Modelling Aggregate
Non-Life Underwriting Risk:
Standard Formula vs Internal Model

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The aim of this paper

– to analyse the risk profile of a multi-line non-life insurer regarding the **Premium risk**

– to obtain a **sensitivity of Internal Risk Model** for different insurers according to volume and claim variability

– to discuss on **dependency** among different lines of business

– to analyse a consistent comparison of capital requirement for *Premium Risk* obtained by either *Internal Model* and "Solvency II-QIS3" *Standard Formula*

Collective Risk Simulation Model

- A **Collective Risk Model** is here applied with the aim to quantify the capital required _**for premium risk**_ **for a multi-line non-life insurer** (with **TH=1** year).

- Following the collective approach, for each line of business the aggregate claims amount is given by a **compound Poisson process**, where:

- **number of claims distribution is the Poisson law**, with a parameter n_0 increasing year by year by the _real growth rate g_ and with a _structure variable q_ distribuited as a Gamma (h;h):

$$\widetilde{n}_t = n_0 \cdot (1+g)^t \cdot \widetilde{q}$$

- **the claim size amounts Z_{it} are assumed i.i.d. with a LogNormal distribution** and to be scaled by _the claim inflation rate i_

Premiums Volume

- The <u>Total Initial Gross Premium Volume</u>, for each LoB, is equal to:

$$B_0 = P_0(1+\lambda) + c \cdot B_0 = (n_0 \cdot m_0)(1+\lambda) + c \cdot B_0$$

Where at time 0:
- n_0 is the expected number of claims
- m_0 is the expected claim cost
- c is the expenses coefficient (% Gross Premiums Volume)
- λ is the safety loading coefficient

- For each line of business both the nominal gross premium volume $B_{t,lob}$ and the risk premium $P_{t,lob}$ increase yearly by *the claim inflation rate* (i) and *the real growth rate* (g):

$$B_t = B_{t-1}(1+i)(1+g) = \left[P_{t-1} \cdot (1+i)(1+g)\right] \cdot (1+\lambda) + c \cdot B_t$$

Internal Model
Case Studies

- **Four different Non-Life Insurance Companies are regarded**
  OMEGA, TAU, TAUHIGH, EPSILON

- All of them **have different dimension and/or claim size coefficient of variability (c_z)**

- All insurers underwrite business in the same <u>**5 Lines of Business (LoBs) with the same weight on the gross written premiums volume:**</u>
  - LoB 1: **Accident**   (**10% Gross Premiums Volume**)
  - LoB 2: **Motor Damages**   (**10%**   "   "   "   )
  - LoB 3: **Property**   (**15%**   "   "   "   )
  - LoB 4: **MTPL**   (**55%**   "   "   "   )
  - LoB 5: **GTPL**   (**10%**   "   "   "   )

- The **Total Initial Gross Premiums Volume** of the four insurers:
  - Comp. **OMEGA**   **1000 mill** (Euro)
  - Comp. **TAU** and **TAUHIGH**   **500 mill** (Euro) – <u>**differ for the claim size CV only**</u>
 - Comp. **EPSILON**   **100 mill** (Euro)

Market Combined Ratios

It is to be emphasized that:

a) MTPL combined ratios have been **reduced under 100%** in the recent time because of mainly the frequency reduction;

b) Motor Damages showed Combined Ratios higher than 100% in the first '90s. Afterwords, premium rating increase permitted CR **values lower than 80%.**

NOTE: Comb. Ratios are net of run-off result

Combined Ratios
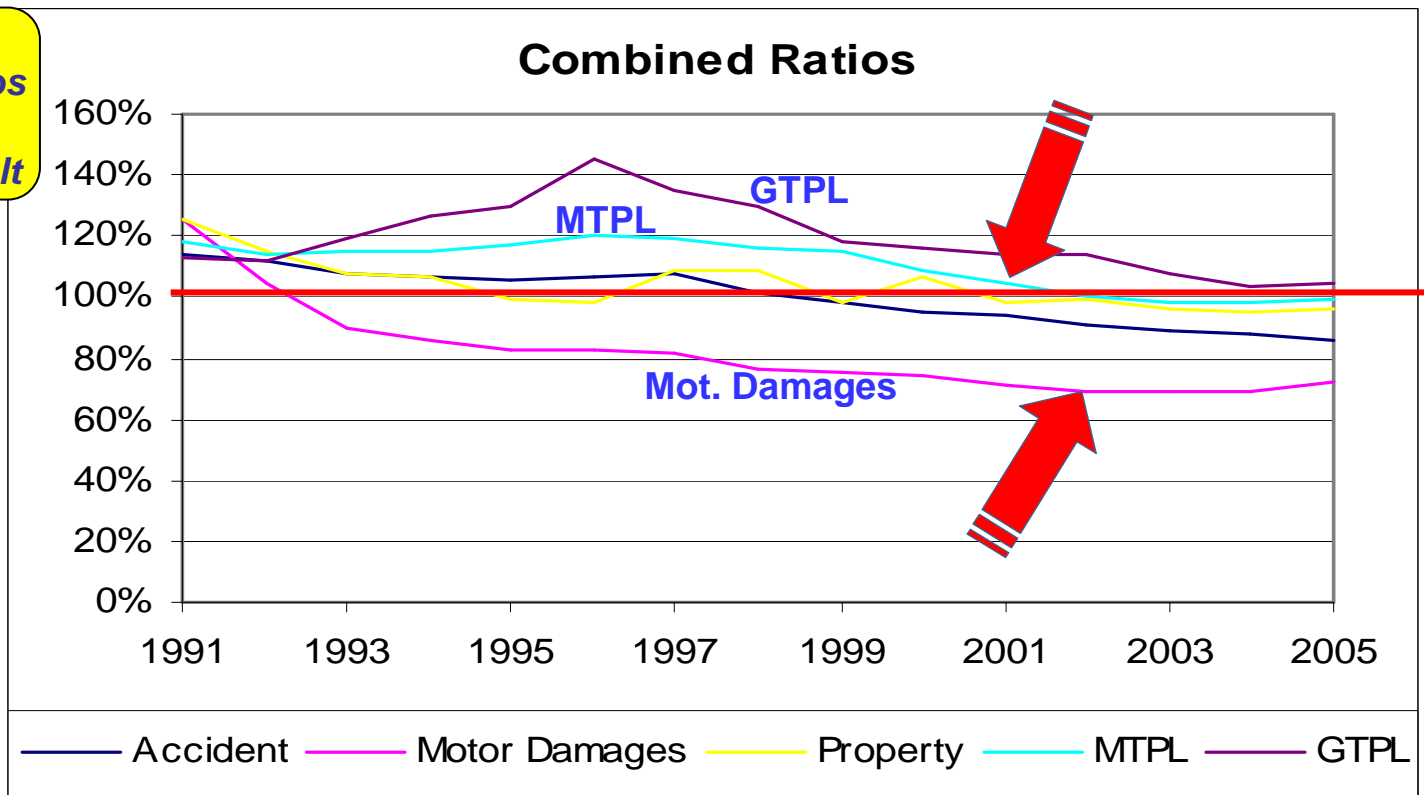
Parameters for premiums and claims

	LoBs	n_0	$\sigma(q)$	g	m_0	c_z	i	λ	c
OMEGA	LoB1	**17.374**	14,0%	1,9%	3.200	3	3%	22,40%	31,95%
	LoB2	**18.515**	28,9%	1,9%	2.500	2	3%	64,25%	23,98%
	LoB3	**16.580**	11,2%	1,9%	6.000	8	3%	6,28%	29,51%
	LoB4	**111.316**	8,7%	1,9%	4.000	4	3%	1,88%	17,52%
	LoB5	**7.721**	13,9%	1,9%	10.000	12	3%	-7,03%	28,22%
TAU	LoB1	**8.687**	14,0%	1,9%	3.200	3	3%	22,40%	31,95%
	LoB2	**9.258**	28,9%	1,9%	2.500	2	3%	64,25%	23,98%
	LoB3	**8.290**	11,2%	1,9%	6.000	8	3%	6,28%	29,51%
	LoB4	**55.658**	8,7%	1,9%	4.000	4	3%	1,88%	17,52%
	LoB5	**3.861**	13,9%	1,9%	10.000	12	3%	-7,03%	28,22%
TAUHIGH	LoB1	8.687	14,0%	1,9%	3.200	**4,5**	3%	22,40%	31,95%
	LoB2	9.258	28,9%	1,9%	2.500	**3**	3%	64,25%	23,98%
	LoB3	8.290	11,2%	1,9%	6.000	**12**	3%	6,28%	29,51%
	LoB4	55.658	8,7%	1,9%	4.000	**6**	3%	1,88%	17,52%
	LoB5	3.861	13,9%	1,9%	10.000	**18**	3%	-7,03%	28,22%
EPSILON	LoB1	**1.737**	14,0%	1,9%	3.200	3	3%	22,40%	31,95%
	LoB2	**1.852**	28,9%	1,9%	2.500	2	3%	64,25%	23,98%
	LoB3	**1.658**	11,2%	1,9%	6.000	8	3%	6,28%	29,51%
	LoB4	**11.132**	8,7%	1,9%	4.000	4	3%	1,88%	17,52%
	LoB5	**773**	13,9%	1,9%	10.000	12	3%	-7,03%	28,22%

Acc
MDam
Prop
MTPL
GTPL

n_0 = expected number of claims (t=0) $\sigma(q)$ = std structure variable   g = real growth rate

m_0 = expected claim cost (t=0) c_z = claim size CV ($\sigma(Z)/E(Z)$) i = claim inflation rate

λ = safety loading coefficient c = expenses coefficient

The Capital Requirement
(Independence Assumption)

➢ The Required Capital **for the 4 Insurers with the 5 LoBs** is obtained, **for the moment**, <u>under LoB independence assumption.</u>

In case of independence among the claim amount of all the lines, **the total aggregate amount of claims will be clearly the sum of single LoB claim amount X$_i$ with an aggregate RBC amount** (SCR^{Agg,IM}) obtained for TH=1 by:

$$SCR_{\alpha}^{Agg,IM} = VaR_{\alpha} - \sum_{i=1}^{L} P_i(1+\lambda_i)$$

being clearly minor than the sum of single RBC requirements.

