MORTALITY OF ASSURED LIVES

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INTRODUCTION

THE standard table of mortality most generally used in Britain at the present time for life assurance calculations is the A 1924–29 table, which was derived from the experience of assured lives during the period 1924–29. During the quarter of a century that has elapsed since that time there have been substantial changes in mortality rates and the A 1924–29 table is to-day out of date. The publication of the A 1924–29 Light table, based upon the experience of certain selected offices whose mortality was lighter than average, has no doubt been helpful in providing a table which goes some way in the direction of the lower mortality rates of to-day. However, this table suffers from the disadvantage that, while the mortality rates at young ages are not low enough for current experience, at some of the older age-groups the rates are too low, with the consequence that the shape of the mortality curve does not accord with present conditions. The greatest proportionate reductions in mortality since 1924–29 have occurred at the younger ages, and there can be little doubt that a new table is needed to provide offices with an efficient and up-to-date instrument for life assurance calculations.

CHOICE OF DATA FOR A NEW TABLE

2. Statistics relating to the mortality experience of male assured lives accepted at ordinary rates are assembled from year to year by the Joint Mortality Investigation Committee from returns submitted by the contributing offices. The data are subdivided into eight classes of business, namely:

Whole Life With-Profit Medically Examined.
Whole Life Non-Profit Medically Examined.
Endowment Assurance With-Profit Medically Examined.
Endowment Assurance Non-Profit Medically Examined.
Whole Life With-Profit Non-Medical.
Whole Life Non-Profit Non-Medical.
Endowment Assurance With-Profit Non-Medical.
Endowment Assurance Non-Profit Non-Medical.

In considering a basis for a new table (or tables), the first task of the Joint Mortality Investigation Committee was to select the period and the classes of business for which the data should be utilized and to consider whether the data should be subdivided in any way—e.g. between 'light' and 'heavy' offices—with a view to producing alternative tables to suit varying needs.

3. Over the period 1940–47, the data in the C.M.I. were abnormal in so far as they included deaths due to the Second World War. Some war deaths were recorded in 1946 and 1947 because claims under policies on the lives of deceased prisoners-of-war were still being reported in those years, and consequently 1948 was the first year for which the data represented normal civilian mortality. In 1947, however, mortality in Britain among the general

population was heavy, whereas in 1948 it was exceptionally light; and there is reason to presume that these two features were interdependent in so far as the adverse conditions in the early months of 1947 may have precipitated many deaths which otherwise might not have occurred until the following year. Since 1947 had to be excluded it seemed wise to exclude 1948 as well and to commence with the experience of 1949. At the time when the Committee began to consider the construction of the new table, the experience for 1952 had just become available and the period 1949–52 commended itself as a suitable basis. The years 1949 and 1950 had a very similar experience, intermediate between 1947 and 1948. In 1951, mortality at the middle and older ages was heavy on account of an influenza epidemic. The year 1952 was lighter than 1949–50 but not so light as 1948. Epidemics and 'heavy years' occur from time to time, and one heavy year combined with one fairly light and two moderate years seemed a reasonable and prudent basis for tables designed for use in the conduct of life assurance.

4. When determining a suitable period, it is proper to consider the observed statistics in the light of long-term secular trend. At younger ages, male mortality has been steadily declining for many decades, and there is no reason at present to suppose that the decline has ceased. At ages over 55, however, there has been little variation in male mortality rates since 1942, and this is true both for the general population and for assured lives. The C.M.I. data (all classes combined, durations 3 and over) yielded the following percentages of actual deaths to deaths expected by the A 1924–29 table during the years 1949–52:

Year	Age-group		
	Under 45	45–75	Over 75
1949	54·1	82·2	88·0
1950	53·1	83·7	87·4
1951	49·5	87·5	93·5
1952	50·5	81·4	85·4
1949–52	51·7	83·7	88·5

While the two older groups in this table reflect what has been said in paragraph 3 about the relative experience of the four years, the youngest group exhibits a different trend, which suggests that improvement in mortality rates at young ages is still continuing.

5. The basic period of the table having been determined, the next step was to decide whether the whole of the data should be employed, or whether the experience should be limited to certain sections. Tables 1–8 show, for each of the eight classes of lives during the four-year period, the exposed-to-risk at durations 3 and over, the actual deaths and the observed rates of mortality; while Table 9 gives similar information for all classes combined. To facilitate comparison, table 10 has been compiled and shows, in age-groups, the percentages of actual deaths to deaths expected by the A 1924–29 table. From this table it can be readily seen that the Whole Life is heavier than the Endowment mortality and that the Non-Medical is heavier than the Medical mortality. It would be difficult to construct separate complete tables for Whole Life and Endowment assurances, because the data are insufficient in

the Whole Life class at the young ages and in the Endowment class at the older ages. The most promising basis for differentiation would thus appear to be a subdivision of the data into Medical and Non-Medical business.

6. There is, however, a further method of analysis which the Committee felt it to be essential to examine. Investigation of the experience of individual offices showed beyond doubt that differences between offices provide by far the largest source of variation,* and, moreover, that when comparisons are made between the Medical and Non-Medical business of individual offices the margin between the mortality experienced by these two classes of business greatly diminishes. It seems, in fact, that a high proportion of the total Non-Medical business is transacted by offices whose mortality experience is heavier than average in all classes. Consequently, one reason why the Non-Medical experience is heavier than the Medical experience is that it contains a higher proportion of data from 'heavy' offices.

7. Differentiation according to participation or non-participation in profits was also considered. Within the Endowment class there seems to be a significant difference between the With-Profit and Non-Profit experience, the Non-Profit mortality being considerably lighter. In the Whole Life class, however, the With-Profit and Non-Profit experiences resemble each other more closely. While it may be that Non-Profit Endowment assurances represent a specially select class of business, it is probable that the differences between offices noted above in paragraph 6 would have a significant influence on all forms of inter-class variations and there seemed little advantage in further pursuing the question of differentiation according to participation or non-participation in profits.

8. In view of the facts discussed in the three preceding paragraphs, the Committee decided that a subdivision by class of business, or even a subdivision between Medical and Non-Medical data, was not justifiable, and that the only differentiation which should be considered was a subdivision between 'light' and 'heavy' offices or between 'light', 'medium' and 'heavy' offices. This would reflect what was eventually done in the A1924–29 tables. A twofold subdivision would not, perhaps, be very helpful to offices which lie near the average because neither a 'light' nor a 'heavy' table would fit very closely to their needs, and accordingly a threefold subdivision was probably preferable. It was felt, however, that in the first place a general table based upon the whole of the data, and therefore reasonably representative of a medium table, should be constructed, and that the question of constructing 'light' and 'heavy' tables should be considered later. In the event it may be that offices requiring a heavier table will continue to use the A1924–29, even though the mortality rates will be rather high at young ages. Thus, it may be sufficient if later on an up-to-date 'light' table is constructed based upon the experience of the 'light' offices either during the period 1949–52 or, since the experience of subsequent years will by then have accumulated, during some later period. Another alternative which may be considered is the construction of a hypothetical 'light' table bearing some arbitrarily determined relation to the basic table. But whichever of these suggestions may eventually be adopted, the individual actuary will always need to employ his personal judgment when determining a mortality basis to suit a given set of circumstances.

* See Appendix 1.

9. Having decided to use the whole of the data for the period 1949–52, the Committee had to consider two problems of a statistical character. It has already been pointed out that Whole Life mortality is in general somewhat heavier than Endowment mortality. At young ages, where in any case the Whole Life data are relatively scarce, this feature is not of great moment. Between ages 50 and 70, however, it becomes more important, since the sums at risk in life office portfolios are at these ages generally much greater for Whole Life assurances than for Endowment assurances. Consequently it was felt that the rates of mortality in the published table should, at the ages in question, bear a closer relationship to the Whole Life rates than to the rates for all classes combined. Above age 70 the Endowment data virtually disappear, so that the question of differential mortality ceases to be relevant. However, at the advanced ages a further consideration arises. It is known that for many years the Whole Life Non-Profit 'in force' data contained paid-up policies under which all contact had been lost with the life assured. In many cases the assured person had probably died and no claim had been lodged. In consequence there was an understatement of deaths and an inflation of the exposed-to-risk. In 1951 offices were requested to exclude Non-Profit paid-up policies from their returns to the C.M.I., so that the position has recently been greatly improved. The data for 1949–52, however, contain a substantial number of these policies, and the understatement of mortality rates in extreme old age is probably appreciable. It was therefore decided that at ages over 80 attention should be focused upon the Whole Life With-Profit (Medical) class (there are few data in the corresponding Non-Medical class at these ages); while between 50 and 80 the final table should give increasing weight to the Whole Life With-Profit (Medical) experience and, correspondingly, diminishing weight to the experience of all classes combined.

THE DURATION OF SELECTION

10. As on previous occasions when tables of assured lives' mortality have been under construction, the problem of selection has given difficulty. The C.M.I. data are collected in a form which permits of analysis by duration up to a period of five years from issue. Owing, however, to the form of the A1924–29 table it has been customary to employ a three-year select period in preparing the exposed-to-risk for purposes of statistical analysis. It is on this account that all the preliminary work of investigation was carried out on the data at durations '3 and over'. For examining selection, however, the actual deaths at each duration from 0 to 4 were compared with the expected deaths according to the contemporaneous experience at durations '3 and over'. The results are shown in Tables 11–13, from which it can be seen that after the first two years of duration there is no evidence of a rapid progression of the select rates of mortality towards the ultimate level. For all ages and classes of business combined the percentages of actual to expected deaths are as follows:

Duration	100 A/E
0	54·7
1	69·4
2	80·3
3	83·8
4	84·4

Since the expected deaths are based on the experience at '3 and over' the ratios of actual to expected deaths in durations must ultimately reach a level appreciably higher than 100%—perhaps 110%. Three features immediately present themselves for consideration, viz.:

(i) There is no rapid approach of the select rates towards the ultimate limit.

(ii) After an initial rise from duration 0 to duration 2, there is a marked flattening in the progress of the percentages from duration 2 to duration 4.

(iii) In order to reach the ultimate level, which must necessarily be higher than 100%, the percentages must take a sharp upward turn at some point beyond duration 4.

11. The failure of the select mortality to approach the ultimate level at a reasonably short duration repeats earlier history, for a similar position arose both with the O^M and the A1924–29 tables. Among the factors which can contribute to a situation of this kind are (*a*) the prolongation of initial temporary selection, (*b*) the operation of selection by withdrawals and (*c*) spurious selection caused by an inherent inconsistency between the data at short and long durations. With regard to (*c*), offices vary greatly in the average term of their business, and the proportions contributed by individual offices to the '5 and over' data are very different from the corresponding proportions at durations 0–4. Some of the 'heavier' offices are, in fact, among those which bulk proportionately larger at duration '5 and over'. No doubt, if the analysis could be pursued to longer durations, the differences would be still more pronounced and this presence of 'heavy' data at long durations appears to be a likely source of spurious selection.

12. The length of time during which temporary initial selection persists has long been a controversial topic. In spite of the heterogeneity of the data, it might be expected that a comparison of ratios of select to ultimate mortality within the Medical and Non-Medical sections would throw light on this question. The true ultimate rates cannot, of course, be known; but ratios of the actual deaths at durations 0–4 to the deaths expected on the basis of the corresponding mortality at durations 3 and over are as follows:

Duration	Medical 100 A/E	Non-Medical 100 A/E	All classes 100 A/E
0	50·6	58·9	54·7
1	66·8	72·6	69·4
2	74·1	87·6	80·3
3	81·9	86·4	83·8
4	80·2	88·9	84·4

Thus the same pattern, in which the curve shows a steep increase at first and then levels off, occurs in both sections of the experience, and in both there must eventually come a sharp upward turn so that the select may merge into the ultimate rates—although the 'gap' to be spanned is greater for the Medical than for the Non-Medical business. The percentages based upon the '3 and over' rates probably give a fair guide to the relative degree of selection within each section, and they suggest that medical examination may

have the effect at the earlier durations of lowering the mortality by about 8 % of the ultimate rate.

13. The remaining suggestion made in § 11 as a possible explanation for the prolonged duration of selection is the selection exercised by policy-holders who lapse or surrender. Unfortunately, no evidence exists which would throw light on this vexed question; but if proportionately fewer unhealthy lives withdraw by lapse or surrender and if, moreover, the withdrawals are fairly heavy at the earlier and the medium term durations, the result would be a form of selection which would persist to a high duration and which would cause a substantial worsening of the ultimate experience. Traditionally it is usual to think of the 'ultimate' level of mortality as being the experience of an average body of lives to which a select group will normally revert after the effects of temporary initial selection have worn off. If, however, the 'ultimate' experience is worse than the average, owing to the negative selection exercised by withdrawals, it becomes possible to contemplate selection as lasting indefinitely, the residue of lives remaining in force being continually weakened by the withdrawal of healthy individuals.

14. In view of the situation disclosed in previous paragraphs and the lack of information about the trend of mortality at the higher durations, the Committee felt that, in deciding a basis for the select rates of mortality, attention must be concentrated on the relationship between the select and ultimate rates in the combined data for all classes of business. It was clearly impossible, on the data available, to produce a select table in which the 'ultimate' column of q_x represented the true ultimate rates of mortality in the experience and in consequence there was much to be said for discarding selection altogether and producing an aggregate table. However, the Committee thought that the general climate of opinion might not be favourable to an aggregate table, and they therefore decided that the new table should be constructed on a select basis but that in view of the flattening in the progression of the ratios of select to ultimate mortality which had been observed to set in from duration 2, the select period should be reduced from three to two years. Thus the proposed new table will give recognition to the fact that mortality at durations 0 and 1 has been substantially lighter than at subsequent durations, but that from duration 2 onwards the increase in mortality with duration appears to have been slow and gradual and indefinitely extended, thus making further differentiation by duration difficult to justify. At the same time it will be readily appreciated that the difference between $q_{[x]}$ and q_x is not a measure of the full amount of selection inherent in the data.

GRADUATION

15. It being decided to construct a table with a two-year period of selection, the next step was to add the data for duration 2 to the '3 and over' data already assembled and then to determine a suitable basis for producing graduated '2 and over' rates of mortality. It was the view of the Committee that, as arguments for adhering closely to the data were necessarily modified by the decision to give increasing weight at the older ages to the experience in the Whole Life With-Profit (Medical) class, the construction of a smooth series of rates was more important than achieving a degree of fidelity to the data which would satisfy standard significance tests. It was hoped, therefore,

that a mathematical formula could be found which would give a satisfactory representation of current assured lives' mortality and so provide a good working instrument for life office calculations.

16. The discovery of a suitable mathematical formula did not prove easy. It was necessary to decide whether certain major features of the ungraduated rates, when regarded as a series, should be retained in the graduated table. Between ages 20 and 30 there is a 'hump' in the mortality rates which is doubtless due primarily to the incidence at young ages of deaths from accidents. Although it was felt that, in a working table for practical use, it would be permissible to remove this 'hump', over a considerable range of ages in the early part of the table q_x must nevertheless remain nearly flat.

17. The next difficulty was encountered in the neighbourhood of ages 45–50. Below age 40, the reduction in male mortality rates has been proportionately greater than above age 55 where, owing largely to the effect of respiratory cancer and coronary disease, the downward trend has been slow. Consequently a considerable 'hollowing out' has taken place in the mortality curve and a steeper rise is now needed between 40 and 55 to bridge the interval between youth and later middle-age. This development was already apparent in the 1947–48 experience, and it can also be observed in recent mortality statistics for the male population. It must therefore be regarded as a genuine characteristic of male mortality at the present time and one which should be reproduced in the graduated curve. Above age 55 there are various minor waves in the ungraduated curve, but none of them is sufficiently distinctive to be retained. The 1947–48 experience showed a different set of variations, and consequently it was felt that irregularities within this upper range should be eliminated. At the advanced ages, of course, the curve of q_x begins to grow flatter as it moves towards unity.

18. Thus the main features required in the graduated curve are as follows:

(i) An almost flat level of q_x at young ages.
(ii) A sharp upward turn between 40 and 55.
(iii) A flattening off in the upper part of the q_x curve.

The third of these features suggests that one of the Perks family of curves might prove suitable. After carrying out various experiments, Mr R. E. Beard evolved a formula which, although it has an obvious affinity with the formulae of Perks, is new in actuarial work. Beard's formula is

$$q_x = A + Bc^x/(Ec^{-2x} + 1 + Dc^x).$$

Thus it has five parameters,* and its distinguishing feature is the presence in the denominator of a term involving c^{-2x}. The value of q_x tends to A as x tends to $-\infty$ and to $(A+B/D)$ as x tends to $+\infty$.

19. The fitting of the formula was achieved empirically. Even if a 'best' fit based upon accepted techniques of statistical estimation had been desired, the complicated nature of the graduation formula would have rendered the mathematical work intractable. However, as already explained, a close fit conforming to statistical standards was not required, the primary consideration being to produce a smooth curve which would serve as a

* See discussion and Committee's reply on p. 80. Eds. *J.I.A.*

satisfactory working instrument. By trial and error the following values were determined for the parameters in the formula:

$$A = \cdot 00111,$$
$$B = \cdot 0218623,$$
$$D = \cdot 0272978,$$
$$E = \cdot 01846,$$
$$c = (1 \cdot 0525)^2,$$

Origin at $x = 62\frac{1}{2}$.

The resulting rates of mortality will be found in Table 18. In Table 14 will be found a comparison of actual and expected deaths for the All Classes 1949–52 data (durations 2 and over), while a similar comparison for the Whole Life With-Profit (Medical) data is given in Table 15. (Ages under 40 have been omitted from Table 15 owing to paucity of data.) As already indicated, rigorous statistical tests of the graduation are not appropriate, but the function $|A-E|/\sqrt{A}$ given in the final column of each table is a fair measure of the deviations age by age and the distribution of the values of this function is summarized in the following table:

Distribution of $\frac{|A-E|}{\sqrt{A}}$ at individual ages

(Summarized from Tables 14 and 15)

Value of $\frac{\lvert A-E \rvert}{\sqrt{A}}$	All Classes (ages 21–95)		Whole Life With-Profit (Medical) (ages 41–100)	
	No. of ages	% of total	No. of ages	% of total
0·0–0·9	42	56	37	62
1·0–1·9	21	28	17	28
2·0–2·9	10	16	4	10
3·0–3·9	2		1	
4·0–4·9	—		1	
Total	75	100	60	100

Note. Ages over 95 are neglected in the All Classes column because of the unreliability of the data (see text).

Thus, whilst the graduation is an attempt to blend the All Classes experience at younger ages, with the Whole Life With-Profit (Medical) experience at older ages, the deviations between actual and expected deaths are not unduly large even when each experience is considered separately. In the All Classes experience the total deviations, irrespective of sign, amount to 2553, and this may be compared with the total of $\sqrt{A}$, which amounts to 2433. Corresponding figures for the Whole Life With-Profit (Medical) experience are 995 and 1129 respectively. In fact, although primary consideration has throughout been given to the achievement of a smooth curve with as few parameters as possible, there need be no concern that fidelity to the data has been sacrificed unduly.

20. Closer inspection of Tables 14 and 15 shows that in the All Classes experience the deviations between actual and expected deaths change sign frequently up to age 55. Thereafter the expected deaths tend to be mainly in excess of the actual deaths, i.e. the graduated curve is running above the experience. It is at this point, however, that attention should pass to the Whole Life With-Profit (Medical) experience, where it will be seen that changes of sign in the deviations are more frequent, although there is a sequence of nine ages, viz. from 59 to 67, where the values of $(A-E)$ are all positive in the Whole Life table and, with one exception, negative in the All Classes table. In this interval the curve is running between the two experiences and so fulfils one of the conditions laid down earlier. The only remaining feature which may seem to call for comment is that in Table 15 the deviations appear rather large in the range 76–85, the expected deaths being in excess in the earlier part of the range and the actual deaths in the latter part—although the net value of $(A-E)$ over these ten ages is only -2. It would, however, be difficult to find any smooth curve which would not produce a similar result within this range.

21. As a further test of the suitability of the graduation, Spencer's 21-term summation formula—which, it will be recalled, was employed in the graduation of the A 1924–29 table—was applied to the ungraduated rates both for the All Classes data and for the Whole Life With-Profit (Medical) section. The resultant values of q_x at quinquennial intervals from ages $32\frac{1}{2}$ to $87\frac{1}{2}$ are shown below (no values for ages outside this range being given by a 21-term formula) together with the corresponding values from the proposed table:

Age	Spencer graduation of All Classes data	Proposed table	Spencer graduation of W.L.W.P. (Med.) data
$32\frac{1}{2}$	·00116	·00122	*
$37\frac{1}{2}$	·00153	·00152	*
$42\frac{1}{2}$	·00246	·00245	·00255
$47\frac{1}{2}$	·00453	·00446	·00426
$52\frac{1}{2}$	·00799	·00793	·00708
$57\frac{1}{2}$	·01321	·01338	·01361
$62\frac{1}{2}$	·02172	·02201	·02363
$67\frac{1}{2}$	·03449	·03577	·03570
$72\frac{1}{2}$	·05619	·05752	·05643
$77\frac{1}{2}$	·09006	·09111	·09013
$82\frac{1}{2}$	·14278	·14081	·14372
$87\frac{1}{2}$	·20756	·20985	·21035

* Value not available owing to insufficient data.

Thus, from $32\frac{1}{2}$ to $52\frac{1}{2}$ the proposed table bears a close resemblance to the Spencer values for the All Classes experience. After $52\frac{1}{2}$ it moves steadily from the All Classes to the Whole Life With-Profit (Medical) experience, with which it nearly coincides at $67\frac{1}{2}$. Thereafter it lies first above, and later below, the Spencer values for the Whole Life With-Profit (Medical) experience. These features all confirm the conclusions arrived at in the preceding paragraph.

SELECT RATES OF MORTALITY

22. For the select portion of the table it was decided to apply the 'damaged lives' method used by Beard to construct a hypothetical mortality table as described in his paper *Some experiments in the use of the incomplete gamma function for the approximate calculation of actuarial functions* (*Proceedings of the Centenary Assembly of the Institute of Actuaries*, vol. 2, p. 89). For a two years' period of selection the basic equations are:

$$\begin{aligned} l_{[x]} &= l_x - \phi(0)\, f(x)\, d_x, \\ l_{[x]+1} &= l_{x+1} - \phi(1)\, f(x)\, d_x, \\ d_{[x]} &= l_{[x]} - l_{[x]+1}, \\ &= d_x - \{\phi(0) - \phi(1)\}\, f(x)\, d_x, \\ d_{[x]+1} &= l_{[x]+1} - l_{x+2} \\ &= d_{x+1} - \phi(1)\, f(x)\, d_x. \end{aligned}$$

After experiment it was found that satisfactory series for $q_{[x]}$ and $q_{[x]+1}$ could be obtained by putting

$$\phi(0) = 1, \quad \phi(1) = \cdot 415, \quad f(x) = \cdot 4925 + \cdot 007x.$$

The resultant ratios of $q_{[x]}/q_x$ and $q_{[x-1]+1}/q_x$ are given for quinquennial intervals in Table 16, together with the corresponding ratios of actual deaths at durations 0 and 1 in the All Classes experience to the deaths expected by the graduated values of q_x for durations 2 and over. The method does not automatically imply that the ratios of select to ultimate rates will either increase or decrease uniformly with age, and under the proposed basis the ratio $q_{[x-1]+1}/q_x$ has a minimum value at age 36 and a maximum at age 41. In Table 17 will be found the exposed-to-risk and actual deaths in the All Classes experience at durations 0 and 1 at individual ages, and also the corresponding expected deaths by the rates of mortality produced on the basis described above. The results are summarized in the following table:

Comparison of actual and expected deaths for durations 0 and 1 (All Classes)

Age-group	Duration 0		Duration 1	
	Actual deaths	Expected deaths	Actual deaths	Expected deaths
21–25	112	102	103	93
26–30	122	123	171	141
31–35	125	115	143	142
36–40	151	145	177	188
41–45	186	188	265	261
46–50	232	218	330	310
51–55	194	175	267	272
56–60	106	113	216	188
61–80	95	94	128	170
Total	1323	1273	1800	1765

A separate comparison of actual and expected deaths has not been made for the Whole Life With-Profit (Medical) experience because the quantity of data in this group at the durations concerned is insufficient to give results of any significance. Values of $q_{[x]}$ and $q_{[x-1]+1}$ will be found in Table 18, alongside the corresponding values of q_x.

23. An advantage of the 'damaged lives' method of calculating select rates of mortality is that it produces a series of simple relationships between select and ultimate functions. In the proposed new table the following relations hold (for convenience the symbol h_x is used to represent $f(x)\,d_x$):

$$d_{[x]} = d_x - \cdot 585 h_x,$$

$$d_{[x]+1} = d_{x+1} - \cdot 415 h_x,$$

$$D_{[x]} = D_x - v^x h_x,$$

$$D_{[x]+1} = D_{x+1} - \cdot 415 v^{x+1} h_x,$$

$$C_{[x]} = C_x - \cdot 585 v^{x+1} h_x,$$

$$C_{[x]+1} = C_{x+1} - \cdot 415 v^{x+2} h_x,$$

$$a_{[x]} = \frac{a_x - \cdot 415 v f(x) q_x}{1 - f(x) q_x},$$

$$A_{[x]} = \frac{A_x - (\cdot 585 v + \cdot 415 v^2) f(x) q_x}{1 - f(x) q_x}.$$

It is hoped that some of these relationships may prove helpful in practical computation.

