MODELLING MULTI-POPULATION MORTALITY WITH COHORT EFFECTS

Andrew Cairns

Heriot-Watt University, Scotland

and

The Maxwell Institute, Edinburgh

Joint work with

David Blake, Kevin Dowd, Guy Couglan and Marwa Khalaf-Allah

Plan

- Aims
- Period & cohort effects: stylised facts
- 2-population APC model plus MCMC
- Case studies
  - England and Wales versus CMI assured lives
  - Males
  - Females

Aims

- Principles underpinning 2-populations modelling
- Method for dealing with
  - small sub-populations
  - missing data
- short- and medium-term assessment of basis risk

  $\Rightarrow$ e.g. compare bespoke longevity swap versus index-based hedge

Motivation for two-population modelling

A: Risk assessment

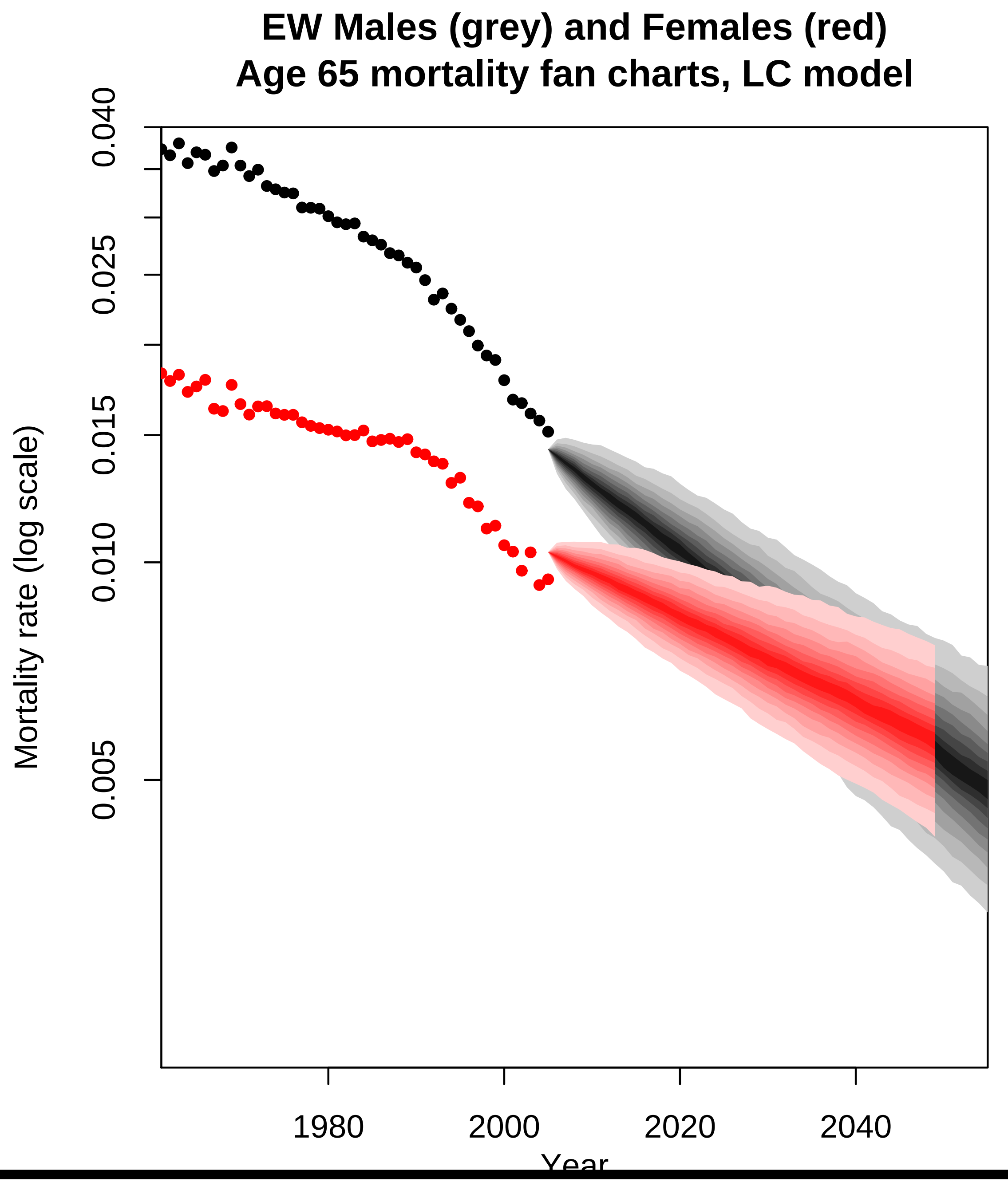
- Males/Females (e.g. consistent demographic projections)
- Blue/White collar (socio-economic)
- Smokers/Non-smokers
- UK/Europe (e.g. consistent demographic projections)
- Annuities/Life insurance
- Limited data $\Rightarrow$ learn from other populations

Motivation for two-population modelling

B: Risk management for pension plans and insurers

- Retain systematic mortality risk; versus:
- ‘Over-the-counter’ deals (e.g. longevity swap)
  - own experience $\Rightarrow$ 100% risk reduction
  - expensive
- Standardised mortality-linked securities
  - linked to national mortality index
  - $< 100\%$ risk reduction
  - less expensive

The problem with single population projections



Two populations

- Linked in some way
- But not identical
- Desire for consistent forecasts
  - distributions
  - individual future scenarios

Measures of mortality

- Crude death rate

$$m(t,x) = \frac{\text{\# deaths in } [t, t+1) \text{ age } x \text{ last birthday}}{\text{avg. population in } [t, t+1) \text{ age } x \text{ last birthday}}$$

Age-Period-Cohort model (APC)

$$\log m(t,x) \;=\; \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_{t-x}^{(3)}$$

- $N = 3$ components
- Origins in medical statistics
- $\beta_x^{(1)}$ age effect
- $\kappa_t^{(2)}$ single random period effect
- $\gamma_{t-x}^{(3)}$ single random cohort effect