➢ **The RBC ratios (given by the RBC amount divided by initial Gross Premiums) is related to three examined confidence levels (α):**
  - 99.00% (corresponding to a S&P rating BB approx.)
  - **99.50%** (adopted in QIS3/QIS4, and roughly equivalent to a S&P rating BBB-)
  - 99.97% (corresponding to a S&P rating AA).

  and regarding 99.50% level as our benchmark:

  The RBC is obtained for the **Premium Risk only** and without any consideration of **Reinsurance.**

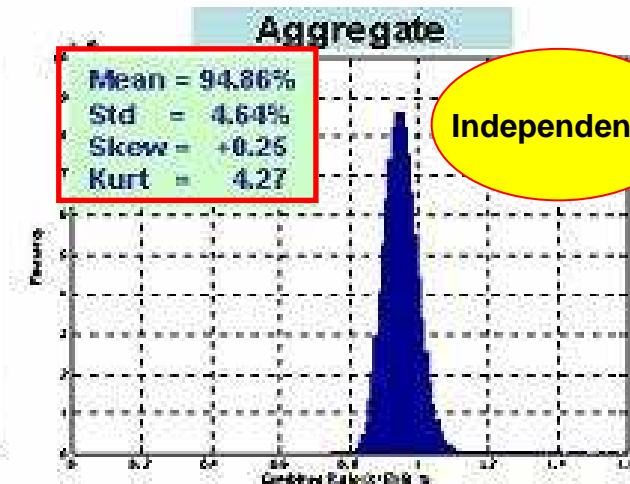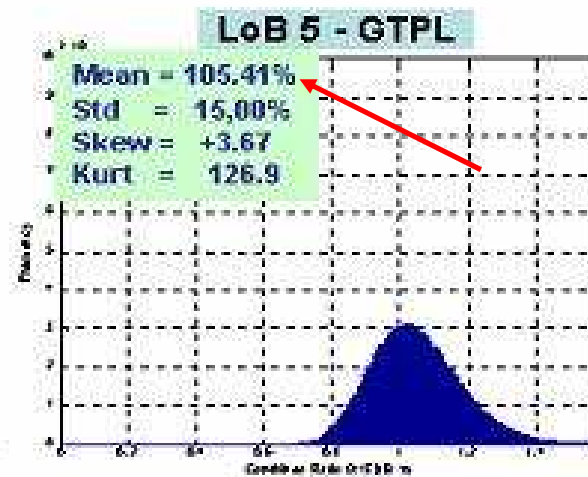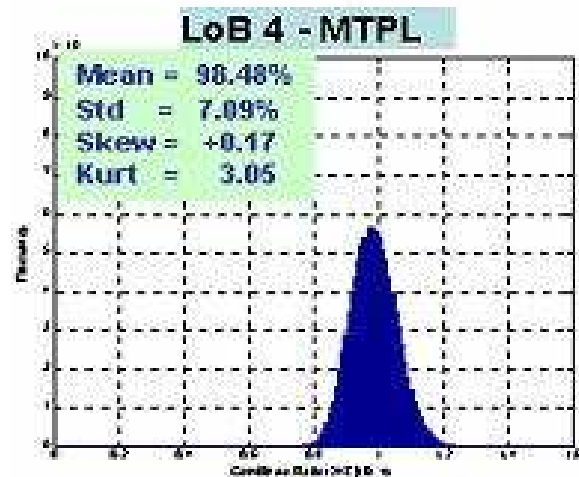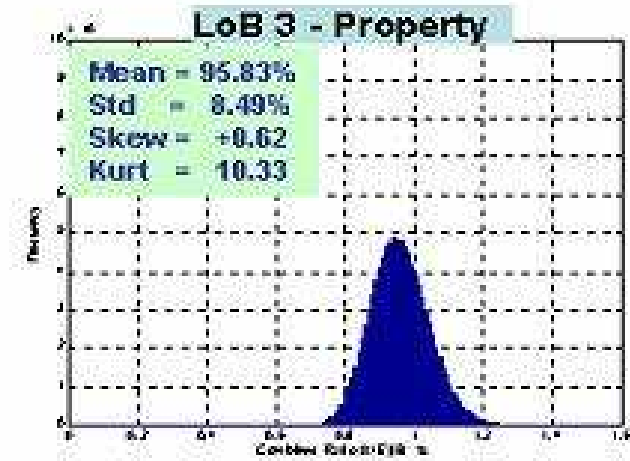# Company OMEGA
Claim Amount Distributions: X(1)

LoB 1 - Accident
Mean =	58,37 mln
Std =	8,28 mln
STD/Mean = 14.18%	
Skew =	+0.28

LoB 2 – Motor Damages
Mean =	48,58 mln
Std =	14,04 mln
STD/Mean = 28.91%	
Skew =	+0.58

LoB 3 - Property
Mean =	104,44 mln
Std =	13,37 mln
STD/Mean = 12.80%	
Skew =	+0.62

LoB 4 - MTPL
Mean =	467,54 mln
Std =	40,94 mln
STD/Mean = 8,75%	
Skew =	+0.17

LoB 5 - GTPL
Mean =	81,06 mln
Std =	15,75 mln
STD/Mean = 19,43%	
Skew =	+3.67

Independence

Aggregate
Mean = 760,01 mln	
Std =	48,70 mln
STD/Mean = 6.41%	
Skew =	+0.25
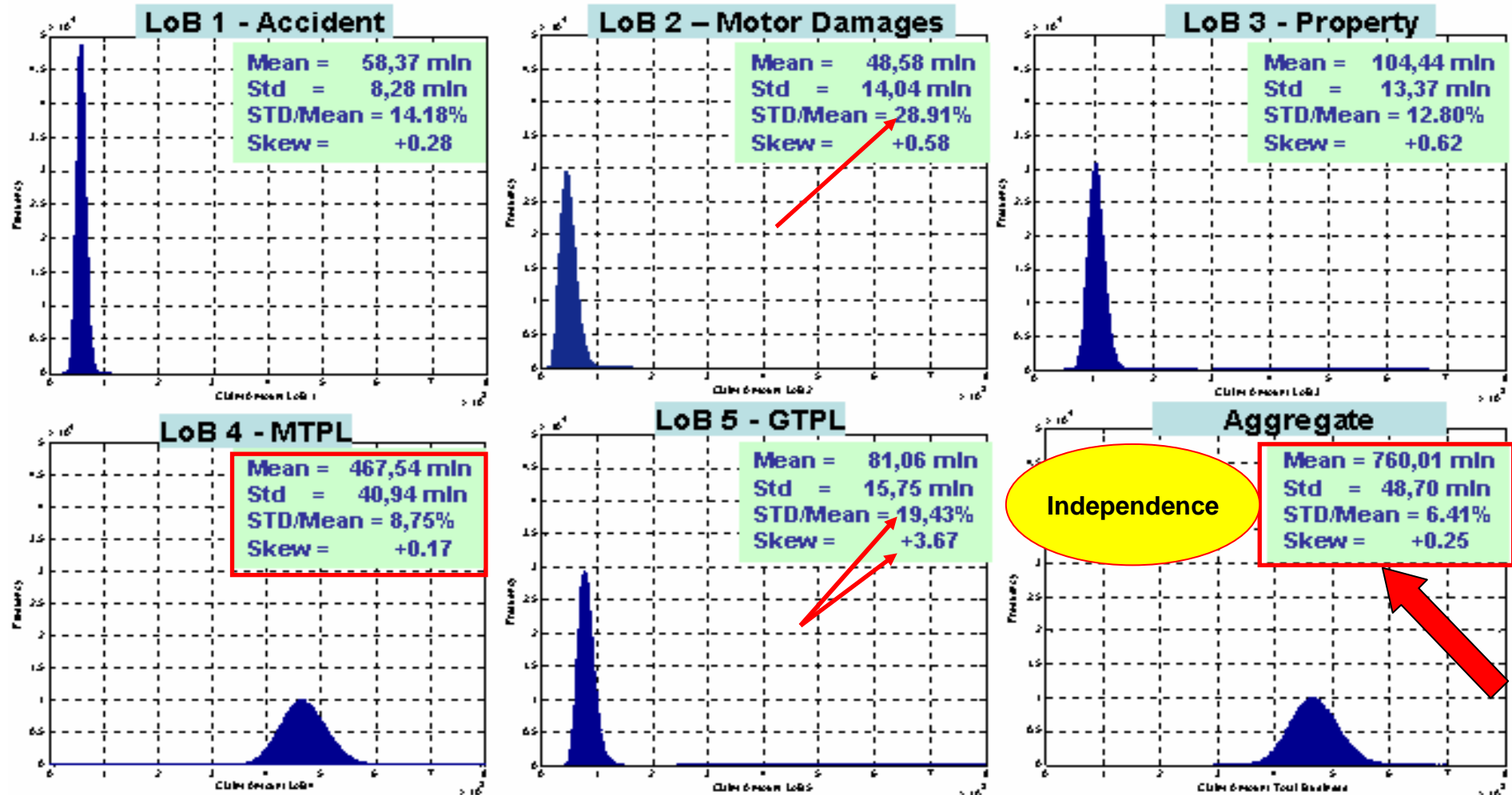
Company OMEGA
Combined Ratios Distributions (X/B)

