

METHODS OF FORECASTING MORTALITY USING AUSTRALIAN DATA

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(This paper, suitably modified, together with several others in the field of demography, constituted a thesis approved by the University of London for the award of the Ph.D. degree.)

1. INTRODUCTION

SEVERAL methods of forecasting mortality have from time to time been suggested. Some of them have, however, only been applied to the statistics of one or two countries and, while they have been found to be satisfactory in these cases, we cannot justifiably say that the assumptions on which they are based are universally valid. In §§ 3-7 these standard methods of mortality forecasting will, with necessary modifications, be applied to Australian data. These sections do not, therefore, involve anything in the way of new principle, but, being applications of old methods to new data, they should make a further contribution to our knowledge of the validity of the assumptions on which these standard methods are based. In § 9 a further method of forecasting will be described, and in § 10 the results obtained by the several methods will be compared.

The methods used in §§ 3-7 have been described in detail in the original papers. As some of these papers are in the journals of kindred societies and are not readily available to all members, it was considered desirable, for the sake of readers, and to bring out the underlying assumptions, to give a brief description of the methods in this paper.

2. DATA

Cramér and Wold⁽¹⁾ were fortunate in having available for testing their methods of graduation and forecasting the unique statistics of the Swedish population. For this country elaborate mortality data are available at regular intervals from 1750 to the present time. Satisfactory results were not obtained when their investigations were limited to the experience of the shorter period 1850-1930; their method was satisfactory, however, when applied to the period 1800-1930. Rhodes's investigations⁽²⁾ into the mortality of England and Wales covered the period 1841-1930.

The main difficulties in forecasting Australian mortality result from the lack of data and from the irregular periods between censuses. Rates of mortality are available for the Commonwealth for only five irregularly spaced periods, namely, 1881-90, 1891-1900, 1901-10, 1920-22 and 1932-34. Consideration was given to using A. F. Burridge's statistics⁽³⁾ for 1870-81 for New South Wales, Victoria and Queensland, and the rates of mortality obtained by dividing the Australian deaths for the years 1938-40 by the estimated population at June 1939. Certain unusual features in both these experiences made their inclusion unjustified, and it was necessary, therefore, to carry out the

forecasts by the standard methods on the basis of the five experiences centred in the years 1885-86, 1895-96, 1905-1906, 1921 and 1933—less than 50 years in all.

3. GRAPHICAL PERIOD FORECAST (4)

This, the simplest method of forecasting, consists in plotting the various values of $\mu(x, t)$ and t for a constant value of x (x_1 say), drawing a smooth curve through the points and extending the curve to give values of $\mu(x_1, t)$ for future values of t . In carrying out the extrapolation the forecaster is guided by the general shape of the curves in other countries and by the shape of the curves for neighbouring ages. This method is shown in Fig. 1, where past history is recorded by a full line and forecasts by a broken line.

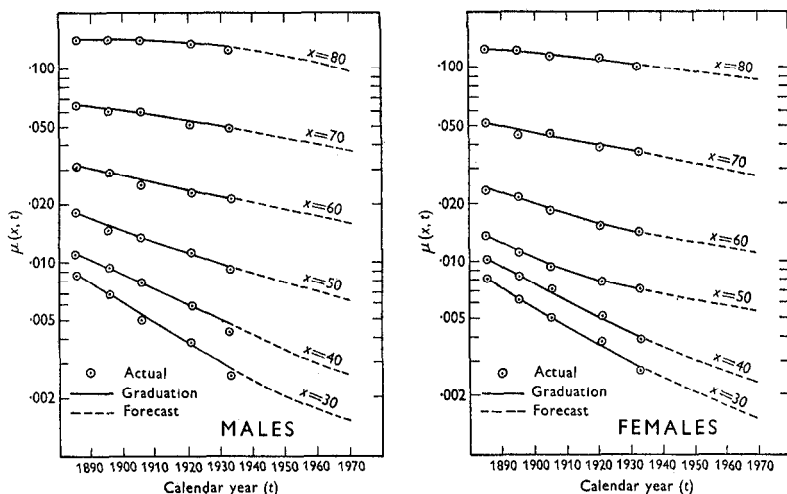


Fig. 1. Actual Mortality and Graphical Period Forecast.

It may be said of this method

- (i) that it is simple,
- (ii) that the basis of forecasting does not justify any further technicalities,
- (iii) that some idea of the large possible range of error can be obtained at a glance,
- (iv) that too much is left to the discretion of the forecaster,
- (v) that it can be used for every age, and
- (vi) that it would be theoretically satisfying if the probability of a person dying at age x were independent of his previous history and depended only on the sanitary conditions, medical knowledge, etc., of the year in question.

4. GRAPHICAL GENERATION FORECAST (4)

Let us express $\mu(x, t)$ in terms of the year at birth $t - x$ ($=\theta$ say) as $_{\theta}\mu_x$ which, therefore, represents the force of mortality at age x for a person born in the year θ . The Graphical Generation Forecast involves plotting the various

values of ${}_t\mu_x$ for a constant value of x (say x_1), using the year of birth as abscissa, drawing a smooth curve through the points and extending the curve to give the force of mortality at age x_1 for later years of birth. The force of mortality at various ages throughout the lifetime of a person born in a given year may be obtained from the intersections of a vertical line with the several curves. The method is shown in Fig. 2, which suggests that the slight reduction in mortality at the older ages is due to the fact that these ages have only just entered the reducing-mortality years of birth.

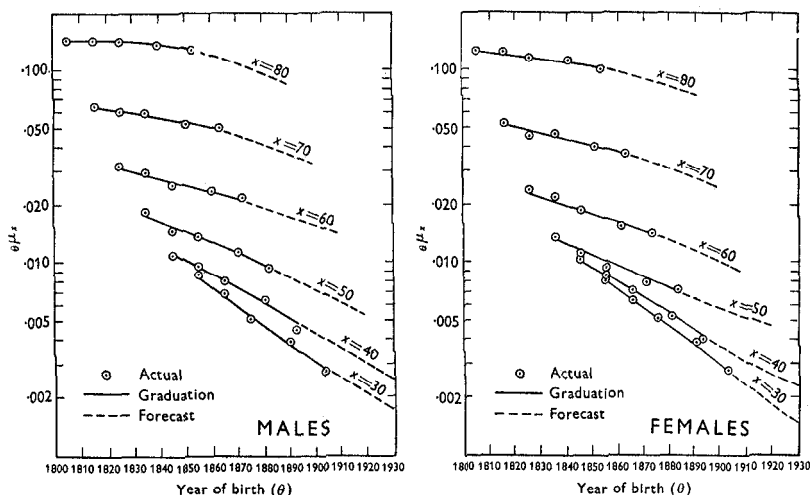


Fig. 2. Actual Mortality and Graphical Generation Forecast.

It may be said of this method

- (i)-(v) as in § 3,
- (vi) that its theoretical appeal depends on whether a person's mortality is determined mainly by the constitution built up in early years, for then his mortality will be a function of age and year of birth and curves for higher ages can be built up from curves actually experienced at lower ages. There is some statistical evidence which supports this hypothesis (England and Wales, Sweden, Scotland (5) and Australia (§ 5.1)).

It must, however, be said

- (vii) that if use is made of data for a year when some important factor independent of age came temporarily into play (e.g. 1919 influenza epidemic) every curve would show a discrepancy at some year of birth and graduation would be more difficult.

5. RHODES'S METHOD (2)

5.1. The validity of the Kermack, McKendrick and McKinlay assumption (5, 6)

These authors showed that for some countries it was reasonable to assume that ${}_t\mu_x$ depended on two factors: (i) the age x and (ii) the year of birth θ , and they deduced that

$${}_t\mu_x = Q(x) \cdot R(\theta), \quad (1)$$

where $Q(x)$ was a function of age only and $R(\theta)$ a function of the year of birth only.

A simple method of checking the truth of this hypothesis for any given data is to select a base year of birth, say 1860, and to calculate, for each value of θ , $\theta\mu_x \div 1860\mu_x$ for several values of x . For a given value of θ this expression should be constant for all values of x , if the hypothesis be true. For persons born in 1860, Australian mortality statistics only tell us the mortality at the appropriate ages at the time of the five censuses, that is, we have values of the denominator of the above test expression for values of x of $25\frac{1}{2}$, $35\frac{1}{2}$, $45\frac{1}{2}$,

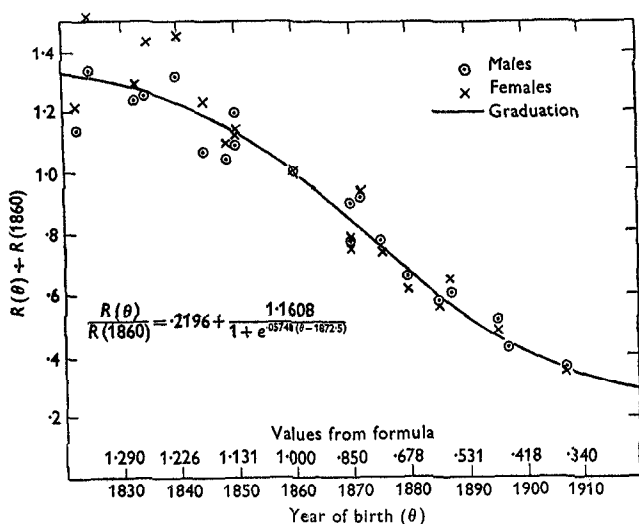


Fig. 3. Values of $R(\theta) \div R(1860)$.

61 and 73 only. In addition to this limitation to five values of x , we are further limited in that, for each of these values of x , we have only five values of the numerator $\theta\mu_x$, namely, the values of μ_x at each census. Hence Australian mortality statistics yield but 25 values of the test expression covering all years. These are shown for males and females separately in Fig. 3. It will be noticed that the irregular intercensal periods prevent us from obtaining several values of the test expression for any year of birth, and hence in the case of Australia a check on the hypothesis is difficult. With regular decennial censuses (e.g. England and Wales, and Sweden) this difficulty disappears.

Since

- (i) one abnormal value of $1860\mu_x$ affects five values of the test expression,
- (ii) where, in the Australian data, two values of the test expression occur for a given birth year they are approximately the same,
- (iii) values for neighbouring years of birth are consistent and the values show a definite trend, and
- (iv) the hypothesis is satisfactory for the more elaborate data of Sweden and England and Wales,

it is reasonable to assume the hypothesis to be true for Australian data. Comparing male and female data, it will be seen that a remarkable agreement occurs. This brings out the reasonable but not otherwise obvious result that the secular changes in male and female mortality in Australia have been the same. We can, therefore, consider together the male and female test expressions and effectively we have doubled our data, and the combined data allow us to accept the Kermack, McKendrick and McKinlay hypothesis with greater confidence.

5.2. $R(\theta) \div R(1860)$.

Having established the validity of this hypothesis we may assume the values of Fig. 3 to be values of $R(\theta) \div R(1860)$. The method of graduation adopted by Dr Rhodes proceeds by determining mathematical expressions for $R(\theta) \div R(1860)$ and $Q(x) \cdot R(1860)$, the product of the two expressions giving the desired result.

