

MORTALITY OF ASSURED LIVES IN SOUTH AFRICA

CONTINUOUS INVESTIGATION 1956-58

[The following is a slightly abbreviated version of a report by the Mortality Standing Committee of the Actuarial Society of South Africa, which has been made available by courtesy of that Society. The report was discussed at meetings of the Society in Johannesburg on 28th November 1960 and in Cape Town on 23rd January 1961; abstracts of the discussions and a written reply thereto on behalf of the Committee have been placed in the Faculty Library.—Editor *T.F.A.*]

(1) Origin and scope of investigation

At the 1956 Annual General Meeting of the Society it was decided to ask life offices to contribute appropriate data to a Continuous Mortality Investigation to be conducted under the auspices of the Society.

Data was called for in respect of ordinary life business, i.e. industrial and funeral business, annuity business, group assurance schemes and deferred annuity pension schemes were to be excluded. The subdivision of the data was basically as follows :

- (i) Territory : Union, Central African Federation, East Africa.
- (ii) Race : Europeans, Africans, Indians and Coloured lives.
- (iii) Medical and Non-medical separately.

Cases not accepted at ordinary rates for the race concerned were to be excluded. Separate returns were to be made in respect of female lives unless the proportion of female lives was small. Offices were asked if possible to make returns both on a sum assured basis and on a number of policies basis. Selection was to be traced for three years.

(2) Response to investigation

The first returns received related to the census date falling in the year 1956. Eleven offices commenced contributing at that stage, one further office commenced in 1957, and one in 1958. The thirteen

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offices represent some 70% of the industry, judged by premium income. It is hoped eventually to cover at least 90% of the industry.

(3) Selection of data for first investigation

Table 1 sets out the totals of the available data in the various subsections.

It was decided to concentrate on the subsection with the largest amount of data, as this was most likely to produce reliable results. Other portions of the experience will be investigated later as time permits, and as the volume of data becomes sufficiently large to produce reliable results.

The subsection of the data chosen for the first investigation was therefore in respect of Europeans in the Union. It was decided to produce a table of graduated ultimate (3 years duration and over) rates of mortality, combining medical and non-medical business, and to use these rates of mortality as a standard table to compare the medical experience with the non-medical experience. Time did not permit of the investigation of the select portion of the data.

It should be noted that while the subsection is described as being in respect of Europeans, some offices found it impracticable entirely to exclude non-European lives. A small proportion of non-European lives is therefore included in the experience. The experience is also preponderantly in respect of male lives, the proportion of female lives being small.

All offices participating in the investigation made returns on a policies basis, whereas five offices did not make a sum assured basis return, and one other office did so for only part of the period. The investigation was therefore made on a policies basis.

(4) Period of investigation and volume of data

Seven offices had made returns in respect of four census dates, i.e. 1956 to 1959 inclusive. Of the other six offices, five made three returns and one made two returns.

A decision had to be made on whether to adopt a period for the investigation such that all offices contributed for the same period. An analysis of the combined experience of all offices for the 1956, 1957 and 1958 census returns separately showed a sufficient degree of correspondence for the whole of the data to be adopted as the basis of the investigation. The experience therefore covers the census dates falling in the calendar years 1956, 1957 and 1958 together with the associated deaths.

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The volume of the data may be judged from the following figures:

| | Exposed to risk | Deaths |
|-------------|--------------------|--------|
| Medical | 786,033·5 | 8,688 |
| Non-medical | 744,253·0 | 2,992 |
| Total | 1,530,286·5 | 11,680 |

The detailed figures are set out in Tables 2-4.

TABLE 1

*Totals of exposed to risk and deaths for various
subsections of the experience*

| | Durations 0-2 years | | Durations 3 years and over | |
|-------------------------------|---------------------|--------|----------------------------|--------|
| | Exposed to risk | Deaths | Exposed to risk | Deaths |
| Europeans : Medical | | | | |
| Union | 162,915 | 425 | 786,034 | 8,688 |
| Central African Federation | 20,800 | 56 | 69,488 | 492 |
| East Africa | 8,062 | 11 | 18,304 | 123 |
| Europeans : Non- medical | | | | |
| Union | 422,890 | 850 | 744,253 | 2,992 |
| Central African Federation | 45,526 | 84 | 52,012 | 168 |
| East Africa | 9,193 | 9 | 5,781 | 7 |
| Coloureds | 9,413 | 18 | 10,543 | 50 |
| Natives | 1,488 | 2 | 11,972 | 47 |
| Indians | 24,091 | 61 | 46,380 | 346 |

(5) Graduation

Ungraduated rates of mortality for half-ages were derived using the "modified policy year" method of Haynes (*J.I.A.*, vol. 69, p. 154). Ultimate rates of mortality for the policy year commencing in financial year N are given by

$$q_{x-\frac{1}{2}} = [\text{Deaths after policy anniversary in year } N \text{ aged } x \text{ nearest birth-} \\ \text{day at end of year } N \text{ plus deaths before policy anniversary in year } \\ (N+1) \text{ aged } x \text{ nearest birthday at end of year } N] \div [\text{In force at}$$

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end of year N aged x nearest birthday *plus* deaths after policy anniversary in year N aged x nearest birthday at end of year N].

The ungraduated rates of mortality (medical and non-medical combined) for durations 3 years and over, so arrived at, are set out in Table 4. The ungraduated values of q_{x-1} for x from 17 to 96 inclusive were used to produce graduated values of q_{x-1} for x from 27 to 86 inclusive, using the Spencer 21-term summation formula. The calculations were done on an electronic computer. The graduated values for x from 77 to 82 inclusive were adjusted by inspection to improve the run of the figures.

The usual tests of graduation were applied over the range from ages $26\frac{1}{2}$ to $85\frac{1}{2}$. The progression of the rates is not ideally smooth, but is certainly smooth enough for practical purposes. Expected deaths over the range are 11,264.8 as against actual deaths of 11,265. The individual deviations seldom exceed twice the standard error of the deviations. The successive deviations change sign frequently as do the accumulated deviations. The graduation was considered satisfactory and was adopted.

