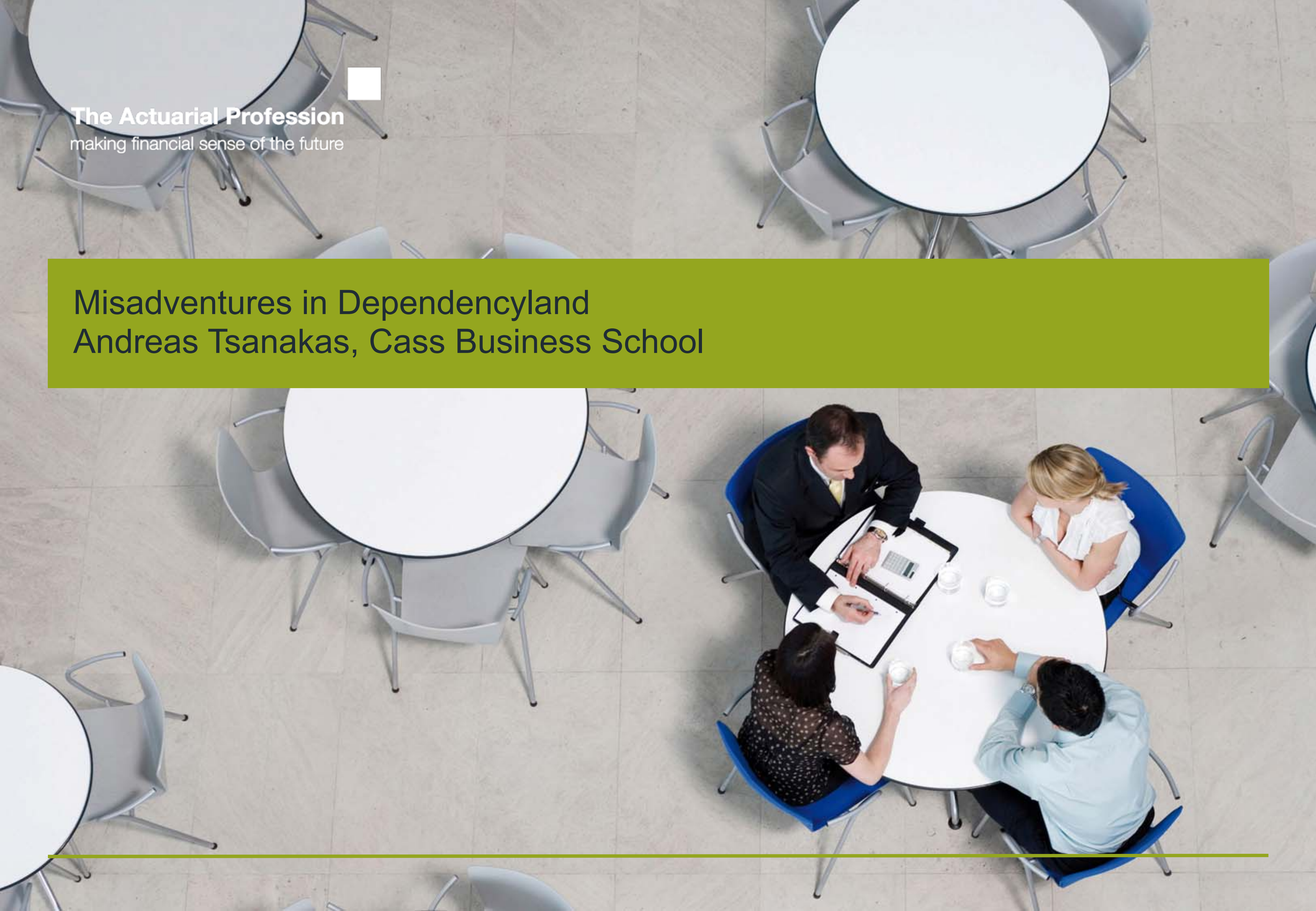
The Actuarial Profession
making financial sense of the future



Misadventures in Dependencyland

Andreas Tsanakas, Cass Business School

Some questions

- ~~Is the model correct?~~
- ~~Is the model reasonable?~~
- Are model inputs reasonable?
  - Statistical assumptions
- Are model outputs reasonable?
  - Diversification effects
- What do we mean by *reasonable*?
  - Consistent with experience
  - Consistent with expectations

Inputs – experience

- Typically not enough data to do sophisticated statistics
  - Why?
- What is *enough*?
  - Depends on the question we asked
  - Sensitivity of output on parameter error depends on the nature of the output
  - There are some outputs for which there might *never* be enough data
    - 99.5%-VaR?

Inputs – experience

- Statistical analysis with what we've got!
- Metrics
  - Correlation
    - Measureable, inconvenient, intuition fails
  - Rank correlations (Spearman, Kendall)
    - Measurable, convenient, intuition fails
  - Asymptotic tail dependence
    - Not measureable in practice
  - Conditional exceedance probabilities
    - Measureable, convenient, intuition (maybe) does not fail
- Model selection

Expert judgement

- Big issues around elicitation process
- Some bad questions:
  - What is the value of the correlation coefficient?
  - Does “tail correlation” exist for these risks?
- A slightly better question:
  - What is the probability that Loss 1 exceeds its 1:10 years level, given that Loss 2 exceeds its 1:10 years level?
  - Still don’t know the answer, but at least understand the question…

Conditional exceedance probabilities

- The copula gives us the probability of X_1, X_2 being at the same time below a fixed percentile.

$$C(p_1,p_2) = P\left(X_1 \leq VaR_{p_1}(X_1) \text{ and } X_2 \leq VaR_{p_2}(X_2)\right)$$

