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Mortality and Smoking Prevalence

A Mortality Model for Multiple Populations

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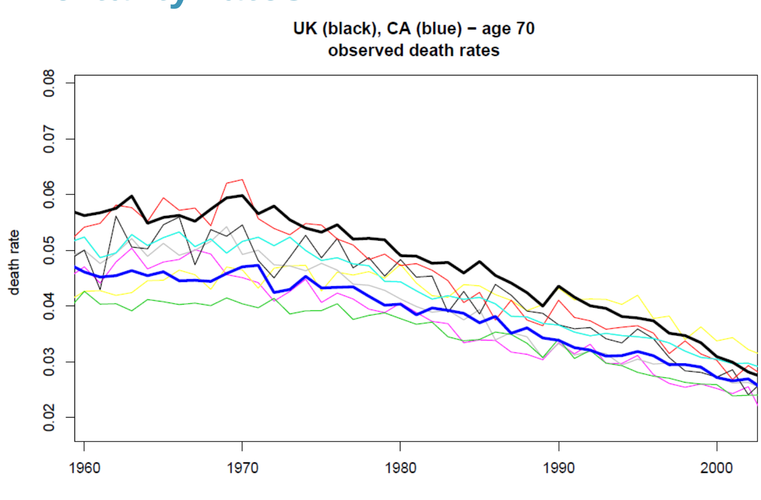
Mortality rates



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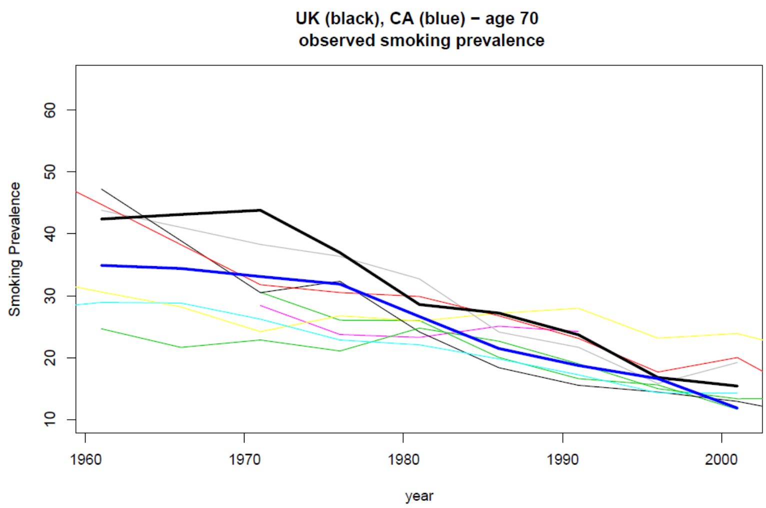
Smoking prevalence



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Objectives

- Develop a consistent model for the mortality experience of multiple populations that can be used for scenario generation
- Explain differences between mortality rates in different populations
- Identify common factors influencing mortality in a number of populations

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Building a model – Available Data

$D_i(x,t)$: Number of deaths,
$D_i^N(x,t)$, $D_i^S(x,t)$ for non-smokers, smokers (not observed)
$D_i(x,t) = D_i^N(x,t) + D_i^S(x,t)$

$E_i(x,t)$: Exposure-to-risk

$m_i(x,t)$: $= D_i(x,t)/E_i(x,t)$,
$m_i^N(x,t)$, $m_i^S(x,t)$, death rates

$s_i(x,t)$: Smoking prevalence, in $[0,1]$,
the number of smokers is $s_i(x,t)E_i(x,t)$

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Building a model

Total number of deaths:

$$\begin{aligned} D_i(x,t) &= D_i^N(x,t) + D_i^S(x,t) \\ &= m_i^N(x,t)\left[1 - s_i(x,t)\right]E_i(x,t) + m_i^S(x,t)s_i(x,t)E_i(x,t) \end{aligned}$$

Death rates:

$$m_i(x,t) = \frac{D_i(x,t)}{E_i(x,t)} = m_i^N(x,t) + \left[m_i^S(x,t) - m_i^N(x,t)\right]s_i(x,t)$$

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Building a model

Modelling assumptions:

- Smoking prevalence has the same effect on mortality rates in all observed countries.
- Total mortality in country *i* is the weighted average of non-smokers' and smokers' mortality adjusted by a “country effect”

$$m_i(x,t) = \left[\mathbf{m^N(x,t)} + \left[\mathbf{m^S(x,t)} - \mathbf{m^N(x,t)}\right] s_i(x,t)\right] \exp(\Gamma_i(x,t))$$

