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MORTALITY AT THE HIGHEST AGES

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The pattern of mortality at the highest ages has been considered by many authors, including Redington (1969), Humphrey (1970) and Benjamin (1964, 1982). The questions raised have included the following:

(a) Is there a definite upper limit to the span of human life, so that q_x reaches unity at a finite age? Or does q_x tend gradually to unity as age tends to infinity, as happens under the Gompertz and Makeham laws? Or does q_x tend to a constant less than unity, as under the Perks formula or the formula which was used to graduate the English Life Tables No. 11 and 12?

(b) Has the fall in mortality rates at lower ages been accompanied by a similar fall at the very highest ages? Has the upper tail of the curve of death ($\mu_x l_x$) shifted?

(c) Does the lower mortality of females compared with males extend to the very highest ages, or do the rates eventually tend to converge?

In earlier discussions, notably following the paper by Redington (1969), the hope was expressed that more work on the numbers of registered deaths at the highest ages, or using the D.H.S.S. sample of pensioners as an extra source of data, might help to throw light on these questions. The present paper attempts to make a further contribution to the debate by examining the data which are now available for England and Wales.

ENGLISH LIFE TABLES

Because the numbers reaching the highest ages are so small, their mortality rates can only be studied from national data. Like previous writers on this subject, we therefore begin with the English Life Tables. The rates of mortality (q_x) from the last five E.L.T.s, at five-yearly intervals of age, are shown in Table 1. These rates are illustrated graphically in Figures 1 and 2, though omitting E.L.T. No. 11 to avoid visual confusion. In E.L.T. Nos. 10 and 11 the published rates for males stop at q_{104}; the values of q_{105} shown in Table 1 are the extrapolations given by Humphrey (1970).

The traditional method of estimating mortality rates in the E.L.T.s is to divide the numbers of registered deaths by an exposed to risk based on the census of population. Unfortunately this method runs into difficulties at the highest ages because the numbers of very old persons are exaggerated in the censuses. This is partly because the form-fillers sometimes enter the wrong age or date of birth and partly because of coding or keying errors. Over most of the age range such errors

Table 1. *1000q from English Life Tables and Gompertz curves*

		Males					Females				
		Age					Age				
Table	Years	85	90	95	100	105	85	90	95	100	105
E.L.T. No. 10	1930–32	210	286	376	484	604	179	251	336	441	562
E.L.T. No. 11	1950–52	207	293	376	440	483	167	241	313	368	404
E.L.T. No. 12	1960–62	187	256	324	380	420	147	221	303	378	434
E.L.T. No. 13	1970–72	173	241	320	405	493	129	198	288	405	546
E.L.T. No. 14	1980–82	166	227	290	381	524	119	185	249	323	478
Gompertz	1942–57	202	276	370	483	610	159	225	313	424	556
Gompertz	1981	165	227	308	409	528	130	185	260	358	479