LoB 1 - Accident

Mean = 87.55%
Std = 7.88%
Skew = +0.28
Kurt = 3.11

LoB 2 – Motor Damages

Mean = 70.24%
Std = 13.38%
Skew = +0.58
Kurt = 3.63

LoB 3 - Property

Mean = 95.83%
Std = 8.49%
Skew = +0.62
Kurt = 10.33

LoB 4 - MTPL

Mean = 98.48%
Std = 7.09%
Skew = +0.17
Kurt = 3.05

LoB 5 - GTPL

Mean = 105.41%
Std = 15.00%
Skew = +3.67
Kurt = 126.9

Aggregate

Mean = 94.86%
Std = 4.64%
Skew = +0.25
Kurt = 4.27
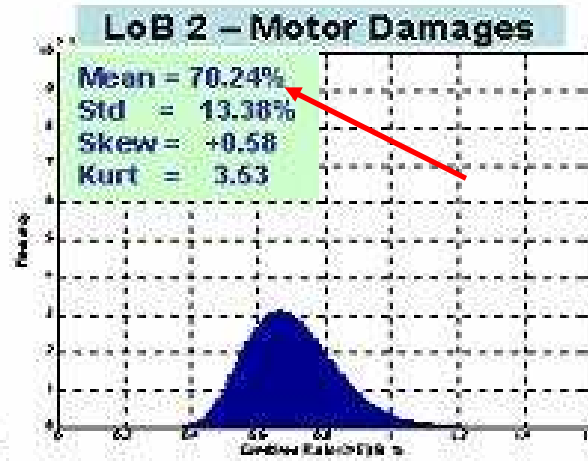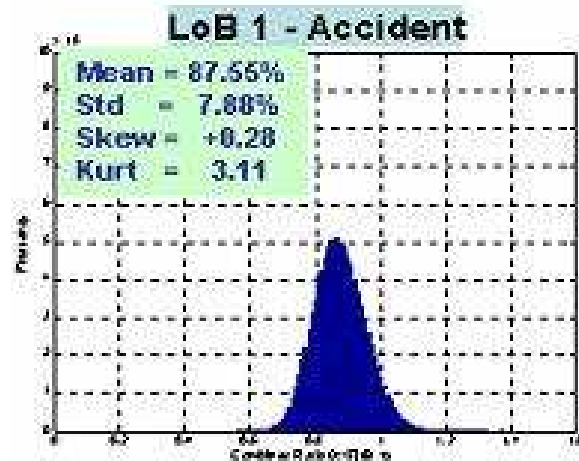
Independence

Company OMEGA
RBC e RBC ratios

RBC Amount

RBC ratio

7.96%
(no correlation)
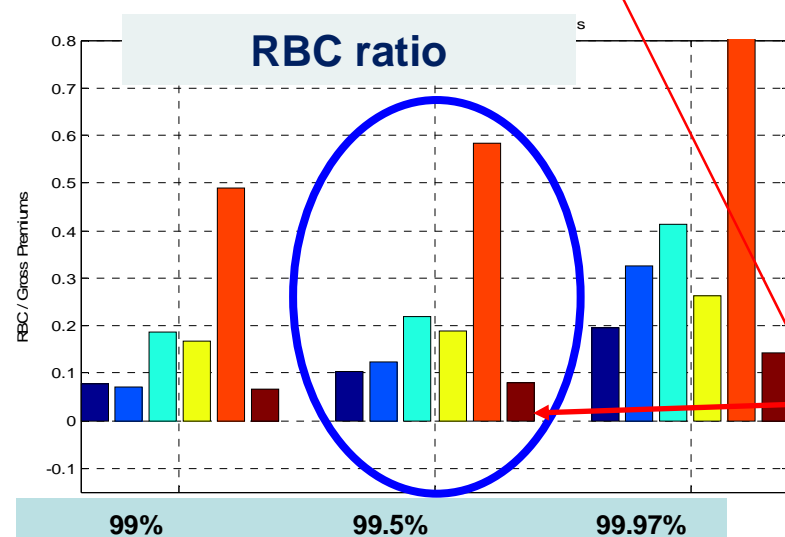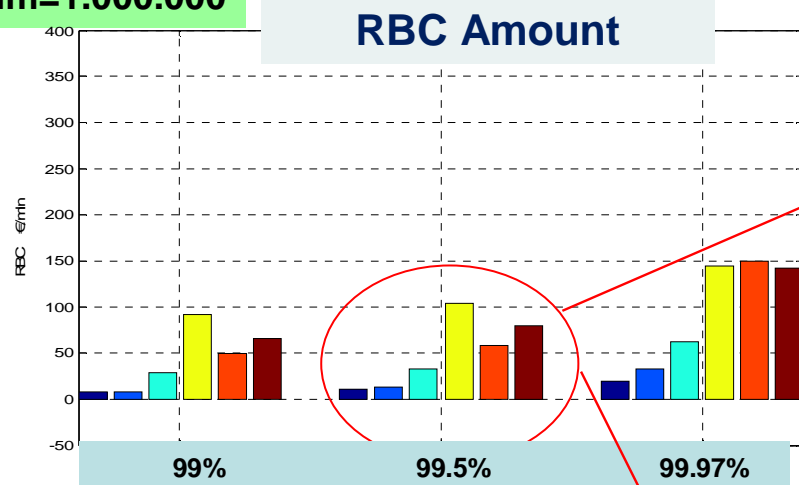
✓ The total capital requirement ($RBC_{99.5\%}$) for the whole <u>company Omega</u> is equal to **7.96%** of gross premiums in case of independence (almost 77 million of Euro).

✓ As expected **the highest ratio is registered for the line GTPL** (58.4%) due mainly to its large claim size CV. Property Line shows a high ratio too (21.8%), while **Line MTPL (18.8%)** and Accident (10.4%) has lower ratios. <u>Motor Damage has a 12.5% ratio, notwithstanding the large safety loading λ, because of the large standard deviation of q.</u>

Accident	10,40%
Motor Damages	12,47%
Property	21,82%
MTPL	18,84%
GTPL	58,39%
Aggregate	7,96%

The Capital Requirement (Linear Correlation)

The Capital Requirement ($SCR^{Agg,Matr}$), under linear correlation assumption, **is derived rescaling the RBC obtained from Internal Model in case of independence** ($SCR^{Agg,IM}$).

This appear necessary because SCR^{IND} gives an approximate estimation of diversification effect between different LoB compared to aggregated IM:

$$SCR^{Agg,Matr} = SCR^{Agg,IM} + \frac{\left(SCR - SCR^{IND}\right)}{\left(SCR^{Full\,Corr} - SCR^{IND}\right)}\left(SCR^{Agg,Full} - SCR^{Agg,IM}\right)$$

Using only aggreg. IM by indep.assump. w/o Matrix corr.