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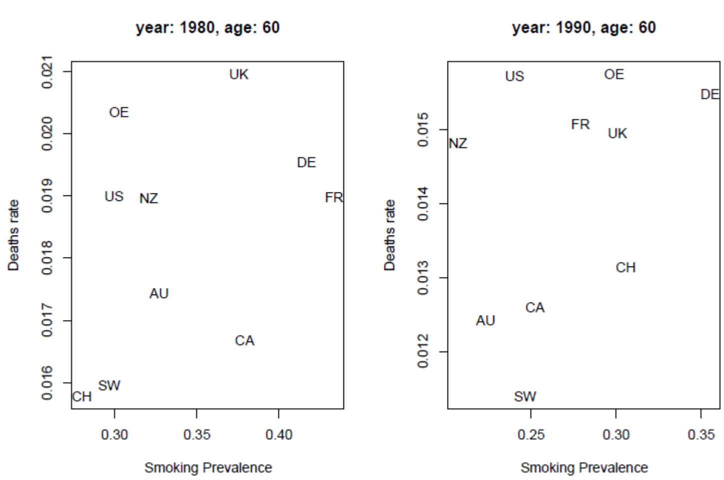
Linear Regression



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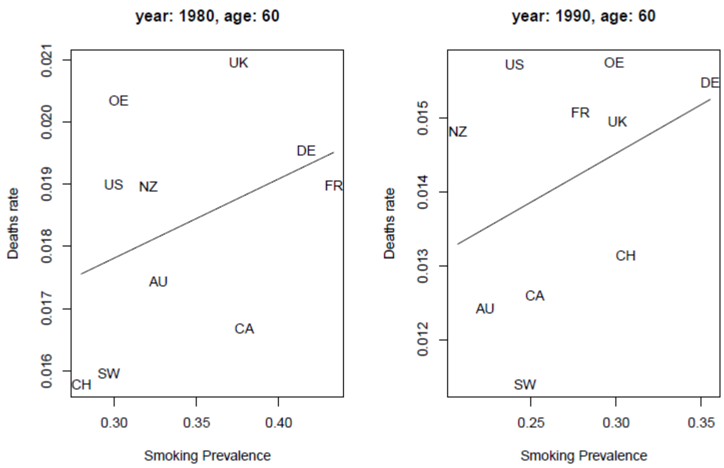
Linear Regression



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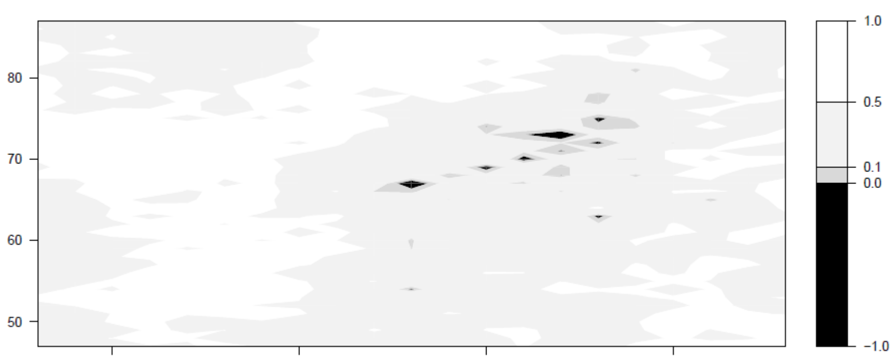
Correlation between smoking and mortality



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Adjusted Maximum Likelihood Estimation

Model:

$$m_i(x,t) \;=\; \left[\mathbf{m}^{\mathbf{N}}(\mathbf{x},\mathbf{t}) + \left[\mathbf{m}^{\mathbf{S}}(\mathbf{x},\mathbf{t}) - \mathbf{m}^{\mathbf{N}}(\mathbf{x},\mathbf{t})\right] s_i(x,t)\right] \exp(\Gamma_i(x,t))$$

Assume country effects are normally distributed

$$\Gamma_i(x,t) \sim N\left(0, \sigma^2_\Gamma(x,t)\right)$$

We assume that annual changes in the log mortality rates μ^S and μ^N for a given cohort are positively correlated.

