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ASSET & LIABILITY MODELLING – THE WAY AHEAD?

by

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Asset & Liability Modelling - The way ahead?

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1 Introduction

1.1 Used to its full potential, cash-flow modelling is a very powerful tool. Furthermore, I believe that it is an area in which we shall see significant developments over the next decade. As understanding of the practical benefits of using such models increases, coupled with the inevitable continuing improvements in the speed of personal computers, it is interesting to speculate whether it will, by the end of the decade, be quite normal to carry out a cash-flow modelling exercise in place of the traditional, discounted cash-flow actuarial valuation.

1.2 The term asset & liability modelling is not clearly defined and may mean many different things to different people. Although the precise definition is not critical to this paper, in order to avoid confusion I will start with the following (almost certainly not uncontroversial) definitions:

- **Cash-flow modelling/Emerging costs valuation**

  A process whereby a mathematical model is constructed to generate a set of cash-flows to represent a particular set of liabilities. A similar model is constructed for the assets.

- **Asset & liability modelling**

  A particular type of cash-flow modelling whereby the model for the assets (and possibly that for the liabilities) includes a random parameter. The interaction between assets and liabilities can be investigated by means of a large number of simulations.

1.3 It is perhaps unfortunate that the term asset & liability modelling has become synonymous in many people's minds with a mathematical means of trying to determine the proportion of a fund which should be placed in different asset classes. This is simply one application of a asset & liability modelling and indeed of a cash-flow approach in general. There are many other interesting areas which can be explored.

1.4 Asset & liability modelling has been used in the UK since the mid 1980s. The techniques have been (both correctly and incorrectly) widely applied in the United States for in excess of ten years.

Many, like myself, believe that they provide a useful insight into the inner-workings of long-term funds. On the other hand, many remain sceptical, believing that the results owe more to the assumptions used than to any underlying reality.

Amongst those with experience of the development of such models in the United States, there appears to be a perception that the work which has been done in the UK is generally of a very high standard. Certainly most of the pension consultants have invested a great deal of resources into the development of such models. Why then have so few UK pension plans commissioned asset & liability modelling exercises? Why do so many actuaries remain sceptical? Is an asset & liability modelling exercise simply an excuse for a consultant to charge higher fees? What are the benefits?

1.5 Whilst I am convinced of the merits of the approach I am all too aware of some of the many pitfalls which lie along the route to the general application of such models. The truth is almost certainly that whilst modelling is appropriate in some circumstances, in others a broad-brush approach is more helpful.

1.6 First and foremost I believe that cash-flow models are best viewed as a powerful education tool for all involved in funding to meet long-term liabilities. Used properly the techniques can help to de-mystify the processes involved.

A recent report commissioned by the NAPF(1) made the following call to the trustees of UK pension plans:

> *"Trustees of each pension fund have a responsibility to know their own scheme and to consider whether it has any special characteristics which require it to be invested to a degree differently from the generality of pension funds."*

Such an understanding can, I believe, come from the use of cash-flow models.

The reaction of some to the above call was along the lines of "The advisers too have similar responsibilities". In many cases I would agree with this. Looking at the funding of a pension plan in a different way forces many, perhaps well established, preconceptions to be re-examined. The fund actuary is not (and should not be) immune from the education process.

1.7 In summary, I am convinced that in funding for long-term liabilities we all have a lot to learn. Perhaps most importantly we, as a profession, have a responsibility to educate and advise those legally responsible for the assets.

The 1990's needs to see a continued unravelling of the mystery which so often seems to surround the methods adopted in advice connected with the pre-funding of long-term liabilities. In terms of funding to meet the liabilities of a UK defined benefit pension plan I believe we have to move from a position where many trustees base decisions on the following tenets:

> *"Equities provide a good match for inflation linked liabilities."*
>
> *"The aim should be to maximise the return on the fund with an acceptable degree of risk."*

moving perhaps to a world in which they are replaced by the following:

> *"Long-term investment policy is framed by reference to both the liabilities and our overall objectives. Short-term policy is framed so as to ensure that the objectives are met and that those investment opportunities open to us are capitalised upon."*
>
> *"The performance of all our advisers is measured by the extent to which we are able to meet our objectives."*

1.8 Appendix 1 to this paper offers an outline presentation of the results of an asset & liability modelling exercise used to determine long-term (ie strategic) asset allocation. I have included the example partly for the benefit of those readers who are unfamiliar with asset & liability modelling and partly as a practical example of presentation.

1.9 A list of references is included in Section 10. In particular I would like to thank my colleague Peter Lockyer for his help and advice. Whilst acknowledging the debt to my colleagues and other members of the profession with whom I have discussed, at various times, the issues involved, I must take full responsibility for the views expressed in this paper.

1.10 This paper has been put together over a period of several months. Over this time I have become increasingly aware that many of the issues are neither black nor white; in fact, in many instances, one has difficulty distinguishing between the various shades of grey!

What has emerged is, I believe, a description of asset & liability modelling, what it hopes to, and what it can achieve, together with a number of more detailed asides covering those areas which are of particular interest to me. I make no more apologies. Let us begin!

2 Why look at cash-flows?

2.1 As stated in the introduction to this paper, asset & liability modelling essentially involves an investigation of the cash-flows inherent in both the assets and liabilities. What are the advantages of such an approach?

In this Section I want to put forward a number of reasons why looking at the actual cash-flows in both the assets and liabilities is important.

2.2 The undertaking of a scientific study of the cash-flows inherent in both the assets and liabilities is not a new approach. Many financial institutions have investigated the possibility of matching liabilities with assets which generate known cash-flows at predetermined times. Initially this was done in an attempt to find investment portfolios which precisely matched the institution's liabilities, both in the timing and the amount of their cash-flows.

2.3 Such ideas, although perhaps theoretically attractive, are in practice very difficult to devise and implement. Cash-flows (from both the assets and liabilities) are often not determined at the outset both in timing and amount. Faced with the impracticalities of the complete matching of cash-flows many financial institutions (in particular life insurance companies) moved on to consider immunisation. The theory of immunisation was first put forward by Redington[(2)] in 1952.

2.4 An immunized portfolio is one in which, discounting all cash-flows at a particular interest rate, the present value of the assets equals the present value of the liabilities. Furthermore for a small change in interest rates, either up or down, the value of the assets exceeds the value of the liabilities.

2.5 The theory and concepts underlying immunisation appear to have led to a number of ideas which seem to be prevalent within many financial institutions, namely:

- The fluctuating payments received from equity type investments are inappropriate for matching liabilities.
- Fixed interest assets are held with average term to redemption below that which might otherwise be held in order to minimise losses caused by a general rise in interest rates.
- Assets should be denominated in the same currency as the liabilities in order to eliminate currency risk (which is not easily included in either immunisation or matching theories).

Many institutions are, today, pursuing investment policies which have their roots in the concepts of matching or immunisation.

2.6 Devising an immunized portfolio for many financial institutions presents a number of fundamental problems.

- Considerable uncertainty often surrounds the cash-flows inherent in both the assets and liabilities. Furthermore, inflation has now become a much more significant factor than was the case in the 1950's; whilst it is clear, in many cases, that inflation will have a significant effect on both assets and liabilities the interaction is far from clear.

- The term of the liabilities, in many cases, exceeds the term of the available assets.

- For a UK defined benefit pension plan a large proportion of the liabilities will increase in line with earnings; there are however no assets which increase in the same way.

2.7 The problem of matching assets and liabilities within the context of a UK defined benefit pension plan has been addressed in recent actuarial literature, in particular in a number of papers by Wilkie and Wise(3).

Arthur and Randall(4) consider a concept termed 'partial matching'. Here the stated aim is to discover the most nearly matched portfolio or, to put it another way, that with the strongest degree of immunisation. For example, it is stated that for a man aged 60 due to retire at 65 with a non-increasing, non-commutable pension payable for 15 years precisely, then, under a few assumptions given in the paper (including that of an investable universe comprising of conventional gilts and index-linked securities only), the most nearly matched portfolio is one consisting of 53% conventional gilts and 47% index-linked securities.

In practice this sort of minimum risk position may well be almost as likely to be unacceptable to the trustees as what we might term a 'maximum risk' position. The former perhaps reducing the volatility of the contribution rate to a minimum, but at the price of higher expected contribution rate due to investment returns foregone; the latter minimising the expected contribution rate, but at the expense of an unacceptably wide range of possible outcomes.

The practical problems faced by those charged with the investment of long-term funds do not however lend themselves to rigorous analysis. In essence this is because we are dealing with subjective perceptions of an uncertain future. Cash-flow models do, I believe, provide a framework within which certain characteristics inherent in the assets and liabilities can be established; objectives can be determined and informed judgements can be made.

2.8 Let us leave to one side for the moment what has gone before and concentrate on some basic reasons as to why cash-flows are important.

The traditional discounted cash-flow approach to valuing long-term liabilities essentially involves reducing the 'information' (for want of a better word) inherent in a set of disparate liabilities into one number. Whilst the method is powerful, much of the information is lost. If we consider the following example, discounting at an interest rate of 10% between time periods, each of the following series of cash-flows has a present value (at time 0) of approximately 100:

Time	1	2	3	4	5	6	7	8	9	10
Series 1	110	-	-	-	-	-	-	-	-	-
Series 2	16.3	16.3	16.3	16.3	16.3	16.3	16.3	16.3	16.3	16.3
Series 3	3	5	10	15	20	25	30	35	40	-
Series 4	-	-	-	-	-	-	-	-	-	259

2.9 Consider what you would do if offered 100 as an asset to hold against any of the above cash-flows. For simplicity we will restrict the asset classes in which you can invest to cash and equities and assume they have their usual characteristics.

One of the first thing you would need to know is which set of cash-flows actually represent your liabilities. If you are investing to meet the liabilities in the first series, equities are probably a risky investment and cash a safer one. Trying to frame any sort of policy without knowing which set of liabilities you were trying to meet would seem somewhat strange.

The incidence of the cash-flows is important in framing an investment policy.

What happens if we can only guarantee a return of 8% on our investments? Assuming we hold 100 at the start then the funding levels at time zero, on the assumption of 8% returns, are 98%, 91%, 89% and 83% respectively.

The incidence of the cash-flows will affect the stability of funding levels (and future contribution rates if the liabilities are not fully pre-funded).

Assume now that the return on equities is 12% per time period with probability 0.8 and 2% with probability 0.2. The expected return on the asset is 10%, however this is not very helpful.

In order to make decisions about how much to invest in the 'risky' asset one needs to have some sort of analysis showing the possible outcomes. Furthermore if there are constraints on the funding level which must be satisfied (and/or for example any revealed deficit above a certain threshold requires the injection of more capital) then the need for such analysis is even more important.

Details of the cash-flows in the assets and liabilities are required for an analysis of the sensitivity of results to variations in experience.