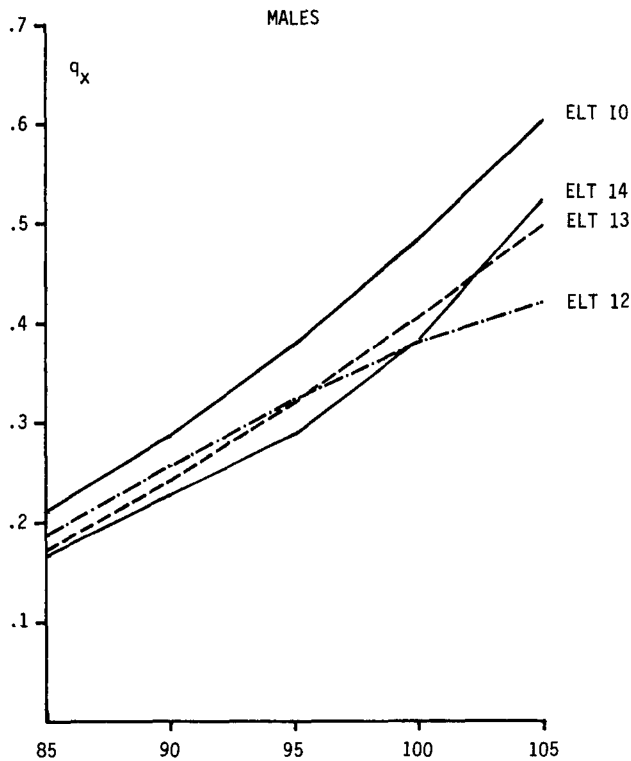

Figure 1. Rates of mortality for males from English Life Tables

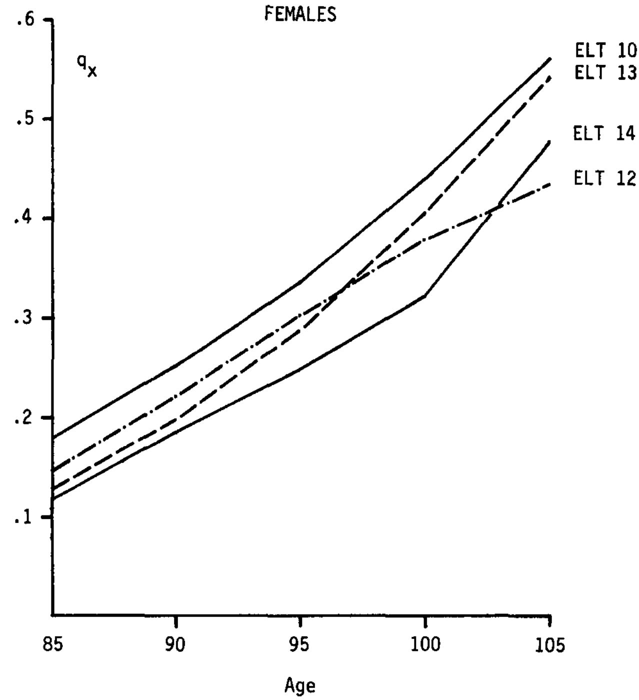

Figure 2. Rates of mortality for females from English Life Tables

can go either way and may tend to cancel out, but at the extreme upper tail of the age distribution they do not. A recent examination (O.P.C.S., 1984) showed that the census numbers were too high by 2·7% in the age range 95–99 and were much more seriously defective for centenarians.

This difficulty has long been recognised and in the E.L.T.s the mortality rates at the very highest ages have been estimated by extrapolating the graduated values at younger ages. The effect, however, as remarked by Humphrey (1970), is that eventually the method of graduation becomes more important than the underlying data. In E.L.T. No. 10, mortality rates at the highest ages were found by assuming a Gompertz curve. In E.L.T.s No. 11 and 12, a method of graduation was used which assumed that m_x follows a combination of a logistic curve with a normal (Gaussian) curve, with the effect that at high ages q_x tends to a constant less than unity. In E.L.T. No. 13, the method of graduation assumed that the limiting age for both males and females was 110. In E.L.T. No 14 it was assumed that m_x follows a cubic curve. These differences of method largely account for the different shapes of the curves illustrated in Figures 1 and 2.

MORTALITY RATES BASED ON DEATHS ONLY

It is possible to avoid using the census data at high ages by constructing the exposed to risk entirely from the data on registered deaths. As an example of the method, those who were living and aged 100 in 1970 must have died at age 100 in 1970, or at age 101 in 1971, or at age 102 in 1972, and so on. If we wait until all those who were aged 100 in 1970 have died, we can work backwards from the deaths to the numbers who were living. This is the 'method of extinct generations' which was pioneered by Vincent (1951). It has been applied to deaths in England and Wales by Humphrey (1970) and more recently by Thatcher (1981).

Humphrey used as his data all deaths at age 85 and over in England and Wales from 1910–62 and he split these into periods. He produced graduated rates of mortality at ages 86–104 based on the experience of 1942–57 and found that these followed the Gompertz curve. His findings are discussed further below.