TABLE 2

*South African Assured Lives 1956-58 (durations 3 years and over)
Exposed to risk, deaths and observed rates of mortality*

Europeans—Union—Medical

| Nearest age x | Exposed to risk | Deaths | q_x | Nearest age x | Exposed to risk | Deaths | q_x |
|--------------------|--------------------|--------|--------|--------------------|--------------------|--------|---------|
| 15 | 46.75 | — | .00000 | 60 | 10834.5 | 212 | .01957 |
| 16 | 82.75 | — | .00000 | 61 | 9782.75 | 184 | .01881 |
| 17 | 157.75 | 1 | .00634 | 62 | 9007 | 232 | .02576 |
| 18 | 482.75 | 2 | .00414 | 63 | 8493.75 | 228 | .02684 |
| 19 | 710.75 | 2 | .00281 | 64 | 7969.5 | 180 | .02259 |
| 20 | 1132.25 | 4 | .00353 | 65 | 7220 | 187 | .02590 |
| 21 | 2052.5 | 2 | .00097 | 66 | 6734.5 | 222 | .03296 |
| 22 | 3069.5 | 4 | .00130 | 67 | 6300.25 | 264 | .04190 |
| 23 | 4039.5 | 8 | .00198 | 68 | 5875.5 | 231 | .03932 |
| 24 | 5032 | 5 | .00099 | 69 | 5482.5 | 251 | .04578 |
| 25 | 6304.5 | 5 | .00079 | 70 | 5010 | 199 | .03972 |
| 26 | 7955.25 | 9 | .00113 | 71 | 4357.25 | 190 | .04361 |
| 27 | 9749.25 | 8 | .00082 | 72 | 3874.75 | 217 | .05600 |
| 28 | 11355.25 | 7 | .00062 | 73 | 3656.5 | 190 | .05196 |
| 29 | 12511.25 | 20 | .00160 | 74 | 3507.75 | 192 | .05474 |
| 30 | 13733.25 | 16 | .00117 | 75 | 3341.5 | 219 | .06285 |
| 31 | 14921.75 | 25 | .00168 | 76 | 3050.5 | 266 | .08720 |
| 32 | 16082.75 | 32 | .00199 | 77 | 2676 | 229 | .08558 |
| 33 | 16993.25 | 24 | .00141 | 78 | 2345.75 | 192 | .08185 |
| 34 | 17785.25 | 21 | .00118 | 79 | 1941.5 | 185 | .09529 |
| 35 | 19041.25 | 36 | .00189 | 80 | 1649.5 | 155 | .09448 |
| 36 | 20111 | 45 | .00224 | 81 | 1394 | 168 | .12052 |
| 37 | 20613.25 | 49 | .00238 | 82 | 1145.75 | 131 | .11434 |
| 38 | 20612 | 50 | .00243 | 83 | 886.75 | 119 | .13120 |
| 39 | 20712.25 | 53 | .00256 | 84 | 656.5 | 74 | .11272 |
| 40 | 21242.5 | 50 | .00235 | 85 | 533.25 | 60 | .11252 |
| 41 | 21844.25 | 81 | .00371 | 86 | 409.75 | 71 | .17328 |
| 42 | 22598.75 | 88 | .00389 | 87 | 301.75 | 49 | .16239 |
| 43 | 23573 | 73 | .00310 | 88 | 211.5 | 31 | .14657 |
| 44 | 24593.5 | 114 | .00464 | 89 | 166 | 33 | .19880 |
| 45 | 24741.5 | 94 | .00380 | 90 | 122.5 | 12 | .09796 |
| 46 | 24973 | 113 | .00452 | 91 | 102 | 23 | .22519 |
| 47 | 25097.75 | 147 | .00586 | 92 | 55.5 | 13 | .23123 |
| 48 | 25059.5 | 171 | .00682 | 93 | 40 | 12 | .30000 |
| 49 | 24986 | 177 | .00708 | 94 | 19.5 | 3 | .15385 |
| 50 | 24150.25 | 174 | .00720 | 95 | 14.75 | 3 | .20339 |
| 51 | 23718.5 | 186 | .00784 | 96 | 14.5 | 6 | .41379 |
| 52 | 23137 | 226 | .00977 | 97 | 8 | 1 | .12590 |
| 53 | 21579 | 205 | .00950 | 98 | 6 | — | .00000 |
| 54 | 19435 | 180 | .00926 | 99 | 2 | 2 | 1.00000 |
| 55 | 16093.75 | 191 | .01187 | 100 and over | 3 | — | .00000 |
| 56 | 14348.25 | 194 | .01352 | | | | |
| 57 | 14013.75 | 157 | .01120 | | | | |
| 58 | 13624.25 | 199 | .01461 | | 786,033.5 | 8,688 | |
| 59 | 12740.75 | 213 | .01672 | | | | |

TABLE 3

South African Assured Lives 1956-58 (durations 3 years and over)
Exposed to risk, deaths and observed rates of mortality
 Europeans—Union—Non-medical