How does the presence of inflation affecting both the assets and the liabilities influence matters? In a defined benefit pension plan (and for example in a General Insurance fund) this simply adds uncertainty to the future likely values of both assets and liabilities. In my view this makes it even more important to have some form of sensitivity analysis.

The effect of inflation on both sides of the balance sheet may well be such that it does not affect the important variables (funding level for example) by as much as one might initially expect (see 3.16).

2.10 Does the above offer any new insights? The three conclusions highlighted in 2.9 are unlikely to come as much of a surprise to anyone. Each can be arrived at by a mixture of common-sense and traditional methods. I contend though that a cash-flow approach to valuing long-term liabilities offers a way in which those not involved with the issues on a day to day basis can make informed judgements as to the appropriateness of different strategies.

2.11 If cash-flows are so important when looking at liabilities why bother with the traditional discounted cash-flow approach?

There are a number of advantages in adopting the traditional approach to an actuarial valuation of a long-term liabilities.

- The results can be presented as a simple balance sheet; the liabilities being encapsulated into one number.

- There is no need to make allowance for options in the liabilities which may be exercised whereby for example different benefits, of equal value, may be chosen. The most obvious example in a defined benefit pension plan being the option to surrender part of the pension at retirement for cash. The exercise of this option will clearly have an impact on the timing of cash-flows from the plan, but it may have little effect upon the present value of benefits to be provided.

- Payments to be made in the relatively distant future have, because of the effect of discounting, less effect on the overall balance sheet than payments made over the more immediate future.

- In the case of a pension plan the traditional approach does not require any explicit assumption regarding new entrants to the plan. Rather the assumption is implicit in the funding method being adopted.

2.12 To summarise the position as I see it.

The cash-flows of a typical UK defined benefit pension plan inherent in both the assets (largely equity type investments) and the liabilities (essentially final salary related) are uncertain. Traditional valuation methods are very powerful, although a great deal of information is lost.

Some of this information is vital to those who are responsible for taking decisions concerning investment and funding policy.

Although the theoretical development of matching and immunisation is sound, the results are, in my view, of little practical help in investigating the interaction of the assets and liabilities of a UK pension fund.

A cash-flow approach to valuing long-term liabilities does not provide the answers to all the problems. It can, however, make the problems more accessible to those who must take ultimate responsibility for the decisions which are made.

3 Cash-flow and asset & liability modelling

3.1 Cash-flow modelling involves projecting, through time, both the assets and liabilities of any financial organisation, looking at the cash-flows emanating from both sides in future years.

3.2 Initially a model must be constructed which encapsulates the behaviour of both assets and liabilities and, very importantly, the interaction between them. In the case of a pension fund, this interaction is likely to come in some way from the fact that both the assets and liabilities of the fund at any future point in time will depend, at least in part, upon the level of inflation from now until that time.

3.3 The following provides a brief description of how the assets and liabilities of a UK defined benefit pension plan might be modelled in practice.

Starting from the membership at the valuation date one can project forward, allowing typically for deaths, withdrawals, retirements and disability (usually assumed to be in accordance with the actuary's valuation assumptions). Payments from the plan such as pension and commutation payments would be calculated from the actual membership data. An allowance for new entrants needs to be made.

As well as the demographic elements of the basis one needs to make assumptions concerning the financial elements of the basis. The traditional approach to valuing liabilities would suggest that the actuary choose a fixed set of long-term return assumptions to apply to the assets of the plan, the level of general salary escalation and, where appropriate, the assumed level of price inflation.

3.4 One could simply take as a starting point such a fixed set of assumptions regarding the returns on, and income from, various classes of assets and apply them to the particular portfolio held by the plan. This would effectively replicate what is being done in the conventional valuation: the only difference being that the cash-flows are not being discounted to the valuation date.

3.5 Projecting forward the assets and liabilities of the plan in this manner provides a cash-flow approach to a valuation and might in itself be a useful exercise. There are a number of questions which one might ask. How does the return implied by the actual portfolio held compare with the interest rate you would use for the valuation? How do the decrements and new entrant assumptions affect the population? How does the average age distribution affect the contribution rate which will emerge at future valuations on the current funding method.

It would be unwise to read too much into the results of such an exercise, particularly over very long projection periods. It can be instructive nonetheless.

3.6 What can we say about the economic assumptions to which the above approach would lead? The UK accounting standard SSAP24 effectively requires disclosure by the actuary of a set of economic assumptions which he/she believes to be a 'best estimate'. One has only to look through the published accounts of many blue-chip companies to appreciate that the range of such assumptions adopted by actuaries is extremely diverse. Most actuaries, whilst being able to put together a well reasoned argument for their own preferred best estimate assumptions, would accept that there is a great deal of uncertainty attached to each individual item.

3.7 Accepting that the behaviour of the financial elements of the basis is one of the greatest uncertainties affecting pension plan funding leads one to consider whether it is possible to reduce this uncertainty. Figure 1 shows how the annual rate of price inflation in the UK has varied since 1919.

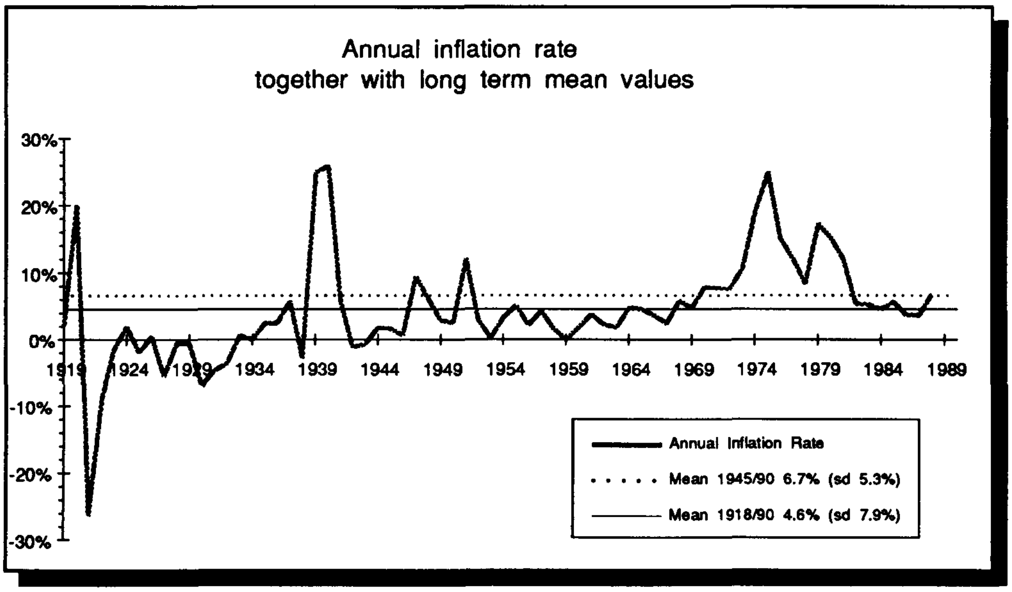

Figure 1

Looking solely at historical experience one could make the following hypotheses about future inflation:

i) Based on the experience from 1919-1990 a long-term mean rate of 4.5% per annum with a standard deviation of 8% per annum.

ii) Based on the experience from 1945-1990 a long-term mean of 6.5% per annum with a standard deviation of 5% per annum.

Alternatively, given our entry to the Exchange Rate Mechanism, one could hypothesise a low mean, low standard deviation scenario.

The essential point is that each scenario will affect differently the future development of the fund. Furthermore, simply carrying out valuations on the assumption of both high and low inflation levels overlooks the differing volatilities.

The traditional valuation approach treats each economic variable as fixed whereas in practice it is known that they will vary through time. Furthermore, one cannot be certain just how they will vary. Is there a way in which we can model this uncertainty? Moreover, by modelling the unknown elements can we gain useful insight into the impact of the financial elements of the basis on the plan?

3.8 The construction of such models is considered in more detail in Section 4. Assume, however, for the moment that we do use such a model to determine asset returns, salary and price inflation for each future time period. We are now in a position to project the development of the fund for as many years as are required for our purpose. At regular intervals a valuation on the actuary's long-term basis or some other basis (perhaps depending on financial conditions at the time of the valuation) can be carried out and a contribution rate calculated, solvency levels investigated etc.

3.9 Furthermore, pre-empting a little of the discussion in Section 4, we will assume that, in view of the uncertainties in the future economic variables, a random component is included in each variable which it is required to model.

The projection process described in 3.3 to 3.5 then represents one simulation of how the financial state of the plan will evolve over time. The next stage would be to repeat the simulation progress. What we have now is an asset & liability model. We can carry out any number of simulations, each one projecting the development of the plan over as many years as we want.

3.10 The number of simulations required in order to produce credible results needs to be established. This will depend on, amongst other things, the properties of the equations governing the behaviour of the economic variables.

3.11 It is also important to keep in mind the time period over which the projection can be expected to provide meaningful results. For time periods of less than a few years the projection is unlikely to accord with reality unless current investment conditions are taken into account. After periods in excess of ten years the results will be significantly affected by factors such as the new entrant and withdrawal assumptions.

3.12 Up to now we have been looking at the position where the asset distribution between the various asset classes held at the outset remains unchanged over future time periods. What would be the effect if the assets held by the plan were different? We can use the model to investigate the behaviour of the plan (in terms of on-going contribution rates or funding levels emerging at future simulated valuations) with different assumed investment portfolios.

3.13 What sort of information is of practical use to us? If we were to focus upon the valuation of the plan in say five years' time then the lowest expected contribution rate might result from putting 100% of the assets into the highest returning asset class. Such a strategy is also likely to lead to the biggest range of returns and therefore likely to be unacceptable to almost any group of trustees. One can similarly look at the portfolio giving the lowest variability in the expected contribution rate. Whilst this portfolio is likely to be more acceptable to the trustees there will be a price to pay in terms of loss of returns. The example in Appendix 1 considers these points in more detail.

3.14 There are one or two aspects of the liability modelling process which, I believe, merit further attention.

The decrements in general

It may be necessary to make additional assumptions to those that would be used in a traditional discounted cash-flow valuation. If early retirements take place on a relatively cost-neutral basis then they may have little impact on the present value of benefits. They will however affect the timing of cash-flows and therefore an assumption as to the number of such retirements may well be appropriate.

It may indeed be the case that the investigation is undertaken largely to investigate the effect of a large change in the membership of the plan.

New entrant assumptions

The new entrant assumptions, both in terms of numbers and age distribution, can only be arrived at after discussion with the employer. If there is a great deal of uncertainty it may be sensible to make two different assumptions to consider whether the results of the investigation are significantly affected.

The interaction of the employer's assumed new entrant level and the actuary's assumed level of exits can lead to a population which changes significantly. The actuary needs to ensure that the assumptions are self consistent and that they do not contradict the rationale behind the preferred funding method.

Should the decrements themselves be stochastic?

