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# MODELLING A WITH-PROFITS LIFE OFFICE

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## 1. INTRODUCTION

1.1 In this paper I intend to concentrate on the application of mathematics and the problems faced and (sometimes) solved by such application, rather than on the mathematics itself. That is not to belittle the mathematics in any way, and I would like to acknowledge at the start that the stochastic financial model which underlies the results given in this talk is based on Professor Wilkie's model as detailed in *T.F.A.* **39,** 341.

## 2. INITIAL PROBLEMS FACED

2.1 Those who have been involved with modelling an office will know what I mean when I say it is a salutary experience. The computerized actuary (and investment manager) need to be told *exactly* how they are to operate in a very wide range of financial conditions over a very long period of time. Certainly in my own office we have not yet attempted to program in a great deal of judgement; on the other hand the computerized actuary does not come face to face with the computerized marketing manager!

2.2 It may be useful to set down the main questions which have to be answered and this I have done with a few comments. I shall not consider all these questions here, but a possible approach is outlined in the Appendix for use in producing office model results which are discussed in Section 4.

*Data*

2.3.1 A profile of the existing business portfolio is required—whether this is the office's whole portfolio or a representative sample will probably depend on how an office's own approach to modelling has developed, the scope of research being undertaken and so on.

2.3.2 A profile of the existing assets is also required—this is not difficult for the whole portfolio, but it has to be related to the degree of sophistication of the financial model used. It is not easy, for example, to examine financial scenarios for a mix of overseas equities, allowing also for currency fluctuations. In the case of my own office this was regarded as a lesser priority aspect and assets have been categorized as either fixed interest securities (gilts) or United Kingdom equities. Where a large property portfolio, for example, is held this might dictate development of a financial model for this type of investment.

2.3.3 Policy asset shares are required—this can pose problems on a number of counts. Firstly, historical data may make it difficult to build up asset shares for all types of contract, for example paid-up policies and those which have been varied. Secondly, what *is* an asset share? Here it is necessary to link in with the office's bonus distribution policy, although complex asset share definitions can certainly complicate the programming for the office model!

2.3.4 New business profile—the main factors here are the distribution of the new business by term, options granted, and the rate of expansion in new business envisaged. This last point may well be a variable whose impact on the office the actuary is seeking to examine.

*Bonus Policy*

2.4.1 What is the target relationship between guarantees (basic benefit plus reversionary bonus) and terminal bonus (non-guaranteed)? Does it vary by individual policy terms or is it a global target? This aspect is a fundamental one and yet not one which has received much attention in published actuarial literature. Its selection impacts on other things, namely investment policy, an office's ability to expand and its solvency position. If the terminal bonus proportion is set too low all three may be unduly restricted; if set too high the merits of with-profits contracts (as opposed to unit-linked) may be jeopardized long-term. Inevitably this has to be a balancing act, taking into account, *inter alia*, an office's historical stance and market position.

2.4.2 How will the computerized office actuary decide the reversionary bonus rate to be declared each year in a wide range of conditions? Normally an actuary would carry out a number of investigations to assist him in formulating a recommendation on the level of reversionary bonus rates to be declared, often involving both a retrospective and a prospective analysis. We have found this impracticable to program and a relatively simple yet robust approach seems necessary. One approach is described in the Appendix.

2.4.3 How will the computerized actuary decide his terminal bonus rates? In terms of operating a computerized model the most readily available yardstick will be a policy's accumulated asset share, i.e. premiums invested after charges for expenses and life cover. The asset mix inherent in the asset share is fundamental, and for the modelling to be useful the asset mix needs to follow the underlying principles adopted by the office in its real world declarations, albeit without what might be termed marketing considerations. If these principles involve some form of asset matching as maturity approaches the programming for the model can become quite complex because the asset mix for outgoing policies (and one has to remember surrenders as well) does not reflect the total asset mix of the office. If a pro rata or global managed fund approach is being adopted this eases the problem, but then leads on to the necessity to consider what charge should be made for the guarantees provided.

As will be demonstrated later, with-profits guarantees can be valuable and a balancing act is required between guarantees on the one hand and investment

freedom on the other. A stochastic office model gives the actuary an opportunity to explore the trade-offs in some detail as an aid to formulating or modifying bonus policy.

Smoothing is an additional factor. How does the office actually intend to smooth out short-term fluctuations? The point for the model is that a specific rule has to be set down and it is difficult trying to find a successful balance which does not over-smooth and produce results too far away from current conditions—some forms of averaging can result in payouts moving in the opposite direction to financial markets. A possible approach (consistent with Wilkie's model) is to use a long-term dividend yield over-ride as a base for smoothing. From the financial standpoint, the basic point which needs to be allowed for is that smoothing of payouts increases the dispersion or range of results within which the office might lie.

*Investment Policy and Solvency*

2.5.1 The questions are simple—what is the office's policy and how will it vary over time? For the purposes of computerizing the policy the solutions are not easy. Bearing in mind that monetary guarantees are given, a life office will normally seek some stability in asset terms—just how much and when is linked in with its bonus policy as referred to in §2.4. Indeed, with a global matching philosophy, the types of asset held may be thought to be of less significance, but they cannot be judged in isolation from the office's bonus policy, the size of its free assets (the unallocated excess over asset shares), and, when looking ahead, its rate of new business growth. Solvency (what this really means, or should mean, is another major issue), bonus policy, investment policy and expansion are the four points of the actuarial compass in the with-profits world. Touch one and the others are all affected.

2.5.2 Solvency must be considered. Theoretically this could be defined as the office's continuing ability to give fair asset shares, after allowing for the effect of guarantees (which may require paying out more than the asset share in certain financial conditions). In reality, however, solvency is defined as an office's ability to meet the statutory minimum valuation basis plus any margins the actuary thinks necessary and to have sufficient additional assets to meet E.C. solvency margin requirements. As will be demonstrated later the peculiarities of this minimum basis, coupled with a net premium valuation method, can have a significant impact on an office's investment strategy.

2.5.3 The research work with which I have been associated has so far used Wilkie's investment model with assets confined to fixed interest securities (gilts) and U.K. equities. Early research started off with a fixed investment strategy, e.g. as used for the purpose of this paper every year assets are switched to give an overall holding (for the with-profits fund) of 80% equities/20% gilts. In the stochastic simulations generated by the model the statutory minimum valuation basis proved very severe, in particular the disallowance of future equity dividend growth for the valuation yield. (I should add, however, that Wilkie's model can produce significant dividend cuts.) The 7·2% restriction can also prove quite

severe. The office model needs to react to these aspects. On general grounds anyway it can be argued that the lower the margin between assets and liabilities the more restricted the investment policy needs to be. A relatively simple investment policy has been used for the production of the analyses referred to later on and this is described in the Appendix.

2.5.4 The choice of investment model is, of course, also fundamental. The advantage of Professor Wilkie's model is that it is an integrated one, providing not only gilt and equity yields and prices but inflation as well, which is very useful for dealing with future new business and maintenance expenses.

*Miscellaneous*

Other important components which need to be dealt with are now referred to briefly.

2.6.1 Should solvency reserves allow for mismatching requirements as well? This is a difficult point. In the real world an Appointed Actuary will be expected to deal with this, which will normally require reserves additional to the statutory minimum basis using current yields. On the other hand, to examine the office stochastically is to examine its ability to withstand financial shocks; to pile mismatching reserves on top could be far too severe a test. In my own office we have carried out our investigations without the incorporation of additional mismatching reserves.