Thatcher used data on the deaths of 4,404 centenarians who died in 1960–79. His paper gave data for males and females separately and a life table for persons (i.e. males and females together); from the data we can easily construct the life tables for males and females separately and these are given in Table 2. In this, for example, the top line shows that 648 males reached the age of 100 during the period and of these 292 died before reaching the age of 101. Thus for males $q_{100} = \cdot 451$ and this estimate, being based on a binomial sample of size 648, has a standard error of ·020.

Despite the standard errors, there are some conclusions which can be drawn with reasonable confidence from Tables 1 and 2:

(i) At ages 100–105 the observed values of q_x in Table 2 (i.e. the values based on the actual numbers of deaths, without any graduation or extrapolation) are *higher* than E.L.T. No. 12, for both males and females. Since

Table 2. *Life tables based on deaths (only) in 1960–79*

	Males				Females			
Age	Deaths at this age d_x	Exposed to risk l_x	Rate of mortality q_x	Standard error of q_x	Deaths at this age d_x	Exposed to risk l_x	Rate of mortality q_x	Standard error of q_x
100	292	648	·451	(·020)	1,463	3,756	·390	(·008)
101	175	356	·492	(·026)	931	2,293	·406	(·010)
102	86	181	·475	(·037)	534	1,362	·392	(·013)
103	46	95	·484	(·051)	370	828	·447	(·017)
104	26	49	·531	(·071)	198	458	·432	(·023)
105	10	23	·435	(·103)	142	260	·546	(·031)
106	6	13	·462	(·138)	54	118	·458	(·046)
107	3	7	·429	(·187)	30	64	·469	(·062)
108	0	4			18	34	·529	(·086)
109+	4				16			

mortality rates have been falling, and the observed values relate to a later period than E.L.T. No. 12, we conclude that E.L.T. No. 12 underestimated the rates of mortality at ages 100–105.

(ii) At least at ages 100–104, the observed values tend to be higher than E.L.T. No. 13 for males but lower for females.

(iii) E.L.T. No. 14 is consistent with the observed values at around age 105 but is significantly below them at age 100.

These points are discussed further below.

THE D.H.S.S. SAMPLE OF PENSIONERS

In the discussion following the paper by Redington (1969), the hope was expressed that the D.H.S.S. 5% sample of pensioners might produce useful new data on mortality at high ages. This sample, which has since been increased in size to 10%, has indeed proved useful for estimating the aggregate population of centenarians (O.P.C.S., 1984). As a specimen illustration of the kind of information which the sample can contribute to the estimation of mortality rates, the relevant figures for females at ages 95 to 100 in 1981 are given in Table 3. In this table the deaths (in the first column) are from the registration data. The initial estimate of the exposed to risk (in the second column) is from the D.H.S.S. sample, grossed up by a factor of 10. This grossed up figure was then reduced by 3% (in the third column) to allow for delays (in 1981) in removing deceased persons from the sample. The resulting death rate m_x was then converted to a rate of mortality q_x by using the approximate formula $q_x = 1 - \exp(-m_x)$, which is appropriate at high ages.

Of course, the estimate that there were (for example) 880 females aged 100 was found by grossing up a sample of only 88 females, so the resulting estimates of q_x for centenarians have larger standard errors than those in Table 2. For males the standard errors are much larger still. However, we must remember that Table 3 is based on only a single year and moreover relates to the extreme tail of the age distribution. If it becomes possible to repeat such analyses on a more regular basis in the future, to graduate the resulting population estimates from year to

Table 3. *Mortality rates for females in 1981 using the D.H.S.S. sample of pensioners*

Age x	Registered deaths in 1981 d_x	Living in June 1981: DHSS sample $\times 10$	Living in June 1981: Adjusted L_x	Central death rate m_x	Rate of mortality q_x
95	2246	7860	7624	·295	·255
96	1679	5010	4860	·345	·292
97	1252	3550	3444	·364	·305
98	864	2390	2318	·373	·311
99	583	1320	1280	·455	·366
100	398	880	854	·466	·373

year and to take account of them in calculating the mortality rates for individual ages 85–94 as well as 95 and over, the result could be a useful addition to the data for England and Wales.