Key hypothesis

- $m_1(t, x) =$ pop. 1 death rate in year t at age x
- $m_2(t, x) =$ pop. 2 death rate in year t at age x
- Hypothesis (e.g. Li and Lee, 2005):

  For each age x, $\dfrac{m_1(t, x)}{m_2(t, x)}$ does not diverge over time

Age-Period-Cohort model (APC)

$m_k(t,x) =$ population k death rate

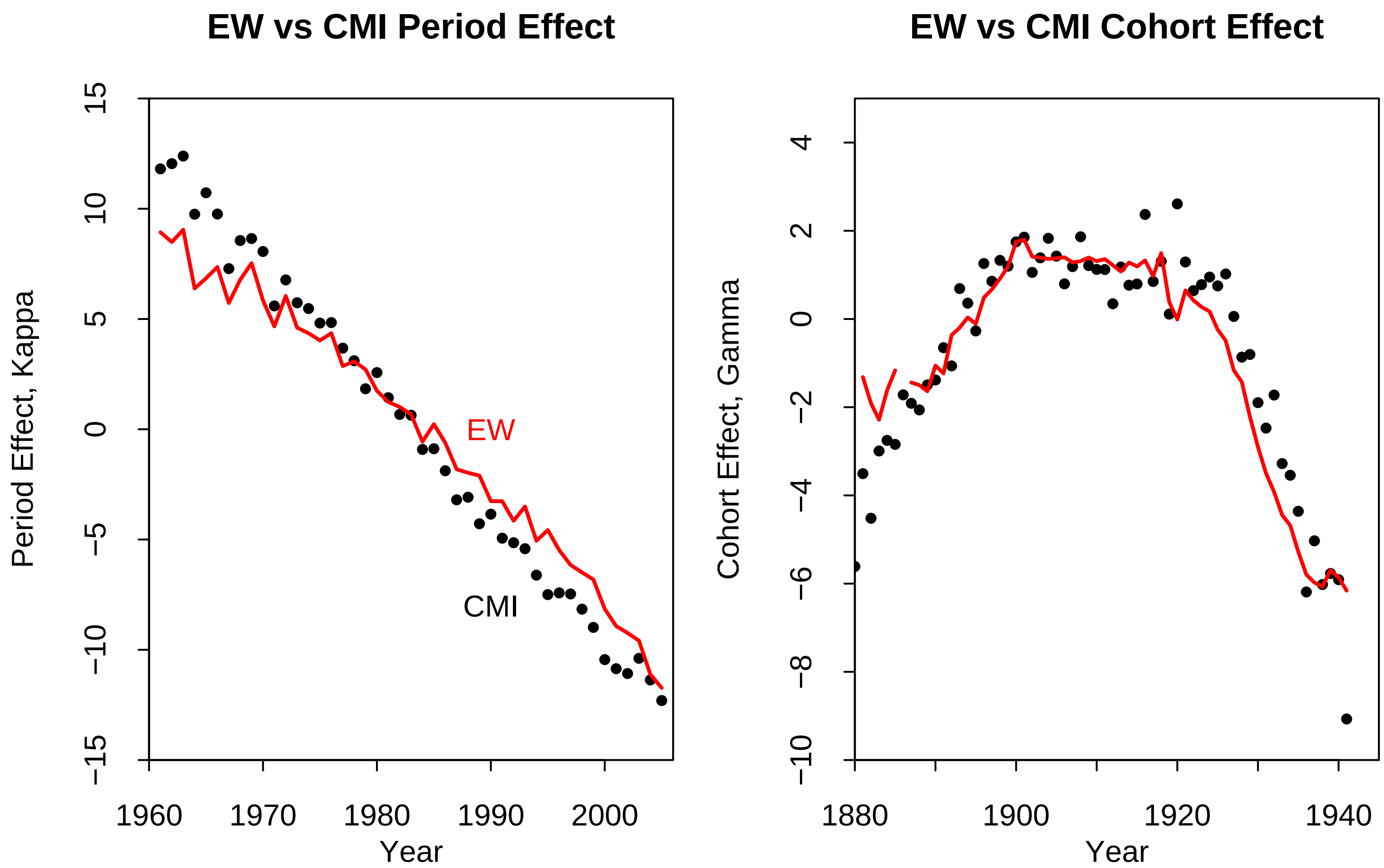
$n_a =$ number of age groups

$$\log m_k(t,x) = \beta^{(1k)}(x) + n_a^{-1}\kappa^{(2k)}(t) + n_a^{-1}\gamma^{(3k)}(t-x)$$

Hypothesis $\Rightarrow$

- $\kappa^{(21)}(t) - \kappa^{(22)}(t)$ mean reverting
- $\gamma^{(31)}(t-x) - \gamma^{(32)}(t-x)$ mean reverting