COMPARISON WITH EARLIER TABLES

24. In Table 19 will be found values of q_x at quinquennial ages by various tables of assured lives' mortality. In comparison with the O^M Aggregate table, the greatest proportionate reduction is at age 35, where q_x in the proposed new table is only 18% of the corresponding O^M value. Above that age the proportionate improvement in mortality rapidly diminishes and at age 55 the new q_x is 51% of the O^M value, while at age 85 the ratio is 84%. At ages up to 40 the new values of q_x are rather less than 50% of the A 1924–29 values, but between 40 and 55 the two series converge and by 55 the ratio has reached 87%. It remains at approximately this level until age 80, after which it begins to increase again. It is of interest to note that at ages 55, 60 and 65 the values of q_x from the new table are *higher* than the corresponding A 1924–29 Light values. These ages belong to the period of life at which, owing largely to cancer and coronary disease, there has been little reduction in male mortality in recent years. The new table is also heavier than the 1947–48 experience at all ages from 55 upwards (except at age 80); this feature is no doubt largely due to the exceptionally light experience of 1948.

25. A comparison of monetary values between the proposed new table and the A1924–29 table is given below. The rate of interest employed is 3%:

Age x	$A_{[x]}$		$P_{[x]}$		$A_{[x]:\overline{15}\rceil}$		$P_{[x]:\overline{15}\rceil}$	
	24–29	49–52	24–29	49–52	24–29	49–52	24–29	49–52
20	·25110	·22614	·00977	·00851	·64687	·64423	·05335	·05274
30	·31851	·29455	·01361	·01216	·64798	·64474	·05361	·05286
40	·40715	·38569	·02000	·01829	·65284	·64882	·05477	·05381
50	·51523	·49705	·03096	·02880	·66595	·66241	·05807	·05715
60	·63622	·61723	·05094	·04700	·70041	·69314	·06809	·06579

Thus in comparison with the A1924–29 table, the new table produces reductions of 2*s.* 6*d.* at age 20 and 7*s.* 11*d.* at age 60 in the annual premiums per £100 sum assured for Whole Life assurances at 3% interest. The corresponding reductions in annual premiums for 15-year Endowment assurances are 1*s.* 3*d.* at age 20 and 4*s.* 7*d.* at age 60.

CONCLUSION

26. The main difficulty which has been before the Committee throughout its deliberations has been the very considerable variability in the mortality experienced not only by different classes of assured lives but more particularly by different life assurance offices. With such heterogeneity in the data it would have been out of place to construct a precision instrument derived by the application of rigorous statistical techniques. It may reasonably be claimed that the table now being laid before the profession is a serviceable representation of the broad average of modern assured lives' mortality. As has been indicated, the Committee will later on consider supplementing it with a 'light' table. However, the working actuary will continue to need his individual judgment when determining bases for his calculations and occasions will always arise when, whatever standard tables may be available, adjustments to meet particular circumstances will be necessary.

Table 1. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Whole Life With-Profit (Medical)

Nearest age x	Exposed-to-risk	Deaths	q_{x-1}	Nearest age x	Exposed-to-risk	Deaths	q_{x-1}
11–20	270	—	—	61	15,272	306	·02004
				62	15,961	353	·02212
21	260·5	—	—	63	16,682·5	377	·02260
22	327·5	—	—	64	17,382	459	·02641
23	388	—	—	65	17,885	518	·02896
24	513	—	—				
25	762·5	1	·00131	66	18,324	560	·03056
				67	18,862	626	·03319
26	1,061	—	—	68	19,474·5	672	·03451
27	1,460	—	—	69	19,976·5	766	·03835
28	1,914·5	4	·00209	70	20,245	862	·04258
29	2,480	2	·00081				
30	2,996	4	·00134	71	20,284	946	·04664
				72	19,994	1,054	·05272
31	3,441·5	4	·00116	73	19,526·5	1,135	·05813
32	3,883	2	·00052	74	18,895·5	1,177	·06229
33	4,384	3	·00068	75	18,193·5	1,259	·06920
34	5,108	6	·00117				
35	6,044	7	·00116	76	17,211	1,164	·06763
				77	16,219	1,365	·08416
36	6,981·5	11	·00158	78	14,854·5	1,377	·09270
37	7,711	17	·00220	79	13,168	1,324	·10055
38	8,319·5	9	·00108	80	11,188	1,177	·10520
39	8,890·5	15	·00169				
40	9,423·5	17	·00180	81	9,819·5	1,158	·11793
				82	8,611·5	1,176	·13656
41	9,981	21	·00210	83	7,269	1,004	·13812
42	10,520	15	·00143	84	6,262	977	·15602
43	10,920·5	36	·00330	85	5,220·5	945	·18102
44	11,065	40	·00362				
45	11,094·5	31	·00279	86	4,221	754	·17863
				87	3,472·5	672	·19352
46	11,036·5	41	·00371	88	2,783	579	·20805
47	10,983	38	·00346	89	2,189	497	·22704
48	11,046	51	·00462	90	1,674	377	·22521
49	11,238·5	53	·00472				
50	11,326·5	67	·00592	91	1,247·5	333	·26693
				92	890·5	259	·29085
51	11,281·5	57	·00505	93	610	178	·29180
52	11,221·5	78	·00695	94	428·5	138	·32205
53	11,089·5	93	·00839	95	282	85	·30142
54	11,238	73	·00650				
55	11,561·5	118	·01021	96	202·5	82	·40494
				97	106	33	·31132
56	11,989·5	112	·00934	98	70·5	23	·32624
57	12,611·5	147	·01166	99	36	13	·36111
58	13,341	175	·01312	100	18·5	4	·21622
59	14,031	229	·01632				
60	14,639	270	·01844	Over 100	34	7	·20588

Table 2. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Whole Life Non-Profit (Medical)

Nearest age x	Exposed-to-risk	Deaths	q_{x-1}	Nearest age x	Exposed-to-risk	Deaths	q_{x-1}
11–20	167	—	—	61	9,153·5	158	·01726
				62	9,229·5	176	·01907
21	187·5	—	—	63	9,273·5	184	·01984
22	212	1	·00472	64	9,216·5	226	·02452
23	216·5	—	—	65	8,998·5	250	·02778
24	336	—	—				
25	547	—	—	66	8,757·5	238	·02718
				67	8,540	275	·03220
26	826	—	—	68	8,295·5	302	·03641
27	1,153·5	2	·00173	69	8,017	291	·03630
28	1,508·5	1	·00066	70	7,736	307	·03968
29	1,871	4	·00214				
30	2,279	1	·00044	71	7,374	363	·04923
				72	6,864	370	·05390
31	2,670·5	4	·00150	73	6,306·5	350	·05550
32	3,108	6	·00193	74	5,861·5	347	·05920
33	3,473	2	·00058	75	5,401	380	·07036
34	3,879·5	2	·00052				
35	4,663	9	·00193	76	4,885·5	351	·07185
				77	4,443	343	·07720
36	5,487	9	·00164	78	3,917·5	351	·08960
37	6,268·5	12	·00191	79	3,387·5	318	·09387
38	7,047·5	12	·00170	80	2,758·5	283	·10259
39	7,741	12	·00155				
40	8,411	11	·00131	81	2,217	267	·12043
				82	1,811·5	231	·12752
41	8,980	17	·00189	83	1,463	203	·13876
42	9,509·5	13	·00137	84	1,164	167	·14347
43	9,976	24	·00241	85	902	142	·15743
44	10,315	20	·00194				
45	10,476	42	·00401	86	693	113	·16306
				87	556	108	·19424
46	10,393·5	46	·00443	88	450	73	·16222
47	10,277	56	·00545	89	349	73	·20917
48	10,179	46	·00452	90	246	54	·21951
49	10,079·5	54	·00536				
50	9,991·5	59	·00591	91	173	42	·24277
				92	123·5	30	·24291
51	9,834	74	·00752	93	85·5	23	·26901
52	9,634	94	·00976	94	48	9	·18750
53	9,336·5	82	·00878	95	37	4	·10811
54	9,167	90	·00982				
55	9,137	95	·01040	96	37	8	·21622
				97	27·5	2	·07273
56	9,165·5	81	·00884	98	22·5	2	·08889
57	9,283·5	128	·01379	99	17	1	·05882
58	9,314·5	127	·01363	100	14·5	—	—
59	9,232	136	·01473				
60	9,126	144	·01578	Over 100	21·5	4	·18605

Table 3. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Endowment Assurance With-Profit (Medical)

Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$	Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$
11–20	5,898	6	·00102	61	48,084	886	·01843
				62	43,599·5	810	·01858
21	4,532	4	·00088	63	40,032	848	·02118
22	5,870	7	·00119	64	35,879	844	·02352
23	7,613	7	·00092	65	26,180	640	·02445
24	10,951	13	·00119				
25	15,572	16	·00103	66	17,060·5	500	·02931
				67	13,060·5	394	·03017
26	20,208·5	29	·00144	68	10,470	334	·03190
27	25,146·5	28	·00111	69	8,337	289	·03466
28	31,071·5	45	·00145	70	5,841·5	233	·03989
29	37,488	48	·00128				
30	43,112	29	·00067	71	4,090·5	170	·04156
				72	3,036·5	149	·04907
31	47,293	66	·00140	73	2,210	102	·04615
32	50,607·5	50	·00099	74	1,550	78	·05032
33	54,774	68	·00124	75	854·5	51	·05968
34	62,138	80	·00129				
35	72,193·5	89	·00123	76	456·5	33	·07229
				77	332	31	·09337
36	81,744·5	107	·00131	78	253·5	11	·04339
37	89,702·5	124	·00138	79	170·5	8	·04692
38	97,551	138	·00141	80	120·5	—	—
39	105,244·5	178	·00169				
40	111,436·5	191	·00171	81	24·5	1	·04082
				82	17·5	3	·17143
41	117,483·5	215	·00183	83	9·5	—	—
42	122,509	251	·00205	84	10·5	1	·09524
43	126,000	283	·00225	85	3·5	—	—
44	127,920	375	·00293				
45	126,685·5	426	·00336	86	2	—	—
				87	2·5	—	—
46	124,816·5	425	·00340	88	3	1	·33333
47	123,206·5	453	·00368	89	·5	—	—
48	121,452·5	585	·00482	90	—	—	—
49	118,513·5	574	·00484				
50	110,831·5	682	·00615	91	·5	—	—
				92	1	—	—
51	103,312·5	673	·00651	93	1	—	—
52	97,916·5	669	·00683	94	1	—	—
53	93,598·5	677	·00723	95	·5	—	—
54	90,643	773	·00853				
55	84,546·5	806	·00953	96	—	—	—
				97	—	—	—
56	80,168	900	·01123	98	—	—	—
57	77,557·5	818	·01055	99	—	—	—
58	74,936	951	·01269	100	—	—	—
59	70,306·5	956	·01360				
60	57,468·5	874	·01521	Over 100	—	—	—

Table 4. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Endowment Assurance Non-Profit (Medical)

Nearest age x	Exposed-to-risk	Deaths	q_{x-t}	Nearest age x	Exposed-to-risk	Deaths	q_{x-t}
11–20	383·5	1	·00261	61	11,112	163	·01467
				62	9,866	166	·01683
21	445·5	—	—	63	8,648	161	·01862
22	667·5	1	·00150	64	7,355·5	143	·01944
23	904·5	1	·00111	65	4,707·5	95	·02018
24	1,375	2	·00145				
25	2,110·5	3	·00142	66	3,331·5	61	·01831
				67	2,618·5	62	·02368
26	2,978	3	·00101	68	2,084	44	·02111
27	3,998	3	·00075	69	1,629	45	·02762
28	5,270·5	6	·00114	70	1,160·5	43	·03705
29	6,676	7	·00105				
30	8,029	9	·00112	71	802·5	33	·04112
				72	584·5	25	·04277
31	9,242	5	·00054	73	399	16	·04010
32	10,180·5	15	·00147	74	266	12	·04511
33	11,130	10	·00090	75	147	4	·02721
34	12,684·5	14	·00110				
35	14,868	18	·00121	76	92·5	9	·09730
				77	58	4	·06897
36	16,926	13	·00077	78	33·5	—	—
37	18,981	28	·00148	79	26·5	2	·07547
38	21,005·5	25	·00119	80	13	1	·07692
39	22,728·5	31	·00136				
40	24,305·5	52	·00214	81	5	—	—
				82	3·5	—	—
41	25,764·5	51	·00198	83	1·5	—	—
42	27,097	41	·00151	84	1	—	—
43	27,996	59	·00211	85	1	—	—
44	28,403	62	·00218				
45	28,338	90	·00318	86	·5	—	—
				87	·5	—	—
46	27,980	87	·00311	88	2	—	—
47	27,718	100	·00361	89	—	—	—
48	27,349	104	·00380	90	1	—	—
49	26,773·5	104	·00388				
50	25,433	144	·00566	91	1	—	—
				92	·5	—	—
51	23,694·5	147	·00620	93	—	—	—
52	22,263·5	136	·00611	94	—	—	—
53	21,250	142	·00668	95	—	—	—
54	20,530	158	·00770				
55	19,309	159	·00823	96	—	—	—
				97	—	—	—
56	18,319·5	200	·01092	98	—	—	—
57	17,543·5	176	·01003	99	—	—	—
58	16,781	178	·01061	100	—	—	—
59	15,658·5	203	·01296				
60	13,152·5	209	·01589	Over 100	—	—	—

Table 5. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Whole Life With-Profit (Non-Medical)

Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$	Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$
11–20	174·5	—	—	61	3,712	83	·02236
				62	3,683·5	90	·02443
21	157·5	—	—	63	3,648	87	·02385
22	174	1	·00575	64	3,588	99	·02759
23	240	1	·00417	65	3,440·5	95	·02761
24	421	1	·00238				
25	590	3	·00508	66	3,316	107	·03227
				67	3,096	102	·03295
26	704·5	—	—	68	2,881·5	109	·03783
27	837	—	—	69	2,667·5	114	·04274
28	1,013·5	2	·00197	70	2,436	113	·04639
29	1,198·5	1	·00083				
30	1,443	3	·00208	71	2,190·5	122	·05570
				72	1,958·5	118	·06025
31	1,594	2	·00125	73	1,749	110	·06289
32	1,742	4	·00230	74	1,584	125	·07891
33	1,891·5	1	·00053	75	1,420·5	109	·07673
34	2,211·5	2	·00090				
35	2,639·5	—	—	76	1,292	109	·08437
				77	1,166·5	101	·08658
36	3,133	5	·00160	78	990	107	·10808
37	3,622	4	·00110	79	815	73	·08957
38	4,067	4	·00098	80	627·5	79	·12590
39	4,526	13	·00287				
40	4,899·5	6	·00122	81	531·5	74	·13923
				82	428·5	60	·14002
41	5,324	19	·00357	83	351	39	·11111
42	5,582·5	32	·00573	84	295	43	·14576
43	5,749	10	·00174	85	233·5	39	·16702
44	5,849·5	8	·00137				
45	5,964·5	18	·00302	86	186	28	·15054
				87	151	30	·19868
46	5,964	23	·00386	88	113	20	·17699
47	5,871	29	·00494	89	86	17	·19767
48	5,673·5	34	·00599	90	71·5	15	·20979
49	5,423·5	31	·00572				
50	5,205·5	38	·00730	91	45·5	11	·24176
				92	28	4	·14286
51	4,853	27	·00556	93	16·5	8	·48485
52	4,604·5	31	·00673	94	7·5	4	·53333
53	4,413	49	·01110	95	4·5	—	—
54	4,223·5	50	·01184				
55	4,112	48	·01167	96	5·5	3	·54545
				97	3·5	—	—
56	4,049	48	·01185	98	4·5	—	—
57	3,968	54	·01361	99	4	2	·50000
58	3,906·5	57	·01459	100	·5	1	2·00000
59	3,908	64	·01638				
60	3,804	67	·01761	Over 100	—	—	—

Table 6. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Whole Life Non-Profit (Non-Medical)

Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$	Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$
11–20	66·5	—	—	61	2,348	53	·02257
				62	2,153	49	·02276
21	101	—	—	63	1,946	46	·02364
22	142	—	—	64	1,791·5	46	·02568
23	163·5	—	—	65	1,615	45	·02786
24	220	1	·00455				
25	354	—	—	66	1,453	45	·03097
				67	1,317	47	·03569
26	496·5	—	—	68	1,169·5	32	·02736
27	679	3	·00442	69	1,018·5	54	·05302
28	891·5	—	—	70	852·5	36	·04223
29	1,096	—	—				
30	1,317	4	·00304	71	729	41	·05624
				72	644·5	29	·04500
31	1,477	—	—	73	564	45	·07979
32	1,575·5	3	·00190	74	475·5	33	·06940
33	1,704	1	·00059	75	415	31	·07470
34	1,964	1	·00051				
35	2,428	5	·00206	76	357	28	·07843
				77	306·5	20	·06525
36	2,965	5	·00169	78	261	23	·08812
37	3,424·5	8	·00234	79	221	27	·12217
38	3,891·5	7	·00180	80	158·5	16	·10095
39	4,323	9	·00208				
40	4,728	9	·00190	81	124	12	·09677
				82	115	8	·06957
41	5,012	10	·00200	83	89·5	15	·16760
42	5,196·5	9	·00173	84	63	11	·17460
43	5,308	15	·00283	85	51	12	·23529
44	5,305	14	·00264				
45	5,264·5	14	·00266	86	32·5	4	·12308
				87	24	6	·25000
46	5,198·5	27	·00519	88	12·5	1	·08000
47	5,100	36	·00706	89	11·5	2	·17391
48	4,980·5	32	·00643	90	11	3	·27273
49	4,801·5	33	·00687				
50	4,634	31	·00669	91	7	—	—
				92	4·5	1	·22222
51	4,440·5	29	·00653	93	5	—	—
52	4,264·5	26	·00610	94	4	—	—
53	4,098	27	·00659	95	2·5	—	—
54	3,864·5	34	·00880				
55	3,588·5	38	·01059	96	1·5	—	—
				97	·5	—	—
56	3,341·5	39	·01167	98	·5	—	—
57	3,170	51	·01609	99	1	—	—
58	2,992	51	·01705	100	—	—	—
59	2,771	44	·01588				
60	2,530	47	·01858	Over 100	—	—	—

Table 7. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Endowment Assurance With-Profit (Non-Medical)

Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$	Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$
11–20	17,705·5	28	·00158	61	37,381·5	753	·02014
				62	32,302	701	·02170
21	14,934·5	21	·00141	63	27,834	644	·02314
22	19,619·5	24	·00122	64	22,549·5	537	·02381
23	24,540·5	36	·00147	65	12,557	343	·02732
24	32,432·5	38	·00117				
25	41,756	42	·00101	66	7,218·5	200	·02771
				67	5,216·5	184	·03527
26	49,642	59	·00119	68	3,978	149	·03746
27	56,670	54	·00095	69	2,856·5	87	·03046
28	63,954·5	55	·00086	70	1,856·5	67	·03609
29	72,156	106	·00147				
30	78,543·5	105	·00134	71	1,332	62	·04655
				72	973·5	42	·04314
31	81,716	87	·00106	73	622	29	·04662
32	83,591·5	100	·00120	74	335·5	18	·05365
33	85,616·5	135	·00158	75	153·5	8	·05212
34	92,485	77	·00083				
35	105,148·5	141	·00134	76	92	4	·04348
				77	57·5	6	·10435
36	117,281	161	·00137	78	45	4	·08889
37	126,586	195	·00154	79	36	5	·13889
38	134,314·5	222	·00165	80	25·5	5	·19608
39	141,048·5	275	·00195				
40	145,084·5	285	·00196	81	28·5	3	·10526
				82	24	4	·16667
41	149,759·5	338	·00226	83	20·5	2	·09756
42	154,495	366	·00237	84	13·5	4	·29630
43	158,238·5	374	·00236	85	6	—	—
44	159,630	448	·00281				
45	157,430·5	500	·00318	86	5·5	3	·54545
				87	6	1	·16667
46	156,495·5	575	·00367	88	5·5	—	—
47	156,934·5	629	·00401	89	3·5	—	—
48	157,977	705	·00446	90	2·5	—	—
49	154,479·5	794	·00514				
50	143,347	840	·00586	91	1	—	—
				92	—	—	—
51	133,809·5	996	·00744	93	—	—	—
52	125,687	948	·00754	94	—	—	—
53	118,482	1,061	·00895	95	—	—	—
54	108,628·5	1,047	·00964				
55	95,332·5	1,003	·01052	96	—	—	—
				97	—	—	—
56	85,298·5	933	·01094	98	—	—	—
57	77,701·5	1,021	·01314	99	—	—	—
58	71,364·5	954	·01337	100	—	—	—
59	63,406	963	·01519				
60	46,734	814	·01742	Over 100	—	—	—

Table 8. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

Endowment Assurance Non-Profit (Non-Medical)

Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$	Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$
11–20	668·5	2	·00299	61	3,929	67	·01705
				62	3,265	61	·01868
21	862·5	—	—	63	2,732·5	51	·01866
22	1,353·5	3	·00222	64	2,162	40	·01850
23	1,784·5	4	·00224	65	852	26	·03052
24	2,436	6	·00246				
25	3,236	6	·00185	66	359·5	13	·03616
				67	225	9	·04000
26	3,993·5	7	·00175	68	168	6	·03571
27	4,748	7	·00147	69	104·5	2	·01914
28	5,637·5	6	·00106	70	52	3	·05769
29	6,679	6	·00090				
30	7,637	9	·00118	71	33·5	—	—
				72	27	—	—
31	8,088·5	10	·00124	73	18	—	—
32	8,471·5	10	·00118	74	13	2	·15385
33	8,873·5	9	·00101	75	7·5	—	—
34	9,844·5	9	·00091				
35	11,617·5	14	·00121	76	4·5	1	·22222
				77	4·5	—	—
36	13,565·5	20	·00147	78	3·5	1	·28571
37	15,220	19	·00125	79	3	1	·33333
38	16,782·5	29	·00173	80	1·5	—	—
39	18,328	33	·00180				
40	19,380·5	41	·00212	81	·5	—	—
				82	1	—	—
41	20,264·5	49	·00242	83	—	—	—
42	21,156·5	43	·00203	84	—	—	—
43	21,880	52	·00238	85	—	—	—
44	22,363	58	·00259				
45	22,070·5	83	·00376	86	—	—	—
				87	—	—	—
46	21,801·5	88	·00404	88	—	—	—
47	21,536	85	·00395	89	—	—	—
48	21,207	85	·00401	90	—	—	—
49	20,594	96	·00466				
50	19,282	101	·00524	91	—	—	—
				92	—	—	—
51	18,157·5	107	·00589	93	—	—	—
52	17,080	101	·00591	94	—	—	—
53	16,049	150	·00935	95	—	—	—
54	14,542·5	148	·01018				
55	12,657	107	·00845	96	—	—	—
				97	—	—	—
56	11,155	120	·01076	98	—	—	—
57	9,954	130	·01306	99	—	—	—
58	8,951	119	·01329	100	—	—	—
59	7,797	103	·01321				
60	5,329	86	·01614	Over 100	—		—

Table 9. Assured Lives, 1949–52 (durations 3 and over): exposed-to-risk, deaths and observed rates of mortality

All Classes combined

Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$	Nearest age x	Exposed-to-risk	Deaths	$q_{x-\frac{1}{2}}$
11–20	25,333·5	37	·00146	61	130,992	2,469	·01885
				62	120,059·5	2,406	·02004
21	21,481	25	·00116	63	110,796·5	2,398	·02164
22	28,366	37	·00130	64	99,924	2,394	·02396
23	35,850·5	49	·00137	65	76,235·5	2,012	·02639
24	48,684·5	61	·00125				
25	64,928	71	·00109	66	59,820·5	1,724	·02882
				67	52,935·5	1,699	·03210
26	79,910	98	·00123	68	48,521	1,648	·03396
27	94,692	97	·00102	69	44,606·5	1,648	·03695
28	111,262	119	·00107	70	40,180	1,664	·04141
29	129,644·5	174	·00134				
30	145,356·5	164	·00113	71	36,836	1,737	·04715
				72	34,082·5	1,787	·05243
31	155,522·5	178	·00114	73	31,395	1,787	·05692
32	163,159·5	190	·00116	74	28,981	1,792	·06183
33	171,846·5	229	·00133	75	26,592·5	1,842	·06927
34	190,315	191	·00100				
35	219,602	283	·00129	76	24,391	1,699	·06966
				77	22,587	1,870	·08279
36	248,083·5	331	·00133	78	20,358·5	1,874	·09205
37	271,515·5	407	·00150	79	17,827·5	1,758	·09861
38	292,979	446	·00152	80	14,893	1,561	·10481
39	312,830	566	·00181				
40	327,669	612	·00187	81	12,750·5	1,515	·11882
				82	11,012·5	1,482	·13457
41	342,568·5	720	·00210	83	9,204	1,263	·13722
42	356,066	770	·00216	84	7,809	1,203	·15405
43	366,068	853	·00233	85	6,417·5	1,138	·17733
44	370,850·5	1,025	·00276				
45	367,324	1,204	·00328	86	5,140·5	902	·17547
				87	4,212·5	817	·19395
46	363,686	1,312	·00361	88	3,369	674	·20006
47	361,626	1,426	·00394	89	2,639·5	589	·22315
48	359,864·5	1,642	·00456	90	2,006	449	·22383
49	351,903·5	1,739	·00494				
50	330,051	1,962	·00594	91	1,475·5	386	·26161
				92	1,048	294	·28053
51	309,383	2,110	·00682	93	718	209	·29109
52	292,671·5	2,083	·00712	94	489	151	·30879
53	278,316·5	2,281	·00820	95	326·5	89	·27259
54	262,837	2,373	·00903				
55	240,244	2,374	·00988	96	246·5	93	·37728
				97	137·5	35	·25455
56	223,486·5	2,433	·01089	98	98	25	·25510
57	211,789·5	2,525	·01192	99	58	16	·27586
58	201,586·5	2,612	·01296	100	33·5	5	·14925
59	187,110	2,698	·01442				
60	152,783	2,511	·01644	Over 100	55·5	11	·19820

Table 10. Assured Lives, 1949–52 (durations 3 and over): percentages of actual to expected deaths on the basis of the A 1924–29 table

(*Note.* Percentages for groups in which the actual deaths are less than 100 are shown in italics.)