THE GOMPERTZ CURVE

Before assessing the above results, we need to compare them with the Gompertz graduation made by Humphrey. We recall that Gompertz found that the force of mortality μ_x was related to the age x by the formula

$$\mu_x = Ac^x \qquad (1)$$

and that this implies that

$$\log p_x = -Bc^x \qquad (2)$$

This in turn implies that if $\log(-\log p_x)$ is plotted against x, the resulting graph should be a straight line. Here c is the Gompertz constant and A and B are constants of proportionality which are determined as soon as we know p_x at a single point.

Humphrey (1970) found that his estimates of mortality rates based on deaths only, in 1942–57, could be well graduated in the age range 86–104 by the Gompertz curves with constants $c = 1{\cdot}0740$ for males and $c = 1{\cdot}0804$ for females, though he excluded age 85 (which did not fit so well) and he found that the actual deaths fell slightly below the graduations at ages 100–104. He also fitted Gompertz curves to data given by Vincent (1951) for deaths over age 85 in Sweden in 1901–45 and France in 1920–38; all the Gompertz constants were between 1·065 and 1·090. (We may remark that these constants are somewhat lower than those normally found in Gompertz graduations at younger ages such as 35–85. One theory, discussed by Redington (1969), is that in mixed populations there may be a 'survival of the fittest' effect which produces an apparent reduction in the Gompertz constant at the highest ages.)

In Figures 3 and 4 the values of $\log(-\log p_x)$ given by the mortality rates in E.L.T. No. 14 are plotted against x as dashed lines. (Here the logarithms are to the base 10). The straight lines marked H show Humphrey's graduations for 1942–57. For comparison, the Figures also show straight lines (marked G) which have been drawn to have exactly the same slopes as Humphrey's graduation (viz given by the Gompertz constants $c = 1{\cdot}0740$ for males and 1·0804 for females) but shifted downwards so as to pass through the points given by E.L.T. No. 14 at age 90. No other data have been used to construct these straight lines and it is not of course suggested that they represent the actual mortality rates in 1981. However, they show how relatively close the rates in E.L.T. No. 14 are to Gompertz curves and also show the ages at which they differ most. The equations of the lines marked G are:

$$\text{Males} \quad -\log p_x = {\cdot}11178 \times 1{\cdot}0740^{(x-90)} \qquad (3)$$
$$\text{Females} \quad -\log p_x = {\cdot}08867 \times 1{\cdot}0804^{(x-90)} \qquad (4)$$

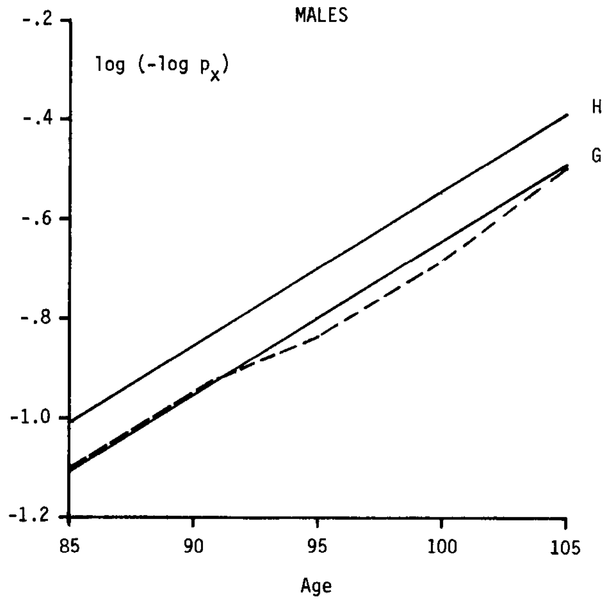

Figure 3. log ($-$log p_x) for males from E.L.T. No. 14 (dashed line), Humphrey's graduation (H) and a straight line (G) drawn parallel to (H)

Values of q_x derived from (3) and (4) are shown in the bottom line of Table 1. The penultimate line shows q_x from Humphrey's graduations, here extended to ages 85 and 105.

