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MODELLING THE OPERATIONS OF A GENERAL INSURANCE COMPANY BY SIMULATION

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1. INTRODUCTION

1.1 The traditional approach to examining the financial status of a company is to look at the balance sheet and the profit and loss account. Such information is usually publicly available, it is certified by the auditors as having been drawn up according to relevant accounting standards and it is generally presumed to communicate reliable information.

1.2 In the case of a manufacturing or trading company the profit and loss account records purchases and sales and the balance sheet will include a valuation of stock in hand, since it is anticipated that this will give rise to future sales income. Working capital is required because products have to be manufactured or purchased before they can be sold. Profit is realized when the product is sold for more than it cost to buy it or to make it.

1.3 A general insurance company runs a different type of operation. Customers pay in advance for the service they expect to receive and the company does not normally have to make payments out until after premiums have been received (except for marketing costs). The profit and loss account records premiums received and claim payments made, but there will also usually be a substantial item of investment income in respect of the investment of policyholders' money prior to the settlement of claims. The balance sheet features these investments and, on the other side, liabilities in respect of premium monies received, but where the cover has not yet been provided, and in respect of claim events which have already occurred but where payment has not yet been made. Profit can only be established accurately once all the claim payments have been made relating to the period in question. This can take many years in some classes of business.

1.4 Drawing up the balance sheet of a general insurance company presents considerable difficulties because of the need to set a value on the outstanding liabilities. Difficulties arise because of the uncertain amount of these liabilities and because of uncertainty about the timing of them. There is also the question of whether to take into account the investment income which will be available on the assets during the period up to when the payments are made, that is to say whether or not the future liabilities should be discounted.

1.5 A further question arises in respect of the assets. These can be valued in a number of different ways, for example at cost, at market value, or at a value obtained by discounting the expected future proceeds of the assets.

1.6 These factors create a significant level of uncertainty about the picture shown by the balance sheet, and, correspondingly, about the profit declared by the company. Some of this uncertainty is inherent in the nature of the business being carried on by the company. Some, however, is imposed by the requirement to set single values on the assets and the liabilities.

1.7 A further problem is that the balance sheet does not tell you anything about the effect of future business. A company that appears to be solvent from the balance sheet may be writing unprofitable business at such a rate that it may become insolvent within a very short space of time.

1.8 Balance sheets are obviously necessary to meet accounting requirements and to satisfy regulations laid down by the supervisory authority. However, if management wants to understand more fully what is going on, or if a statutory requirement were to be introduced for an actuarial report on the reserves or on the financial strength of the company, an alternative approach may be desirable. A logical way to tackle the problem is to unbundle the inflows and outflows in each future year into a cash flow projection and then to model the uncertainty in each item.

2. EMERGING COSTS PARADIGM

2.1 The current and future financial strength of the company will be determined by the aggregate effect of a large number of different inflows and outflows. As far as the liabilities are concerned, it is clearly not the balance sheet provision for outstanding claims which matters, but the actual payments which will be required in future. Similarly, it is not the market value of the assets at the year end which matters, nor even the value for which they could actually be realized at a current date, but the stream of interest income and dividends which the assets can be expected to generate. Even in a run-off situation assets will only need to be realized in the future.

2.2 We need to consider each of these flows in turn. As inflows to the company we have:

premium income;
income from interest and dividends on assets;
reinsurance and other recoveries.

The outflows are:

claim payments;
reinsurance premiums;
expenses and commission;
tax;
dividends.

2.3 We do not intend to say any more about reinsurance in this paper, although this can be an issue of considerable practical importance. Ignoring reinsurance, we can characterize the pattern of flows in any year t by an equation as follows:

$$A(t) = F(t)A(t-1) + B(t) + I(t) - C(t) - E(t) - T(t) - D(t)$$

where $A(t)$ is the amount of the assets (at market value) at the end of year t,
F(t) is a factor representing the proportionate change of asset values during year t,
$B(t)$ is the premium income in year t,
$I(t)$ is the investment income in year t,
$C(t)$ is the claim expenditure in year t,
$E(t)$ is the expense cost in year t,
$T(t)$ is the tax payment in year t,
$D(t)$ is the dividend payment in year t.

2.4 For most practical purposes we can ignore variations in each of these quantities over the financial year—indeed some, such as tax and dividends, will normally be single annual payments.

3. SOLVENCY CONSIDERATIONS—RUN-OFF

3.1 A useful starting point is to consider pure solvency. For this purpose it is reasonable to assume a cessation of premium income, and of tax and dividend payments. The concern of the supervisory authority is that insurance companies should be solvent, in order to provide justification for allowing them to remain in business. This is interpreted as meaning that the existing portfolio of assets should be sufficient to permit all liabilities already assumed to be met, without any continuation of new business.

3.2 We can then reduce the equation of flow to the following:

$$A(t) = F(t)A(t-1) + I(t) - C(t) - E(t).$$

In order to model this situation we need to model the income from investments and changes in the value of assets, the payment of claims and the future level of expenses. In practice in such a run-off situation the main expenses would be the expenses of settling claims, and these can conveniently be incorporated in the claim expenditure item $C(t)$.

Claim Variability

3.3 The main involvement of actuaries in general insurance over the last 20 years has been in analysing the liabilities item, particularly that in respect of outstanding claims. The distinctive contribution of actuaries to this subject was to emphasize the need to relate claim payments to the exposure-to-risk. Aggregate claim payments year by year offer limited possibilities for meaningful analysis, but if these payments are subdivided by the year of origin of the business (the year in which the premium was paid or the year in which the risk was covered), and are related to the respective premium income, some usable patterns emerge. The raw data of claim payments made in the past can be expressed in the form of a so-called run-off triangle:

Year of origin	Year of development						
	1	2	3	4	5	6	7
1982	$x(1,1)$	$x(1,2)$	$x(1,3)$	$x(1,4)$	$x(1,5)$	$x(1,6)$	$x(1,7)$
1983	$x(2,1)$	$x(2,2)$	$x(2,3)$	$x(2,4)$	$x(2,5)$	$x(2,6)$	
1984	$x(3,1)$	$x(3,2)$	$x(3,3)$	$x(3,4)$	$x(3,5)$		
1985	$x(4,1)$	$x(4,2)$	$x(4,3)$	$x(4,4)$			
1986	$x(5,1)$	$x(5,2)$	$x(5,3)$				
1987	$x(6,1)$	$x(6,2)$					
1988	$x(7,1)$						

3.4 The classical problem is to extrapolate what will happen to each of these horizontal sequences of claim payment figures in years of development yet to be observed.

3.5 In order to provide a model for future claim payments we postulate that each class of business can be characterized by a run-off pattern $\{p(i); i=1,2,3 \ldots N\}$, where

$$\sum_i p(i) = 1$$

and where all claim payments in respect of the tranche of business have been met by the end of the Nth year of development. The run-off factors, $p(i)$, represent the proportion of total claim payments in respect of a tranche of business that are made in development year i. The run-off factors are assumed to be independent of the year of origin and assume no inflation of future claim amounts. Inflation is introduced at a later stage.