- **<u>SCR is estimated joining the single capital charge</u> CC$_i$ (equal to VaR$_i$-P$_i$ obtained by IM)** with a correlation matrix

these value are not equal 0-5% of Prem. approx.

$$SCR = \sqrt{\sum_{i=1}^{L}\sum_{j=1}^{L} Corr_{i,j} \cdot CC_i \cdot CC_j - \sum_{i=1}^{L} \lambda_i P_i}$$

L: number of LoBs

-SCR^{IND} and $SCR^{Full\,Corr}$ are derived in either independence and full correlation assumptions and are respectively given by:

$$SCR^{IND} = \sqrt{\sum_{i=1}^{L}\left[CC_i\right]^2 - \sum_{i=1}^{L} \lambda_i P_i}$$

$$SCR^{FullCorr} = \sum_{i=1}^{L}\left[CC_i\right] - \sum_{i=1}^{L} \lambda_i P_i = \sum_{i=1}^{L} SCR_i = SCR^{Agg,Full}$$

Company OMEGA
Aggregation and Diversification

QIS3 Correlation Matrix

LOB	Accident	Mot. Damages	Property	MTPL	GTPL
Accident	1	0,25	0,25	0,25	0,25
Mot. Damages	0,25	1	0,25	0,5	0,25
Property	0,25	0,25	1	0,25	0,25
MTPL	0,25	0,5	0,25	1	0,5
GTPL	0,25	0,25	0,25	0,5	1

For those 5 lines the capital requirement **is increasing to 13.96% in case the QIS3 matrix correlation is assumed**, and 11.49% with QIS2 correlation.
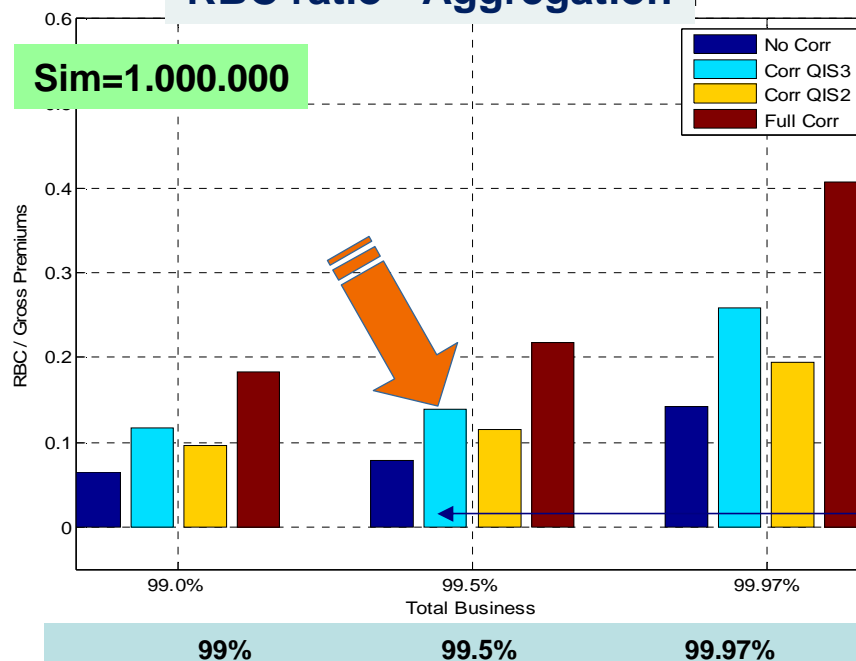
Finally in the extreme case of **full correlation** the ratio is rising to 21.76% (18.33% and 40.81% for the other two confidence levels).

RBC ratio - Aggregation

Sim=1.000.000

	rbc ratio		
	99%	99,50%	99,97%
NoCorr	6,51%	7,96%	14,21%
Corr QIS3	11,63%	13,96%	25,87%
Corr QIS2	9,61%	11,49%	19,43%
Full Corr	18,33%	21,76%	40,81%

Company TAU
RBC ratios according to correlation

Sim =1.000.000

RBC ratio

Aggregation

| No Corr |
| Corr QIS3 |
| Corr QIS2 |
| Full Corr |
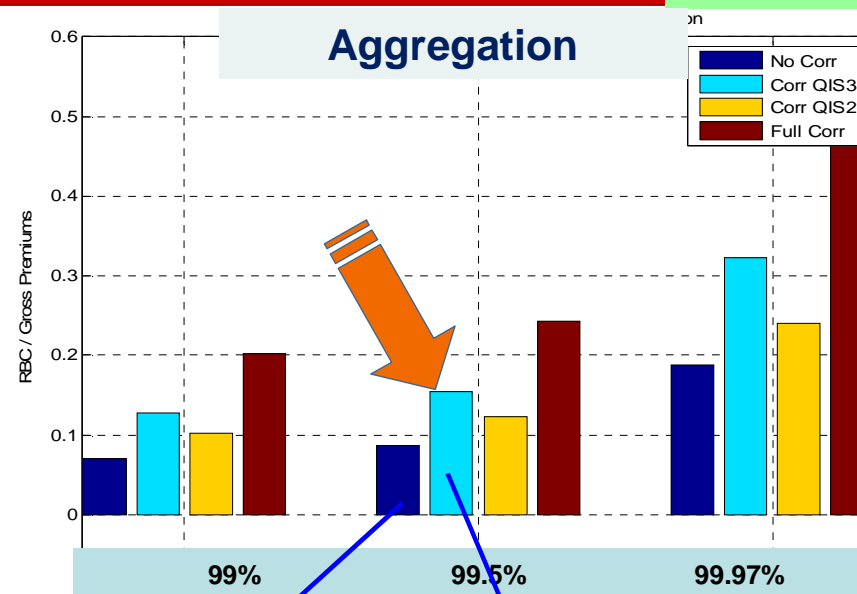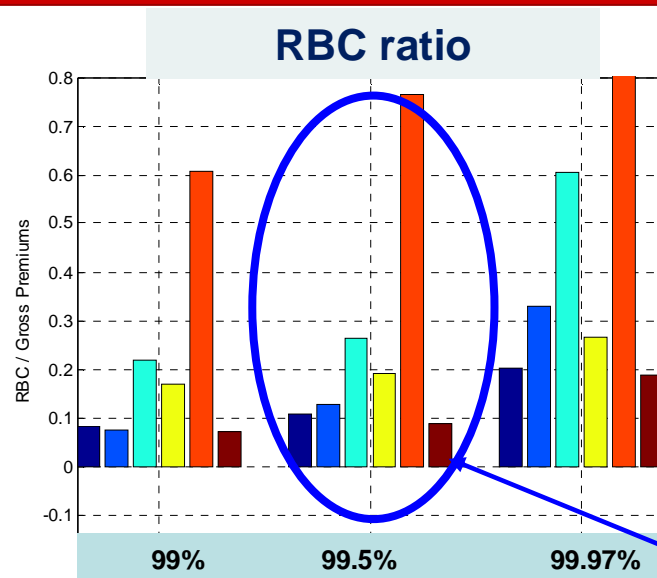
8.68 %
(no correlation)

Accident	10,78%
Motor Damages	12,69%
Property	26,35%
MTPL	18,99%
GTPL	76,51%
Aggregate	8,68%

	rbc ratio		
	99%	99,50%	99,97%
NoCorr	7,06%	8,68%	18,82%
Corr QIS3	12,75%	15,53%	32,32%
Corr QIS2	10,24%	12,32%	23,99%
Full Corr	20,17%	24,39%	50,35%

Company TAU HIGH
RBC ratios according to correlation

Sim=1.000.000

RBC ratio

| 99% | 99.5% | 99.97% |

Aggregation

Legend: No Corr, Corr QIS3, Corr QIS2, Full Corr

| 99% | 99.5% | 99.97% |

**10.53 %
(no correlation)**
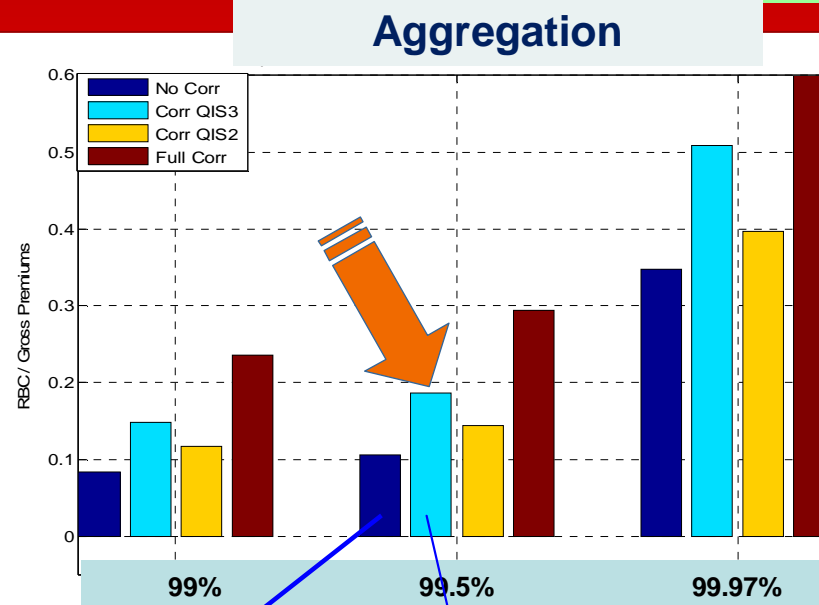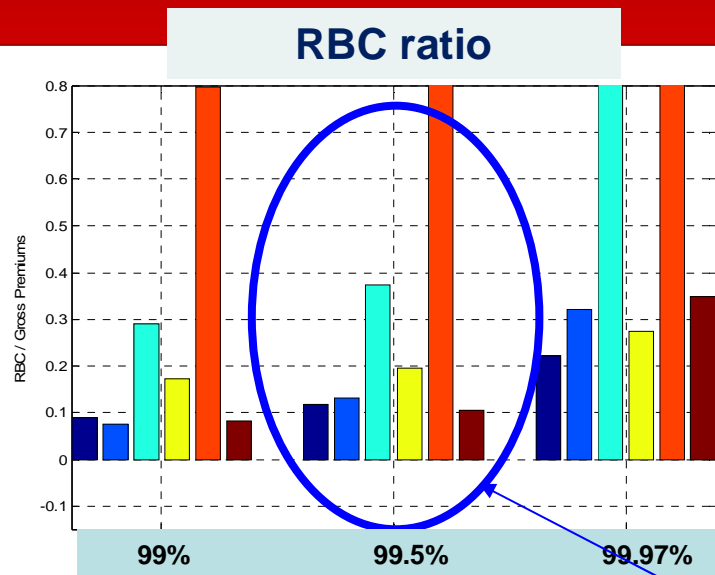
Accident	11,71%
Motor Damages	12,99%
Property	37,35%
MTPL	19,52%
GTPL	106,53%
Aggregate	10,53%

	rbc ratio		
	99%	99,50%	99,97%
NoCorr	8,32%	10,53%	34,79%
Corr QIS3	14,89%	18,69%	50,86%
Corr QIS2	11,65%	14,36%	39,63%
Full Corr	23,52%	29,46%	74,66%