2.6.2 New business will be written on in the future in financial conditions which may be markedly different from current conditions. What premium rates will apply? For unitized with-profits business the only variant may be the guaranteed interest rate roll-up. For conventional with-profits business, including single premium business, the position is more complex. Changing premium rates gradually to reflect changing conditions is difficult to program precisely! In practical terms it is probably necessary to set an extreme position beyond which premium rates would change, probably to support a minimum bonus level and to allow bonus rates to alter over time. Single premium business, if significant, may require a more active rating policy.

2.6.3 It is important to be able to incorporate lapse assumptions into the model—i.e. to allow for policies being surrendered or made paid-up, particularly when testing the effect of varying rates of new business expansion. Past lapse rates can provide a useful guide for this, but surrender bases (and paid-up policy terms) can be more difficult where terms will alter to reflect financial conditions. A percentage of asset shares, possibly with minima at short durations, is the simplest approach from the modelling standpoint. Paid-up policies, particularly for conventional with-profits business, can significantly increase the number of business cohorts in the model—again the office's approach to payouts on such policies is relevant.

2.6.4 Expenses and mortality need also to be considered and allowance made for how these might change in the future (e.g. inflation of expenses). Relevant here also is the assumption about average premiums per policy.

## 3. PURPOSES OF MODELLING

3.1 Section 2 (with apologies to Redington) provides a 'motorway dash' through the with-profits modelling environment rather than a ramble through the actuarial countryside. My own experience has been that many of the problems faced are interlocking and one can find oneself seemingly unable to break through this circle of problems. It is important, in order to progress, to knock these problems off in a sensible order, often finding apparently workable solutions which subsequently have to be re-examined or challenged.

3.2 Bearing in mind the nature of the joint seminar for which this paper is being written it seems worth recording some of the things an actuary might be wanting to explore and produce recommendations on. Some of these have already been referred to earlier.

3.2.1 Bonus strategy and the likely future pattern of bonus rates need to be considered. Over much of the 1980s we have enjoyed very high rates of investment return. It seems unrealistic to anticipate the continuance of such high rates of return into the future. Using an office model we can examine possible trends in both reversionary and terminal bonus rates and this can be very helpful when considering one's annual bonus declaration.

3.2.2 Investment strategy— what is, for example, the maximum proportion it is sensible for an office to hold in equities (or more volatile assets)? Seeking a precise answer in this respect is difficult. However, using an office model in conjunction with a stochastic financial model can be helpful.

3.2.3 How robust is the office to future shocks in the financial system? What is the impact on this of expansion, particularly expansion of with-profits business? Is, for example, unitized with-profits business much better than conventional with-profits business from a solvency standpoint? Some argue that it is, but I would sound a note of caution.

3.2.4 How much new business is it sensible for an office to take on? This is an essential question to examine—for example, solvency may not be an issue but investment policy could be adversely affected by too high a rate of expansion.

3.2.5 I referred earlier to the fact that with-profits contracts provide guarantees yet offices generally try to pursue as free an investment policy as possible. What price should be put on the guarantees and what is the trade-off between this and the likely gain from a freer investment policy? Varying charges can be tested against the theoretical position assuming negative terminal bonuses could apply.

3.2.6 For a proprietary company, variations in profits release can be examined, and indeed progress of what might be termed the office's 'embedded value'.

3.3 These and other aspects can be examined by running the office forward into the future. But if this is done purely in what I might call a 'deterministic' way the picture which emerges may appear unrealistically serene. It is very important, therefore, to be able to examine the future stochastically. On entering this world,

the problems referred to in Section 2 can pale into insignificance as one tries to grapple with all the information and results produced. Great care is usually needed in interpreting the results, but I would like to illustrate the sort of information and results which one can see from running an office forwards and these are considered in the next section.

## 4. SOME OFFICE MODEL RESULTS

4.1 Firstly, the office model demonstrated here is *not* an image of my own office, but has been selected to highlight some of the problems that emerge when one examines an office in a stochastic environment.

4.2 The choice of financial model can be regarded as crucial. I have used here Professor Wilkie's model, although some of the parameters have been altered to reflect more closely post-World War 2 experience. It is easy to reject many of the individual simulations or future scenarios derived from the model as unrealistic, just as one might well have rejected a prediction of the actual financial conditions over 1972 to 1975 made by someone in 1971. This point may seem trivial, but we have to be careful that we do not end up rejecting the simulations we do not like and only using the ones we do! Wilkie's model has been used for other actuarial research work, most notably by the Faculty of Actuaries Working Party on the Solvency of Life Assurance Companies (see *T.F.A.* **39,** 251) and by the Faculty's Bonus and Valuation Research Group (see *T.F.A.* **40,** 490). An actuary using this model may wonder what to do with his results. For example a probability of insolvency (however defined) emerges; it is unlikely to be Nil. What should he do?—what might be an acceptable risk of ruin by his peers? If set too severe, in the absence of much discussion about it, for example within the profession, he could advise severe restraints unnecessarily on his own office's actions. If he is too relaxed about some of the results and things go badly. . . .

This is an area where further debate would be welcomed. One can look back, for example, to the introduction of guidelines on the reserving for unit-linked maturity guarantees, using a stochastic approach. Of course many offices withdrew such guarantees from new contracts as a consequence. This may imply severity at too great a level. There seems much that needs to be debated in this area. I shall return to this aspect later.

4.3 Clearly the detail of the model office I have used is not wholly irrelevant to a full discussion of the results demonstrated, but here I have confined myself to describing the more important aspects in the Appendix, concentrating on the main problem areas referred to in Section 2. For simplicity I have shown results for an office writing conventional with-profits business only in the pension fund; I do not believe the generality of the results presented is affected by this simplicity of structure, although normally one would be looking at the office's portfolio as a whole.

4.4 Although one can examine results in a wide range of conditions the central trend line chosen is important when considering, for example, the likely level of

bonus rates in the future and the relative investment returns under gilts and equities. It makes sense, therefore, to examine the future under different trend lines; this increases the range of results to be considered. I have confined myself here to a single trend line, using a relatively higher average rate of return. The main parameters used are as set out in the Appendix. Results are shown looking over a 40-year period. The figures show 9 dotted lines, representing the worst simulation in any one year, the 5th, 10th and 30th worst, the median, and the 30th, 10th, 5th and highest result. Individual lines do not therefore represent any one scenario, a point which *always* has to be borne in mind when considering results this way. The solid line in each graph represents the zero variance experience—the 'serene' one I referred to earlier. Ninety-nine stochastic simulations are considered. (Normally a higher number would be preferable, but the total running time can become quite long for a complex model.)

*Figures 1–4 inclusive*

4.5 Figures 1–4 show the range of inflation rates, dividend yields, dividend growth rates and gilt yields generated by the model and the central trend lines. The periods of deflation can certainly be challenged, although the dividend cuts generated in some simulations have much more impact on results.

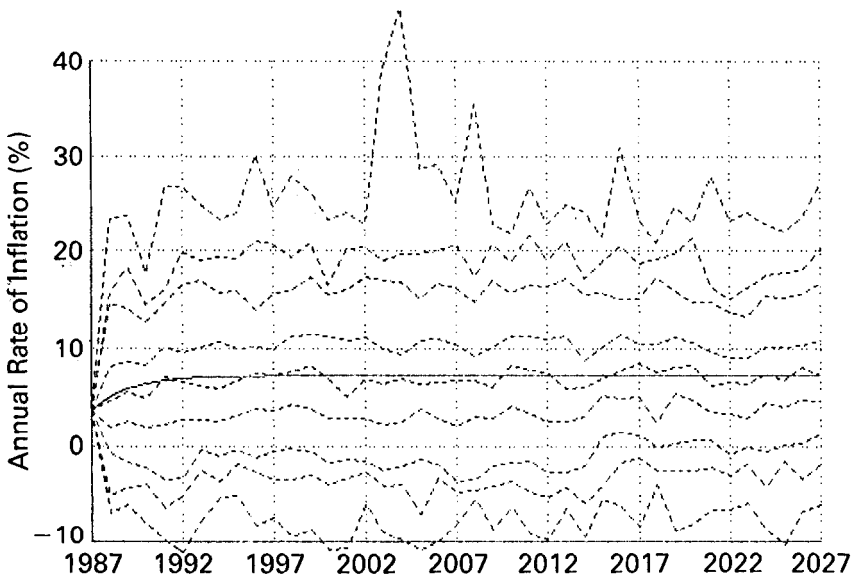


Figure 1.