3.6 Assumptions have to be made about the pattern of past premium income and the total claims generated by those premiums, which have given rise to the portfolio of outstanding claims at the date of assessment. The outstanding claims can then be expressed as a projection of future claim payments in each future year for each past year of origin. In terms of the nomenclature introduced above for the run-off triangle we want to estimate, for the year 1989 say,

$$\sum_{s=1}^{7} x(s, 9-s) = \sum_{s=1}^{7} X(s)p(9-s) \qquad (p(i) = 0 \text{ for } i > N)$$

where $X(s)$ is the total claim outgo expected in respect of year of origin s, and then to apply an inflation factor to bring the claim outgo up to the level appropriate for 1989.

3.7 This brings us to a fundamental problem in modelling the future cash flows—the incorporation of variability. The uncertainty of future cash flows has already been identified as a major difficulty in interpreting the financial status of a company. The run-off patterns which we have assumed represent only one estimate of what might happen. Any projection needs to recognize a range of possible outcomes. In principle this range should apply to the incidence of payments as well as to their size.

3.8 In an early version of our model we assumed a distribution of values for the claim payment projected for each cell of the run-off triangle. We generated this distribution by simulation. In each realization of the simulation we sampled a single value for the claim payment for each cell from the relevant distributions. These separate cells were then aggregated to derive, for that realization, the total claim payment in that year in respect of all past years of origin.

3.9 In the later stages of our work we moved away from this individual cell approach and applied the randomization procedure to the aggregate claim payment for the financial year in question. This can be justified because of the very large numbers of claims involved in any one year. We take the total expected claim outgo for the year as being the sum of the expected outgo for the individual cells (for example

$$\sum_{s=1}^{7} X(s)\,p(t-1980-s) \qquad (p(i)=0 \text{ for } i>N)$$

in year t in the run off introduced above). If we denote this by $C(t)$, we then assume that $C(t)$ is normally distributed with mean $\bar{C}(t)$, and standard deviation $a\bar{C}(t)+b\sqrt{\bar{C}(t)}$ to provide variability which in part reflects the aggregation of large numbers of independent claim payments, but in part reflects secular variability which is likely to affect all risks in a similar manner, at least within the separate liability types.

3.10 This approach can be criticized for being oversimplistic. However, we believe that it provides a realistic representation of claim payment uncertainty in aggregate, although clearly the values adopted for the parameters a and b in the standard deviation are critical to the degree of variation generated.

Asset Variability

3.11 Of far greater importance to the analysis than the assumptions about claim variability is the model adopted for future inflation and future asset movements. Developing suitable models for this purpose is a major project in its own right, which we did not feel lay within our particular area of competence. We have, therefore, drawn heavily on work by Wilkie (1986)[(3)]. We intend only to give a broad overview of the structure of the models and comment on some further work we have done to produce a model which reflects more satisfactorily the experience of inflation and asset movements since the beginning of the 1950s, and which we feel is more suitable for making projections for the future.

3.12 Wilkie (1986)[(3)] established a set of interrelated autoregressive equations to generate multiple realizations of future scenarios, including the year-by-year rate of inflation, the movement of asset values, the level of dividends, etc. There is clear evidence of linkages between the behaviour of inflation, equity prices, gilt prices, etc., but it is not possible to demonstrate a unique relationship which can be represented by a simple formula. Without having to identify specific causal relationships, time series techniques can be used to obtain models with appropriate lags, linkages, damping factors and feedbacks.

3.13 The models will reflect the broad characteristics of the data set used to calibrate them. Wilkie[3] used data for the whole period from 1919 to 1982 to derive the parameters of his model, but one might take the view that some of the earlier parts of this period should be given little weight in determining the shape of an asset model for the future.

Modelling Inflation

3.14 Let us begin with the model for future inflation, since the other models depend, to a greater or lesser extent, on this one. The model for the retail price index $q(t)$ can be expressed in the form

$$\nabla \ln\{q(t)\} - \mu_q = \alpha_q \, [\nabla \ln\{q(t-1)\} - \mu_q] + \sigma_q \, z_q \, (t)$$

where the backwards difference operator ∇ is defined by

$$\nabla x(t) = x(t) - x(t-1)$$

and z_q (t) is a sequence of independent identically distributed unit normal variates. μ_q, α_q and σ_q are constants which define the average rate of increase of the retail price index, the extent to which the previous year's rate of inflation influences this year's inflation and the amplitude of the random 'white noise' factor. The equation may be a little less opaque if expressed in the following form:

$$\ln\{i(t)\} - \mu_q = \alpha_q[\ln\{i(t-1)\} - \mu_q] + \sigma_q z_q(t)$$

where $i(t)$ is the increase in the retail price index from year $t-1$ to year t, i.e. the rate of inflation.

3.15 Wilkie[3] suggested values of ·05 for μ_q, ·6 for α_q and ·05 for σ_q. What does this imply for the distribution of future RPI movements? Figure 1 shows the distribution of rates of inflation in year 11 of the projection, generated by 10,000 simulation realizations, together with the distribution of year-on-year movements in the RPI (for the month of December) from 1951 to 1988. The most important difference is that the model generated about 21% of realizations with negative inflation and only some 28% of cases with positive inflation of less than 5%, whereas the actual experience of this period has been that over half of the years showed inflation of between zero and +5%.

3.16 Although the distribution obtained from the model reflects the experience of the period 1919 to 1982, it does not closely reflect the experience since 1950, which has not shown any periods of falling inflation. It is a matter of

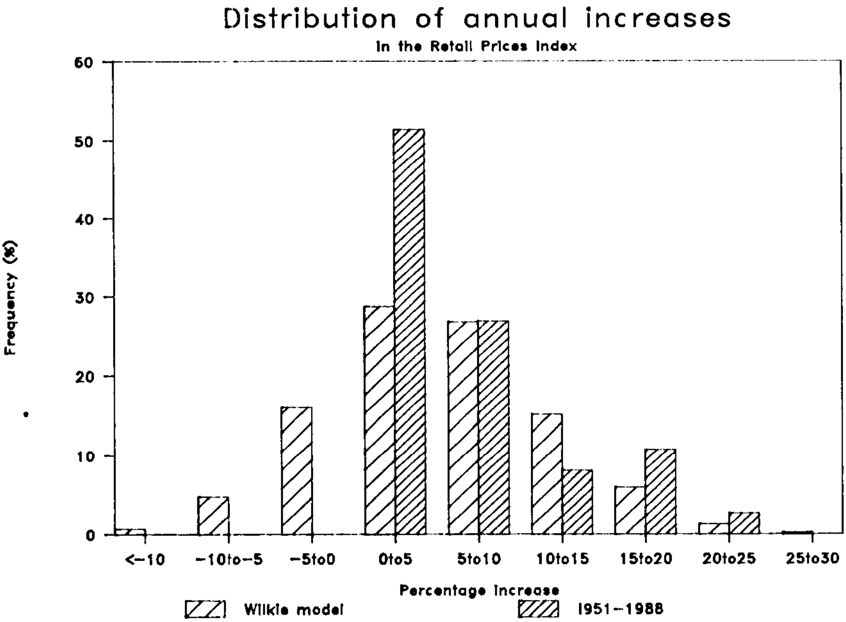

Figure 1.

judgement and opinion whether it is appropriate to use a model for future inflation which generates such a high proportion of realizations with negative inflation. This can be mitigated by increasing the mean value of the inflation model, μ_q, and by reducing the standard deviation of the random element. Figure 2 shows how the distribution of rates of inflation in year 11 of the projection is changed if we take values of ·7 for μ_q, ·6 for α_q and ·03 for σ_q. This reduces the proportion of cases with negative inflation to less than 2%. This seems a more reasonable pattern, but it may reduce variability too much.