The data of Fig. 3 would suggest a graduation by the logistic curve

$$R(\theta) \div R(1860) = A + \frac{B}{1 + e^{k(\theta - \tau)}}, \quad (2)$$

which has frequently been found to be very adaptable to population data. Several methods, all lengthy, have been suggested for fitting a logistic curve to given data. If there are ample data and a definite logistic trend, and if all points are of equal importance, a method such as the least squares method of Cramér and Wold (1) should be used.

Our data, however, are unusual in that they are scanty, and some points are far more important than others. For example, the point (1860, 1.0) *must* lie on the curve which should also pass between the two points plotted for the year 1907.5; and, generally, more weight should be given to the points for the years after 1860 than to the scattered points before 1860. For these reasons a special short method of graduation was adopted, namely, selecting from the graphical representation of the data four points through which the curve should pass and solving for the four constants. If we substitute in (2) the four points (1835, 1.26), (1860, 1.0), (1885, .6) and (1910, .34) and equate the difference of the first two and the last two resulting equations, we readily obtain $\tau = 1872.5$. Substituting back in the quotient of the difference of the first and second and first and third equations gives k , and simple substitution A and B.

The resultant graduation is

$$R(\theta) \div R(1860) = .2196 + \frac{1.1608}{1 + e^{.05748(\theta - 1872.5)}} \quad (3)$$

5.3. *Formulae and results*

If we divide the census values of $\mu(x, t)$ by $R(t - x) \div R(1860)$ from (3), the result, on the Kermack, McKendrick and McKinlay assumption (1), should be dependent on age only and should equal $Q(x) \cdot R(1860)$. The values obtained for males and females are given, with average values, in Table 1. In determining average values it is advisable to ignore extreme values which have probably arisen from some abnormal circumstances, and thus, if there were sufficient values, the median would be a suitable average. With only five values, the average chosen here as most suitable is the arithmetic mean of the three central values.

These average figures increase rapidly with age and, being proportional to μ_x , a Makeham graduation is suggested for the ages we are considering, namely, over 30. (For Swedish data Dr Rhodes required two Makeham curves for ages over 30.) For ages under 30, special treatment, or merely recording the values of $Q(x) \cdot R(1860)$ for each value of x , would be required.

Table 1. Values of $\mu(x, t) \div \frac{R(t-x)}{R(1860)}$

(a) Males

Period	Age										
	30	35	40	45	50	55	60	65	70	75	80
1881-1890	·00820	·00840	·00945	·01018	·01453	·01888	·02440	·03340	·04870	·06870	·10400
1891-1900	·00750	·00818	·00906	·01036	·01268	·01680	·02350	·03420	·04660	·06780	·10660
1901-1910	·00660	·00740	·00870	·01071	·01291	·01590	·02150	·03080	·04810	·07500	·10860
1920-1922	·00750	·00790	·00915	·01105	·01366	·01651	·02370	·03300	·04670	·07070	·11240
1932-1934	·00685	·00773	·00915	·01136	·01315	·02022	·02720	·03660	·05200	·07570	·11750
Average	·00728	·00794	·00912	·01071	·01324	·01740	·02390	·03350	·04780	·07150	·10920

(b) Females

Period	Age										
	30	35	40	45	50	55	60	65	70	75	80
1881-1890	·00783	·00795	·00885	·00951	·01095	·01380	·01795	·02515	·03933	·05550	·09300
1891-1900	·00696	·00798	·00802	·00804	·00953	·01230	·01730	·02460	·03500	·05640	·09325
1901-1910	·00680	·00726	·00772	·00808	·00887	·01107	·01575	·02385	·03680	·08000	·08800
1920-1922	·00742	·00756	·00789	·00804	·00947	·01179	·01549	·02223	·03553	·05725	·09315
1932-1934	·00708	·00766	·00812	·00904	·01144	·01400	·01785	·02581	·03854	·05950	·09300
Average	·00715	·00772	·00801	·00839	·00998	·01263	·01697	·02455	·03696	·05772	·09305

Carrying out a Makeham graduation by the minimum- χ^2 method described in § 6 below and multiplying by (3) above, we obtain

Males:

$$\mu(x, t) = \left[\cdot 006087 + \cdot 0012274 (1 \cdot 5567)^{\frac{x-30}{5}} \right] \left[\cdot 2196 + \frac{1 \cdot 1608}{1 + e^{0 \cdot 5748(t-x-1872 \cdot 5)}} \right], \quad (4)$$

Females:

$$\mu(x, t) = \left[\cdot 006688 + \cdot 00042669 (1 \cdot 7012)^{\frac{x-30}{5}} \right] \left[\cdot 2196 + \frac{1 \cdot 1608}{1 + e^{0 \cdot 5748(t-x-1872 \cdot 5)}} \right] \quad (5)$$

Fig. 4 gives a comparison of the graduated values from these formulae with the actual census values and also the expected values in the future. It will be seen that most of the improvement in mortality is expected at the higher ages.

5.4. Application to practical problems

Forecast rates of mortality have not been introduced generally into financial problems when, in theory, they should be used. This is mainly due to the fact that two variables are involved—age and time—and no satisfactory method has been found of simply tabulating, for these two variables, the many elementary and commutation functions on which such financial computations are based. Any simple solution to the problem seems impossible. If we limit ourselves to ages over 30 we may attempt a solution in the case of Rhodes's method of forecasting.

According to this method ${}_t\mu_x$ is obtained from ${}_t\mu_x (= A + Bc^{x-30})$ simply by multiplying by a constant r depending only on θ . But ${}_t\mu_x$ is a Makeham curve. Hence, if complete monetary tables based on ${}_t\mu_x$ are available a number of actuarial functions based on ${}_t\mu_x$ may be obtained from these tables by using the well-known facts that for a Makeham curve (i) a constant increase in the

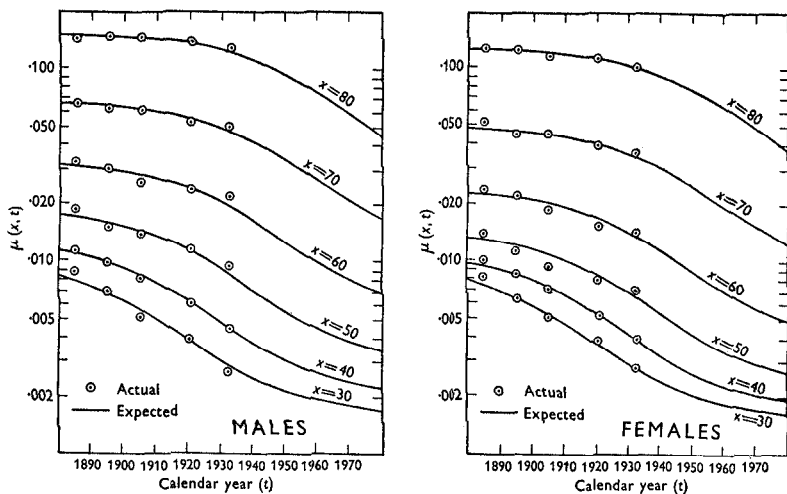


Fig. 4. Comparison of Actual and Expected Mortality using Rhodes's method.

force of mortality has the same effect as an equal increase in the force of interest, and (ii) an increase in the constant B is equivalent to an increase in age. Using an obvious notation, we may show that

$$\left. \begin{aligned} {}_tL_x &= {}_tD_{x+\alpha}^j, & \text{where } j &= e^{(r-1)A} - 1 & \text{and } r &= c^\alpha, \\ {}_tD_x^i &= {}_tD_{x+\alpha}^j \\ {}_tN_x^i &= {}_tN_{x+\alpha}^j \\ {}_t a_{x:\overline{n}|}^i &= {}_t a_{(x+\alpha):\overline{n}|}^j \end{aligned} \right\}, \quad \text{where } j = (1+i)e^{(r-1)A} - 1 \quad \text{and } r = c^\alpha. \quad (6)$$

$A_{x:\overline{n}|}^i$ may be obtained from conversion tables or graphs.

For any year of birth we may therefore obtain the l_x column, values of annuities and assurances, using the complete set of tables based on ${}_t\mu_x$ simply by altering the age and the rate of interest. $j-i$ for all practical purposes is constant for different values of i (including zero, for l_x) and varies from 0 to .5% over the important years of birth. A simple graph giving the transformation is found in Fig. 5. Our problem is only solved provided we can derive from the tables the required monetary functions at *any* rate of interest. These functions are usually tabulated only at integral and half-rates of interest. We shall therefore have to devise some satisfactory method of interpolation.

We can readily obtain interpolation formulae such as

$$a_x^{i_1} = a_x^{i_0} \left[1 - \frac{i_1 - i_0}{.005} \left(1 - \frac{a_x^{i_0+1}}{a_x^{i_0}} \right) \right]$$

or more accurate and more complicated formulae. Similar formulae can also be derived for other functions. It is obvious, however, that all of these formulae (and even linear interpolation) are too complicated to be used in everyday problems. We shall therefore consider a graphical method of interpolation.

The usual argument against graphical tabulations, namely, their limited accuracy, is not an important one here, for the accuracy obtainable with a large graph is better than the accuracy to which we are working when forecasting mortality.

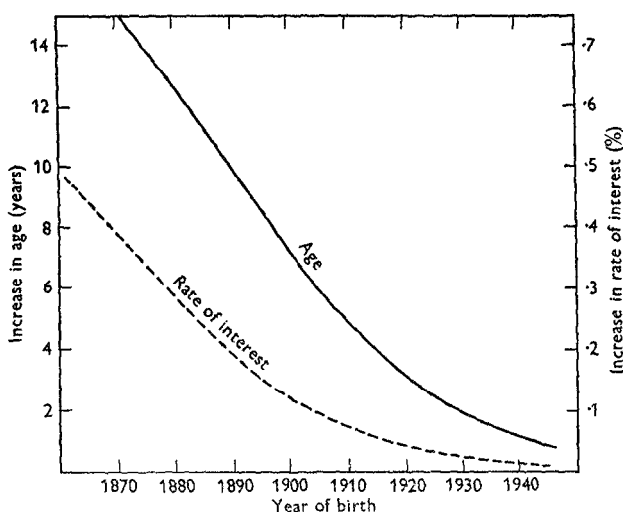


Fig. 5.

If we graph on logarithmic paper the values of D_x , N_x and M_x for rates of interest from 0% to 6% for all values of x , and use a separate 'slide rule' marked logarithmically also, we may immediately read off the graph for any rate of interest and any age such functions as

Probability of surviving $l_{x+n} \div l_x$,

Whole-life annuities $a_x = N_x \div D_x$,

Whole-life assurances $A_x = M_x \div D_x$,

Deferred whole-life annuities or assurances $\frac{N_{x+n}}{D_x}$, $\frac{M_{x+n}}{D_x}$.

Temporary annuities may be obtained by subtracting two readings since

$$a_{x:\overline{n}|} = \frac{N_x}{D_x} - \frac{N_{x+n}}{D_x}.$$

Similarly with temporary assurances, endowment assurances, etc.