| Nearest age x | Exposed to risk | Deaths | $q_{x-\frac{1}{2}}$ | Nearest age x | Exposed to risk | Deaths | $q_{x-\frac{1}{2}}$ |
|-----------------|-----------------|--------|---------------------|-----------------|-----------------|--------|---------------------|
| 15 | 162.5 | — | .00000 | 60 | 2238.25 | 38 | .01699 |
| 16 | 272.5 | — | .00000 | 61 | 1963 | 46 | .02343 |
| 17 | 470.25 | — | .00000 | 62 | 1732.5 | 41 | .02367 |
| 18 | 2000 | 5 | .00250 | 63 | 1552.75 | 41 | .02640 |
| 19 | 3052.75 | 5 | .00164 | 64 | 1302.25 | 37 | .02841 |
| 20 | 5182 | 10 | .00193 | 65 | 1091.75 | 27 | .02473 |
| 21 | 8766.25 | 18 | .00205 | 66 | 920.25 | 30 | .03228 |
| 22 | 12521 | 28 | .00223 | 67 | 842.25 | 31 | .03681 |
| 23 | 15077 | 21 | .00139 | 68 | 701.25 | 42 | .05989 |
| 24 | 17490.5 | 32 | .00183 | 69 | 550 | 26 | .04727 |
| 25 | 19958.25 | 32 | .00160 | 70 | 425.25 | 26 | .06114 |
| 26 | 22460.25 | 34 | .00151 | 71 | 330.25 | 22 | .06662 |
| 27 | 24524.5 | 27 | .00110 | 72 | 279.75 | 12 | .04290 |
| 28 | 25754.75 | 35 | .00136 | 73 | 214.5 | 17 | .07925 |
| 29 | 25972 | 30 | .00116 | 74 | 147.5 | 14 | .09492 |
| 30 | 25854.25 | 29 | .00112 | 75 | 92 | 9 | .09783 |
| 31 | 25647.25 | 39 | .00152 | 76 | 75 | 8 | .10667 |
| 32 | 25560.5 | 34 | .00133 | 77 | 60.25 | 2 | .03320 |
| 33 | 25420.5 | 46 | .00181 | 78 | 39 | 5 | .12821 |
| 34 | 25312.75 | 32 | .00126 | 79 | 20.75 | 1 | .04819 |
| 35 | 25526 | 48 | .00188 | 80 | 9.5 | 4 | .42105 |
| 36 | 25904.75 | 59 | .00228 | 81 | 3 | — | .00000 |
| 37 | 25485.5 | 69 | .00271 | 82 | 1.75 | — | .00000 |
| 38 | 24748.25 | 60 | .00242 | 83 | 4.25 | 2 | .47059 |
| 39 | 24006.5 | 51 | .00212 | 84 | 3 | 1 | .33333 |
| 40 | 23792.75 | 69 | .00290 | 85 | 2 | — | .00000 |
| 41 | 23158.25 | 58 | .00250 | 86 | 1 | — | .00000 |
| 42 | 22805.5 | 86 | .00377 | 87 | 1 | — | .00000 |
| 43 | 22745.5 | 86 | .00378 | 88 | — | — | — |
| 44 | 22619.75 | 85 | .00376 | 89 | — | — | — |
| 45 | 21335.25 | 104 | .00487 | 90 | — | — | — |
| 46 | 20418.25 | 93 | .00455 | 91 | — | — | — |
| 47 | 19676.25 | 109 | .00554 | 92 | — | — | — |
| 48 | 18550.25 | 109 | .00588 | 93 | 1 | — | .00006 |
| 49 | 17033.75 | 136 | .00797 | 94 | — | — | — |
| 50 | 15011.75 | 126 | .00839 | 95 | — | — | — |
| 51 | 13469.75 | 114 | .00846 | 96 | — | — | — |
| 52 | 12128.25 | 96 | .00791 | 97 | — | — | — |
| 53 | 10422.75 | 92 | .00883 | 98 | — | — | — |
| 54 | 8236.75 | 94 | .01141 | 99 | — | — | — |
| 55 | 5814.75 | 75 | .01290 | 100 | 1 | — | .00000 |
| 56 | 4689.5 | 66 | .01407 | and over | 744,253.0 | 2,992 | |
| 57 | 4034.75 | 59 | .01462 | | | | |
| 58 | 3494.25 | 59 | .01688 | | | | |
| 59 | 3039.25 | 50 | .01645 | | | | |

TABLE 4

South African Assured Lives 1956-58 (durations 3 years and over)
Exposed to risk, deaths and observed rates of mortality
 Europeans—Union—Medical and Non-medical combined