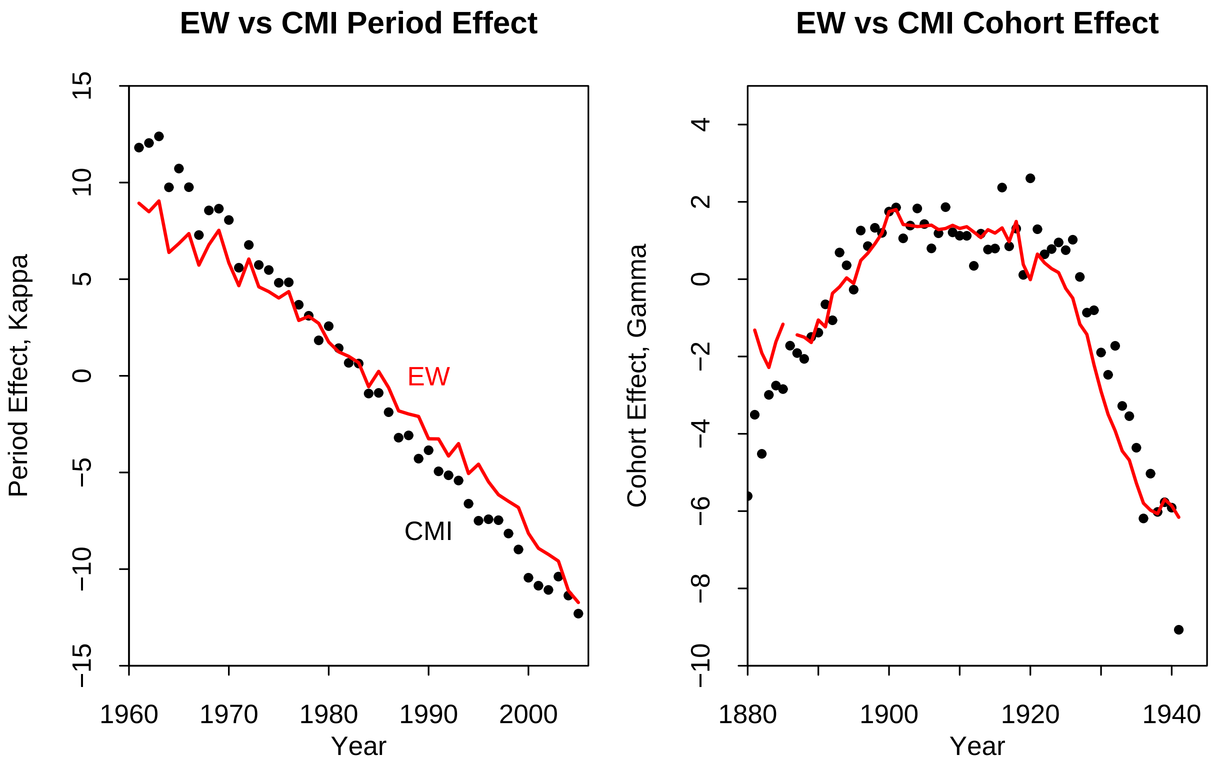
EW versus CMI (assured lives) males



Source: Output from LifeMetrics software

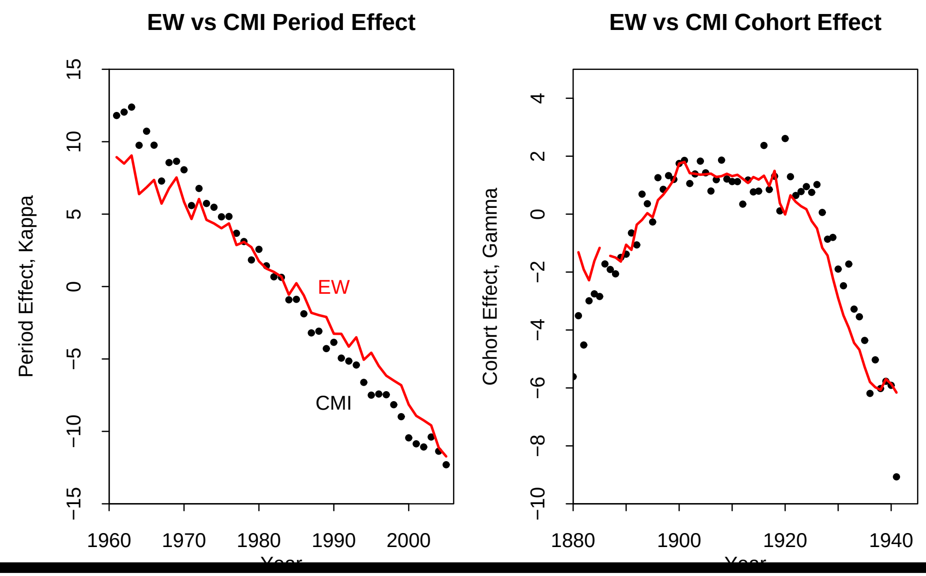
Stylised facts: period effects

- Annual innovations in $\kappa^{(21)}(t)$ and $\kappa^{(22)}(t)$: moderate correlation
- Limited data $\Rightarrow$ proving mean reversion difficult



Stylised facts: cohort effects

- Annual innovations in $\gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$ NOT highly correlated
- Longer-term shapes of $\gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$ very similar
- Small population 2 $\Rightarrow \gamma^{(32)}(c)$ noisy



Stylised facts: inferences

- “True” $\kappa^{(21)}(t)$ and $\kappa^{(22)}(t)$ incorporate significant correlated randomness

Stylised facts: inferences

- “True” $\gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$:
  - Relatively smooth in the short term
    - Accumulation over lifetime of environmental factors
    - cohort-related lifestyle e.g. smoking
  - Stochastic trends
  - (+ hypothesis) Mean reverting spread
- Estimated $\gamma^{(3k)}(c)$ affected by Poisson noise

A 2-population model (one large, one small)

- $\beta^{(11)}(x),\ \ \beta^{(12)}(x)$: no model
- Large population 1
  - $\kappa^{(21)}(t)$: random walk with drift
  - $\gamma^{(31)}(c)$: AR(2) around linear drift ($\longrightarrow$ ARIMA(1,1,0))
- Spreads:
  - $S_2(t) = \kappa^{(21)}(t) - \kappa^{(22)}(t)$: AR(1)
  - $S_3(c) = \gamma^{(31)}(c) - \gamma^{(32)}(c)$: AR(2)

Small population

- Learn from large population dynamics
- Similar levels of variability
- Similar long-term trends

Bayesian Priors

- Mostly weak uninformative priors

Enhanced priors for the cohort effect

Biological reasonableness $\Rightarrow \gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$ have