This is an interesting point. In practice it is likely that the uncertainties in the economic elements of the basis will far outweigh those connected with the decrements. However, the effect of a large number of withdrawals from pension plans during the 1980s was a significant contributory factor to the surpluses which arose.

It might be thought desirable to allow for different new entrant and/or withdrawal assumptions depending on the economic climate prevailing during the course of a particular economic scenario. One should be wary, however, of making the process too complex.

3.15 Returning to the point made earlier concerning the uncertainty in the future financial development of the fund arising out of the uncertainty in asset returns, inflation etc. I believe that it is possible to model the uncertainty referred to above, at least in part, through the use of econometric models and that, in doing so, one can reduce the overall uncertainty inherent in the traditional approach. This is essentially because the uncertainty is not as great as you would expect when you allow salaries to be related to the same inflation levels as investment returns. It also makes it possible to see what effect the uncertainty has upon your plan and make informed decisions against this backdrop.

3.16 The following example is far too simplistic. It does however illustrate the point I want to make.

Assume that inflation can be 10%, 6% or 4% over a certain time period (perhaps the 90th, 50th and 10th decile of a probability distribution). Assume further that the assets increase by 60% of the inflation rate plus a constant 3%. The liabilities are assumed to increase in line with inflation. We now look at two different cases. In the first case we simply take the liabilities as pre-determined, based on a mean anticipated inflation rate of 6%; in the second we assume that the growth in the liabilities depends on the actual level of inflation.

What can we say about the funding levels at the end of the time period?

In the first case we have:

Inflation	Assets	Liabilities	Funding Level
10%	1+.6x.10+.03=1.090	1+.06=1.060	102.8%
6%	1+.6x.06+.03=1.066	1+.06=1.060	100.6%
4%	1+.6x.04+.03=1.054	1+.06=1.060	99.4%

In the second case we have:

Inflation	Assets	Liabilities	Funding Level
10%	1+.6x.10+.03=1.090	1+.10=1.100	99.1%
6%	1+.6x.06+.03=1.066	1+.06=1.060	100.6%
4%	1+.6x.04+.03=1.054	1+.04=1.040	101.3%

Looking at the first case (where the liabilities are calculated by reference to an assumed long-term mean inflation rate) the variability of the funding level is greater than in the second case. Furthermore high levels of inflation appear to provide a favourable environment for the fund. The second case gives the 'true position'; the funding level is not as dependent on inflation as we had been led to believe, furthermore the fund actually performs better in a low inflationary environment.

What does the example tell us? In the first case above only half the problem has been 'captured' by the model. The interaction between assets and liabilities has been lost; the results can be extremely misleading. Over long time periods projections of the first type could lead to a much greater level of uncertainty than would be shown if both assets and liabilities were projected simultaneously.

3.17 The future behaviour of economic variables is uncertain. Whilst this is true, it is important to bear in mind that there is some additional information not captured by the traditional discounted cash-flow techniques.

Most actuaries are content to assume that, over the long-term, investment returns will exceed the general level of earnings growth which will in turn exceed the level of price inflation. Is it not equally valid to expect that the nominal returns on cash will vary less from year to year than the returns available from equities? Similarly, over long time periods one would expect the real returns from index-linked gilts to be less than those available from equities.

The next Section looks at ways one might go about capturing this additional information in a useful format.

4 Econometric modelling

4.1 In Section 3 it was suggested that modelling the uncertainty in the economic variables which affect the development of a pension fund might provide a better insight.

Moving away from the choice of fixed assumptions concerning the rate of growth in economic variables over future time periods a first approach might be to agree on a set of assumptions relating to the future returns, standard deviations of, and correlations between, the various asset classes. This approach, however, ignores the fact that the economy is made up of a series of highly interrelated processes. A high level of inflation in one year is likely to be linked to a high level of inflation in the following year.

Moreover, this method does not easily allow the liabilities to be treated in a consistent manner. In a period of high inflation nominal earnings increases are likely to be high. Similarly it is increasingly common to find the level of annual increase to pensions in payment being related to price inflation (the Limited Price Indexation proposals of the Social Security Act 1990 being just one example).

4.2 An alternative approach, which attempts to overcome this difficulty, is to use a more detailed model of the behaviour of the economy, which allows for inflation, and which can be used to project the future behaviour of the various asset classes. Such a model will incorporate an allowance for the behaviour of asset values and other financial variables to, in part, depend on their behaviour in the preceding year. The key inputs might be the long-term expected returns from each asset class and the long-term expected rate of inflation.

4.3 The model must take account of both past data and fundamental relationships between the different variables being modelled. For example the assumed rate of price inflation in any one period might be modelled as follows:

Current period inflation	= 50% (prior period inflation)
	+ 20% (two periods prior inflation)
	+ 20% (long-term inflation)
	+ constant
	+ random element

In incorporating a random parameter into the equations we are acknowledging that no deterministic relationship exists between the variables being modelled.

Other series, for example wage inflation, will depend upon, amongst other things, the level of price inflation.

4.4 It is a far from trivial exercise to build such a model and it is an area on which I hope the 1990's will see much debate. Models which are appropriate for predicting the returns on asset classes over a twelve month time horizon may be very different to those which are appropriate to a twelve year time horizon. In stark contrast to the twelve month time horizon, there are very few people involved in trying to build econometric models appropriate to the longer time frame. It is an area where there is plenty of scope for actuarial input.

It is important not to lose sight of what you are trying to achieve by the use of such a model and to be aware of the strengths and weaknesses of your chosen model.

4.5 Many early models were based solely on observations of past data. This approach is likely to be flawed for two distinct reasons.

First, because known historical effects (which are not being directly or indirectly modelled) will have distorted past data. For example if we do not intend to model the money supply then it is unreasonable to expect our model of the gilt market to pick up changes in the Government's attitude to funding Public Sector debt.

4.6 The second reason is a statistical point.

Readers may be familiar with the dangers of data-mining, whereby a model is chosen which provides the best fit to the observed data and is then validated by reference to the same data as was used to produce the model. Unsurprisingly such a model can be shown to provide an excellent fit to historical data. Despite the obvious fallacy of the approach it is still quite common for financial models to be built up and tested in such a way. Needless to say they do not behave anywhere near as well as expected when tested out-of-sample.

4.7 Data-mining is something which should be avoided at all costs. There is however a slightly more subtle way in which the sample can influence the choice of model. This is known as data-snooping; it arises when the properties of the data series influence the researcher's choice of model. A couple of examples may help to demonstrate the point:

i) Discovering spurious correlations between variables and building these into a model. An extreme example might be a link between the level of the stock market and weather patterns.

ii) The use of the long-term mean of the series being predicted. The price inflation model proposed in 4.3 is an example.

4.8 Both these approaches are likely to lead to an overstatement of the predictive powers of the model when looking at statistical tests which relate back to the sample data.

Avoiding (i) is not always easy. For example, historic data may suggest a strong link between nominal earnings growth from one time period to the next. Analysing the growth in Gross Domestic Product in the same way may give rise to the same conclusions regarding GDP. If it is now discovered that there appears also to be a strong link between nominal earnings growth in each period and GDP growth in the same period how should one proceed? It is only necessary to build two of the three links into our model for the third to follow automatically.

The position may be complicated if two variables, the behaviour of which appears to be correlated, are in fact both influenced by a third, unmodelled variable. Historic data for this third variable might not be available.

In the case of (ii) above one could attempt to get round the problem by using, at each point in time, the mean of the inflation rate prior to that point in time. The result of adopting such an approach is that the models often behave much less well than had been expected and less well than much simpler alternative models.

For the interested reader the subject is covered in more detail in an article by Dimson and Marsh[5].

4.9 The best practical solution to the problem of data-snooping seems to me to be a combination of two things. The first is to start off with some reasonably firm premises concerning the inter-dependence of the economic variables being modelled; the second is to allow for (and this can only be done in a subjective manner) known distortions to the observed data (eg the Government's policy for funding Public Sector debt when looking at the price of gilt edged securities).

Further considerations in model building

4.10 **Best estimate v. other approaches**

Actuarial bases can be chosen to be optimistic concerning future investment returns, pessimistic or realistic. In producing a model for the future behaviour of economic variables I believe that it is important to be as realistic as possible. The long-term characteristics of the basis should not therefore be too dissimilar from the actuary's own best estimate basis.

There are two important points arising from this.

First, if valuations are carried out on an actuarial basis which is pessimistic about future returns (perhaps a prudent approach) then a projection is likely to lead to falling contribution rates and/or rising funding levels since 'reality' (as predicted by the model) will, on average, be more favourable than the actuarial basis (which determines the contributions required) predicts.

Secondly, there is scope for discussion, particularly with the investment manager (or managers) concerning the choice of appropriate long-term assumptions. These should be agreed at an early stage.

4.11 Chaos

The mathematics which forms chaos theory has been with us for just over a decade now. It is interesting whether there are any consequences of direct relevance to econometric modelling. For the interested reader Appendix 2 to this paper puts forward "some chaotic thoughts".

5 Asset valuation techniques

5.1 Up until the early 1970's it was commonplace to find the assets of a UK pension fund being taken into the balance sheet at either book value or market value. The counter arguments to both approaches are well known (although both apparently continue to be in widespread use when considering the assets of certain other long-term funds).

5.2 Although the subject of asset valuation techniques may be thought to be somewhat incidental to asset & liability modelling, I think it merits some attention in this paper. The chosen method can greatly affect the outcome of an asset & liability modelling exercise which is intended to frame strategic asset allocation policy.

5.3 There are still many unresolved problems with the methods used for placing a value upon the assets of a UK pension fund for the purposes of an on-going valuation and it is an area which I believe has not been written about sufficiently in actuarial literature.

Heyward and Lander[6] writing in 1961 advocated an approach based on looking at the expected future income from the portfolio held. Nowadays the most frequently adopted approaches are based either on looking at smoothed market values or the cash-flows expected to be received from the assets held (or more frequently a notional portfolio).

5.4 What, if any, constraints are placed on, or guidance is given to, the actuary in valuing the assets?

i) GN9, as issued in 1984, requires the actuary to place a value on the assets which is "consistent with the method of valuing the liabilities". This is open to a wide range of interpretations.

ii) The Government Actuary's basis, specified for the purposes of determining whether or not there is an excessive surplus within the plan under the terms of the Income and Corporation Taxes Act 1988, prescribes a method for placing a value upon the assets. The method is based on looking at the discounted cash-flows arising from either the assets actually held or from a notional portfolio.

The answer therefore appears to be very little, although the discounted cash-flow approach is now enshrined in legislation.

5.5 For those unfamiliar with the techniques the following sets out an example of the discounted income approach as might be applied to a portfolio of equities.