3.17 Alternatively, the random element could be made asymmetric, with a damped level of fluctuations when below the mean, or an effective barrier to negative inflation. Yet another approach might be to reduce substantially the standard deviation of the random element and to superimpose an additional random element, which creates sharp upwards fluctuations of inflation from time to time. There is a lot more work to be done on developing suitable models for inflation and for the assets.

Modelling Fixed Interest Yields

3.18 Wilkie[3] modelled fixed interest securities by taking the yield on $2\frac{1}{2}$% Consols, an irredeemable government security which has been available for many years. This gives access to a large run of past data and avoids the problems

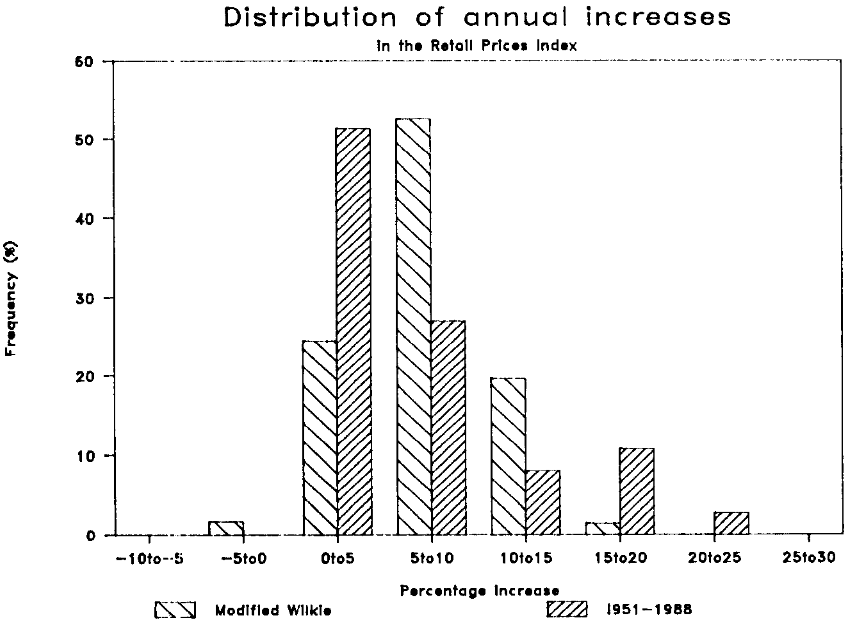

Figure 2.

of maturity dates and the consequential need to reinvest in new securities. His model for the yield on Consols is as follows:

$$c(t) = \omega_c m_c(t) + n_c(t)$$

where
$$m_c(t) = \delta_c \nabla \ln \{q(t)\} + (1-\delta_c)\, m_c(t-1)$$
$$= \delta_c \ln \{i(t)\} + (1-\delta_c)\, m_c\,(t-1)$$

and
$$\ln\{n_c(t)/\mu_c\} = [\alpha_c \mathrm{B} + \beta_c \mathrm{B}^2 + \gamma_c\, \mathrm{B}^3] \ln\{n_c(t)/\mu_c\} + \phi_c\, e_y(t) + \sigma_c\, z_c(t)$$

with B representing the backwards step operator, such that

$$\mathrm{B}\,x\,(t) = x\,(t-1)$$

and hence
$$\mathrm{B}^n x(t) = x\,(t-n).$$

The $m_c(t)$ factor represents the carried-forward effect of inflation on yields, whereas the $n_c(t)$ factor introduces a link with the values of the yield in the previous 3 years and a random 'white noise' element.

3.19 For our purposes it was not sufficient to rely on a model for irredeemable securities. We needed to assume investment in dated fixed interest securities. We postulated a simple yield curve such that 20-year stocks yielded the same as irredeemables and cash yielded 1 percentage point less, with a linear progression of yields for intervening values of the term of the stock (subject to an overall minimum of ·5%).

3.20 In our latest model we allow for investment in fixed-interest securities of three different terms to redemption and assume that stocks are sold after a year and replaced by new stocks of the original term. If we look at the year-on-year price movements generated for a 10-year stock, purchased at a yield of ·5 percentage points less than the Wilkie model yield on $2\frac{1}{2}$% Consols, and sold a year later with 9 years to run, on a yield basis of ·55 percentage points less than the yield on Consols, we get a distribution of ratios of prices to those a year earlier, from 10,000 realizations on the Wilkie model, as shown in Figure 3. Alongside is shown the distribution based on actual movements of the yield on Consols from December to December in the period 1951 to 1988 (still with the same artificial yield curve assumptions). The actual figures show a similar mean but rather greater dispersion than the model results.

Modelling Equity Prices and Dividends

3.21 Equity prices are modelled by means of the dividend yield, $y_e(t)$, and an index of total share dividends, $d_e(t)$, with the price given by

$$p_e(t) = d_e(t)/y_e(t).$$

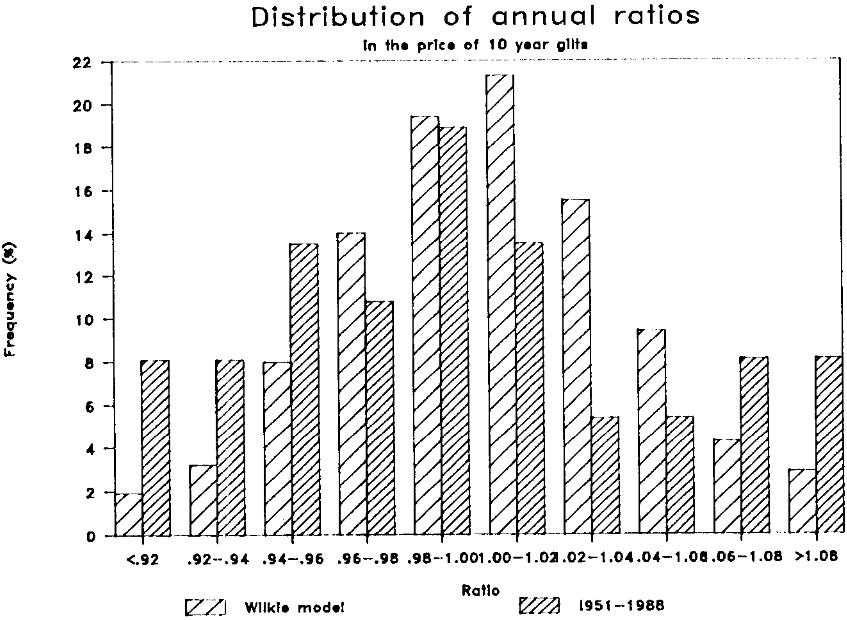

Figure 3.

The model for the dividend yield is

$$\ln\{y_e\} = \omega_{ye} \ln\{i(t)\} + n_{ye}(t)$$

where
$$n_{ye}(t) - \ln(\mu_{ye}) = \alpha_{ye}[n_{ye}(t-1) - \ln(\mu_{ye})] + \sigma_{ye} z_{ye}(t)$$

with ω_{ye}, α_{ye}, μ_{ye} and σ_{ye} constants and $z_{ye}(t)$ a sequence of independent, identically distributed unit normal variates.