- similar short term volatility
- similar long term variability

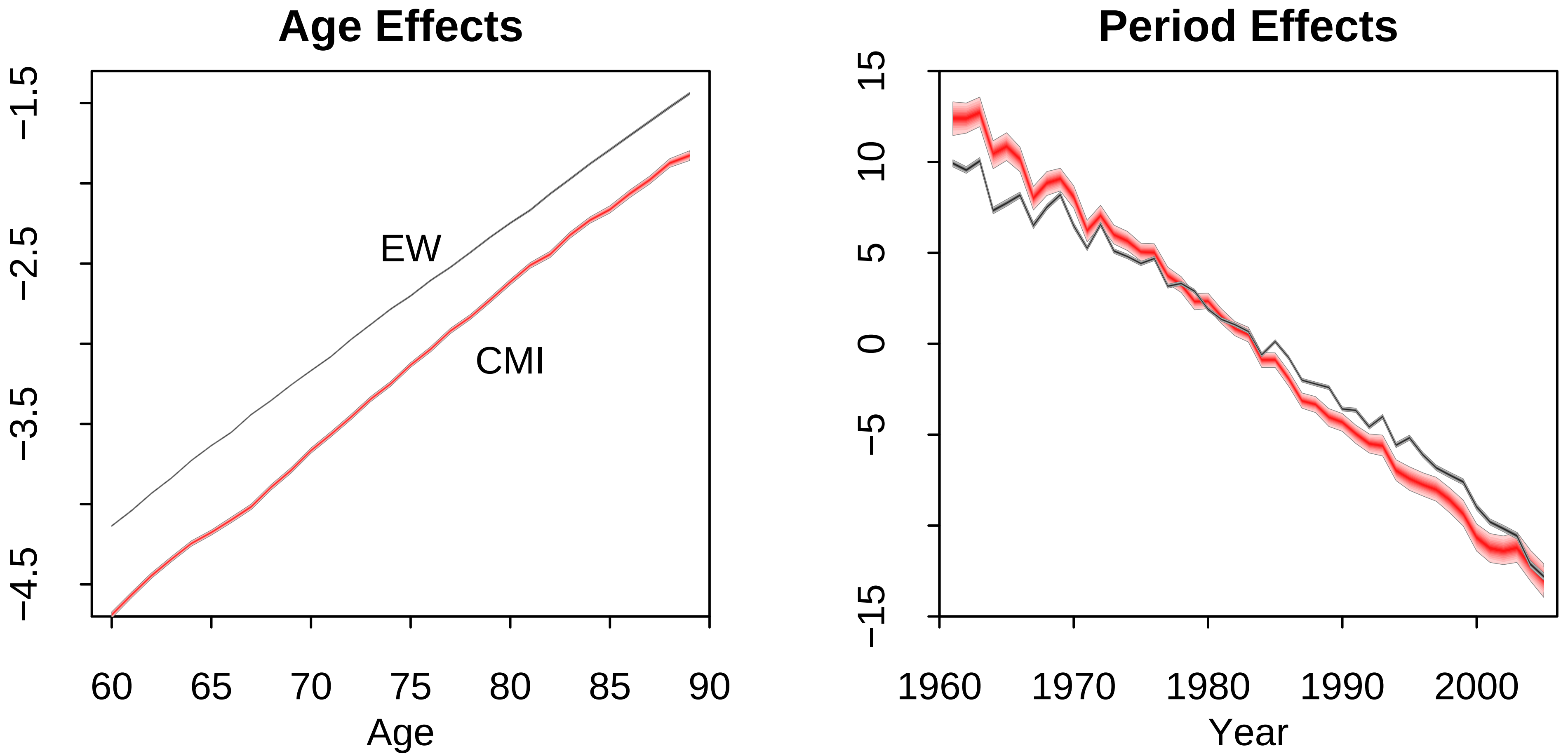
Markov chain Monte Carlo

- $\theta =$ parameter vector
  - process parameters
  - latent processes
    $\beta^{(11)}(x), \beta^{(12)}(x), \kappa^{(21)}(t), \gamma^{(31)}(c), S_2(t), S_3(c)$
- Metropolis-Hastings algorithm
- update elements or blocks of θ
  $\Rightarrow$ Markov chain $\theta(u)$ with stationary dist'n $=$ posterior