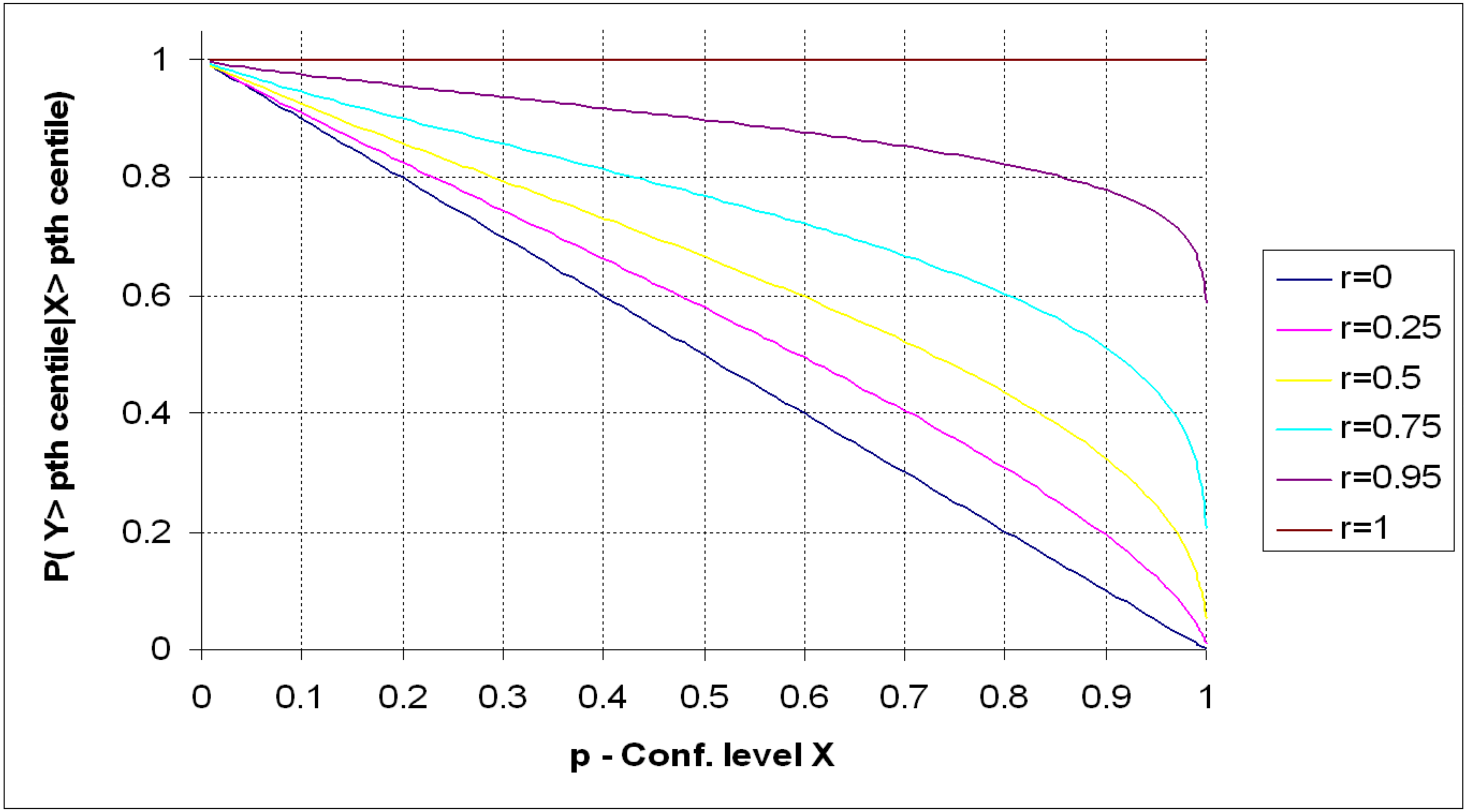
- Consider the function of p

$$CEP(p) = P\left(X_1 > VaR_p(X_1) \mid X_2 > VaR_p(X_2)\right)$$

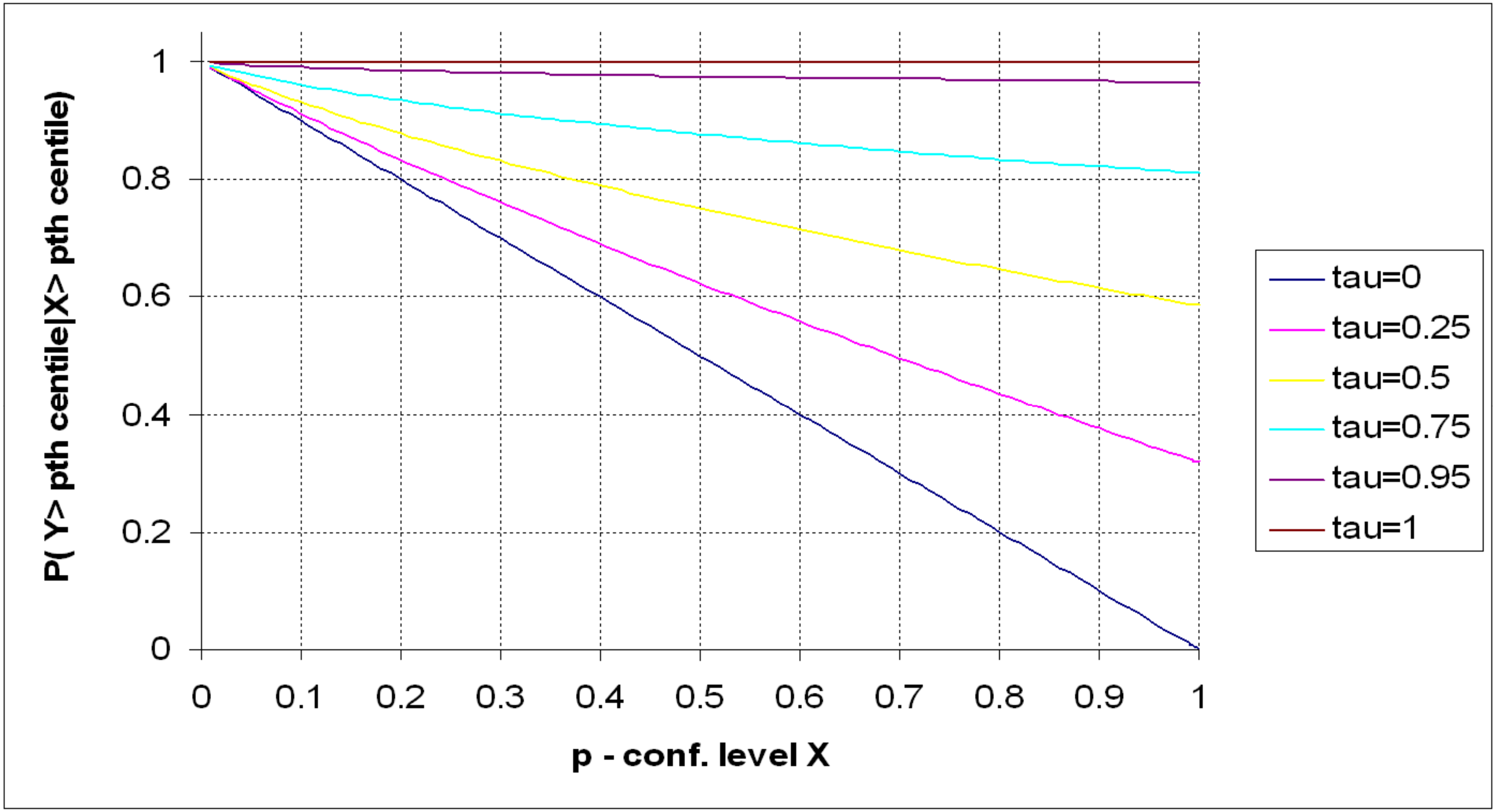
- Can show that

$$CEP(p) = \frac{C(p,p) - p}{1-p} + 1$$

Gaussian copula



Gumbel copula



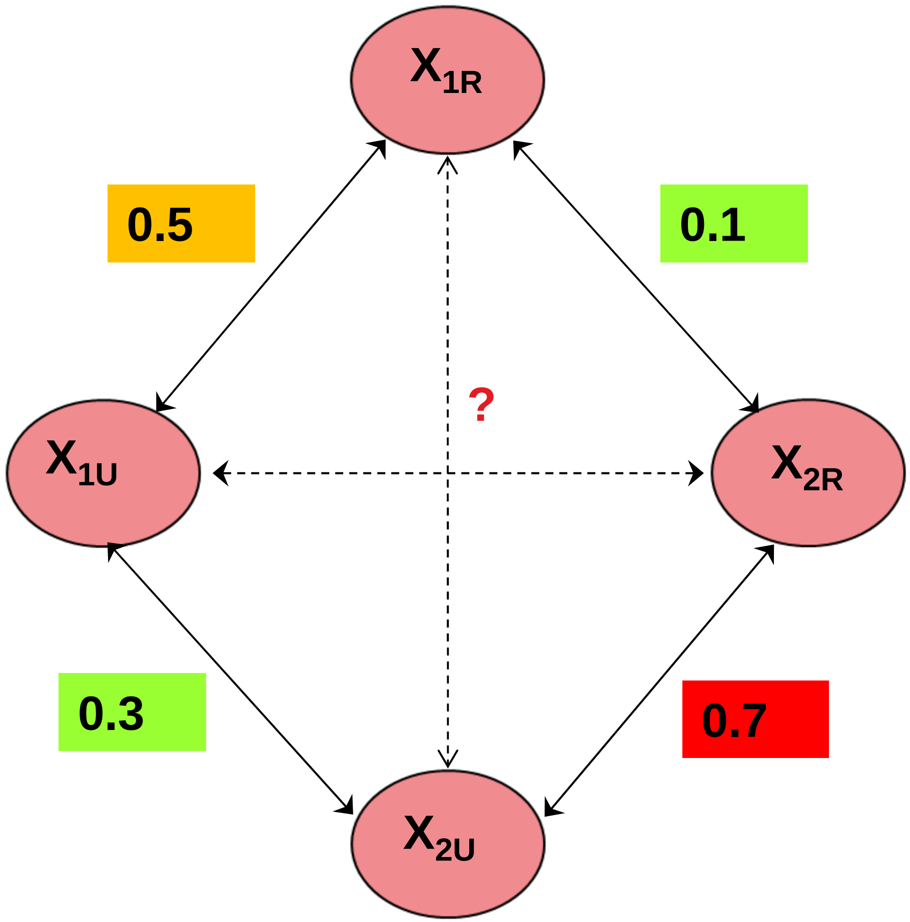
Implicit assumptions

- All statistical models contain implicit assumptions
  - In the past: independence, Gaussian copula...
- Extra assumptions made to populate high dimensional models
  - Large correlation matrices (Gaussian, t)
  - Graphical dependence models (Gumbel and friends)