It will be seen from Figures 3 and 4 that the largest difference between E.L.T. No. 14 and the Gompertz line G occur at age 100, for both males and females. Bearing in mind our earlier finding that the observed values of q_x in Table 2 are higher than in E.L.T. No. 14 at age 100 for both sexes, it seems reasonable to suppose that the actual rates of mortality may well have been even closer to the Gompertz curves than were the rates in E.L.T. No. 14; though we need to take account of the difference in the dates between the observed values, which relate to 1960–79, and E.L.T. No. 14 which relates to 1981.

Although we do not have enough data to make a formal graduation, it is possible to test whether the observed data in Table 2 are significantly different from the Gompertz curves, either in their levels or in their slopes, after allowing for differences in dates. As regards the levels, the most reliable values in Table 2 are those for q_{100}. For males, the estimated value of q_{100} in Humphrey's graduation was ·483 in 1942–59, centred on about 1950. Equation (3) gives $q_{100}=\cdot 4088$ relating to 1981. Interpolating between these, we would expect a value of $q_{100}=\cdot 435$ for 1970, which is the central year for the period of 1960–79

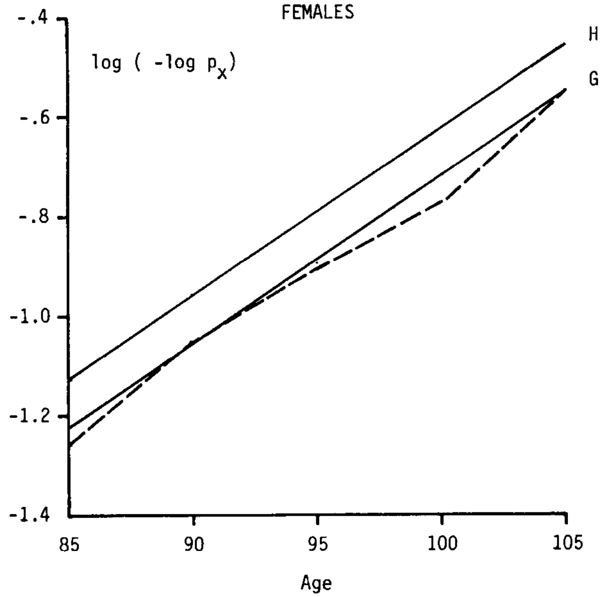

Figure 4. log $(-\log p_x)$ for females from E.L.T. No. 14 (dashed line), Humphrey's graduation (H) and a straight line (G) drawn parallel to (H).

covered by Table 2. The observed value in Table 2 is $q_{100} = \cdot 451$ with a standard error of ·020, which is clearly consistent with expected value. For females, Humphrey's graduation gives $q_{100} = \cdot 424$ centred on 1950, equation (4) gives ·3575 for 1981, interpolation gives ·381 for 1970, and the observed value in Table 2 is ·390 with a standard error of ·008. Again the observed figure is consistent with the expectation.

As regards the slopes of the mortality rates, Table 2 shows that out of 648 males who reached the age of 100, 23 reached 105 and 4 reached 108. According to the Gompertz curve (3) we would expect 30·8 to reach 105 and 2·8 to reach 108. For females, Table 2 shows that out of 3,756 who reached the age of 100, 260 reached 105 and 34 reached 108. According to the Gompertz curve (4) one would expect 279·6 to reach 105 and 33·7 to reach 108. Thus the mortality rates above age 100 show the same apparent drop below the slope of the Gompertz curve as has been observed by previous writers, though the numbers of deaths are too small to establish full statistical significance.