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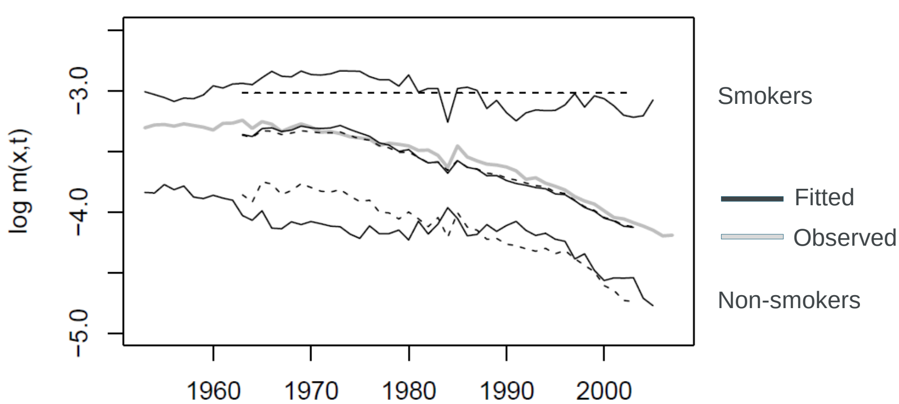
Log death rates at age 65 in the UK



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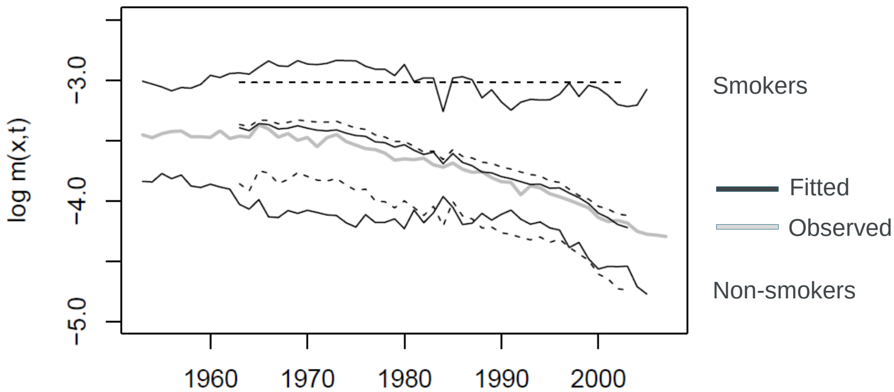
Log death rates at age 65 in the Canada



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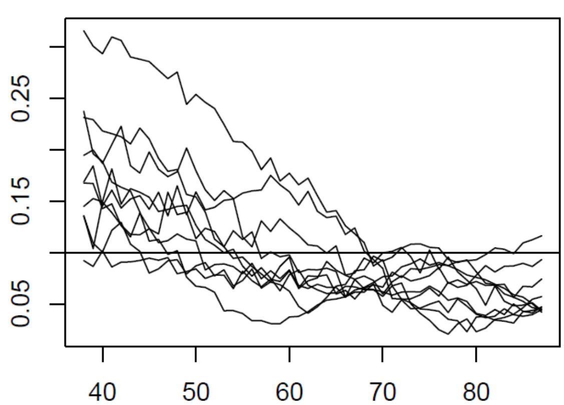
Average country effect for different ages

$$m_i(x,t) \;=\; \Big[\mathbf{m}^{\mathbf{N}}(\mathbf{x},\mathbf{t}) + \big[\mathbf{m}^{\mathbf{S}}(\mathbf{x},\mathbf{t}) - \mathbf{m}^{\mathbf{N}}(\mathbf{x},\mathbf{t})\big]\, s_i(x,t)\Big] \exp(\Gamma_i(x,t))$$



Average absolute value of the country effect for all ten countries as a function of age. The average is taken over all years for which smoking prevalence data are available.

Smoking explains mortality at old ages well compared to young ages

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Possible Applications

Scenario generation for future mortality using stochastic models for non-smokers' and smokers' mortality rates, and assumptions (or models) for smoking prevalence

- Risk management of pension funds
- Assessment of basis risk
- Quantifying hedge ratios for longevity hedges as the mortality experiences of two populations can be modelled simultaneously
- …

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Conclusions

- The empirical results indicate that there is a significant downward trend in non-smokers' mortality rates while the rates for smokers seem to fluctuate around a constant level.
- model fits the observed mortality rates rather well
- Impact of country effect is lowest for high ages
- However, we have also considered implied cessation rates, and they indicate that we underestimate high age non-smokers' mortality
- The methodology can be applied to other factors

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Papers

- Mortality and smoking prevalence: An empirical investigation in ten developed countries, 2013, *British Actuarial Journal / FirstView Article*, pp 1-15
- Mortality and Smoking Prevalence, A Mortality Model for Multiple Populations, 2013, unpublished

spin-off's (in preparation):

- A Common Age Effect Model for the Mortality of Multiple Populations
- Modelling England & Wales Mortality with Smoking Prevalence as a Covariate

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