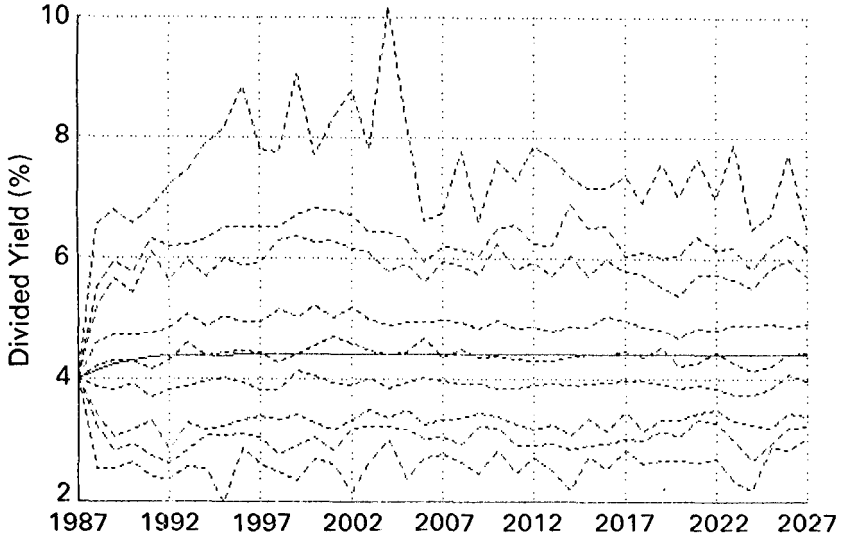


Figure 2.

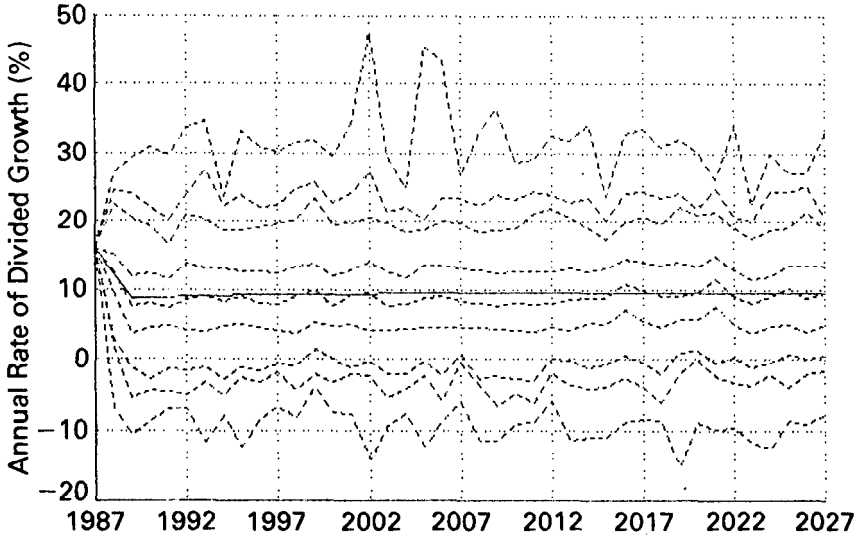


Figure 3.

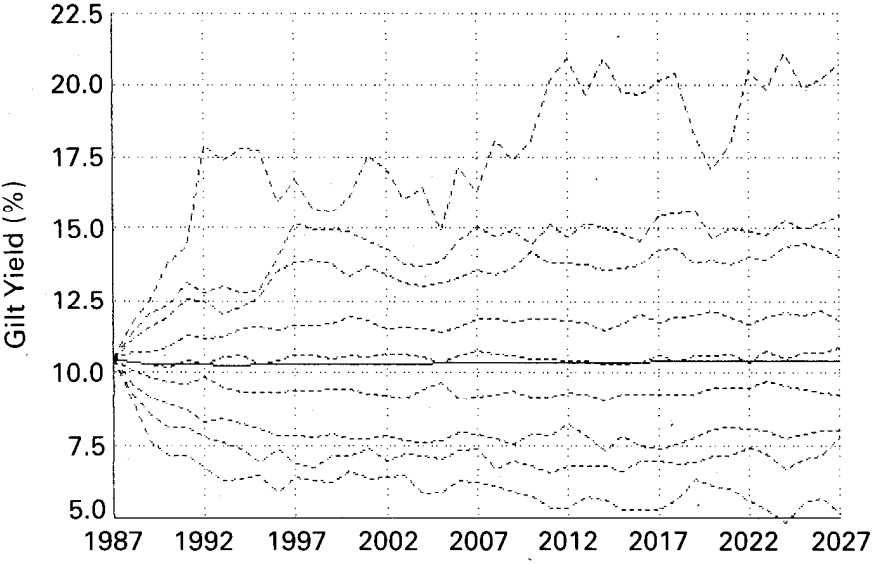


Figure 4.

*Figures 5–7 inclusive*

4.6 Figures 5–7 are based on the assumption that the office's investment policy is fixed (see Appendix), i.e. 80% of assets are held in U.K. equities and 20% in gilts and assets switched each year to restore this mix. I have chosen these proportions as not too dissimilar from the generality of leading with-profits offices' asset mixes for their with-profits funds and free or unallocated assets.

4.7 Figure 5 shows the progression of the asset to liability ratio again assuming the minimum valuation basis is used. There is a fall in $A/L$ initially for most simulations, mainly because of past new business growth. Much more relevant, however, is the rapid descent of the poorer simulations—the 10th percentile falling below 1·05 (rather than 1 to allow for the E.C. solvency requirements) around 1992, giving a new meaning to 1992! For most of the 1990s the 5th percentile remains below 1·05. I shall return to $A/L$ ratios later.

4.8 Figure 6 shows the ratio of total asset shares to liabilities, $AS/L$, and is an interesting illustration of the impact of bonus policy and valuation regulations on the need for free assets, since at least 10% of the time $AS/L$ is below 1. Remember, too, that this office has an overall terminal bonus target of 40% in mind.

4.9 Figure 7 looks at the office's ratio of total assets to total asset shares and should best be considered in relation to the earlier $A/L$ figure. $A/AS$ remains above 1. Remember that the office is effectively charging $1\frac{1}{2}$% of asset shares for

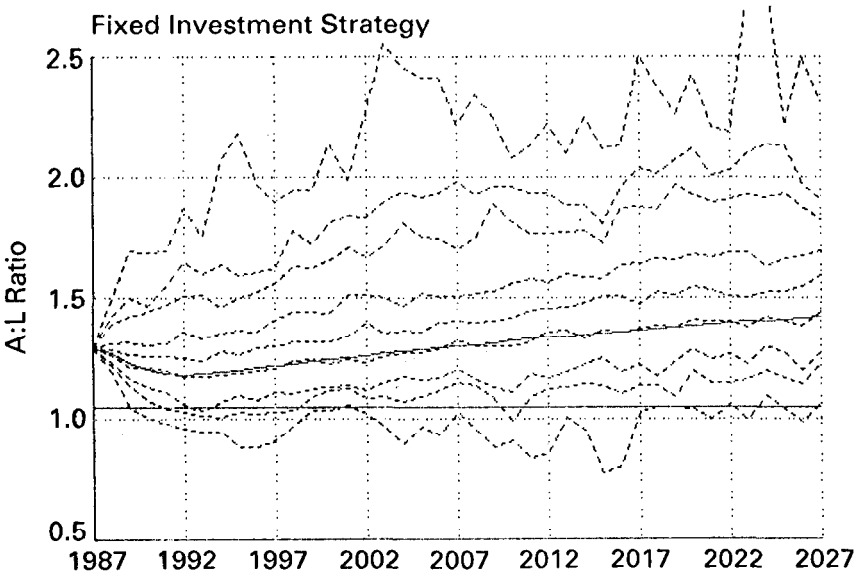


Figure 5.