3.22 The model for equity dividends is given by

$$\ln\{d_e(t)\} = \omega_{de}\, m_{de}(t) + \alpha_{de} \ln\{i(t)\} + \mu_{de} + \beta_{de}\, e(t-1) + e_{de}(t) + \gamma_{de}\, e_{de}(t-1)$$

where
$$m_{de}(t) = \delta_{de} \ln\{i(t)\} + (1-\delta_{de})\, m_{de}(t-1)$$

and
$$e_{de}(t) = \sigma_{de} z_{de}(t)$$

with ω_{de}, σ_{de}, μ_{de}, α_{de}, β_{de} and γ_{de} constants and $z_{de}(t)$ is a sequence of independent, identically distributed unit normal variates.

3.23 Figure 4 shows the distribution of year-on-year equity price movements from 10,000 realizations of this model, using Wilkie's parameters, alongside the actual year-on-year movements (for the month of December) from 1951 to 1988. Dramatic falls in equity prices have been less common in this period than the

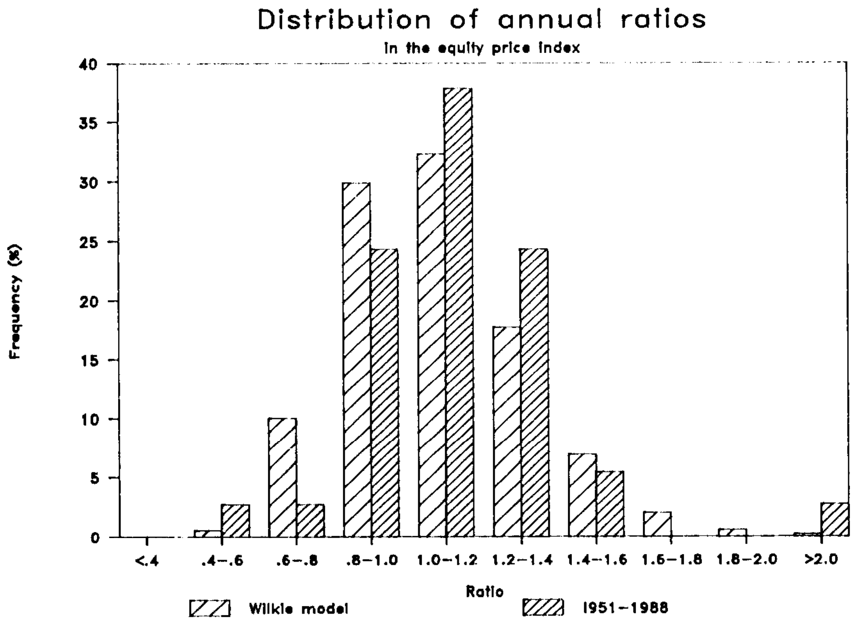

Figure 4.

model might suggest and the average rate of increase of prices has been significantly higher than indicated by the model.

Simulating the Run-Off

3.24 We have described the inflation and asset models in some detail, partly because of their inherent interest and partly because of their great importance for the whole exercise. It is vital to have robust models for these aspects, on which it is reasonable to rely, since much of the variability in a company's projected results derives from this element.

3.25 Let us now combine the asset and inflation models with the model for the variability of claim payments and examine a simulation of the run-off of a general insurance company. Figure 5 depicts 100 realizations, with investment assumed to be wholly in fixed interest securities. Investment in equities would, of course, yield a very much more variable picture, but such investments would normally form only a small part of the asset portfolio.

3.26 Figure 6 shows the distribution of assets remaining at the end of the run-off, i.e. when all claim payments have been met. The cases with remaining assets of less than zero indicate the simulation realizations that have given rise to insolvency. In this particular case, there were 9 insolvencies out of 1,000 simulations, and the mean assets remaining at the end of the run-off, discounted for the effect of inflation, were 172% of the net written premiums in the year before the date of assessment, having started off with an apparent asset margin of 40% of net written premiums (together with a hidden margin of up to 40% because outstanding claims were not discounted). The standard deviation of the distribution was 106% of the net written premiums.

3.27 Further details of results from such simulations, and how they can be used to illustrate differing probabilities of adequacy for companies with different levels of solvency margin and differing asset and liability characteristics, were given in Daykin *et al.* (1987).[1]

4. MANAGEMENT MODEL

4.1 We would like to go on now to describe briefly how this approach has been generalized to the situation of a company operating as a going concern, that is continuing to write business and generating further premium income, with associated claims and other expenses. A major difficulty in effecting this generalization is to model the premium-setting process in an adequate way, and to find ways of reflecting the operation of a free insurance market, whereby higher premium rates may increase margins of profitability, but also result in a loss of business volume, unless the rest of the market is behaving in a similar way.

4.2 We have developed a model which we see being used by actuaries as a tool for exploring the consequences of uncertainty in the operations of a general insurance company. For this purpose it needs to provide adequate representation of the behaviour of the company as it operates in the wider market. It needs to

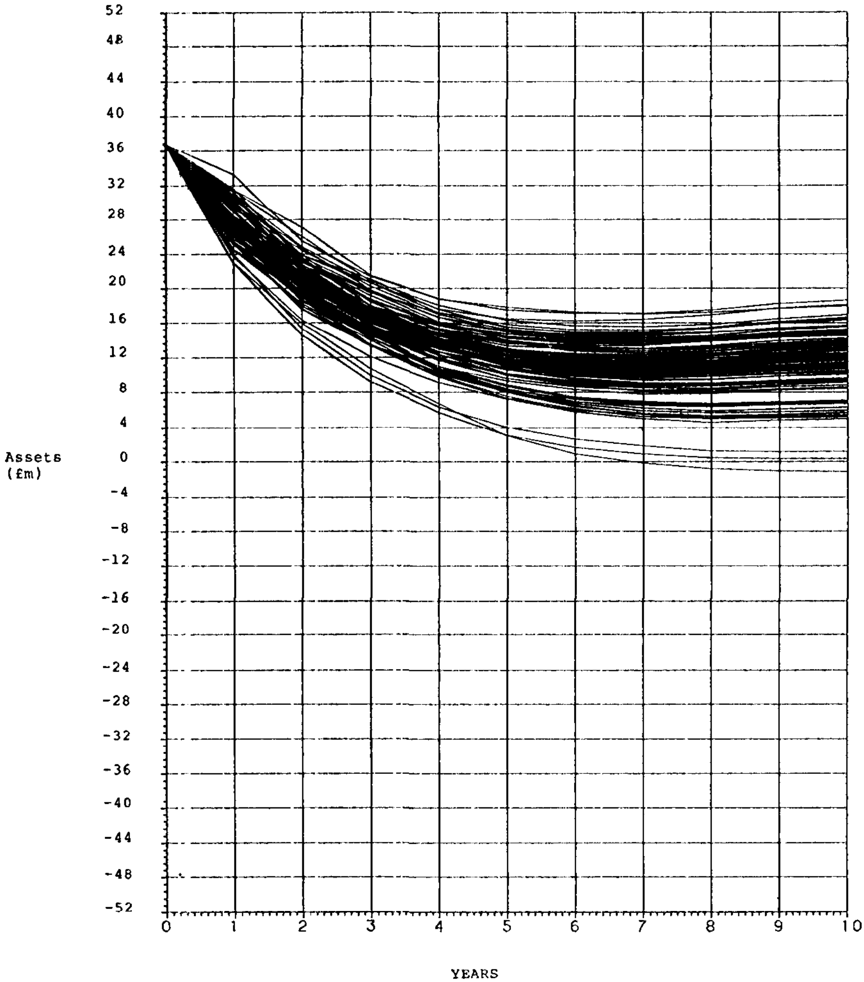

Figure 5. Run-off of assets assuming no new business (100 simulations).

allow realistically for tax and dividend payments to shareholders, and it needs to give effect to likely feedback mechanisms, whereby the company's actions are subject to modification in the light of what has happened.