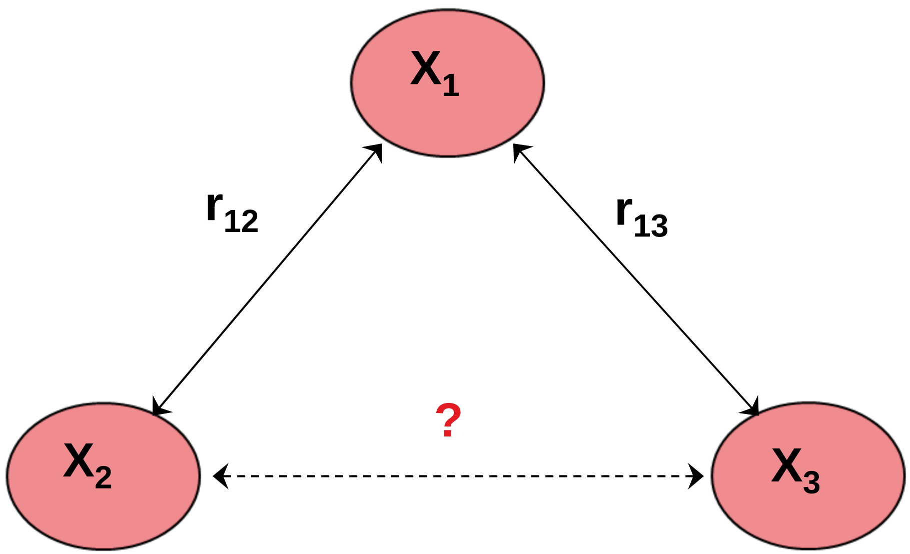
Example

	X_{1R}	X_{1U}		
X_{1R}	1	0.5		
X_{1U}	0.5	1		
X_{2R}	0.1	?	1	0.7
X_{2U}	?	0.3	0.7	1

Specifying “indirect correlations”



A simpler problem



Admissible correlation values

- From the properties of correlation matrices:

$$r_{12}r_{13} - \sqrt{\left(1-r_{13}^{\ 2}\right)\left(1-r_{12}^{\ 2}\right)} < r_{23} < r_{12}r_{13} + \sqrt{\left(1-r_{13}^{\ 2}\right)\left(1-r_{12}^{\ 2}\right)}$$

- For example:
  - $r_{12}=0$, $r_{13}=0$ → $-1 \leq r_{23} \leq 1$
  - $r_{12}=0.5$, $r_{13}=0.5$ → $-0.5 \leq r_{23} \leq 1$
  - $r_{12}=0.9$, $r_{13}=0.9$ → $0.62 \leq r_{23} \leq 1$

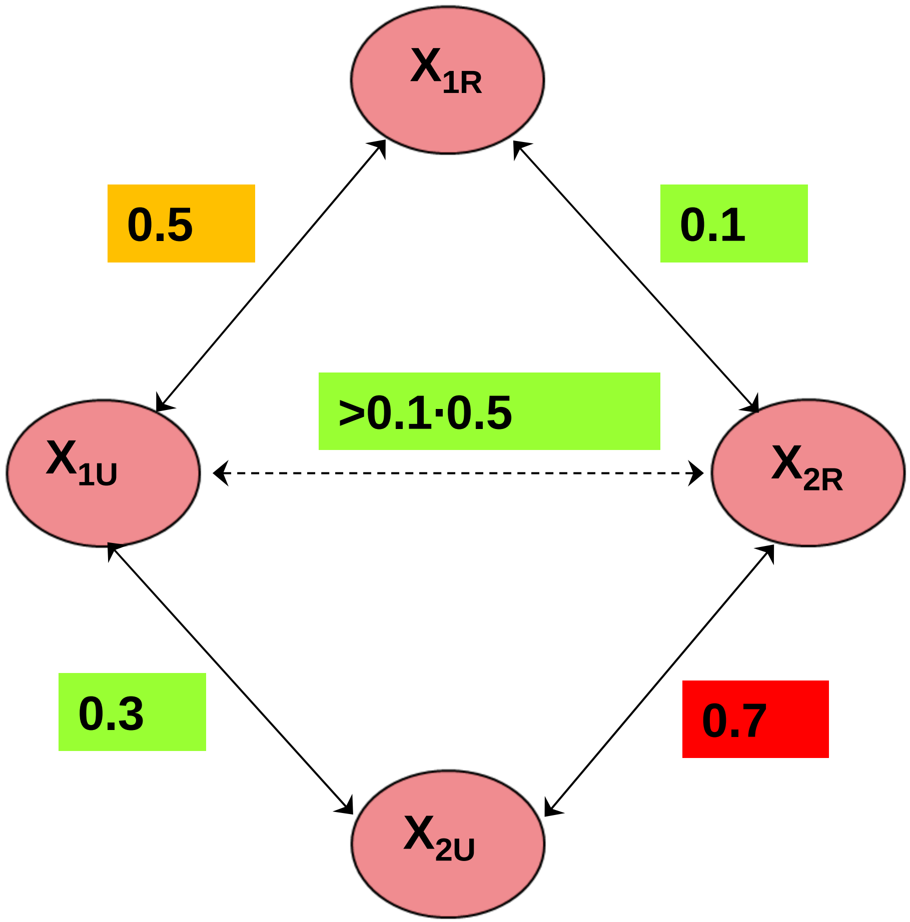
- Bounds are very wide!

Conditional independence

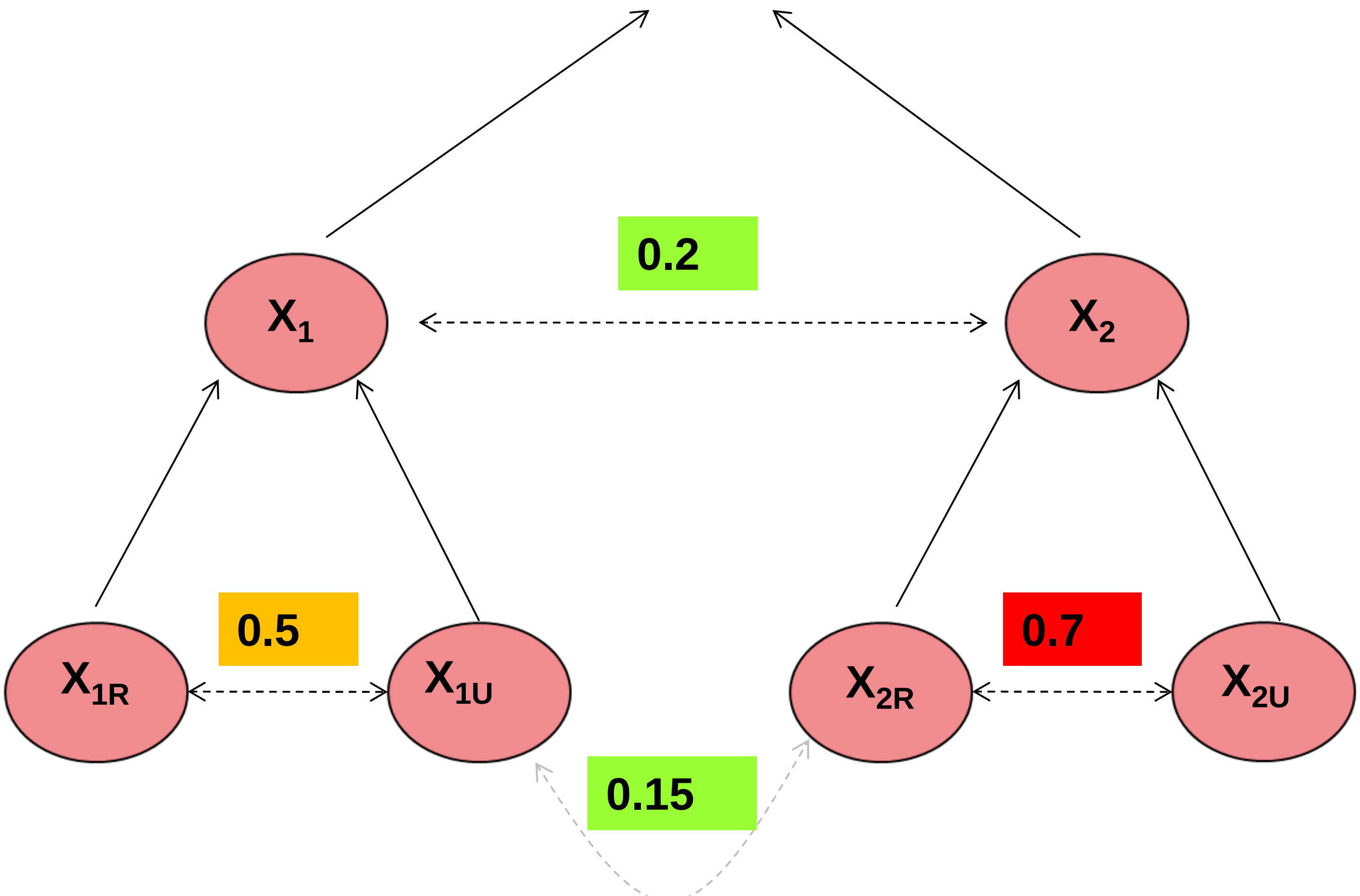
- Sometimes an additional assumption is made
  - "X_2, X_3 correlated only via X_1"
  - *Conditional independence* given X_1
- A reasonable approximation then is:

  $r_{23} = r_{12} \cdot r_{13}$
- In higher-dimensional problems this approach will produce correlations that are too low
  - Presence of additional "routes" to correlation between risks

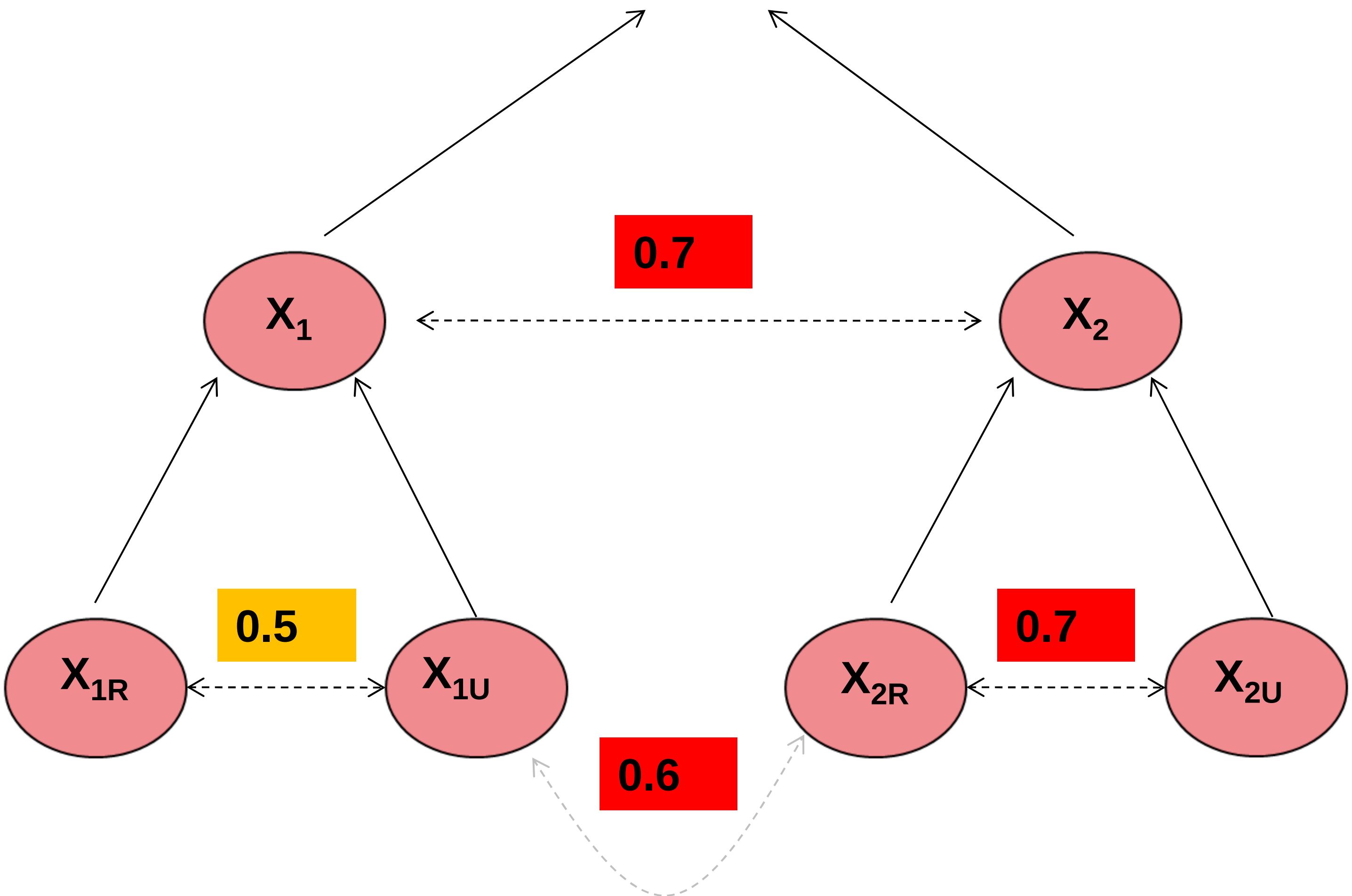
Specifying "indirect correlations"



Hierarchical structures



Hierarchical structures



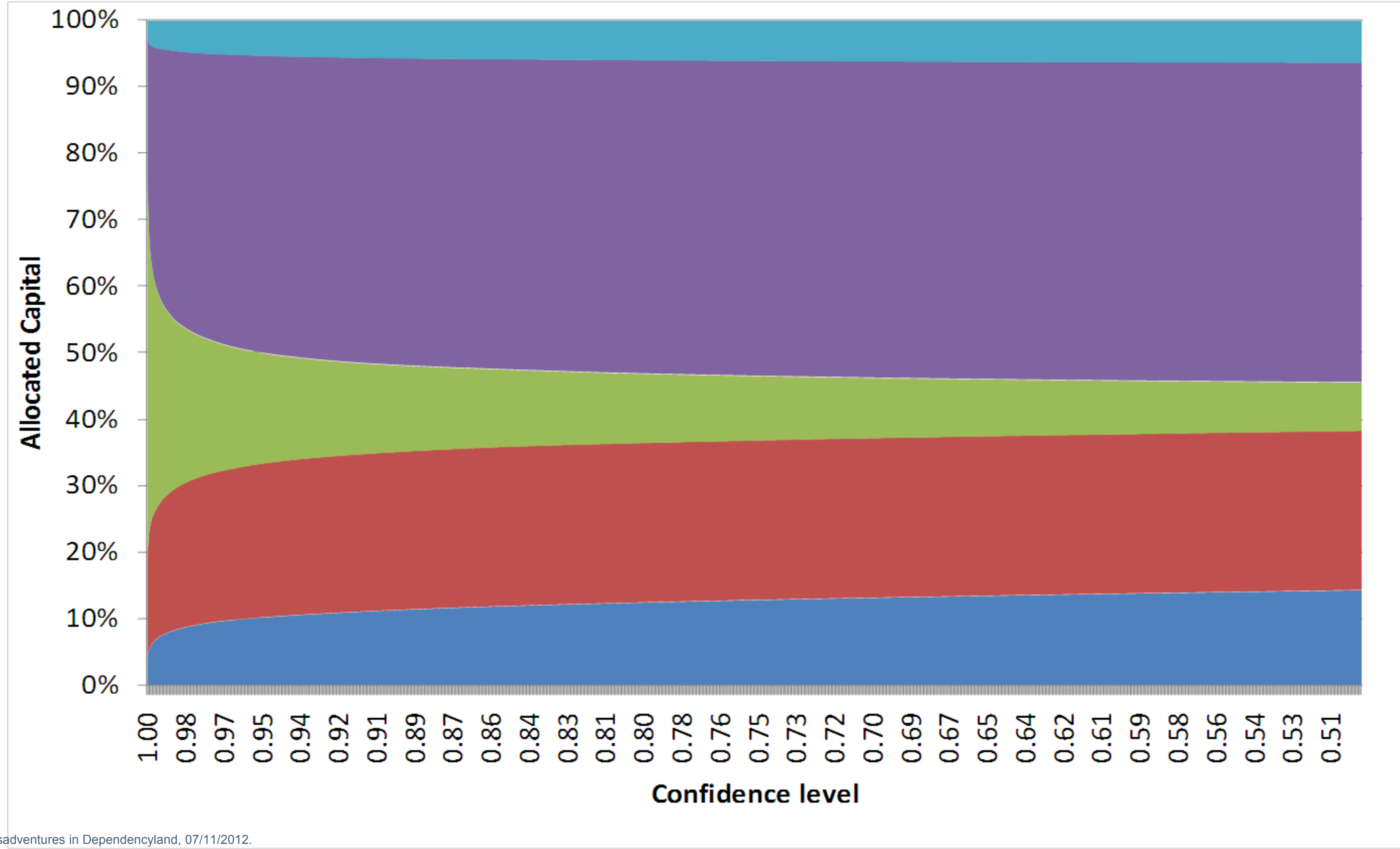
Outputs – experience

- Comparing model outputs to scenarios
  - Scenarios can be historical or made up
  - Tricky – compare what with what?
- Compare historical losses with (local) model outputs
  - Dependence not the only loss driver
- Compare structure of composite scenarios with that of dependence model
- Reverse stress tests with Euler/marginal capital allocations