The main conclusion which we draw, however, is that to the extent that mortality rates at ages 85–105 can be approximated by Gompertz curves, it appears from Figures 3 and 4 that the slopes of these lines in 1981 (and hence the Gompertz constants c) were still remarkably similar to those found by

Humphrey. But the levelof the lines has shifted downwards, which would imply a more or less uniform percentage reduction in the force of mortality at these ages. We must remember, of course, that at these ages uniform percentage reductions in μ_x do not imply uniform percentage reductions in q_x; the relationship between μ_x and q_x can be derived from equations (1) and (2).

RATIOS OF FEMALE TO MALE MORTALITY RATES

Ratios of female to male mortality rates (q_x) have shown remarkable declines at ages up to 85 (Daykin, 1985), but at ages 90–105 the changes have been less spectacular. Table 4 shows the ratios derived from the last five E.L.T.s. It also shows the ratios given by Humphrey's Gompertz graduation (his Table 10, here extended to include ages 85 and 105), and for comparison the ratios from the Gompertz curves (3) and (4) above. We note that the latter fail to reflect the latest fall to ·72 in the observed ratio at age 85.

The figures in Table 4 are largely self-explanatory but we may add two small comments. The first is that the rather high ratios at ages 100 and 105 in E.L.T. No. 13 are consistent with our earlier findings from Table 2, which suggested that the extrapolated rates in E.L.T. No. 13 may have been too low for males and too high for females at these ages. This could explain the high ratios in Table 4.

Secondly, we observe that the Gompertz curves for males and females have different constants, and so long as these differ in the direction at present observed, then the mortality rates for males and females will continue to converge as age increases. However, such patterns may not be immutable. For example, at lower ages, the ratio of female to male mortality has been considerably affected by the changing incidence of smoking and lung cancer in males. In 1981 the generation of males born in 1901–10, who may represent the peak of lung cancer mortality (see *J.I.A.*, **90,** 244), had not yet reached the highest age ranges.

EXPECTATIONS OF LIFE

In Table 5 we bring together the expectations of life from E.L.T.s No. 10–14,

Table 4. *Ratio of female to male mortality rates* (q_x)

	Age 85	90	95	100	105
E.L.T. No. 10	·85	·88	·89	·91	·93
E.L.T. No. 11	·81	·82	·83	·84	·84
E.L.T. No. 12	·79	·86	·94	·99	1·03
E.L.T. No. 13	·75	·82	·90	1·00	1·11
E.L.T. No. 14	·72	·81	·86	·85	·91
Gompertz (1942–57)	·79	·81	·85	·88	·91
Gompertz (1981)	·79	·81	·84	·87	·91

Table 5. *Expectations of Life from English Life Tables and Gompertz curves*

		Males					Females				
		Age					Age				
Table	Years	85	90	95	100	105	85	90	95	100	105
E.L.T. No. 10	1930–32	3·50	2·63	1·97	1·48		4·00	2·98	2·22	1·65	1·22
E.L.T. No. 11	1950–52	3·48	2·56	1·99	1·67		4·20	3·12	2·47	2·09	1·88
E.L.T. No. 12	1960–62	3·90	2·97	2·38	2·00		4·58	3·32	2·49	1·99	
E.L.T. No. 13	1970–72	4·14	3·09	2·34	1·84		5·02	3·57	2·52	1·79	
E.L.T. No. 14	1980–82	4·34	3·33	2·57	1·86	1·24	5·38	3·95	3·02	2·21	1·38
Gompertz	1942–57	3·61	2·68	1·96	1·42	1·02	4·41	3·24	2·33	1·66	1·17
Gompertz	1981	4·34	3·24	2·39	1·75	1·27	5·22	3·87	2·82	2·02	1·43

together with values calculated from the Gompertz graduation for 1942–57 by Humphrey and the Gompertz curves (3) and (4) for 1981. These last estimates are of course conditional on the assumption that the Gompertz curves can be extended beyond age 105. The technical method used to obtain expectations from the Gompertz curves was to calculate l_x at 6-monthly intervals and to apply Simpson's rule within each year of age.