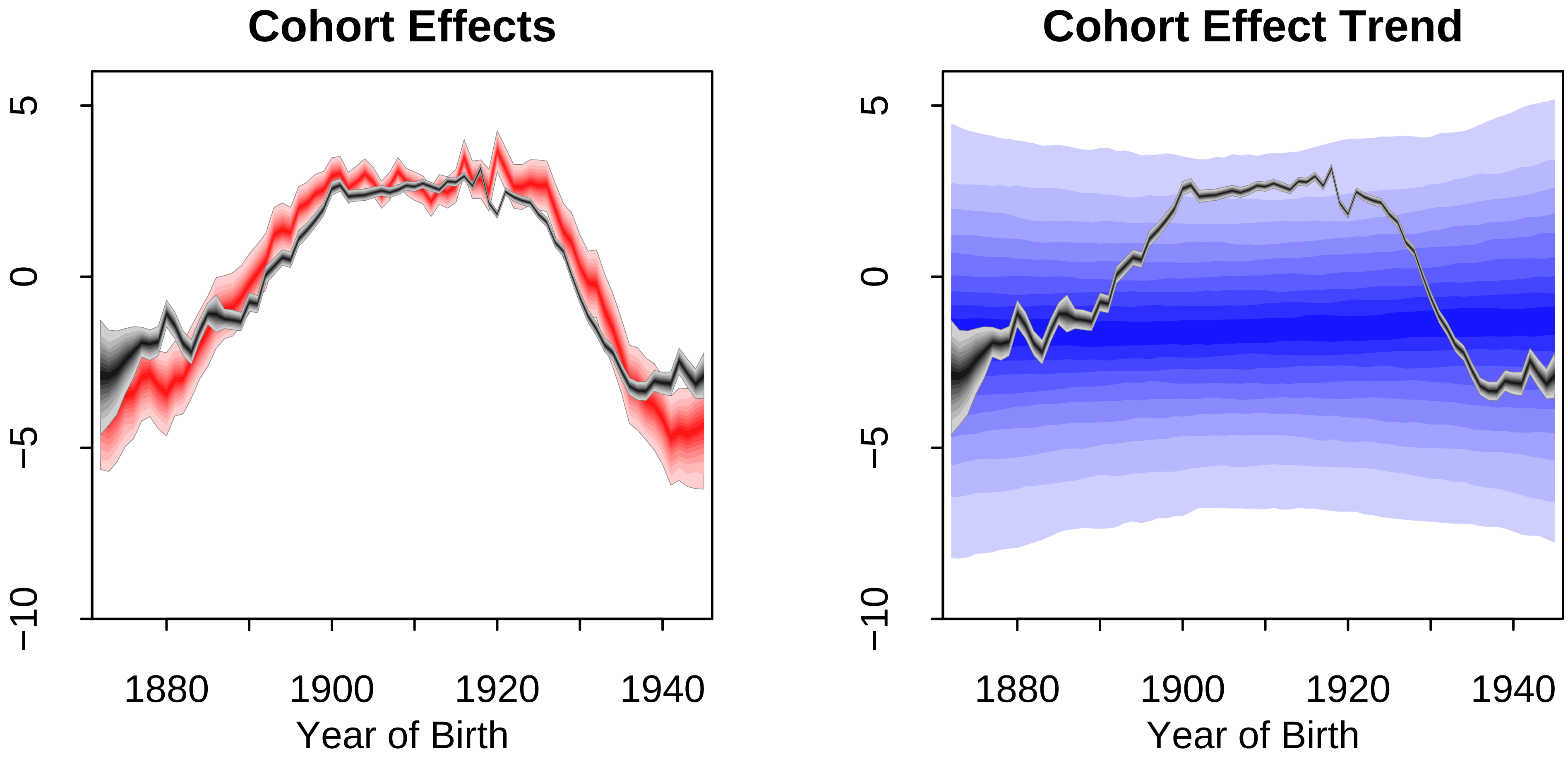
Case Study: EW versus CMI males

- Data: 1961-2005
- Ages: 60-89
- CMI exposures $\sim 10\%$ of EW
- EW missing data:
  - 1886 cohort
  - 1961-1970, ages 85-89

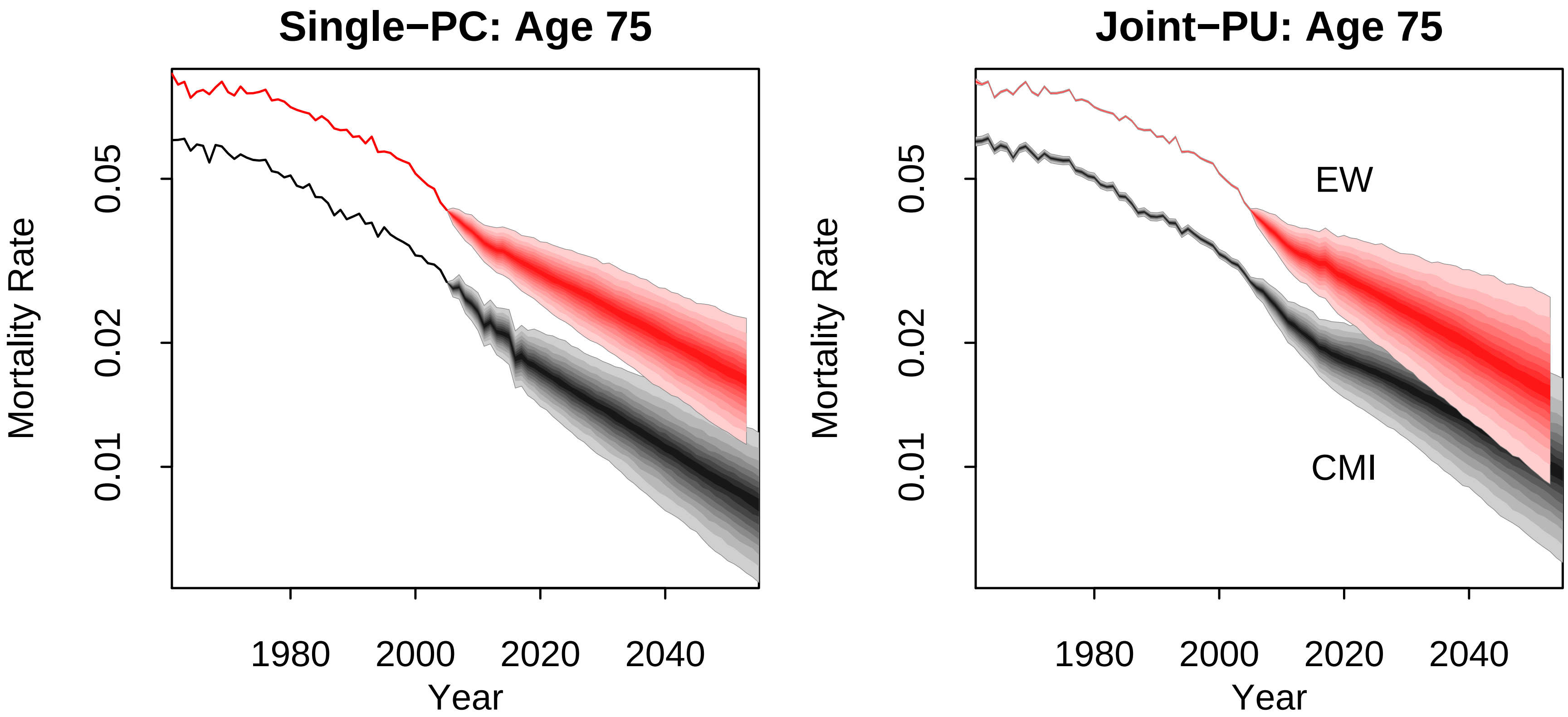
EW (grey) and CMI (red) Age and Period Effects