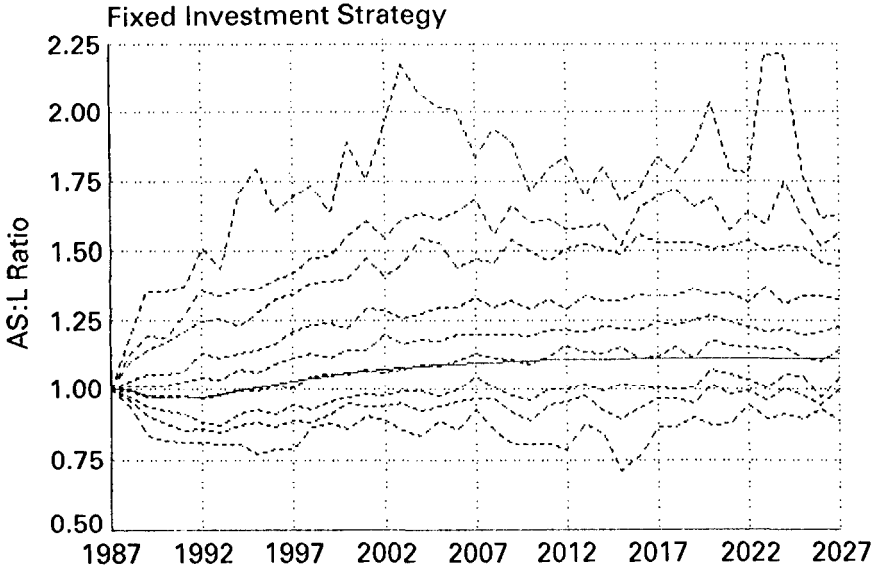


Figure 6.

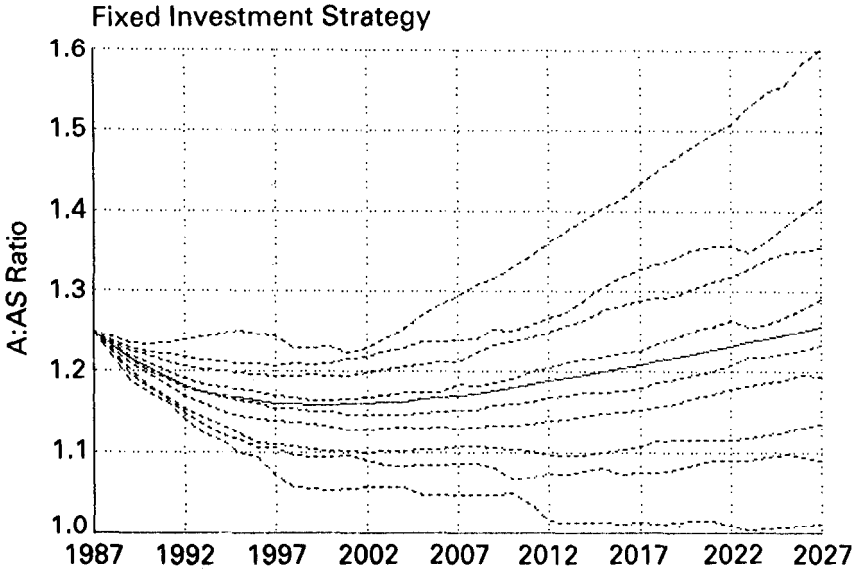


Figure 7.

Table 1.

| *Year* | *Number of simulations where office has been insolvent* | *Number of simulations where A/AS has been below 1* | *Year* | *Number of simulations where office has been insolvent* | *Number of simulations where A/AS has been below 1* |
|---|---|---|---|---|---|
| 1987 | 0 | 0 | 2008 | 32 | 0 |
| 1988 | 0 | 0 | 2009 | 32 | 0 |
| 1989 | 0 | 0 | 2010 | 34 | 0 |
| 1990 | 1 | 0 | 2011 | 34 | 0 |
| 1991 | 5 | 0 | 2012 | 35 | 0 |
| 1992 | 10 | 0 | 2013 | 35 | 0 |
| 1993 | 15 | 0 | 2014 | 36 | 0 |
| 1994 | 17 | 0 | 2015 | 36 | 0 |
| 1995 | 19 | 0 | 2016 | 37 | 0 |
| 1996 | 20 | 0 | 2017 | 37 | 0 |
| 1997 | 22 | 0 | 2018 | 38 | 0 |
| 1998 | 25 | 0 | 2019 | 39 | 0 |
| 1999 | 26 | 0 | 2020 | 39 | 0 |
| 2000 | 26 | 0 | 2021 | 39 | 0 |
| 2001 | 26 | 0 | 2022 | 39 | 0 |
| 2002 | 29 | 0 | 2023 | 39 | 0 |
| 2003 | 29 | 0 | 2024 | 39 | 0 |
| 2004 | 30 | 0 | 2025 | 39 | 0 |
| 2005 | 31 | 0 | 2026 | 39 | 0 |
| 2006 | 31 | 0 | 2027 | 39 | 0 |
| 2007 | 32 | 0 | | | |

the guarantees granted (see Appendix). True insolvency, it can be argued, certainly occurs if $A/AS$ falls below ·985. The ratio could climb back up from a lower figure as a result of receiving future premiums on existing business and surrender profit and also if the office is allowed to write on new business—it probably would not be. Of course the office may have to pay out guarantees for existing business worth more than ·985 of asset shares so it may have effectively passed the point of ruin earlier. Even so, the difference between $A/AS$ and $A/L$ is extremely marked. The poorest simulations never fall below 1 let alone ·985, although in some of the simulations a part of the free assets is eaten away by over generous guarantees in conjunction with the *rigid* investment policy.

4.10 I have commented already that the interpretation of results in this format must be handled with great care. If we look again at the $A/L$ ratios, but consider the number of individual simulations which actually fall below 1·05, 10 have done so by 1992, a total of 26 by 2000, 32 by 2007, 37 by 2017 and 39 by 2027. Out of 100 scenarios considered, the office would be insolvent in 39 of them—yet this office started in 1987 with free assets of 25% of the asset shares. What is even more significant is that $A/AS$ does not fall below 1 at any time over the 40-year period. Table 1 shows the progression of $A/L$ failures.