Using this simple graphical interpolation and the transformation of Fig. 5 we can easily use mortality forecast by Rhodes's method in the solution of everyday practical problems.

5.5. Comments

The following comments about this method of graduation and forecasting may be made.

- (i) It depends on the Kermack, McKendrick and McKinlay assumption.
- (ii) It yields a particularly simple formula.
- (iii) For ages over 30 it may easily be applied to practical financial problems, and in a given case we could test to see if the same transformation would give (as it probably would) sufficiently accurate results for younger ages.
- (iv) Once adopted it leaves little to the discretion of the forecaster.
- (v) A relatively large amount of work is involved.
- (vi) The difficulties at the younger ages detract from the simplicity of the final result and increase the numerical work.
- (vii) A single Makeham graduation may not be satisfactory for

$$Q(x).R(1860).$$

6. MAKEHAM PERIOD METHOD

This method was first used by R. Blaschke in 1923 and has since been used by other writers including an application to Swedish statistics in an excellent paper by Cramér and Wold (1). In this method, for each calendar year t_1 for which $\mu(x, t)$ is available, we graduate $\mu(x, t_1)$ by the Makeham curve $A + Bc^{x-30}$ and consider the constants so obtained as functions of time.

Cramér and Wold used the actual deaths and exposed to risk in fitting a Makeham curve. We have used their method—called minimum- χ^2 method—but have applied it to μ_x . The partial differentiations resulting from a rigorous application of the minimum- χ^2 principle immediately lead to a complicated set of equations, and the simpler solution, set out in steps below, has been shown by Cramér and Wold to give the same result and has therefore been adopted. The steps in the method are:

- (i) obtain an approximate value of c , e.g. from $c^{20} = \frac{\mu_{75} - \mu_{55}}{\mu_{55} - \mu_{35}}$,
- (ii) assuming this value of c , determine A and B by equating (for quinquennial ages from 30 to 80 inclusive) actual and expected first and second summations of μ_x , that is by solving

$$\left. \begin{aligned} 11 A + B (c^{55} - 1) (c^5 - 1)^{-1} &= \Sigma \mu_x, \\ 66 A + B [(c^{60} - 1) (c^5 - 1)^{-1} - 1] (c^5 - 1)^{-1} &= \Sigma \Sigma \mu_x, \end{aligned} \right\} \quad (7)$$

- (iii) determine, for this value of c , the expected values of μ and hence χ^2 ,
- (iv) repeat (ii) and (iii) using three neighbouring values of c ,
- (v) fit a cubic equation through these values of χ^2 and determine, by differentiation, the value of c which gives minimum χ^2 ,
- (vi) find the corresponding values of A and B from (7), thus obtaining the Makeham curve with minimum χ^2 .

160 *Methods of Forecasting Mortality using Australian Data*

As an illustration consider the 1932-34 female data for which our first approximation to c^5 is 1.67. We obtain

c^5	A	B	$10^5 \chi^2$
1.65	.0019848	.00065238	22.338
1.67	.0022399	.00058199	10.340
1.69	.0024777	.00051993	7.908
1.71	.0026994	.00046513	13.294

Hence, using 1.65 as origin and 0.02 as unit,

$10^5 \chi^2 = 22.338 - 17.36367 (c^5 - 1.65) + 5.657 (c^5 - 1.65)^2 - .29133 (c^5 - 1.65)^3$, which, differentiating and solving the resulting quadratic, gives, for minimum χ^2 , $c^5 = 1.6856$. Solving (7) then gives $A = .002427$ and $B = .00053293$. Therefore, for 1932-34 females ages 30-80, the minimum- χ^2 Makeham is

$$\mu_x = .002427 + .00053293 \left(\frac{x-30}{5} \right)^5.$$

[The tabulated values of χ^2 are, as the author states, based on the graduated and ungraduated values of μ_x without allowing for the exposed to risk, i.e. the exposed to risk is effectively assumed to be unity at each age. This point is of no importance for the author's purpose but explains why the tabulated values of χ^2 are so small. These values are, of course, not appropriate for the purpose of a probability statement on the basis of the χ^2 -test. For such a purpose χ^2 must be based on the actual and expected deaths, in other words the exposed to risk must be allowed for. Eds. *J.I.A.*]

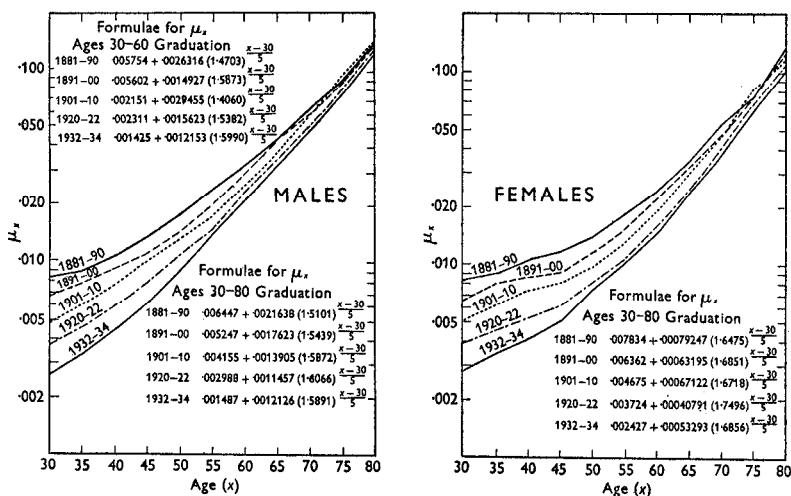


Fig. 6. Graphs showing actual values of μ_x ; also formulae for χ^2 -minimum graduations.

Having obtained a Makeham graduation for each calendar year for which data were available Cramér and Wold proceeded to graduate the resultant constants as a function of time. They found, for Swedish data, that logistic curves gave good graduations of $\log c$ and $\log B$ and the straight line a good graduation of A .

When applied to Australian statistics this method did not give satisfactory results, as can be seen by looking at the results given for males and females in Figs. 6 and 7. Had we ignored the 1933 Census figures a graduation of the male (30-80) data could have been attempted. This graduation would not have forecast the 1933 census death-rates. If we include the 1933 census it is impossible to make a forecast, as the future direction of the curves depends mainly on the irregular result of this last census. With a further census or two a forecast by this method may become possible. A study of the male data suggested that the lack of continuity may have been due to irregular movements in the rates at higher ages, and accordingly the investigations were repeated using statistics for ages 30-60 only where the data appeared to run smoothly with regular improvement from census to census. The results of this latter investigation are included in Figs. 6 and 7. In all three cases the para-

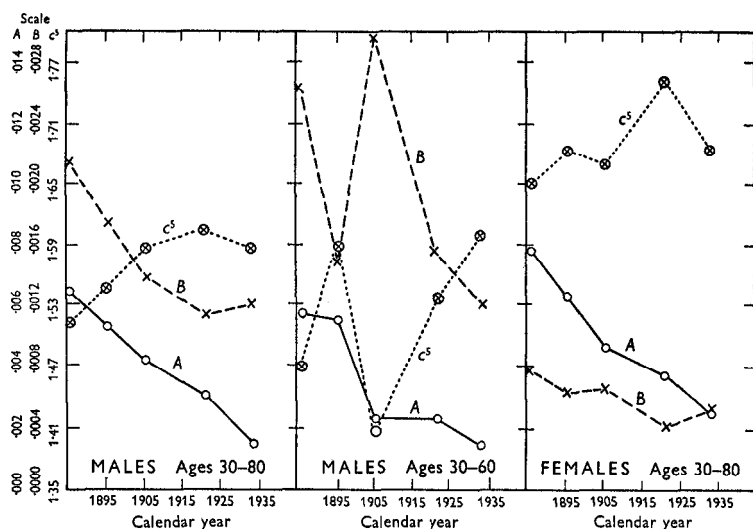


Fig. 7. χ^2 -minimum Makeham constants for Australian censuses.

meters do not show a sufficiently systematic variation with time to justify a forecast. This confirms an opinion expressed by Bernstein in 1933. The position might be different if we had statistics for a further quarter or half-century.

The following comments on the method may be made.

- (i) It has a certain theoretical appeal.
- (ii) The Australian results suggest that, even though a systematic decline in mortality occurs at all ages, the variations in the Makeham constants may not be systematic.
- (iii) Very extensive data are required (as is the case in Sweden) if the method is to be used with any confidence—frequently (e.g. Australia) it cannot be used at all.
- (iv) It involves a very considerable amount of numerical work. (The results of Fig. 6 actually involve 89 Makeham graduations with given values of c and the corresponding lengthy determinations of χ^2 in each case.)

- (v) The method is in any case limited to ages over 30 where the Makeham formula applies; it may not even apply to these ages if the curve departs sufficiently from the Makeham form—recent Australian data suggest a movement in this direction.

7. MAKEHAM GENERATION METHOD

In this method, for each year of birth θ , we graduate ${}_0\mu_x$ by a Makeham curve and then consider the resulting Makeham constants as a function of θ . Davidson and Reid (7), in applying this method to English data, made the (probably unjustified) assumption that c is independent of year of birth and hence greatly reduced the numerical work. Cramér and Wold (1) used the technique of § 6 in applying this method to Swedish data. Statistics for well over a century are required if the trend is to be obtained by this method. Davidson and Reid admit the 'slender nature' of *their* statistics! There are, therefore, very few countries to which this method can as yet be applied. Australian statistics scarcely give us sufficient information for one year of birth, and hence this method cannot be applied to Australia for another half-century.

8. OTHER FORMULAE

The following are examples of other formulae which have been suggested or used:

$$q(x, t) = A + Bc^t \quad (\text{Mortality of Annuitants}),$$

$$q(x, t) = q(x, 1896) A^{\frac{t-1896}{a+B}} \quad (\text{Sacks}),$$

$$q(x, t) = A + Bt + Ct^2 + Dt^3 \quad (\text{Esscher}),$$

$$\left. \begin{aligned} \mu(x, t) &= D^{A+Bt+Ct^2} \\ \mu(x, t) &= \mu(x, 0) e^{(A+Bt+Ct^2)t} \end{aligned} \right\} \quad (\text{Knibbs}).$$

In the above A , B , C and D are functions of x but not of t .

9. CAUSE OF DEATH METHOD

9.1. *General remarks*

Most countries now publish the deaths for each calendar year analysed according to age and cause of death. This information, together with the estimated population each year in age-groups, enables us to determine the rates of mortality according to sex, age, and cause of death. Whatever method of forecasting is used it is likely that this information will help us to make a better forecast. If we use a method based on the Kermack, McKendrick and McKinlay assumption, it is likely that more accurate results will be obtained if we exclude mortality from causes of death for which this assumption is not likely to be valid, e.g. accident, and epidemic diseases such as influenza. If we use a graphical method, the exclusion of influenza will eliminate one cause of variation and permit a better graduation, as can be seen by glancing at Fig. 8. We could divide causes of death into two groups and apply a period method to one group and a generation method to the other.