Age-group	Medical					Non-Medical					All Classes combined
	W.L.W.P.	W.L.N.P.	E.A.W.P.	E.A.N.P.	Total	W.L.W.P.	W.L.N.P.	E.A.W.P.	E.A.N.P.	Total	
10½–19½	*0·0*	*0·0*	*44·2*	*112·4*	*45·2*	*0·0*	*0·0*	*68·4*	*129·9*	*69·8*	*63·3*
20½–24½	*18·9*	*28·3*	*44·9*	*54·1*	*44·3*	*161·3*	*43·5*	51·4	*83·6*	54·7	51·9
25½–29½	*42·6*	*44·2*	48·1	*43·8*	47·1	*48·7*	*65·9*	49·9	*51·5*	50·2	49·1
30½–34½	*36·7*	*49·4*	47·1	*40·8*	45·6	*34·1*	*41·8*	46·2	*42·5*	45·5	45·6
35½–39½	*49·8*	*47·6*	45·3	42·6	45·2	*46·9*	*58·3*	51·2	50·7	51·2	48·5
40½–44½	59·3	51·7	54·9	48·4	54·0	*67·1*	*52·3*	57·2	58·1	57·5	55·8
10½–44½	52·2	49·6	50·5	45·6	49·8	58·2	53·5	53·1	54·3	53·3	51·7
45½–49½	70·9	81·1	72·1	63·2	71·1	87·5	102·1	73·0	69·2	73·8	72·5
50½–54½	77·6	96·9	80·7	73·1	80·4	97·2	80·2	92·2	82·9	90·9	85·7
55½–59½	89·8	86·5	82·1	78·0	82·8	95·9	102·9	90·5	86·5	90·8	86·2
60½–64½	94·9	85·6	84·2	71·5	85·5	99·6	97·5	92·7	79·0	92·7	87·8
65½–69½	86·0	82·6	80·9	59·6	82·6	92·4	90·9	83·4	*95·0*	87·8	83·5
70½–74½	84·7	84·6	73·2	*64·4*	83·5	98·1	95·2	74·2	*31·5*	92·0	84·3
45½–74½	86·3	85·2	80·1	70·8	81·8	95·8	94·8	86·5	79·6	87·1	83·7
75½–79½	84·8	82·1	*62·0*	*72·8*	83·9	93·3	85·2	*93·2*	*172·4*	91·9	84·4
80½–84½	92·3	88·0	*51·4*	*0·0*	91·5	91·0	*86·4*	*93·7*	*0·0*	90·2	91·4
85½–89½	93·7	85·8	*59·2*	*0·0*	92·6	84·7	*82·4*	*80·8*	—	84·3	92·2
90½–94½	101·0	81·6	*0·0*	*0·0*	98·5	*94·7*	*15·0*	*0·0*	—	*79·0*	97·9
95½–99½	98·4	*29·4*	—	—	83·3	*89·6*	*0·0*	—	—	*75·1*	83·0
Over 99½	*41·2*	*37·2*	—	—	*39·6*	—	—	—	—	—	*39·6*
75½–	89·8	83·6	*60·8*	*63·9*	88·5	91·5	82·8	*91·5*	*153·1*	89·9	88·5

Table 11. Assured Lives, 1949–52, All Classes combined: comparison of actual deaths at durations 0–4 with the expected deaths according to the corresponding mortality at durations 3 and over

Age-group	Duration 0			Duration 1			Duration 2			Duration 3			Duration 4		
	A	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*
Under 20½	42	55	76·4	25	39	64·1	24	29	82·8	15	19	78·9	13	11	118·2
20½–24½	112	186	60·2	103	143	72·0	95	99	96·0	68	67	101·5	34	48	70·8
25½–29½	122	209	58·4	171	201	85·1	169	193	87·6	159	163	97·5	125	113	111·6
30½–34½	125	195	64·1	143	197	72·6	168	198	84·8	167	179	93·3	110	144	76·4
35½–39½	151	270	55·9	177	278	63·7	240	285	84·2	212	264	80·3	176	217	81·1
40½–44½	186	354	52·5	265	373	71·0	318	392	81·1	316	384	82·3	304	342	88·9
45½–49½	232	428	54·2	330	450	73·3	408	480	85·0	449	492	91·3	418	470	88·9
50½–54½	194	359	54·0	267	411	65·0	341	455	74·9	403	489	82·4	423	504	83·9
55½–59½	106	233	45·5	216	279	77·4	247	326	75·8	282	370	76·2	319	401	79·6
60½–64½	67	136	49·3	76	166	45·8	147	199	73·9	169	235	71·9	224	271	82·7
Over 64½	28	69	40·6	54	96	56·3	68	116	58·6	115	147	78·2	137	183	74·9
All ages	1,365	2,494	54·7	1,827	2,633	69·4	2,225	2,772	80·3	2,355	2,809	83·8	2,283	2,704	84·4

Table 12. Assured Lives, 1949–52, Medically Examined Classes: comparison of actual deaths at durations 0–4 with the expected deaths according to the corresponding mortality at durations 3 and over

Age-group	Duration 0			Duration 1			Duration 2			Duration 3			Duration 4		
	A	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*
Under 20½	14	10	140·0	9	8	112·5	7	6	116·7	2	4	50·0	3	2	150·0
20½–24½	32	47	68·1	31	39	79·5	34	28	121·4	21	18	116·7	7	12	58·3
25½–29½	46	78	59·0	69	80	86·3	74	79	93·7	70	66	106·1	43	44	97·7
30½–34½	54	84	64·3	62	92	67·4	82	96	85·4	77	88	87·5	51	70	72·9
35½–39½	53	110	48·2	75	123	61·0	114	132	86·4	93	123	75·6	95	100	95·0
40½–44½	63	147	42·9	115	168	68·5	136	183	74·3	144	181	79·6	129	159	81·1
45½–49½	101	190	53·2	149	211	70·6	154	226	68·1	209	227	92·1	198	209	94·7
50½–54½	98	202	48·5	152	226	67·3	164	242	67·8	200	245	81·6	169	234	72·2
55½–59½	68	155	43·9	131	177	74·0	136	202	67·3	173	226	76·5	173	242	71·5
60½–64½	52	113	46·0	51	126	40·5	110	143	76·9	114	159	71·7	149	176	84·7
Over 64½	26	63	41·3	47	83	56·6	51	96	53·1	89	119	74·8	99	143	69·2
All ages	607	1,199	50·6	891	1,333	66·8	1,062	1,433	74·1	1,192	1,456	81·9	1,116	1,391	80·2

Table 13. Assured Lives, 1949–52, Non-Medical Classes: comparison of actual deaths at durations 0–4 with the expected deaths according to the corresponding mortality at durations 3 and over

Age-group	Duration 0			Duration 1			Duration 2			Duration 3			Duration 4		
	A	*E*	100 *A/E*	*A*	*E*	100 *A/E*	*A*	*E*	100 *A/E*	*A*	*E*	100 *A/E*	*A*	*E*	100 *A/E*
Under 20½	28	46	60·9	16	31	51·6	17	23	73·9	13	15	86·7	10	9	111·1
20½–24½	80	137	58·4	72	103	69·9	61	70	87·1	47	48	97·9	27	36	75·0
25½–29½	76	130	58·5	102	120	85·0	95	113	84·1	89	97	89·7	82	68	120·6
30½–34½	71	111	64·0	81	105	77·1	86	101	85·1	90	91	98·9	59	74	79·7
35½–39½	98	160	61·3	102	154	66·2	126	152	82·9	119	139	85·6	81	116	69·8
40½–44½	123	208	59·1	150	206	72·8	182	209	87·1	172	203	84·7	175	183	95·6
45½–49½	131	239	54·8	181	240	75·4	254	254	100·0	240	265	90·6	220	262	84·0
50½–54½	96	152	63·2	115	180	63·9	177	209	84·7	203	242	83·9	254	270	94·1
55½–59½	38	75	50·7	85	100	85·0	111	122	91·0	109	142	76·8	146	158	92·4
60½–64½	15	22	68·2	25	39	64·1	37	55	67·3	55	75	73·3	75	96	78·1
Over 64½	2	·6	33·3	7	12	58·3	17	19	89·5	26	29	89·7	38	41	92·7
All ages	758	1,286	58·9	936	1,290	72·6	1,163	1,327	87·6	1,163	1,346	86·4	1,167	1,313	88·9

Table 14. Assured Lives, 1949–52, All Classes combined, durations 2 and over: exposed-to-risk and comparison of actual deaths with expected deaths according to the proposed new table

Age (nearest)	Exposed-to-risk	Actual deaths	Expected deaths	$A-E$		$\sqrt{A}$	$\frac{\lvert A-E\rvert}{\sqrt{A}}$
				+	−		
21	28,211·5	44	31	13	—	6·6	2·0
22	36,353	49	40	9	—	7·0	1·3
23	52,371·5	67	59	8	—	8·2	1·0
24	72,432·5	80	81	—	1	8·9	·1
25	91,170	98	102	—	4	9·9	·4
26	108,815	128	122	6	—	11·3	·5
27	126,136	127	143	—	16	11·3	1·4
28	144,894·5	150	164	—	14	12·2	1·1
29	164,805	215	188	27	—	14·7	1·8
30	180,717·5	201	208	—	7	14·2	·5
31	188,986·5	210	221	—	11	14·5	·8
32	195,921	214	233	—	19	14·6	1·3
33	204,555·5	275	250	25	—	16·6	1·5
34	223,037·5	225	279	—	54	15·0	3·6
35	253,345	315	329	—	14	17·7	·8
36	282,785·5	360	382	—	22	19·0	1·2
37	308,046	456	441	15	—	21·4	·7
38	329,179·5	498	500	—	2	22·3	·1
39	347,528	623	570	53	—	25·0	2·1
40	361,463·5	665	647	18	—	25·8	·7
41	376,416	779	742	37	—	27·9	1·3
42	390,795	836	856	—	20	28·9	·7
43	398,459·5	903	976	—	73	30·0	2·4
44	400,104·5	1,092	1,100	—	8	33·0	·2
45	394,420	1,280	1,223	57	—	35·8	1·6
46	388,800	1,386	1,361	25	—	37·2	·7
47	387,158	1,529	1,533	—	4	39·1	·1
48	381,634·5	1,720	1,702	18	—	41·5	·4
49	370,278·5	1,805	1,863	—	58	42·5	1·4
50	346,413·5	2,049	1,961	88	—	45·3	1·9
51	324,229·5	2,188	2,056	132	—	46·8	2·8
52	307,858·5	2,152	2,186	—	34	46·4	·7
53	289,714	2,343	2,297	46	—	48·4	1·0
54	271,470·5	2,441	2,397	44	—	49·4	·9
55	247,742·5	2,438	2,433	5	—	49·4	·1
56	230,151	2,497	2,509	—	12	50·0	·2
57	218,517·5	2,590	2,642	—	52	50·9	1·0
58	206,786	2,652	2,767	—	115	51·5	2·2
59	190,991·5	2,737	2,829	—	92	52·3	1·8
60	155,801	2,550	2,550	—	—	50·5	0·0

Table 14 (*cont.*)

Age (nearest)	Exposed-to-risk	Actual deaths	Expected deaths	$A-E$		$\sqrt{A}$	$\frac{\lvert A-E\rvert}{\sqrt{A}}$
				+	−		
61	133,559	2,502	2,415	87	—	50·0	1·7
62	122,722	2,443	2,448	—	5	49·4	·1
63	112,703	2,440	2,481	—	41	49·4	·8
64	101,149	2,412	2,456	—	44	49·1	·9
65	77,238	2,029	2,067	—	38	45·0	·8
66	60,641	1,737	1,788	—	51	41·7	1·2
67	53,847·5	1,714	1,750	—	36	41·4	·9
68	49,150·5	1,664	1,758	—	94	40·8	2·3
69	44,936	1,662	1,769	—	107	40·8	2·6
70	40,360·5	1,665	1,748	—	83	40·8	2·0
71	36,992	1,740	1,762	—	22	41·7	·5
72	34,178	1,789	1,790	—	1	42·3	0·0
73	31,461	1,788	1,810	—	22	42·3	·5
74	29,005·5	1,792	1,832	—	40	42·3	·9
75	26,612	1,844	1,844	—	—	42·9	0·0
76	24,403	1,700	1,854	—	154	41·2	3·7
77	22,593	1,870	1,880	—	10	43·2	·2
78	20,363	1,874	1,855	19	—	43·3	·4
79	17,828·5	1,758	1,776	—	18	41·9	·4
80	14,895	1,561	1,621	—	60	39·5	1·5
81	12,752·5	1,515	1,514	1	—	38·9	0·0
82	11,014·5	1,482	1,425	57	—	38·5	1·5
83	9,204·5	1,263	1,296	—	33	35·5	·9
84	7,809	1,203	1,195	8	—	34·7	·2
85	6,417·5	1,138	1,065	73	—	33·7	2·2
86	5,140·5	902	924	—	22	30·0	·7
87	4,212·5	817	819	—	2	28·6	·1
88	3,369	674	707	—	33	26·0	1·3
89	2,639·5	589	597	—	8	24·3	·3
90	2,006	449	487	—	38	21·2	1·8
91	1,475·5	386	384	2	—	19·6	·1
92	1,048	294	292	2	—	17·1	·1
93	718	209	214	—	5	14·5	·3
94	489	151	155	—	4	12·3	·3
95	326·5	89	110	—	21	9·4	2·2
96	246·5	93	88	5	—	9·6	·5
97	137·5	35	52	—	17	5·9	2·9
98	98	25	39	—	14	5·0	2·8
99	58	16	24	—	8	4·0	2·0
100	33·5	5	15	—	10	2·2	4·5
Total	11,102,329·5	92,286	93,079	880	1,673	2433·0	—
					−793		

Table 15. Assured Lives, 1949–52, Whole Life With-Profit (Medical), durations 2 and over: exposed-to-risk and comparison of actual deaths with expected deaths according to the proposed new table

Age (nearest)	Exposed-to-risk	Actual deaths	Expected deaths	$A-E$		$\sqrt{A}$	$\frac{\lvert A-E\rvert}{\sqrt{A}}$
				+	−		
41	10,588	21	21	—	—	4·6	0·0
42	11,133	16	24	—	8	4·0	2·0
43	11,418·5	36	28	8	—	6·0	1·3
44	11,567·5	40	32	8	—	6·3	1·3
45	11,528·5	32	36	—	4	5·7	·7
46	11,452·5	43	40	3	—	6·6	·5
47	11,376	40	45	—	5	6·3	·8
48	11,449	51	51	—	—	7·1	0·0
49	11,597	53	58	—	5	7·3	·7
50	11,704	67	66	1	—	8·2	·1
51	11,661	57	74	—	17	7·5	2·3
52	11,610·5	79	82	—	3	8·9	·3
53	11,450·5	93	91	2	—	9·6	·2
54	11,566·5	75	102	—	27	8·7	3·1
55	11,865	121	117	4	—	11·0	·4
56	12,313·5	115	134	—	19	10·7	1·8
57	12,949·5	148	157	—	9	12·2	·7
58	13,661·5	178	183	—	5	13·3	·4
59	14,354	230	213	17	—	15·2	1·1
60	14,930·5	272	244	28	—	16·5	1·7
61	15,568·5	312	281	31	—	17·7	1·8
62	16,332·5	355	326	29	—	18·8	1·5
63	17,012·5	383	374	9	—	19·6	·5
64	17,607	462	427	35	—	21·5	1·6
65	18,053·5	522	483	39	—	22·8	1·7
66	18,476	563	545	18	—	23·7	·8
67	19,063	628	619	9	—	25·1	·4
68	19,615	672	702	—	30	25·9	1·2
69	20,044	769	789	—	20	27·7	·7
70	20,289·5	863	879	—	16	29·4	·5
71	20,347	946	969	—	23	30·8	·7
72	20,030	1,055	1,049	6	—	32·5	·2
73	19,553	1,136	1,125	11	—	33·7	·3
74	18,910	1,177	1,194	—	17	34·3	·5
75	18,201·5	1,260	1,261	—	1	35·5	0·0
76	17,218·5	1,165	1,308	—	143	34·1	4·2
77	16,221	1,365	1,350	15	—	36·9	·4
78	14,857	1,377	1,354	23	—	37·1	·6
79	13,169	1,324	1,312	12	—	36·4	·3
80	11,188	1,177	1,217	—	40	34·3	1·2
81	9,819·5	1,158	1,166	—	8	34·0	·2
82	8,611·5	1,176	1,114	62	—	34·3	1·8
83	7,269	1,004	1,024	—	20	31·7	·6
84	6,262	977	958	19	—	31·3	·6
85	5,220·5	945	867	78	—	30·7	2·5
86	4,221	754	759	—	5	27·5	·2
87	3,472·5	672	675	—	3	25·9	·1
88	2,783	579	584	—	5	24·1	·2
89	2,189	497	495	2	—	22·3	·1
90	1,674	377	407	—	30	19·4	1·5
91	1,247·5	333	325	8	—	18·2	·4
92	890·5	259	248	11	—	16·1	·7
93	610	178	182	—	4	13·3	·3
94	428·5	138	136	2	—	11·7	·2
95	282	85	95	—	10	9·2	1·1
96	202·5	82	72	10	—	9·1	1·1
97	106	33	40	—	7	5·7	1·2
98	70·5	23	28	—	5	4·8	1·0
99	36	13	15	—	2	3·6	·6
100	18·5	4	8	—	4	2·0	2·0
Total	647,347	28,565	28,560	500	495	1128·4	—
				+5			

Table 16. Comparison of ratios of select to ultimate rates of mortality in the new table with ratios of the actual deaths in the All Classes, 1949–52, experience at durations 0 and 1 to the expected deaths according to the ultimate rates of mortality

Age-group	Duration 0		Duration 1	
	$q_{[x]}/q_x$ at mid-point of age-group	Ratio of actual deaths to expected deaths by ultimate q_x	$q_{[x-1]+1}/q_x$ at mid-point of age-group	Ratio of actual deaths to expected deaths by ultimate q_x
20½–24½	·607	·671	·723	·798
25½–29½	·602	·598	·721	·877
30½–34½	·578	·622	·705	·708
35½–39½	·563	·585	·714	·668
40½–44½	·539	·533	·710	·722
45½–49½	·520	·551	·701	·745
50½–54½	·501	·557	·683	·669
55½–59½	·483	·449	·667	·763
60½–64½	·466	·493	·652	·461
65½–69½	·451	·357	·636	·519

Table 17. All Classes, 1949–52: exposed-to-risk and actual deaths at durations 0 and 1 with expected deaths according to the select rates of mortality in the proposed new table

Age (nearest)	Duration 0			Duration 1		
	Exposed-to-risk	Actual deaths	Expected deaths	Exposed-to-risk	Actual deaths	Expected deaths
21	23,477·5	24	16	8,323·5	8	7
22	29,325·5	17	20	19,972·5	14	16
23	30,178	27	21	27,056·5	27	22
24	32,539·5	24	22	28,891·5	24	23
25	34,212·5	20	23	30,907·5	30	25
26	35,319·5	28	24	32,758·5	36	27
27	34,799·5	19	24	33,996	37	28
28	35,518·5	29	24	34,172·5	38	28
29	36,698·5	22	25	35,479·5	29	29
30	37,515	24	26	35,450·5	31	29
31	35,969	23	24	34,808·5	28	29
32	32,934·5	23	23	33,684·5	30	28
33	30,656·5	17	21	31,793	37	27
34	31,000	30	22	31,369	15	28
35	33,279·5	32	25	32,936	33	30
36	34,430·5	26	27	35,257·5	31	34
37	33,627	29	27	35,524	38	36
38	32,976	37	28	34,305·5	35	37
39	32,742	35	30	33,537	33	39
40	33,397	24	33	33,308	40	42
41	31,932	27	34	34,182	57	48
42	29,490·5	40	35	32,447	38	51
43	27,836	35	37	29,558·5	61	51
44	26,243·5	37	39	27·712	38	54
45	26,204·5	47	43	25,885·5	71	57
46	22,834·5	40	42	25,995	75	64
47	19,500·5	42	40	22,432·5	64	62
48	17,942	45	42	19,074·5	57	60
49	16,668	42	43	17,361	65	61
50	17,582	63	51	15,952	69	63
51	12,819·5	40	41	16,395	68	71
52	9,542	40	34	12,073	50	59
53	8,199	25	33	9,075·5	53	49
54	7,182·5	36	32	7,851·5	53	47
55	7,198·5	53	35	6,920·5	43	46
56	5,520	29	29	6,977	65	51
57	3,935	22	23	5,347·5	44	43
58	3,083·5	13	20	3,921·5	33	35
59	2,713	22	19	3,101	27	30
60	2,836	20	22	2,684	47	29
61	1,993	19	17	2,781	31	33
62	1,331	18	12	1,962·5	15	26
63	1,079·5	13	11	1,269	8	18
64	913	8	10	1,023	17	16
65	1,026	9	13	866·5	5	15
66	713·5	10	10	978·5	21	19
67	407	4	6	678·5	7	14
68	230·5	4	4	379·5	8	9
69	187·5	—	3	205·5	1	5
70	149	2	3	186·5	5	5
71	78	2	2	121	7	4
72	42·5	1	1	72	1	2
73	32·5	1	1	28·5	—	1
74	25·5	—	1	27·5	—	1
75	12	1	—	16·5	2	1
76	10·5	2	—	12·5	—	1
77	3·5	—	—	6·5	—	—
78	7	1	—	3	—	—
79	9	—	—	4	—	—
80	4·5	—	—	6	—	—
Total	968,113·5	1,323	1,273	959,106	1,800	1,765

Table 18. Values of $q_{[x]}$, $q_{[x-1]+1}$ and q_x according to the proposed table for Assured Lives, 1949–52

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x	$q_{[x]}$	$q_{[x-1]+1}$	q_x
10–19	·00068	·00081	·00111	60	·00815	·01132	·01720
				61	·00893	·01244	·01899
20	·00068	·00081	·00111	62	·00979	·01367	·02096
21	·00068	·00081	·00111	63	·01073	·01501	·02312
22	·00068	·00081	·00111	64	·01175	·01647	·02549
23	·00068	·00081	·00112				
24	·00068	·00081	·00112	65	·01286	·01807	·02809
				66	·01408	·01981	·03095
25	·00068	·00081	·00112	67	·01542	·02171	·03409
26	·00068	·00081	·00113	68	·01688	·02378	·03753
27	·00068	·00081	·00113	69	·01848	·02604	·04130
28	·00068	·00082	·00114				
29	·00068	·00082	·00115	70	·02022	·02849	·04543
				71	·02213	·03115	·04995
30	·00068	·00083	·00116	72	·02422	·03403	·05489
31	·00069	·00084	·00118	73	·02650	·03716	·06028
32	·00070	·00085	·00120	74	·02899	·04053	·06616
33	·00071	·00087	·00123				
34	·00073	·00090	·00127	75	·03171	·04417	·07257
				76	·03469	·04808	·07953
35	·00075	·00093	·00132	77	·03795	·05227	·08709
36	·00079	·00099	·00139	78	·04150	·05676	·09528
37	·00083	·00105	·00148	79	·04539	·06155	·10414
38	·00088	·00112	·00158				
39	·00095	·00121	·00171	80	·04963	·06661	·11369
				81	—	·07197	·12397
40	·00103	·00134	·00188	82	—	—	·13500
41	·00113	·00148	·00208	83	—	—	·14681
42	·00125	·00164	·00231	84	—	—	·15942
43	·00139	·00184	·00259				
44	·00156	·00207	·00292	85	—	—	·17282
				86	—	—	·18704
45	·00175	·00233	·00330	87	—	—	·20205
46	·00195	·00261	·00372	88	—	—	·21785
47	·00219	·00294	·00420	89	—	—	·23440
48	·00245	·00331	·00474				
49	·00274	·00371	·00534	90	—	—	·25168
				91	—	—	·26963
50	·00305	·00413	·00599	92	—	—	·28817
51	·00340	·00461	·00671	93	—	—	·30730
52	·00377	·00513	·00750	94	—	—	·32688
53	·00417	·00570	·00837				
54	·00461	·00630	·00931	95	—	—	·34683
				96	—	—	·36706
55	·00508	·00698	·01035	97	—	—	·38747
56	·00560	·00770	·01148	98	—	—	·40796
57	·00616	·00849	·01272	99	—	—	·42840
58	·00676	·00936	·01408				
59	·00743	·01030	·01557				

Table 19. Values of q_x by various Mortality Tables for Assured Lives

x	OM Aggregate	A 1924–29	A 1924–29 Light	1947/48	1949–52
20	·00404	·00235	·00172	·00150	·00111
25	·00481	·00235	·00190	·00150	·00112
30	·00595	·00241	·00207	·00150	·00116
35	·00738	·00286	·00231	·00164	·00132
40	·00915	·00388	·00310	·00220	·00188
45	·01153	·00527	·00426	·00371	·00330
50	·01504	·00764	·00635	·00616	·00599
55	·02045	·01190	·00983	·01003	·01035
60	·02887	·01973	·01574	·01652	·01720
65	·04196	·03188	·02662	·02699	·02809
70	·06207	·05327	·04666	·04135	·04543
75	·09264	·08497	·07856	·06728	·07257
80	·13844	·12910	·12019	·11414	·11369
85	·20569	·18676	·17254	·16680	·17282
90	·30075	·25611	·24164	·23584	·25168
95	·42473	·33675	·33475	·32005	·34683

APPENDIX 1

Note on the Variations between Offices

In paragraph 6 reference is made to variations in mortality among the offices contributing data to the Continuous Mortality Investigation. It is not possible to give full information on this subject because the statistics of individual offices are confidential. However, an indication of the range of variation is given in Tables A and B, on pp. 35–6. These tables are based on the 1949–52 data at durations 3 and over within age-group 45–75 (i.e. the range of ages at which variations are most important). The percentages of the actual deaths to the expected deaths by the A 1924–29 table have been calculated for each office, separately for Medical and Non-Medical business. The deviations from the corresponding percentages for all offices combined (i.e. 81·8% for Medical business and 87·1% for Non-Medical business) are summarized in Table A. Where there were fewer than 100 deaths, the office concerned was omitted; as a consequence there are fifty-four offices in the Medical column of the table and twenty-five offices in the Non-Medical column.