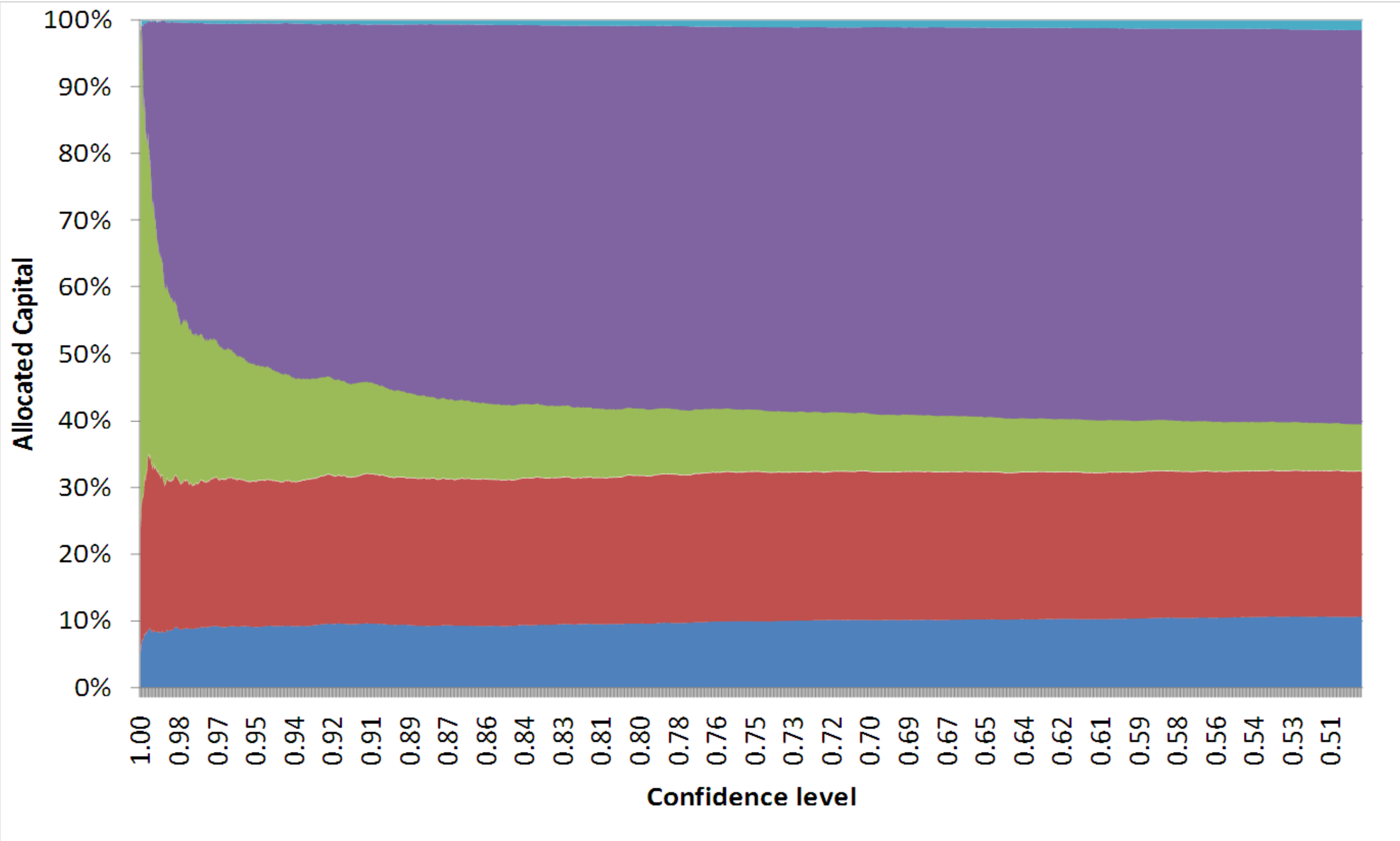
Reverse stress tests and capital allocation

- Reverse stress test
  - Start from business failure scenario
  - Decompose scenario into loss components
- Capital allocation
  - Start from simulations that drive capital
  - Work out average loss for each portfolio risk on those simulations
- The two are in a sense the same thing!
  - Broad comparisons can be made

TVaR – standalone capital allocation



TVaR – marginal capital allocation



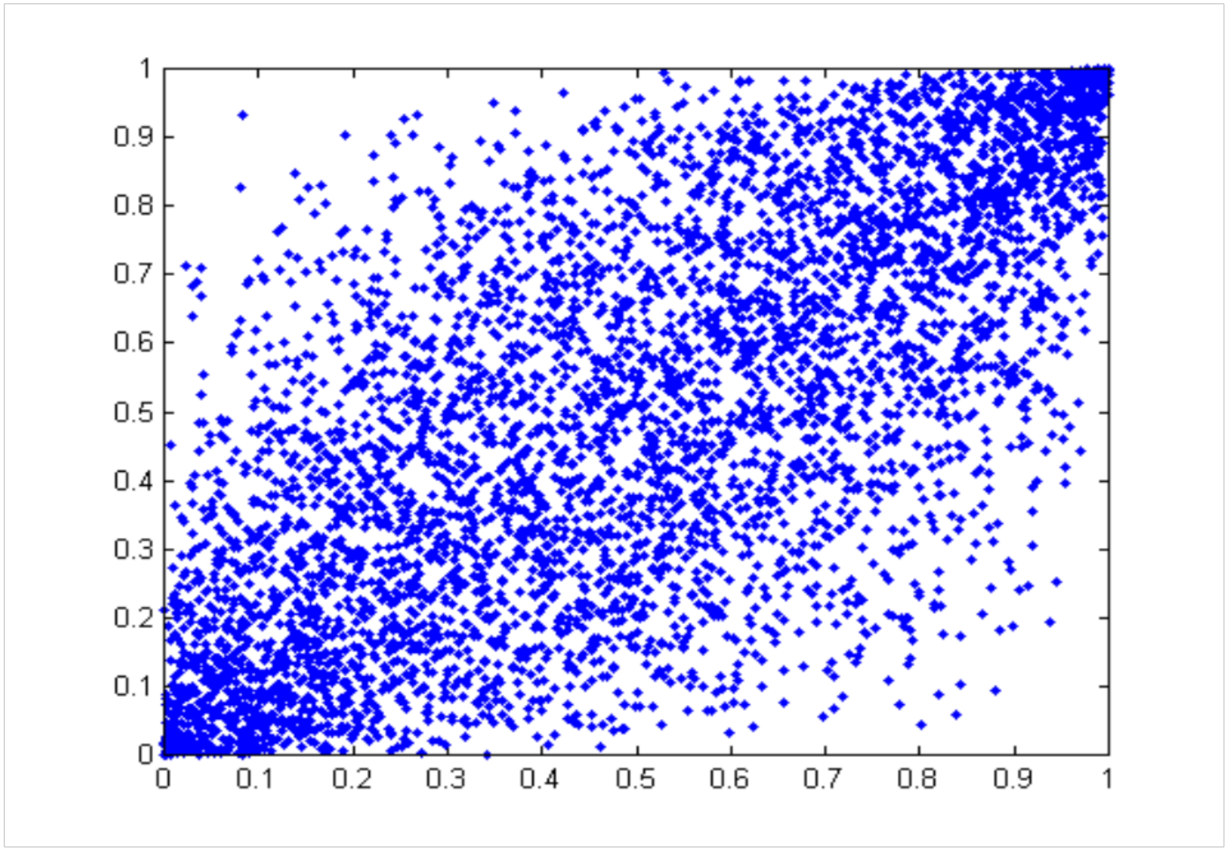
Outputs – reasonableness

- Intuition about diversification very hard to obtain!
  - "Diversification benefit": a truly dodgy concept
- Sensitivities to
  - Copula choice
  - Strength of dependence
  - Granularity
  - Business interactions
  - Other statistical assumptions
  - ...