The unsatisfactory nature of the death statistics according to cause of death is well known. The actual rates of mortality from various causes will vary considerably according to whether the deaths are tabulated according to 'final

cause' of death or 'underlying cause'. Changes in medical diagnosis and in medical 'fashions' make comparisons over a number of years very difficult. For some causes of death (e.g. accident) these objections can be neglected and in others the objections can be reduced by appropriately grouping causes of death. If, however, we are only concerned with *total* mortality as obtained by forecasting rates for individual causes of death and adding the results the 'fundamentally dubious character' of the statistics is not so important. We shall be concerned with this method of forecasting in this section.

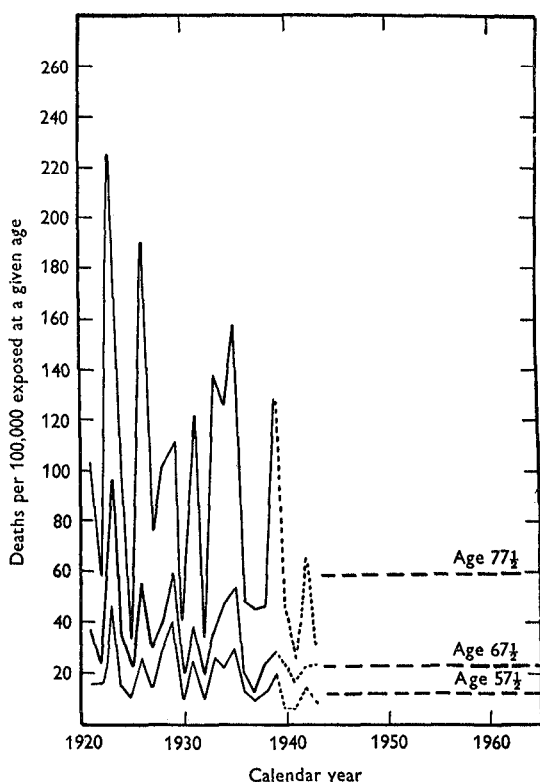


Fig. 8. Male rates of mortality from influenza.

We can obtain a better estimate of the deaths in the next few years from certain causes by studying the trends of the last few years rather than those of forty years ago. The effects of car and aeroplane accidents, penicillin, mass radiography, sulpha drugs, etc., can be satisfactorily allowed for only by studying the year to year variation in the mortality from various causes. If we use decennial censuses, the latest of which may be 15 years old, we are not able to allow for recent tendencies which may be of major importance. The forecast should be carried out by, or in conjunction with, a medical expert.

I think that only causes of death for which there is a definite explainable trend, and epidemic diseases, should be considered separately in this way. Examples are accident, tuberculosis, influenza and pneumonia. Causes of

164 *Methods of Forecasting Mortality using Australian Data*

death for which the mortality trend is indefinite (except epidemic diseases) should, I think, be combined and dealt with as 'other causes'. It should be mentioned, however, that this rule has not been followed here where most major causes of death or groups of causes have been considered separately.

It is possible in this way to detect significant tendencies which would be overlooked if we relied on data for census years only and generally a more intelligent forecast is obtained. For young countries like Australia, where few censuses are available, results for the past twenty years only give a satisfactory forecast.

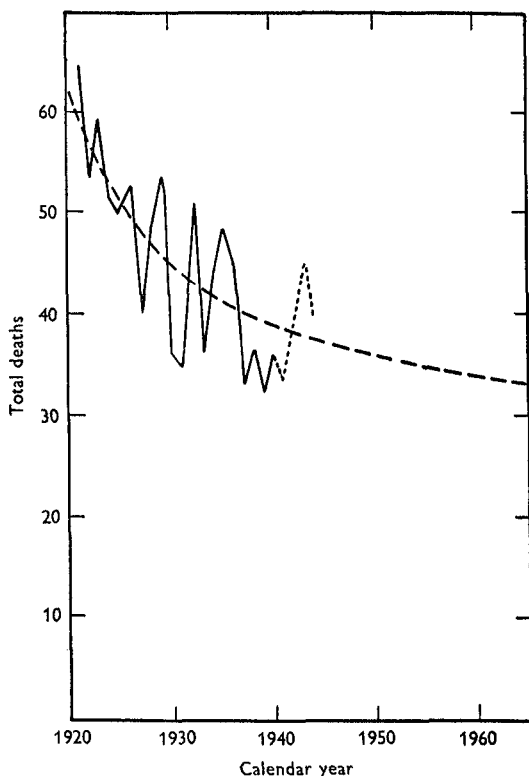


Fig. 9. Total male deaths from epilepsy, assuming 100,000 exposed in each age-group 15-19 to 65-69.

The war years introduce a difficulty in forecasting mortality at this time. Some use should, however, be made of the statistics for these years. Rates have been determined for both sexes and used at ages above the ages for military service. This gives a good deal of information for the female sex and, for both sexes, some idea of the trend which may be expected at younger ages.

9.2. *Results*

Rates of mortality were determined for each year from 1921 to the present time for each sex separately for quinary age-groups for the following causes of death: influenza, pulmonary tuberculosis, epilepsy, bronchitis and pneumonia,

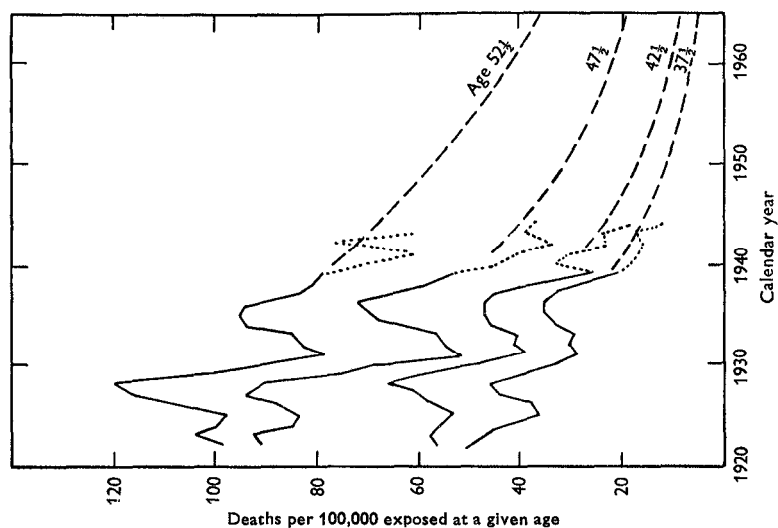


Fig. 11. Male rates of mortality from bronchitis and pneumonia.

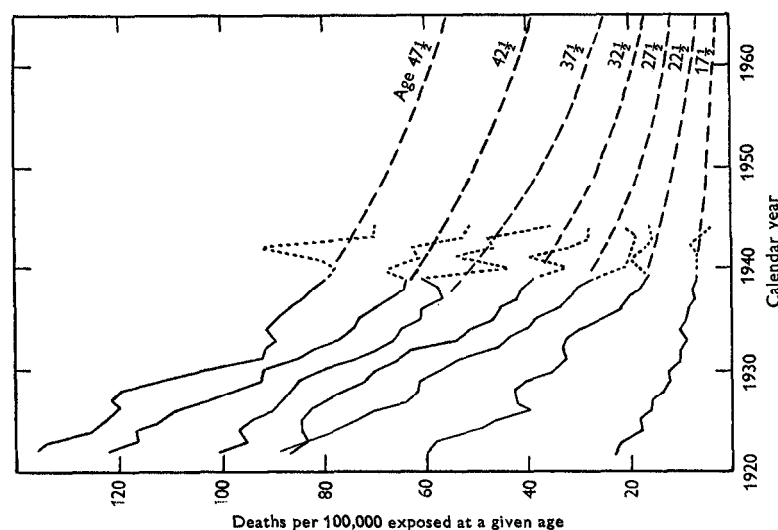


Fig. 10. Male rates of mortality from pulmonary tuberculosis.

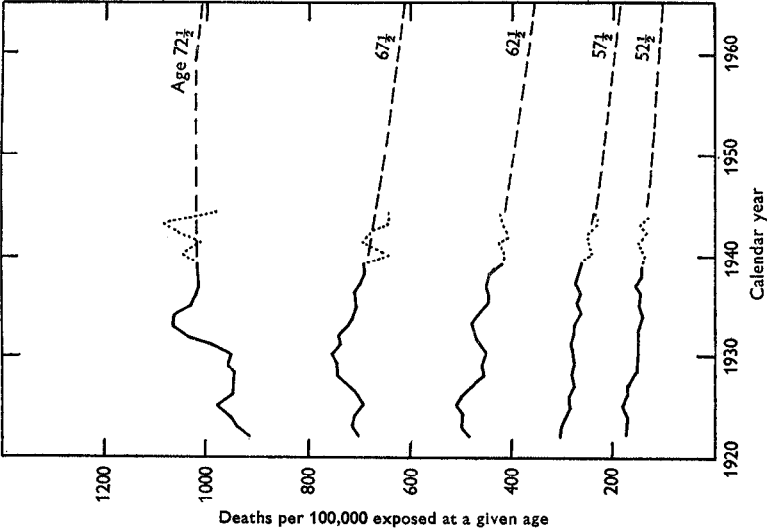


Fig. 13. Male rates of mortality from 'growths'.

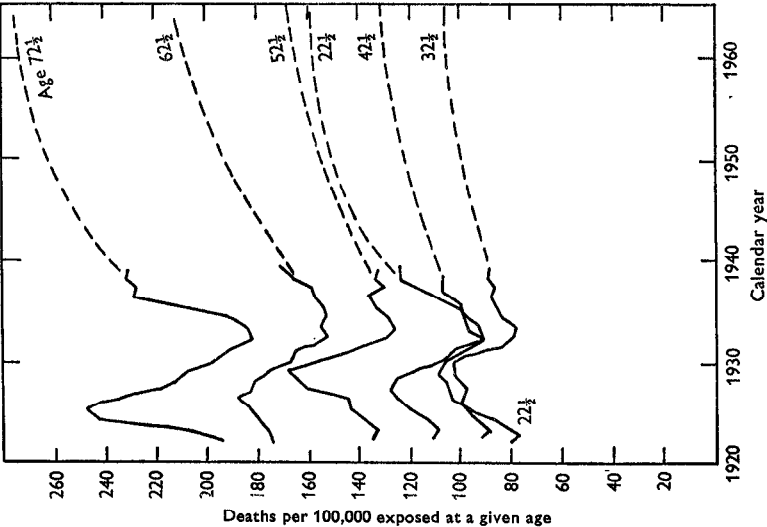


Fig. 12. Male rates of mortality from accident.