Table A shows that in the Medical section the offices were widely and evenly spread. Although there is some concentration in the immediate neighbourhood of the mean—thirteen out of the fifty-four offices differ from the mean percentage by less than 2%—there are eight offices which differ by more than 12%. In actual fact, the observed percentages range from 68·2 to 97·7.

The Non-Medical section shows a highly unusual distribution. Not a single office comes within 2% of the mean. The dispersal is very wide indeed, and the actual percentages range from 53·9 to 101·9. Out of the total of twenty-five offices, seven offices differ from the mean by more than 14%. On the whole the distribution, though very ragged, has a bi-modal character—the offices above the mean forming one group and those below the mean forming another.

Table A. Contributing offices analysed according to the observed deviation of the percentage of actual to expected deaths by A 1924–29 within age-group 45–75 from the corresponding percentage for all offices combined

Deviation from mean percentage	Medical (mean percentage = 81·8 %) No. of offices	Non-Medical (mean percentage = 87·1 %) No. of offices
Positive deviations		
14·0–16·0	3	3
12·0–14·0	1	—
10·0–12·0	2	1
8·0–12·0	2	2
6·0– 8·0	3	1
4·0– 6·0	5	3
2·0– 4·0	3	1
0·0– 2·0	6	—
Negative deviations		
0·0– 2·0	7	—
2·0– 4·0	3	2
4·0– 6·0	6	3
6·0– 8·0	4	—
8·0–10·0	3	—
10·0–12·0	2	1
12·0–14·0	4	4
14·0–16·0	—	—
16·0–18·0	—	1
18·0–20·0	—	1
20·0 and over	—	2
Total	54	25

The observed deviations, however, are subject to different standard errors, depending upon the size of office. Accordingly, Table B has been prepared in which the actual deviations have been standardized by dividing them by their corresponding standard errors. Out of fifty-four offices in the Medical section, four have deviations greater than four times their corresponding standard errors, while seventeen differ from the mean by more than twice their standard errors.

In the Non-Medical section, eleven offices out of twenty-five differ from the mean by more than twice their corresponding standard errors. Eight of these eleven have negative deviations, i.e. they are 'light' offices. It happens that some of the heavier offices are among the larger offices, and their proportionate weight causes the mean percentage to be appreciably higher than the median. As a result there is a longer 'tail' below the mean than above it.

The Non-Medical distribution in Table B is, in fact, ragged and widely dispersed and, as in Table A, there is a deficiency in the neighbourhood of the mean. All these observations tend to support the view that inter-office variations are inherent rather than due to chance fluctuation.

Of the twenty-five offices included in the Non-Medical columns of Tables A and B, one office did not contribute data to the Medical section. A comparison has been made between the Medical and Non-Medical percentages for the

remaining twenty-four offices, and it has been found that for ten of these offices the Non-Medical experience was actually *lighter* than the Medical experience. The heavier mortality of the Non-Medical experience for all offices combined derives more from the proportionately greater weight of data contributed by the heavier offices in this section than from the intrinsic differences between medically examined lives and lives accepted under non-medical schemes.

Table B. Contributing offices analysed according to the standardized deviation of the percentage of actual to expected deaths within age-group 45–75 from the corresponding percentage for all offices combined

Standardized deviation	Medical No. of offices	Non-Medical No. of offices
	Positive deviations	
7·0–8·0	—	1
6·0–7·0	—	—
5·0–6·0	1	—
4·0–5·0	1	—
3·0–4·0	—	1
2·0–3·0	4	1
1·5–2·0	5	2
1·0–1·5	5	4
0·5–1·0	3	2
0·0–0·5	6	—
	Negative deviations	
0·0–0·5	7	1
0·5–1·0	3	2
1·0–1·5	4	1
1·5–2·0	4	2
2·0–3·0	5	2
3·0–4·0	4	2
4·0–5·0	2	3
5·0–6·0	—	—
6·0–7·0	—	1
Total	54	25

APPENDIX 2

Note on functions proposed for inclusion in the published tables

In June 1955 a circular was addressed to the offices which contribute to the Continuous Mortality Investigation setting out proposals for functions to be included in the published tables. The text of these proposals is reproduced below.

(1) *Mortality functions*

$$l_{[x]} \qquad l_{[x]+1} \qquad l_{x+2}$$

(N.B. The radix to be $l_{10}=999{,}999$)

$$d_{[x]} \qquad d_{[x]+1} \qquad d_{x+2}$$

$$q_{[x]} \qquad q_{[x]+1} \qquad q_{x+2}$$

$$\mu_{[x]} \qquad \mu_{[x]+1} \qquad \mu_{x+2}$$

$$e_x \qquad \mu_x l_x \qquad \log l_x$$

(2) *Single-life monetary functions (other than policy values)*

It is proposed to tabulate single-life monetary functions (other than policy values) at the following rates of interest:

1%	1½%	2%	2¼%	2½%
2¾%	3%	3¼%	3½%	3¾%
4%	4½%	5%	5½%	6%

The commutation functions proposed are:

$$D^{-1}_{[x]} \qquad D^{-1}_x$$

$$D_{[x]} \qquad D_x \qquad N_{[x]} \qquad N_x$$

$$C_{[x]} \qquad C_x \qquad M_{[x]} \qquad M_x$$

$$S_{[x]} \qquad S_x \qquad R_{[x]} \qquad R_x$$

It is felt that functions at duration 1 are seldom used. When required they can be easily computed from other tabulated functions by a simple series of formulae which will be given in the published volume.

The assurance and annuity functions proposed are:

$$a_{[x]} \qquad A_{[x]} \qquad P_{[x]} \qquad a_{[x]:\overline{n}|}$$

$$a_x \qquad A_x \qquad P_x \qquad a_{x:\overline{n}|}$$

(N.B. Temporary annuities are to be tabulated for *every* age.)

$$\left.\begin{array}{ll} A_{[x]:\overline{n}|} & A_{x:\overline{n}|} \\ {}_nE_{[x]} & {}_nE_x \\ P_{[x]:\overline{n}|} & P_{x:\overline{n}|} \end{array}\right\} \text{For quinquennial values of } (x+n) \text{ from 40 to 75}$$

It is not proposed to tabulate values of ${}_tP_{[x]}$ and ${}_tP_x$. This decision will, however, be re-considered if a number of offices should express a need for these functions.

(3) *Single-life policy values*

It is proposed to tabulate single-life policy values at the following rates of interest:

2%	2½%	3%	3½%
4%	4½%	5%	6%

The functions to be tabulated are:

$100_t V_x$ for all values of t and x.

$100_t V_{x:\overline{n}|}$ for quinquennial values of x and individual values of t and n.

(4) *Joint-life functions*

The following rates of interest are proposed for the tabulation of joint-life functions:

1%	1½%	2%	2½%	3%
3½%	4%	4½%	5%	6%

The functions which it is proposed to tabulate are:

$D^{-1}_{[xx]}$	D^{-1}_{xx}		
$D_{[xx]}$	D_{xx}	$N_{[xx]}$	N_{xx}
$M_{[xx]}$	M_{xx}	$R_{[xx]}$	R_{xx}
$a_{[xx]}$	a_{xx}	$A_{[xx]}$	A_{xx}
$P_{[xx]}$	P_{xx}		

a_{xy} for alternate values of x and y

$a_{[xxx]}$	a_{xxx}	$a_{[xxxx]}$	a_{xxxx}

The offices were invited to comment on these proposals and their replies have been analysed. Suggestions having the support of at least three offices are shown below.

(1) *Mortality functions*

Include $e_{[x]}$	(3 offices)
Omit $\log l_x$	(5 offices)

(2) *Single-life monetary functions (other than policy values)*

Rates of interest: Include 1¼%	(7 offices)
1¾%	(11 offices)
Omit 6%	(5 offices)
5½%	(3 offices)
Include $C_{[x]+1}$	(3 offices)
Include temporary annuities alongside $A_{x:\overline{n}|}$ and $P_{x:\overline{n}|}$ for quinquennial maturity ages	(8 offices)
Omit ${}_nE_{[x]}$ and ${}_nE_x$	(5 offices)
Tabulate annuities due instead of immediate annuities	(5 offices)
Include ${}_tP_{[x]}$	(6 offices)
Include ${}_tP_x$	(9 offices)
Include continuous functions $\bar{A}$, $\bar{P}$ and $\bar{a}$	(4 offices)

(3) *Single-life policy values*

Rates of interest: Include	$2\frac{1}{4}\%$	(3 offices)
	$2\frac{3}{4}\%$	(3 offices)

(4) *Joint-life functions*

Rates of interest: Include	$2\frac{1}{4}\%$	(8 offices)
	$2\frac{3}{4}\%$	(8 offices)
	$3\frac{1}{4}\%$	(3 offices)
Omit	6%	(4 offices)
Include $a_{[xy]}$		(6 offices)

Other suggestions

Among suggestions advanced by not more than two offices were the following:

(i) Tabulation of certain functions at half ages.

(ii) The use of four decimal places in the tabulation of annuities.

(iii) The tabulation of temporary increasing assurance functions for quinquennial maturity ages.

(iv) The use of even ages for x and odd ages for y in the tabulation of a_{xy} at alternate ages.

Conclusion

Inevitably, most of the suggestions involve the publication of more material and thus increase the size and cost of the volume. It would not be practicable to give effect to all the suggestions reproduced above, and it is hoped that the Institute and Faculty discussions may help the Committee in the decisions which will have to be made when determining the contents of the published tables.

ABSTRACT OF DISCUSSION AT THE FACULTY OF ACTUARIES*

Mr R. E. Beard, in presenting the report, remarked that it was hard to think of a better subject for discussion in the centenary year of the Faculty than that of Assured Lives' Mortality. The actuarial profession had its origins in the construction of mortality tables and the use thereof, and despite efforts and excursions into other fields, or widening spheres, the technique and training developed from the use of mortality tables still formed the foundation of actuarial science.

It was important to keep clearly in mind what they were doing when they investigated mortality statistics and from them prepared tables for practical use. They were devising instruments for practical application, and while some might enjoy speculating on the reasons for any specific mathematical expressions designed to describe the statistics, such thoughts should not be allowed to cloud the practical issues involved in the construction of the tables. That night they were to discuss the fundamental tools of the profession, not the philosophy behind them; therefore they were concerned with the practical end.

Mortality rates were not static; the shape of the mortality curve and its relative levels were dynamic, and they had been varying ever since any reliable statistics had been tabulated. There was no such thing as an absolute level of mortality and rates depended much on the manner of selection of the group of lives whose experience was being examined. That absence of an absolute standard was one of the reasons for the actuarial profession, because it implied the need for judgment in any particular application of the technique of the actuary, who had to determine the appropriate instrument to use and the consequences of deviations from the standard adopted.

In no sphere was that lack of an absolute standard so marked as in the comparative mortality among insurance offices and the classes of policy they issued. There was no mortality table which could be stated to be right for any particular group, either because of the method of selection of the group or because of changes in time. In devising a standard table, therefore, they had to produce one which, in some sense, was right for no one section.

Reference was made in the memorandum to a concept that had been dormant for 50 years in the discussions of mortality tables, namely, the effect of selective withdrawal. That factor could explain some of the features of select mortality experience that had been observed. If that were one of the main reasons behind the variations shown it had to be remembered that the mortality rates were those of a selected group of the population subject to withdrawal—a variation factor of some considerable power and one which had to be kept in mind when a particular mortality table was applied in a given set of circumstances.

It was against that background of variation and general principles that the Committee had endeavoured to produce a new standard table, and there were five main decisions which had led them to put forward particularly the table they proposed.

The first point was the general level of the mortality rates—that was discussed in § 3. There were reasons for excluding 1947 and 1948; although 1948 was a very light year, in the opinion of the Committee it did link up with 1947 and accordingly they finally decided to use 1949–1952. 1951 was a little heavier

* A full report of the discussion in Edinburgh is to be found in *T.F.A.* 23, 212.

than usual owing to an influenza epidemic, but the other three years were probably a little lighter; on the average the four years selected were probably reasonably representative of the immediate past.

Having fixed the period to be used, they had had to decide what sort of table they should produce out of the heterogeneous mass of data. In §§ 5–8 they had given some of the reasons which had influenced them in aggregating all the data. They knew that the level of mortality was largely influenced by variation among the offices making up the statistics, and those differences were reflected in the experience by classes; accordingly it was thought best to use a level of mortality which went right through the middle of the data; it would not be right for any particular office but, on the other hand, it would be broadly representative of the group as a whole.

It was necessary next to decide whether the general level so obtained should be modified in any way for the production of the final table, and for the reasons given in § 9 they had decided that it was proper to give greater weight at the later middle ages to the whole life experience.

They had had also to discuss and decide how long a period of selection they should use, and although they had put little in the memorandum on the question of selection there had been a long discussion on the subject. The evidence from the figures was that selection persisted for a long time. There were various reasons for that, and there was also the question of selective withdrawals. The trouble was that it was impossible to say where the ultimate table should be; they had not been able to analyse it beyond five years anyhow—even if they had there would still have been some difficulty in knowing what should be the proper ultimate level. The final decision had been that the best thing to do was to use the ultimate table based on the combined experience, knowing full well that it included a lot of unexpired selection, and to use a two-years select period only.

The graduation process chosen was a simple one. The Committee had pooled their experience and decided what mortality rates they required and fortunately they had been able to find a logistic or Perks curve with an adjustment which gave a suitable result. It was smooth and it had a little bump in the middle of it which they thought reflected what was in the data, but beyond that it was a curve which the tests that had been made showed to be a reasonable representation. They did not claim perfection for their particular table; it was a practical instrument for practical purposes. It provided a modern standard against which each actuary could measure his own particular requirements.

Mr G. F. Menzies, in opening the discussion, said that a new mortality table was bound to provoke argument and criticism and the Committee had, no doubt, felt tempted to give lengthy reasons for their every action in order to forestall criticism. They had resisted that temptation. The result was an admirably lucid memorandum which left plenty of room for critics and he proposed to take full advantage of the opportunity.

The decision to present a hotchpotch table was, he thought, inevitable in view of the success of the A 1924–29 Table. Any other decision would have raised a storm of criticism. It was interesting, however, to consider the Committee's justification for it.

The reasons given in § 3 for the choice of the period seemed unanswerable. The group under age 45 showed mortality significantly different from that of the two other groups. That body corresponded to the military service ages and for obvious reasons it could be imagined that during the period under review there

would have been a growing weight of new entrants at durations three, four and five, which by reason of selection might have influenced the result. Moreover it was possible that that post-war generation was completely different in character from the pre-war generation. The over-45 group might also have suffered to some extent from the delayed effect of war strain. From the statistics given it was not easy to pursue that line of thought, but it would have been interesting to know if there was any evidence of piling up of the exposed-to-risk at the earlier durations when examined in separate calendar years.

§5 reminded him of the defence of the small boy who was accused of breaking his pal's bat. 'In the first place', he said, 'I never took the bat, in the second place it was broken when I got it, and in the third place it was all right when I put it back'—like the Committee's, a complete defence!

Whole life mortality was heavier than endowment assurance mortality, but the data were insufficient for a table at the earlier ages in one case and at the later ages in the other. Non-medical was heavier than medical mortality but that was due to the offices who contributed the data rather than to any inherent difference. In the endowment class, non-profit mortality was lighter than with-profit, but in the whole life class they were approximately equal. There again it was suggested that differences among the offices were responsible. It might be suggested also that the rapid post-war growth of non-profit business had some connexion. However, the only consistent conclusion which emerged was that the data were hopelessly heterogeneous and the Committee could not be blamed for producing a hotchpotch table.

He wondered sometimes if it was altogether to the credit of the Census method that it had focused attention so closely on the heterogeneity as between offices. After all, if they took the largest office in the heavy group and examined it by geographical or occupational grouping they would again, no doubt, find heterogeneity. For practical reasons they had to use the same premium rates for both sexes, for all parts of the country, and for widely differing occupations. That, he thought, was the real defence for the hotchpotch and it had to be coupled with the remarks made by the Committee in their final paragraph, 'The working actuary must continue to use his individual judgment.'

The Committee, in Appendix 1, had raised a small corner of the security curtain covering the sources of data, but on the question of light and heavy offices, he wished they would go just a little further. Had there been through the years any alteration in the constitution of light and heavy groups or was it a case of once a heavy always a heavy office?

Having decided to use all the data—he was still thinking of his small boy—the Committee next decided that parts of it could not altogether be trusted and that consequently between 50 and 70 the published table should run closer to the whole life rates between those ages. He accepted their judgment, but he was not altogether convinced by their reasoning. They had suggested also the possibility of constructing later a hypothetical light table bearing an arbitrarily determined relation to the basic table. If there was progress much further along those lines, they might even in the future be spared the necessity of collecting and tabulating data!

The treatment of selection and the select period was perhaps the most controversial issue raised by the memorandum. He was rather disappointed that the Committee had not had the courage of their convictions and produced an aggregate or a truncated aggregate table. As it was, the select period had been reduced from three to two years and the difference between the select and ultimate net

premiums given in § 25 appeared to run from 1*d.* % to 2*s.* 3*d.* % on whole life, and from about 2*d.* % to 2*s.* 8*d* % on endowment assurance premiums. The differences between the select and aggregate premiums would presumably be less. Relative to the premium itself those reductions could only be important for term assurance and there, in practice, heavy loadings were customarily imposed. For the sake of them the calculations and printing which the new table involved were practically doubled. He, for one, would have welcomed an aggregate table.

Having said that, he wished to examine in more detail the treatment adopted. In § 10 it was suggested that the ratio A/E might reach 110% at higher durations. That might be so, but since durations 3 and 4 were necessarily small in weight compared to the remaining durations for the years 5 and over, he was not convinced that the ratio was bound to rise appreciably higher than 100% and doubted that it would become as high as 110%. Had the rates for durations 5 and over been given it would have thrown some light on the point.

Of the reasons given in § 11 for the prolonged select period, spurious selection appeared most credible to him, particularly since durations 0–5 contained post-war new entrants who might well be different in character from pre-war assured lives. On the following page the Committee suggested that medical examination might have the effect of lowering mortality by about 8% of the ultimate rate. He found that reasoning a little difficult to follow, since earlier they had said that heavy offices contributed a higher proportion of the non-medical data. He was not at all impressed by reason (*b*)—withdrawal selection. If the deliberate selection exercised by the offices was so trivial in effect, he found it difficult to believe that self-selection by policyholders could affect and continue to affect mortality at high durations.

In the consideration of all those questions there was bound to be a feeling of frustration that the data, as far as duration was concerned, stopped at the end of 5 years. Beyond that period actuaries continued to indulge, as they had for the past thirty years, in airy speculations and theories which could not be tested.

He presumed that the Committee were satisfied, particularly since the period of investigation had been cut to four years, that the census method was sufficiently accurate to cope with the early durations. Since the war, the end of the fiscal year had introduced another influence to disturb the even distribution of entrants over the calendar year. If new business were increasing over the four years, could that affect the accuracy of the data? However, he had no wish to add his personal speculations to those already in vogue, but he suggested that it was time that they examined the question of selection more fully. Punched-card equipment had become almost universal for the contributing offices, and he thought that the Committee might consider the merits of some sort of policy-year investigation which would enable them to go beyond the five-year limit.

Of the graduation he said that fitting by trial and error covered a multitude of figures. A little information on how the first trial values were obtained would be helpful to students and instructive to their elders. Did the Perks formula fail? He had tried a tentative experiment with the Perks formula, but unfortunately the first trial ended in disaster and he had had no time to repeat the experiment.

The formulae used in the graduation of the select period were elegant, but the graduated ratios in Table 16 present a ragged appearance. He presumed that the maximum and minimum values referred to in § 22 were simply an accident of the method, but what he found really difficult to accept was the fact that for both durations the expected deaths were less than the actual; that excess of the

actual as measured by the accumulated deviation ran right down the table. It would be seen from the grouped totals in §22 that the apparently closer total agreement of duration 1 was achieved by a large negative deviation in the last age-group 61–80. No doubt, even if the graduation were adjusted to bring equality between expected and actual deaths, the effect on monetary values would be slight, but, so far as he was concerned, the graduation made him even more sceptical of the practical value of the two select columns.

He thought that the Committee were somewhat niggardly in their presentation of monetary values. The table in §25 was too scamped for a true impression of the effect of the new table to be obtained.

Having, as he said, exercised fully his privilege of carping, he conceded to the full the Committee's modest claim to have presented a serviceable working table. He was sure that the new table would prove a sound successor to the A 1924–29 Table and he congratulated the Committee on the completion of a difficult piece of work.

Mr D. W. A. Donald agreed that the new tables would be a welcome aid to the modern actuary though, as the Committee pointed out, they would have to be used with discretion. If a study of the memorandum gave the impression that disproportionate importance had been given to answering traditional questions that was not a reason to criticize the Committee. They had sought to answer the questions that the profession asked, and with the data available they had produced a workmanlike table which was a reasonable representation of the average experience of assured lives' mortality in recent average years. Whether that information was the most useful aid to actuaries faced with the problem of the prudent conduct of their offices' business in modern conditions was more open to doubt. So many questions were left unanswered—unasked even—and had to be left unanswered while the offices and the profession were content with what, in his opinion, were the outmoded methods by which the continuous investigation was carried out. Was there not a danger that, viewed against the comfortable background of continuously improving vitality, actuaries as a profession had been too content to feel that time was on their side, that for traditional assurance business any table based on the experience of the past was more or less bound to be 'on the right side' for the future and that therefore they could ignore many questions which it would certainly be troublesome and expensive to investigate, particularly when it was not known whether all the trouble and expense would produce any useful answers? So long as the bulk of the liabilities underwritten had been of the same type as those reflected in the experience investigated, the doubts whether any more elaborate investigation would produce any more definite answers had been, if not a worthy, at least a sufficient reason for not undertaking a possibly fruitless quest for knowledge. Was that still true? Many offices were undertaking large liabilities annually under various forms of pension scheme and there were questions of what were the likely rates of mortality in active service and of mortality of lives not subject to rigorous medical selection; what were the effects on mortality of rates of withdrawal (particularly in times of trade depression)? There were questions also of the mortality of term assurance business and of whether there was any significant variation of mortality with size of sum assured. Those had all become of far greater and more direct importance financially than ever before. Some of those questions would require new investigations; others would require new methods to be applied to existing investigations. It might be that more time and money spent on at least

trying to answer them would be amply repaid in the future. For a large part of the new liabilities assumed each year it was no longer true that time was on their side.

To him the most interesting feature of the new data was not so much the rates of mortality produced as the movements in those rates since the 1924–29 experience. It was not obvious why mortality should vary as much as it did with class of policy, but much the same features were shown by the new experience as were apparent in the previous one and it was reasonable to assume that that was no chance result. If that were true the rates derived by combining all the classes into the 1949–52 experience were not strictly comparable with the similar results in 1924–29; they had to be careful what conclusions they drew from the apparent reductions in rates of mortality between the two experiences. The proportions of each class of business contributing to each experience had altered considerably. Taking the ages from 21–81 as being the most important financially, the exposures in the two experiences might be compared in the following way.

In 1924–29 26% of the data related to whole life and 74% to endowment assurance policies. For 1949–52 the figures were 14% and 86%. In the previous experience 85% of the data came from with-profit business. In the new data the figure was 82%—a surprisingly small change. The biggest change had been in the proportion of non-medical business, from 16% to 49·7%. On comparing the rates experienced in each class it seemed that secular improvements had occurred at approximately the same rate and, as the differences between various classes, though significant, were small, the effect in the new experience of its completely different composition was masked. That might not continue to be true, and if an office's own experience was based on a different blend of the different classes, or if by its new business policy it was consciously trying to influence the growth of any one class, then the fact that it was at that time experiencing lighter mortality than the rates proposed in the new table did not necessarily mean that the new table would be safe for that office in the future.