| Nearest age x | Exposed to risk | Deaths | $q_{x-\frac{1}{2}}$ | Nearest age x | Exposed to risk | Deaths | $q_{x-\frac{1}{2}}$ |
|-----------------|-----------------|--------|---------------------|-----------------|-----------------|--------|---------------------|
| 15 | 209·25 | — | ·00000 | 60 | 13072·75 | 250 | ·01912 |
| 16 | 355·25 | — | ·00000 | 61 | 11745·75 | 230 | ·01958 |
| 17 | 628 | 1 | ·00159 | 62 | 10739·5 | 273 | ·02542 |
| 18 | 2482·75 | 7 | ·00282 | 63 | 10046·5 | 269 | ·02678 |
| 19 | 3763·5 | 7 | ·00186 | 64 | 9271·75 | 217 | ·02340 |
| 20 | 6314·25 | 14 | ·00222 | 65 | 8311·75 | 214 | ·02575 |
| 21 | 10818·75 | 20 | ·00185 | 66 | 7663·75 | 252 | ·03288 |
| 22 | 15590·5 | 32 | ·00205 | 67 | 7142·5 | 295 | ·04130 |
| 23 | 19116·5 | 29 | ·00152 | 68 | 6576·75 | 273 | ·04151 |
| 24 | 22522·5 | 37 | ·00164 | 69 | 6032·5 | 277 | ·04592 |
| 25 | 26262·75 | 37 | ·00141 | 70 | 5435·25 | 225 | ·04140 |
| 26 | 30415·5 | 43 | ·00141 | 71 | 4687·5 | 212 | ·04523 |
| 27 | 34273·75 | 35 | ·00102 | 72 | 4154·5 | 229 | ·05512 |
| 28 | 37110 | 42 | ·00113 | 73 | 3871 | 207 | ·05347 |
| 29 | 38483·25 | 50 | ·00130 | 74 | 3655·25 | 206 | ·05636 |
| 30 | 39587·5 | 45 | ·00114 | 75 | 3433·5 | 219 | ·06378 |
| 31 | 40569 | 64 | ·00158 | 76 | 3125·5 | 274 | ·08767 |
| 32 | 41643·25 | 66 | ·00158 | 77 | 2736·25 | 231 | ·08442 |
| 33 | 42413·75 | 70 | ·00165 | 78 | 2384·75 | 197 | ·08261 |
| 34 | 43098 | 53 | ·00123 | 79 | 1962·25 | 186 | ·09479 |
| 35 | 44567·25 | 84 | ·00188 | 80 | 1650 | 159 | ·09636 |
| 36 | 46015·75 | 104 | ·00226 | 81 | 1397 | 168 | ·10206 |
| 37 | 46098·75 | 118 | ·00256 | 82 | 1147·5 | 131 | ·11416 |
| 38 | 45360·25 | 110 | ·00243 | 83 | 891 | 121 | ·13580 |
| 39 | 44718·75 | 104 | ·00233 | 84 | 659·5 | 75 | ·11372 |
| 40 | 45035·25 | 119 | ·00264 | 85 | 535·25 | 60 | ·11210 |
| 41 | 45002·5 | 139 | ·00309 | 86 | 410·75 | 71 | ·17285 |
| 42 | 45404·25 | 174 | ·00383 | 87 | 302·75 | 49 | ·16185 |
| 43 | 46318·5 | 159 | ·00343 | 88 | 211·5 | 31 | ·14657 |
| 44 | 47213·25 | 199 | ·00421 | 89 | 166 | 33 | ·19880 |
| 45 | 46076·75 | 198 | ·00430 | 90 | 122·5 | 12 | ·09796 |
| 46 | 45391·25 | 206 | ·00454 | 91 | 102 | 23 | ·22549 |
| 47 | 44774 | 256 | ·00572 | 92 | 55·5 | 13 | ·23423 |
| 48 | 43609·75 | 280 | ·00642 | 93 | 41 | 12 | ·29268 |
| 49 | 42049·75 | 313 | ·00744 | 94 | 19·5 | 3 | ·15385 |
| 50 | 39162 | 300 | ·00766 | 95 | 14·75 | 3 | ·20339 |
| 51 | 37188·25 | 300 | ·00807 | 96 | 14·5 | 6 | ·41379 |
| 52 | 35265·25 | 322 | ·00913 | 97 | 8 | 1 | ·12500 |
| 53 | 32001·75 | 297 | ·00928 | 98 | 6 | — | ·00000 |
| 54 | 27671·75 | 274 | ·00990 | 99 | 2 | 2 | 1·00000 |
| 55 | 21908·5 | 266 | ·01214 | 100 and over | 4 | — | ·00000 |
| 56 | 19037·75 | 260 | ·01366 | | | | |
| 57 | 18048·5 | 216 | ·01197 | | | | |
| 58 | 17118·5 | 258 | ·01507 | | 1,530,286·5 | 11,680 | |
| 59 | 15780 | 263 | ·01667 | | | | |

TABLE 5

South African Assured Lives 1956-58 (durations 3 years and over)
Graduated rates of mortality and comparison of actual deaths with
expected deaths computed from graduated rates of mortality

European—Union—Medical and Non-medical combined

| Nearest age x | Graduated $q_{x-\frac{1}{2}}$ | Actual deaths | Expected deaths | Actual—Expected deaths | | Accumulated deviation | | Standard error |
|--------------------|----------------------------------|------------------|--------------------|---------------------------|------|--------------------------|------|-------------------|
| | | | | + | — | + | — | |
| 15 | ·00140 | — | ·3 | — | ·3 | — | ·3 | ·5 |
| 16 | ·00162 | — | ·6 | — | ·6 | — | ·9 | ·8 |
| 17 | ·00192 | 1 | 1·2 | — | ·2 | — | 1·1 | 1·1 |
| 18 | ·00207 | 7 | 5·1 | 1·9 | — | ·8 | — | 2·3 |
| 19 | ·00210 | 7 | 7·9 | — | ·9 | — | ·1 | 2·8 |
| 20 | ·00206 | 14 | 13·0 | 1·0 | — | ·9 | — | 3·6 |
| 21 | ·00198 | 20 | 21·4 | — | 1·4 | — | ·5 | 4·6 |
| 22 | ·00186 | 32 | 29·0 | 3·0 | — | 2·5 | — | 5·4 |
| 23 | ·00173 | 29 | 33·1 | — | 4·1 | — | 1·6 | 5·8 |
| 24 | ·00160 | 37 | 36·0 | 1·0 | — | — | ·6 | 6·0 |
| 25 | ·00146 | 37 | 38·3 | — | 1·3 | — | 1·9 | 6·2 |
| 26 | ·00135 | 43 | 41·1 | 1·9 | — | — | — | 6·4 |
| 27 | ·00126 | 35 | 43·2 | — | 8·2 | — | 8·2 | 6·6 |
| 28 | ·00123 | 42 | 45·6 | — | 3·6 | — | 11·8 | 6·8 |
| 29 | ·00123 | 50 | 47·3 | 2·7 | — | — | 9·1 | 6·9 |
| 30 | ·00128 | 45 | 50·7 | — | 5·7 | — | 14·8 | 7·1 |
| 31 | ·00135 | 64 | 54·8 | 9·2 | — | — | 5·6 | 7·4 |
| 32 | ·00146 | 66 | 60·8 | 5·2 | — | — | ·4 | 7·8 |
| 33 | ·00158 | 70 | 67·0 | 3·0 | — | 2·6 | — | 8·2 |
| 34 | ·00172 | 53 | 74·1 | — | 21·1 | — | 18·5 | 8·6 |
| 35 | ·00188 | 84 | 83·8 | ·2 | — | — | 18·3 | 9·2 |
| 36 | ·00204 | 104 | 93·9 | 10·1 | — | — | 8·2 | 9·7 |
| 37 | ·00222 | 118 | 102·3 | 15·7 | — | 7·5 | — | 10·1 |
| 38 | ·00241 | 110 | 109·3 | ·7 | — | 8·2 | — | 10·5 |
| 39 | ·00261 | 104 | 116·7 | — | 12·7 | — | 4·5 | 10·8 |
| 40 | ·00282 | 119 | 127·0 | — | 8·0 | — | 12·5 | 11·3 |
| 41 | ·00306 | 139 | 137·7 | 1·3 | — | — | 11·2 | 11·7 |
| 42 | ·00334 | 174 | 151·7 | 22·3 | — | 11·1 | — | 12·3 |
| 43 | ·00367 | 159 | 170·0 | — | 11·0 | ·1 | — | 13·0 |
| 44 | ·00407 | 199 | 192·2 | 6·8 | — | 6·9 | — | 13·9 |
| 45 | ·00454 | 198 | 209·2 | — | 11·2 | — | 4·3 | 14·5 |
| 46 | ·00506 | 206 | 229·7 | — | 23·7 | — | 28·0 | 15·2 |
| 47 | ·00564 | 256 | 252·5 | 3·5 | — | — | 24·5 | 15·9 |
| 48 | ·00625 | 280 | 272·6 | 7·4 | — | — | 17·1 | 16·5 |
| 49 | ·00689 | 313 | 289·7 | 23·3 | — | 6·2 | — | 17·0 |
| 50 | ·00754 | 300 | 295·3 | 4·7 | — | 10·9 | — | 17·2 |
| 51 | ·00822 | 300 | 305·7 | — | 5·7 | 5·2 | — | 17·5 |
| 52 | ·00891 | 322 | 314·2 | 7·8 | — | 13·0 | — | 17·7 |
| 53 | ·00963 | 297 | 308·2 | — | 11·2 | 1·8 | — | 17·6 |
| 54 | ·01044 | 274 | 288·9 | — | 14·9 | — | 13·1 | 17·0 |