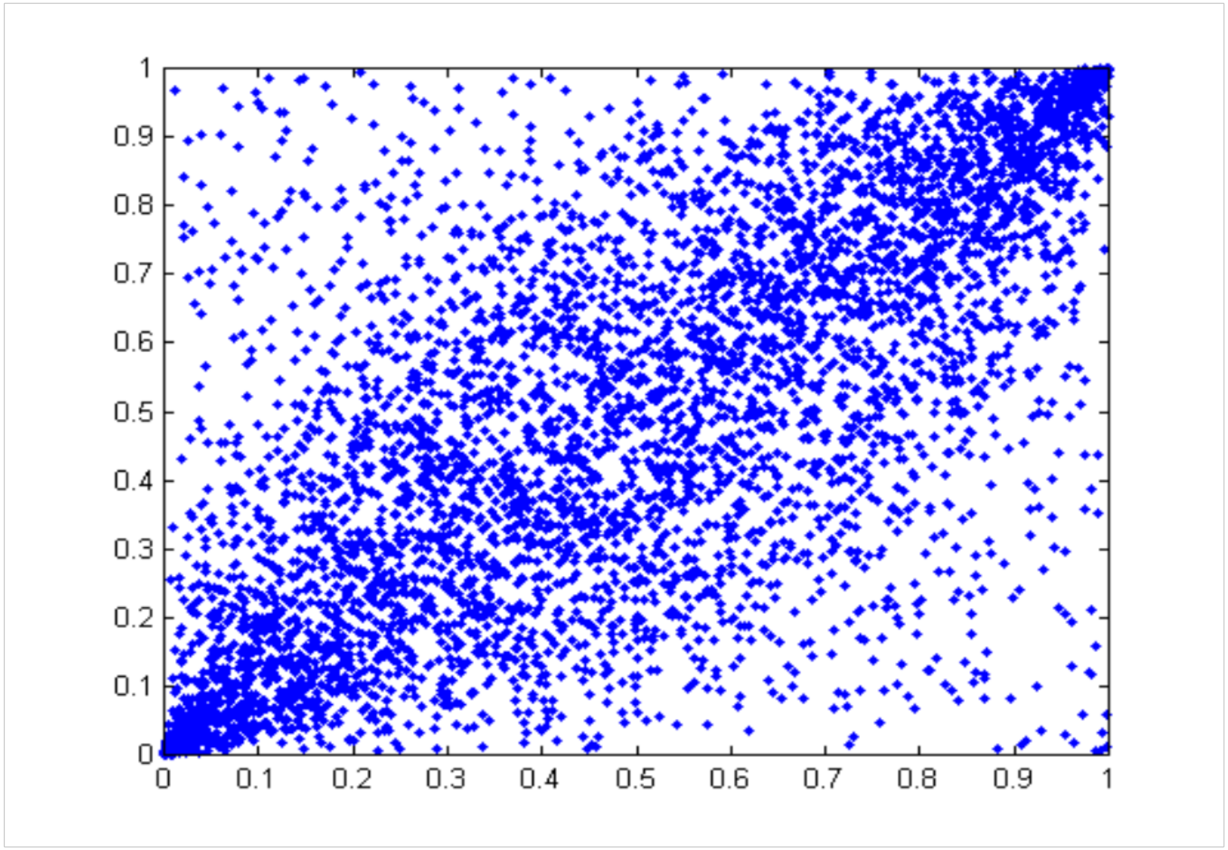
How is diversification affected by copula choice?

- Simulation experiment
  - Portfolio of 10 risks, LogNormal with $\mu=100$, $\sigma=25$
  - Correlation between each pair set by $\tau = 0.5$.
- Simulate using
  - Gaussian, t (dof=3), and flipped Clayton copulas
  - Discrete normal mean-variance mixture
  - Benchmark formulas