That, he thought, was particularly true of the mortality of policies not subject to medical examination and the statement in §6 possibly went further than the Committee intended in giving the impression that medical selection might not have much influence on mortality rates. When he read it he wondered which came first, the hen or the egg. In other words were the non-medical rates shown heavy because they came mainly from 'heavy' offices, or were those offices 'heavy' because they did a large amount of non-medical business and might therefore also be less selective in their medical underwriting standards? He had never seen any experience of policies of the same class effected at the same time that did not show lighter mortality for medically examined lives than for those accepted without medical examination. Indeed, the most striking evidence of the effects of searching medical requirements had been in the recent American investigation into the mortality experience of large policies. Over a considerable period such policies had shown lighter mortality, though before 1932 that was not true. The explanation given was first the additional specialist examinations (X-ray, E.C.G. and so on) required as a matter of course in America for large policies, and second an attempt by the companies, after their experience in 1929–32, to stiffen their standards of selection and to be less influenced by the competitive attractions of large sums assured in reaching record new business figures. Those might not be the actual reasons, but they were at least plausible. In current conditions of competition in Britain it was, he thought, a pity that what might be taken to be official backing should be given to the idea that medical examinations did not really matter. On the other

side there seemed to be some evidence that personal selection might count for more than medical selection, for the rates of mortality exhibited by male annuitants in 1946–48 (again average years) at duration 5 and over were lighter at most ages from 61 to 75 than those for assured lives at durations 3 and over in 1949–52. The reasons for that might repay study—possibly on the lines of an analysis of causes of death. Did the reference in § 17 to the effects of respiratory cancer and coronary disease indicate that some such investigation was already under way?

All that was only to emphasize the warnings the Committee had given in § 8 that 'the individual actuary will always need to employ his personal judgment when determining a mortality basis to suit a given set of circumstances.' The circumstances demanding the exercise of that judgment had changed so greatly that he suggested for the future that the individual actuary would be helped more if the traditional lines of investigation were left to look after themselves. With the new tables, and possibly a set based on the heaviest and lightest rates exhibited in different classes of policy, it might be that they could feel that they had received sufficient guidance about assured lives' mortality for many years to come and that the Committee might be asked to consider, with the co-operation of the offices in time and expense, some of the other important problems of mortality rates then confronting them.

Mr D. Begg said that under modern conditions assurance companies were being called on to provide pension schemes for clerical staff, the members of which had formerly to provide for their own old age by investment. What better form of investment was there than an endowment assurance? With the aid of the percentages given in Table 10 he had compared the whole life with the endowment assurance business. In doing so, of course, he had had to weight the experiences, which he had done by using the exposed-to-risk in Tables 1 to 8. For example, he had found that if he took the group $45\frac{1}{2}$–$49\frac{1}{2}$ the weighted average for whole life was 81·9%, while the endowment assurance figures, similarly weighted, gave 73%, a difference of 8·9%. The interesting thing was that when the investigation was carried through what he had called the comparable ages there was a steady difference of some 5%–7% taking the data as a whole, i.e. the actual to the expected for whole life were about 5%–7% higher than the actual to expected for endowment assurance. In the medically examined class the differences were rather more striking, because they were almost constant at a figure of about 6%–7%. The detailed results were:

Percentages of actual to expected deaths

Age-group	W.L. Total	E.A. Total	Diff.	W.L. Med.	E.A. Med.	Diff.
$35\frac{1}{2}$–$39\frac{1}{2}$	50·3	48·2	2·1	49·1	44·6	4·5
$40\frac{1}{2}$–$44\frac{1}{2}$	57·1	55·6	1·5	55·7	53·7	2·0
$45\frac{1}{2}$–$49\frac{1}{2}$	81·9	73·0	8·9	75·8	70·3	5·5
$50\frac{1}{2}$–$54\frac{1}{2}$	87·1	85·7	1·4	86·7	79·5	7·2
$55\frac{1}{2}$–$59\frac{1}{2}$	91·1	85·5	5·6	88·7	81·2	7·5
$60\frac{1}{2}$–$64\frac{1}{2}$	93·0	85·6	7·4	90·8	81·9	8·9
$65\frac{1}{2}$–$69\frac{1}{2}$	85·4	79·0	6·4	85·1	77·4	7·7

The differences between the endowment assurance standard of mortality and whole life standard showed no tendency to disappear towards the end of the

comparable ages; they kept on, indeed, slightly increasing. It appeared to him, therefore, as though *a priori* the lower rates of endowment mortality would continue after the comparable ages. It was indeed sad that the actual experience was, of course, quite unobtainable.

He saw no objection to combining the medical and non-medical classes for, while the rates of mortality might differ, there was a comparatively stable ratio of exposed-to-risk in each class. Up to age 60 only 10% or less of the experience was of what he would call the heavier mortality class and 90%, at least, of the endowment assurance class; over age 70 the position was completely reversed and at least 90% was of the heavy class of mortality and only 10% of the light. While, therefore, he complimented the Committee on their mathematical adjustments and graduation which was clearly demonstrated in Table 16, he wondered whether they had not succeeded in mating a lion with a tigress.

Keeping in mind the small amount of the whole life data, which was definitely less than 10% up to age 60, he would hesitate to suggest graduating the whole life data alone. The question in his mind was rather whether a satisfactory graduation could be obtained by using the endowment data alone or by using the complete data up to say age 65 and omitting all the data thereafter. The idea of using only part of the data was not a new one. He had taken it from George King who at that point was referring back to Makeham, whose paper was even earlier.

There was no doubt about the validity of combining different types of mortality, such as with-profit and non-profit, in order to obtain a thoroughly practical working table on which the working actuary might make such adjustments as in practice he felt were called for; but where there were two demonstrably different types of mortality the validity of combining those two curves seemed open to question, especially in view of the increasing importance of the small pension fund in life assurance practice.

He had raised a similar question in discussing the 1924–29 tables and he thought it was one of special importance in modern conditions.

Mr A. A. MacDonald remarked that the heterogeneity of the data was particularly noticeable in the non-medical experience. He thought it should be remembered, however, that non-medical business really consisted of two different types, what might be called true non-medical business and the business that was non-medical because the office refused or decided not to take an examination. In 1924 the Committee in a memorandum addressed to contributing offices had asked them to state whether the business had been accepted after receipt of a full and satisfactory statement as to personal health and family history or whether it had been accepted under schemes for staffs, etc., without any evidence of health. It seemed to him that that might be the division in the non-medical experience and it seemed likely that the ten offices whose non-medical experience was lighter than the corresponding medical experience belonged to the first of the above categories. The Committee did not indicate whether those ten offices belonged to the light group, and it would be interesting to know if that were the case.

He suggested that non-medical business should be re-defined to include only business accepted without evidence of health, and that cases where the office reserved the right to examine but did not do so because it considered examination unnecessary in the light of the medical evidence already available should be treated as medical business. By that means they would remove one source of

heterogeneity from the non-medical data. He felt that any comparison such as that in § 12 using the non-medical data was bound to be open to suspicion as that experience seemed to be merely an averaging of two entirely dissimilar ones. If the duration of selection had been examined with the data split between the light and heavy offices that might have given some measure of any spurious selection due to cause (*a*) mentioned in § 11.

In dealing with the question of withdrawal selection there were two problems. The first was the withdrawal rate and the second the question of mortality after withdrawal. Withdrawal rates could easily be obtained and he had recently had access to an investigation, which unfortunately only covered the first five years of duration, and there was no indication that the rates had reached their ultimate level by the end of that period. It seemed from that that the withdrawal rates could prolong selection beyond the end of five years.

The unknown quantity was the withdrawal selection itself and until there was further information on that point he did not see that they could make progress. He wondered if it would be possible to institute an investigation into a sample of withdrawals with the co-operation of the Ministry of National Insurance and the use of the policyholder's National Insurance number. The greatest difficulty there would probably be in obtaining that number because policyholders surrendering or lapsing were not very co-operative, but it might be possible for the Ministry of National Insurance to trace any particular case from full particulars of name, address, and date of birth.

Of the table itself he said that the Committee had given a modern mortality curve, and they had promised also a light table based on the data. They implied, further, in § 8 that a heavy table would follow a somewhat similar shape, and he felt that it would be an advantage to have that heavy table as well, to complete a series of tables at different levels of mortality following the modern shape of the curve.

Like the opener, he felt that an aggregate table might have been produced in view of the unsatisfactory nature of the select data; he thought, however, that that should only be produced up to, say, age 55 where for all practical purposes the select and aggregate tables would be the same. Over that age selection did begin to have an effect and modified select rates could be introduced at that point.

In the practical use of the new table the life-office actuary was concerned with the effect on reserves, and he had calculated some reserves for whole life policies from the data given in § 25, converting the select values to ultimate values by means of the formulae in § 23. Compared with the A 1924–29 values the ultimate reserves by the new table were approximately 1 % lower at the younger ages and earlier durations rising to about 3 % lower at the older ages available. By a more laborious process he had calculated some values for endowment assurances. Those showed that the reserves for sums assured on the new table were about 1 % higher for policies maturing at age 60, tending towards the A 1924–29 values at older maturity ages, except for longer terms.

There would, of course, be a release of bonus additions on a transfer to the new table, but it seemed to him that no general conclusions could be drawn as to the result of a transfer from a valuation on the A 1924–29 to the new table, as each office would require to investigate its own position, taking into account the distribution of the business between whole life and endowment assurances.

Mr W. Perks was rather surprised that nowhere in the memorandum did the word 'duplicate' appear. The Committee had shown some interest in the effect

of duplicates on the random variations in mortality rates, and the memorandum did give values for certain standard errors; in particular there were measures of standard errors in relation to the variations between offices. He thought it would be found that, if reasonable allowance were made for the effect of duplicates on the standard errors, the number of offices whose experiences were more than two standard deviations away from the average would be considerably reduced.

In comparing individual offices' mortality with the average the Committee had used the A 1924–29 table as the standard. That, he thought, tended to exaggerate the difference between offices because the average ages for different offices varied and the average durations of their business in a given age-group varied. It seemed that the amount of real mortality variation between offices was much less than would appear from the tables in the memorandum.

With regard to the graduation formula he mentioned that a certain general formula had been given in the last paragraph of his paper written twenty-four years earlier (*J.I.A.* 63, 12). That was a general formula of which the particular formula that had been used for the new table was a particular case. The interesting feature of the formula adopted was the term c^{-2x} in the denominator. The memorandum suggested that that particular formula involved five parameters—it was his belief that it implied at least six since c^{-2} was really a parameter which had been arbitrarily fixed. It had been introduced in order to reproduce a wave in the rates between ages 40 and 60; between those ages the average duration of the business was bound to increase very substantially. At age 40 it could be expected that the average duration would be perhaps about six years. At age 60 the average duration was probably something like fifteen years. So, with data showing selection over a very long period, whatever might be the cause of that selection, there would be an effect on the mortality rates, excluding the first two years, between ages 40 and 60 such that for any curve that went through those rates there was bound to be some parameter to give expression to that changing duration factor, and Ec^{-2x} should really be expressed as Ew^x where w was another parameter.

Mr J. G. Wallace supported the remarks made by Mr Donald. As the Committee had pointed out in their conclusion, there was a considerable variability between rates of mortality, particularly as between different offices as well as between different classes of assurance, and also with relation to time. In the face of that heterogeneity he supported the Committee in their decision to produce an average table fit for practical use, though he looked forward to the appearance of the promised light table. He felt that when they had put all that work into the production of those actuarial tools it seemed almost ungracious to ask if they had considered extending their activities along research lines rather than along lines of practical work. For instance, one question had been raised at the meeting concerning the non-medical mortality, and it seemed to him that some research could be carried out, possibly with the existing data, to estimate the value of obtaining a private medical attendant's report in underwriting non-medical business.

He asked if the question of the census method had been re-examined. It seemed to him that sampling techniques could well be used for carrying out subsidiary investigations, and another system of collection of data might be justified on that account.

Mr Donald had referred to variation in mortality rates with sum assured, and he thought that an investigation there would be instructive, particularly if the

correlation of the cause of death from suicide and accident with sum assured could be examined.

In the case of annuitants care was taken to examine female mortality; could they not have an up-to-date sample investigation into female assured lives?

Mr M. D. Thornton pointed out that, when the work on the new tables had been completed, actuaries would have as the principal tools for their trade for the following twenty years or so the $a(55)$ for annuitants and the A 1949–52 table for assured lives. It was obviously sensible that the male annuitants over age 46 should show lighter mortality than the (predominantly male) assured lives. It was unfortunate that the reverse should be the case under age 46. For years they had had the inconvenience in the calculation of deferred annuities of taking D_{x+n}/D_x from one table and a_{x+n} from another. It seemed that the same inconvenience was likely to continue for years to come. When the $a(55)$ males table was prepared it had been necessary of course to derive the rates for ages under 50 from assured lives' data, helped no doubt by extrapolation from the ages at which annuitant data were significant. Assured lives' experience in the years 1947–48 at ages under 50 appeared in retrospect to have been heavy and unsuitable as a guide. He suggested that when the tables were next revised, perhaps in twenty years time, the assured lives' table should be revised first. Then, perhaps, future actuaries might have the advantage of having, as working tools, tables for assured lives and for annuitants which bore a reasonable relationship to each other, not merely at the older ages, but throughout the whole length of the tables.

He had a second point which concerned the graduation. The Committee had described their decision to remove the 'hump' at ages 20–30, to follow a steeper rise after age 40, and from ages 55 to 70 to approach more closely to the whole life curve, and so on. Those were matters of high judgment. He imagined that the Committee, in forming their judgment, had been helped by a graph of the various sectional curves, among which their recommended curve pursued its course. He would have welcomed such a graph in the memorandum. Once their decision on those important matters had been made they were, he thought, probably in a position to write down their chosen q's throughout the whole length of the table to two significant figures and that completed the fundamental work of graduation. There remained the relatively less important job of smoothing the chosen curve and providing values to the normal five decimal places. Up to that point the graduation, by whatever method it was actually done, bore a strong affinity to a graphical graduation, and it seemed to him illogical that the final smoothing had not been done in the traditional manner associated with graduation by that means. It had been described by Dr T. B. Sprague in a classic paper as far back as *J.I.A.* **26**, 77. Perhaps it had been felt that the climate of actuarial opinion which was believed to be against an aggregate table was also against a graphical graduation.

The method actually adopted, perhaps owing to the Committee's presentation of it, had the appearance of contributing more to the final result than it actually did. It did not choose the curve; what it did was to smooth the chosen curve, and while it did that well there were roughnesses still remaining in some places. Those could well be removed by the process of 'hand-polishing'. It was an advantage of the graphic method that by continuing the 'hand-polishing' process any desired degree of smoothness could be achieved. It was a disadvantage of the method adopted that there was a tendency to assume that when one curve had been successfully fitted to the entire table, undoubtedly a great achieve-

ment, the result was immune from further examination as to smoothness; yet the only object of the whole process was the comparatively humble one of achieving that smoothness. From the graduated rates the values of $10^5\Delta^2 q_x$ from age 51 were 8, 7, 10, 9, 11—not the acme of smoothness—while from age 86 the values of $-10^5\Delta^3 q_x$ were 4, 2, 6, 8, 0, 14, 8. The latter example was from an unimportant part of the table, but it was a pity to leave even that part in that state. It would hardly affect the life table and not affect at all the practical results of using the table, but that applied in a lesser degree to the whole process of graduation after the decision had been taken on where the curve should lie. The vital thing was to choose the curve and that alone. It would be a great pity if the generations of students who would study the memorandum failed to realize that the Committee's decision as to which features of the collected data the curve should reproduce would have a major effect on premiums and reserve values for years to come, and that compared with that the effect of the method of smoothing the chosen curve was not of great importance. It was indeed the technical brilliance of the curve fitting which was most likely to blind students to its true place in the scale of values. Perhaps had it been less brilliant the Committee might have taken action to remove the little roughnesses it had left.

Mr A. T. Haynes wished to reassure Mr Thornton that things were really going in the right direction; he believed, speaking from memory, that in the 1920's the assured lives and annuities tables crossed about 62. If, as he assured them, the new curves crossed at 46 it was an improvement, an improving trend.

He felt that the Committee really had had a tremendous number of anxious hours in which they had had to give consideration to a most difficult problem for which they simply had not got the material to find a solution. The moral to be drawn was that there should be a great deal of anxious thought given to the material that should be obtained for the future and the form in which that material should be obtained. He found it rather disturbing that so little was known of the trend of selection and of the true ultimate level of mortality. He certainly did not favour the idea of an aggregate table and he was not sure that he favoured the two-year period of selection, because he was not happy that it should be thought that the true ultimate rate of mortality might be as much as 10% more than the published rate in the tables. What effect might there be, for instance, on reserve values for whole life assurances effected many years earlier, perhaps in a closed fund, if the assurance factors ought to be based on mortality up to 10% more than the mortality rates shown in the table? For many of the problems that they had to deal with in practice ultimate mortality rates were so important, and aggregate mortality rates might give them a wrong answer, especially when they were dealing with things like reserve values which depended more on the trend of mortality than on the level of mortality.

He thought the Committee deserved the deepest sympathy in the task that they had had to face, and he would not like to think that next time they would have to meet the same difficulties.

Mr J. Inglis spoke as one who had been a gentle critic of the Mortality Committee for a long time. Although still quite unrepentant, he did appreciate the tremendous amount of work which the Committee had carried out and the value of that work to the profession and he paid his own humble but sincere tribute to them and expressed his admiration for their labours.

He had long regarded the paper by Elderton and Fippard (*J.I.A.* **46**, 260) as one of the classics of actuarial literature. That the census method, as applied to life assurance data, had certain objections was well known but the fact that the method had persisted as the basic method for official tables for over thirty years could not be entirely due to the difficulties of setting up an alternative method. Those difficulties were not insuperable and he suggested that the original critics of the census method were satisfied that the objections did not outweigh the advantages. (See *T.F.A.* **11**, 105, *Notes on the Census Method of Obtaining Rates of Mortality among Assured Lives*, by R. Ll. Gwilt.)

The proposed table was only the second based on life assurance data (excluding the Light/Heavy Tables which were based on the same data as the first) produced in the thirty-five years or so during which the Joint Mortality Investigation Committee had existed in one form or another. While he appreciated that the years 1940–47 were unsuitable for the purpose, he felt that it was unfortunate that no really reliable guide as to the trend of life assurance mortality had been published for any part of the 25-year period since 1929. He made a plea for the publication of the next table at no greater interval than 10 years. As it was, he thought it was correct to say that offices had been working very much in the dark since the 1924–29 table was published. He did not regard it as much consolation that, in spite of the difficulties, the offices, largely owing to the force of competition, had been fairly successful in anticipating the reductions of premium rates at last shown as justifiable.

No doubt, in the same circumstances, their American friends would have produced a new table every two years or so during the past thirty-five years; but, whereas the Americans would probably have simplified matters by making 16 of the 17 tables illegal, British actuaries preferred the sinful life of freedom without benefit of Parliament and they would have had all 17 in use at the same time, with consequent headaches to actuaries and students alike. There seemed, however, to be a happy medium.

Further, the A 1924–29 table was on the average about eight years out-of-date on publication. He guessed that the A1949–52 was unlikely to be available before the end of 1956, making it approximately seven years out-of-date. From statistics relating to one contributing office which he had examined, it was highly probable that current mortality was already departing from the rates shown by the new table. The old familiar tendency was apparent, namely for the lower end of the curve to become 'unhinged' from the curve of the standard table while the upper end remained more or less attached. The office in question was clearly in the 'Light' category and that had no doubt the same 'unhinging' effect on the comparative mortality; even allowing for that, however, the post-1952 mortality was appreciably lighter.

He did not suggest that they should go so far as to produce a forecast table for assurance purposes but he did suggest that the case for a more frequent production of a standard table was a strong one.

He had been a little shocked at first by the apparently slender reasons for selecting 1949–52 as a suitable period for inclusion but on further consideration the choice seemed a sound one in the circumstances. In any event if the 1952 statistics were the latest available, if the period 1930–39 was too early (or impossible to deal with during the war period) and if the period 1940–48 was unsuitable for the reasons given by the Committee, there did not seem to have been much choice!

He did not propose to attempt any criticism of the graduation but he noted

with interest that it was probably the first occasion, so far as assurance statistics were concerned, that practical considerations had outweighed the desirability of a more or less rigid adherence to the ungraduated data. He had often thought that the function of graduation had been somewhat over-stressed in the past. Certainly so far as office premiums were concerned, it was usual, after the addition of the necessary loadings, to adjust the gross premiums slightly in order to achieve a smooth progression. He thought it was possible to obtain in that way exactly the same table of rates of premium no matter which method of graduation was applied to the q_x column. Apart from premium rates, no other practical use of the tables required a meticulous graduation.

He thought it a pity that the Committee had not produced sufficient information to enable offices to judge the effect of the proposed table on reserves. That aspect was probably more important than the effect on premium rates. From some tentative experiments, it seemed likely that the new table would produce slightly lower reserves for whole life assurances. Data for examination of endowment assurance reserves were not readily available. Did the Committee agree that the general effect might be to produce a modest reduction, if the rate of interest were not changed, and that the reduction was not likely to be more than the equivalent of a difference of, say, $\frac{1}{8}\%$ in the rate of interest in an average case?

It would have been interesting to know the effect of the new table on the Z's under Lidstone's method but the few available factors did not supply the answer. It might well be that the A 1924–29 Z's could be used with reasonable accuracy to get the mean ages. The work involved in a change of Z factors was not of great consequence if punched cards were available, for such a change could be easily carried out with the assistance of a multiplying punch or an electronic computer.

He had noticed an interesting point in examining the tables included in the memorandum. In the 11 tables 1–9 and 14 and 15, the maximum exposed-to-risk was with two exceptions at ages 43, 44 or 45 (indeed 43 only just managed to creep in instead of 44), the two exceptions being Whole Life With-Profits—Medical (Tables 1 and 15) where the modal value was at age 71 although there were subsidiary modes at 45 and 44 respectively and at 50 in both cases. No doubt the point was quite familiar to the Committee. He had not had time to consider that feature carefully but at first sight it did not seem to him that a common maximum was to be expected.

He was glad to learn of the intention to publish values of temporary life annuities at all ages. The A 1924–29 tabulation at even ages had undoubtedly been the cause of an enormous amount of irritation, not to mention the additional work falling on individual offices. He did not think that more than three decimal places were worth while and he suggested that pure endowment factors were unnecessary, even at quinquennial maturity ages. The differences of the temporary annuities (available at all practical ages) gave those factors to three decimal places, which was sufficient for most purposes. He thought the quinquennial values should be as before, i.e. including the temporary annuities, or annuities-due. He felt that possibly the suggested substitution generally of annuities-due for ordinary annuities would lead to more errors than did the familiar arrangement.

He made a strong plea for the additional rates of interest suggested; they were almost essential. That probably did not apply to the same extent to policy values. He also asked the Committee to consider whether the whole life

'duration 1' factors and commutation columns should not be given in full, including $R_{[x]+1}$ which was useful for computation of increasing and decreasing benefits and sometimes for policy alterations.

He fully appreciated that the greater the volume of published material the greater the cost. Surely, however, the addition of a guinea or two per volume would be much less costly to the offices than the time and labour involved in extending the tables for themselves.

With regard to joint-life annuities he thought that something better than the existing arrangement was very desirable. Something on the lines of the suggestion quoted in the memorandum would be an improvement. Select values should be given as well as ultimate.

The differences in mortality between offices had received much less attention than they apparently deserved. He did not envy the Committee when it had to consider the two extremes (a composite table on the one hand and a table for each individual office on the other) and to decide just where the happy medium fell—if such a thing existed. Probably a separation of 'light' and 'heavy' offices, as before, would be a partial solution.

Mr A. T. Jamieson said that the form in which selection appeared in the data given could be partially explained by the assumption that each office's experience was, in fact, not homogeneous. The criterion for admission to the experience was acceptance by the office underwriter. The underwriter could only bring history and present condition into his assessment, so that his evaluation might be regarded as considering the curve of q_x for the life up to that date. If the curve in question ran higher than the underwriter's hypothetical curve of good lives, or even if it showed a tendency to go higher, the life was excluded from the experience. If the lives selected were divided into smaller and more homogeneous groups and the curves of q_x applicable to each group were considered, it would be seen that some of those curves would cross the underwriter's hypothetical curve later on, while others would remain below it to the ultimate age in the table. That was, of course, simply a restatement of the fact that some lives at a later stage became uninsurable while others were acceptable as long as the underwriter was prepared to consider persons of their age. The office's experience, therefore, consisted of a stream of curves all of which lay under the underwriter's mortality curve until they had passed the selection date. It would be observed that the select table curve ran close to the underwriter's curve and the aggregate curve somewhat above it.

Of the curves belonging to groups selected in the past year, practically none would have crossed over the underwriter's curve; but, of the curves selected 20 years earlier, quite a number might be expected to have crossed over, i.e. become unacceptable at ordinary rates. It followed that if the number in a particular duration that had crossed over and the degree to which they had crossed over were less proportionately than in the aggregate, then the mortality at that duration would be lower than the aggregate, and *vice versa*. That explained the features numbered (i) and (iii) in §10.

There appeared to be no means of testing that hypothesis directly, as data for sufficiently long durations were not available. Since the medical and non-medical experiences both showed that pattern, however, there was at least some ground for assuming that the phenomenon might be expected within individual offices, rather than that it was due to the combination of the different experiences. It was also noteworthy that part of the pattern appearing in Tables 11, 12 and 13

was the tendency to more rapid approach to the ultimate at young ages, as evidenced by the 100 A/E columns at durations 2, 3 and 4. It would be observed that a higher proportion exceeded the column average among the younger groups. That trend might be expected from the type of mortality experience under consideration.

The conclusion he was forced to draw from that was that the only complete mortality table was a double-entry one for age and duration and clearly that was uselessly cumbrous. Therefore he rather sympathized with the remarks in § 14 regarding the much more satisfactory appearance of a completely aggregate table. It was unfortunate, but true, that the climate of opinion was against such a venture and probably the final method decided on was the most satisfactory at the time of speaking.

