopted meant paying for the spread up front. In reality, a market price will be charged for this. As the market uses Black-Scholes theory in pricing options, this was the natural choice. We could have removed any reference to Black-Scholes if we had got live prices from an investment bank and used them.

There would then be no consistency issue, just a price now for the options and a stochastic model of the future based on the current view of future economic conditions. The prices that we calculated used live market feeds as at August 1999, and, to all intents and purposes, were live prices. However, the prices of options will vary in the future. Our strategy was a simple passive strategy, so this was not a key concern for us. However, this is something that we would have to consider in a more dynamic options approach to deal with the like of regular premiums, increasing guarantees, and so on.

The main question appears to be: "Is the apparent similarity between a capital-backed-charge approach and a derivative alternative a fluke?" The simple answer to this is 'yes', since we chose the 1% charge in advance of the derivative modelling, as an arbitrary common-sense number. Nevertheless, this result was surprisingly robust to sensitivity testing. We looked at two ranges of sensitivities: one involving higher average equity returns; the other a series of sensitivities on option prices. Although higher equity returns reduced the amount of capital backing required, and therefore the impact of a 1% p.a. charge, this reduction in capital requirements also moves a put spread further out of the money and reduces the cost of the derivative strategy. The reductions in yield for the two options still remain reasonably close.

Option price sensitivities included changing the risk-free rate of return, the implied volatility and the skew between implied volatilities at different strike levels. The worst case scenario is for this skew to reduce dramatically, as this reduces the proceeds from the put that we are selling, and, in relative terms, increases the costs of the put that we buy. Again the reduction in yield is not materially changed. The broad equivalence is probably not unreasonable, when you consider that the two approaches to charging are both based on the amount of capital backing required. In addition, the expected return within the option pricing is a risk-free rate, based on market swap rates which are a credit spread of around 100 basis points over gilts. This means that the expected return on the options sits right in the middle between our assumptions for gilts and equities.

It is comforting to know that work on a different stochastic model gave similar results, although it took us substantially longer than several hours to produce ours! It is also interesting to hear many comments on benchmarking rather than actually going out and buying derivatives. We feel that the market for these options is extremely small in comparison to the demand of life offices if everyone were to adopt these strategies. We are quite clear that this method is used for pricing guarantees as a benchmark, and not necessarily for buying a put spread.

One of the simplifications that we adopted, which has been highlighted as a potential weakness, is the use of a deterministic gilt assumption. We would agree that introducing gilt volatility would be a natural step in the analysis, but we also feel that it would not substantially change the results. It was easier to interpret results in the light of only one stochastic parameter, and a derivative alternative is a lot simpler to manufacture and present. In addition, the autoregressive nature of the Wilkie model means that great care has to be taken in using a stochastic projection of returns on two asset classes, especially if you allow switching between them. The added complication of adopting stochastic gilt returns would not have aided understanding at this stage, and would be unlikely to alter our core conclusion.

It is, perhaps, a useful topic for further research, although there is a range of other simplifications likely to be more significant than the approach for gilts, which is also worthy of investigation, not least portfolio studies and the potential for smoothing over time. Other options include adopting other models — for example, the Resilience Test Reserve Working Party have included market jumps within their models. Alternatively, you could look at a more dynamic approach to derivative studies, allowing for the likes of regular premiums, new business and exits. Looking at these more complex portfolio studies, we may be able to help answer some of the questions raised by Dr Macdonald, and we could, perhaps, also look at some of the reserving suggestions raised by Professor Wilkie.

We are delighted that so many of you have suggested areas for further research, and that these have coincided with those that we have already identified. Some of the members of the

Bonus and Valuation Research Group are likely to move on in the near future, so we are pleased to see that there will be so many keen volunteers to replace them and to take this groundwork forward.

The President (Mr C. W. F. Low, F.F.A.): All members of the Faculty owe the Working Party a debt. The paper has been most timely when the whole viability of with-profits is being questioned. The strength of the Scottish life assurance industry has, over nearly two centuries, been built on with-profits with guarantees. We who advise it have shared in that prosperity. The question of the viability of with-profits is now a fact which we have to address. It has been mentioned by some speakers in the discussion, it has been mentioned in the press, and, indeed, we have the Government considering whether or not with-profits is an acceptable method for provision of stakeholder pensions, when to me (and I suspect to many actuaries) it would seem a natural method. The life assurance industry has a problem in describing how with-profits can, and should, work. We have a task to advise them how it may be done.

The Working Party has shown that our thinking on the subject has, as always, moved forward from one working party to another. Perhaps the industry's product and its product description has not moved forward, and is in need of radical redesign. Some ideas, well-meaning and very useful to actuaries, may not, in fact, be the way forward. I suspect that transparency of design and communication in clear English is not to explain inter-generational transfers by means of put option trading off market! While with-profits is being questioned as a way forward for stakeholders, apparently indexed products, supported entirely by derivatives, are seen as the preferred route. There is food for thought there.

On the wider horizon, we are expecting that an issues paper from the International Accounting Standards Committee on financial reporting for life assurance will be published early in December 1999. The driver behind that is common worldwide reporting standards. It is to be hoped that worldwide regulators may agree to moving to regulation based on a worldwide accounting standard, with, obviously, prudential margins added.

That brings regulation into the debate, and an opportunity for us to propose fundamental changes to the European Commission, which was the driver behind our existing valuation regulations, which actuaries know are an impossible mismatch of asset and liability valuation techniques. The time is right for the profession to be advising its market on how to redesign and explain its products, and for us, as professionals, to advise governments and regulators as to how these markets could be regulated in the public interest of the consumer.

The paper has been a vital building block for that project. I would like you all to thank the Working Party, and, in particular, the one member who is a shining example to us all, of going well beyond the call of duty, in remaining a member of the Working Party despite the most difficult personal circumstances, and to the industry for the support his employer gave him at the time.