*Figures 8–10 inclusive*

4.11 Figure 8 shows the range of reversionary bonus levels. The steep early

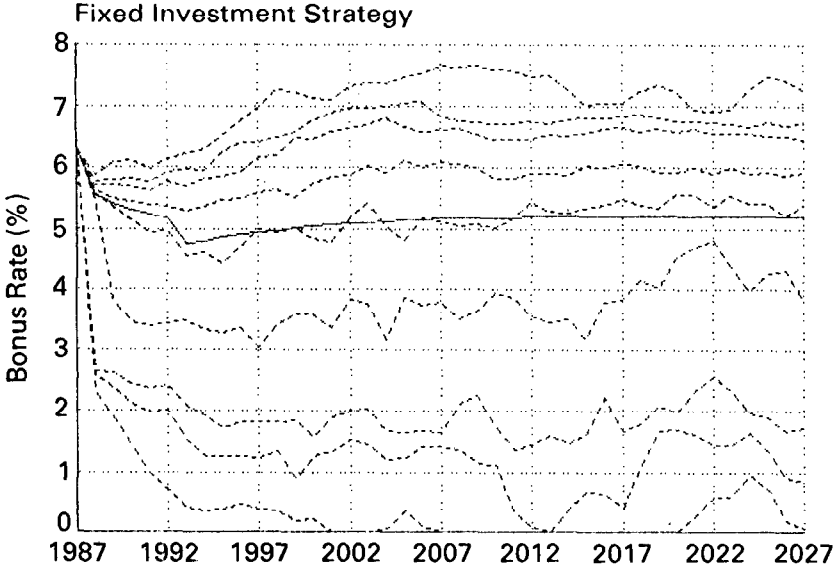


Figure 8.

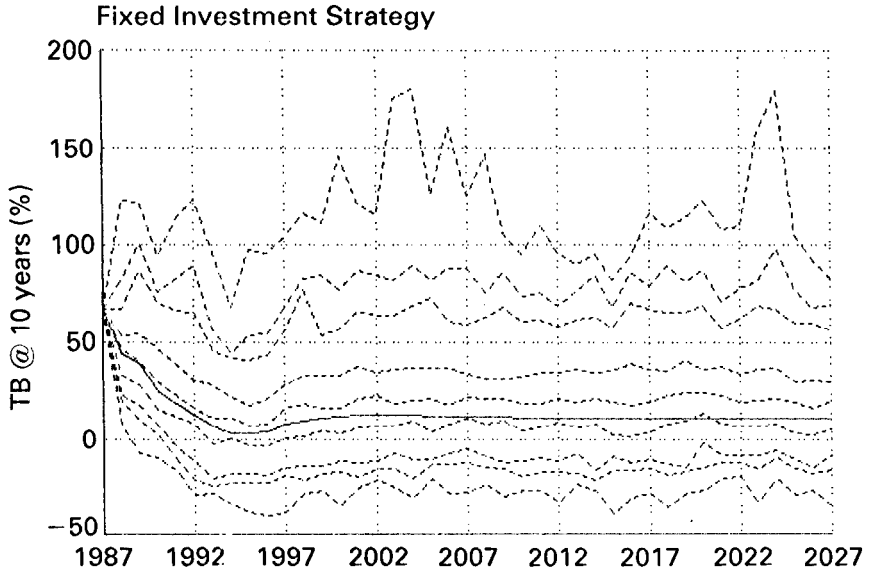


Figure 9.

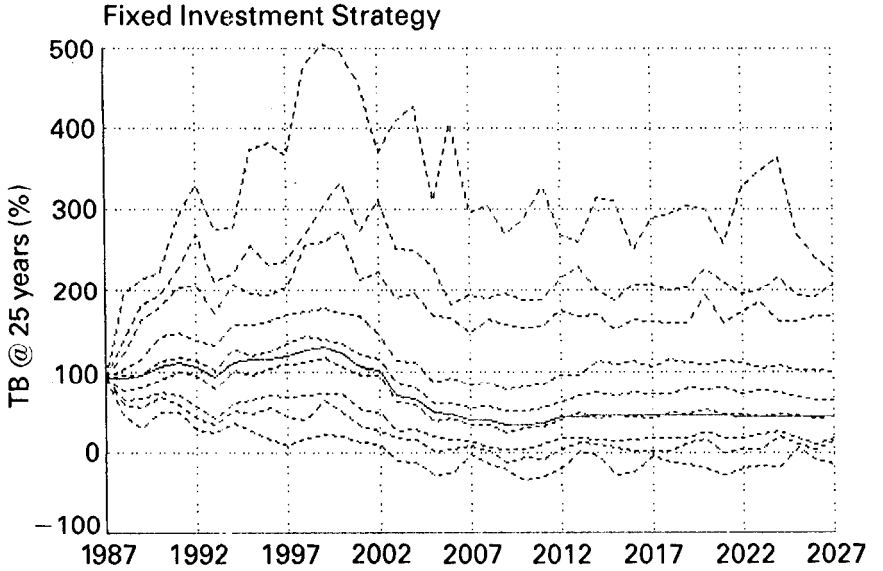


Figure 10.

decline for most simulations is interesting—the subsequent range of rates is very wide, confirming at least that consideration of the maintenance of a particular rate of reversionary bonus is somewhat pointless and perhaps foolish. Also underlying this graph is the relative volatility of bonus rates with the application of a very rigid rule, yet smoothing may increase the likelihood of insolvency.

4.12 Figures 9 and 10 show, for 10-year and 25-year terms respectively, the terminal bonus rates applicable (as a percentage of basic benefit plus attaching bonus). The frequency of negative (theoretical) terminal bonus rates seems alarming for the short-term contract, much less so for the longer-term contract. Obviously it explains how the office can run into difficulties and must lead one to question to some extent at least the use of the pro rata asset shares (or managed fund) approach to payouts unless the effective terminal bonus target can be kept at a reasonable level for all terms of with-profits business.

*Variable Investment Strategy: Figures 11 to 13 inclusive*

4.13 Clearly an office would not follow such a rigid investment policy as used in the preceding examination. Investment managers might anticipate some of the disasters—of course if all investment managers in all offices do so the market presumably collapses earlier rather than later. The other element, of course, concerns the level of the bonus declaration where in the real world judgement

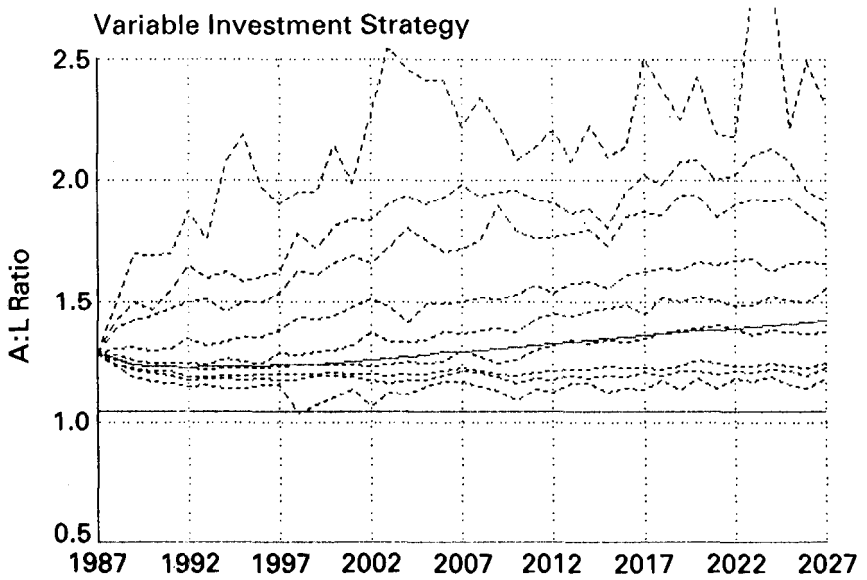


Figure 11.