The discounted income approach requires an assumption as to the rate of dividend growth in future years. If we take this to be g: the current dividend yield to be given by d and all future payments are to be discounted at interest rate i. The present value of an infinite stream of dividend income might then be expressed as:-

$$d \times \left(\left(\frac{1+g}{1+i}\right)^{\frac{1}{2}} + \left(\frac{1+g}{1+i}\right)^{1\frac{1}{2}} + \left(\frac{1+g}{1+i}\right)^{2\frac{1}{2}} + \cdots \right)$$

This can be simplified to the following expression:-

$$d \times \left(\frac{1}{i-g}\right) \times (1+g)^{\frac{1}{2}} \times (1+i)^{\frac{1}{2}}$$

This is equivalent to assuming that the par yield for the equity market is given by:-

$$\frac{i-g}{(1+g)^{\frac{1}{2}} \times (1+i)^{\frac{1}{2}}}$$

In other words, when the yield on the equity market is equal to the above the actuarial value will equal the market value. Typical values for i and g might be 9% and 4% respectively leading to a par yield (on the above basis) of around 4.7%.

It is relatively common for the actuary to assume that the entire portfolio is invested in the Financial Times Actuaries All-Share Index and multiply the market value of the portfolio by the ratio of the yield on the Index to the assumed par yield.

5.6 Although there is much to commend the above approach outlined in 5.5, one can be left feeling that this part of the valuation is a little ad hoc. In practice mightn't one expect dividend growth to fluctuate as we move through various stages of the economic cycle? Surely the level and volatility of future inflation has some bearing on the future value of assets?

5.7 Perhaps 1990 provided one of the clearest demonstrations of the lack of public understanding of the way in which actuaries incorporate the assets of a pension fund into the valuation process. Whilst the stock market fell by some 9.7% (as measured by the Financial Times Actuaries All-Share Index after allowing for reinvestment of dividend income) the actuarial value rose by some 16.5%. The difference being attributable to a growth in dividends of some 10.5% over the year.

As a result trustees are, on the one hand presented with articles forecasting "the end of the decade of the equity" whilst on the other, the actuary is reporting a healthy investment profit for the year. On this occasion the result may be a pleasant surprise for the uninformed sponsor/trustee; if dividend growth slows dramatically during the early 1990's many funds could be in for one or two nasty and unexpected surprises.

5.8 Is the question of the value placed on the assets by the actuary always covered in sufficient detail in the presentation of the results of an on-going actuarial valuation? In my experience it is not uncommon for the trustees of a fund have little understanding of how the value of the assets that appears in the valuation balance sheet is derived.

5.9 The asset valuation method is just one of those areas which an asset & liability model forces us to review closely.

In placing an actuarial value on a particular asset class it is helpful to think of the actuary as doing two distinct things. First he/she is smoothing out fluctuations in market values. Secondly he/she is implicitly indicating whether, on the long term assumptions employed in the actuarial basis, a particular asset class appears under or over valued at the valuation date. Figure 2 indicates the effect of using actuarial values for a portfolio of equities (as set out in 5.5) compared to the market value.

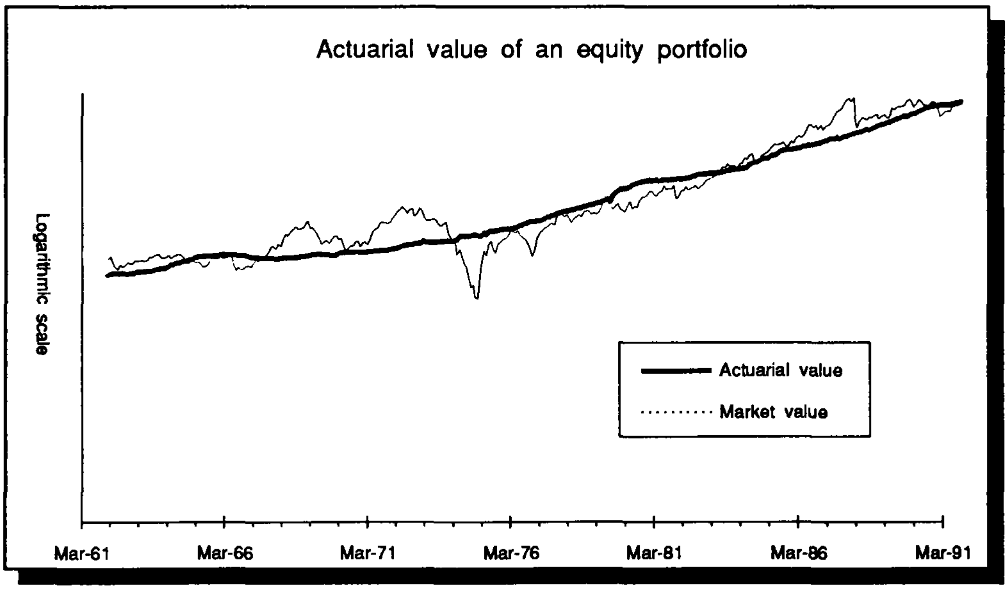

Figure 2

The actuarial value can be seen to have a strong smoothing effect. The positioning of the line showing actuarial value is however dependent on the assumed par yield.

5.10 Now consider the position where we are using an asset & liability model to determine the strategic asset allocation policy which gives the highest probability of meeting specific objectives. What value should be placed on the plan's assets at future points in time when it is necessary to compare the assets and liabilities of the plan? Let's consider a number of possible approaches:-

i) Market values

In this case the model will tend to lead to portfolios which have a higher proportion of cash and fixed interest securities than one might perhaps otherwise expect. This is because (as can be seen from figure 2) the smoothing methods used by the actuary are very powerful. They have the effect of reducing significantly the volatility of the price over time of those asset classes exhibiting a steady growth in income. As the time period on which you are focussing increases cash and fixed interest investments will, however, become less appropriate since over these periods the effect of lower expected returns from these asset classes will begin to predominate.

Since, in reality, the majority of actuaries do adopt such smoothing techniques the use of market values in such an exercise is likely to be misleading.

ii) Discounted cash-flow approach (notional portfolio)

This might mirror the approach adopted in the actuary's current valuation basis. If we assume a notional portfolio consisting entirely of UK equities then, to some extent, we artificially reduce the volatility of UK equities relative to the other asset classes. Figure 3 shows both the market value and actuarial value for a portfolio of gilts.

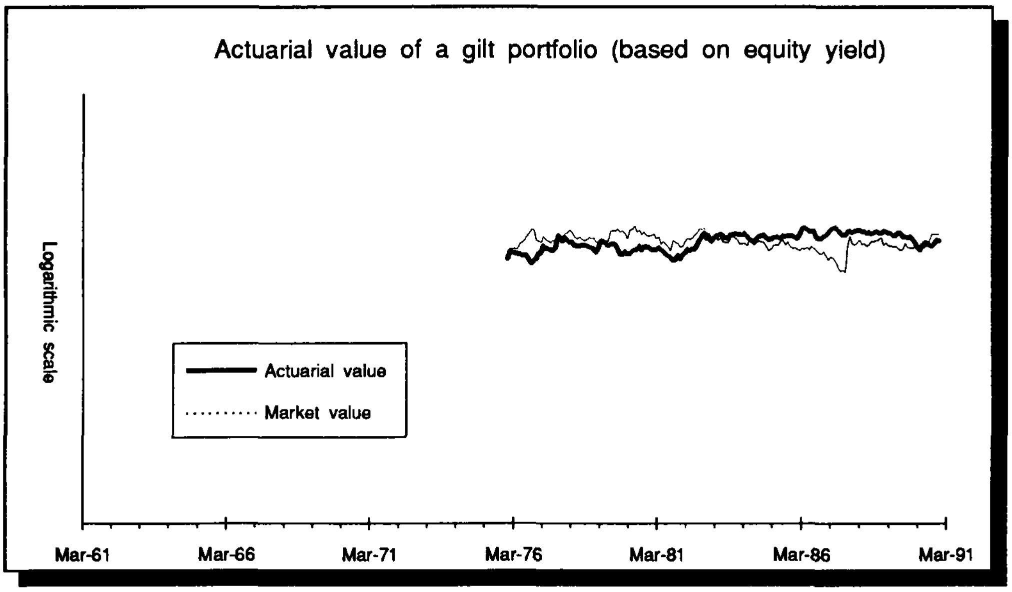

Figure 3

The actuarial value of gilts has however been calculated by assuming a notional reinvestment in equities; volatility is not reduced. The same would apply to other asset classes.

This may however reflect what the actuary is doing in the on-going valuation. If such an approach is likely to continue then it could be argued that this method is appropriate.

There is a further subtle point concerning the use of the discounted cash-flow approach to valuing assets within an asset & liability model. In 5.9 I suggested that as well as smoothing returns the actuary was effectively indicating whether, at the time of the valuation, a particular asset class was under or over valued relative to the assumed long term basis. The econometric model underlying the model will also define a long-term economic basis and relative to this particular asset classes may appear under or over-valued from time to time. It is important that, on average, each asset class is valued in such a way that its 'average value' relative to the long-term assumed economic basis is equal to the correct par value. If this isn't the case then one can find that for example every £1 invested in gilts is worth, on average, £1.10. This will inevitably lead to distortions.

iii) Discounted cash-flow approach (actual portfolio)

This presents a number of practical problems. It is, for example, difficult to value a portfolio of Japanese equities on this basis. The current dividend yield may well be below 1% per annum; what assumption should one make concerning future dividend growth? It is difficult not to build in personal prejudices at this stage.

iv) Average market values

If we value each asset class by looking at the average market value over a three or five year period then the smoothing effect can be similar to that obtained through using actuarial values.

This method also has the advantage that no subjective judgements concerning the growth of dividends or rental income is required.

5.11 I do not intend to consider the above points further here. The above is an attempt to give an indication of some of the pitfalls and some of the issues which should be addressed by those using such models.

I believe that the valuation of assets for long-term funding purposes is an area where there is tremendous scope for actuarial input.

6 Asset & liability modelling as a means of determining strategic investment policy

6.1 Throughout this paper I have tried to stress that asset & liability modelling does not, by any means, provide all the answers to the questions one might like to pose concerning the pre-funding of long-term liabilities.

That said, the methods do form a particularly powerful technique for devising a long-term investment strategy. This is an area where I am concerned that we, as actuaries, have perhaps failed to grasp the nettle. We are, I think, uniquely qualified to make sensible comments on the appropriateness of the assets as a match for the liabilities, indeed GN9 explicitly requires the actuary to comment in a valuation report if he/she feels the investment policy to be inappropriate. In 1986 Gardner in a paper presented to this Society(7) similarly criticised the profession for its reluctance to give positive advice in the area of advising on a coherent long-term investment strategy. I'm not sure that we've come all that far in the six years that have elapsed since those remarks were made.

6.2 For many years actuaries have provided limited advice on long-term or strategic asset allocation, either formally or informally. Historically this was based on the actuary's general knowledge of the particular plan's liabilities, the policy being pursued by other similar plans and the actuary's general knowledge of investments.