Company EPSILON
RBC ratios according to correlation

Sim=1.000.000

RBC ratio

Aggregation

14.76 %
(no correlation)
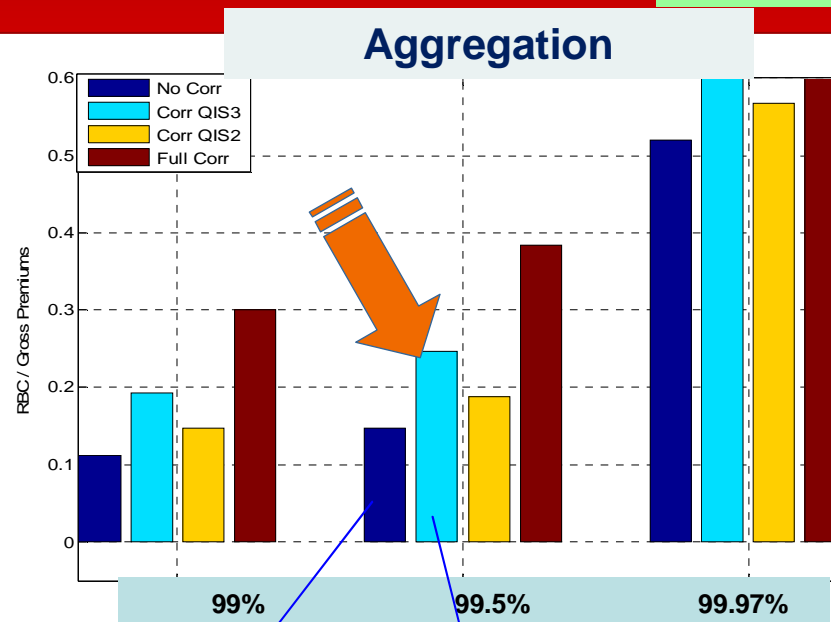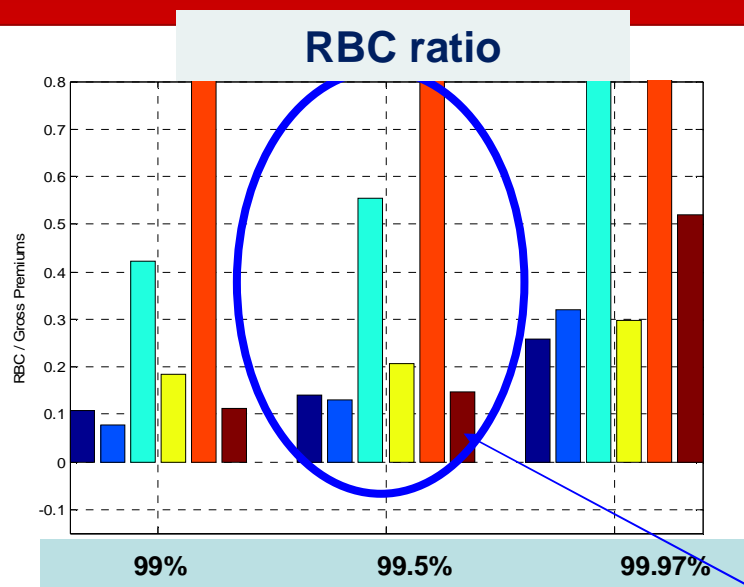
Accident	13,91%
Motor Damages	13,04%
Property	55,34%
MTPL	20,78%
GTPL	159,08%
Aggregate	14,76%

	rbc ratio		
	99%	99,50%	99,97%
NoCorr	11,21%	14,76%	51,97%
Corr QIS3	19,23%	24,73%	70,96%
Corr QIS2	14,72%	18,74%	56,79%
Full Corr	30,03%	38,34%	100,91%

Combined Ratios (Total LoB)
for the examined Insurers

Comp. OMEGA

Mean = 94.86%
Std   =   4.64%
Skew =   +0.25
Kurt   =   4.27

Comp. EPSILON

Mean = 94.86%
Std   =   6.13%
Skew =   +3.68
Kurt   = 150.28

Independence

Comp. TAU

Mean = 94.86%
Std   =   4.83%
Skew =   +0.70
Kurt   =   18.57

Comp. TAU HIGH

Mean = 94.86%
Std   =   5.24%
Skew =   +1.79
Kurt   =   67.23
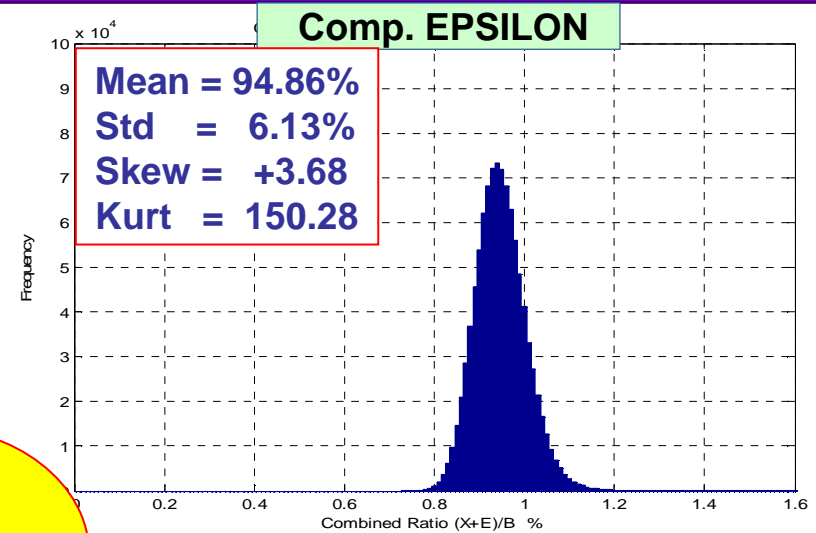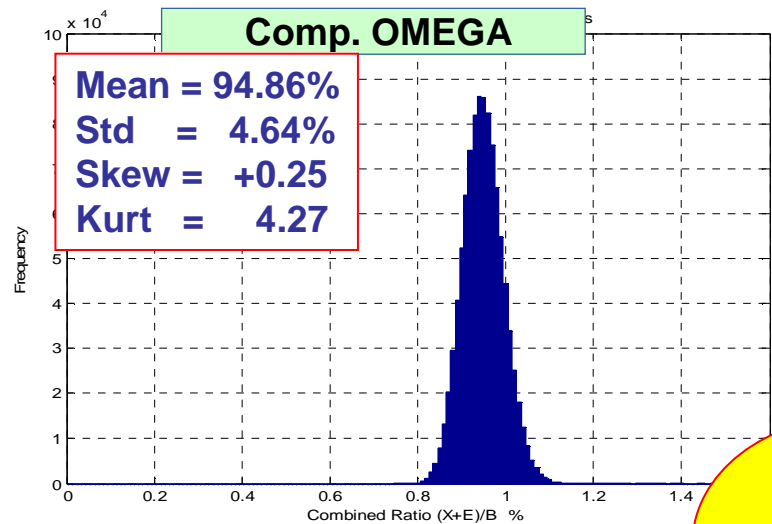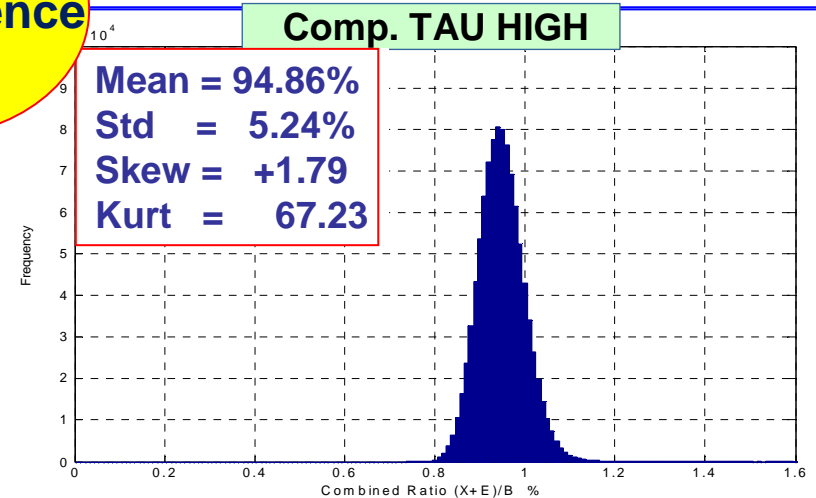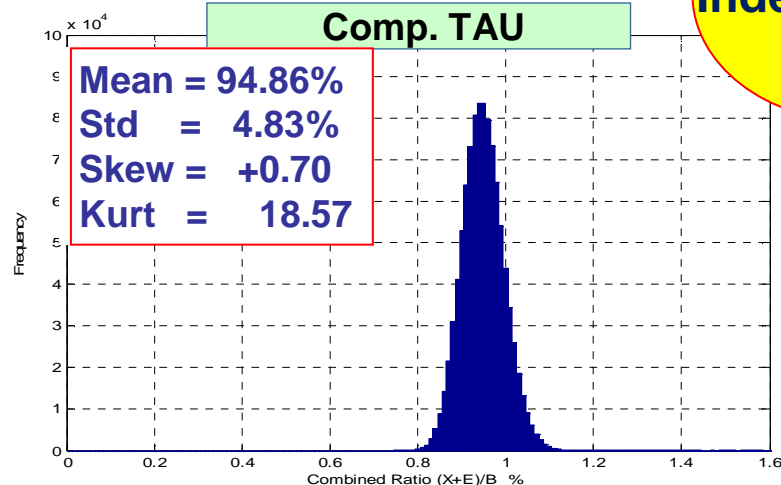
A summary of RBC ratios