4.3 The management model is designed to simulate a realistic market situation, by modelling the possible reaction, in terms of the volume of business written, to increases or decreases in the premium rates charged by a particular

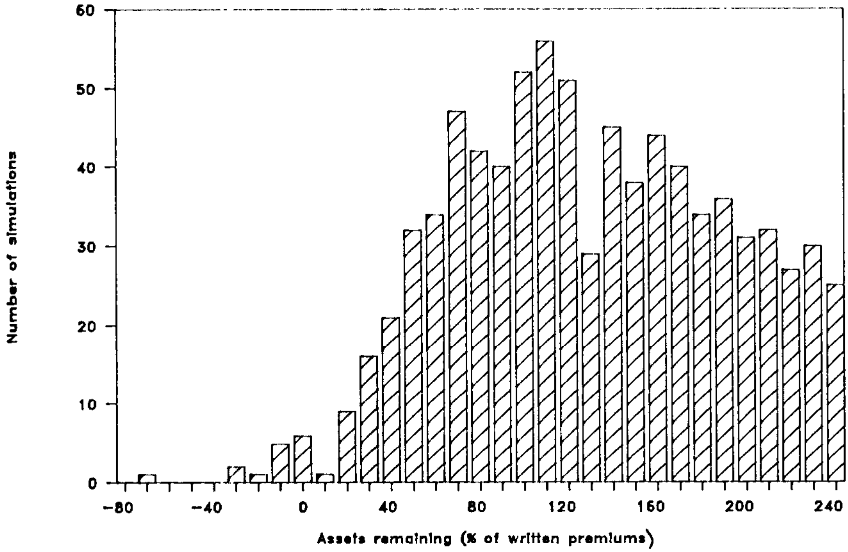

Figure 6. Distribution of assets at end of run-off from 1000 simulations on standard basis. No further business.

company relative to the average market level. The model involves projecting the behaviour of the market as a whole alongside the behaviour of the particular company under investigation and allows the user to test the behaviour of the company against that of the market.

4.4 Profit margins may be set for each type of business (see § 4.7), to represent both the average market situation and the experience of the particular company under investigation. Initial premium volume is specified both for the market and for the company, as is the extent to which the market and the company set premium rates with a view to recovering past losses or rewarding policyholders in respect of past profits. The company rates can then be set at a specified percentage above or below the market rates and the volume of business written by the company is assumed to increase or decrease according to whether premiums are lower than or higher than the market rates, with appropriate gearing factors to reflect the elasticity of demand.

4.5 As in Daykin *et al.* (1987)[1], the claim ratios experienced by the company each year on the business being written are assumed to be modelled stochastically. The management model, however, allows also for cyclical variation in both claim experience and premium rates. Stochastic variation is allowed for in the aggregate claim payments made year by year, as described in § 3.9. The rate of inflation and asset movements and returns are modelled using the autoregressive stochastic processes which have already been described in §§ 3.14 to 3.22.

4.6 The model allows for business to have been written for up to 20 years prior to the date of investigation. This is to enable an initial portfolio to be established based on the outstanding claims from past years' business and the asset margin at the start of the projection. The initial distribution of assets has to be specified. Expenses are allowed for explicitly and realistic allowance is made for the payment of tax and for dividends to shareholders. Although the model is based on the emerging costs paradigm, accounting quantities are monitored as well as cash flows, in order to be able to demonstrate solvency margins, profits after tax, outstanding tax liabilities, etc.

Premium Income

4.7 The model allows for six types of business to be written. These need not correspond on a one-to-one basis with particular classes of business or lines. They are characterized by a series of parameters, including claim settlement pattern, volume growth rate, claim variability and the fraction of the market held by the company. A key feature is the specified profit rate for the liability type. This is the percentage margin for profit in the gross premiums, allowing for expenses and for the expected run-off of claims in accordance with the claim settlement pattern, discounted at a realistic rate of interest.

4.8 The target profit margin, PF, is related to the claim ratio, CR, and the run-off factors, $p(s)$, by the following equation:

$$PF = 1 - e - CR \sum_{s=t}^{t+20} p(s-t)/(1+i)^{s-t-.5}$$

where e is the proportion of the premium absorbed in expenses, and i is the rate of discount.

4.9 The company under investigation is assumed initially to have had a specified share of the market by numbers of policies and to have premium rates exactly equal to those of the market. The market volume is specified separately for each liability type, as is the fraction of that particular part of the market held by the company.

4.10 The premiums written by the market are assumed to be the product of the total volume in terms of numbers of policies and the average market premium in £ sterling. Changes in market volume are determined by specified growth rates, which are specified both for the past and for the future, with flexibility to allow for different growth rates in two separate past periods and in two separate future periods. Thus premiums for the market can be derived as

$$BM_T(t) = NM(t) \cdot RM(t)$$

where $NM(t)$ is the number of policies written in year t

$$NM(t) = NM(t_0)(1+\theta_1)^{t'}(1+\theta_2)^{t''}$$

where θ_1 is the rate of growth of volume for the first t_1 years,
θ_2 is the rate of growth of volume after the first t_1 years,
$t' = \min(t, t_1)$,
$t'' = \max(t - t_1, 0)$,
and $RM(t) = RM(t-1)(1 + i(t))$

gives the average market premium allowing for inflation but not for any other adjustments.

4.11 Market premiums are assumed to be set in order to achieve the specified target profit for each liability type. Since the market premium levels are specified by the user, the profit assumption establishes the corresponding basic level of claims. This is done using notional 'true' market premiums, which assume perfect knowledge of current as well as past inflation. The actual market premiums are assumed to be set on the basis of a formula estimate of the current rate of inflation, based on the increases in the RPI over the past four years. This involves replacing $i(t)$ in the equation above for $RM(t)$ by $i_m(t)$, where

$$i_m(t) = \sum_{k=1}^{4} IM_k \, i(t-k)$$

i.e. $i_m(t)$ is the market's estimate of inflation in year t based on inflation in the four previous years and the weighting factors for the market (IM_k).

4.12 This formula is specified by the user and may involve factors which are positive or negative and greater or less than one. Thus it is possible to project the past trend of inflation linearly, quadratically or using a weighted or unweighted average of the past four years' RPI ratios. As the simulation moves forward, the starting point for the calculation of projected inflation includes the estimated and actual rate of inflation for previous years.

4.13 Premium rates in the market may be assumed to vary cyclically according to a cycle of specified length and shape. The amplitude and phase of the cycle may be specified separately for each liability type. Furthermore, the market may be assumed to set premiums higher than would be implied by the profit targets, in order to attempt to recoup past shortfalls relative to target profits, or to set premiums lower in order to pass on to policyholders the benefit of past surpluses relative to target profits. The actual premiums written by the market in year t can then be defined (for each liability type) as follows:

$$BM_w(t) = NM(t)RM'(t)\phi(t) - \pi_m[PM'(t-1) - PM_T(t-1)]$$

where $\phi(t)$ is the premium cycle function, specified by cycle length, shape, amplitude and phase at $t = t_0$

π_m is the proportion of the past year's excess profits which is to be returned to policyholders in the year (or shortfall to be recouped if $PM'(t-1)$ is less than $PM_T(t-1)$)
$PM'(t-1)$ is the actual market profit for year $(t-1)$

$PM_T(t-1)$ is the target market profit for year $(t-1) = PRFM.\ BM_T(t-1)$
$BM_T(t-1)$ is as defined in §4.7
$PRFM$ is the target profit margin per unit premium
and $RM'(t) = RM(t-1)\ (1 + i_m(t))$.