6.3 The development of asset & liability modelling can, in part, be attributed to the desire by a number of actuaries to advise pension plan sponsors and/or trustees on strategic asset allocation based on a more scientific approach to the problem. Modern Portfolio Theory is the progenitor of most such approaches.

6.4 Modern Portfolio Theory in turn has its foundations in the work of Harry Markowitz(8). Markowitz described how an investor might go about choosing between different asset mixes within a mean-variance framework. The aim being to derive an efficient frontier of portfolios having, for a given expected return, the minimum variance. Much further work has been done in this area by, amongst others, academics and quantitative teams within investment houses who have, over the past decade, also tried to address the question of the trade-off between investment risk and return.

6.5 Economic theory has moved on from the mean-variance framework as set out by Markowitz to look at other possible 'environments' within which it is assumed that investors make their decisions. The Capital Asset Pricing Model and Arbitrage Pricing Theories provide two such examples. Some of the assumptions underlying these theories are quite wide ranging and it is a pity that, to date, these theories do not appear to have been picked up and developed by the actuarial profession in the UK. There is plenty of scope for further research in this area.

6.6 It is still the case today that a large amount of pension fund money is given to investment managers with little more instruction than "Do your best!"

I shall return to this point in the following Section; needless to say that, with such limited guidance, it is only natural that the following two things should happen:

i) Investment managers will inevitably seek to limit their commercial risk by following closely the investment policies being pursued by their competitors.

ii) Investment managers will, try to rationalise the chosen approach and reassure their clients that their funds' money is safe in their hands. Hence phrases such as "minimising risk whilst maximising return" and "long-term approach" predominate in investment management marketing literature.

6.7 The results of an asset & liability model which are used to investigate the effect on funding levels and/or contribution rates of adopting a variety of different investment strategies can be of considerable benefit.

At the outset it is clear which investment policy will lead to the lowest expected contribution rate in say five years' time. One has simply to put all the fund's assets into the asset category which has the highest expected return. This is however likely to lead to the possibility of an unacceptably wide range of likely contribution rates emerging (high returning asset classes being associated with high volatility). Choosing the portfolio which appears to give the greatest certainty over the contribution rate emerging in five years' time is, however, likely to necessitate an unacceptable sacrifice in investment returns foregone. There is an infinite range of portfolios in-between and the results of the modelling can provide a useful framework for a discussion on attitudes to both risk and reward as well as the overall objectives.

6.8 The results of the modelling exercises should be seen as providing a framework for discussion. They are not providing the answer to the question of what should be the fund's strategic investment policy. Inevitably the final decision must be one with which those responsible for the investment of the assets are comfortable. No two groups of people are likely to reach the same conclusion, even given identical circumstances. The majority of the asset & liability exercise should, I believe, take the form of an educative process looking at the trade-off between the risk of failing to meet particular objectives relative to the possible rewards.

6.9 **Implementation issues**

Whilst there has been much discussion concerning the use of asset & liability modelling as a technique for deciding on the appropriate long-term asset mix, the short-term problem of moving from the existing asset mix to the preferred long-term mix is often overlooked.

If the long-term asset mix has been arrived at using an asset model which is based on long-term expected rates of return on various asset categories with no allowance for current market conditions, then the implementation of the policy needs careful consideration.

If the investment policy itself is viewed as a strategic investment decision, then the implementation of the policy should probably be viewed as a tactical decision. In order to implement the policy the advice and cooperation of the investment manager is required. Only he is in a position to provide knowledge of the short-term day to day movements of the markets which will assist in implementing the new policy. Often the implementation of the results of a liability driven asset allocation exercise is accompanied by the reorganisation of the way in which the pension fund is managed. This adds to the complexity of the implementation.

Ideally the investment manager will already be aware that the exercise is being carried out and may have already had some input into the process regarding his views on long-term rates of return. In such circumstances the time-table for the implementation of the new policy can often be agreed relatively quickly.

During and after implementation it is vital that the investment manager is aware of how performance is being measured.

7 The investment manager's perspective

7.1 What of the investment management community's attitude to the involvement of consultants in determining strategic investment policy?

In practice views appear to remain divided. Whilst many welcome the combination of plan sponsor and/or trustees together with the actuary providing specific guide-lines for the strategic asset allocation mix, there are many who remain to be convinced.

7.2 Currently the assets of the majority of self-administered large UK pension plans are managed by one or more of up to 100 investment managers on a balanced or discretionary basis. The investment manager is responsible not only for the individual stocks chosen, but also the division of the fund between different asset categories such as equities and bonds.

7.3 During the 1980s the predominance of league tables of performance has tended to lead to a degree of homogeneity in asset allocation across the spectrum of investment managers. This is, I believe, partly the result of the lack of positive guidance which has been given to them as mentioned in 6.6.

As an aside it is interesting to speculate as to what the composition of the industry average portfolio might have looked like if index-linked gilts had been used to finance Government borrowing some ten years earlier or perhaps even if the Government had chosen to issue National Average Earnings linked bonds.

7.4 It is unfortunately all too common to come across a situation where the trustees are unhappy with the performance of an investment manager because of his failure, in their opinion, to stick to the strategic investment guide-lines which they believe they have given. The investment manager for his part may have been given a few vague objectives, often amounting to little more than "We want above average performance for our fund". Often the root cause is that the trustees are not sufficiently clear as to what their objectives are. Investment managers must however take some of the blame here for not insisting on realistic and rational objectives at the outset.

7.5 The Financial Services Act requires investment managers to obtain a written statement of the client's investment objectives. Unfortunately in the majority of cases this has simply lead to phrases such as "maximise returns with an acceptable degree of risk" becoming more firmly established. In possession of such a mandate it is hard to see how one can be faulted, in the majority of cases, for simply following the industry average approach on asset allocation.

7.6 Let us assume that the maximisation of returns with an acceptable degree of risk can be skilfully transposed by the investment manager into a practical investment policy. There still remains a fundamental problem in that one is intuitively led to think of risk in terms of the variability of the market value of the assets.

A policy derived in such a way may therefore be inappropriate for two reasons:

i) The example in 3.16 showed that the important fund variables (eg funding level for example - as opposed to the market value of the assets) may behave very differently from the way one would expect from considering the assets alone.

ii) In 5.10 we concluded that the use of market values may well be misleading. In determining contribution rates and on-going funding levels the actuary is likely to look at the actuarial value of the assets. In this case the overall returns from equities may not be as volatile as one might otherwise expect.

The essential point here should be that the actuary has an important part to play in explaining both these points and in advising on the setting of long-term investment objectives.

7.7 The end of the 1980s has seen an increasing trend in the UK towards the use of specialist managers. A manager may, for example, be appointed to manage a portion of the portfolio investing solely in fixed interest securities.

A similar situation can arise when a larger company finds itself able to employ sufficient expertise to manage its own pension plan money in-house. The resources required to run a portfolio of overseas assets however may be more efficiently purchased from an external organisation.

In both situations an asset & liability modelling exercise provides one way in which a decision can be taken regarding the proportion of the fund to be given to the specialist or external manager.

One drawback to the division of the fund into a number of specialist funds is that the proportion of the fund in each asset class will vary over time due to differing returns on the various classes. There is a consequent need for either regular rebalancing or some additional expertise regarding the relative proportions to be held by each manager over short-term periods.

The short-term variation of the proportion of the fund in each asset class is known as Tactical Asset Allocation. At the present time there are relatively few investment managers in the UK with an established track record in tactical asset allocation. This means that, if the specialist route is adopted, potential gains from tactical asset allocation are often foregone. It is an area where several investment managers are currently attempting to establish credibility. Success may facilitate more funds adopting a specialist manager route with a consequent increase in the demand for asset & liability modelling exercises.

7.8 On a final note, it seems to me that investment management in a league table dominated environment must be a relatively unrewarding task. By definition only half of the investment managers can be in the top half of the league table at any one time.

If, at the other extreme, we were to move to a world in which each fund had its own customised benchmark and objectives then (at least in theory) it is possible that all investment managers could be regarded as successful.

7.9 In summary, the close involvement of the investment manager in either the setting of long-term investment policy or, at the very least, in the implementation of a policy which has been agreed with the trustees, can only help to improve the level of understanding between investment managers and those responsible for the performance of long-term funds.

Actuaries too have an important role to play in providing advice on the establishment of rational long-term investment objectives. Cash-flow models can provide a powerful catalyst both for the discussion of the issues and for the framing of these objectives.

8 The trustees' and/or the sponsor's perspective

8.1 Trustees come from a variety of different backgrounds. Many have little prior, direct experience either of the investment of large amounts of money or with the funding of a defined benefit pension plan.

8.2 Practical experience of asset & liability modelling would indicate that there are a number of important benefits to be gained by the end-user of the models outlined in this paper:

i) A clearer understanding of the nature of funding to meet long-term liabilities and the nature of risk.

ii) Better returns for a lower degree of risk.

Whilst this maybe true it is a subtle point to get across. It is (not surprisingly) very hard to explain to a group of trustees that although the performance of the fund was third quartile there was less chance of fourth quartile performance had inflation been higher.

iii) A framework in which to establish customized objectives.

Possible examples include:

- Minimising the variability of expected future contributions.
- Maximising the probability that pension increases at or above a certain specified level can be maintained.
- Maintaining the ratio of assets to liabilities above a specified level.
- Control of SSAP24 or FAS87 costs.

iv) A practical framework for determining asset allocation decisions.

It must, however, be remembered that the model is not a black box; you must be happy that you can live with whatever policy is finally implemented.

v) A significant 'comfort factor' gained from having been through the learning process.

8.3 The area of setting objectives is an important one. There are many large pension funds which set inconsistent or unrealisable objectives for their fund managers. An example might be the following:

- An annual real return of at least 4% per annum.
- Asset allocation to be within prescribed band-widths.
- Performance to be above the industry average.

8.4 It is also important that the trustees' or sponsor's objectives are not only rational but also potentially realisable. A company wishing to control its SSAP24 or FAS87 costs within a very narrow target range may find that the results of an asset & liability study indicate that, on the assumptions employed, such control is not possible.

If satisfying such an objective is important then the best approach may well be to use an asset & liability study in the first instance to investigate the variability of the funding level. The next stage might be to investigate ways of reducing the variability in the figures (without reducing the return on the portfolio unacceptably). The remainder of the protection required might then be obtainable through for example the use of futures and/or options.

8.5 Nowadays the average pension fund trustee is likely to be a company executive who is completely at home with projections of revenues and expenditure. These projections will be based on a number of assumptions concerning various parts of the business and will be used to determine the allocation of resources within the company. Why should the pension fund (which with the advent of accounting standards may be significant item in the company balance sheet and/or profit & loss account) be treated in a different way?