TABLE 5 (contd.)

| Nearest age x | Graduated q_{x-1} | Actual deaths | Expected deaths | Actual—Expected deaths | | Accumulated deviation | | Standard error |
|--------------------|------------------------|------------------|--------------------|---------------------------|-------|--------------------------|------|-------------------|
| | | | | + | — | + | — | |
| 55 | ·01139 | 266 | 249·5 | 16·5 | — | 3·4 | — | 15·8 |
| 56 | ·01251 | 260 | 238·2 | 21·8 | — | 25·2 | — | 15·4 |
| 57 | ·01382 | 216 | 249·4 | — | 33·4 | — | 8·2 | 15·8 |
| 58 | ·01527 | 258 | 261·4 | — | 3·4 | — | 11·6 | 16·2 |
| 59 | ·01684 | 263 | 265·7 | — | 2·7 | — | 14·3 | 16·3 |
| 60 | ·01853 | 250 | 242·2 | 7·8 | — | — | 6·5 | 15·5 |
| 61 | ·02040 | 230 | 239·6 | — | 9·6 | — | 16·1 | 15·4 |
| 62 | ·02247 | 273 | 241·3 | 31·7 | — | 15·6 | — | 15·5 |
| 63 | ·02481 | 269 | 249·3 | 19·7 | — | 35·3 | — | 15·8 |
| 64 | ·02745 | 217 | 254·5 | — | 37·5 | — | 2·2 | 16·0 |
| 65 | ·03033 | 214 | 252·1 | — | 38·1 | — | 40·3 | 15·9 |
| 66 | ·03329 | 252 | 255·1 | — | 3·1 | — | 43·4 | 16·0 |
| 67 | ·03621 | 295 | 258·6 | 36·4 | — | — | 7·0 | 16·1 |
| 68 | ·03907 | 273 | 257·0 | 16·0 | — | 9·0 | — | 16·0 |
| 69 | ·04192 | 277 | 252·9 | 24·1 | — | 33·1 | — | 15·9 |
| 70 | ·04490 | 225 | 244·0 | — | 19·0 | 14·1 | — | 15·6 |
| 71 | ·04819 | 212 | 225·9 | — | 13·9 | ·2 | — | 15·0 |
| 72 | ·05204 | 229 | 216·2 | 12·8 | — | 13·0 | — | 14·7 |
| 73 | ·05661 | 207 | 219·1 | — | 12·1 | ·9 | — | 14·8 |
| 74 | ·06197 | 206 | 226·5 | — | 20·5 | — | 19·6 | 15·0 |
| 75 | ·06810 | 219 | 233·8 | — | 14·8 | — | 34·4 | 15·3 |
| 76 | ·07483 | 274 | 233·9 | 40·1 | — | 5·7 | — | 15·3 |
| 77 | ·08191 | 231 | 224·1 | 6·9 | — | 12·6 | — | 15·0 |
| 78 | ·08904 | 197 | 212·3 | — | 15·3 | — | 2·7 | 14·6 |
| 79 | ·09592 | 186 | 188·2 | — | 2·2 | — | 4·9 | 13·7 |
| 80 | ·10263 | 159 | 169·3 | — | 10·3 | — | 15·2 | 13·0 |
| 81 | ·10920 | 168 | 152·6 | 15·4 | — | ·2 | — | 12·4 |
| 82 | ·11572 | 131 | 132·8 | — | 1·8 | — | 1·6 | 11·5 |
| 83 | ·12222 | 121 | 108·9 | 12·1 | — | 10·5 | — | 10·4 |
| 84 | ·12882 | 75 | 85·0 | — | 10·0 | ·5 | — | 9·2 |
| 85 | ·13560 | 60 | 72·6 | — | 12·6 | — | 12·1 | 8·5 |
| 86 | ·14288 | 71 | 58·7 | 12·3 | — | ·2 | — | 7·7 |
| 87 | ·15102 | 49 | 45·7 | 3·3 | — | 3·5 | — | 6·8 |
| 88 | ·16004 | 31 | 33·8 | — | 2·8 | ·7 | — | 5·8 |
| 89 | ·16994 | 33 | 28·2 | 4·8 | — | 5·5 | — | 5·3 |
| 90 | ·18072 | 12 | 22·1 | — | 10·1 | — | 4·6 | 4·7 |
| 91 | ·19238 | 23 | 19·6 | 3·4 | — | — | 1·2 | 4·4 |
| 92 | ·20492 | 13 | 11·4 | 1·6 | — | ·4 | — | 3·4 |
| 93 | ·21834 | 12 | 9·0 | 3·0 | — | 3·4 | — | 3·0 |
| 94 | ·23264 | 3 | 4·5 | — | 1·5 | 1·9 | — | 2·1 |
| 95 | ·24782 | 3 | 3·7 | — | ·7 | 1·2 | — | 1·9 |
| 96 | ·26388 | 6 | 3·8 | 2·2 | — | 3·4 | — | 1·9 |
| 97 | ·28082 | 1 | 2·2 | — | 1·2 | 2·2 | — | 1·5 |
| 98 | ·29864 | — | 1·8 | — | 1·8 | ·4 | — | 1·3 |
| 99 | ·31734 | 2 | ·6 | 1·4 | — | 1·8 | — | ·8 |
| 100 | ·33692 | — | ·3 | — | ·3 | 1·5 | — | ·5 |
| 101 | ·35738 | — | ·7 | — | ·7 | ·8 | — | ·8 |
| 102 | ·37872 | — | ·4 | — | ·4 | ·4 | — | ·6 |
| | | 11,680 | 11,679·6 | 426·0 | 425·6 | — | — | 887·6 |