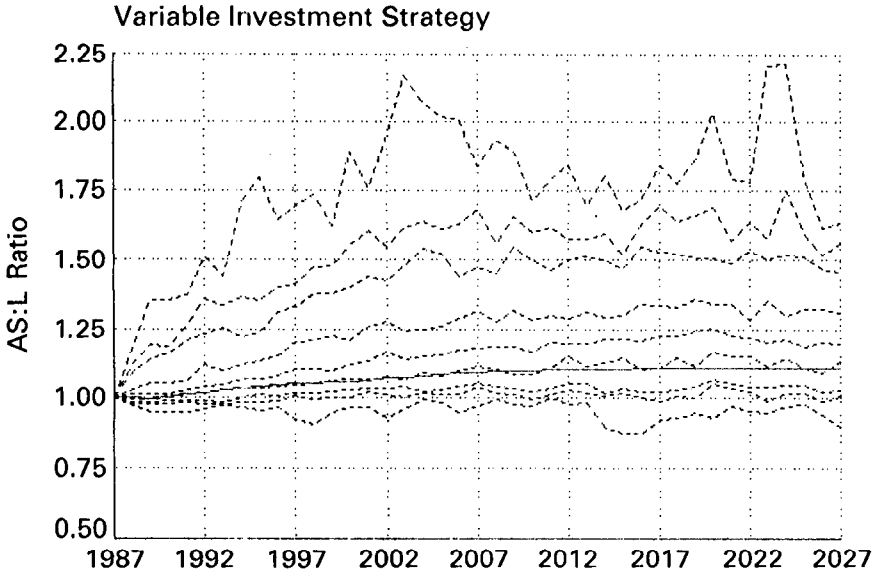


Figure 12.

would be exercised, but one might argue that judgement can be made badly, just as it can be made wisely.

4.14 However, we have to react in some way to the fact that in 39 of the simulations the office would fail the current statutory minimum valuation basis plus E.C. solvency margins. There are various ways of doing this, e.g. adopting a more rigid asset matching process, but I consider here the adjustment of the overall investment policy with the retention of the pro rata asset share approach to maturity payouts. The approach adopted, as described in the Appendix, is to switch progressively from equities to gilts. There are two reasons for this. Firstly gilts are presumed to be less volatile than equities and, the lower the free asset ratio, the more sensible it seems to be in less volatile assets. However, more importantly, under current valuation regulations $92\frac{1}{2}\%$ of the full gilt redemption yield can be used for the determination of the value of the liabilities, $L$. This may sound a little cavalier, bearing in mind that the net premium valuation method does not include an explicit allowance for future bonus. However, the very wide gulf between $A/L$ and $A/AS$ failures demonstrated already leads me (at least) to seriously question the validity of the minimum standard and hence argue that there are circumstances in which one has no option but to use the minimum basis to the full extent permitted. A better basis would be preferable, but that is too wide an issue to be addressed in detail here.

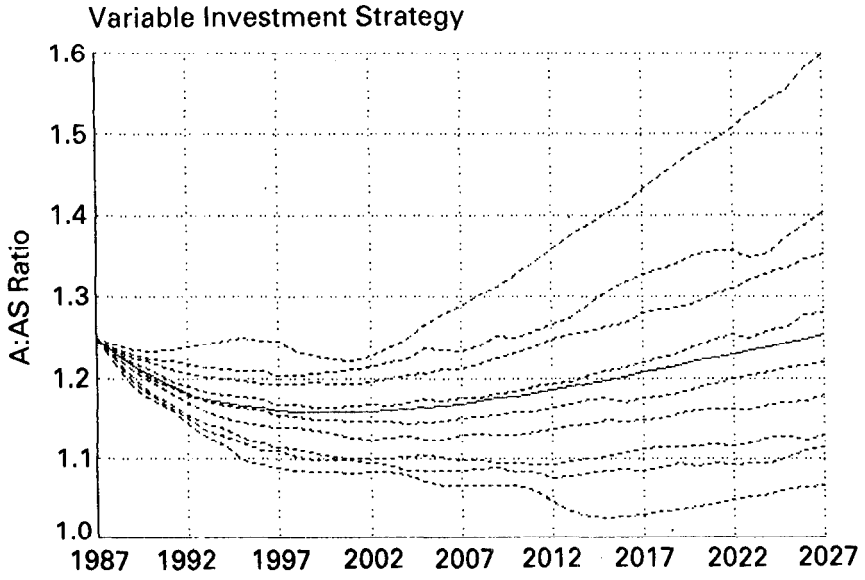


Figure 13.

4.15 In looking at Figure 11 we see an immediate improvement in the position. In fact only one of the simulations falls below 1·05 (just) and none below 1. Interestingly, though, $AS/L$, as seen in Figure 12, can fall below 1, so that even on this revised basis an office still needs some free assets to survive. The $AS/L$ position is very much improved, but again it must be remembered that we are aiming for a 40% terminal bonus target overall.

4.16 Figure 13, showing the progression of $A/AS$, again shows an improvement with no cases of 'true' insolvency. These figures tend to suggest that it is not the fact that the office is following a more broadly based matching philosophy which enables it to operate satisfactorily but rather that it is the yield restrictions in the valuation basis which tend to operate too severely too often. I accept, of course, that we cannot move to no rules at all.

4.17 The major problem is that at some stage the office is constrained in its investment policy, not by investment considerations but by the need to increase the minimum valuation yield in order to pass the solvency test. There are some situations where fixed interest securities should be held for what I call true solvency purposes but very many other situations where the need actually has appalling results for future payouts to policyholders and the future size of the office's free assets. That cannot be right.

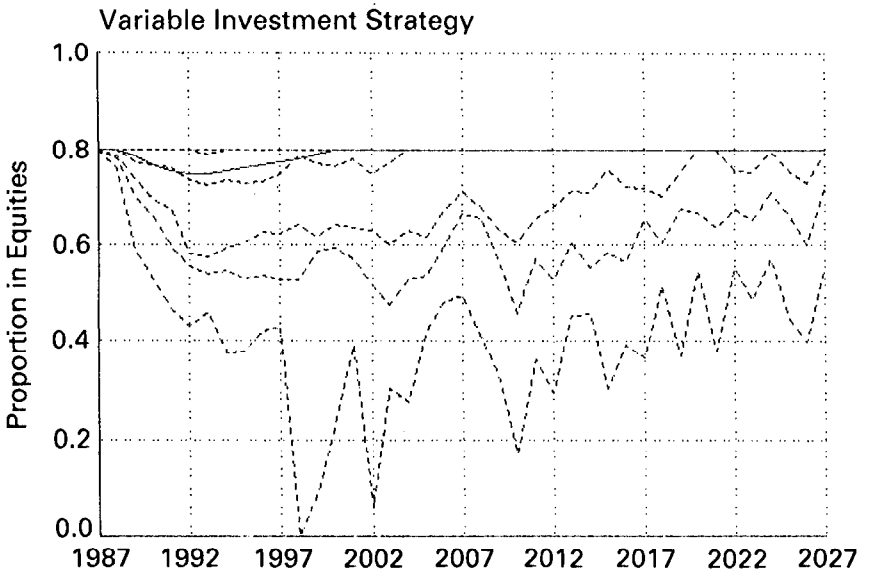


Figure 14.

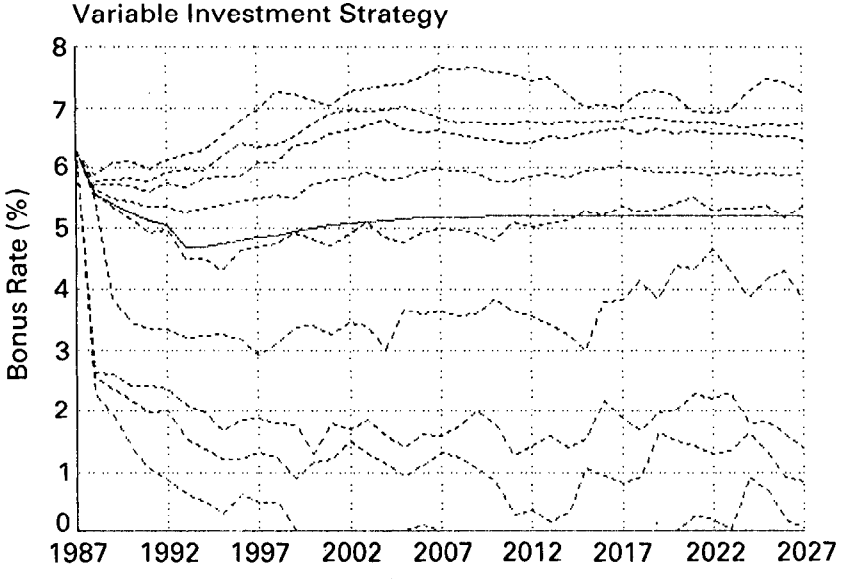


Figure 15.