Main results are summed up:

RBC ratio 99.5%

	OMEGA	TAU	TAUHIGH	EPSILON
Accident	10,40%	10,78%	11,71%	13,91%
Motor Damages	12,47%	12,69%	12,99%	13,04%
Property	21,82%	26,35%	37,35%	55,34%
MTPL	18,84%	18,99%	19,52%	20,78%
GTPL	58,39%	76,51%	106,53%	159,08%
Aggregate (Independence)	7,96%	8,68%	10,53%	14,76%

- RBC ratio at 99.5% confidence level **for each LoB** and for company under **independence assumption**

Aggregate RBC ratio 99.5%

	OMEGA	TAU	TAUHIGH	EPSILON
NoCorr	7,96%	8,68%	10,53%	14,76%
Corr QIS3	13,96%	15,53%	18,69%	24,73%
Corr QIS2	11,49%	12,32%	14,36%	18,74%
Full Corr	21,76%	24,39%	29,46%	38,34%

- Aggregate RBC ratio at 99.5% level **under different dependence assumptions**

RBC ratio under Correlation (QIS3)

	OMEGA	TAU	TAUHIGH	EPSILON
RBC ratio 99%	11,63%	12,75%	14,89%	19,23%
RBC ratio 99.5%	13,96%	15,53%	18,69%	24,73%
RBC ratio 99.97%	25,87%	32,32%	50,86%	70,96%

- Aggregate RBC ratio with linear correlation (QIS3 Correlation Matrix) **for different confidence levels**

Premium Risk:
Standard Formula in QIS3

- Some analyses to assess **the impact of the Standard Formula proposed in QIS3** are performed restricted to the Premium Risk.

- The Premium Risk could be estimated according two different approaches, the first based on a **market-wide approach** and the second one taking into account the specific technical data of the company by the loss ratios (**undertaking-specific approach**) :

$$NL_{pre}^{mark} = \rho(\sigma_M) \cdot P \quad and \quad NL_{pre}^{us} = \rho(\sigma_U) \cdot P$$

P: Next year net premium volume

- $\rho(X)$ is the 99,5% VaR of a probability distribution with standard deviation x:

$$\rho(x) = \frac{\exp(N_{0.995}\sqrt{\log(x^2+1)})}{(x^2+1)} - 1$$

$N_{0.995}$: 99,5% quantile of the Normal distribution

- The overall volatility (σ) is obtained joining the single-LoB volatility with a correlation matrix.

$$\sigma = \sqrt{\frac{1}{P}\sum_{r \, x \, c} CorrLoB^{rxc} \cdot P_r \cdot P_c \cdot \sigma_r \cdot \sigma_c}$$

Premium Risk:
Market Wide vs Undertaking Specific

- The differences between market-wide and undertaking-specific approach are noticeable **in the single-LoB volatility valuation**. Market Approach <u>is based on a market-wide estimate of the standard-deviation for premium risk, obtained by a specific volatility factor given as input by CEIOPS:</u>

Premium Risk (QIS3 Standard Formula)	Accident	Mot. Damages	Property	MTPL	GTPL
Volatility Factor	5%	10%	10%	10%	10%

- The undertaking-specific estimate of the standard deviation for premium risk is determined on the basis of the volatility of historic loss ratios. **In this second approach the volatility for premium risk in the individual LoB is derived as a credibility mix of the undertaking-specific estimate and of the market-wide estimate as follows:**

$$\sigma_{U,lob} = \sqrt{c_{lob}\sigma^2_{LR,lob} + (1 - c_{lob})\sigma^2_{M,lob}}$$

- The credibility factor depends on number of loss ratios. **If insurer has all 15 loss ratios requested by CEIOPS, c_{lob} will be almost 79%.**

$$\begin{cases} c_{lob} = \dfrac{n_{lob}}{n_{lob}+4} & if \ n_{lob} \geq 7 \\ c_{lob} = 0 & otherwise \end{cases}$$

- We refer **to the same 4 theoretical companies having 5 LoBs**.

- To those data, related mainly to premium and claims, **some data are now added concerning the historical series of the loss ratios**.

- Different historical patterns are assumed for these 4 insurers **with the aim to compare consistently Internal Model results and undertaking-specific approach of Standard Formula** and <u>to consider that a smaller company would obviously report a more volatile distribution of the loss ratios</u>.

- Finally it is to be emphasized **that Market-Wide Standard Formula gives the same RBC ratio for all insurers because of the lack of a size factor** (as well known in **QIS2 the presence of size factors** increased the RBC ratios for small size insurers).
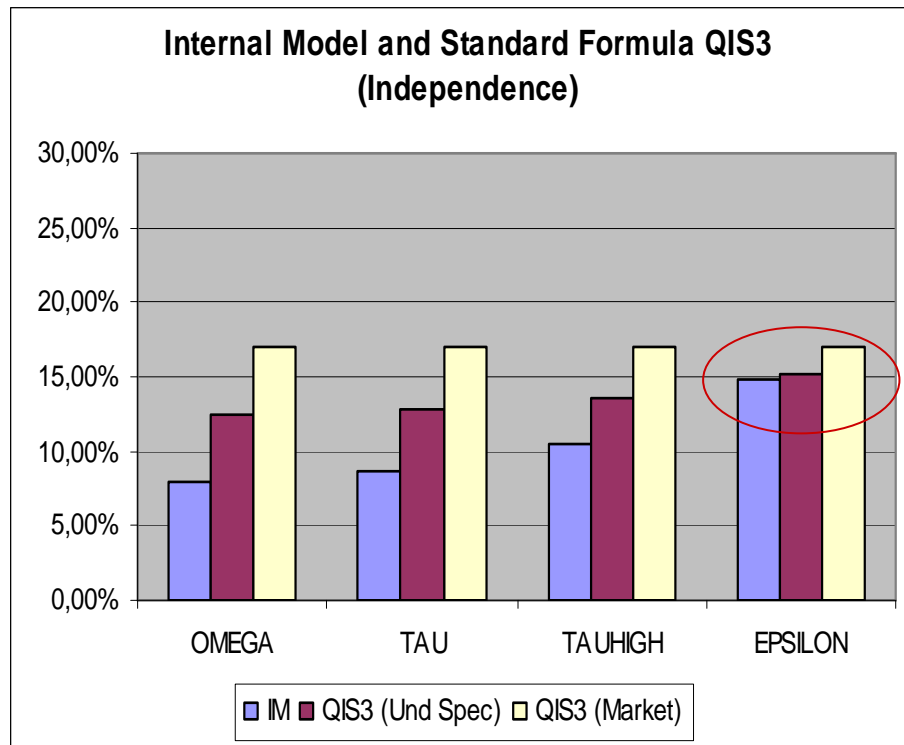
Loss Ratio Patterns
(MTPL and GTPL)

- In particular, line by line loss ratio patterns for each company are determined with the double assumptions that:
  - **the mean of last 3 Loss Ratios and the standard deviation of last 15 Loss Ratios coincide with the exact mean and standard deviation obtained under the Compound Mixed Poisson Process**.