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The remaining graduation problems concerned the completion of the two ends of the curve. At the young ages the data shows a "hump" in the curve round about ages 18-22, presumably due to accidental deaths. There is a corresponding "hollow" about age 28. It was decided to complete this portion of the curve by graphic graduation, and to preserve the features shown by the crude rates of mortality. The usual tests of graduation were applied, and showed the graphic graduation to be satisfactory.

At the high ages an attempt at a graphic graduation showed that a second degree polynomial would be appropriate. The formula

$$q_{85\frac{1}{2}+h} = q_{85\frac{1}{2}} + \cdot 0077h + \cdot 00044h^2$$

was adopted after it had been established that it would fit the data satisfactorily.

Table 5 shows the graduated rates of mortality for ages $14\frac{1}{2}$ to $101\frac{1}{2}$, together with the actual and expected deaths, deviations, accumulated deviations and the standard errors of the deviations. The standard error has been taken as the square root of the expected deaths.

(6) *Comparison with standard tables*

For the purposes of identification the mortality table arrived at in this investigation is designated the S.A.56/58 Ultimate Table.

The following table compares ultimate rates of mortality from the S.A.56/58 Ultimate Table (see Table 6) with those from various standard tables. The figures in parentheses are the S.A.56/58 rates of mortality as a percentage of the standard table rates of mortality.

q_x *Ultimate*

| Age | S.A.56/58 | A1924-29 | A1924-29 Light | A1949-52 | C.S.O.58 Basic |
|-----|-----------|--------------|----------------|--------------|----------------|
| 15 | ·00150 | ·00206 (73) | ·00164 (91) | ·00111 (135) | ·00056 (268) |
| 20 | ·00203 | ·00235 (86) | ·00172 (118) | ·00111 (183) | ·00084 (242) |
| 25 | ·00140 | ·00235 (60) | ·00190 (74) | ·00112 (125) | ·00093 (151) |
| 30 | ·00131 | ·00241 (54) | ·00207 (63) | ·00116 (113) | ·00108 (121) |
| 35 | ·00196 | ·00286 (69) | ·00231 (85) | ·00132 (148) | ·00141 (139) |
| 40 | ·00294 | ·00388 (76) | ·00310 (95) | ·00188 (156) | ·00236 (125) |
| 45 | ·00479 | ·00527 (91) | ·00426 (112) | ·00330 (145) | ·00403 (119) |
| 50 | ·00788 | ·00764 (103) | ·00635 (124) | ·00599 (132) | ·00671 (117) |
| 55 | ·01193 | ·01190 (100) | ·00983 (121) | ·01035 (115) | ·01093 (109) |
| 60 | ·01944 | ·01973 (99) | ·01574 (124) | ·01720 (113) | ·01756 (111) |
| 65 | ·03182 | ·03188 (100) | ·02662 (120) | ·02810 (113) | ·02761 (115) |
| 70 | ·04648 | ·05327 (87) | ·04666 (100) | ·04543 (102) | ·04330 (107) |
| 75 | ·07142 | ·08497 (84) | ·07856 (91) | ·07257 (98) | ·06380 (112) |
| 80 | ·10592 | ·12910 (82) | ·12019 (88) | ·11369 (93) | ·09564 (111) |
| 85 | ·13913 | ·18676 (74) | ·17254 (81) | ·17282 (81) | ·14012 (99) |
| 90 | ·18644 | ·25611 (73) | ·24164 (77) | ·25168 (74) | ·19838 (94) |

N.B. The C.S.O.58 Basic Table is a recent American table that is stated to represent the average company ultimate mortality on standard ordinary business for the period 1950 to 1954.

TABLE 6

S.A.56/58 Ultimate Table (durations 3 years and over)