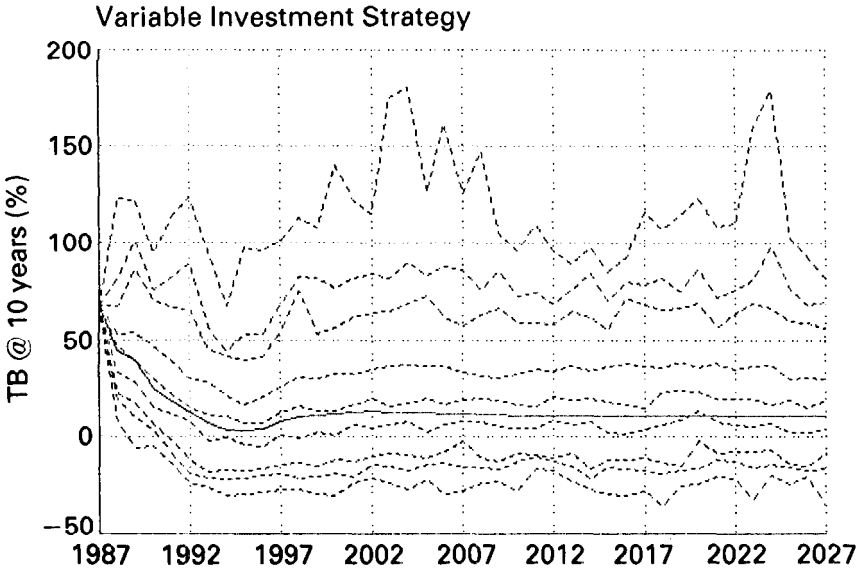


Figure 16.

*Figure 14*

4.18 The degree of investment constraint underlying the foregoing is illustrated in Figure 14. Between 1988 and 2004 around 30% of the simulations in any one year are restricted below 80% and around 10% in any one year restricted below 70%. This happens even although an overall terminal bonus target of 40% is being aimed at. (One wonders how many reversionary bonus declarations currently have such a high implicit margin, at least for business currently and recently written on.)

*Figures 15–17*

4.19 Figures 15–17 correspond to the earlier Figures 8–10. Interestingly, the frequency of zero reversionary bonus rates is increased on the variable investment policy, the marketing implications of which the computerized actuary conveniently ignores. The frequency of negative terminal bonus rates is little changed.

## 5. CONCLUSIONS

5.1 In considering various aspects of an office's with-profits strategy, bonus philosophy and future development plans the actuary can make much use of a

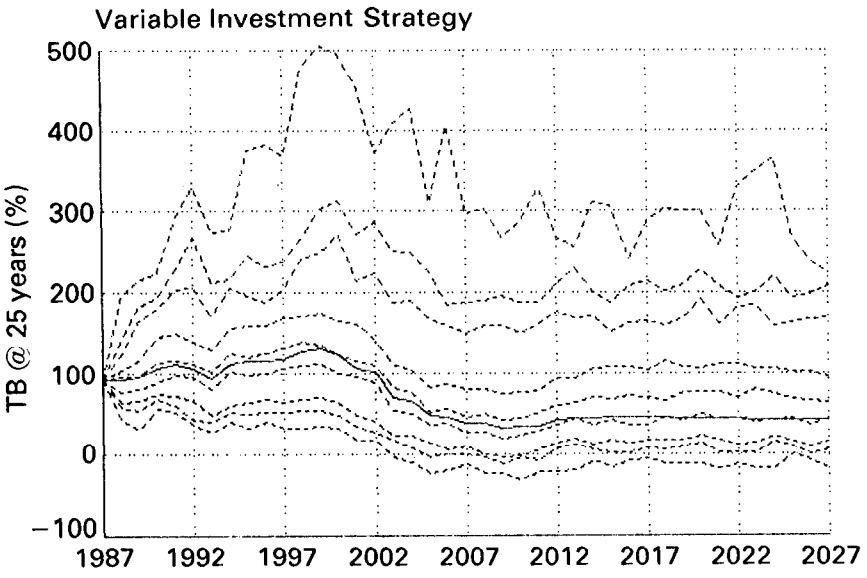


Figure 17.

model office. If future analyses are restricted to deterministic assumptions the actuary is likely to get too serene a picture and consequently a misleading vision of the future. Examining the future stochastically gives him a range of possible results. Although this is heavily dependent on the underlying financial model chosen it nevertheless gives him the opportunity to explore the consequences of various different bonus philosophies and rates of expansion. The consequences for solvency and investment policy are central to this, but here the actuary faces a number of difficult problems. For example, what degree of investment constraint might be acceptable—here the relative position of the competition is important but impossible to determine. This can be particularly difficult when considering possible rates of expansion, for example an office's ability to absorb increased new business flows as a result of recent developments in the pensions market. Is the pro rata asset share (or managed fund) approach really tenable?—if not, what should an office do as regards *present* distribution policy?

5.2 I hope this somewhat brief look at a with-profits office model has demonstrated one vital area where mathematics can be put to good use, even if some of the problems faced seem intractable. The actuary is certainly not lacking in information and this must enable the decision-making process to be improved. I think this applies not just to the financial management of an individual life office but can also apply, for example, to the more general considerations of solvency standards. Finally, it is important to continue to challenge and re-

examine one's approach as the research work is extended. For example, the impact on results of some aspects of the determination of reversionary bonus rates each year can be very significant and 'refinement' is necessary before relatively firm ground can be reached. Even then, the real world's practical constraints may make it very difficult, if not impracticable, to do what should be done. I would certainly welcome more discussion on many of these topics.

# APPENDIX

## MAIN FEATURES OF OFFICE MODEL

### A1 *General*

A1.1 Gross Pension Fund business only; conventional with-profits business only (mixture of annual and single premium business).

A1.2 Mutual office with reasonable new business growth, increased somewhat in the mid-1980s. Start period is 1987 with new business annual premiums approximately 20% of in-force premiums. Single premiums approximately 194% of annual premiums. The average term of annual premium new business is 18 years and of single premium new business is 13 years, ranging from 1 year upwards.

A1.3 From 1987 new business grows in line with inflation plus 5% real growth per annum; expenses rise in line with inflation (except new business expenses which are premium related).

A1.4 Liabilities valued on the minimum basis permitted by Insurance Companies Regulations, but no additional mismatching reserved held—see also Section A4. The office has free or unallocated assets. The ratio of total assets ($A$) to liabilities on the minimum basis ($L$) is 1·29 at 1987. The ratio of total assets to total asset shares ($AS$) of the in-force at 1987 is 1·25.

A1.5 Asset shares are calculated assuming a pro rata investment mix across the whole fund (i.e. including the free assets), less expenses and mortality costs.

### A2 *Bonus Policy*

A2.1 The office regards terminal (or final claims) bonus as an integral part of its system. In its reversionary bonus declaration it is aiming for a global terminal bonus target of 40% of basic guaranteed benefits plus reversionary bonuses, i.e. on average, the terminal bonus element of final payouts is about 29% of the total payout. The premium rate structure (not unlike most offices' conventional premium rates) is such that the likely terminal bonus content will vary by original term, being small at short terms and higher at long terms.