- Hence, we have four different patterns for each LoB (here only MTPL and GTPL are figured out), where OMEGA Company has the behavior more similar to market data:
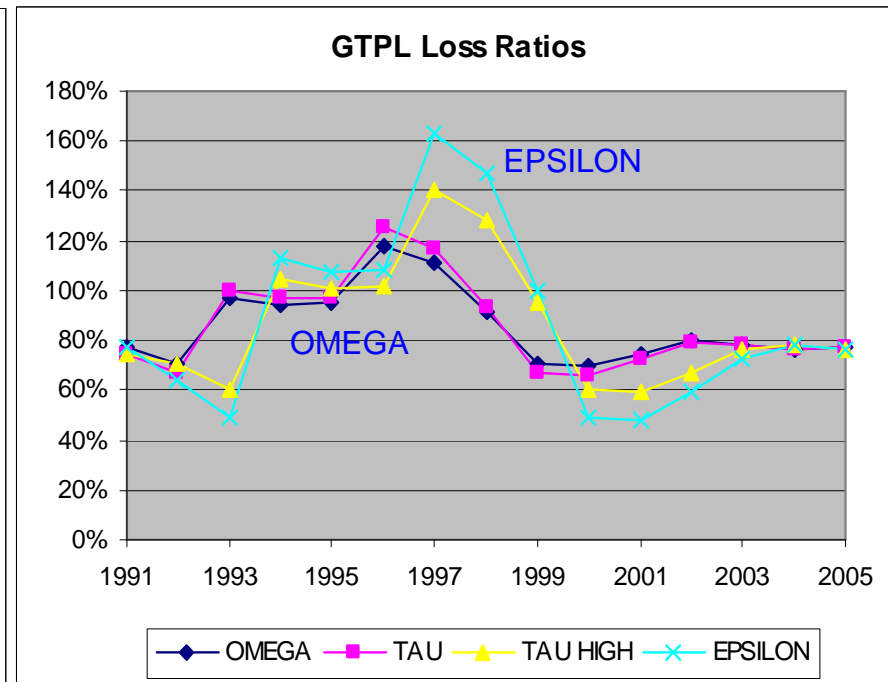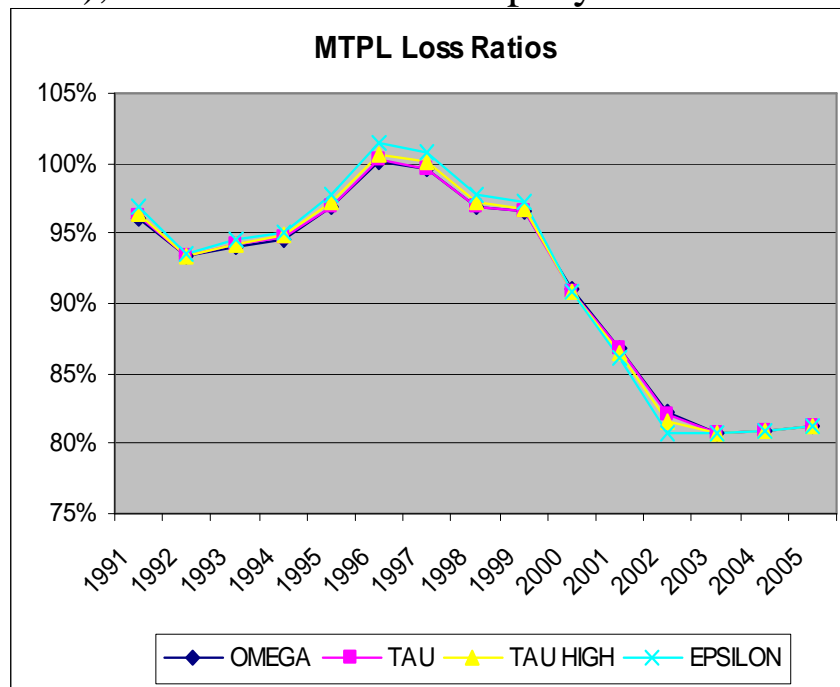
Internal Model vs Standard Formula
(Independence Assumption)

Internal Model and Standard Formula QIS3 (Independence)

The differences **are due to some assumptions of the Standard Formula**:

a. **the credibility factor, with 15 loss ratios, is less than 100%.** Then the higher market-wide volatility factors have some impact on the undertaking-specific approach too;

b. **SCR QIS3 formula does not take into account the technical expected profits/losses** while Internal Models regard safety loading in Risk Based Capital as a reducing factor;

c. **QIS3 Aggregation Formula considers less than Internal Model the diversification effect under independence assumptions**. In fact, Internal Model determines the Capital Requirement on the Aggregate Claims distribution, while QIS3 derives it, joining single-line Capital Charges by an approximation formula;

d. **moreover the "QIS3 $\rho(x)$ transformation" is calibrated with the assumption of a LogNormal distribution for the aggregate claims**. This assumption produces a standard deviation multiplier underestimated for small or highly variable LoBs

	OMEGA	TAU	TAUHIGH	EPSILON
$(IM^{IND}/SCR^{IND}_{PRE,U.S.})-1$	-36,5%	-32,4%	-22,4%	-2,2%

Difference c)

The next figure shows how **QIS3 Aggregation Formula** *(SCR^{IND}* obtained by Matr. Corr. Formula using individual LoB Capital Charges coming from IM) **presents higher RBC ratios than Internal Model** *(SCR^{Agg,Im}* obtained directly by aggregated IM) **for all Companies and confidence levels** *describing only approximately the diversification effect.*

For instance, regarding the target confidence level of 99.50% as to Omega Company the Standard Formula obtains a required ratio of 8.54% instead of 7.96% by Internal Model :

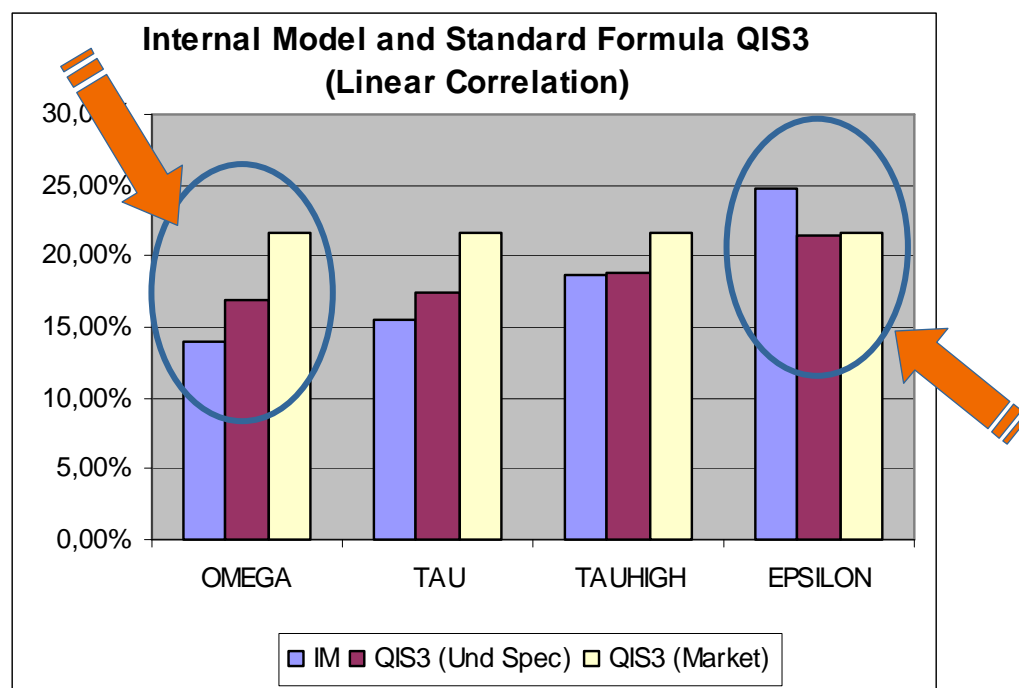|		OMEGA	TAU	TAUHIGH	EPSILON
99%	SCR^{Agg,Im}	6,51%	7,06%	8,32%	11,21%
	SCR^{IND}	6,87%	7,57%	9,04%	12,34%
99,50%	SCR^{Agg,Im}	7,96%	8,68%	10,53%	14,76%
	SCR^{IND}	8,54%	9,59%	11,97%	16,83%
99,97%	SCR^{Agg,Im}	14,21%	18,82%	34,79%	51,97%
	SCR^{IND}	17,84%	23,24%	38,72%	56,51%

- The multiplier obtained from the Internal Model results and under the assumption of LogNormal distribution of aggregate claims are both figured out.

- **For OMEGA Company LogNormal assumption is not so far from frequency-severity results** (i.e. Internal Model) while **in case of EPSILON Company the LogNormal assumption underestimates by far the skewness of aggregate claims obtained by simulations** (0.25 against an exact skewness of 3.68) and it drives to a multiplier lower than Internal Model (2.82 instead of 3.13):

INTERNAL MODEL		OMEGA	EPSILON	OMEGA	EPSILON	LOGNORMAL ASSUMPTION
		$(RBC+\lambda*P)/\sigma(X)$		$(RBC_{LogN}+\lambda*P)/\sigma(X)$		
	Accident	2,84	2,91	2,99	3,04	
	Mot. Damage	3,11	3,10	3,43	3,45	
	Property	2,94	3,84	2,95	3,23	
	MTPL	2,74	2,77	2,83	2,85	
	GPTL	3,35	4,19	3,15	3,91	
	Aggregate	2,74	3,13	2,76	2,82	

Internal Model vs Standard Formula
(Linear Correlation)

**Internal Model and Standard Formula QIS3
(Linear Correlation)**

Bar chart axis values: 0,00% / 5,00% / 10,00% / 15,00% / 20,00% / 25,00% / 30,0...