In regard to the methods used to obtain the select rates from the ultimate table, they were indebted to the Committee and to Mr Beard for the simplicity and closeness of fit achieved simultaneously, which should prove of great value in practice.

Mr F. M. Redington, speaking as a member of the Committee, referred to the remark made by a previous speaker that he could not see how ultimate mortality could possibly rise to 110% of the aggregate. That was an important point and one which he thought could have been more forcibly put across than in the memorandum; it was a purely numerical problem. The last known mortality at duration 4 was, say, 84% of the aggregate of that for 3 and over. They knew that the average for 3 and over was 100, and it had got to march up by steps from 84 to a series of figures which averaged 100; it was going up so slowly it obviously was going to take a long time to get to 100, and it had got to advance a lot longer and higher after that to average 100.

The only other point he wished to comment on was the criticism of the census method. There was one place where they had suspected the census method, and that was in the pensioners' mortality at the retirement ages of 65, 66 and so on, where there were obviously considerable dangers in the method. A thorough investigation was made which examined the experience by taking careful account of months of age and calendar months. Even in those extreme circumstances the census method was vindicated by producing results that came very close indeed to the results obtained by more precise methods.

Mr R. Ll. Gwilt, in closing the discussion, said that as the Committee would reply in writing after considering the remarks made at the meetings in Edinburgh and London he proposed that evening to offer only a few general comments.

Mr Beard, in presenting the memorandum, had pointed out that one of the most important facts that influenced the Committee in reaching its conclusions was the wide variation in the experience of the individual offices. Mr Perks had suggested that, as duplicates were not eliminated, those variations might have been exaggerated. That might have been so, but there were wide variations in the statistics which emerged, and for that reason the Committee, as in the 1924–29 experience, had decided to base the table on the combined statistics for all classes of assurance, medical and non-medical—the mixture as before—although for the new table it had been decided to give more weight to the whole life experience at the older ages as had been explained in the memorandum and by Mr Beard.

He had been interested to find that several speakers would have been quite content with an aggregate table for, as indicated in the memorandum, the

Committee—or at any rate some members of the Committee including himself—would have been content to have an aggregate table for the 1949–52 experience for reasons apart from its simplicity. First there were the wide variations in mortality experience among the individual offices which probably contributed in differing proportions to the select and ultimate experience; secondly there were withdrawals which undoubtedly had an influence and which probably also varied considerably from office to office. Incidentally, the Committee some 25 years earlier had given a great deal of thought to the subject of withdrawals and had done a lot of investigation, but unfortunately, in the absence of information about the mortality of lives who withdrew, they had been unable to reach any conclusions that gave them complete satisfaction. They felt then, however, as they still felt, that withdrawals were bound to have a considerable influence.

There were other reasons why he would have been willing to accept an aggregate table, including the fact that there were no means of discovering the true duration of temporary selection, which possibly varied considerably from office to office. On the subject of temporary selection he had once before referred at a Faculty meeting to an investigation carried out some years previously in the U.S.A. which had led to the conclusion that the effect of temporary selection was probably of little importance after the first two or three years but that class selection had a material effect. He did not recollect whether the subject of withdrawals was specifically referred to in that investigation.

Another argument in favour of an aggregate table, was that, as Mr Beard and others had stressed, no standard table could be appropriate to all offices; each office had to have regard to its own experience and to decide what adjustments, if any, to make in using a particular table.

Finally, the general level of mortality had become so low that under the age of 60 considerable percentage variations had little effect from the monetary point of view, at any rate in calculating premiums, and if they ignored selection entirely under the age of 60 it could not make very much difference. Over the age of 60 temporary selection was more important, but it was, he thought, rather more difficult to measure because the data became more scanty and class selection more important.

He had expected that at the meeting there would have been many suggestions from members about the nature of the tables which should be included in the published volumes, and he was pleased to note that eventually Mr Inglis had had something to say about that, though no other speaker had mentioned the subject. No doubt there would be various other suggestions in a week's time, and when the Committee had seen all the proposals they would give them careful consideration. It would, however, be wrong for him to give the impression that the Committee would give effect to every suggestion made, because the tables had to be kept within a reasonable compass.

There was one other matter to which he should, perhaps, refer and that was the suggestion that had been made by several speakers, that instead of, or in addition to, the continuous investigation of mortality of lives assured at ordinary rates of premium various other types of investigation might be carried out. The Committee had in hand, at the time of speaking, consideration of the arrangements for an investigation of the mortality of impaired lives. That was a very big undertaking in itself and would involve all the offices in a great deal of work. Further investigations on the lines of some which had been mentioned—and which many of them would have liked to have—would have meant asking offices for more information which they might have difficulty in supplying before the

impaired lives investigation had got well under way. The Committee would, however, keep in mind the various suggestions.

The Committee was greatly appreciative of all the valuable work Mr Beard had done in carrying out the graduation and devising a suitable modification of the Perks formula for the purpose. The speaker also expressed the warm thanks of the Committee to its indefatigable Secretary, Mr R. D. Clarke. He thought there was not the slightest doubt that had it not been for Mr Clarke's efforts the Committee—and most certainly the Chairman—would have had to bear a much heavier burden. Mr Clarke had undertaken an enormous amount of work in connexion with the assured lives' investigation—in the analysis of the data, in collaborating with Mr Beard in the graduation and finally in drafting the memorandum.

The President (Mr K. K. Weatherhead), in proposing a vote of thanks to the Joint Mortality Investigation Committee, said that they were very grateful to the Committee for all the work that they did, and for the memorandum that they had prepared and on which they had had such an interesting discussion. He also thanked Mr Beard on behalf of those present for opening the discussion. They knew how much Mr Beard did for the Committee; they were delighted to have him at the meeting and hoped he would soon visit them again.

ABSTRACT OF DISCUSSION AT THE INSTITUTE

The President (Mr J. F. Bunford) said that there would be general agreement that it was a happy and fortunate thing that the first sessional meeting in the reconstructed Staple Inn Hall should be concerned with the mortality of assured lives, an important subject to which the Joint Mortality Investigation Committee had devoted so much time and thought over the years. They were honoured by the presence of a number of Faculty members, led by their President, Mr Weatherhead, who, as always, were very welcome at the Institute.

Mr A. E. Bromfield, F.F.A., in presenting the memorandum, said that it was the first time that he had had the pleasure of speaking at Staple Inn and he felt particularly fortunate to take part in the first sessional meeting since the Institute came back to its old home.

From his experience on the Mortality Committee, he had an impression that each time an actuary was faced with the problem of constructing a mortality table from a set of observed data he started off with the hope that on that occasion the statistics would prove friendly, so that the results of his labours could be both practically useful and statistically satisfying. Unfortunately things seldom worked out in that way. In Scotland they had an old saying 'Facts are chiels that winna ding', which might be freely translated into English as 'Statistics are intractable'. It had been said that statistics were always intractable if an attempt was made to drag them in a direction in which they did not want to go; but, even though there might be no wish for violence of that kind, the question usually arose of whether to have a table which would be of the most practical use or to follow the statistics exactly.

The Joint Committee had been forced to admit that it had taken certain small liberties with the statistics in order to produce a workable table—which, after all, was the most important end-product of their investigations. He had the

impression that other Mortality Committees in the past had had to make the same compromise.

The problem of constructing a new set of mortality tables fell into certain divisions, each with its own problems. First it had to be established if a need for the new tables existed, and on that occasion the answer was simple. In view of the time which had elapsed since the last tables had been prepared, and of the changes which had taken place in mortality since then, there could be no question that an up-to-date set of tables would be of tremendous use to the profession as a whole. If it had not been for the war, something would probably have been done earlier.

The second decision which had to be taken was to fix the period of exposure which should form the basis of the tables and the Committee felt confident of the choice they had made. The use of a short period had definite advantages if it was possible to be sure first that the data were sufficient, secondly that they were dealing with a random sample of year-to-year experience, and thirdly that the period chosen fitted reasonably well into the long-term secular trend.

With total deaths of over 90,000 the data appeared to be adequate, and in §3 of the memorandum it was shown that the period chosen was a fair average of year-to-year experience. With regard to the secular trend, some interesting graphs had been produced by the Committee a few years earlier showing mortality rates for durations 5 and over for each year from 1924 to 1948 in 5-year age-groups, and they gave a fair picture of the secular trends in the various groups. They would be extended for later years in due course, but in the meantime it was possible to look on the 1949–52 ultimate rates as a guide to the position of the new tables in the secular trend.

The graphs showed some interesting features. From 1924 to 1939 there was a fairly steep downward trend in mortality at all ages, the slope being considerably steeper at the younger ages. After 1939 there were quite different results in the various age-groups. For ages up to 40 the war years showed very high mortality, followed by a steep drop in the years 1946–48 to a point which was a reasonable extension of the pre-war secular trend. The period 1949–52 showed a further extension of the same trend. Above age 40 there was a curious feature, which became rather more obvious as age increased. The initial increase in mortality at the beginning of the war was followed by a steep reduction in 1940–41 to a lower level than the secular trend indicated and that lower level persisted up to 1948. For ages 40–50 the period 1949–52 showed a continuation of the low level of mortality but above age 50 the tendency seemed to have been for mortality to increase again and to take its place, roughly speaking, as an extension of the pre-war downward trend. A study of the curves was most interesting, and seemed to confirm that, although the period 1949–52 might show a slightly higher mortality than the average, it did take its place reasonably well in the secular trend.

The third problem was whether differences between certain subdivisions of the data justified the use of only part of the data in the construction of the tables. For the reasons set out in the memorandum, the Committee decided that no distinction could be made between whole life and endowment assurances, between with-profit and non-profit, or between medical and non-medical. The question of segregating 'light' and 'heavy' offices had been postponed, though there was no doubt that material differences did exist. As mentioned in §8, the Committee would subsequently be considering the alternatives of producing 'light' and 'heavy' tables from the observed data, or perhaps of

preparing hypothetical tables based on certain adjustments to the basic table. They would welcome expressions of opinion on that point.

An allied point, not mentioned in the memorandum as no statistics were available, was the question of basing an investigation on sums assured instead of lives. From some figures which he had seen in connexion with pensioners' mortality, it seemed that material differences might emerge from an investigation based on amounts for that particular class of life, and the same thing might happen with assured lives. That might be a line for future consideration, but in the absence of statistics for Britain it was comforting to find that in America the mortality for high sums assured tended to be lighter, since proposals for high sums assured were underwritten very severely on the medical side.

On the question of medical and non-medical statistics, he emphasized a most important point which was sometimes overlooked, namely the meaning of the term 'non-medical' in the current investigation. The statistics emerging from the assured lives' experience had led to the theory being put forward that there was no material difference between medical and non-medical mortality, and the remarks in §6 might be interpreted as strengthening that theory. That had not been the Committee's intention. It seemed to him, at any rate, that it would be unnatural if all the trouble that actuaries took in selecting lives on the medical side produced no difference in the mortality.

There was a further point. He believed that the term 'non-medical' was sometimes used rather loosely. It had to be remembered that the current investigation referred only to those cases where a non-medical proposal form had been obtained in which at least some information regarding the proposer's state of health had been given. In endowment assurance scheme business, on the other hand, there was a completely different type of proposal which was becoming common, and which was sometimes referred to as non-medical, though it would more properly be called automatic. That was the case where the proposer was accepted irrespective of his state of health, the only protection to the office being that he had to be actively at work at the time. Those cases were not included in the current investigation, and any inferences which might be drawn from a comparison of the medical and non-medical sections had no bearing at all on those other 'automatic' proposals. Expressing a personal view, he thought that the pressure of competition had forced offices to take much too lenient an attitude to those automatic proposals, and a separate investigation into the mortality of automatic proposals might well be undertaken.

The next general problem involved the duration of selection, and there it had to be admitted reluctantly that no correct solution seemed possible. From all the statistics available it appeared to be fairly certain that for one reason or another the effect of real or spurious selection persisted over a long period of years, so that even if they had data to examine that selection over a long period it was doubtful if they would be able to produce an exact select table which would be workable in practice. The chief ground for regret was that they did not know the true level of ultimate mortality, and the corollary was that the ultimate section of the new tables should not be mistaken for true ultimate mortality, nor the difference between the select and ultimate regarded as a measure of the true effect of selection.

With regard to graduation, while he did not wish to add to the memorandum there was a special point which arose from the comparison of various mortality tables in Table 19. One of the important gaps in their knowledge related to the appropriate mortality to be used for the active-service period of members of

pension schemes and, apart from American statistics, their only guides were the results taken out by individual offices. Those showed that active-service mortality was very light indeed, and it had been difficult in calculating pension rates to choose a suitable mortality table. One approximation was to deduct $\frac{1}{4}\%$ from q_x under the A 1924–29 table; that gave a good fit up to age 55 but was not always convenient to use in practice. Another was to take the A 1924–29 Light table with two years deducted from age. For an average scheme that gave a reasonable result in total, but the shape of the curve was not appropriate, as mortality was over-estimated at the younger ages and under-estimated above age 50.

The new 1949–52 table seemed to be much closer to active-service mortality than any previous table, and if a 'light' table was subsequently produced it might give an even closer fit. In the meantime, it would appear that offices would be able to use the new table, perhaps with a deduction from age, when calculating group pension rates. If one year was deducted from the age a good fit was obtained at ages over 45, and, although the mortality was over-estimated at the younger ages, the financial effect at those ages was not important.

The Committee hoped that the discussion would give them some guidance on the general views of the profession. The fact that two discussions were being held was a token of the importance which the Committee attached to the matter, and it emphasized that their aim was to produce a workable table in a form acceptable to actuaries throughout the country. Attention might be drawn in the discussion to the gaps in their knowledge and to the many interesting investigations which could be carried out if only the extra statistics were available; but the source of the statistics was the life offices, and they had other things to do besides producing material for the Continuous Mortality Investigation.

Mr C. M. O'Brien, in opening the discussion, quoted the words 'It would be fair to suggest that the profession should soon be invited to consider whether they should ever again have an investigation like this last one.' That, he said, was not his opinion, but a quotation from the discussion which had taken place in Staple Inn Hall some 22 years earlier on a similar memorandum presented before the publication of the A 1924–29 tables. From the new memorandum it was clear that the Committee had rejected that suggestion, and in his view rightly so.

He differed a little from what the President had said in his opening remarks about the importance of mortality. He thought that perhaps in talking about normal assured lives it was not quite so important as it had been in their earlier states of knowledge, for he was sure that no actuary would deny that mortality was the most stable of the factors involved in premium calculations. Perhaps a symptom of that was that only two meetings were being held to discuss the subject, one in Scotland and one in London, whereas 22 years earlier two meetings had been held in each place.

That being so, it was his view that the table produced by the Committee was sufficient for the needs of the profession, whatever criticisms of detail, and possibly even of theory, might be raised against it. They had provided, as they had emphasized in the text, merely a standard or yardstick. He did not say 'merely' in any sense of belittling their work; they had given their full reasons for reaching that particular standard and for the methods they had adopted. That was as much as could be expected, in view of the nature of the problem.

Naturally the memorandum left many questions unanswered, primarily because of limitations of data. That was to be regretted. The search for truth was,

he thought, a desirable thing in itself, quite apart from the fact that it was usually that sort of search which led to practical advances. But the cost of such a search would be large, and there was no justification for asking the offices to spend a great deal of additional time and labour in producing more data, and particularly more subdivisions of data, when with the existing methods of collecting data a tool adequate for their needs could be provided. In his opinion the table which had been submitted was a good one and adequate for their needs and it followed that the method of collecting the data was also sufficient.

There were, of course, many points in the memorandum which individual actuaries would find of interest or which they would wish to criticize. He proposed to confine himself to two. Table 19 was of great interest to him. As would be expected, the curve of the A 1924–29 lay mainly between those of the O^M and the A 1949–52, but its position relative to the other two varied considerably with age. It would be seen that at age 50 the mortality had improved approximately half as much in the past 25 years as it did in the previous 50 years. At younger ages the A 1924–29 lay much closer to the new table, and the rate of improvement in mortality was much less in comparison with that in the earlier period. At the older ages, however, the position was quite different. At age 70 mortality had improved as much in the past 25 years as in the preceding 50 and from age 70 to age 85 the curves were approximately equidistant. Over age 85 the curves converged but the significance of the high ages would not be large. That was an interesting result, particularly in view of Mr Bromfield's remarks about the position of the new table in terms of the secular trend. It seemed that the slope of the mortality curve was changing appreciably, and if that continued it would clearly have a big effect on net-premium reserve values, which depended, as was well known, rather on the slope than on the absolute size of mortality rates.

The second question to which he wished to refer was selection. As was to be expected, the Committee had once again found that the select rates failed to run into the ultimate. They gave various reasons for that, and among them the possibility that withdrawals were selective. That was a reason which had frequently been given in the past, but he did not think that there was, or indeed could be, any practical evidence to show whether that was so or not. Marginally he thought that it was bound to be so; a man who was on his death-bed would obviously not surrender his policy if he could possibly avoid doing so. Generally speaking, however, his impression was that financial reasons were far more relevant in cases of withdrawal than health, and he did not feel that the latter was likely to be a significant factor. What he thought was a possibility—though he was afraid that he could produce hardly any evidence to support his view—was the existence of duplicate policies. Those in the ultimate experience could be divided into two parts: (*a*) those who, if they proposed again, would be accepted at the normal rates, and (*b*) those who would not be so accepted. If it were supposed that a batch of section (*a*) did in fact propose, and their mortality were studied, he failed to see any good reason why it should run into the mortality of the combined group (*a*) + (*b*) for at any rate a very long time. There were large numbers of duplicate policies in ordinary business, and he thought that that might well be one of the factors which caused the failure of the select rates to run into the ultimate.

He had been particularly interested in the information given in the memorandum about the difference between the experience of offices. It produced answers on the lines of what would be expected from the analysis which preceded the

publication of the A 1924–29 Light and Heavy tables, but he hoped that the Committee would print the information in the volume as finally published. They emphasized in the text of the memorandum that the table was a standard and that it did not absolve actuaries from deciding whether or not it was suitable for any particular problem; but if information showing the great variation between offices were included in the published volume it would serve as a salutary reminder of that point.

He was surprised that there was no information on policy values, though the Committee did include a short paragraph dealing with the financial effect on premiums. No doubt their answer would be that it was outside their sphere, and that was probably a fair answer. He hoped, particularly in view of the difference in the slope of the curves, that later in the discussion some figures on the effect of changing from the A 1924–29 standard or 'light' table to the new table would be given.

It was interesting, in that connexion, to look back at earlier discussions. Following the publication of the O^M tables, the discussions at the Institute had consisted largely of actuaries giving the results of their calculations of reserve values. That subject had featured little in the discussions following the publication of the A 1924–29 table, and he suspected that relatively little would be heard of it now. The reason, he imagined, was that with the increased use of electric and electronic calculating machines it was much simpler for the actuary to do the calculations of reserve values in his own office than it had been in the past.

There was one small plea which he would like to make. The Committee had asked offices what they thought should be included in the final published volume. It was no longer a great labour to calculate a complete commutation column for a particular job, and he suggested that the Committee should eschew in the final volume any 'fancy' functions. The addition of a number of functions which were seldom needed made the book physically larger and got in the way when normal day-to-day calculations were being done involving only the more usual functions.

Mr H. Dicken said that, in dealing with the difficult question of selection, the Committee's decision to take a select period of two years was firmly based on the trend of the figures shown in the table in § 10, and he did not wish to criticize that decision. At the same time, it seemed to him to be significant that if a table similar to that in § 10, but limited only to medically examined classes and to ages over 45, was prepared from the figures in Table 12 a very different progression of figures was obtained:

Duration	100 *A/E*
0	47·7
1	64·4
2	67·7
3	80·4
4	78·5

It appeared that in that important section of the data there was a wide gap of 13 % of the ultimate between the ratios for duration 2 and duration 3, as compared with a gap of only 3½ % in the table in § 10, and only about 8 % in the column headed Medical of the table in § 12.

A further aspect of what was virtually the same point, the value of medical selection at the later ages, was obtained if, with the help of Tables 12 and 13, the experience in the first five years was aggregated and the resulting ratios in the Medical as against the Non-Medical were differenced. It would be found that at the younger ages up to 45 the difference between Medical and Non-Medical averaged only 2% of the ultimate mortality, while at ages over 45 it averaged 11%, compared with the average of 8% for all ages mentioned in §12.

Turning to the question of the ultimate mortality, and noting the closeness of the fit and the method of graduation, he thought that the contentious point was the decision of the Committee to introduce a deliberate over-weighting of the Whole Life With-Profit Medical class. It seemed to him that, for the reasons advanced by the Committee, their action was fully justified, and the only question raised in his mind was exactly how much over-weighting of that one class had been introduced into the data summaries before the five constants had been obtained in graduating the ultimate q_x column.

With regard to the monetary values, a very meagre ration of which was given in §25, he felt, like the opener, that it was odd that no mention should have been made of policy values. The criterion as to changes in policy values lay in a comparison of the slopes of the two q_x curves, namely the A 1924–29 and the A 1949–52. It would be found that throughout, until after age 90, the slope of the A 1949–52 curve was below that of the A 1924–29. The two curves diverged up to about age 40, and then, as mentioned in the memorandum, converged between 40 and 55, diverged a second time between 55 and 80, and converged again after 80, finally crossing at 92.

He thought that from those shapes it might be concluded that the effect on policy values was necessarily that over the range of ages where the main weight of the liabilities rested, the whole life policy values would be lower than those of the A 1924–29 table, while the endowment assurance policy values would be higher. To check that he had done some rough calculations, which appeared to confirm that view and to suggest that possibly in the aggregate, given a normal distribution of business in-force, the whole life policy values might be 1½% below the A 1924–29 level, and those for endowment assurances about ½% above. After allowance for bonuses and paid-up policies, which would alter the picture, and with the whole life and endowment assurances added together, it would seem that for most offices there would not be any considerable difference in the aggregate reserves on the new table compared with those on the basis of the A 1924–29 table.

Mr M. E. Ogborn had been interested to see what could be learnt from the select data. The Seventeen Offices table, which was the first combined experience of assured lives, was an aggregate table of the experience to 1838, although select material was available. The H^M table, based on the experience to 1863, was an aggregate table, but Sprague examined selection for 5 years; the O^M, for the 1863–93 experience, had a 10-year select period, the A 1924–29 a 3-year period, and the 1949–52 experience a period of 2 years. The select period seemed to take the familiar bell-shaped form!

Lest there should be any impression that that was a feature of the data, he had had some tests made to see what the effect of selection actually was, and he had been interested to find that it seemed to be very much the same in at least each of the last three experiences, the 1863–93, the 1924–29 and the 1949–52. The alteration in the select period was not so much a difference in the data as in the

way of handling the data, a difference in what was considered desirable for practical work.

Taking the experience of medically-examined lives alone, the percentages of the select mortality to the mortality at durations 3 and over were, for the age-range of 20½ to 59½:

Duration	1863–93 %	1924–29 %	1949–52 %
0	47·1	49·7	50·8
1	68·5	69·0	70·3
2	76·9	79·5	75·3
3	83·3	79·1	84·1
4	86·4	88·5	80·8

Although there were variations between the experiences, he thought it would be agreed that they showed a substantially similar run of figures and a similar reluctance to approach the ultimate mortality. The figures, of course, related to medically-examined lives; the non-medical business took a different course during the select period.

On general grounds it would be expected, he thought, that the effect of selection would be different at different ages. At the younger ages the advances in medical science had reduced the rate of mortality virtually to an accident rate, so that it was to be expected that at the younger ages the percentage of the select mortality to the ultimate would have risen; the two rates of mortality were approaching each other and becoming more nearly an accident rate. On the other hand, at the older ages it was to be expected that the advances of medical science would produce a build-up, as it were, of impaired lives, many people continuing to live—and to be exposed to risk—who at an earlier period would have died at younger ages. Some of them, of course, would be normal lives, but a proportion would be still living who were impaired lives. At the older ages, therefore, it was to be expected that the effect of selection would be greater than formerly.

In fact the statistics did show just that trend. Taking the percentages given on the next page,* it would be found that in the 1863–93 experience the percentage of the mortality at duration 0 to the mortality at durations 3 and over was increasing with age, whereas in 1924–29 the percentage was decreasing with age. The simplest way of showing the increase or decrease was to fit a straight line to the percentages. He did not suggest that that was necessarily appropriate, but it gave the run of the figures compactly. In the 1863–93 experience the percentages on that basis were increasing by 1·65 for each 5 years of age; in 1924–29 the percentages were decreasing by 1·73 for each 5 years of age; and it was interesting to find that in 1949–52 the percentages were also decreasing, but at a greater rate than in 1924–29, the decrease in the percentage being 3·10 for each 5 years of age. Those were the figures for duration 0. He was sorry to say that the figures for the other select durations were not quite so clear.

Although he saw the reasons which had led the Committee to adopt a two-year period for the select table, he would be sorry personally if the offices were to give

* The Committee have supplied the tables set out on pp. 83 and 84 giving the percentages for the three experiences for each duration up to 4. Eds. *J.I.A.*

up making their returns on the basis of the five-year select period. He felt that there was something to be learnt from the statistics, even if that information was not used when the tables were constructed; and even though on a future occasion an aggregate table might be produced, he would still hope that the data would be submitted in a select form, so that something might be learnt from the data which could not be obtained from the aggregate statistics.