4.14 The company under observation is assumed to operate within the framework of the market which we have described, starting initially from a neutral position, with a specified market share. It is then assumed to deviate from the market in one or more of the following ways:

(i) as a matter of policy, with a view to increasing profits or in order to grow or contract, it will aim to set its general premium levels at a stated percentage below or above the market level;
(ii) it will estimate the current rate of inflation on the basis of past changes in the retail price index, using a formula which may differ from that used by the market as a whole;
(iii) it may attempt to recoup a past shortfall in profit by charging higher premiums, or pass on the benefits of past excess profits to policyholders in terms of lower premiums, to a different extent than the market.

4.15 In addition, the model allows for a random variation in the level of company premiums relative to the market, because of all the uncertainties in the premium-setting process. The equations follow those for market premiums with appropriate modifications.

4.16 Corresponding to the 'true' market premium, a 'true' company premium is also calculated as part of the process of deriving the related claim expenditure. This allows for the actual volume of business written, but for a premium level which assumes perfect knowledge of inflation, no premium cycle and no adjustments to increase or decrease profitability.

4.17 In practice §4.14 (i) and (iii) can be regarded as alternative strategies. If the company is aiming to set premiums at a specified percentage above the market level, it cannot also attempt to recoup past losses at a rate independent of that applying to the market as a whole.

4.18 The extent to which the company sets its premiums above the market level can be made to vary directly according to the asset margin at the end of the previous year. This is intended to provide a feedback mechanism so that a worsening solvency position is responded to by increasing premium rates. The user can specify whether or not this is to apply, the asset margin level below which it applies and the factor of proportionality.

4.19 The rate at which the company's volume is allowed to grow or contract can be controlled so as to produce damped growth. This is done by taking somewhere between the current and the previous year's fractions of the market as the starting point from which to assess the effect of gearing in the next year. Limits can also be placed on the ultimate extent of any growth or contraction in market share.

4.20 Facility is also given for the introduction of intervention strategies. This is triggered by the asset margin falling below the required level and corresponds to the system of supervision in the E.C., whereby the supervisor can require the company to produce a plan for the restoration of a sound financial position, once it has failed to maintain the required solvency margin. The facility can be used with a higher trigger point than the statutorily required solvency margin, on the assumption that management would introduce similar measures to those which might be required by the supervisor, and that they would do so before the point was reached where the supervisor could intervene. As far as premiums are concerned, the intervention strategy permits new market share targets to be specified for each liability type, e.g. to restrict the volume of unprofitable or risky lines of business, and allows for the company's premium levels to be reset at a different percentage above the market level. Further provision is made for intervention in the area of asset distributions.

4.21 If the company charges higher premiums than the market, it may expect to lose business volume. Similarly, if it charges lower premiums than the market, it will expect to gain business volume. The relationship between premium differentials and volume response, or the elasticity of demand, is controlled by gearing factors which are specified by the user, separately for premiums above and below the market level. The company is assumed to be small enough for its behaviour not to have a feedback effect on the market as a whole.

We have the following:

$$\frac{NC'(t)-NC(t)}{NC'(t)}=g_a\frac{RC^*(t)-RM^*(t)}{RM^*(t)}$$

where $NC(t)$ is the actual volume of business written
$NC'(t)$ is the premium volume which would have been written in the absence of a gearing effect
g_a is the gearing factor
$RC^*(t)$ is the average premium actually charged by the company in year t $[=BC_w(t)/NC(t)]$
and $RM^*(t)$ is the average premium charged by the market in year t $[=BM_w(t)/NM(t)]$.

4.22 Up to 20 years' future business can be assumed. There would be no problem in principle in extending this, but it is difficult to obtain stable results over long periods without incorporating additional feedback mechanisms, so it is doubtful whether useful results would be obtained on a longer timescale. Experience shows that for most purposes no more than 10 years should be considered.

4.23 Expenses other than claim settlement expenses are taken into account explicitly. Claim settlement expenses are assumed to be included in the cost of settling claims. Separate provision is made for fixed expenses, which are assumed to increase in future years in line with inflation. In addition, the fixed expenses can be assumed to go up in steps, triggered by a specified percentage increase in the volume of business. This could be to reflect the need to increase staff numbers and office space, once business has increased by a certain amount.

4.24 The variable component of expenses includes brokerage or commission and expenses related to the volume of business. These expenses are taken to be proportional to gross written premiums.

The expenses can be expressed as

$$E(t) = e_1\, B(t_0)q(t)(1+\alpha\theta)^n + e_2 B(t)$$

where n is an integer chosen such that:

$$N(t_0)(1+\theta)^n \leqslant N(t) < N(t_0)(1+\theta)^{N+1}$$

$N(t)$ is the volume of business in year t

θ is the proportionate increase in volume which triggers an increase in the fixed expenses, and

α is the multiplier which gives the corresponding increase in fixed expenses for a volume increase of θ.

5. CONCLUSION

5.1 It has not been our intention here to offer a complete description of the model, but to outline some of its key characteristics. A fuller treatment is given in Daykin and Hey (1989),[2] which was presented at the ASTIN Colloquium in New York, November 1989, and in Daykin and Hey (1990)[4]. The model is to be made available commercially on a diskette for use on an IBM PC or compatible microcomputer, together with a detailed instruction manual.

5.2 We will conclude, however, by presenting a few results to show the sensitivity of the outcome to particular features of the model and particular assumptions (see the Appendix).

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APPENDIX

ILLUSTRATIVE RESULTS FROM THE MANAGEMENT MODEL

1. We present here some illustrative results from the model. In view of the very large number of parameters and the variety of options open to the user, it is not possible to give more than a preliminary indication of a few selected outcomes.

2. Figure A1 illustrates a non-stochastic scenario with an initial asset margin of 60% of written premiums. The parameters have been chosen to give a reasonably stable development for the market as a whole, with the asset margin rising slowly to 63·5% of written premiums after 10 years. The company has 10% of the market by premium income in each liability type, an identical asset distribution to the market as a whole and the same characteristics in other respects, except that the company aims to set premiums 2% higher than the market level. As a result of this the company loses market share, but improves its profitability relative to the market. The company's asset margin grows to 73·9% of written premiums over 10 years.