Categories: OMEGA, TAU, TAUHIGH, EPSILON

Legend: ☐ IM ☐ QIS3 (Und Spec) ☐ QIS3 (Market)

	OMEGA	TAU	TAUHIGH	EPSILON
$(IM/SCR_{PRE,U.S.})-1$	-17,3%	-11,3%	-0,8%	15,7%

✓ In case of linear correlation, **Internal Model results show again** (but not for Epsilon Company) **a lower ratio than Standard Formula**

✓ The Internal Model capital reduction is less than **the independence case: Omega Company obtains a decreasing of 17% respect to undertaking-specific approach (it was 36.5% in case of independence)** and Tau Company reduces the requirement of only 11%.

✓ Finally it is worth to point out how Internal Model approach obtains a higher Capital Requirement than by the undertaking-specific for Epsilon Company, **mainly due to the inappropriate use of the LogNormal distribution in the Standard Formula for a small size company**

The Aggregation formula

- The different way to consider diversification effect, between Internal Model and Standard Formula, has a lower impact in correlation case.

- Moreover, formula, used with the aim to determine RBC ratio by Internal Model under dependence assumptions, **represents indeed an approximation formula defined in a similar way than QIS3 Aggregation Formula**.

- Under these assumptions (dependence), **Internal Model and QIS3 Standard Formula aggregation structures describe almost in the same way the diversification impact**.

- Some analyses show that the "aggregation formula" gives Capital Requirement not so far from the RBC obtained using the multivariate aggregate claims distribution with a dependence structure described by a **Gaussian Copula**. Obviously. If another copula function is used, the differences arising from aggregation will result quite significant (and then less comparable) because of the tail dependence.

Aggregate RBC ratios by Copulas
(Company OMEGA)

RBC ratio (Omega Company)

Legend: Linear Correlation, t-Student (30), t-Student (3), Gaussian
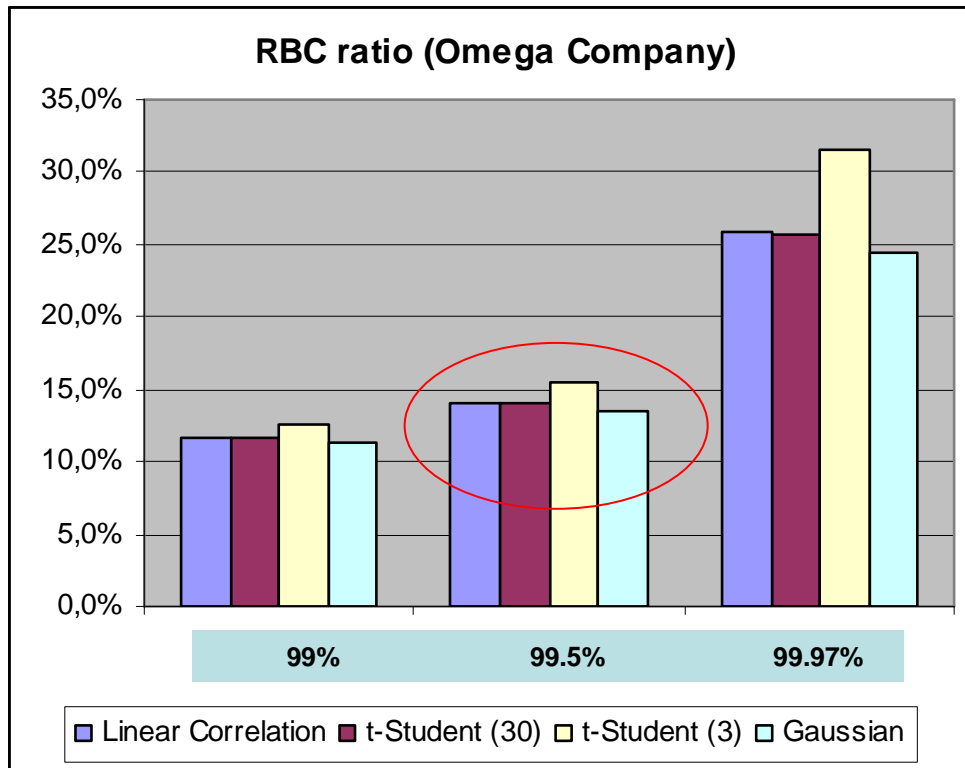
- It is compared RBC ratio obtained by Internal Model and by different dependence assumptions.

- For the Omega Company, it can be observed how **Gaussian Copula** gives rise to a lower Capital Requirement than linear correlation.

- **t-Student**, with few degrees of freedom **(3)**, **presents a high dependence on both tails**, causing a skewed Aggregate Claims (0.45 against 0.33 under independence) and the highest RBC ratio (15.5% at 99.5% level). These results confirm the positive tail dependence assumed by Student Copula.

- Finally a **t-Student Copula, with 30 degrees** of freedom, has a distribution rather close to a Gaussian (tail dependence decreases for raising degrees of freedom) and shows results similar to linear correlation.

Aggregate RBC ratios by Copulas
(4 Insurers)

- Moving to the others companies, the results are almost the same.

- **Gaussian** shows the lowest Capital and **Student Copula with 3** degrees has the highest RBC ratio.

- **t-Student, with 30 degrees** of freedom, presents the same RBC ratio of linear correlation for medium-large size companies, while it has lower Capital Requirements than linear correlation for small or highly variable Companies (Epsilon and Tau High)

	Linear Correlation	t-Student (30)	t-Student (3)	Gaussian
OMEGA	14,0%	14,0%	15,5%	13,5%
TAU	15,5%	15,5%	17,1%	14,9%
TAU HIGH	18,7%	18,3%	20,5%	17,9%
EPSILON	24,7%	24,1%	26,8%	23,8%

Some remarks on QIS4

- As well known CEIOPS has published on 31 March 2008, the Technical Specifications to be used for the 4th Quantitative Impact Study.

- Focusing our attention only above **Underwriting Risk Non-Life module**, QIS4 has developed a modular structure like QIS3. Some Premium Risk parameters and some formulas has been modified:
  - LoBs segmentation is the same as the segmentation applied in QIS3 valuation, **excluding health and accident**, which for the purpose of SCR calculation are treated in Underwriting Risk Health module with the same formulation as Non-Life LoBs.
  - Premiums and provisions should be allocated between different geographical areas. It's calculated, for each line of business, the Herfindahl index **quantifying the diversification effect** only if undertaking has less than 95% of its non-life activities in the same geographical area.
  - **The market-wide estimate** of the standard deviation for premium risk **has been modified** (from 10% to 9% for MTPL and Motor Damages, and from 10% to 12.5% for GPTL)
  - **Companies can not use more than 5 loss ratios for Property, Motor Damages and Accident**. Furthermore the credibility factor depends on number of Loss Ratios, but the volatility will never be determined only using the standard deviation of loss ratio (if undertaking has the maximum number of Loss Ratios, it should calculate the standard deviation using a credibility factor equal to 0.79 like in QIS3)

QIS4 Stand. Formula: some results

Using undertaking-specific approach, QIS4 gives lower standard deviation than QIS3, except for GTPL line. In fact, the credibility mix with lower volatility factor and the behaviour of last five Loss Ratios reduce undertaking-specific standard deviation and Capital Requirement.

Figure shows the RBC ratios for all 4 Companies with QIS3 and QIS4 Standard Formula. **For large Insurer QIS4 shows again higher Capital Requirement than Internal Model**. Companies with small dimension or with high variability gives RBC ratio higher than Internal Model.

	QIS3	QIS4
OMEGA	16.88%	14.63%
TAU	17.51%	15.14%
TAU HIGH	18.85%	16.22%
EPSILON	21.37%	18.25%

However it could be observed that **it is not appropriate a full comparison** between QIS4 Standard Formula and Internal Model results.
In fact undertaking-specific approach uses only five Loss Ratios for some LoBs, while Internal Model parameters have been calibrated considering all 15 Loss Ratios with a higher standard deviation

Final Comments

- The use of Internal Models **show significant reduction of required capital for large and medium size companies**.

- It can be observed how some Standard Formula assumptions can conduct at a **similar requirement** between Undertaking-Specific and Internal Model **for small and highly variable companies**.

- It should be emphasized that the Risk Theoretical Model here applied is only a simplified version of the complex practical risk management process and furthermore all valuations have been made **without** considering **reinsurance**. It is worth to emphasize how reinsurance (XL in particular) should have a high impact on aggregate claims variability and on safety loading, with a more flat scale of capital requirements according to company size.

- Finally, when simulation models are used **great attention need to be paid to avoid as much as possible the three classical modelling risks** (model/parameter/process risk).

  In particular **the risk of assessing inappropriate parameters, used in the model, plays a relevant role for the high impact of some parameters** on Capital Requirement. For example, it could be useful **to introduce a structure variable on claim size distribution too** with the aim to consider the parameter uncertainty.

  The **calibration of these systematic parameters** are crucial in order to get appropriate results by IM.

- For a full comparison with Standard Formula, **reserve risk** will be also introduced in future improvements of the model, having regard to the large capital required by QIS3 (approx. 35-40% of premium volume).

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Thank you for the attention !