One could argue that the pension fund really is, and should remain, separate from the company, or that the liabilities must be viewed in the context of a much longer time framework. Company plans are however formed in the shorter term; revenues, including pensions' costs, need to be controlled. I believe that the mystery which so often surrounds the pension fund should be unravelled: a cash-flow approach may provide the only meaningful way in which to do this.

8.6 It is important to remember throughout this paper that there is no one right answer. What is right for one group of individuals will be regarded by another group, faced with the same set of circumstances, as a risky strategy. What we are interested in throughout is developing a framework within which informed judgements can be made.

The paper inevitably contains many references to pension fund trustees and/or the plan sponsor. Whilst in specific instances the views of the two parties may (for very sound reasons) differ, I do not differentiate between the two in this paper. If we treat the primary aim of an asset & liability modelling exercise as being one of education then it does not seem unreasonable to expect both parties to profit. The root cause of many of the worst battles in history can be attributed to ignorance on the part of one or more of the protagonists!

9 Conclusions

9.1 As stated in the introduction to this paper, I firmly believe that the 1990's will see the use of cash-flow modelling exercises become much more widespread. This will I believe cover not only pension funds, but all financial institutions investing to meet long-term liabilities. The models will not only be used to determine investment strategy but also as a means of investigating the development of the fund over time and to effectively provide some sort of sensitivity analysis.

9.2 Computer technology will obviously play a major part here. Simulation exercises which took some two hours on the latest machines widely available eighteen months ago can now be done in under twenty minutes on machines costing no more than their predecessors.

9.3 The introduction of SSAP24 has brought the area of pension plan funding to the attention of the finance director and brought funding issues to the fore when setting strategic business objectives. This development can only be beneficial for asset & liability modelling in the long run.

9.4 If the techniques are to be used to their full potential then I belive that it is necessary for us to address ourselves to the following areas of research and understanding:

Technical framework

This is required both in order to build up a mathematical model for the liabilities and in order to produce a coherent framework for placing a value upon the assets; a mathematical model for the assets perhaps? Many actuaries have been doing the former for many years, but have, up to now, given very little consideration to the second part of the problem. It was an interesting observation at a recent conference I attended that a session dealing with asset valuation techniques was able to put forward very few new ideas as to how the assets of a pension fund should be brought into the balance sheet. This is an area in which we must see further ideas and debate.

In this paper I have consciously not strayed too far into the technicalities of asset & liability modelling. As an introduction to these areas I recommend two papers by my colleague Peter Lockyer(9).

Explanation and Education

The ideas underlying a modelling exercise are complex. Furthermore the results of several thousand simulations cannot be said to make interesting bed-time reading! A clear grasp of the concepts together with good presentational skills is a basic requirement for those who wish to succeed in this area. Many members of the profession are currently sceptical of the merits of asset & liability modelling as an actuarial tool: perhaps the education process should begin here!

'Quality Control'

We have a responsibility to ensure that the work carried out in this area is technically sound and that it is presented in an informative way. It is counter-productive if misleading claims are made as to its worth. Asset & liability modelling is a complicated subject; it is perhaps all too easy to blind the end-user with science (to the detriment of the understanding of all concerned and perhaps to the short-term financial benefit of the consultant alone).

9.5 I do not believe that it is an exaggeration to say that asset & liability modelling will, over the coming decade, provide the actuarial profession with some of its greatest challenges. In many instances I have focussed on the application of asset & liability modelling to a UK defined benefit plan, however, a large number of the points made are equally valid in considering the assets and liabilities of any financial institution. In particular many of the ideas in this paper are relevant to a General Insurance environment[10].

9.6 Throughout the paper I have repeatedly referred to the need to educate and inform those with ultimate responsibility for the investment of funds to meet long-term liabilities. Whilst this education could take place alongside the current valuation methods, I am convinced that a cash-flow approach provides a powerful catalyst for the addressing and debating of such issues.

I firmly believe that, for those involved in the funding of long-term liabilities, cash-flow modelling techniques and asset & liability models provide the way ahead.

10 References

(1) National Association of Pension Funds (1991), Committee of enquiry report into investment performance measurement.

(2) Redington F M (1952), Review of the principles of life office valuation. JIA 78(3), pp 286-315.

(3) Wise A J (1984), The matching of assets to liabilities. JIA 111(3), pp 445-485.

Wilkie A D (1985), Portfolio selection in the presence of fixed liabilities. JIA 112(2), pp 229-277.

Together with subsequent papers.

(4) Arthur T G and Randall P A (1989), Actuaries, pension funds and investments. JIA 117(1), pp 1-27.

(5) Dimson E and Marsh P R (1990), Volatility forecasting without data-snooping. Journal of Banking and Finance.

(6) Heyward G and Lander M (1961), Pension fund valuations in modern conditions. JIA 87(3), pp 314-338.

(7) Gardner T J A (1986), Pension fund performance measurement - the way ahead? JSS 30, pp 163-179.

(8) Markowitz H M (1959), Portfolio selection: efficient diversification of investments. John Wiley & Sons Inc., New York.

(9) Lockyer P R (1990), Further applications of stochastic investment models. Presented to SIAS 6 March 1990.

Lockyer P R (1989), The implementation of liability driven asset strategies. Unpublished.

(10) Daykin C D, Bernstein G D, Coutts S M, Devitt E R F, Hey G B, Reynolds D I W and Smith P D (1987), Assessing the Solvency and Financial Strength of a General Insurance Company. JIA 114, pp 227-310.

Coutts S M and Clark G J (1991), A stochastic approach to asset allocation within a general insurance company. Transactions of 2nd AFIR international colloquium V4, pp 95-112.

Appendix 1 - A simple case study

The following is a simple example of, and comments on, the simulated results of an asset & liability model study as they could be presented to both the plan sponsor and the trustees. The example is based loosely on a number of such exercises carried out for a number of medium/large UK pension funds.

The primary objective is to review the current investment strategy. Currently the fund is held on the trustees' behalf by one investment manager who believes that the trustees are looking for him to provide above median performance in each year.

The first stage of the process is to establish the trustees' objectives in determining investment policy. This is not as easy as it may first seem. The input required at this stage varies enormously between different groups of people. The investment of time is however well spent; failure to explore fully the objectives at this stage can result in much back-tracking at a later stage. An initial response to the question from the trustees might be along the lines of:

> *"To maximise the return on the fund, thereby maximising the funds available either for pension increases or for benefit improvements."*

If this were the whole story then the trustees would simply invest the plan's assets in the asset class which they expected to produce the highest return.

The sponsor for his part may frame an objective along the following lines:

> *"To maintain a relatively stable annual contribution to the plan. This should be as low as possible without prejudicing a competitive level of benefit provision from the plan."*

One problem with the trustees preferred approach is that the level of contributions required from the sponsor is likely to vary unacceptably. Recognition of this often leads to a statement of objectives being expressed as:

> *"To maximise the return on the fund with an acceptable degree of risk."*

Combining the above and rephrasing slightly we arrive at our statement of objectives for the purposes of this example:

"To maximise the surplus that can be expected to emerge in average conditions in the plan on an on-going basis, thereby maximising the funds available either for a reduction in the level of contributions required, pension increases or for benefit improvements."

"At the same time to minimise the variation in the sponsor's likely contribution in such circumstances."

These objectives are of course to some extent contradictory. A major part of the exercise is to explore the issues sufficiently in order to allow the plan sponsor and trustees to decide on an optimal balance between these two objectives. The views of the two parties may well differ as to the weight which should be given to the two primary objectives. The resolution of any conflict is also an important goal; compromise may well be necessary.

Besides a clear statement of the objectives there are two further items which will have a significant impact upon the analysis. The first of these is the time horizon over which the policy is designed to meet the stated objectives. In this case the position immediately following the actuarial valuation in six years' time has been chosen. The second is the method used to value the assets. We will consider the effect of using both a three year average of market values and market values.

The liabilities, together with the associated cash-flows (eg regular pension payments, lump sum commutation payments and contributions received from both the members and the sponsor), of the plan need to be projected as described in Section 3. It is assumed that the plan will provide an annual pension increase of the lesser of 5% or the increase in the Retail Price Index over the previous year.

The returns and cash-flows from the plan's investments are also projected based on an econometric model.

Several thousand results are simulated and tabulated. The final stage of the process is optimisation. In this case the optimisation centres on the expected contribution rate required by the sponsor. For each expected contribution rate, it is possible to determine the combination of investments in various asset classes which produces the lowest standard deviation (variability) in this contribution rate. The totality of all such portfolios forms an efficient frontier. Figure 4 shows this in graphical form.

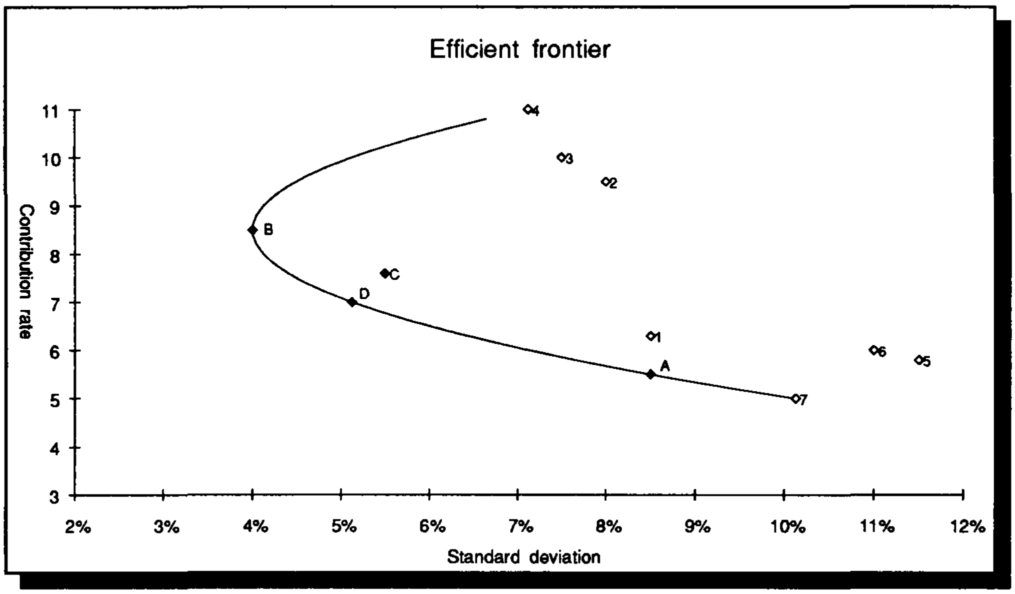

Figure 4

The points labelled 1 to 7 indicate the expected contribution rate emerging from the valuation if the entire portfolio had been invested in each of 7 different asset classes in turn.

The points labelled A to D represent the four portfolios discussed below.

Figure 5 shows, for each of the portfolios A to D, not only the expected contribution rate but also a graphical indication of the variability in the contribution rate which can be expected on the assumptions employed.