Percentages of duration 0 to durations 3 and over

Age-group	Actual values of 100 A/E			Calculated values of 100 A/E, on assumption of a straight line		
	1863–93	1924–29	1949–52	1863–93	1924–29	1949–52
20½–24½	41·5	59·2	68·1	42·44	56·25	64·35
25½–29½	43·8	48·8	59·0	44·09	54·52	61·25
30½–34½	48·7	52·3	64·3	45·74	52·78	58·15
35½–39½	43·6	56·6	48·2	47·39	51·05	55·05
40½–44½	50·0	48·1	42·9	49·04	49·32	51·95
45½–49½	47·2	42·9	53·2	50·69	47·58	48·85
50½–54½	55·2	52·9	48·5	52·34	45·85	45·75
55½–59½	55·7	40·7	43·9	53·99	44·11	42·65

One other thought arose from the figures which he had given. He believed that in the study of any mortality experience it was necessary to consider that build-up of impaired lives to which he had referred; in particular, when the experience was used in their calculations, it had to be considered whether the mortality realized in practice would be over the same body of lives, including the impaired ones, as existed in the data. He had particularly in mind policies of the pure endowment type. The assumption of the same table of assured lives for those classes assumed that their contracts would be maintained in force until death, whereas he could not help feeling that some of those impaired lives would escape before death, and the mortality shown by those classes would be lighter than that shown in the A 1949–52 tables.

Mr F. Burden remarked that he had been particularly interested in that part of the memorandum which dealt with the choice of the ultimate data on which the final mortality table had been calculated. The Committee had explained clearly why they had chosen not to graduate the whole of the combined data but to weight the data in favour of Table 1, Whole Life With-Profit (Medical), from age 50 onwards. They said in §9 that at ages over 80 attention should be focused upon the Whole Life With-Profit (Medical) class, and that between 50 and 80 the final table should give increasing weight to it. He thought that that attitude was fully justified on the ground that the table which it was desired to achieve was for practical use, but they should look underneath it to see whether it was really necessary to follow that course.

Before adopting it, they ought to be satisfied that the two reasons which were given did in fact hold good. He thought that the two reasons were that life office portfolios were said to have much more at risk under whole life than under endowment assurance policies, and that the whole life without profits mortality was unreliable, there being an over-statement of the exposure and an under-statement of the claims, because paid-up policies without profits often stayed on the books, the claims not being made when they should be made.

It was interesting to note that in Table 1 the peak of the exposed-to-risk was at age 71, while in each of the other seven tables the highest figure for exposed-to-risk was at age 45 or earlier. If, therefore, all the data were combined and graduated as they stood, decreasing weight would be given automatically to Tables 2 to 8 as compared with Table 1, where the peak of exposed-to-risk was not reached until the age of 71.

Moreover, before agreeing that the artificial procedure suggested by the Committee was justified they required to be satisfied that the mortality under Table 1 was significantly higher than that under the other tables. He noticed that while the comparison of mortality shown in Table 10, where percentages of actual to expected deaths were in five-year age-groups, showed that the whole life mortality was heavier than the endowment assurance mortality, it also showed that the two non-medical whole life tables, Tables 5 and 6, disclosed significantly higher mortality than Table 1. It would appear that if the graduation of the final table had been based on all four whole life tables a different shape of curve might have been obtained with higher rates of mortality than under the graduation which had been adopted. In the case of endowment assurance mortality, he thought that it was only in the two non-profit tables that the mortality was significantly lighter; in the two with-profit tables the mortality at ages 45 to 80 did not differ much from that of the complete graduation.

Some support for his views was to be found in the table in § 21. It was satisfying to see that the Spencer graduation of the all-classes data was almost identical with that of the proposed table up to and including age $52\frac{1}{2}$, and differed quite materially from Table 1. In fact Table 1, the Whole Life With-Profit (Medical) table, showed values of q_x over that range of ages significantly lighter than those of all classes combined. At age $52\frac{1}{2}$ Table 1 gave a q_x of ·007, as against ·008 in the other two graduations or something like a 14% difference, the biggest difference that occurred between Table 1 and the combined data. At age $62\frac{1}{2}$ the q_x for the Spencer graduation of Table 1 was greater than for the combined data by its biggest amount, just under 10%; but at that point the proposed table was much nearer to the combined data than to the Whole Life. At that particular age, $62\frac{1}{2}$, where perhaps weight should have been given to Table 1, sufficient weight had not been given to it if it was desired to achieve a table which would be safe for actuaries to use. Later there was little difference between all three q_x's, but he thought it would be found that the all-classes data and the Table 1 data were nearer to each other in many places than to the proposed graduation.

While his remarks might sound like criticism of a very fine and ingenious graduation of artificially distorted data, they had to be kept in proper perspective. The curve was smooth and useful, and, apart from a small range of ages, did not seriously understate q_x; but he thought that there could have been an equally satisfactory result, with no artificial distortion of the data, had the whole of the data been graduated as they stood.

There was one small point which he would like to make with regard to selection. It was made clear early in the memorandum that one of the reasons why the mortality experience of the non-medical business was greater than that of the medical was that those offices which did the bulk of the non-medical business appeared to experience heavy mortality. That had to be accepted as a reasonable explanation, and, if so, on turning to the table in § 12, where there was an analysis of selection in the first five years split into medical and non-medical, he could not see why the conclusion that the 8% difference might be a

measure of the effect of medical examination need reasonably be drawn. Why was the conclusion not the same as before, that the q_s from the non-medical data reflected the heavier mortality which those offices were experiencing? It was not, it seemed, a question of the medical examination but of the heavier mortality experienced by those offices which did the bulk of the non-medical business.

Mr J. B. H. Pegler endorsed the opener's general blessing of the proposals put forward by the Committee and his view that they would produce a workable table of the type which was wanted. He supported his plea that there should not be any 'fancy' tables, but just the main tables which they were all going to require, the functions less often needed being left to be calculated as and when they were required.

He was glad that the Committee had decided to use the fitting of mathematical formulae as their means of graduation. He could remember in his student days his bitter disappointment, when the A 1924–29 tables were discussed, at finding that the Spencer summation formula had been used for its graduation. In view of the considerable amount of research on graduation which had been done by Perks, Beard and others, it was particularly pleasing to see that the results of those researches had been used in such an important table.

Nevertheless, he asked why it was thought necessary to test the mathematically-fitted graduation against the Spencer graduation. He understood that the two important tests for graduation were smoothness and fidelity to the original data. A mathematical formula automatically provided smoothness and he would have thought that the test of fidelity to the original data was better decided by comparison with the original data, and not by reference to another, and in his view inferior, graduation.

He did not understand the statement in § 12, where the medical and non-medical select rates at durations 0 to 4 were compared, that 'in both there must eventually come a sharp upward turn so that the select may merge into the ultimate rates'. He could not see any theoretical or practical reason why the select rates should merge into the ultimate rates before the limit of the table, and in particular he could not see why they should merge with a sharp upward turn.

He seemed to remember being told in his student days that large cases experienced a higher mortality than the average. He had never seen any statistical evidence put forward for that point of view, and he had always privately suspected it to be a myth. The American data referred to by Mr Bromfield seemed to point a little towards lower mortality for large sums assured but, as Mr Bromfield had suggested, the same conditions might not apply in Britain. That was a matter of some importance, and he would ask the Committee to consider carefully whether, if they had not the data already, they should collect data which would give some information on the subject.

He was impressed by the extraordinary lucidity with which the information and the argument had been put forward by the Committee. The subject of the memorandum might easily have become heavy and indigestible, but in fact he had found it a real pleasure to read.

Mr W. Perks said that the opener had quoted from the discussion on the A 1924–29 tables but had not disclosed the authorship of the quotation. He, the speaker, confessed that the remarks were his, but added that he was completely unrepentant about what he had said.

The memorandum should, he thought, be considered from two quite different points of view, the practical and the theoretical. From the practical point of view the main question was, he thought, whether the new table was a suitable instrument for life office work. To that his answer would be a definite 'Yes', although he thought that, apart from war risk, it was a safe-side table, based as it was on three average years in the recent past and one bad year. Its construction from the point of view of selection and class differences also seemed to have veered towards the heavy side, despite the evidence of continuing improvement in mortality. For his part he preferred the 1947–48 table, but he would not be content until they were supplied with a set of hypothetical tables.

Incidentally, he would not accept the statement in § 1 of the memorandum that the rates by the A 1924–29 Light table were too low in some of the older age groups. He was not sure, either, that for practical purposes they were too high at the younger ages, particularly in view of the war risk. At ages 55 and 65 the A 1924–29 Light table was about 5 % lower than the 1949–52 table, and at age 60 about 10 % lower. Those were not large differences and were easily explained away by the choice of the years and by the prevalence of lapse selection.

The biggest question about male mortality was what was going to happen between ages 45 and 75. It was important to keep in mind the very different course of female mortality in Britain at those ages, and also the fact that in Scandinavia the male mortality at those ages was 20–25 % lighter than in Britain. Was the British position to be ascribed to the privations of the first world war? If so, why did Western Germany and Italy show lower mortality in middle life than was shown in Britain? Was it due to dietary differences or had it something to do with the difference between rural and urban life? If so, why did not the women show similar differences? It was a puzzling business and no one knew the answer, but he would hazard a guess that male mortality in that important age-range, both in Britain and in the United States, was going to show a fall.

On the theoretical side, there were many points of interest in the memorandum in the interpretation of the data and in the construction of the new table. He felt, as probably the Committee did, exasperated by the limitations imposed by the method of bulk collection of the data. Whether it would be commercially worth while to go over to an individual policy system he was not quite sure. Presumably it could be done only for new entrants, but theoretically there could be no doubt that it ought to be done. The current methods seriously hampered research.

There was, however, an intermediate course that would not involve the offices in much additional work, and that was to write a card for each policy becoming a claim by death, recording as much information as possible, including cause of death, date of entry and such indications as would enable all duplicates to be linked up. The analysis of the deaths that would then be possible would throw a great deal of light on many of the unanswered questions in the memorandum.

Since making his remarks at the Faculty meeting he had begun to wonder how the Committee had computed the standard errors quoted in Table B of Appendix 1. In view of the other evidence in the memorandum they might have taken the square root of the actual deaths or they might have worked on the expected deaths by the A 1924–29 table—that would provide some offset to the effect of duplicates. The only sound course would be to base the comparisons and the standard errors on the new table and to make some allowance for duplicates. That, however, would not eliminate the effects of duration.

There was a similar point in the difference between medical and non-medical percentages of actual to expected deaths for the age-group 45–75 in Table A, which might easily be explained by different age and duration distributions. Most offices had limiting entry ages in their non-medical schemes, with obvious effects on average durations at later attained ages.

The most interesting feature of the memorandum from the theoretical point of view was the evidence that duration effects—he did not say 'selection'—lasted a very long time. That feature was also shown by the 1946–49 experience of assured lives in America, where for entry ages over 40 the select rates for duration 10 years were still only about 70–80% of the ultimate after 15 years. There could be little doubt that in the data under consideration there was a mixture of at least three effects: true long-term selection, lapse selection, and difference in class of life between the long and short durations. On all three grounds it could be argued that the new table would produce premiums that were too high, at least for entrants at ages over 45, quite apart from the possibility of further improvement in mortality.

He found § 14 most unconvincing. If the select period was to be only two years, the ultimate rates between ages 45 and 65 ought to have been reduced, partly to eliminate the effect of lapse selection, which should not be charged for in the premiums, and partly to offset the effects of the different class of lives in the ultimate data, which they might well presume would not be present in the future experience in respect of the business then being written. The Committee's difficulty had presumably been to know by how much to reduce the observed rates. Failing that, he suggested that selection ought to have been shown in the table for at least five years. He emphasized at that point that he was speaking from the strictly theoretical point of view. Incidentally, it was only where there was a long period of selection that there could be any significant arithmetical advantage in using the 'damaged-lives' technique adapted from the paper submitted by Bulina to the Rome Congress. He was not sure that he liked the precise form of that technique that had been used for the new table, but there was no time that evening to go into that.

As mentioned in the memorandum, the mortality rates, excluding the first two years, showed a marked wave between ages 40 and 60, and there was a similar wave in the national statistics. There was no doubt, however, that in the new table it had been exaggerated by the element of neglected selection. The average duration at age 40 was probably little over 5 years, whereas at age 60 it might be as much as 15 years. That might explain the need for the term c^{-2x} in the graduation formula. There was an interesting analogy between the parameter c^{-2x} and r in the select graduation formula in his 1931 paper. He suggested that there were at least six parameters in the formula used, and not five as stated in the paper, because he regarded c^{-2x} as an arbitrary parameter fixed in the light of the data. His own view was that to the extent to which the wave between 40 and 60 was due to neglected selection it should not have been fully retained in the table for future use, and some process of cutting through it would have been desirable.

Mr R. Ll. Gwilt had been interested in the number of references made to the question of selection. Some actuaries, apparently, preferred a table tracing selection for somewhat longer than the two years which the Committee had chosen. Interestingly enough, at the Faculty meeting there were several who would have been quite content to have an aggregate table, and he thought that

he would put himself in that class. He agreed entirely with Mr Perks that on theoretical grounds it was desirable to trace selection, or perhaps it would be better to say variation in mortality by duration, for far more than five years, but they did not possess the information for more than five years. He had struggled hard for many years to get it, but eventually gave up the fight. He hoped, however, that they would not decide to drop the collection of the data in durations for, at any rate, the first five years.

Another point which interested him was the reference to policy values, and the suggestion by one speaker that with a steeper slope of mortality than in the A 1924–29 table rather higher policy values could be expected. He remembered thinking that himself when he saw the statistics for the years 1947–48 (*J.I.A.* **77**, 103), which showed a distinctly steeper slope than the A 1924–29 table; and yet when he had examined policy values and compared them with A 1924–29 values he had found that they were lower, not higher. The answer was that it was not only the slope of the mortality curve that mattered but the slope as compared with other things.

Mr H. A. R. Barnett said that he was much in agreement with what had been done, but he would like to make one or two comments on the collection of data. First of all he would accuse the Committee of hiding their light under a bushel in one respect. There was one great advantage in their choice of the period 1949–52 that they had not mentioned, and that was that it facilitated the comparison of assured lives' experience with national data. It should be quite easy to construct a national table for 1949–52 without having to make hazardous approximations for the exposed-to-risk, because the Census date was close to the centre of the period.

Having referred to a comparison with the national data, he felt that he should mention a point which might be regarded as a 'hardy perennial' of his. Mr Perks had already touched on it. He referred to the question of some information as to cause of death in the data collected for the Committee. He had already tried to indicate a possible means by which that could be collected without making it too onerous for the offices giving the information. If it were desired to offset the additional work involved, he felt that much could be said about selection. He agreed with Mr Perks about the theoretical desirability of having more information, but he did not see that in practice they were going to gain anything by continuing to have information in respect of the fourth and fifth years of duration.

Mr N. Benz also referred to the question of initial selection. He had hoped to hear something from Mr Gwilt on the subject, because at long last there seemed to be an opportunity to find out a little more about that vexed question. The question was not merely theoretical, because the ultimate level of mortality, whatever it might be, affected the valuation of liabilities. At the time of speaking they did not know the answer, but even if they were to have only some kind of pilot investigation it would be very helpful.

The selection of the period of the investigation was discussed in §3. It struck him as most fortunate that 1949–52 represented a proper quadrennium for the purpose in mind. Was it possible that the Committee might even then be waiting for a new assured lives' table? Most actuaries had for most of their working lives used an assured lives' table which they had known contained substantial margins, though of degrees varying with age, but that would not always continue. He

would like to know whether the Committee would at some time give consideration to the possibility of what he expected would be called by the critical a hypothetical table. They had actual mortality rates for the whole body of offices over something like 25 years, albeit marred by events such as wars, and he would not have thought that it was beyond their very considerable powers to produce a table which would be generally acceptable, bearing in mind the professional responsibility which had to be carried by each actuary when determining a mortality basis.

In 1848 not only was the Institute founded, but various revolutionary movements broke out. He hoped that he had not been guilty of another one.

Mr R. E. Beard said that at least one speaker had referred to hypothetical tables. In preparing a hypothetical table the compiler assembled his knowledge and belief of what the table should be and endeavoured to find a curve to represent those ideas. Something of that kind had happened in the case under discussion; the Committee had decided what they wanted from the data before them and from general considerations, and had tried to produce a table which reflected those ideas. If the result was not a hypothetical table it was a near relation to it.

To some extent that procedure cut through the point which had been made about the weighting of the data. Once the details had been decided it was not necessary to work with the mass of data and add extra weight to some of the exposed and deaths, but the work proceeded by the 'feel' of the arithmetic to produce an answer. That was one reason why comparisons were made of rates of mortality at the different points. A comparison had of course also been made of the actual and expected deaths to see whether or not the curve did produce what the Committee thought that they should have.

He had found in working on the data that the question of selection bedevilled the whole problem, and he had been constrained to turn up old issues of the *Journal*, going back over some fifty years. For anyone interested in the question of lapse selection those papers were worth reading, and it was interesting to see the attempts made by actuaries many years earlier to measure the effect of that factor.

That led to the question of what was a select rate. It seemed to him that all mortality rates depended to a certain extent on something which probably could not be measured, so that nowhere did they stand on firm ground and there was bound always to be a certain lack of precision about what had been determined. That fact disturbed him, because he did not know where his feet were and it was a further argument for departing altogether from anything like a graduation based on observed data and for using rates of mortality or a table that seemed proper to use.

The question of analysing the deaths was interesting and, of course, had been raised some time earlier when duplicates were being discussed. It looked as though they could usefully spend some effort on such analyses. They would not get the answers to all questions, but they might get the answers to some, including facts about what happened at the longer durations, which the current method of collecting data did not reveal.

Mr J. M. Beattie, in closing the discussion, said that the memorandum which the Committee had prepared fell naturally into a number of different parts, and he proposed to deal with some of the points which had been raised in the

discussion under corresponding headings; but before doing so he thought that it would be only fair to say that the general feeling of the meeting seemed to be that the Committee had done an extremely satisfactory job. Hardly anyone had put forward any real alternative to the Committee's suggestions, either in regard to their general conclusions or even in regard to their detailed methods. Mr Perks and Mr Beard had suggested collecting more data and using those data in different ways, but he felt that that fell a little outside the province of what the Committee had been asked to do with the data with which they had been provided.

A point of great importance was that on those occasions the Committee had a more or less impregnable defence: they could always challenge any member of the Institute to produce a better table. On past occasions they had used that defence, and no one, so far as he knew, had responded. The only suggestion for producing a different table which had been made that evening had been Mr Burden's that the table might have been constructed by graduating the data for all classes. It might be of great interest if that line were pursued by the speaker who had suggested it, so that they could at a later stage see the results of such a graduation.

Dealing with more detailed points under the headings suggested by the Committee's memorandum, he said that the first and most difficult problem with which they had been faced was the choice of data to use for the construction of the new table. After all, mortality varied a good deal in different years, as was well known, and there were also important variations in the mortality for different classes of assurance and also for the medically examined lives and the non-medical lives. The first question which they had to ask themselves was whether those differences in the mortality of different classes were really due to intrinsic differences in mortality or whether, as the Committee had suggested, they were due in some instances to mortality in offices being light or heavy. The Committee had suggested that the heavier non-medical mortality was due to the greater proportion of the non-medical data drawn from the 'heavy' offices, and also that heavier mortality at longer durations might be due to the same cause. Possibly on further investigation the difference between endowment assurance mortality and whole life mortality might be put down to the same reason.

On the question of what data to use in actually constructing the graduated table, the Committee had, he thought justifiably, thrown overboard the idea of producing a graduated table which purported to show the mortality of a particular class of life. They had provided a table which was meant to be an up-to-date instrument for life assurance calculations. Mr Bromfield had suggested that there might be something to be said for making future investigations according to sums assured. At one time it had been commonly suggested that investigations should be made according to amounts at risk, and he, the speaker, thought that what the Committee had done was some kind of approximation to a table according to amount at risk.

He thought that the most interesting topic in the paper, and certainly the most interesting in the discussion, had been the tracing of selection. He had some sympathy with Mr Pegler's surprise at the statement by the Committee that at some stage the rates progressing by duration were bound to take a sharp upward turn, and he had been rather taken aback by the suggestion that they might at some time reach a level which would make the actual deaths as much as 110% of the expected according to the table for durations 3 and over. He was not sure that there was not some element of statistical fallacy in that. Presumably the average age of lives at duration 4 was about 40 years, and the

number of actual deaths was correspondingly low. At duration 25 or 30, of course, the number of actual deaths and the number of expected were considerably higher, and it would not be necessary to go as high as 110%; probably something a little over 100% for longer durations would satisfy this problem.

Mr Perks had suggested that the table which the Committee had produced would tend to give premiums which were too high at the older ages. The Committee had thought at one time of producing an aggregate table, and he believed that the effect of the aggregate table would be, especially at the younger ages, to produce premiums which were on the whole too low. It seemed to him that the so-called ultimate table produced by the Committee had some of the character of an aggregate table, and he would have thought that the rates of premium brought out by the new table were perhaps on the low side.

Many speakers had dealt with the question of the tracing of selection. Mr Dicken had given some interesting figures arrived at by investigating the data in a different way, and Mr Ogborn also had referred to that point in some detail, but neither they nor anyone else had given any real reason different from the one given by the Committee to explain why the rates of mortality should continue to increase by duration long after the period of 5 years' selection traced by the Committee.

On the question of graduation there had been surprisingly few comments, and on that he would make a personal comment. It defeated his imagination to think how the five constants in the formula, some of them involving six significant figures, could have been produced by trial and error. He thought that that might call for a little more explanation by the Committee. The late Mr A. E. King had made a considerable reputation for himself by publishing an explanation of the graduation of the 1863–93 annuitants' experience, and the way was open for Mr Benz or some other member of the Institute.

Some speakers had commented on the comparison of the results of the new tables with the results from past experience. The last experience published by the Committee, on which no standard table was made up, was contained in a note published in 1950 dealing with the experience of the years 1944–48. One interesting feature of that experience was that at age 70 the premium based on ultimate mortality of the years 1947–48 was as much as £1. 2*s*. 6*d*.% less than the premium according to the A 1924–29 table. That was comforting to offices which were currently, and had been for some years, accepting a high proportion of lives at old ages. On the new table the premiums at the old ages were not as low as those based on the 1947–48 experience, but he had worked out what the reduction in the A 1924–29 select premium would be at age 70, and it was as much as 15*s*. 9*d*.%, so that there was still a considerable margin at the old ages. Probably many offices had been giving away some of that margin in advance of the publication of the experience. He expected that few had been inclined to go so far as to use the A 1924–29 Light experience complete in itself. Even if that experience had been used, he thought that the premiums at the older ages were close to the premiums brought out by the new experience.

Several speakers had referred to the reserves brought out by the new table, and it had been suggested by the opener that the reserves by the new table were likely to be heavier than the reserves by other tables exhibiting a less steep rise in rates of mortality towards the older ages. As Mr Gwilt had suggested, the reserves depended on a number of things as well as the shape of the mortality curve. Probably by a curve such as was brought out by the new table reserves were a greater proportion of the net premiums, but if the net premiums

themselves were reduced it was possible, as had been suggested, that the reserves themselves might be reduced for certain classes of policy.

Various speakers had made suggestions about further data which might be provided and further investigations of the data already provided. Probably the most fruitful field would be an investigation of offices exhibiting light and heavy mortality, and he would make a plea for that to be done as early as possible. It might be a formidable task for the Committee in view of the work on their hands, but there was undoubtedly a vast amount of data available for research to any members of the Institute who felt inclined to undertake it.

He noticed that the discussions on the A 1924–29 tables in 1933 had been concluded both at the Institute and at the Faculty by Colonel H. J. P. Oakley, who had been a prominent member of the Committee which conducted the mortality investigations at that time. Colonel Oakley would probably have felt, if he could have been present that evening, that the bombardment conducted by the members had not been formidable, and that the main structure of the Committee's work remained intact and few outposts had been carried.

Mr F. M. Redington, replying on behalf of the Committee, said that it was the first sessional meeting in the new Hall. The new Hall was very similar to the old one, but it was not the same. The recollections of many of them suggested that the differences were greater than the facts showed to be the case. He had been re-reading the previous report on the 1924–29 experience. His recollection was that the shape of the old report was very similar to that of the new, but that the spirit was different; when, however, he came to check the facts, he found that the differences, though they existed, were less than his memory had recorded. In 1933 he had been a critic; in 1955 he had become a producer, and things looked different. The previous Committee had had the same problems, they had had a similar outlook and they had come to very similar conclusions, but they presented them a little differently. There had been, he would like to believe, a growth of understanding within the profession, and he felt that the Committee of which he was a member had been able to rely more than could their predecessors on their audience having a broad and tolerant approach. The discussion that evening had provided evidence of that.

There was a little more openness in the new report, and less mystery in it; but the first conclusion of the Committee, as he suspected it had been of the previous one, had been that it was futile and, what was worse, philosophically wrong to search for *the* mortality of assured lives. There were many mortalities, and their duty had been to find and construct not *the* mortality table but a representative one. That had been their first conclusion, and it was their last. They knew that if they said it clearly enough the members would agree. That was the main point, and he believed that it was the answer to many of the comments which had been made that evening and in Edinburgh.

His function in replying to the discussion was not to answer the comments in detail; the Committee would consider them carefully and give their answers in writing later. There were, however, some aspects of the discussion on which he felt he should comment, because the atmosphere of the meeting that evening would live longer in the memory than the written reply. The points which he would take up were not necessarily the most important.