3. If we then introduce variability of claim ratios and of total claim outgo each

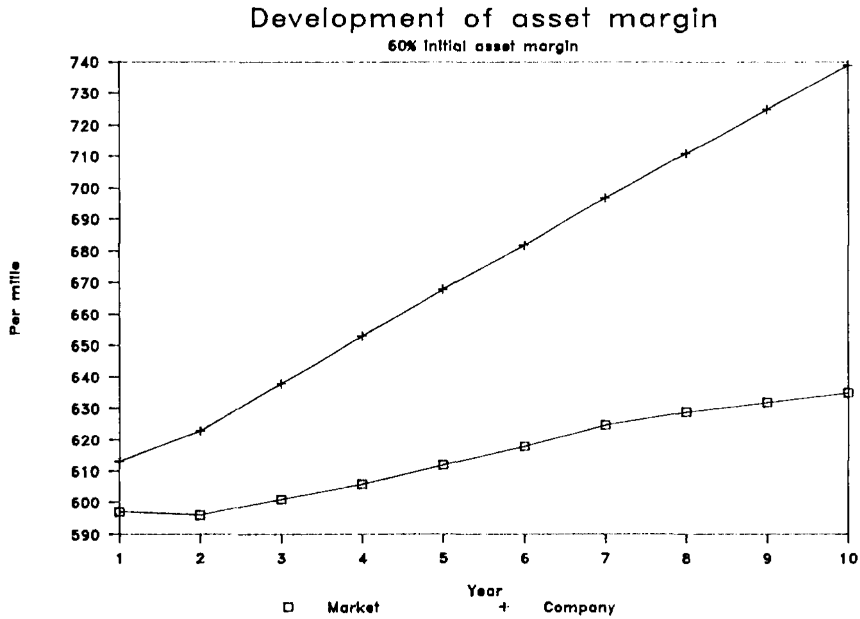

Figure A1.

year (IFSTOC = 1), the projected mean asset margin differs very little from the non-stochastic version. The mean asset margin for the market rises from 60% of written premiums to 63·2% over 10 years, whilst the mean asset margin for the company rises to 73·6% of written premiums. However, with 500 simulations we have a distribution of outcomes, with a standard deviation for the asset margin which rises to 2·9% of written premiums at the end of 10 years for the market as a whole and 4·2% for the company.

4. The mean level of declared profit in year 10 is much the same as for the non-stochastic case, at 9·5% of written premiums for the market as a whole and 11·5% for the company, but the distributions of profit arising from 500 stimulations give respective standard deviations of 1·2% and 1·6% of written premiums. Mean company worth falls marginally, from £4·71m in the non-stochastic case to £4·69m, with a standard deviation of £·37m.

5. Much more dramatic changes come when we introduce variability in the rate of inflation and in asset values. The models which have been described in the paper follow Wilkie (1986)[(3)] but with some changes to the parameters. In particular we have increased the mean and reduced the standard deviation in the inflation model, so as to reduce significantly the frequency with which negative inflation occurs in the projected realizations.

6. With stochasticity not only of claim ratios and claim outgo (IFSTOC = 1), but also of inflation and asset values (IFWILK = 1), the mean asset margin rises over 10 years to 68·1% of written premiums for the market and 79·6% of written premiums for the company, with respective standard deviations of 45·5% and 50·6%. The development of the mean asset margin is shown in Figure A2, together with the paths of the mean plus or minus 1 standard deviation. Figure A3 shows the distribution of the asset margin in year 5 for both the company and the market and Figure A4 shows the corresponding distribution for the overall profit. Figure A5 shows the development of the asset margin distribution for the company at the end of years 1, 3, 5 and 7.

7. Introduction of variability at this level produces a not insignificant probability of insolvency. On the basis of the statutory solvency requirement (16% of premium income), 2·2% of realizations for the company produce technical insolvency in year 5 and 6·4% in year 10. Over 12% of realizations produce a technical insolvency at some time in the first 10 years, but half of these have recovered by year 10. The figures for Companies Act insolvency are ·2% in year 5 and 1·8% in year 10.

8. Mean company worth is higher in the fully stochastic case, at £5·16m, compared to £4·71m in the non-stochastic case, but the variability is very high, with a standard deviation of £3·10m.

9. In Table A1 we give some summary results for a number of different parameter sets. Figure A6 shows the development of the mean asset margin over 10 years on a few of these scenarios (Nos. 1, 6, 7 and 8). Recovering past shortfalls (scenario 6) refers to the facility described in §§4.13 and 4.14(iii) whereby both the company and the market generally seek to recover shortfalls in

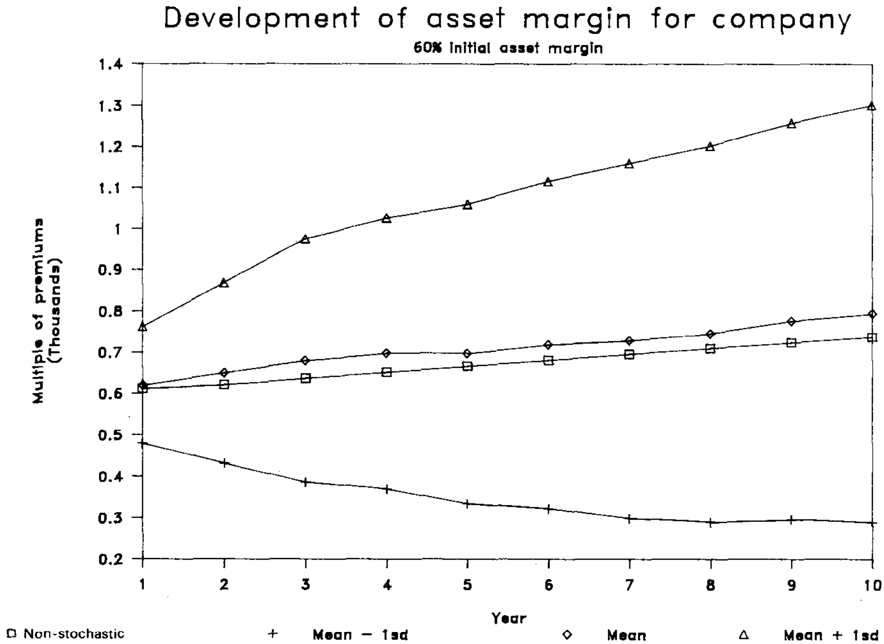

Figure A2.

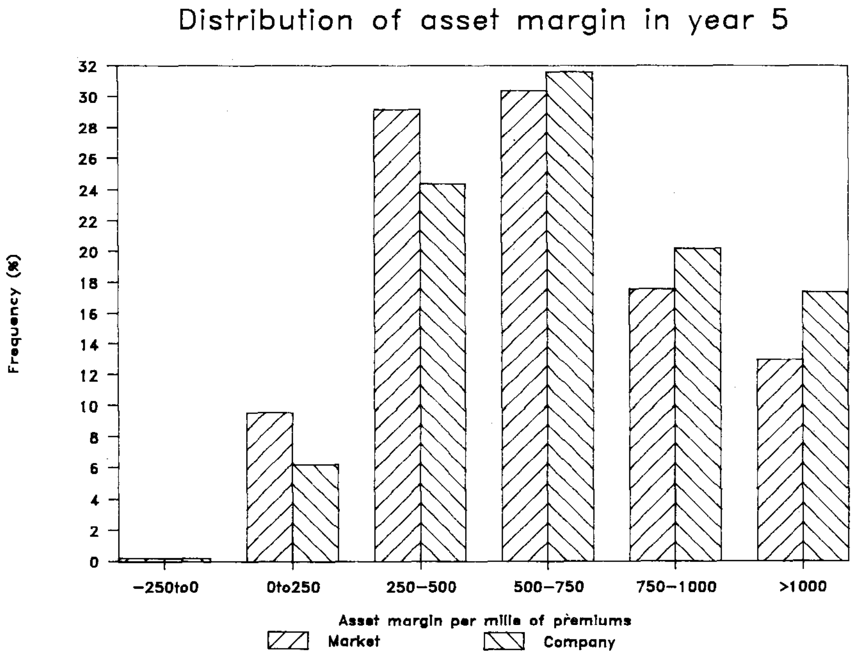

Figure A3.