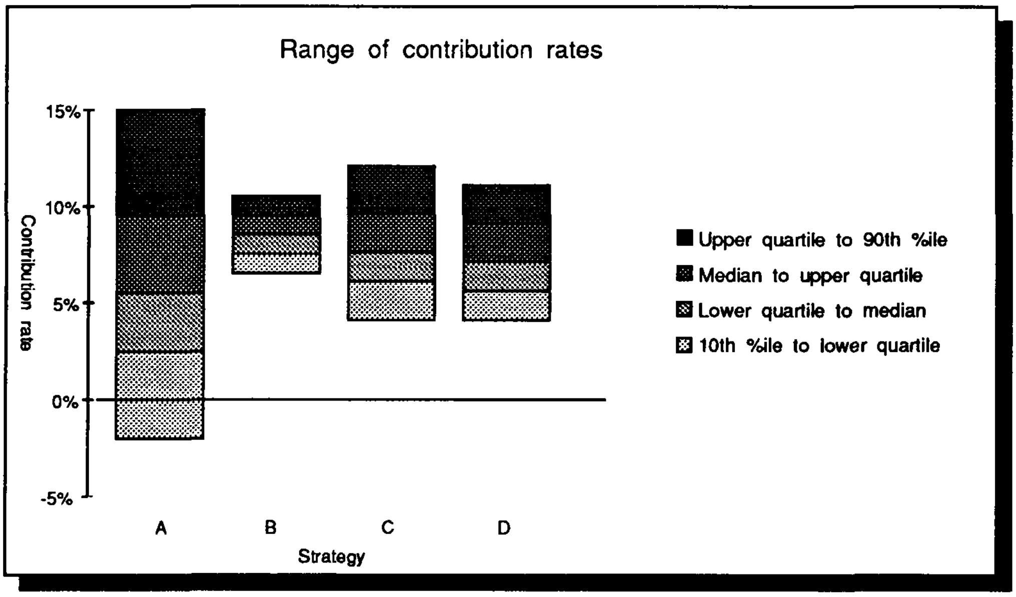

Figure 5

Outline of Results

The first objective may be expressed as maximising reward whereas the second objective represents the need to minimise risk. A key feature of interpreting the results of the modelling process is that of determining the optimal trade-off between 'risk' and 'reward'.

Strategy A: This is the strategy which gives the first objective total priority, consisting of an investment of 100% in overseas equities.

Strategy B: This is the strategy which gives the second objective total priority, simply seeking to minimise the volatility in the level of the sponsor's contribution. It consists of 3% in UK equities, 41% in fixed interest securities, 55% in index linked securities, and 1% in cash.

Strategy C: This distribution represents the approximate average asset allocation of a UK pension fund. Namely a portfolio consisting of 60% in UK equities, 23% in overseas equities, 9% in fixed interest securities, 6% in cash and 2% in index linked securities.

Strategy D: This strategy represents the portfolio which minimises the sponsor's contribution 75% of the time. It consists of a portfolio of 68% in UK equities, 5% in index linked securities and 27% in overseas equities.

The bar for strategy A shows that solely meeting the first objective of maximising the surplus results in the highest possible variation of the contribution rate.

Strategy B produces the lowest likely variation in the contribution rate. However, it also gives rise to the highest level of contribution rate at the 50th percentile.

Strategy C, the asset distribution of the average UK pension plan, provides a lower contribution in favourable conditions than strategy B. This is shown by a lower contribution rate at the 10th decile. However this is at the expense of a higher potential contribution in less favourable conditions, represented by the 90th decile.

Strategy D was chosen to illustrate the trade-off between risk and reward. Out of all the possible strategies it has the smallest contribution rate 75% of the time immediately after the valuation in six years' time, represented by the upper quartile line on the graph.

Time horizons

Strategy D was chosen so as to produce the lowest contribution rate after the valuation in six years' time in 75% of the simulations. If this were the agreed objective, but it was also required to impose the same conditions on the contribution rate emerging from the valuation conducted in either three or nine years' time, then the optimum strategies would be respectively strategies E and F:

Strategy E: 55% in UK equities, 5% in fixed interest securities, 17% in index linked securities, 3% in cash and 20% in overseas equities.

Strategy F: 75% in UK equities and 25% in overseas equities.

These results illustrate that, if it is possible to take a longer term investment view, then it is possible to adopt a more aggressive investment strategy. Adopting strategy F would lead to lower levels of contributions (or additional funds for pension increases and/or benefit improvements) but would however expose the sponsor to greater variability in the contribution rate in the shorter term.

Asset valuation method

The above analysis has been carried out based on a three year average of the market value of the investments held for the purposes of determining the contribution rate This approach reduces the effect of short-term fluctuations in market values on the sponsor's contribution rate.

Repeating the analysis but with the market value of the plan's investments being used to establish the contribution rate leads to a somewhat different picture.

The optimum strategy (ie minimising the expected contribution rate in 75% of the simulations in this example) at the valuation in six years' time is:

Strategy G: 50% in UK equities, 9% in fixed interest securities, 16% in index linked securities, 8% in cash and 17% in overseas equities.

The result of adopting this approach is a marked increase in the variability of the contribution rate. It demonstrates that, as a result of the smoothing techniques adopted by the actuary, the investment policy can be structured so as to achieve the high investment returns, without the need to invest in lower performing asset classes in order to cushion the plan against volatile returns on its investments.

An alternative way in which to view this is that the use of a smoothed actuarial value for the plan's assets when determining future contribution rates permits the taking of a longer term view when making the investment decision without the full exposure to the shorter term risks. This can be seen, in part, by the similarities between the portfolios in strategies E (a 3 year view based on an actuarial value approach) and G (a 6 year view based on a market value approach).

Conclusions which can be drawn

The above example has been greatly simplified. As I have maintained throughout this paper I believe that the discussion and education process which should accompany such an exercise is of at least as much value as the results themselves. Indeed, a grasp of the principals involved is almost certainly a necessary pre-requisite to the practical implementation of the results of the exercise.

The conclusions which can be drawn from the above analysis however might be summarised as follows:

i) Because of the smoothing that can be achieved by the asset valuation method used for actuarial valuations, the stated objectives can best be met by investing in those assets expected to give the highest return in the long-term, that is equities.

ii) Virtually all the liabilities of the plan are related to either price or wage inflation, and the most appropriate investment policy is therefore one which involves investment in those assets with the highest expectation of providing returns which relate to these factors, once again equities.

Several health warnings should be given. It is important to bear in mind that all of the conclusions drawn have been based on an entirely quantitative approach. No allowance being made for any subjective perceptions.

Although the asset valuation method smooths out much of the short-term volatility inherent in equity investment, it cannot smooth out long-term variations in returns arising from changes in fundamental economic conditions. Thus, if poor fundamental investment conditions, such as prevailed in the 1970's, persist a deficit will emerge, and conversely a surplus will emerge in the case of beneficial conditions. Such extremes are in fact represented by the boundaries of the probability range of contribution rates shown by the model.

Both the plan sponsor and the trustees may wish, and indeed may see it as a justifiable part of the balance between risk and reward to make some allowance for the following subjective perceptions:

i) The fact that the quantitative analysis is derived, in part, by a consideration of the relationships which have existed between the various asset classes in the past. Such relationships may not hold in the future.

ii) The perceived risk of committing all the plan's assets to one asset class.

iii) The perceived risk that in following a policy different from that pursued by the majority of UK pension funds, there will undoubtedly be years when the investment policy proposed will produce a return below that of the average fund.

iv) A concern on the part of both the trustees and the sponsor that they are taking on a responsibility which in the past has been undertaken by the investment managers.

Appendix 2 - Some chaotic thoughts!

What is chaos? Let's start with a fairly innocuous looking equation:

X(t+1) = k x X(t) x (1 - X(t))   where 0 X(t) 1   X(0) = 0.5

How does X(t) behave as we change the value for k? As a starting point let's take k = 2.5 and look at X(t) for the first 40 time periods. This gives the, relatively uninteresting, picture shown in figure 6.

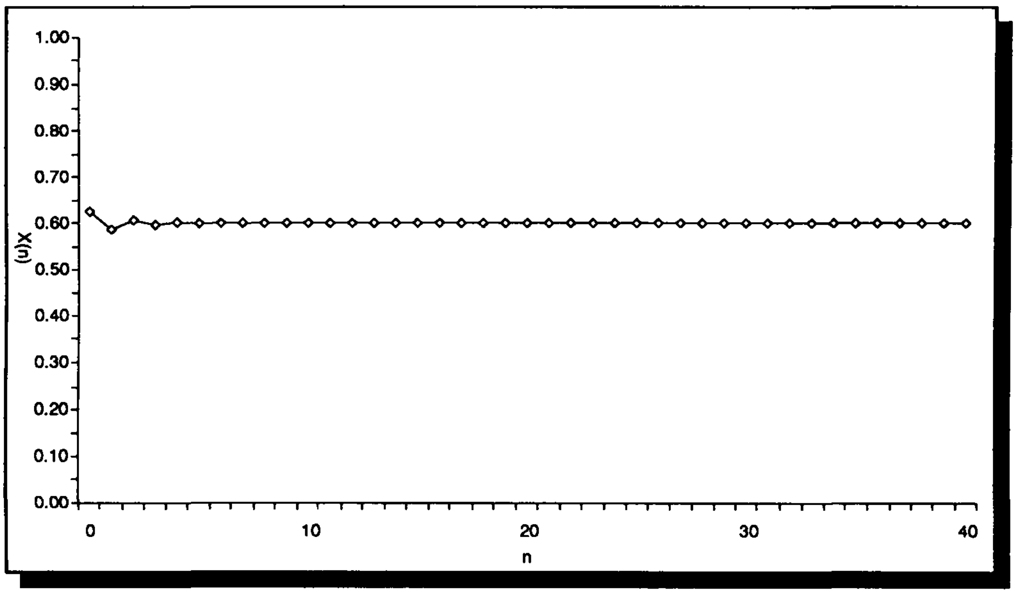

Figure 6

X(t) very quickly settles down to a constant value. If, however, we had chosen a value for k of 3.0 then X(t) would again settle down to a constant value, albeit over a much longer period of time as shown in figure 7.