A2.2 The office follows an overall distribution policy based on 'pro rata' asset shares (see § A1.5). Smoothing of results has been ignored for simplicity (model investigations have shown that smoothing does increase the dispersion or range of results for the office). A charge of $1\frac{1}{2}\%$ is deducted from asset shares before calculating terminal bonus rates for payouts (subject to paying a minimum of guaranteed amounts); this charge is deemed to meet the 'cost' of the guarantees granted, bearing in mind that individual matching of assets to liabilities by term is not allowed for in the asset share calculations. Nothing else is retained for any benefits obtained from the presence of the free assets (e.g. from a freer investment policy).

A2.3 A single-tier reversionary bonus system is operated. Determination of

the reversionary bonus rate to be declared is automatic each year. Accumulated asset shares ($AS$) are determined each year for the in-force assuming premiums net of expenses have been invested in assets in the same proportions as apply for the fund as a whole. The asset shares are valued on a long-term basis, using for fixed interest stocks the long-term gilt yield assumed appropriate for the whole of the projection period, i.e. using the trend-line gilt yield used in the financial model—see Section A4, or, if lower, the average gilt yield over the preceding 5 years. Equity stocks are valued using dividends averaged over the five-year period preceding the valuation date (but not earlier than 1987), indexed to the valuation date at the average rate of dividend growth fixed at the outset for the financial model (or, if lower, the average rate of growth applicable over that five-year period) in conjunction with the long-term dividend yield assumption inherent in the model (see Section A4).

In discounting future cash flows (premiums less claims and expenses) the long-term yield assumptions inherent in the model are used, based on the actual asset mix at the date of the valuation. Thus for an asset mix 80% in U.K. equities and 20% in gilts the valuation yield used is 13·5%. However, as for the asset share valuation, this yield is modified to make some allowance for a period of relatively low investment returns. The average (geometric) investment return over the preceding 5 years is used if this is *lower* than the long-term valuation yield. For this purpose the investment returns used are the underlying gilt yields and equity returns allowing for changes in dividend levels but not market prices. Thus an element of judgement is built in to the valuation yield basis. In reality the actuary could use more discretion but, as commented in the paper, programming in discretion and judgement is very difficult.

[$AS$+discounted value (premiums minus expenses)]=[discounted value of future claims allowing for future reversionary bonuses] × 1·40.

Equality to these two terms gives the single-tier reversionary bonus rate to be declared at the current valuation. However, this bonus rate is first tested against the bonus rate supported by business within 5 years of maturity, assuming a *zero* terminal bonus target. If this latter bonus rate is lower, it is declared for all business.

A2.4 Separate terminal bonus scales are calculated for Annual Premium and Single Premium contracts. Paid-up Annual Premium contracts receive the Annual Premium scale.

A3 *Investment Policy*

*Policy I (fixed).* Assets totalling 80% are held in U.K. equities, 20% in fixed interest securities. Stocks with a 15-year term, at the start of each year, and a 10·5% coupon rate have been used in this model; 15-year stocks have been used as broadly representative of fixed interest holdings. Wilkie's model is based on 2½% Consols but I do not believe the results to be misleading because of this.

Indeed use of irredeemables produced similar results. Each year assets are switched to restore the 80%/20% mix.

*Policy II (variable).* This policy is used to illustrate the effect on valuation liabilities (assuming the minimum statutory basis is used) of holding a higher proportion of fixed interest stocks. The 80%/20% mix is permitted as long as the asset to liability ratio ($A/L$) is at least 1·25. If $A/L$ falls below 1·25 assets are progressively switched out of equities into gilts as the ratio falls towards 1·05 at which level 100% of assets are held in fixed interest stocks. This is a relatively simple rule which has been adopted to take account of two things:

(i) the general logic that the lower the margin between assets and guarantees granted the less volatile the assets held should be. Strictly this should require matching of stocks by term where possible, but again this leads to a very complex model indeed;

(ii) the 'peculiar' nature of the statutory minimum net premium basis which allows use of $92\frac{1}{2}\%$ of the gross redemption yield for gilts but only $92\frac{1}{2}\%$ of the (current) equity dividend yield with no allowance for growth. It can be argued that this is far too severe, although conversely the net premium method makes no specific allowance for future bonus. By moving all the way to 100% in gilts at $A/L$ equals 1·05, the yield assumption is maximized. After the bonus declaration liabilities are calculated on an 80/20, and 0/100 equity/gilt mix and the asset liability ratios calculated. If the 80/20 ratio is below 1·25 the proportion in equities is altered to '$p$' where $A/L\ (p) = 1{\cdot}05 + P/4$, subject to a minimum value for $p$ of *nil*.

(It is, of course, necessary to test the practicality of any such switching basis.)

A4 *Financial Model*

The office model simulates future financial conditions using Professor Wilkie's model as described in *T.F.A.* **39,** 341–403.

For the purpose of the model runs demonstrated here the following long-term assumptions have been made:

| | % |
|---|---|
| (Force of) Inflation | 7 |
| Real dividend growth (per annum) | 2 |
| Dividend yield | 4·40 |
| Resulting long-term gilt yield (gross) | 10·5 |
| Resulting long-term equity return (gross) | 14·25 |

As I wrote earlier, for decisions on bonus rate levels a variety of different long-term assumptions may need to be made. (The above assumptions seem optimistic in terms of investment returns.)

Standard deviations have been altered from those on the full standardized basis. The main changes are detailed in Table A1.

Table A1.

| *Item* | *Parameter* | *Value used* |
|---|---|---|
| Dividend Yield | *YSD* | ·16 |
| Dividend Growth | *DSD* | ·05 |
| | (*DMU* | ·02) |
| Consols Yield | *CSD* | ·16 |

A5 *Solvency*

Solvency is judged using the statutory minimum basis allowing for Zillermization, but with no mismatching reserves and no E.C. solvency margin. An $A/L$ ratio below 1·05 is likely to result in failure to meet the E.C. solvency margin requirements.

A6 *Lapses*

Executive Pension policies, about 45% of the portfolio, are assumed to be surrendered at an effective rate of $2\frac{1}{2}\%$ per annum and made paid-up (all annual premium contracts) at a rate of 4% per annum.

Surrender values, broadly, speaking, are taken as 90% of asset share, rising to $98\frac{1}{2}\%$ over the last third of the term.

A7 *Premium Rates*

Annual premium rates remain stable until the bonus earning power (no terminal bonus) falls below 1% per annum, assuming that future investment returns equal the then current gilt yield throughout, in which case the rates are altered to support the 1% bonus level. Single premium rates are recalculated annually on the same assumptions to ensure they support the same level of bonus as annual premiums for the same term.

A8 *Office Model Sequence of Events*

For simplicity of operation the model assumes all income and outgo take place annually and events are cycled around a single transaction instant. The sequence of events is as follows:

(1) Generate values of gilt yields, dividend yields, dividend index, the yield for index linked gilts and the inflation index.
(2) Receive investment income.
(3) Update notional asset share unit prices.
(4) Determine reversionary bonus rate and add reversionary bonuses to the in-force.
(5) Determine terminal bonus scales.
(6) Pay death claims.
(7) Pay maturity and retiral claims.
(8) Pay surrenders.
(9) Calculate asset to liability and asset share to liability ratios, determine asset mix.

(10) Write on new business.
(11) Receive premiums for new in-force (i.e. after all claims off and new business on).
(12) Incur expenses—initial and renewal.
(13) Make asset sales, make asset purchases.
(14) Calculate proportion of assets in equities, etc., ratio of assets to asset shares.*

* There is a timing difference as a result between $A/L$ and $A/AS$ ratios.