We note from Table 5 that in E.L.T. No. 14 the expectations of life at ages 85–105 are all within 2 months of those given by the Gompertz curves. Table 5 also suggests that in the last 30 years the expectation of life has increased by about 9–10 months for males and 10–14 months for females at age 85, but by only about 3 months for both sexes at age 105.

THE SPAN OF HUMAN LIFE

As outlined in the opening paragraph of this paper, there are three main theories about the span of human life: either it has a definite upper limit, in which case q_x reaches unity at a finite age; or q_x tends gradually to unity as age tends to infinity; or q_x tends to a limit less than unity.

The first theory was espoused by Vincent (1951) who believed that human life has an upper limit of, at the most, 113 years. Nevertheless in February 1986 a Japanese, Shigechiyo Izumi, died at the fully authenticated age of 120 years*; so it seems that Vincent's supposed limit was incorrect. Of course this does not rule out the logical possibility that there may still be an upper limit to life; but if so, it must be at least 120 years.

The second theory would apply if, for example, mortality follows the Gompertz curve not only up to age 105 but also beyond age 105. There would then be no absolute upper limit to life, but of course there will always be a highest age observed so far, and this will gradually increase as more and more people survive to become exposed to risk in the highest age range. For several years the

* See the *Guinness Book of Records, 1987.*

highest age observed in England and Wales was 112, but this record has been broken by Mrs Anna Williams who reached age 114 in June 1987. If the Gompertz law continues to apply at these ages, out of every 100 females who reach the age of 113 one may expect to find that about 2 will reach age 116, or perhaps even more if mortality rates continue to fall.

Under the third theory, some people will live even longer. This theory is based on the observation that the mortality curve seems to flatten at the highest ages. Under the Perks formula, and the formula which was used to graduate E.L.T.s No. 11 and 12, the flattening was assumed to continue. It is true that the subsequently observed mortality rates in Table 2 have proved to be higher than those in E.L.T. No. 12, so it now seems that the flattening was over-estimated or perhaps assumed prematurely. Nevertheless, none of the observed values of q_x has so far exceeded ·55 so the hypothesis that q_x may tend to a limit still remains open. Perks himself (*J.I.A.*, **95**, 305) doubted whether the force of mortality would ever be observed to exceed unity; this implies that the expectation of life will never fall below one year, however high the age. But on the Gompertz curves one would not in any case expect the expectation of life to fall noticeably below one year until at least age 111 so it will be some time before the differences between these hypotheses can be fully tested, at least from data for England and Wales.

CONCLUSIONS

The conclusions of this investigation are more definite than the author, at least, had expected. The absolute upper limit of human life, if it exists, appears to have receded and must now be placed at not less than 120 years. On the other hand, the flattening of mortality rates at high ages, which was anticipated in E.L.T.s Nos. 11 and 12, did not occur to the extent expected. Instead, mortality rates at high ages in England and Wales still look remarkably like Gompertz curves. What is more, the Gompertz constants which were found by Humphrey (1970) in his graduation of data for 1942–57 still seem to give a good representation of the increase of the force of mortality between ages 90 and 105.

What has most obviously changed since 1942–57 has been the general level of the force of mortality, which appears to have fallen by as much throughout the age range 90–105 as it has done at age 90, namely by about ·7% per annum. Thus the curve of deaths has definitely shifted, though not by very far at the extreme upper tail. If the Gompertz curves can be assumed to extend to even higher ages, the expectation of life at age 105 has increased since 1942–57 by about 3 months. The mortality rates of males and females continue to tend towards convergence at high ages.

These conclusions are of course, based on present information and like previous assessments of this kind they will need to be reviewed as further data become available. It will be particularly interesting to see whether the force of mortality continues to fall at the highest ages and whether the relationship between male and female mortality rates will be affected when, for example, the generation of heavy-smoking and cancer-prone males who were born in 1901–10 reaches and then moves through the age ranges 90–105, to be followed some 15 years later by a generation of heavy-smoking females. It may well be that the Gompertz-type patterns will then be disturbed, at least temporarily.

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