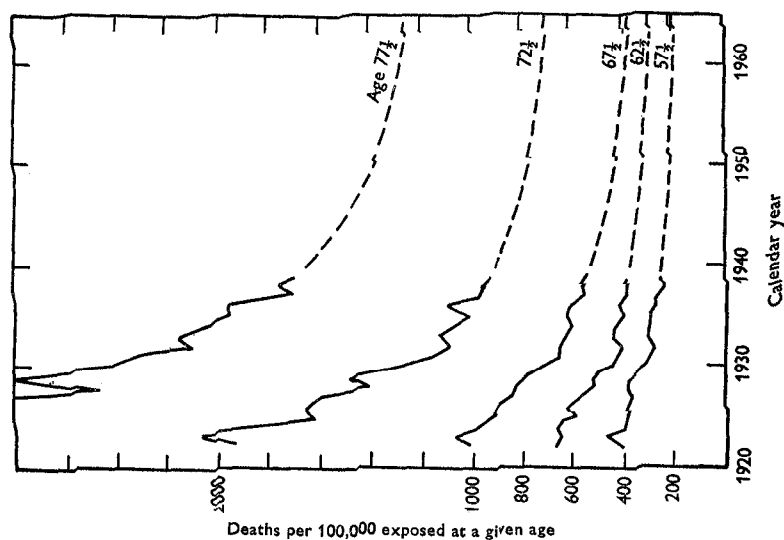


Fig. 13. Male rates of mortality from 'other causes'.

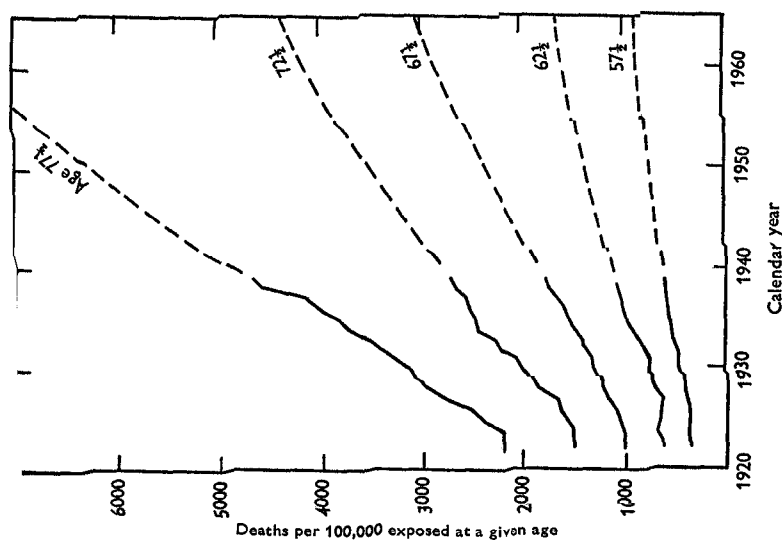


Fig. 14. Male rates of mortality from diseases of the circulatory system.

168 *Methods of Forecasting Mortality using Australian Data*

accident, growths, intracranial lesions, diabetes, nephritis, appendicitis, diseases of the circulatory system, ulcers of the stomach and duodenum and 'other causes'. The results were represented graphically in some four dozen or so graphs, a few selected ones being given for illustration in Figs. 8-15.

The method of extrapolation adopted depended on the facts of the particular case. For example:

Influenza. The epidemic nature (shown by Fig. 8) introduces considerable variability. The rates at all ages have been appreciably lower during the past few years and it was thought this should be considered permanent, particularly in view of the possible use of vaccine inoculations in the future. For ages over 50 the average of the years 1935-44 was used as an estimate of future rates and for ages under 50 (because of the disturbance due to the war) the average of the years 1930-39 was reduced in accordance with the improvement at ages over 50.

Epilepsy. The rates appeared to be independent of age, and extrapolation was carried out graphically on the basis of the total deaths per 100,000 in each quinary age-group (see Fig. 9). It was assumed that the steady fall (partly due to the use of luminal) will continue.

Appendicitis. The rates appeared fairly constant in recent years, and the average rates in each age-group for the years 1930-39 were assumed to continue in the future.

In most cases, however, the usual graphical extrapolation was used. It would take a good deal of space to explain, for each cause of death for each age and each sex, why the extrapolated curve was drawn in a given direction. This is not necessary here where we are concerned only with method. Suffice it to say that all relevant data were considered. For example, with bronchitis and pneumonia the extrapolation was based on a continuation of the fall which began in the mid 1930's and which resulted from the introduction of sulphur drugs. Earlier movements were considered unimportant. With the accident forecast actual causes of accidents were studied (mainly automobile accidents which have increased steadily and considerably since 1933) and the increased use of automobile and aeroplane had to be borne in mind.

10. COMPARISON OF RESULTS

It is interesting to compare the values obtained for the force of mortality at various ages by the several methods of forecasting. These are given in Fig. 16. The variations in the results obtained are remarkable and the reason for the differences should be noted. Dr Rhodes's method assumes that mortality depends on the year of birth and, because of the changes which have taken place over the last century, we can say that according to this method a substantial improvement is expected at the higher ages as these ages enter the lower-mortality years of birth and little further improvement is expected at the lower ages. The results obtained from the cause of death forecast are in marked contrast and the reason for this will now be given, first for the younger ages and then for the older ages.

For males aged 30 the minor causes of death are either constant or slightly decreasing. The only causes of death of importance at age 30 are tuberculosis and accident. The former has shown a very big fall in this century, a fall which will continue but at a slower rate as the number of deaths from this cause approaches zero. The accident deaths passed through a very definite minimum

in 1933 and have since then steadily increased. This steady increase has mainly been due to automobile accidents and will probably be augmented in future years by aeroplane accidents. The steadily increasing use of cars and aeroplanes must cause a steady rise in the accident rate at age 30 unless some very definite policy is introduced to counter the rise. As accidents cause the majority of deaths of males at age 30, and as the present increasing tendency only began in 1933 (the last census year) and is not therefore allowed for in other methods, it is most likely that the 'cause of death' method will give the better results at this age; other methods do not recognize this important and definite recent trend.

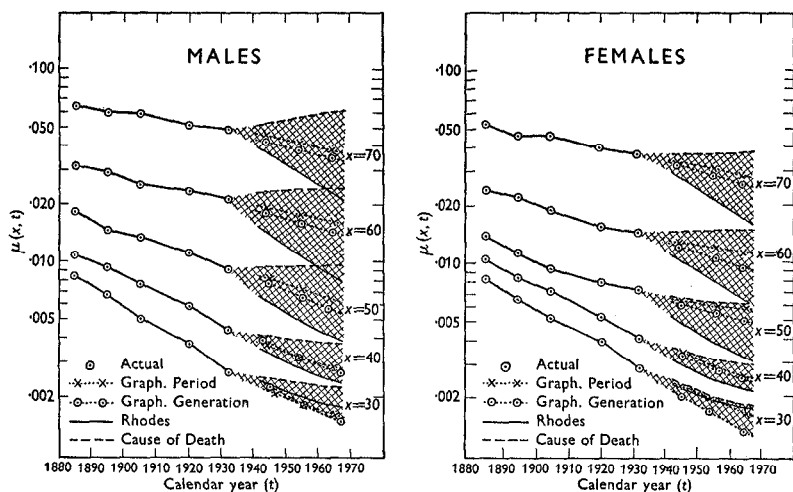


Fig. 16. Various mortality forecasts compared.

For males aged 60-70, apart from very minor causes of death and 'diseases of the circulatory system', the causes of death are constant or show a slight decline. Diseases of the circulatory system, which cause half the deaths, show a definite large steady increase. There is no reason as yet to hope for any alteration in this steady rise which, if it continues, will prove invalid the assumptions on which Dr Rhodes's method is based.

For females at age 30 the accident rate is small and there is little difference between the results of Dr Rhodes's method and the 'cause of death' method. At higher ages the 'diseases of the circulatory system' again cause appreciable discrepancies.

11. CONCLUSION

First, the above application to Australian data suggests doubts about the validity of the assumption underlying the method of Cramér and Wold, namely that Makeham curves which vary steadily with time have constants each of which varies steadily with time. Secondly, as can be seen by a glance at Fig. 16, it should be a warning against the acceptance of the results of a single forecast without careful consideration of the relative merits of the assumptions on which that and other possible methods of forecasting are based.

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ABSTRACT OF THE DISCUSSION

Mr R. D. Clarke, in opening the discussion, said that the forecasting of future mortality rates had always been among the most absorbing of the problems of actuarial science. It was therefore natural that a considerable body of literature on the subject should be found within the pages of the *Journal*. Mortality was changing so rapidly that the need for discussion and exchange of ideas on the subject persisted, and there was plenty of scope for the development of theory to take account of the new facts. The war of 1939-45 had interrupted the progress of research, but since then there had been time to review the position once more, and actuaries realized that, so far as mortality was concerned, they were living in a new era. The inter-war period, which might perhaps be called the $A_{1924-29}$ era, had followed the H^M and O^M eras into limbo, and speculation on future trends was full of interest for every actuarial mind.

The author had submitted an extremely valuable review, in which he had summarized the history to date of the branch of actuarial science with which the paper dealt. He thought it might be safely predicted that the paper would become a standard work of reference for students of the subject. The author had presented the various theoretical points of view with fairness and impartiality, and where his comments on any particular theory were critical they seldom went beyond what the exponents of that theory would themselves admit. This impartiality was maintained to the end, and in § 10, where a comparison was given of the various methods and of their application to Australian data, the author very wisely refrained from arriving at conclusions on the relative validity of this or that method. Nevertheless, the divergences revealed in § 10 were substantial and called for a detailed examination of the underlying hypotheses.

Broadly speaking, methods of forecasting might be resolved into two main categories, namely, those which considered only the total force of mortality and those which proceeded by analysing the force of mortality into component parts. The former category, as was stated in the paper, might be further subdivided into period methods and generation methods. Those methods differed widely in their conceptual bases though they tended to produce very similar results in practical application. It must remain a matter for controversy whether the generation school of thought, which believed that a man's health and vitality were mainly determined by the environment of his childhood, or the period school of thought, which believed, on the other hand, that it was the contemporary environment which had the greater influence, was the more correct in its postulates. In a simple graphical method of forecasting, the two theories produced results which were identical, as would be at once apparent from a glance at Figs. 1 and 2, Fig. 2 being Fig. 1 with a progressive shift to the right. When the more complex assumptions of Kermack, McKendrick and McKinlay were adopted, the situation was, of course, less obvious. If, however, the rate of secular improvement was regular over a fairly long period of time and if, in addition, it did not vary greatly with age, the resultant mortality rates would lend support to either theory. Thus, if the generation theory was correct and the values of $R(\theta) \div R(1860)$, as represented in Fig. 3 of the paper, formed a series declining by fairly regular decrements, as was in fact the case, then the period theory of mortality improvement would also find support from the data. In the same way, provided that the conditions already mentioned were observed, the converse proposition would be equally true.

An adequate test for comparing the two theories could be made only when the rate of secular improvement showed pronounced changes of trend, or when it exhibited appreciable variation with age. The Australian data considered by the author scarcely fulfilled those conditions, although, comparing Figs. 1 and 2, the parallelism of the secular curves was seen to be rather more marked in Fig. 1 than it was in Fig. 2, thus favouring the period theory. However, the test was not a rigorous one and was far from being decisive. Unfortunately, he did not think that any more definite conclusion could be derived from Fig. 4. The small scale of the diagram prevented a just assessment of the goodness of fit obtained on the Kermack, McKendrick and McKinlay hypothesis,

but, so far as he had been able to judge, the fit was not as close as could be wished. In particular, the curves at ages 50 and 60 for both the male and the female lives seemed to show considerable divergence from the data, and he had been left with the feeling that many different sets of curves could have been drawn through the given points with equal success. It would be altogether more satisfactory when there were more data at the higher ages in the region of the downward sweep of the curves. In the absence of fuller information, he could only conclude that the results showed no very clear grounds for either accepting or rejecting the Kermack, McKendrick and McKinlay hypothesis in relation to Australian data.