EW (grey) and CMI (red) Cohort Effects

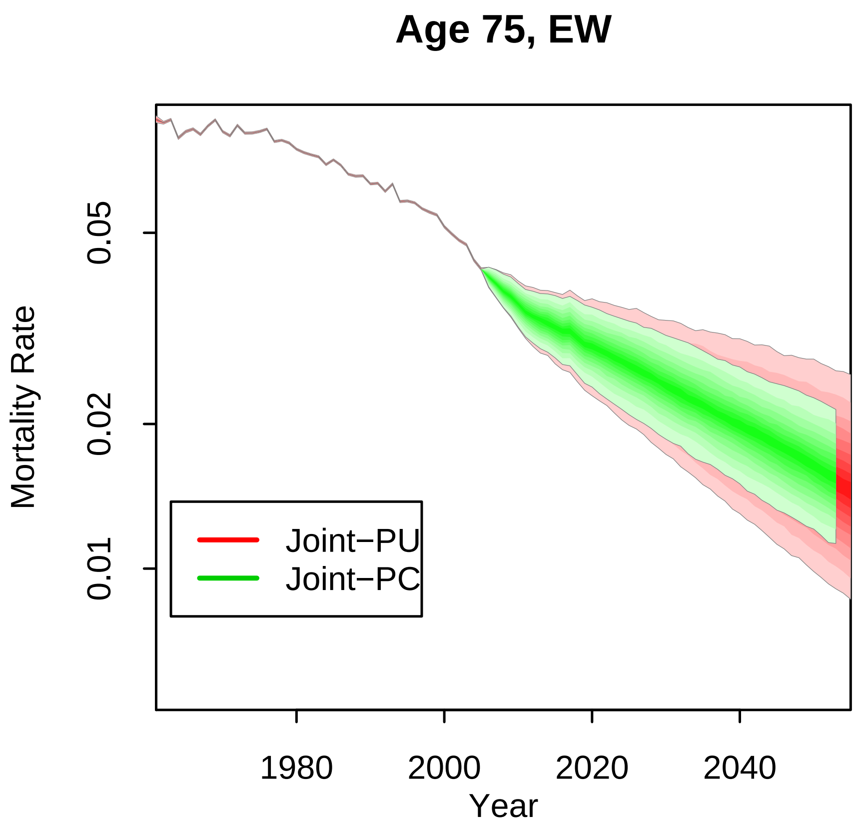


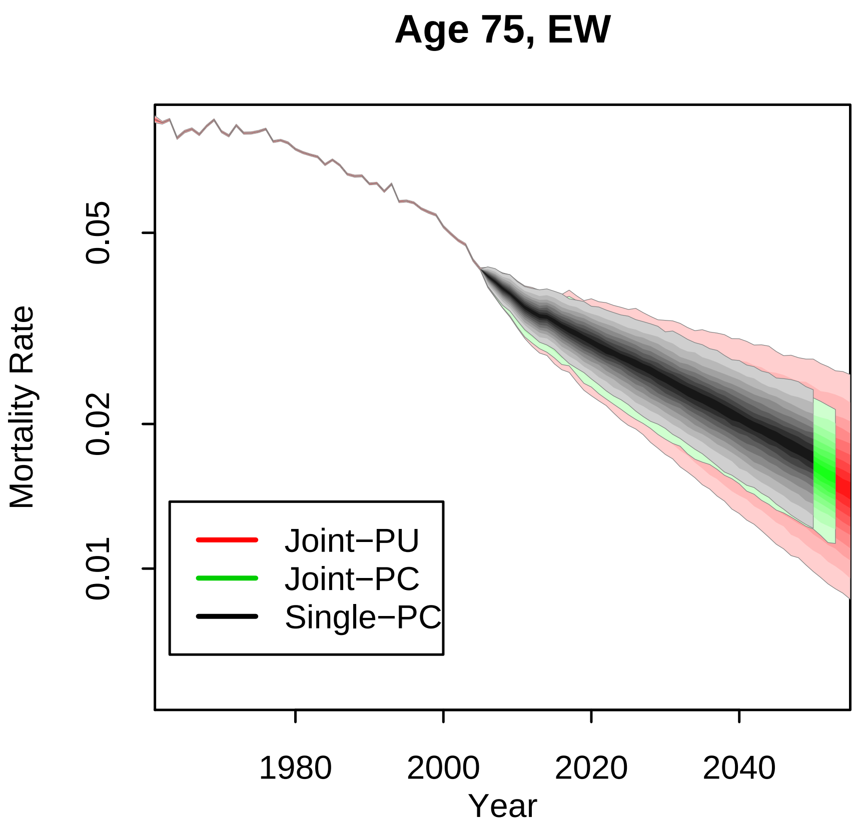
1-population forecasts versus 2-pop MCMC



- Joint-PU: uses the 2-population model with parameter uncertainty
- Single-PC: uses the 1-population model with no parameter uncertainty

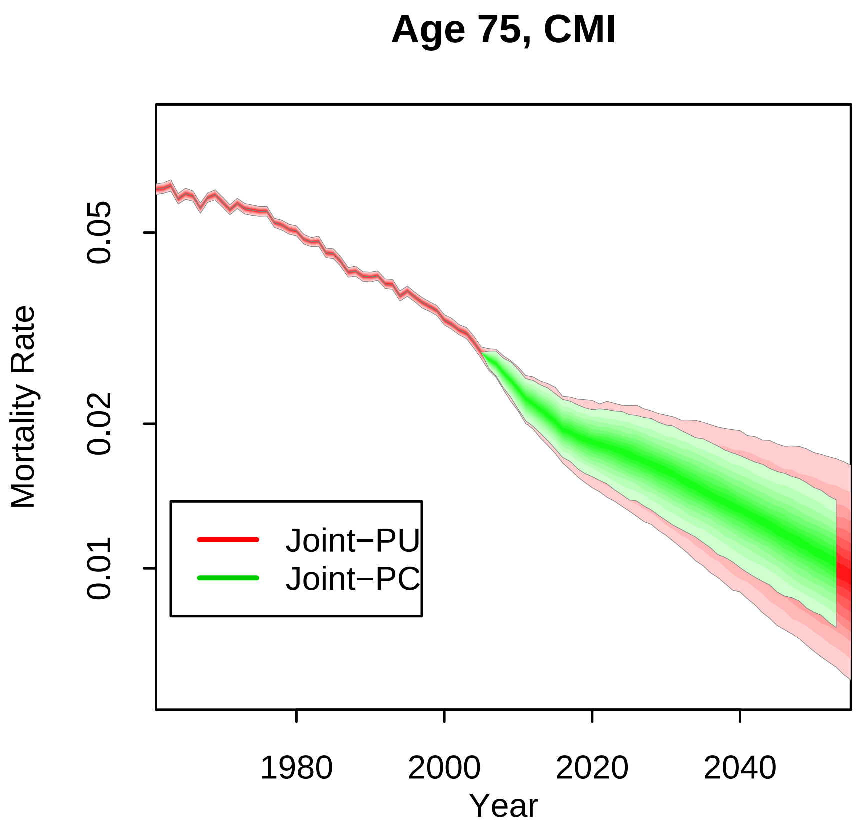
1-population forecasts versus 2-pop MCMC