The first was that made by Mr Pegler and Mr Beattie, who had asked why the select rates should merge into the ultimate with a sharp upward turn. That was primarily an arithmetical problem. They did not know what were the mortalities

at durations 5, 6 and 7; they only knew the average, which they would call 100%. The latest known fact was that the mortality at duration 4 was about 80–84%. There was not only a big gap between duration 4 and durations 5 and over but also there was what had been called in the Committee the 'flat'. At durations 2, 3 and 4 the mortality was flat and the rates showed no effort to get up to the average of 100% which they had to reach. It was very difficult to take a pencil and paper and construct any set of exposed-to-risk and deaths for durations 5, 6, 7, etc., which would give an average of 100%. The q's after duration 4 had to get a move on! When the Committee said in the report that they had to reach 110% in order to average 100%, he felt that that was an under-statement. It was extremely difficult to speculate about causes, because, as had been said, there were different forms of selection, and one of them might be class selection, as for example the different underwriting of the offices throughout the years. But that different underwriting had not always been in one direction; it had not steadily become more severe or less severe, but had changed up and down. He did not find an answer in class selection. It might lie partially in the different weighting of the offices, as mentioned in the memorandum, but the problem remained, and he wished that they had the necessary information.

He would like to take his hat off to Mr Perks, but he feared that the Committee might want him to take his coat off! He agreed largely with what Mr Perks had said, and wished to comment on two things in particular. Mr Perks said that there were at least six parameters in the formula, and he was quite right. It was a matter of no practical importance, but it was, in the conception of those problems, an important point.

The second point was of more practical importance. Mr Perks had spoken of duplicates. The Committee had not overlooked duplicates, and the speaker had thought of making an r_x test of the data, but had concluded that it would be inappropriate. The graduation was not a graduation in the usual sense, but a representation of the mortality, and it had been dropped at that point. Mr Perks's point, however, was of considerably greater importance in Appendix 1. Much of the whole memorandum hung on the fact that there were substantial differences in mortality between the offices. That was what made it inappropriate to search for *the* mortality table. If the conclusions of Appendix 1 were wrong the memorandum would be undermined.

In Table B, Mr Perks was correct in saying that if they had allowed for duplicates the spread between the offices would not have looked so abnormal as it did. The speaker thought, however, that for quite other reasons that Appendix did not over-state the case, but in fact under-stated it. Appendix 1 was based on the experience of four years, but the Committee knew very well that the offices with heavy mortality and with light mortality fell into groups; they were not random. The Committee knew that those groups had persisted for 20 years or more. If they were to extract the data for various groups of offices over the whole period since 1924 and apply a significance test including duplicates there could be no doubt that those groups would show wide and significant variations. He did not say that that disposed of Mr Perks's point—he was quite correct in drawing attention to the importance of duplicates when interpreting Table B.

It was worth referring to the valuation problem which Mr Benz had raised, and which Mr Haynes had mentioned at the Faculty meeting. The new table, which was a little heavy for premiums, could be unduly light for valuation because of the long period of selection unaccounted for. He thought, however, that that possibility was minimized by the fact that it was a table based on policies.

For valuation they ought to use a table based on sums at risk, and the sum at risk on the long-duration policies, where the mortality could be under-stated by this semi-aggregate table, was of little importance. That was a considerable moderator of the fears expressed by Mr Benz and Mr Haynes.

The Committee were appreciative of the interest shown and of the many constructive comments made and of the kindness with which they had been received. He was sure that the rest of the Committee would like him to express their particular appreciation of the work done by their Secretary, Mr R. D. Clarke, who, apart from all his other work, had written what had been rightly described as an extraordinarily lucid memorandum. They would also like to express their appreciation of the work done by their 'parameter-in-chief', Mr Beard.

The President said that there had been a well-sustained and, in parts, provocative discussion. He had had the pleasure of attending also the discussion at the Faculty, and, between the two, he thought that the Committee had a number of good points on which to cogitate. He had been particularly interested to notice the number of speakers at the Faculty who were, or seemed to him to be, protagonists of the aggregate table, whereas at the Institute meeting the number of those speaking in that direction had been less pronounced. The history of past tables of assured life mortality had been one of decreasing select periods, and in time perhaps they would reach the ultimate aggregate table. The Committee, however, had had to consider the uses to which its tables would be put by the working actuary for premium calculations, for policy valuation and so on, and it was easy to understand why the tables had been presented in the form in which they were.

He liked to think that, as they took time to consider the paper and the discussion at leisure, they would find new lines of thought inviting investigation, so that possibly a family of satellite papers or notes might emerge from members who followed up fruitful lines of investigation.

He invited the members present to express their thanks to the Committee for all the consideration and thought which had been crystallized in the report which they had discussed that evening. Mr Redington, in his closing remarks, had referred to the thanks due by the Committee to Mr Clarke and to Mr Beard. They would all particularly applaud those remarks, but he, as President, wished to go further and to express the thanks of them all to the Chairman of the Committee, Mr Gwilt, who was a Fellow both of the Institute and of the Faculty and therefore just the right person for the job.

Mr G. W. Pingstone has written:

I understand that in the discussion at the Institute Mr Bromfield referred to the question of how the ultimate mortality under the proposed table for Assured Lives, 1949–52, compares with that experienced during the service period under group life and pension schemes, and I thought that it would therefore be of interest to indicate the results of an analysis in this connexion carried out by one large office.

Taking the experience in respect of pension benefits under a considerable group of large schemes for the policy years ending in 1954, it was found that the service mortality over the age range 45–59 was very close to that under the proposed Assured Lives, 1949–52, ultimate table, but that at the younger ages it was appreciably lighter and in the age-group 60–65 appreciably heavier.

The lighter mortality at the younger ages is no doubt attributable in part to the effect of new entrants and withdrawals, but in connexion with the heavier mortality experienced in the age-group 60–65, it is rather interesting to note from §20 of the memorandum that it is in this age range that the values of $(A-E)$ are all positive in the whole life table.

The Committee has submitted the following reply to the two discussions: The major part of the discussion falls under three main headings, namely:

(1) Variations in the data. (2) Selection. (3) Graduation.

It will be most convenient to deal with these three major topics first and then to consider the various other matters that were raised.

Variations in the data. Comments on this topic were concerned with variations according to class of policy, size of sum assured and whether medical or non-medical. Much was also said on the important question of variations between offices. As several speakers pointed out, analysis of these different sources of variations is complicated by correlation between them. As explained in the memorandum, a substantial proportion of the non-medical data comes from offices whose mortality experience, both medical and non-medical, is heavier than the average. It follows, therefore, that the heavier non-medical experience is due only in part to the absence of a medical examination, the remainder being a reflection of inter-office variations. Some speakers seem to have misinterpreted the final sentence of §12. The statement that 'medical examination may have the effect at the earlier durations of lowering the mortality by about 8% of the ultimate rate' is not based upon a direct comparison of medical and non-medical mortality. The table reproduced in §12 shows percentages of actual deaths at short durations to those expected on the basis of the '3 and over' mortality, the medical and non-medical percentages being each based on the *corresponding* 'ultimate' rates. Thus the *degree of selection* indicated in the medical section is about 8% greater than in the non-medical section. This conclusion is quite independent of any comparison of the medical 'ultimate' experience with the non-medical 'ultimate' experience. As pointed out by Mr Dicken, the figure of 8% is an overall average. The difference in the degree of selection between the medical and the non-medical sections is less than 8% at the younger ages and more at the older ages.

Speaking on behalf of the Committee, Mr Bromfield referred to possible variations in mortality with sums assured and this subject was also mentioned by Mr Wallace. Although the data collected by the Committee are not classified according to size of sum assured, variations from this source may well be involved both in the inter-office variations and in the differences between medical and non-medical mortality—sums assured under non-medical assurances being limited by the conditions which life offices customarily impose upon non-medical business.

Faced with so many possible sources of variation and with the lack of information which would enable the various factors to be isolated, the Committee felt that a table based on a hotchpotch of all the data was the only practical solution. As, however, inter-office variation is a reality, and as in practice each office needs a table which is appropriate to its own particular experience, there is a case for providing a choice of more than one table. It is for this reason that the Committee has indicated that it will consider the possibility of following the present

table with a 'Light' table. Some speakers asked also for a 'Heavy' table. This, however, is a more debatable matter. The existing A1924–29 series of tables remain available for those who feel the new table to be too light and in any case, as was pointed out in the discussion, the inclusion of 1951 in the base period slightly biases the new table on the heavy side. This is more obvious now, when statistics for 1953 and 1954 have become accessible, than when the Committee began its labours. It seems likely, therefore, that a 'Heavy' table based on 1949–52 would be of limited use.

Three speakers referred to the possibility of publishing a series of hypothetical tables and this, of course, is an extension of the plan of publishing a standard table and a Light table. The limiting factor here is practicability. Life offices require a multitude of functions over a large range of interest rates and these have to be computed, printed and published. It is impossible at this point of time to anticipate decisions which the Committee may take in the future with regard to constructing further tables; but it is questionable whether during any one period it would be justifiable to have more than, say, three standard mortality tables in current use for life assurance work in this country.

Mr Perks referred to Appendix 1, which gives information relating to inter-office variations, and criticized the absence of any reference to the effect of duplicate policies on the calculation of standard errors. No mention was made of duplicate policies in the body of the paper because the graduated curve of q_x was by deliberate policy not submitted to rigorous statistical tests. A special investigation into duplicate policies is at present in progress and the Committee intends to report on this investigation at a later date. Admittedly, however, Appendix 1 is affected by the presence of duplicate policies in the data. The difficulty confronting the Committee was to give some information about inter-office variations without breaking anonymity and Table A and Table B were felt to be the best instruments for the purpose. The Committee is, however, satisfied that there is a consistency in the inter-office variations which places their reality beyond doubt and that even if it were possible to make an allowance for duplicate policies when calculating the standard errors involved in Table B, the various statements made in the memorandum on inter-office variations would not be invalidated.

Selection. In the course of both discussions there were numerous references to the problem of selection and with much of what was said the Committee is in agreement. The basic difficulty is largely due to the fact that the proportions of data contributed by the various classes of business and the various life offices are different at short durations from what they are at long durations. Consequently it is not possible to regard the exposed-to-risk at long durations as homogeneous with the exposed-to-risk at short durations and the failure of the select mortality to advance rapidly towards the ultimate level with increasing duration is not a cause for surprise.

The length of time over which temporary initial selection persists is still a question on which the profession has little information. Even if data were available analysing the mortality of assured lives by duration up to, say, the twentieth year, the variations in the constitution of the data referred to in the preceding paragraph would still obscure the effect of initial selection. Furthermore, there is the additional factor, discussed in the memorandum, of the selective effect of withdrawals. Thus, in the present state of our knowledge, the duration of initial selection must remain a matter for conjecture. It may possibly vary with age; or it may even persist indefinitely. To extend Mr O'Brien's illustration, consider two groups of assured lives all aged 70 of whom one group was selected 40 years

previously at age 30 and the other group 20 years previously at age 50. If initial selection lasts for only a comparatively short time, and if standards of selection remain constant, these two groups should (apart from the operation of any selective influence by withdrawals) show similar mortality. But probably few actuaries would be prepared to commit themselves to this view without further evidence than is at present obtainable.

It is important to remember that 'ultimate' mortality in relation to a table for assured lives is not equivalent to the mortality experienced by the general class of people from whom assured lives are drawn. The ultimate section of the exposed-to-risk is made up of lives who at some time or other in the past were selected for life assurance and who have, therefore, been permanently separated from the population group from which they are drawn. In one sense, therefore, even 'ultimate' mortality can be regarded as 'select' in relation to the section of population to which assured lives belong. Theoretically, however, the existence of an ultimate column in a select table may be justified on the assumption that above a certain limit the rate of mortality for a given attained age ceases to vary significantly with increasing duration.

On a table which has been constructed with a shorter period of selection than is theoretically necessary, the 'ultimate' values of q_x are in reality 'truncated aggregate' values, i.e. they are an amalgam of the true ultimate values and the values at the intermediate durations which do not appear in the table. Clearly they will be affected by the distribution of data according to duration and one reason why whole life mortality tends to be higher than endowment assurance mortality may be that it has a higher average duration. This limitation of truncated aggregate rates has to be borne in mind when considering percentages which relate mortality at particular durations to the '3 and over' experience. At young ages the '3 and over' data necessarily have a short average duration and the percentages of actual deaths at durations 0, 1, 2, etc., to the expected deaths by the '3 and over' rates are for that reason higher than at older ages where the '3 and over' rates have a longer average duration.

There has been a considerable amount of comment on the statements in § 10 that the ratios of actual to expected deaths in durations, where the expected deaths are on the '3 and over' basis, 'must ultimately reach a level appreciably higher than 100%—perhaps 110%'—and that the percentages 'must take a sharp upward turn at some point beyond duration 4'. These statements were not intended to express anything beyond simple arithmetical facts. If, as in the table in § 10, deaths at duration 3 are 83·8% of the expected on the '3 and over' basis and at duration 4 are 84·4%, it seems certain that the percentages for several further durations will continue to lie below 100%. But the average for all durations in the '3 and over' group is 100%. Hence, there must inevitably be several percentages above 100% to balance those which fall below it. The distribution of data by duration will, of course, be important and at older ages, where the average duration is high and a substantial proportion of data concentrated at long durations, it is true—as Mr Beattie indicated—that the percentages need not go much above 100% in order to strike an average of 100% for all durations combined. At younger ages, however, the data are more evenly distributed by duration and in order to strike an average of 100%, the percentages for the longer durations must be considerably above 100% to counterbalance percentages of less than 90% at durations 3 and 4. It is difficult to see how this can be done unless there is a 'sharp upward turn' at some point beyond duration 4 and unless the percentages at longer durations reach a level which is well over 100%.

Mr Dicken made some interesting remarks based upon the variations with age in the percentages of actual to expected deaths for individual durations. It is true that by taking statistics for various groups of ages, a somewhat different picture emerges from that obtained by taking all ages combined. On the figures given by Mr Dicken for a limited section of the data, a three-year select table may seem more appropriate than a two-year select table. However, the Committee felt that the final decision on this question had to be based on the whole of the data rather than on any particular section of it.

Mr Menzies referred to the increases in data at durations 3 and 4 during the period covered by the investigation which would have been caused by the general increase in new life assurances in the years immediately following the war. It is true that as the new business for the years 1946, 1947 and 1948 moved into duration 3 and duration 4 a considerable increase in the exposures occurred and this would have had some effect on the mortality emerging for durations '3 and over'. The effect would vary with age because durations 3 and 4 are more preponderant in the '3 and over' totals at young ages than at older ages. On an average, however, the effect on the emerging mortality is to lighten it by about 1%. If '5 and over' mortality had been used in the calculation of the table in § 10, the percentages would naturally have been slightly lower but the general pattern would have been the same.

The statistics given by Mr Ogborn comparing the percentages of actual to expected deaths at short durations in the 1949–52 experience (medically examined lives only) with similar percentages for previous investigations are of considerable interest, and on pages 83–4 will be found three tables in which the figures are analysed in quinary age-groups. The table for the 1949–52 investigation repeats much of the data appearing in Table 12, but it is reproduced here for completeness. The similarities between the three periods are remarkable; the main difference is the low percentage shown at duration 4 in the 1949–52 investigation compared with the two earlier periods.

Graduation. In the past an appreciable part of any discussion on graduation has usually been directed towards examining the 'goodness of fit' by means of the various recognized graduation tests. In the present instance, the Committee's decision to depart from the traditional approach and to produce a table which is not so much a graduation of the data as a hypothetical representation of modern assured lives' mortality has met with little criticism. A natural result, however, has been almost to eliminate references to 'goodness of fit' and smoothness from the discussion.

Mr Perks pointed out that the formula adopted has, in effect, six parameters, the term c^{-2x} being a special case of r^{-x}. The Committee is in agreement with this viewpoint, although as there was no actual fitting process the term 'parameter' does not have quite its usual significance. Mr Perks expressed concern at the steepness of the q_x curve between ages 40 and 55 and felt that, although the rates of mortality for males over 55 had not been falling in recent years, they might nevertheless be expected to show some further decline when the business now in force at younger ages grew older. He was also concerned with the effect of what he termed 'neglected selection'. At age 60 the average duration in the '2 and over' data is higher than at age 40 and consequently the observed '2 and over' rate of mortality at 60 is not strictly comparable to the observed rate at 40. In other words, there is a greater degree of selection involved in the rate for age 40 than in the rate for age 60.

On the first of these two points it is difficult to speculate. The known facts are

that the decline in the mortality of middle-aged and elderly men which was in evidence up to about twelve years ago appears on the whole to have been halted, and it consequently seems unwise in a table designed for assured lives to assume that the decline will be resumed in the near future. The point regarding 'neglected selection' is, of course, valid; but in the construction of what is, as already pointed out, a truncated aggregate table, an increase in mean duration with advancing age is inevitable. Mr Perks also suggested that a somewhat lower level would have been justified for the graduated table, because the observed rates of mortality had been made heavier by the selective effect of withdrawals. But this effect is speculative and cannot be quantified. Even if it were granted that the observed '2 and over' rates are higher than are needed for the graduated table, it would be extremely difficult to indicate the amount by which they ought to be reduced. The steepening of the slope between ages 40 and 55 is a feature of the data and the Committee felt that it should be retained in the final curve.

Some speakers were troubled by the statement that the values of the parameters were found by trial and error and asked for more information. The fact is, however, that for the formula adopted the recognized techniques of curve-fitting were not the most convenient methods when seeking a suitable mathematical expression. No attempt was made—as Mr Dicken suggested—to employ a series of weights to blend the observed rates for the combined data and the Whole Life With-Profit (Medical) over the transitional ages; nor—as Mr Thornton suggested—were the parameters determined by examining separate sections of the curve. The expression 'trial and error' employed in the memorandum is literally correct as the final values adopted for the parameters were only determined after numerous experiments had been carried out.

It was perhaps unfortunate that in §21 the word 'test' was employed in relation to the comparison between the values of q_x produced by the graduation formula and the corresponding values obtained by applying Spencer's formula to both the combined data and the Whole Life With-Profit (Medical) data. As Mr Pegler remarked, it is not part of the test of a graduation by mathematical formula to compare the results with those obtained by summation methods. At the same time, it was felt that the table given in §21 would be of general interest, particularly as illustrating the transition between ages 50 and 70 from the combined experience to the Whole Life With-Profit (Medical) experience.

Mr Menzies expressed qualms about the graduation of the select rates and felt that the expected deaths were too low in comparison with the actual deaths. The real difficulty in this matter is the uneven progression of the proportions borne at successive age groups by the observed rates of mortality at durations 0 and 1 to the '2 and over' rates. If these uneven series are to be replaced by smooth series, a considerable divergence between actual and expected deaths is unavoidable. The method actually adopted to obtain the select rates was chosen because of its usefulness in simplifying calculations and, if the graduated rates are slightly on the light side, the shortfall is so insignificant as to have no effect on monetary values.

Endowment Assurance Schemes. Mention was made in the course of discussion of the mortality experienced under endowment assurance schemes where policies are issued without evidence of health. It should perhaps be made clear that these policies are excluded altogether from the data which the Committee collects. Only the traditional non-medical business, in respect of which a proposal form is completed which contains a detailed questionnaire on personal and family history, is included in the non-medical data contained in the investigation. It may well

be that for scheme business issued without evidence of health a heavier table would be required. However, the only satisfactory course would be to collect data for this special type of business and this is a matter to which the Committee will give consideration.

Mr Donald suggested that the Committee might investigate the mortality experienced in active service under various forms of pension schemes and he referred to the special effects on this type of experience of the rates of withdrawal. It is true that withdrawals have a large influence on active service mortality and that they vary considerably from year to year, being affected, among other things, by general economic conditions. It is therefore open to question whether rates of active-service mortality are valuable as a guide to future experience. Nevertheless, the Committee will bear Mr Donald's suggestions in mind when considering the planning of any new investigations. Mr Pingstone's account of his own investigations into active-service mortality will be read with considerable interest by everyone who is concerned with the subject.

Deferred Annuities. Mr Thornton expressed concern that the values of q_x at young ages in the new tables were lighter than in the $a(55)$ male table. It has to be remembered, however, that as annuitants' data do not extend to young ages, the values of q_x for young ages in the $a(55)$ tables were hypothetical and were determined by reference to the mortality of assured lives in 1947/8. But there was an appreciable decline in assured lives' mortality at young ages between 1947 and 1952 and in consequence the new table gives lower values for q_x up to age 45 than the $a(55)$ table. The Committee has always emphasized that the $a(55)$ table was not necessarily suitable for deferred annuities and it may well be that many actuaries will in future prefer to use the new table for mortality during the period of deferment and the $a(55)$ table for mortality after deferment.

Frequency of publishing new tables. Mr Inglis submitted a plea for the publication of tables at intervals of ten years. If rapid and major changes in mortality continue to occur, this suggestion would no doubt become a desirable policy to adopt. In other circumstances, however, it is necessary to consider whether the publication of new tables at such frequent intervals is either necessary or economic. At the present level of mortality rates, major changes have to occur before there is any appreciable effect on monetary values. It is impossible to say at the present juncture how long the new table is likely to remain a serviceable instrument for the efficient conduct of life assurance business.

Changing slope of the Mortality Curve. Mr O'Brien drew attention to the changing slope of the mortality curve as revealed by a comparison of the new table with earlier tables. As indicated above when replying to Mr Perks's comments on the graduation, this change of slope has been a major problem in the construction of the new table, the fall in mortality rates in middle life not being comparable to the heavy reductions that have occurred at young ages, but being in fact similar in relative degree to the reductions observed at ages in the range 70–80. Moreover, at the present time the reduction at young ages is continuing whereas the mortality of males over 55 has been approximately stationary since 1942. This means, of course, that the steepening of the slope between ages 40 and 55 may continue. On the other hand, changes in mortality rates tend to occur somewhat erratically, and if they should resume their decline at older ages at any time—possibly as a result of successful developments in the treatment of respiratory cancer or coronary disease—the slope of the q_x curve will again change and the increased steepness between ages 40 and 55 may be smoothed out.

Medically examined assured lives, 1863–93. Comparison of actual deaths at durations 0–4 with the expected deaths according to the corresponding mortality at durations 3 and over

Age-group	Duration 0			Duration 1			Duration 2			Duration 3			Duration 4		
	A	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*
20½–24½	245	591	41·5	316	400	79·0	243	263	92·4	159	165	96·4	94	97	96·9
25½–29½	432	987	43·8	559	850	65·8	584	727	80·3	520	611	85·1	458	496	92·3
30½–34½	498	1022	48·7	631	969	65·1	684	921	74·3	724	876	82·6	737	828	89·0
35½–39½	379	870	43·6	590	854	69·1	626	850	73·6	692	855	80·9	737	853	86·4
40½–44½	360	720	50·0	475	725	65·5	551	736	74·9	607	749	81·0	649	770	84·3
45½–49½	252	534	47·2	373	546	68·3	440	566	77·7	482	593	81·3	493	622	79·3
50½–54½	224	406	55·2	315	426	73·9	342	450	76·0	427	477	89·5	441	509	86·6
55½–59½	156	280	55·7	215	298	72·1	249	322	77·3	285	353	80·7	327	382	85·6
20½–59½	2546	5410	47·1	3474	5068	68·5	3719	4835	76·9	3896	4679	83·3	3936	4557	86·4

Medically examined assured lives, 1924–29. Comparison of actual deaths at durations 0–4 with the expected deaths according to the corresponding mortality at durations 3 and over

Age-group	Duration 0			Duration 1			Duration 2			Duration 3			Duration 4		
	A	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*
20½–24½	77	130	59·2	78	117	66·7	82	110	74·5	113	109	103·7	94	101	93·1
25½–29½	63	129	48·8	92	132	69·7	130	136	95·6	112	150	74·7	165	174	94·8
30½–34½	68	130	52·3	91	139	65·5	128	149	85·9	116	169	68·6	179	197	90·9
35½–39½	86	152	56·6	102	163	62·6	146	176	83·0	143	199	71·9	206	230	89·6
40½–44½	87	181	48·1	121	203	59·6	176	223	78·9	198	255	77·6	228	293	77·8
45½–49½	81	189	42·9	157	209	75·1	172	234	73·5	224	273	82·1	299	320	93·4
50½–54½	135	255	52·9	198	266	74·4	201	278	72·3	243	304	78·9	314	351	89·5
55½–59½	79	194	40·7	156	213	73·2	190	235	80·9	212	261	81·2	251	296	84·8
20½–59½	676	1360	49·7	995	1442	69·0	1225	1541	79·5	1361	1720	79·1	1736	1962	88·5

Medically examined assured lives, 1949–52. Comparison of actual deaths at durations 0–4 with the expected deaths according to the corresponding mortality at durations 3 and over

Age-group	Duration 0			Duration 1			Duration 2			Duration 3			Duration 4		
	A	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*	*A*	*E*	100 *A*/*E*
20½–24½	32	47	68·1	31	39	79·5	34	28	121·4	21	18	116·7	7	12	58·3
25½–29½	46	78	59·0	69	80	86·3	74	79	93·7	70	66	106·1	43	44	97·7
30½–34½	54	84	64·3	62	92	67·4	82	96	85·4	77	88	87·5	51	70	72·9
35½–39½	53	110	48·2	75	123	61·0	114	132	86·4	93	123	75·6	95	100	95·0
40½–44½	63	147	42·9	115	168	68·5	136	183	74·3	144	181	79·6	129	159	81·1
45½–49½	101	190	53·2	149	211	70·6	154	226	68·1	209	227	92·1	198	209	94·7
50½–54½	98	202	48·5	152	226	67·3	164	242	67·8	200	245	82·6	169	234	72·2
55½–59½	68	155	43·9	131	177	74·0	136	202	67·3	173	226	76·5	173	242	71·5
20½–59½	515	1013	50·8	784	1116	70·3	894	1188	75·3	987	1174	84·1	865	1070	80·8