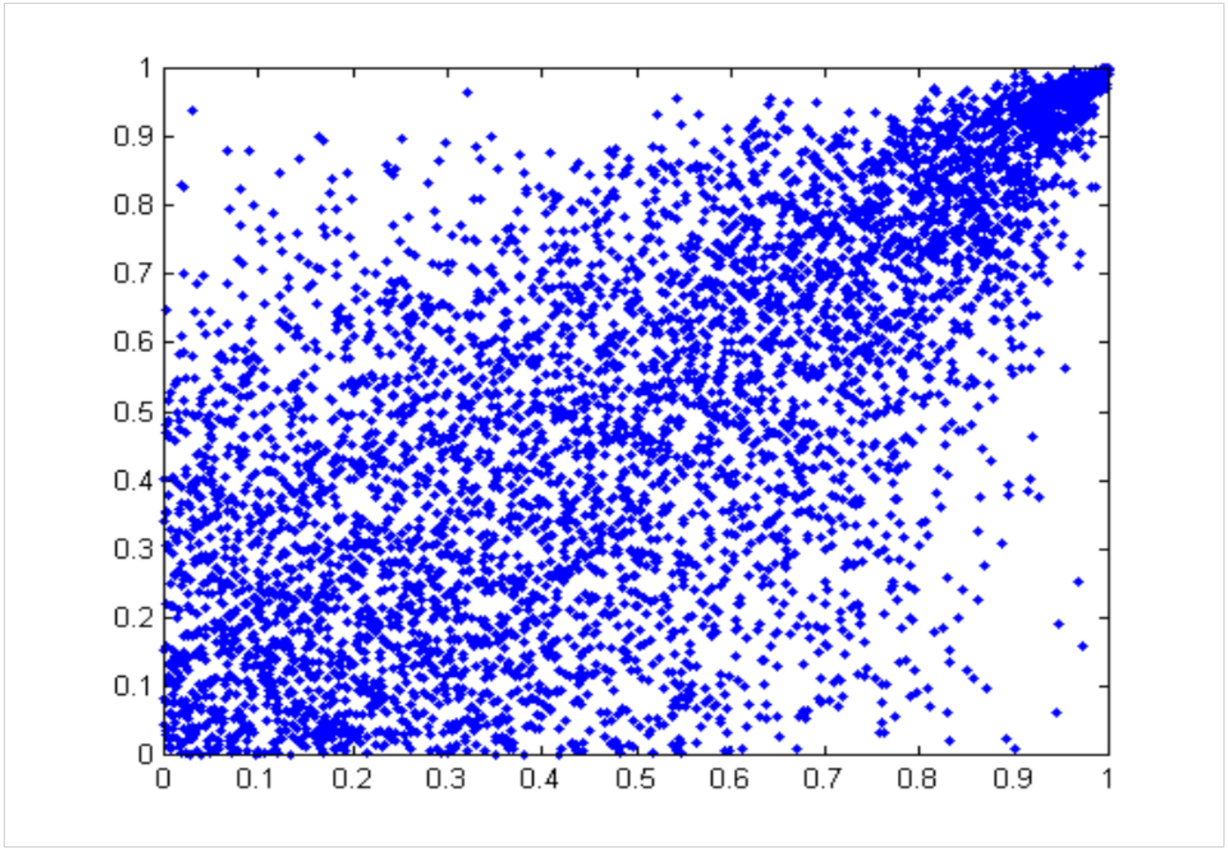
Gaussian copula



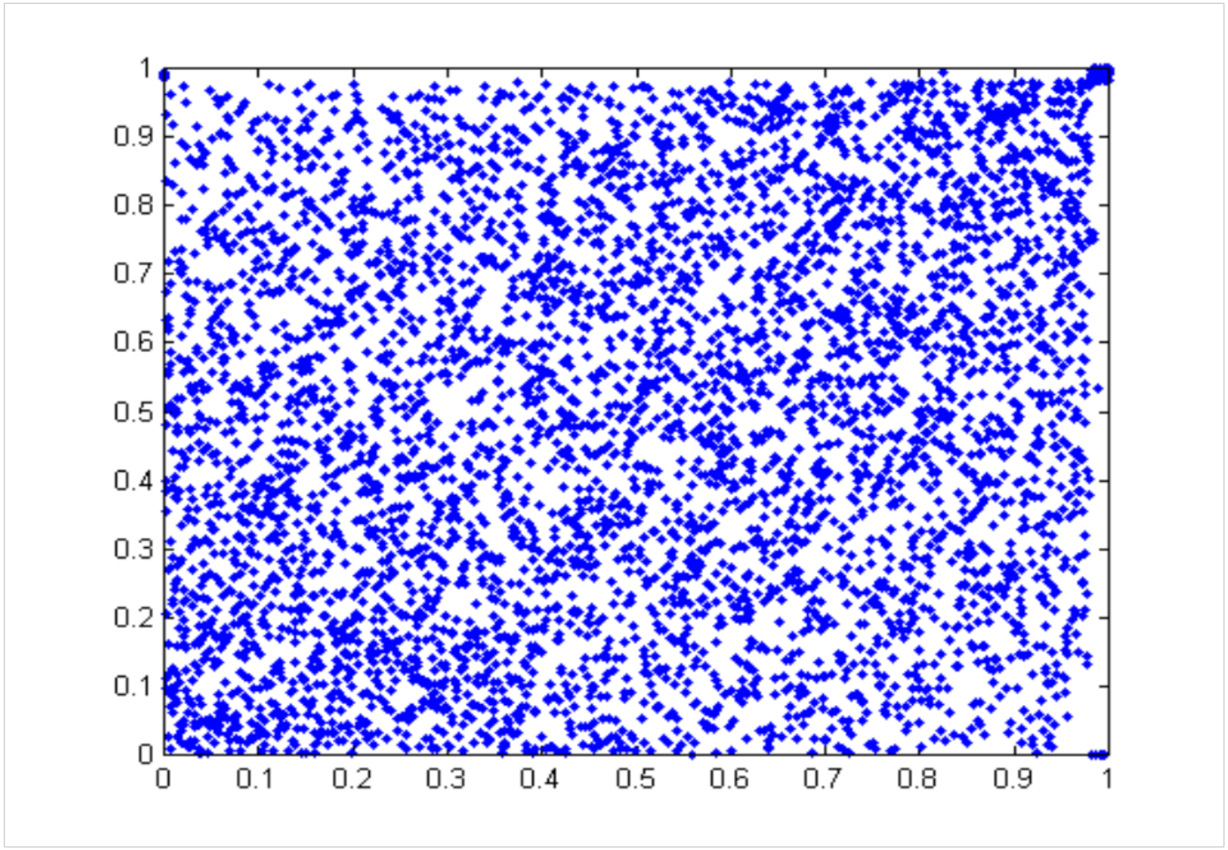
t copula



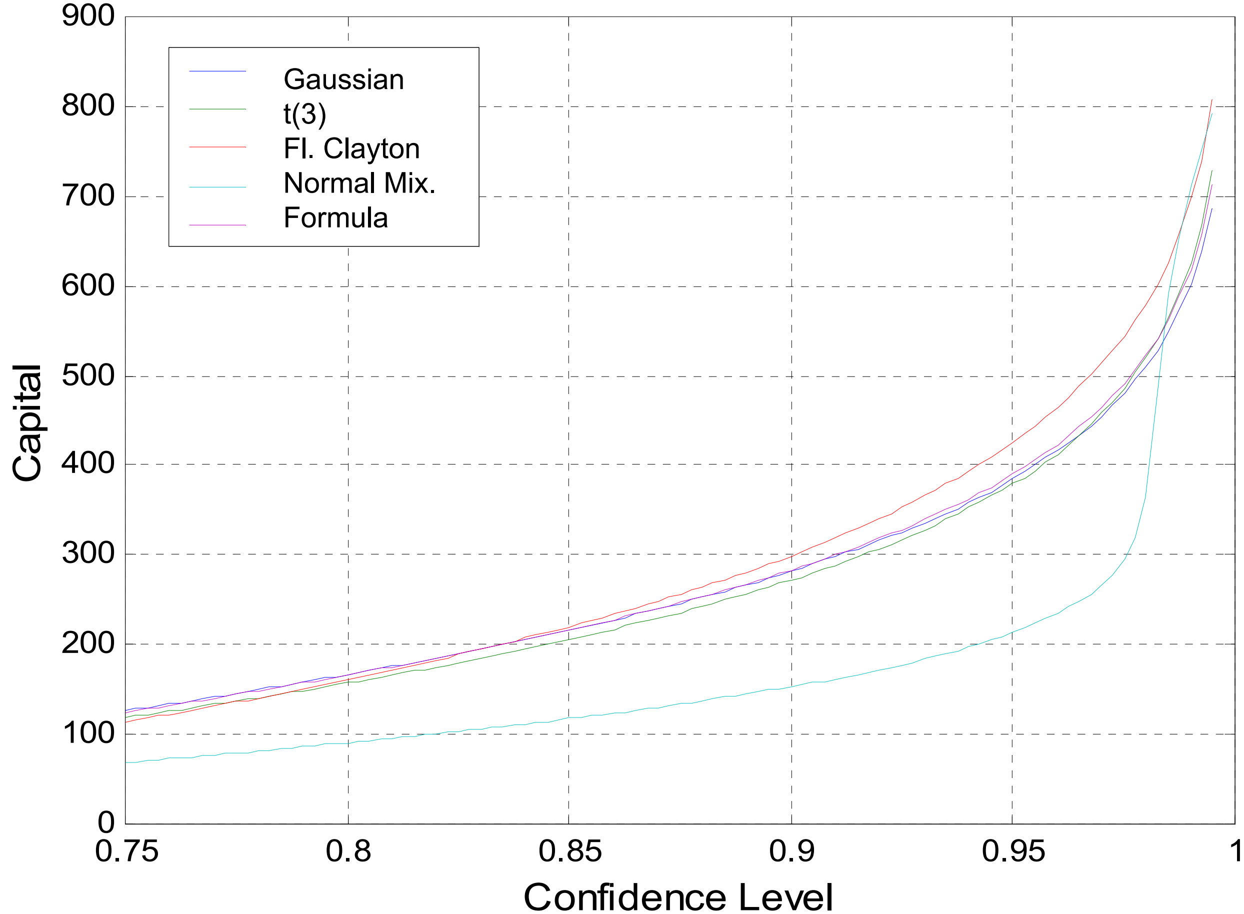
Flipped Clayton copula



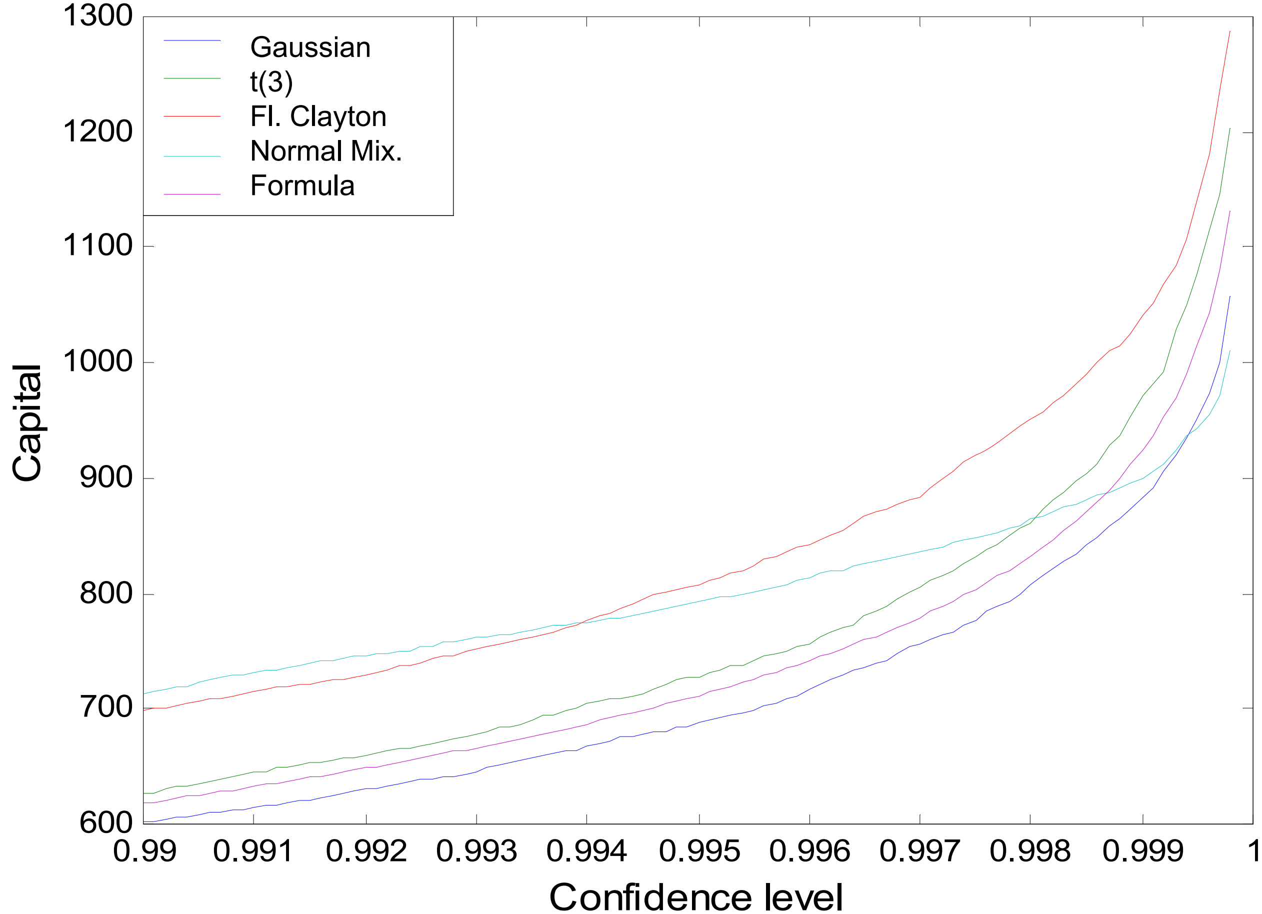
Normal mixture



Capital (from 75th percentile)



Capital (from 99th percentile)



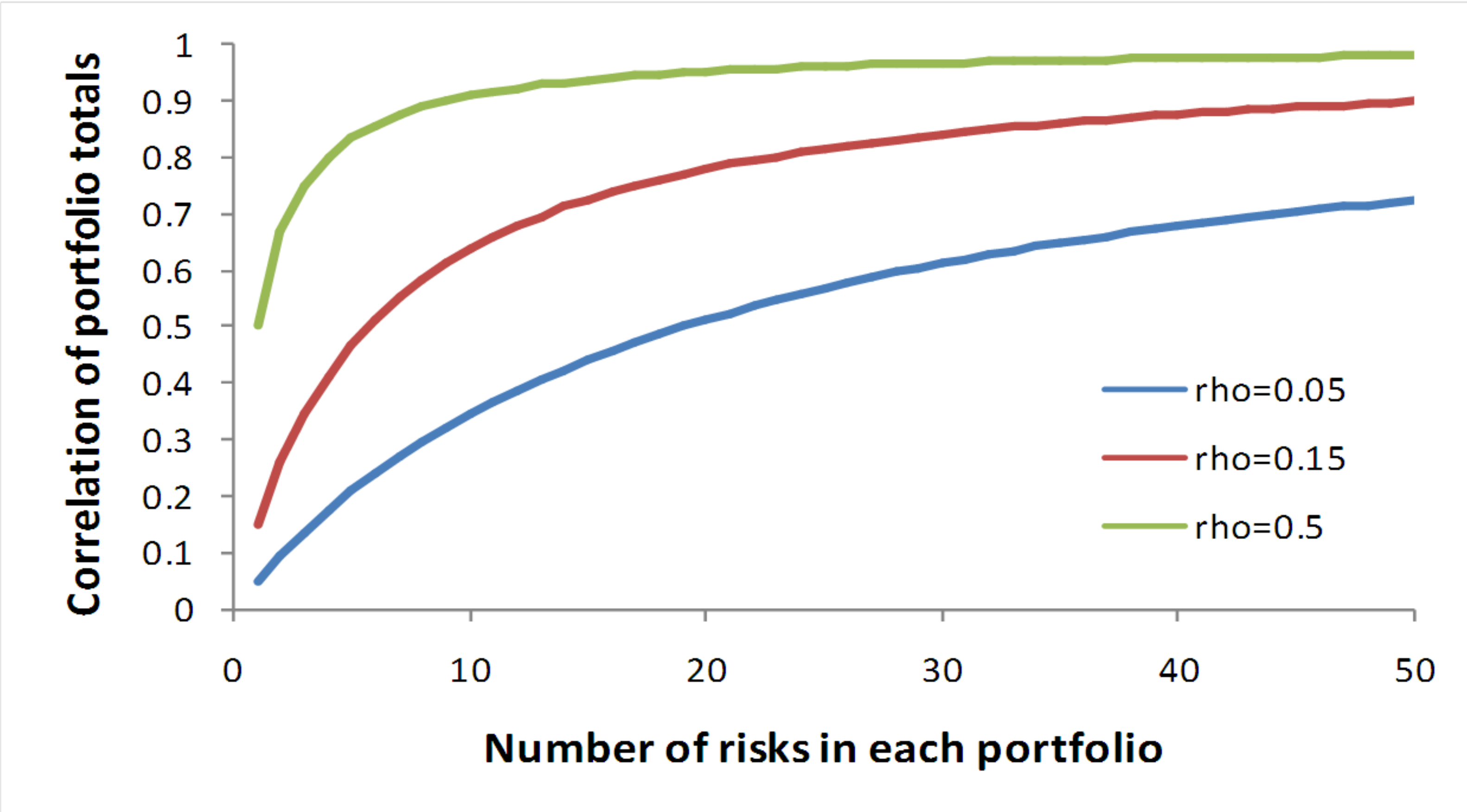
Conditional exceedance probabilities

Correlation *(tau)*	**1 in 10 years Cond. Prob.** *Gaussian*	*t (3 dof)*	**1 in 100 years Cond. Prob.** *Gaussian*	*t (3 Dof)*
5%	0.13	0.21	0.02	0.15
10%	0.15	0.24	0.03	0.18
20%	0.22	0.31	0.06	0.24
50%	0.47	0.54	0.27	0.47
75%	0.73	0.76	0.60	0.72
90%	0.89	0.90	0.83	0.89

Granularity: sensitivity of VaR to default correlation

Size of portfolio	**1000**	**1000**	**1000**	**1000**	**1000**
Marginal default probability	**5%**	**5%**	**5%**	**5%**	**5%**
Default correlation	**0%**	**1%**	**6%**	**12%**	**69%**
99.5%VaR of default number	**69**	**126**	**277**	**431**	**990**

How do input correlations drive output correlations?



Sensitivity analysis

- Vary input parameters and observe relevant outputs
  – Monitor materiality of assumptions
- Some observations
  – Don’t only look at total VaR – examine lower level outputs
  – Link ranges of parameters with uncertainty around them
  – “One-at-a-time” testing practical but ignores interactions

- Is high sensitivity to unknown parameters a risk to be mitigated? Is quantification of model error informative?

Implications of model error

- Even with our best efforts, parameter and model error will dominate our calculations
  - Uncertainty is not a valid excuse for doing bad statistics...
- The importance of evidence is often supplanted by that of power and commercial interest
  - Serving institutional objectives becomes more important than finding out the truth
- A deeply corrosive effect
  - Are we giving up on scientific endeavour?
  - Are we pragmatists or cynics?

On Bullshit (Frankfurt, 1986)

Someone who lies and someone who tells the truth are playing on opposite sides, so to speak, in the same game. [...] [T]he response of the one is guided by the authority of the truth, while the response of the other defies that authority and refuses to meet its demands. The bullshitter [...] does not reject the authority of the truth, as the liar does, and oppose himself to it. He pays no attention to it at all. By virtue of this, bullshit is a greater enemy of the truth than lies are.

Bullshit is unavoidable whenever circumstances require someone to talk without knowing what he is talking about. Thus the production of bullshit is stimulated whenever a person's obligations or opportunities to speak about some topic are more excessive than his knowledge of the facts that are relevant to that topic.

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.