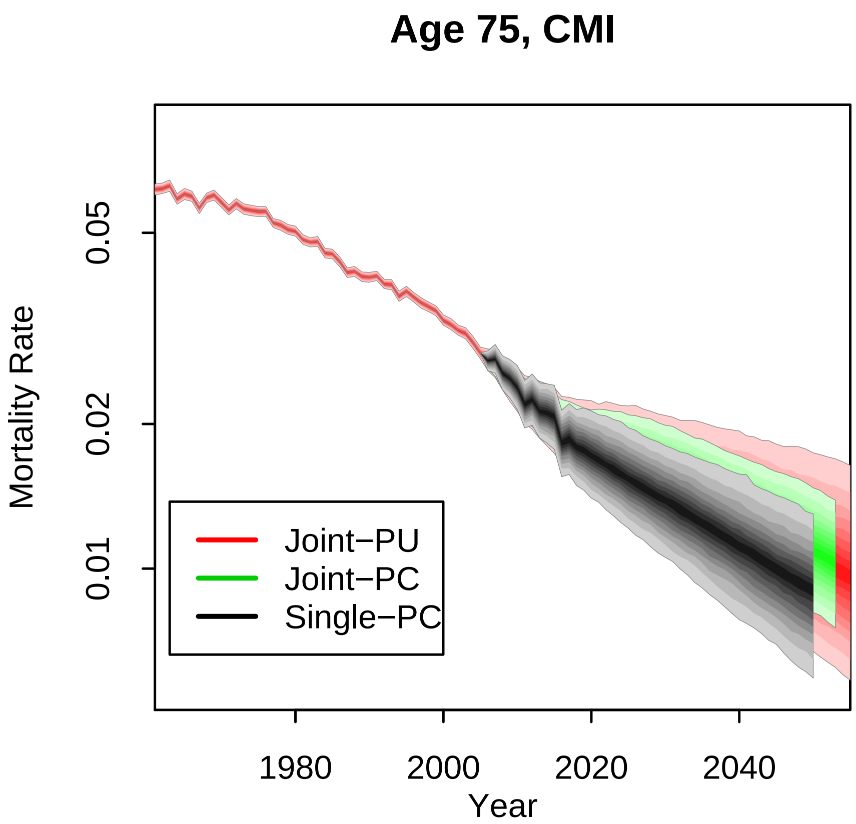




- Joint: uses the 2-population model   Single: uses the 1-population model
- PC: parameters certain   PU: parameters uncertain

1-population forecasts versus 2-pop MCMC





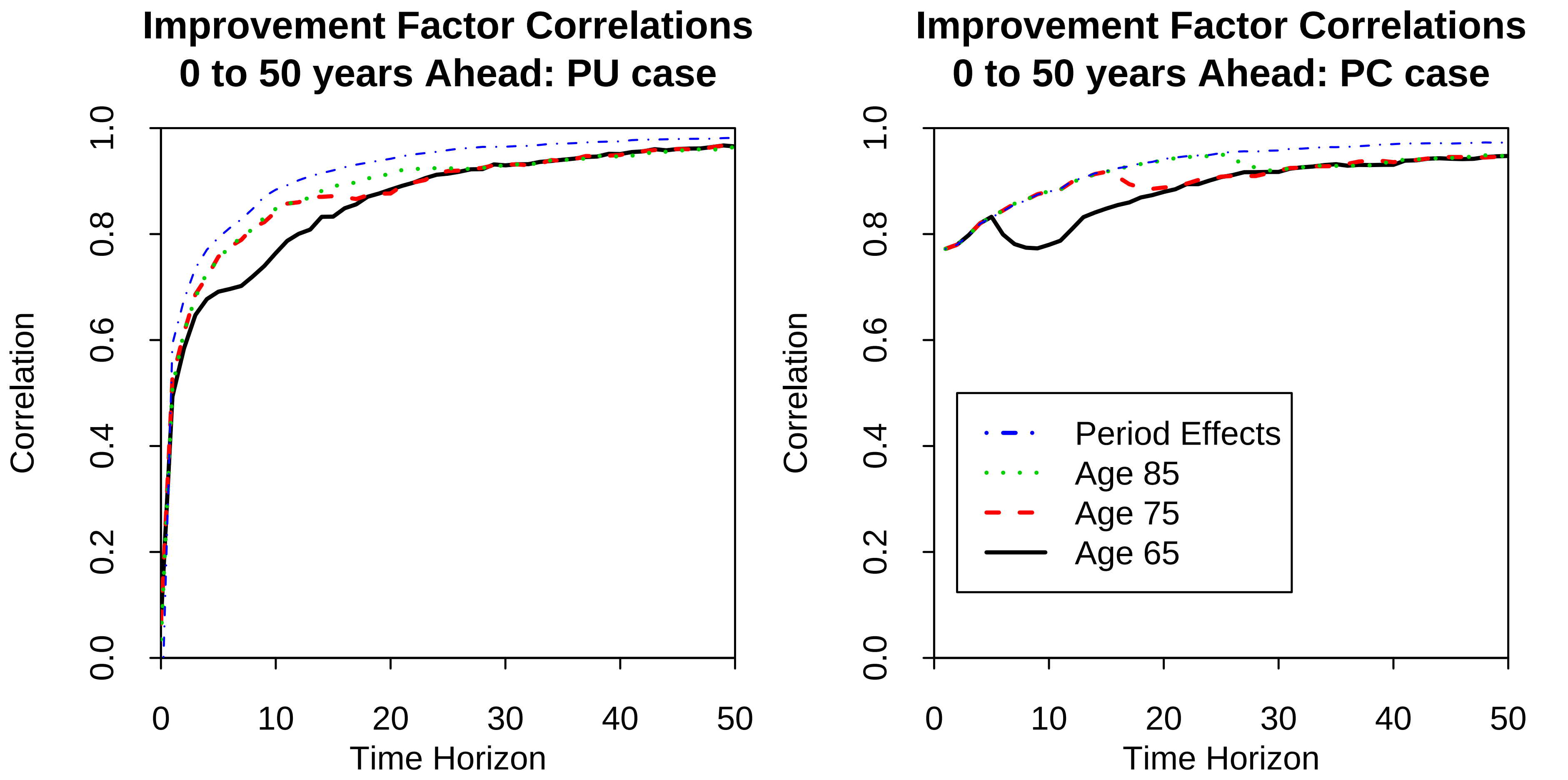
- Joint: uses the 2-population model Single: uses the 1-population model
- PC: parameters certain PU: parameters uncertain