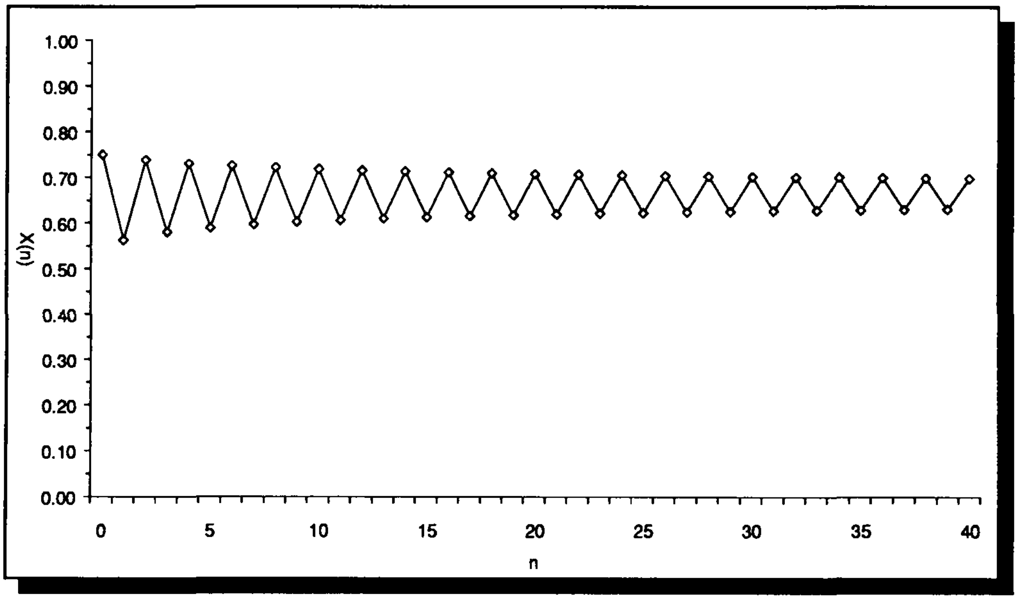

Figure 7

If we move on to consider the position when k = 3.2 something a little unexpected happens. The value of X(t) does not settle down to a constant value but instead oscillates between two values for all time as shown in figure 8.

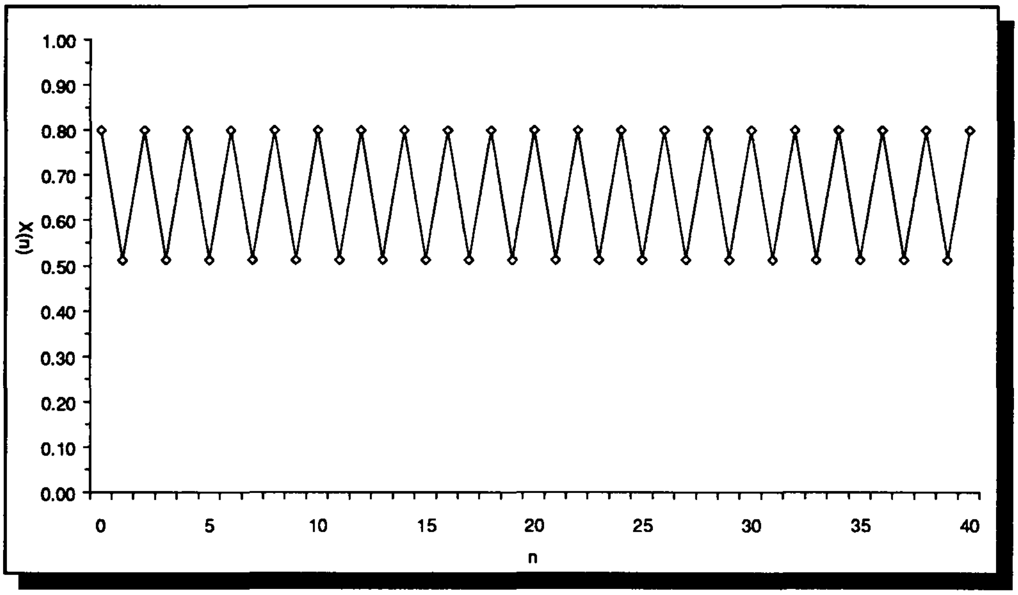

Figure 8

Increasing the chosen value of k still further produces still more interesting results. With a value for k of 3.5 the value of X(t) oscillates between four values for all time as shown in figure 9.

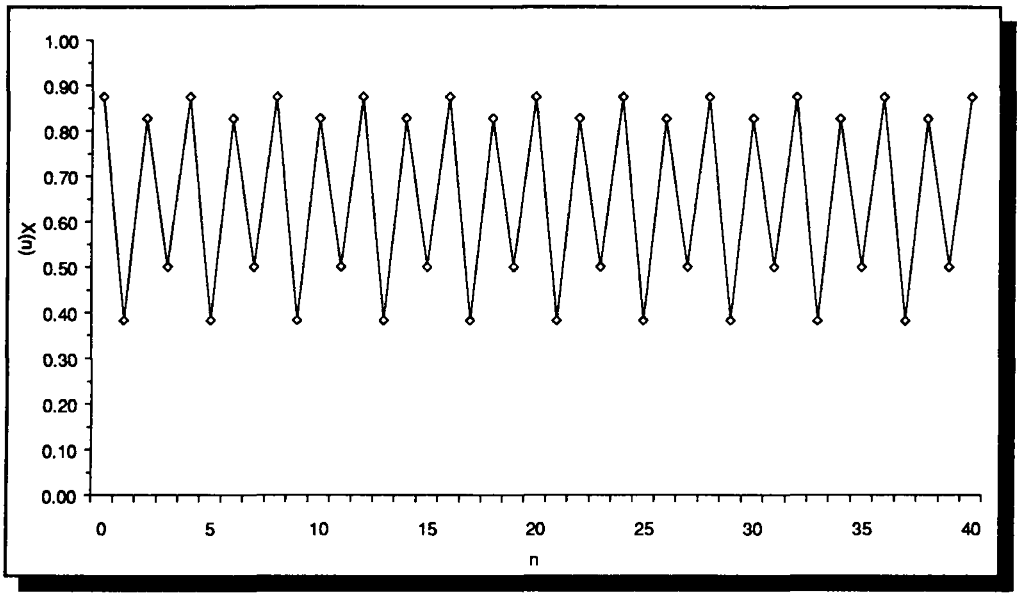

Figure 9

By choosing still larger values for k it is possible to find further cycles which repeat with ever increasing periodicity. At a certain point something strange happens, cycles can no longer be detected and the motion becomes 'chaotic'. Figure 10 shows the value of X(t) over 100 time periods with k = 3.9. Although there is clearly some pattern to the behaviour it would not be possible to, for example, read off from the graph the value of X(20), plug it into the equation and reproduce the rest of the graph. Try it!

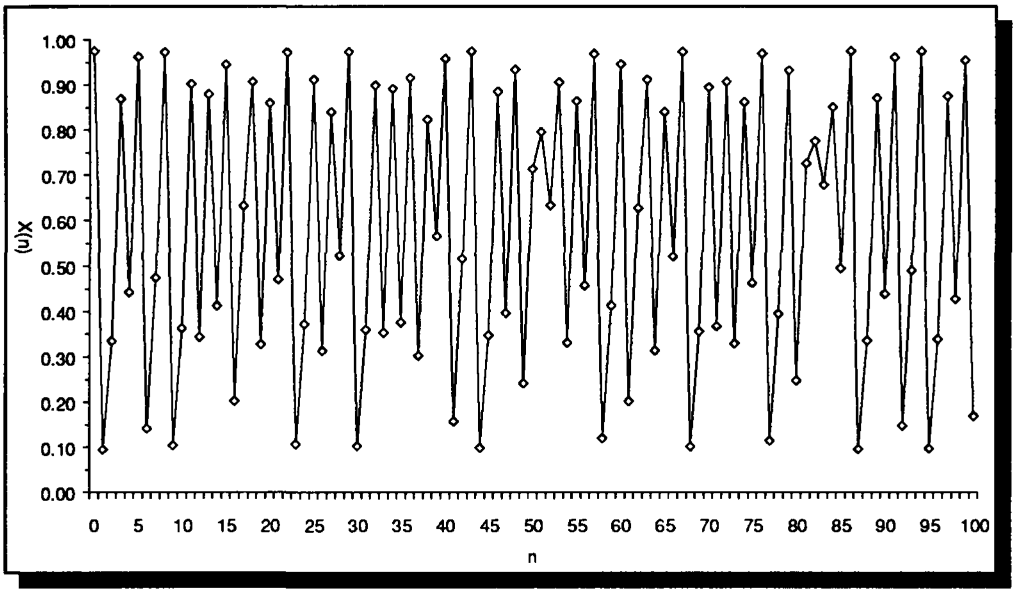

Figure 10

This is what in meteorology has come to be known as the butterfly effect whereby it is said that the effect of a butterfly fluttering its wings in the UK may cause a hurricane to strike Hong Kong.

This observed phenomenon (at least to the extent that weather forecasters are always wrong) can be demonstrated mathematically if you write down some relatively simple equations describing weather patterns. The inputs to the model at any given point in time will be something like a number of surface temperatures at various points around the globe. One finds that they exhibit this type of chaotic behaviour for the parameter values which you want to use in order to fit observed data. The problem is that if one of these temperatures were to be fed into the model with a small difference in rounding then the predictions concerning the UK's weather in three days' time may be vastly different.

What points can be made that are of relevance to us? There are obvious similarities between predicting future weather patterns and predicting future prices of assets. In weather forecasting I believe there is a general consensus that accurate prediction beyond a certain, relatively short, time horizon, is not possible. (Although one can then move on to pattern spotting - as you can see in the last diagram pattern spotting would work quite well although you would have to expect to get it wrong some of the time by a significant margin).

The above fits in quite nicely with the rumbling debate over whether chartism works. The answer would appear to be (if market movements are chaotic) sometimes yes, sometimes no.

There seems to me to be one important theoretical problem, which as yet, it seems, theoretical mathematics is unable to solve. The point is that the 'chaotic' behaviour of the system may result from trying to fit a mathematical model to observed data which is too simple. If an investment manager demonstrated to me that stock prices (or any other economic variable for that matter) exhibited chaotic behaviour, then my initial reaction would be that he should take more factors into account in constructing the model.

What are the implications for asset & liability modellers if our chosen system of equations exhibit chaotic behaviour? This is a very difficult question to answer. Chaos cannot be exhibited by a linear system of equations, but the equations currently used within asset & liability modelling are often non-linear in certain terms.

What would it mean? Suppose we start some equations off from the current time and look at a graph of equity returns over the next 20 years. We then repeat the process, but assuming the initial yield is a fraction different. If the system of equations is chaotic then we may well get a very different graph this time. Intuitively this isn't very satisfying but it ignores the fact that our equations include a significant random component (recognising our belief that no deterministic relationship exists between the variables).

At this point current mathematical theory again seems to run into problems in analysing whether a system is in fact displaying chaotic behaviour or simply stochastic variation. The mathematics falls down because of the number of types of stochastic variation which have to be considered (it could be an additive random component, it could be that the parameters are stochastic or even the form of the equation itself).

Furthermore we are not concerned with accurate prediction of the future levels of the equity market. We are more concerned with the relationship between future inflation, earnings levels, equity prices and equity dividends. Providing these relationships do not vary unexpectedly we might not need to be too concerned.

I'm sure this is a subject on which much more remains to be said!