Europeans—Union—Medical and Non-medical combined

| x | q_x | l_x | e_x | x | q_x | l_x | e_x |
|-----|--------|-----------|--------|-----|--------|-----------|--------|
| 15 | -00150 | 9,999,999 | 55-403 | 60 | -01944 | 8,084,393 | 16-050 |
| 16 | -00179 | 9,984,999 | 54-486 | 61 | -02140 | 7,927,232 | 15-368 |
| 17 | -00201 | 9,967,126 | 53-584 | 62 | -02360 | 7,757,590 | 14-705 |
| 18 | -00209 | 9,947,092 | 52-692 | 63 | -02610 | 7,574,511 | 14-060 |
| 19 | -00209 | 9,926,303 | 51-802 | 64 | -02888 | 7,376,816 | 13-437 |
| 20 | -00203 | 9,905,557 | 50-911 | 65 | -03182 | 7,163,773 | 12-836 |
| 21 | -00192 | 9,885,448 | 50-014 | 66 | -03476 | 6,935,822 | 12-258 |
| 22 | -00180 | 9,866,468 | 49-111 | 67 | -03764 | 6,694,733 | 11-700 |
| 23 | -00167 | 9,848,709 | 48-199 | 68 | -04048 | 6,442,743 | 11-157 |
| 24 | -00153 | 9,832,261 | 47-280 | 69 | -04337 | 6,181,941 | 10-628 |
| 25 | -00140 | 9,817,218 | 46-352 | 70 | -04648 | 5,913,830 | 10-110 |
| 26 | -00130 | 9,803,474 | 45-417 | 71 | -05003 | 5,638,955 | 9-603 |
| 27 | -00124 | 9,790,729 | 44-476 | 72 | -05423 | 5,356,838 | 9-108 |
| 28 | -00122 | 9,778,589 | 43-532 | 73 | -05919 | 5,066,337 | 8-631 |
| 29 | -00125 | 9,766,659 | 42-585 | 74 | -06496 | 4,766,461 | 8-174 |
| 30 | -00131 | 9,754,451 | 41-638 | 75 | -07142 | 4,456,831 | 7-741 |
| 31 | -00140 | 9,741,672 | 40-693 | 76 | -07836 | 4,138,524 | 7-337 |
| 32 | -00152 | 9,728,034 | 39-750 | 77 | -08551 | 3,814,230 | 6-961 |
| 33 | -00165 | 9,713,247 | 38-810 | 78 | -09250 | 3,488,075 | 6-612 |
| 34 | -00180 | 9,697,220 | 37-874 | 79 | -09929 | 3,165,428 | 6-285 |
| 35 | -00196 | 9,679,765 | 36-943 | 80 | -10592 | 2,851,133 | 5-978 |
| 36 | -00213 | 9,660,793 | 36-015 | 81 | -11246 | 2,549,141 | 5-687 |
| 37 | -00231 | 9,640,216 | 35-092 | 82 | -11896 | 2,262,464 | 5-407 |
| 38 | -00251 | 9,617,947 | 34-173 | 83 | -12550 | 1,993,322 | 5-137 |
| 39 | -00271 | 9,593,806 | 33-259 | 84 | -13215 | 1,743,160 | 4-874 |
| 40 | -00294 | 9,567,806 | 32-350 | 85 | -13913 | 1,512,801 | 4-617 |
| 41 | -00319 | 9,539,677 | 31-445 | 86 | -14684 | 1,302,325 | 4-363 |
| 42 | -00350 | 9,509,245 | 30-546 | 87 | -15542 | 1,111,092 | 4-114 |
| 43 | -00386 | 9,475,963 | 29-653 | 88 | -16488 | 938,406 | 3-871 |
| 44 | -00430 | 9,439,386 | 28-768 | 89 | -17522 | 783,681 | 3-635 |
| 45 | -00479 | 9,398,797 | 27-892 | 90 | -18644 | 646,365 | 3-407 |
| 46 | -00535 | 9,353,776 | 27-026 | 91 | -19854 | 525,857 | 3-188 |
| 47 | -00594 | 9,303,734 | 26-172 | 92 | -21152 | 421,453 | 2-977 |
| 48 | -00657 | 9,248,469 | 25-328 | 93 | -22538 | 332,307 | 2-776 |
| 49 | -00721 | 9,187,707 | 24-496 | 94 | -24012 | 257,412 | 2-584 |
| 50 | -00788 | 9,121,464 | 23-674 | 95 | -25574 | 195,602 | 2-400 |
| 51 | -00856 | 9,049,586 | 22-862 | 96 | -27224 | 145,579 | 2-225 |
| 52 | -00926 | 8,972,122 | 22-059 | 97 | -28962 | 105,946 | 2-058 |
| 53 | -01002 | 8,889,040 | 21-265 | 98 | -30788 | 75,262 | 1-896 |
| 54 | -01089 | 8,799,972 | 20-480 | 99 | -32702 | 52,090 | 1-740 |
| 55 | -01193 | 8,704,140 | 19-706 | 100 | -34704 | 35,056 | 1-586 |
| 56 | -01315 | 8,600,300 | 18-944 | 101 | -36794 | 22,890 | 1-428 |
| 57 | -01453 | 8,487,206 | 18-196 | 102 | -38972 | 14,468 | 1-260 |
| 58 | -01604 | 8,363,887 | 17-464 | 103 | -40000 | 8,830 | 1-064 |
| 59 | -01766 | 8,229,730 | 16-749 | 104 | -50000 | 5,298 | -773 |

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In considering the results of the experience an assessment needs to be made of the degree of reliance that can be placed on the results. The A1924-29 and A1949-52 investigations each had about seven times the volume of data of the S.A.56/58 investigation. On the other hand, an exposed-to-risk of some $1\frac{1}{2}$ million policy years is a very considerable amount of data, and bearing in mind that the general level of mortality for the three years taken separately was very similar, it is felt that the table produced gives a good measure of current ultimate assured lives mortality for European male lives in the Union.

The general shape of the mortality curve for the Union is different from the four standard tables with which a comparison is made, although there is a marked similarity (but a different level) over certain wide ranges of ages. For example, compare the mortality rates at ages 50-69 with A1924-29 Light, or at ages 50-84 with C.S.O. 58.

As regards the level of mortality, the S.A. table is generally lower than A1924-29, lower than A1924-29 Light except at the important middle ages, lower than A1949-52 only from the mid 70's, and lower than C.S.O. 58 only at the very high ages.

At the younger ages the S.A. table is perhaps surprisingly in excess of the modern British and American experience. It is estimated that accidental deaths account for more than 40% of the total deaths below age 40 amongst European assured lives in the Union. Comparable figures for Britain and America would establish whether or not the Union's high accident rate is the main reason for the relatively less favourable mortality at the younger ages.

It has been said that the Union is second only to the U.S.A. in the incidence of coronary heart disease amongst males. The high incidence of deaths from this type of disease in the Union may well explain the relationship between the S.A. table and A1949-52 from the middle ages up to the mid 70's, but if the U.S.A. has a still higher incidence than the Union, the relatively lower American experience at these ages is somewhat puzzling.