Though he did not think that actuaries were in a position to pronounce a verdict on the controversy between period and generation hypotheses, there were certain general considerations which, in his opinion, militated against the usefulness of the generation method in practical forecasting. A great deal of mortality improvement arose from advances in science and was therefore related to the contemporary environment. The effects of sulphonamides and penicillin were well known, but an even more striking example might be cited in events which had occurred in British Guiana within the last four years. A triumphant campaign against the mosquito with D.D.T. and similar insecticides had drastically reduced the incidence of malaria throughout the colony. Clearly any attempt to interpret the mortality rates of the population in 1949 as though years of birth were the sole determining factor would be foredoomed to failure.

In Great Britain there had been an important environmental factor operating upon the mortality rates, to which sufficient attention had not always been paid. He referred to the incalculable effects of the economic levelling which had been brought about by the war. He believed that the social revolution which had begun in 1939 and which had resulted in a sharp rise in the standard of living of the least privileged classes had been the major cause of the remarkable mortality improvement that the war years had witnessed. Members of the Institute would recall the President's reference in his *Presidential Address* to the improvements which had occurred and were occurring in old age mortality. Before 1940, rates of mortality at advanced ages among the general male population of England and Wales had remained unchanged for nearly a century. Since 1940 there had been a sudden and quite remarkable decline. At the same time it was of interest to notice from the recent report on annuitant mortality that any improvement during the same period among male annuitants had been very slight, if indeed it could be said to have occurred at all. He thought the conclusion to be drawn was fairly clear. As a result of a widespread improvement in standards of living, the mortality of the general population had been drawing steadily closer to the mortality of the most favoured section of it. In fact, the direct effects of a change in the contemporary environment were being witnessed, and he did not see how, in those circumstances, actuaries could approach the subject of mortality forecasting by means either of theoretical formulae or of extrapolation or by any method whose nature was not essentially empirical.

That was why he believed the most profitable line of approach to be the one considered in the latter part of the paper, which implied an analysis of mortality by cause of death. The author had clearly stated the difficulties which confronted the investigator who took that line of approach, difficulties which arose from such causes as improvements in medical knowledge, leading to more accurate methods of diagnosis. But difficulties of that kind were not arguments against the method; they were merely guides to the future planning of actuarial investigations. To proceed on existing data, however, as the author had done in Figs. 8-15, could lead sometimes to results of very doubtful value. Thus it might, not unreasonably, be surmised that the increase in mortality from circulatory diseases shown in Fig. 14 was directly linked with the decrease in mortality from bronchitis and from 'other causes' shown in Figs. 11 and 15. The apparent trends were, he would suggest, fictitious, and reflected changes in the method of describing the cause of death on death certificates. The feats of extrapolation which the author had performed in those diagrams had filled him with alarm, and he had been forced to the conclusion that the results which had been obtained by those processes and which appeared in Fig. 16 as the mortality rates forecast by the cause of death method were entirely lacking in validity.

In order to compare the trend in the Australian death-rates from circulatory diseases with the corresponding trend for England and Wales he had looked at recent issues of the *Annual Review* of the Registrar-General. He had found that the trend in England and Wales had been steady and downward; quite different from the Australian figures.

If the cause of death method was to be successfully employed—and he believed that there alone lay the main hope of progress in mortality forecasting—fresh data must be assembled by methods involving far more careful statistical control than had hitherto been possible. That inevitably entailed the use of sampling techniques and would require the wholehearted co-operation of a selected number of doctors responsible for signing death certificates, who would be asked to supply complete medical details of all deaths occurring among their National Health patients over a given period. Elaborate as such a scheme might sound, he did not believe it to be beyond the bounds of practicability.

In conclusion, he would like to say how grateful he was to the author for the extremely pleasant hours which the paper had given him. Mortality forecasting was not merely one of the most attractive subjects which engrossed the professional attention of actuaries; it was also one of the most useful. It was true that, over the ages at which the majority of life assurance policies were in force, mortality improvement had already been so extensive that the financial effects of any further improvement were of necessity very limited. Actuaries, however, were also concerned with annuity funds and pension schemes of many types, and in that field the future effects of existing trends were likely to be felt on a large scale. Then there was the wider horizon which all actuaries were bound to survey and which embraced developments to which no actuary could remain indifferent. The changes, both past and future, in the age structure of the population, which were very largely the outcome of falling mortality, had consequences over the whole of the national life of the most far-reaching kind. He thought that actuaries had an obligation to the public to see that the facts of what was occurring and what was likely to occur were accurately and widely disseminated. Current population trends inevitably had a direct effect upon industrial and social planning, upon the kind of houses that people would want to live in, the kind of commodities that they would want to buy and the kind of social services that they would demand.

There were two factors in particular which he might mention. He had already spoken of the fact that the mortality of the population as a whole was moving steadily towards that of its most favoured section. That implied changes in the class structure of the population at the older ages. It must be expected that a larger proportion of old people would be retired manual workers than in the past, a consideration which was not without importance for those whose concern was with the social medicine of old age. The other factor was the tendency towards an older retirement age. It had been suggested—and there was some evidence to support the suggestion—that boredom following retirement was a significant cause of death. If that surmise was correct, then a lengthening of the working period might itself contribute to further mortality improvement.

He had cited those two instances of questions for investigation which arose in the course of examining mortality trends because they demonstrated how pregnant with possibilities was the whole field of research. If the paper proved a stimulus to further endeavours in that field the members might be confident that the author would feel amply rewarded.

Mr P. R. Cox said that forecasting mortality was a subject which actuaries would like to avoid if possible, because so little was known about the precise effects of the various causes of death and how they were likely to develop, but there were times when it might be thought convenient to consider the possible future course of events. One example was in population projection and another was in the preparation of annuity tables. The method adopted and the procedure followed must depend to some extent upon the general aim of the investigation, but very broadly speaking there were two general lines of approach.

The first, which (he thought rightly) had not been followed by the author, was to try to form in advance some idea of the probable ultimate level of mortality and, having done that, to draw a curve between the observed rates, continuing the trend of the past, and the ultimate rates, reaching them at an arbitrary time. That method had recently been adopted in an American investigation. In order to obtain some guidance with regard to the ultimate level, the general procedure was to look at the experience of one of the more favoured sections of the population, perhaps a social class or one of the occupations, or even to look at the mortality of some other country where the death-rates were exceptionally light; that method seemed to be very little different from the arbitrary. It was unlikely that the mortality rates in another country, which might have a quite different social and economic make-up, would ever be experienced in the country in question. Similarly, it did not follow that the rates which were being currently experienced by the very best of the social classes would be the ultimate rates of the country as a whole. Even if, as the opener suggested, there were to be a general levelling up, it was likely that the best classes would experience further improvement, and it was not known where that improvement was going to end.

The idea of an ultimate level was itself rather unsatisfactory, but there was a natural tendency in mortality projection not to pursue the downward trend of the mortality rates very far into the future.

If the method of deducing an ultimate level was not followed, it seemed necessary to try to derive from the observed data some sort of trend curve which might form the basis of a projection. Two fundamental questions had to be asked in doing that. The first was which trend curve best fitted the observed data, and the second was whether such a trend was likely to continue. The author had not gone into those questions very deeply, but he was, of course, handicapped by the nature of the Australian experience.

There was not a very extensive volume of data, and the demographic development of Australia introduced some difficulty. In the 1860's the population of Australia was in the region of a million, and it had grown very rapidly, until now it was about eight million. During that period over a million immigrants had entered the country, and changes in the total mortality experience must be closely associated with the influx of immigrants—depending on the type of people they were, the countries from which they came, the ages at which they entered Australia and so forth. More particularly, the future mortality of Australia would depend to a considerable extent on the type of immigrant admitted into the country thereafter. It was said that, in order to develop the economic resources of the country and to defend its shores, large numbers of fresh people would be required to swell the population; if they were to come from the same countries as in the past, the observed trend might be expected to continue, whereas if they came from different countries, as seemed quite possible, there might be a twist in the curve. It was not known precisely how much the Australian mortality was due to environment and how much to heredity, but the immigrants would carry with them something of their own mortality, and it was very difficult to see how allowance could be made for that factor in any of the methods and formulae described in the paper.

The results at the end of the paper were, as the opener had pointed out, divergent, and it would have been of interest if mortality rates for some later year than 1933 could have been given, even in approximate form, in order to test the various methods and to see which one had turned out the best. He was rather surprised that it had not been possible to get out the approximate Australian death-rates for 1945. All that was required was a rough estimate of the population by age and sex, such as was prepared regularly in Great Britain.

Of the four main methods used in the paper, the first was called the graphical period forecast, which consisted in drawing curves through the data and beyond. The author had relied upon his eye and had drawn his curves freehand through the points. He would like to suggest as an alternative a more mechanical approach, such as fitting a mathematical trend formula to the rates. One advantage of that method was that a number of curves could be tried to see which fitted best, and another was that independent operators could repeat the process for themselves.

The graphical generation forecast seemed a little meaningless owing to the immigrants.

The mortality of a group of people in Australia aged 60 could hardly be closely associated with the corresponding rate in that country at age 30 thirty years earlier if many of them had entered in the intervening period.

The assumption of parallelism between the curves for various age-groups implied that old-age mortality was expected to diminish greatly. It seemed plausible, however, to think that a man aged 100 might have much the same mortality whether he was born in 1700, 1800 or 1900; in other words, that his mortality depended much more upon his age than upon the time when he reached that age. If that were so, there would be some 'wearing off' in the generation effect at the very old ages. It was not known, of course, at what ages that would appear, but there seemed to be at least a possibility that old people's mortality might not diminish quite so fast as had the mortality at the younger ages.

Rhodes's method depended upon the assumption of Kermack, McKendrick and McKinlay that the shape of the curve of deaths was the same at all times. Although that might be approximately the case, it seemed a rather unsatisfactory basis to work on, because medical advances did not go smoothly; they proceeded in jumps from time to time, as new discoveries were made which affected people of different ages. It would be natural to expect a change in the shape of the curve from time to time.

When forecasting by separate causes, the author was a little pessimistic in his approach, because where the death-rate from a particular cause was steadily rising he tended to continue that rise indefinitely into the future, whereas when mortality was falling he tended to flatten the curve fairly quickly. The net result of those two processes taken together was a rise in the mortality at the older ages. In view of the recent experience in Britain, that seemed rather surprising.

If part of the rise in the circulatory diseases was due to more frequent certification of that cause, some other causes must have had less certification; and if the degree of certification of the circulatory diseases were to continue to increase there would be a corresponding reduction in the certification of other causes.

