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MORTALITY DIFFERENCES BETWEEN SMOKERS AND NON-SMOKERS—A REPLY TO BENJAMIN AND MICHAELSON

BY PROFESSOR S. HABERMAN M.A., Ph.D., F.I.A., F.S.S.

THE purpose of this brief note is to comment on the methodology used by Benjamin and Michaelson[1] in their estimation of the expectations of life for non-smokers and smokers and the differences between these expectations of life.

The author of this note has discussed the methodology with the first author of this earlier paper and with their research assistant. Their methodology is as follows:

For males,

let q_x be the mortality rate at age x,

let q_x^S, q_x^{NS} be respectively the mortality rates at age x for smokers and non-smokers,

let k_x be the proportion of the population at age x who are smokers; then it is true that

$$k_x\, q_x^S + (1-k_x)\, q_x^{NS} = q_x.$$

Let the ratio of smoker to non-smoker mortality at age x be r_x, so that $q_x^S = r_x\, q_x^{NS}$. Then by substitution,

$$q_x^{NS} = \frac{q_x}{k_x r_x + 1 - k_x}.$$

In Benjamin and Michaelson's case, q_x is known (based on English Life Tables No 13). k_x and r_x are known and are based on various data sources relating to United States of America experience. Thus, q_x^{NS} and hence q_x^S can be calculated.

The same procedure can be followed for females.

Thus, from these sets of derived q_x^{NS} and q_x^S, life tables may be constructed for male non-smokers, male smokers, female non-smokers and female smokers.

Thus, Benjamin and Michaelson have used a life table approach to estimating the mortality differences between smokers and non-smokers. This implicitly assumes that there are no transfers between the smoker and non-smoker statuses. However, such transfers could be allowed for by using a multi-state model.

Figure 1 introduces the multi-state model that can be used in the estimation of non-smoker and smoker mortality. Three states are identified as are the possible transitions and the 'forces of decrement' or 'transition intensities' so that:

$s_x dx$ can be thought of as the probability that a non-smoker aged x moves to the smoker state in the infinitesimal age interval $(x, x+dx)$.

Similar interpretations would apply to μ_x^{NS}, μ_x^S and g_x.

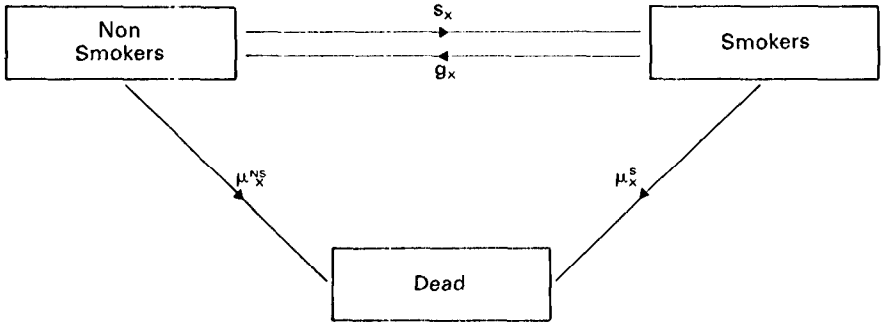

Figure 1.

Statistical evidence from the literature indicates that

$$\mu_x^S > \mu_x^{NS}. \qquad (*)$$

Benjamin and Michaelson's approach is concerned with the indirect estimation of q_x^{NS} and q_x^S (corresponding to μ_x^{NS} and μ_x^S of Figure 1). But they omit consideration of the effect of the transitions from the non-smoker state to the smoker state and vice versa.

Thus, their smoker life tables refer only to individuals who remain in the smoker status *throughout* their lives and their non-smoker life tables refer to individuals who remain in the non-smoker status *throughout* their lives.

More specifically, their computation of $\mathring{e}_x^S$ for smokers assumes that an individual aged x is a smoker at age x and remains a smoker throughout his/her subsequent lifetime. Similarly, their computation of $\mathring{e}_x^{NS}$ for non-smokers assumes that an individual aged x is a non-smoker at age x and remains a non-smoker throughout his/her subsequent lifetime.

Once we allow for the full multi-state model, there is the possibility that a smoker at age x will spend some future time as a non-smoker. In this latter state, his/her force of mortality is *reduced* (*) and hence the true expectation of life $\mathring{e}_x^S$ is *greater* than that calculated by Benjamin and Michaelson.

Similarly, in the full multi-state model, there is the possibility that a non-smoker aged x will spend some future time as a smoker in which state the force of mortality is *increased* (*) and hence the true expectation of life $\mathring{e}_x^{NS}$ is *lower* than that calculated by Benjamin and Michaelson.

The net effect of these errors is that Benjamin and Michaelson *overestimate* the differential in expectation of life between non-smokers and smokers, viz.

$$\Delta_x = \mathring{e}_x^{NS} - \mathring{e}_x^S.$$

These comments parallel those of Bloomfield and Haberman[2] in the context of working life tables.

The above argument would hold even if the transition intensity, g_x, were zero at all ages. Providing that the transition intensity, s_x, were non-zero, it would still be true that the Benjamin and Michaelson approach leads to an overestimate of Δ_x.

Given a cautious approach, for example, to pricing life insurance products, these considerations concerning Δ_x should be borne in mind.

Of course, the models introduced by Benjamin and Michaelson and introduced here are abstractions from the real world. More elaborate models that would provide a closer approximation to reality would allow also for the time spent in the smoking status, the quantity and type of cigarette smoked, and the residual morbid effects of smoking which may remain with an individual who has given up smoking. These more complex models would provide further insight into the magnitude of Δ_x.

REFERENCES

(1) BENJAMIN B. & MICHAELSON R. (1988). Mortality Differences between Smokers and Non-Smokers. *J.I.A.* **115,** 519-525.

(2) BLOOMFIELD D. & HABERMAN, S. (1989). The Assessment of Damages and the Measurement of Work Time Lost to Sickness, Unemployment and Stoppages. London: City University, Department of Actuarial Science and Statistics, *Actuarial Research Paper* No 9.

Professor B. Benjamin subsequently wrote on behalf of himself and Mr R. Michaelson:

We were interested to read the note from Professor Haberman. We were aware that it would have been more useful if we could have provided a multi-state table allowing for transfers from the category of smoker to that of non-smoker and vice versa and we think we could have managed to construct such a total. Our one obstacle was the lack of data relating to the insured population. Any person who remains a smoker at the time of proposal for insurance is probably less likely to change than a member of the general public. Non-smokers in the insured population are probably even less likely to change. At the moment we simply do not know.