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MODELLING AND MEASURING UNCERTAINTY IN MORTALITY FORECASTS AND LEE CARTER

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LEE CARTER STRUCTURE

Note wide acceptance and use of Lee Carter framework e.g. benchmark model used by US Bureau of Census

log (mortality rate) = predictor + error

predictor = age effect + (age effect) x (time effect)

$= \alpha_x + \beta_x \kappa_t$

with identifiability constraints $\sum \kappa_t = 0, \quad \sum \beta_x = 1$

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FITTING

Singular Value Decomposition

Iteration based on weighted least squares

Maximum likelihood with Poisson errors

with adjustments to $\hat{\kappa}_t$ so that

(observed deaths)$_t$ = (predicted deaths)$_x$

GOODNESS OF FIT

- Residuals analysis
  Model enhancements to improve fit
  - Second order terms
  - Cohort effects (see later).

FORECASTING

Time series (ARIMA) models fitted to $\{\hat{\kappa}_t\}$ and then projected

$$\dot{m}_{x,t_n+s} = \hat{m}_{x,t_n} \exp\{\hat{\beta}_x(\dot{\kappa}_{t_n+s} - \hat{\kappa}_{t_n})\},\ s > 0$$

COHORT VERSION

Predictor = age effect + (age effect) x (cohort effect) + (age effect) x (time effect)

$$= \alpha_x + \beta_x^{(0)} \iota_{t-x} + \beta_x^{(1)} \kappa_t$$

There is a problem because cohort = (period – age).
And so we require a two-stage fitting strategy, in which α_x is estimated first, as in the basic Lee Carter Model.

(Data requirements: single year of age data).

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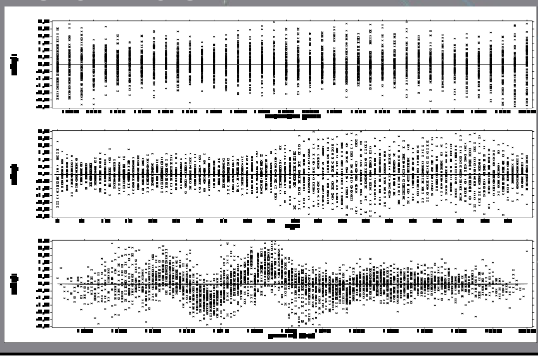
UK FEMALE MORTALITY EXPERIENCE (LC) – RESIDUAL PLOTS



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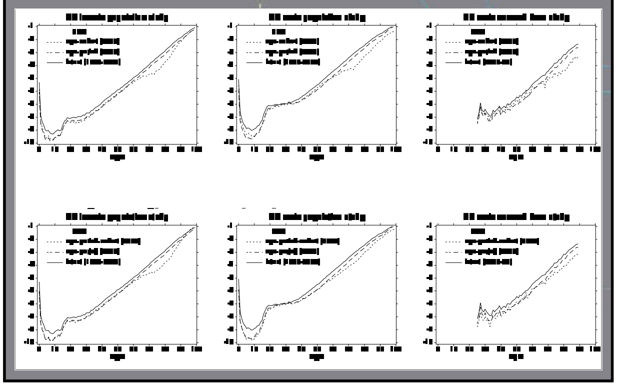
LOG (MORTALITY RATES): PROJECTIONS



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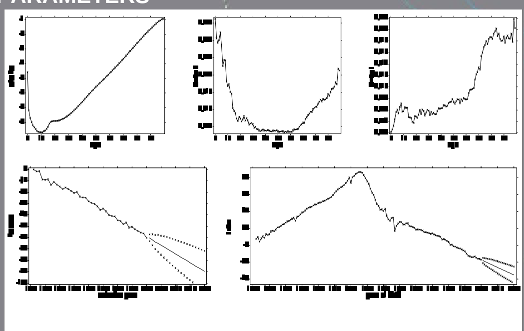
UK FEMALE POPULATION: MODEL M PARAMETERS



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COMMENT ON LEE CARTER AND P SPLINES

Eilers et al (2006): In Lee Carter methods "fitting the data and extrapolating past trends are kept separate… In the authors' opinion, this is an advantage for actuarial applications, since it allows for more flexibility."

Note: Eilers is one of the original advocates of the P Splines methodology.

a) What to do next?
b) Use Lee Carter and P Splines.

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Bayesian Models

Advantages:

- Includes estimation error and process error
- Can incorporate model error
- Enables particular portfolio or risk to be studied
- Flexibility

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Bayesian Lee-Carter

Use the Poisson bilinear model:

$$\mu_{x,t} = \exp\left(\alpha_x + \beta_x \kappa_t\right)$$

Non-informative prior distributions for α_x and β_x with $\sum_x \beta_x = 1$

$$\kappa_t \sim N\left(m_t, \sigma_1^2\right)$$

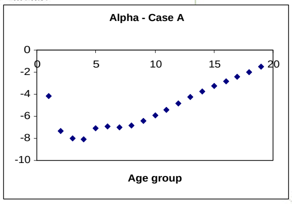
$$m_t \sim N\left(m_{t-1} + b_t, \sigma_2^2\right)$$

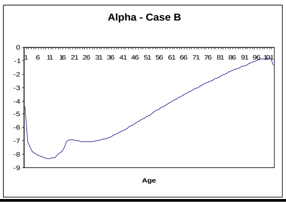
$$b_t \sim N\left(b_{t-1}, \sigma_3^2\right)$$

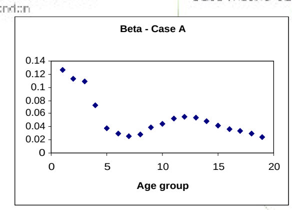
Case A: England and Wales Male Mortality Experience, 1950-1998, inclusive, with a age grouped and classified as {<1, 1-4, 5-9, 10-14, ., 80-84, 85+}.

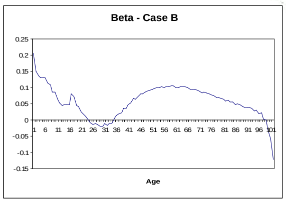
Case B: UK mortality experience, 1961-2003 inclusive, with age classified by individual year, 0-103 inclusive.

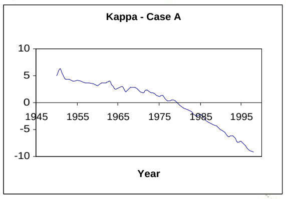
Case C: CMI data for female life office pensioners, 1983-1996 inclusive, with age classified by individual year, 60-95 inclusive.

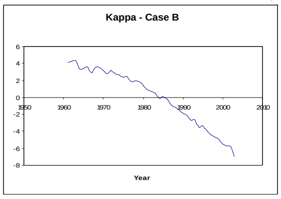


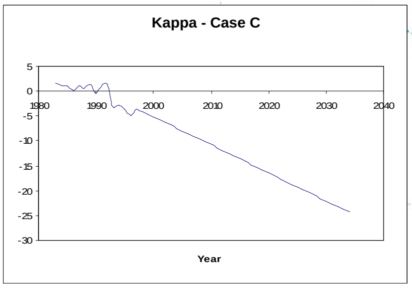












Annuity value

It is possible to look at a wide range of random variables, such as a single life, a portfolio of lives, etc.

Consider a single female life aged 60, IA 6%