Mr P. W. Glassborow wished to put forward a rather different view of the question of forecasting mortality. He would first consider another science. If a physicist decided to determine the acceleration due to gravity from previous experience, which was really statistical experience, he knew very closely what his answer should be and he could therefore forecast his answer. Again, in the science of weather forecasting, if the initial data were known exactly, it would be possible to forecast what would happen because it would follow physical laws. In that case of course, as in the case of mortality, the initial data are no longer under the control of the investigator, but with mortality the position was entirely different, in that changes in mortality did not really follow physical laws at all. Mortality depended on many factors, such as the health of the stock, improvements in sanitary conditions in their widest sense, changes in nutrition, and improvements in medical science which resulted in the prevention of death when a disease had actually started. In forecasting future mortality, the actuary could expect to forecast the improvements due to past changes of the first three of those factors, but, since the improved medical methods were usually discovered suddenly, very often without any warning that they were going to be discovered, he did not think that actuaries could ever expect to forecast all improvements in mortality by purely statistical methods. Therefore, if they were to attempt to forecast future mortality at all, he thought they should try to eliminate the improvement arising from improved medical techniques and to attempt a forecast of the remainder. Any further improvement in medical techniques should be provided for, where necessary, by a purely arbitrary margin.

Sir William Elderton, who was unable to attend the meeting, sent a written contribution to the discussion, in which he said that he thought the paper was valuable and interesting and he hoped the discussion would show the author that the Institute appreciated his work.

He regretted that § 8 appeared in the paper. As it stood, it was difficult to guess how the formulae were used; t seemed to have more than one meaning and not, as

in § 3, to mean merely the calendar year, and he did not think that readers ought to be expected to know without indication where the formulae were given. The first formula was described as 'Mortality of Annuitants' and looked simple until it was remembered that it might be written as follows: $q(x, t) = f(x) + \phi(x) [\psi(x)]^t$. And which set of annuitants? The formula did not remind him of the $a(f)$ and $a(m)$ forecast as it stood, and there had been other annuitants' forecasts, e.g. the Government, the recent German forecast, some American work, etc. Would it not have been better to alter § 8 and say in general terms that other methods had been suggested for forecasting the mortality rates of populations, such as those indicated in the references on p. 170, which might be suitably extended. In connexion with forecasting for special purposes, such as annuity work, it should be remembered that special considerations might arise because of the uses to which the forecast would be put.

In the Swiss Actuarial Journal for 1941 (*Mitteilungen der Vereinigung Schweizerischer Versicherungs-mathematiker*, Vol. XLI, Pt. II, pp. 109-161; Ref. *J.I.A.* Vol. LXXV, p. 111) there was a paper by P. Baltensperger wherein Swiss deaths were analysed with the idea of a subdivision into a part where no improvement was expected and a part where it was assumed that the 1928-35 rates would eventually be halved. A reference to that paper would seem to be appropriate.

All forecasts were guesses at an unknown future, and an examination of Fig. 16 tempted him to the view that, if, when using reasonable methods, actuaries could get such deviations, they might choose for convenience some method which (a) would not take a very extreme view, and (b) would be convenient to use in any work which might be based on it. The figure might also be held to give a warning that something which had not been thought of might make nonsense of the forecast.

A method which might be suitable for one country might give absurd results for another, where different industrial or climatic conditions, for instance, prevailed.

The remarks which he had made might appear critical and in disagreement with the paper, but, except in his views on the author's references to other work, he had done nothing but underline in gratitude remarks made by the author.

Mr W. Perks commented on the use of the word 'forecasting' in the paper. He thought it was undesirable that the actuarial profession should pretend that it could make any reliable forecast of such things as mortality and fertility, and it would be better if the word 'forecasting' was not used in technical papers of the kind under discussion. The processes described in the paper were not forecasting at all; they were merely graduation and extrapolation, and if those words were used instead of the word 'forecasting' misunderstandings would be avoided.

In § 5.1 the author, in referring to Rhodes's method of graduation and extrapolation, used the phrase: 'A simple method of checking the truth of this hypothesis.' Surely it was not a question of 'truth'. For any actuary it should be a question of testing the suitability of a hypothesis for a practical purpose.

The speaker then referred to the question of old-age mortality. He thought he could not do better than quote from a review which he had written in 1945 of Notestein's book on the population of Europe (*J.I.A.* Vol. LXXII, p. 296). In that review he had asked the question: 'How will... old-age mortality change in this country during the next thirty years?' And he gave the reply: 'The plain answer is—nobody knows.' He then asked the question: 'Can we obtain any guidance from the changes during the past twenty-five years?' And he replied: 'The plain answer is—very little.'

The opener had referred to the remarkable fall in old-age mortality in Great Britain since the beginning of the war. The main fall had taken place in the years 1941 and 1942, but he desired to stress the further fall in the mortality at the older ages in the year 1948. The Registrar-General had already reported on that fall, and industrial assurance mortality confirmed it. The combined effect of those two falls at the older ages, taking males and females together, was of the order of 20 %, a remarkable figure for a period of only 10 years.

The opener had also referred to mortality of annuitants. In the last issue of the *Journal* (Vol. LXXIV, p. 126) there appeared a report by the Joint Committee, which

contained some graphs showing the mortality of annuitants up to 1944. In those graphs there was a trend line of the annuitant mortality, and superimposed on that trend line were the actual mortalities of the annuitants and of the whole population of England and Wales. The last annuitant figures were for 1944, and they showed a rise on 1943. He wondered whether that rise was due to the unusual phenomenon of an influenza epidemic in November and December of 1943, with a significant effect on the mortality for those months; he thought that, owing to the delay in reporting deaths of annuitants, it was probable that the mortality shown for the annuitants in 1944 included some of the influenza mortality of 1943. However, the graphs in the report showed that the course of annuitant mortality followed closely that of the mortality for England and Wales for ages over 65, i.e. when one rose the other rose, and when one fell the other fell. In 1944, apparently, there was not that effect. The authors of the report did not carry on the graph for England and Wales past 1942. If they had, they would have found that the low figure for 1942 persisted throughout until 1947 and then dipped again in 1948. The correlation between annuitant mortality and the mortality for the general population of England and Wales, if assumed to persist beyond 1942, would have led the authors of the report to expect the annuitant mortality in the years 1945-48 to be considerably less than appeared to be in their minds when they made recommendations about the adjustments that might reasonably be made to the $a(f)$ and $a(m)$ tables for the purchase prices of annuities. He had gone into that matter at some length because he believed that the report might well prove to be dangerous in the current situation.*

With regard to the method in the paper called the Makeham period method, which had been applied to Swedish statistics by Cramér and Wold, the author had gone through the immense labour of that method and found that the Makeham constants did not lend themselves readily to extrapolation. The author concluded that the method 'may not even apply to these ages'—the ages over 30—'if the curve departs sufficiently from the Makeham form; recent Australian data suggest a movement in this direction'. He thought it was well known that the mortality in the twentieth century had not lent itself to Makeham graduation. He had himself published some work on that matter, and he had seen a number of graduations of sectional experiences of the A 1924-1929 data. The fact was that for that period Makeham's law was unsuitable and at least one more parameter was necessary. He suggested that, if an additional parameter had been used in the application of the Cramér and Wold method, it might well have been that the numerical values of each of the four parameters (instead of the three) would have been such as to lend themselves to extrapolation.

Mr C. D. Sharp said that a good deal of attention had naturally been directed to the improvement in the rates of mortality at the older ages. Most of the work that had been done on forecasting mortality had as its basis the statistics relating to the countries of the West, but in other parts of the world such as India and Egypt, where the population was increasing rapidly, the really interesting parts of the mortality curve were under the age of 30. He would like to support the suggestion made in the paper and again by the opener that an analysis by cause of death was probably the most valuable for that purpose.

The opener had cited the effect of D.D.T. on malaria in British Guiana, and he would like to draw attention to the amazing change which had occurred in Egypt in the winter of 1947. There had been in the summer of that year a widespread cholera outbreak which had been stopped by a series of injections of the population and by the use of D.D.T. It had surprised the more knowledgeable medical officers concerned that there had been no reappearance of the disease in the summer of 1948. For the first time in an Eastern country a very widespread epidemic had been stopped by medical means.

Mr H. W. Haycocks thought that all would agree that a trend was far from being a well-established scientific law and hence extrapolation based on a trend was a very

* A statement by the Joint Committee on the Continuous Mortality Investigation appears on pp. 243-4.

178 *Methods of Forecasting Mortality using Australian Data*

hazardous operation. However, it was not always realized that, even with a well-confirmed law, the fact that the standard errors of the parameters increased with time might make it impossible to forecast with a high degree of confidence. He thought that Mr Perks would agree to the use of the word 'forecast' when a well-confirmed law was being used, for then it was possible to provide an estimate of the standard error of forecast or to make a probability statement about any specified deviation.

Newton's Law, mentioned by Mr Glassborow, was a well-established law and the parameters had been estimated in the first place from an enormous amount of data and subsequently by very accurate experiment.

With regard to mortality the factors involved were numerous and complex, and our knowledge of many of them was very meagre. The mortality law was therefore only a short-hand classification usually depicting either the age-trend or the time-trend. Hence extrapolation could be used only for very limited purposes. The more we knew about the conditions underlying the trend the more confidence we could place in statements about the future trend. Knowledge of medical science and dietetics was of this kind. In practice then the question of purpose was highly relevant. For the actuary the usual purpose was to provide for some safeguard to a fund against the possible effects of future improvement in mortality. The margin could only be an arbitrary one as the results of the different methods of extrapolation indicated.

He had expected that, when the author proceeded to a consideration of causes of death, some evidence would be produced justifying the use of the trends. The evidence was not given, apparently for lack of space. In his opinion a discussion of such evidence could have been one of the most interesting parts of the paper, because the author would then have been attempting to justify the extrapolations. As it was, the freehand curves were obviously highly arbitrary. As used by the author he felt that the method of operating on partial rates only supplemented the method of operating on the total rate. The two methods checked one another, and the differences brought out were so great and contradictory as to suggest that something had gone wrong somewhere.

In connexion with Rhodes's method, Mr Cox had suggested that it would have been interesting if the author had compared his results with the 1945 mortality. If he remembered rightly, when Dr Rhodes read his paper to the Royal Statistical Society, Dr Stocks did make such a test, which showed that the formula misrepresented the observations in that it produced too much improvement at the older ages and too little at the younger. It was rather amusing to notice that Dr Rhodes terminated the discussion by replying to Dr Stocks and ending with the warning that he hoped that no one would use his method for extrapolation.

Mr R. C. B. Lane thought the paper was a very valuable one, because an attempt had been made to discover the real root cause of the improvement in mortality. That improvement was of the greatest importance to the actuarial profession and to the country as a whole, and he thought that any serious attempt (he did not care whether or no it was called 'forecasting') to see where the improvement would lead, particularly at the higher ages, was well worthwhile. The author's work differed from the simple extrapolation of rates, because he tried to see where the medical improvements were leading, where further improvements might come from and where there was room for improvement.