At the high ages the S.A.56/58 experience is good. The question arises whether this is genuine or whether the inclusion of numbers of small paid-up policies where the assured lives have died, but where the policies remain on the books because the deaths have not been notified, has artificially lowered the experience rates of mortality. To guard against this possibility offices were asked to exclude all policies where the sum assured was less than £100, and this was done in respect of at least 60% of the experience. The light experience is

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therefore more likely to be genuine than not, and on general grounds there are two reasons for expecting light mortality at high ages in the Union. First, the climate is more favourable to survival at the older ages than is the case in say Britain. Secondly, the heavier death-rates at the earlier ages can lead to a relative improvement at the older ages on the principle of "the survival of the fittest".

(7) Medical and non-medical experience

The following table sets out a comparison of the medical and non-medical sections of the experience. The expected deaths have been calculated using the graduated values of $q_{x-\frac{1}{2}}$ derived from the combined experience.

| Age-group | Medical | | | Non-medical | | | Medical Non-medical |
|--------------|---------|--------|---------|-------------|--------|---------|------------------------|
| | A | E | 100 A/E | A | E | 100 A/E | |
| 15-19 | 5 | 3.0 | 166.7 | 10 | 12.0 | 83.3 | % 200.1 |
| 20-24 | 23 | 27.2 | 84.6 | 109 | 105.5 | 103.3 | 81.9 |
| 25-29 | 49 | 61.6 | 79.5 | 158 | 153.9 | 102.7 | 77.4 |
| 30-34 | 118 | 118.6 | 99.5 | 180 | 188.7 | 95.4 | 104.3 |
| 35-39 | 233 | 226.4 | 102.9 | 287 | 279.7 | 102.6 | 100.3 |
| 40-44 | 406 | 388.8 | 104.4 | 384 | 389.8 | 98.5 | 106.0 |
| 45-49 | 702 | 709.1 | 99.0 | 551 | 544.7 | 101.2 | 97.8 |
| 50-54 | 971 | 994.0 | 97.7 | 522 | 518.4 | 100.7 | 97.0 |
| 55-59 | 954 | 979.1 | 97.4 | 309 | 285.3 | 108.3 | 89.9 |
| 60-64 | 1036 | 1032.3 | 100.4 | 203 | 194.6 | 104.3 | 96.3 |
| 65-69 | 1155 | 1130.7 | 102.1 | 156 | 145.0 | 107.6 | 94.9 |
| 70-74 | 988 | 1060.9 | 93.1 | 91 | 70.8 | 128.5 | 72.5 |
| 75-79 | 1082 | 1070.2 | 101.1 | 25 | 22.3 | 112.1 | 90.2 |
| 80-84 | 647 | 646.2 | 100.1 | 7 | 2.4 | 291.7 | 34.3 |
| 85-89 | 244 | 238.4 | 102.3 | — | .6 | — | — |
| 90-94 | 63 | 66.3 | 95.0 | — | .2 | — | — |
| 95-99 | 12 | 12.1 | 99.2 | — | — | — | — |
| 100 and over | — | 1.1 | — | — | .4 | — | — |
| Total | 8688 | 8766.0 | 99.1 | 2992 | 2914.3 | 102.7 | 96.5 |

The correspondence between medical and non-medical mortality is somewhat closer than was the case with the A1949-52 experience (durations 3 years and over), where medical mortality was about 94% of non-medical mortality.

The Committee responsible for the A1949-52 investigation pointed out that the aggregate difference between medical and non-medical mortality was largely fortuitous, as the following quotation from their report shows :

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When comparisons are made between the medical and non-medical business of individual offices the margin between the mortality experienced by these two classes of business greatly diminishes. It seems, in fact, that a high proportion of the total non-medical business is transacted by offices whose mortality experience is heavier than average in all classes. Consequently, one reason why the non-medical experience is heavier than the medical experience is that it contains a higher proportion of data from "heavy" offices.

In the present investigation comparisons of figures for individual offices have not been made, but it is likely that the same feature is present. Also, the fact that there are fewer contributing offices, and that one office contributed about 60% of the data, may be the main reason for the closer correspondence of medical with non-medical mortality as compared with the A1949-52 investigation.

The figures in the table given above convey the impression that there is a different relationship between medical and non-medical mortality at the middle and higher ages as compared with the younger ages. Care is needed, however, in interpreting the figures, for somewhat different patterns are revealed by different groupings of the data, as follows :

| Age group | Medical Non-medical | Age group | Medical Non-medical | Age group | Medical Non-medical |
|-----------|------------------------|-----------|------------------------|-----------|------------------------|
| | % | | % | | % |
| 15-24 | 91.5 | 15-29 | 82.2 | 15-34 | 93.4 |
| 25-39 | 98.0 | 30-44 | 104.0 | 35-49 | 100.7 |
| 40-54 | 99.1 | 45-59 | 95.5 | 50-64 | 95.1 |
| 55-69 | 93.6 | 60-74 | 90.0 | 65-79 | 86.6 |
| 70- | 77.1 | 75- | 81.5 | 80- | 51.5 |

(8) *Conclusion*

The mortality table produced by the investigation can be regarded as a useful measure of European male assured life mortality in the Union. It does not purport to represent the experience of any one office, nor of any one class of business, and actuaries will no doubt wish to compare their own offices' experience with the average experience as represented by the table.

Much remains to be done, for no attempt has yet been made to investigate select rates of mortality, nor non-Union mortality, nor the mortality of non-European lives. All of these investigations will be pursued, and the main investigation will be added to as more data becomes available.

As stated earlier, relatively few returns were made on a sum assured basis. For the Committee to make use of the data it will

have to make a comparison, for each office that makes a sum assured return, between the policies basis returns and the sum assured basis returns, or to make a similar comparison for the total of the data for these offices. As some offices prefer not to have their individual results available to anyone other than themselves, the Committee is averse to making comparisons for individual offices. Even the making of the combined comparison might well give an indication of the experience of an individual office. For this reason, and also because of the additional work entailed when so many other aspects still require to be investigated, the Committee would prefer not to pursue any investigation on a sum assured basis. It is felt that returns should no longer be made to the Committee on a sum assured basis, and that those offices that wish to see the effect of adopting this alternative unit of investigation should make their own comparisons from their own data.