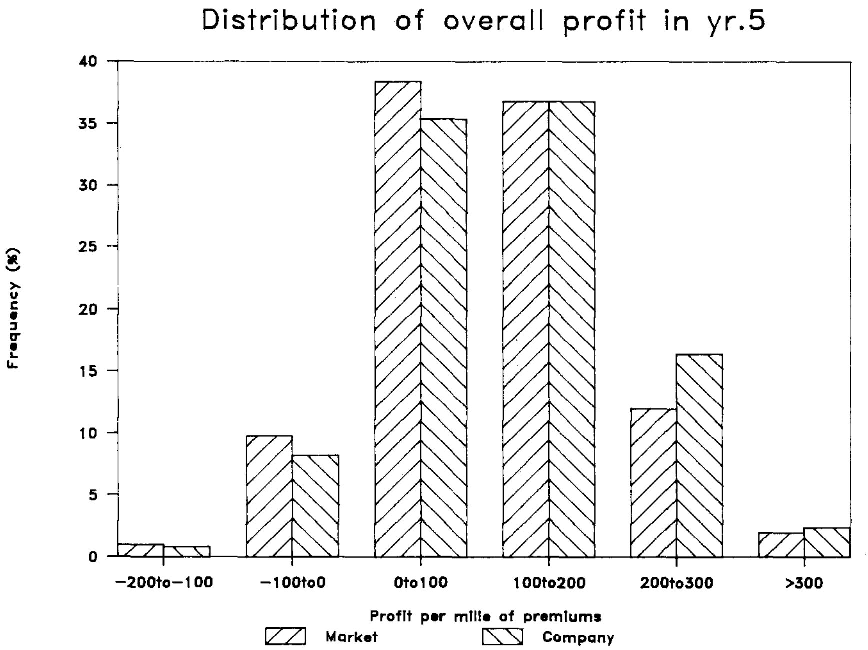

Figure A4.

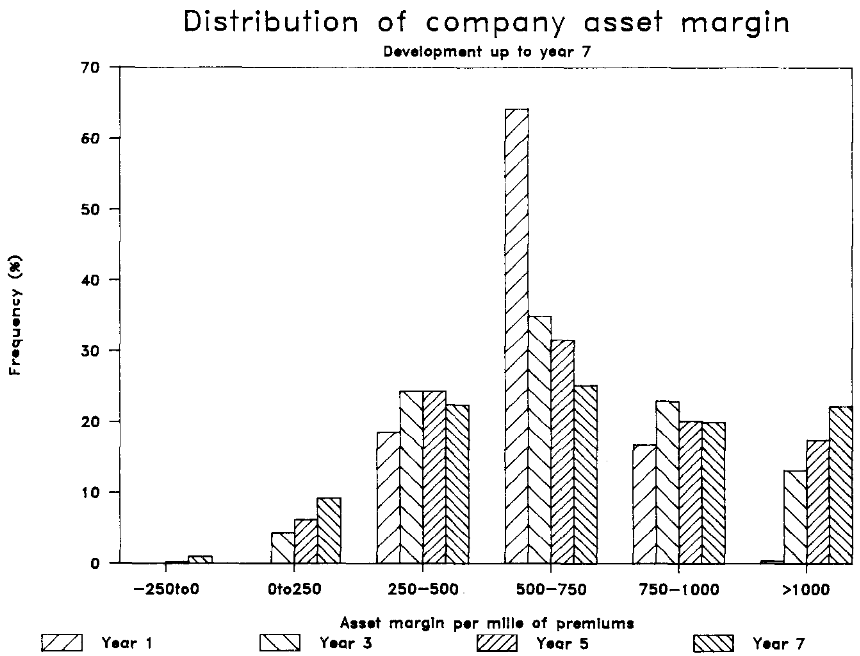

Figure A5.

Table A1. *Summary of key statistics for year 5*

Parameters	IFSTOC	IFWILK	Initial asset margin %	Mean asset margin Market %	Mean asset margin Company %	Frequency of insolvency (company) Statutory %	Frequency of insolvency (company) Cos. Act %	Company worth Mean £m	Company worth S.D. £m
1. Standard	1	1	60	63·9	69·8	2·2	·2	5·16	3·10
2. No asset variability	1	0	60	61·0	66·6	0	0	4·69	0·37
3. No claim variability	0	1	60	63·9	69·9	1·8	·2	5·17	3·09
4. Lower initial asset margin	1	1	40	41·9	46·6	10·4	1·4	3·86	2·79
5. Higher initial asset margin	1	1	80	86·5	93·6	·2	0	6·49	3·43
6. Recovering past shortfalls	1	1	60	64·6	70·5	1·2	0	5·27	2·84
7. Company aims 4% over market	1	1	60	64·6	76·7	·4	0	5·65	2·78
8. More 'conservative' asset distribution	1	1	60	48·8	54·2	9·0	1·4	4·88	3·07

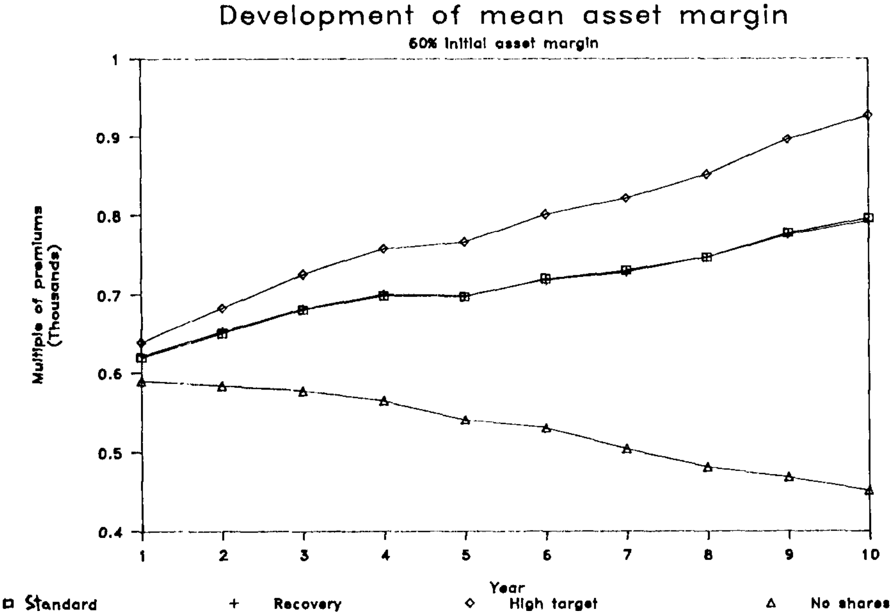

Figure A6.

the past year relative to their target profit and return surpluses relative to the target to policyholders. In this particular run the company seeks to recover (or return) somewhat higher percentages of the shortfalls (or surpluses) than the market.

10. Scenario 7 shows the situation where the company aims to set premiums 4% above the market level instead of 2%. Scenario 8 assumes that both company and market adopt a particularly conservative approach to investment, allowing investment only in cash or quite short-dated gilts to back the technical reserves, but with the asset margin invested in company shares. This produces a steady erosion of financial strength.

11. It is clear that this brief description cannot do justice to the extensive array of results which can be obtained. However, they should provide a glimpse of the possibilities.