He thought that the graph relating to deaths by growths was interesting in that connexion. It would be noticed that there was no tendency there, or hardly any tendency, for the death-rate to increase. (He was referring particularly to the higher ages.) He thought that might perhaps surprise actuaries, because they probably felt that many of those people who had been saved from dying earlier in life would ultimately die of cancer. By contrast, the curve for the diseases of the circulatory system rose very rapidly. It was possible to criticize the author for carrying on that rise, but he thought it must be admitted that there was no reason for doing otherwise. The deaths had gone up steeply for years, and there was no sign of that rise stopping. In some of the other groups the deaths still appeared to be decreasing. It was interesting that, for the higher age-groups, the partial death-rate for mortality from growths remained more or less

the same, whereas the partial death-rate for mortality from circulatory trouble rose rapidly. When those were combined the results shown by the curves in Fig. 16 were obtained, and he thought all the members would agree that where the author showed mortality rates increasing something must be wrong. That was so contrary to general experience that much more evidence than was given in the paper would be required before the forecast would be accepted. He thought it was most unlikely that the author himself expected such an increase. The paper was useful, however, because it showed that there were factors underlying the general decrease in mortality which might ultimately lead to an increase in mortality at some ages. Noticing that, actuaries should not say that one method was desirable and the other not, but they should try to see where the method by cause of death had gone wrong.

One reason had been given already; it had been pointed out that the causes were not independent, that certification alone would have the effect of shifting the cause of a particular death from one category to another, and that in drawing the curves some attention must be paid to what had happened in one curve, before another was completed. That lack of independence might go even deeper. A large number of those people at the higher ages who died from circulatory trouble might be the very people who might, but for modern drugs, have died earlier from pneumonia, simply and solely because they were impaired lives and it would take less to kill them than it would to kill a thoroughly healthy life. Some of the increases which had taken place at the older ages might thus be a direct result of the improvement which had occurred earlier in life over the past ten to twenty years, and which in turn was a direct result of better medical methods.

He thought, therefore, that it was unreasonable to deduce from the results that the mortality rates for the higher ages would increase, but that it was reasonable to deduce that the continuing improvement, which had been going on for so long, might have a definite limit beyond which it would not go, unless and until some further startling discovery was made in the field of medical science.

Mr M. E. Ogborn, in closing the discussion, said that before the author returned to Australia he had had the advantage of talking to him about the paper, and, although he would not attempt to reply to the discussion for him, he would try to speak in his defence as well as to sum up the discussion.

He thought that thanks were due to the author for the opportunity he had given the Institute of having the discussion which had taken place that evening, and the points which had been raised in the discussion justified the author's method of presenting the paper. The author had applied the methods described in the paper to limited data, and in Fig. 16 he had probably deliberately left the divergences rather more extreme than he would, had he been making the extrapolation for a particular purpose.

The speaker questioned, as others had in the discussion, the use of the word 'forecast', which gave an impression of foresight, in contradistinction to the word 'extrapolation', which gave an immediate feeling of possible dangers ahead. As Sir William Elderton had said, the so-called forecast was really a guess at an unknown future.

In this connexion he wished to say a word about the authors quoted in the paper. It was hardly fair to attribute to them the particular methods of forecasting associated with their names when they had not presented their methods for the purpose of forecasting. As had been mentioned, Dr Rhodes had put in a definite qualification that his formula should not be used for forecasting; he was really interpreting past rates. Up to a point the same was true of Cramér and Wold; their main interest was to interpret the past, and when they extrapolated to later years they did so with considerable diffidence. Again, the purpose of the paper by Davidson and Reid was to put forward a new method of computing mortality rates, past observations being brought into account in the computation of rates by current experience. The labels as 'forecasts' might thus be a little unfair to the various authors.

He thought that essentially what was being done was first to investigate the past experience to determine what was significant and secondly to see what happened if the trends deemed to be significant were to continue into later years. For that reason, he

did not think there was really any difference between the various methods of forecasting. The difference lay in the interpretation of the past. That could be seen by looking at the two graphical forecasts in Figs. 1 and 2. The opener had said that there was no difference; he thought that was perhaps going a little far, but he would say there was no reason for making any difference in the dotted lines of those graphs. The only reason why a difference had been made was that in the generation graph parallelism suggested itself.

Fig. 4, in which Rhodes's method was used, brought out the parallelism on which that method was based, and that resulted simply because, in employing the Makeham form, Rhodes had assumed that only the constants A and B varied, and not the constant c . It was interesting to notice that Cramér and Wold, in their extensive investigation, had found that when they used the Makeham assumption all three constants— A , B and c —did in fact vary. Rhodes's investigation gave a good result with a constant c , probably because there was a certain balance of errors, but that made extrapolation more questionable.

The part of the paper which interested him very much was the last section. It was not proper to assume that the trend of partial rates of mortality would necessarily continue, because the partial rate depended on two variables, the rate of mortality from a given cause of death and the proportion of the total that were in fact exposed to risk from that cause. That could be seen by looking at the accident graph in Fig. 12. The author showed increases in the rates of mortality from accident and he gave reasons justifying those increases, but it was questionable whether the increases were due to the larger number of accidents or to the increase of fatalities per accident. The second was more akin to the rate of mortality.

It might be justifiable to expect an increase in the partial rate of mortality from accident because an increase in the number of accidents might be expected. In regard to the mortality from diseases of the circulatory system (Fig. 14), he thought the assumption was rather more questionable. As had been mentioned, the removal of 'other causes' of death at the older ages and perhaps also changes in certification might give an increase which was really spurious. The author tailed off the decreasing partial rates, and there was no reason why the partial rates of mortality that were increasing should not also be tailed off. The fact that the author ran them off more or less in a straight line presupposed that the total rate of mortality would increase. Was there a higher rate of mortality from circulatory diseases amongst those suffering from such diseases, or was a larger number of people put into that group for one reason or another?

The President (Sir George H. Maddex, K.B.E.), in moving a vote of thanks to the author for his paper, said that at times during the discussion he had been reminded of a conversation in which he took part a year or two earlier, with the late Dr Kuczynski and others, on the projection of population. It was suggested during the conversation that the projection of population should be done on the basis of mortality rates assumed to be forecast for a civilian population with negligible migration and with continuance of normal conditions, but Dr Kuczynski, with a sad smile and one of his expressive gestures, just brushed it aside and that basis did not exist any longer. The actuary, of course, could often be dismissed in that way, and the forecasting of mortality could be dismissed as easily as any other of the elements with which actuaries had to deal, but in fact there was a regrettable necessity to make assumptions about the future. Even if actuaries said that they were not going to project at all, they had to review what it was that they were doing when they adopted a life table. It was hardly necessary to tell the youngest member of the Institute that the life table was a thing which did not exist, and, of course, in any actuarial work it was necessary to consider the antecedents of each age or each generation whose current rates were incorporated into a life table, to see what might reasonably be expected to happen with even the most modest assumptions about changes over the next few years. The problem could not be evaded. Therefore, whilst regretting in some ways the concentration on the technicalities of graduation and extrapolation, as the forecasting attempts had been described, he felt with Sir William Elderton that the attempt must be made on a modest basis as to change. Even then, of course, it would remain a very chancy guess. Nobody guessing a few years before the

upward trend of old-age mortality which commenced in about 1922 would have been disposed to allow for that, and certainly no one just before 1942 would have guessed at the discontinuous and substantial drop in old-age mortality. In spite of that, the actuary must not pay too little attention to mechanical forecasting, but he must look at the matter, as the author attempted to do in the final section of his paper, on a different basis.

He had been surprised that, obsessed with the technical aspects of the subject as most of the speakers in the discussion had been, they had made little attempt to differentiate between what seemed to him to be quite different tasks. One was the making of a projection of rates of population over preferably a few years but sometimes of necessity a generation or two, and the other was the quite disinterested attempt to understand what had been happening and, if possible, what was likely to happen in future. He suggested that there was a difference in those tasks and that the actuary approached them from different points of view.

Reference had been made in the discussion to methods based on the fixing of limits for mortality rates. That could lead to the most extraordinary compilations. Not very long since he had seen a mortality table which was used for projection of population and in which the rates at different ages were chosen as being the lowest rates for those ages that had been experienced in a particular country at any time in the past, irrespective of whether the rate for age x came, say, from the year 1900 and the rate for age $x+1$ came from the year 1920. The resulting hotch-potch had been seriously argued by the compiler to be a reasonable ultimate level of mortality. He trusted that actuaries were free of that, but one never knew!

He asked the members to pass a vote of thanks to Dr Pollard for a paper which had involved a very large amount of work and had led to a most interesting discussion which, he thought, Dr Pollard would not regard as being too damaging in criticism.

Dr A. H. Pollard, who was unable to be present at the meeting, sent the following written reply to the discussion:

I have read with considerable interest the transcript of the discussion and would like to comment on one or two of the points raised.

Several speakers objected to the use of the word 'forecasting' in this connexion. The fact is that, whether or not we like it, for some purposes an actuary has to make an estimate of mortality rates at some future date. This estimate may be but an intelligent guess, no more accurate than the guesses of the weather forecaster; but it is, nevertheless, the best estimate that can be made on certain assumptions about the past, and I think it not unreasonable to describe such a process as forecasting. I fully appreciate the arguments raised against the use of this word, but I am not sufficiently impressed to want to be the first to eliminate from the actuarial vocabulary a word which has been used by writers on the subject throughout the past.

Mr Ogborn, in this connexion, thought it was hardly fair to attribute to certain authors particular methods of forecasting as being their methods when, he said, they were not presenting these methods for the purposes of forecasting. There is no doubt I did not treat Dr Rhodes fairly in this regard particularly as I was aware of his remark on this subject. I do not agree, however, that this criticism can be raised in the cases of Cramér and Wold; or of Davidson and Reid. The former authors describe their paper as 'a study in graduation and forecasting' where they attempt to 'estimate future mortality' and the latter authors state they propose to 'make our estimate of forecasted mortality by projecting the curves onwards...'

Sir William Elderton would like to have seen in the paper some explanation of the formulae in § 8 or some journal reference to them and would like to have seen a reference made to a paper by P. Baltensperger in the 1941 Swiss actuarial journal. I heartily agree with Sir William Elderton on this point, but I am sure he will sympathize with Australian actuaries when I explain that I approached all Australian libraries without success for the journals in question. I shall probably never see the paper by P. Baltensperger because the Swiss actuarial journal does not reach these shores. Our knowledge in Australia of European actuarial literature is limited to the *J.I.A.* notes on

the subject with which Sir William Elderton's name is linked. These notes are very useful in wetting the lips of the actuary who has the Institute library to refer to; unfortunately in Australia we are often left with an unquenchable thirst. Actually the present paper resulted first from reading in the *J.I.A.* the note on the paper by Cramér and Wold, and secondly from the coincidence that I was able to obtain the paper itself from an actuary who had gone to the trouble of obtaining for another purpose from Scandinavia the journal in which it appeared.

Mr Cox and Mr Haycocks were surprised that approximate 1945 Australian death-rates were not given for comparison. The reason is that Australian death statistics apply to the civilian population only, deaths on war service being excluded. The estimated population, on the other hand, includes service personnel. The paper was prepared some time ago when the latest statistics available would have been sufficiently affected to make a comparison unjustified.

A paper on this subject is necessarily controversial and I have not commented on the remarks made by several speakers because I am in substantial agreement with them.