Improvement Factors

Population k, t years ahead from 2005

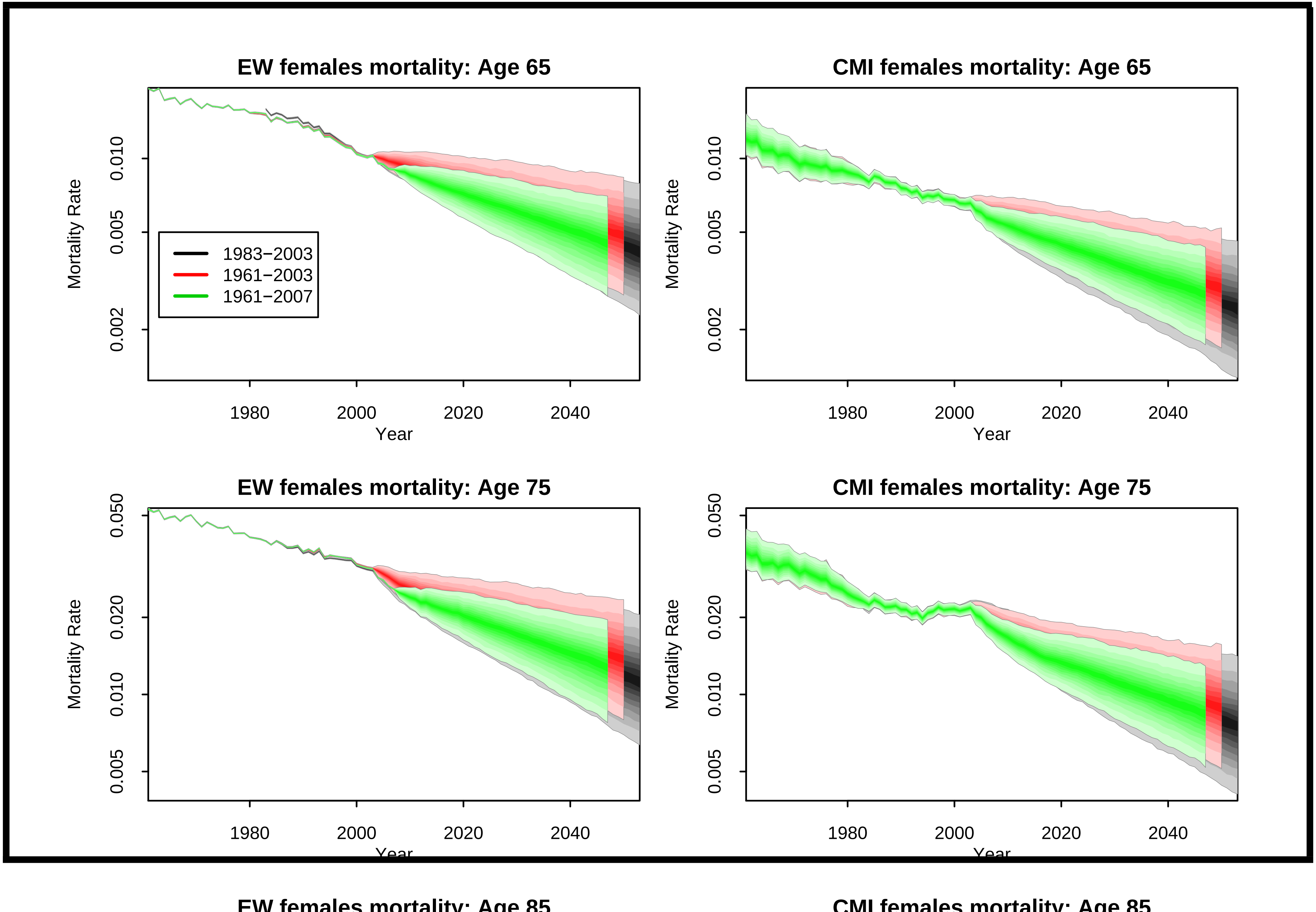
$$IF_k(t,x) = \frac{m_k(2005+t,x)}{m_k(2005,x)}$$

Correlation between $IF_1(t,x)$ and $IF_2(t,x)$



Missing data: extra calendar years

- CMI females: data 1983-2003 (much lower exposures than males)
- EW females:
  - 1983-2003
  - 1961-2003
  - 1961-2007


EW females mortality: Age 65
CMI females mortality: Age 65
EW females mortality: Age 75
CMI females mortality: Age 75
EW females mortality: Age 85
CMI females mortality: Age 85
Mortality Rate
Year
1983−2003
1961−2003
1961−2007
0.010
0.005
0.002
0.050
0.020
0.010
0.005
1980
2000
2020
2040

Missing data: extra calendar years

- Adding 1961-1982 EW data:
  - Small shift in both EW and CMI trend
  - Small changes in forecast uncertainty

- Adding 2004-2007 EW data:
  - – EW now fans out from 2007 instead of 2003
  - – EW generally narrower after 2007
  - – CMI still fans out from 2003 (but less quickly)
  - – CMI generally a bit narrower
  - – EW and CMI small parallel shift in trajectory

Conclusions

- Synthesis of
  - Consistent 2-population projections
  - Bayesian approach
  - Ability to deal with small populations
  - Ability to deal with missing data
  - Full parameter uncertainty
- Full APC model to assess basis risk

Reference:
Bayesian Stochastic Mortality Modelling for Two Populations
LifeMetrics Working Paper, available shortly!
E-mail: A.Cairns@ma.hw.ac.uk