A NOTE ON THE THEORY AND PRACTISE OF MULTIPLICATIVE RATE MAKING

Björn Ajne

0. Introduction

A tarif is a rule for the division of an insurance portfolio into disjoint classes or cells, and an assignment of a premium f_c to each cell c.

The premiums f_c constitute a vector, the dimension of which is equal to the number of cells. A tarif structure or a tarif model is described by specifying a surface in this vector space, on which the premium point with coordinates f_c is required to be situated. The dimensionality of this surface is called the number of free parameters or the number of degrees of freedom of the model.

The simplest structure is the one which assigns the same premium to all cells. Its surface consists of the straight line of points with all coordinates equal. The number of degrees of freedom is obviously equal to one, the free parameter being the common premium.

In the following we will assume that a basic subdivision has already been done.

1. The likelihood function of tarif construction

In general, when we want to construct or modify a tarif the statistical material at hand will (or should) consist of, for each cell c,

- exposure n_c (e.g. number of policy years, or amount insured times years at risk, or premiums)
- claims frequency s_c (number of claims per unit of exposure)
- observed risk premiums p_c (claims amount per unit of exposure)

We denote by

- λ_c the theoretical claims frequency for cell c
- μ_c the expected value of the size of an indivudual claim in cell c (theoretical average claims size)
- π_c the theoretical risk premium in cell c, $\pi_c = \lambda_c \mu_c$.

We assume numbers of claims to bo Poisson distributed. The probability of k claims in cell c is then

$$(n_c\lambda_c)^k \exp(-n_c\lambda_c)/k!$$

We denote by

- σ_c the standard deviation of the individual claims sizes in cell c

and we assume that, given k claims, the total claims amount in cell c has a Normal distribution with mean $k\mu_c$ and s.d. $\sigma_c\sqrt{k}$

The likelihood function L corresponding to the observed frequencies s_c and risk premiums p_c (i.e. the a priori probability of the statistical material) will then be as follows, assuming the cells to be stochastically independent. The observed k-value in cell c is $n_c s_c$, and the observed total claims amount is $n_c p_c$.

$$
(1)\quad \begin{cases}
L(\lambda_c,\mu_c,\sigma_c) = L_1(\lambda_c)\cdot L_2(\mu_c,\sigma_c) \\[2ex]
L_1(\lambda_c) = \prod_c (n_c\lambda_c)^{n_c s_c}\exp(-n_c\lambda_c)/(n_c s_c)! \\[2ex]
L_2(\mu_c,\sigma_c) = \prod_c (2\pi n_c s_c)^{1/2}\sigma_c^{-1}\exp\{-(n_c p_c - n_c s_c\mu_c)^2/2n_c s_c\sigma_c^2\}
\end{cases}
$$

The function L should be maximized w.r.to the parameters λ_c,μ_c,σ_c. This should be done under the constraints imposed on the parameters by the tarif model chosen.

2. Likelihood theory for multiplicative rate making

In multiplicative rate making one specifies for $\pi_c = \lambda_c\mu_c$ a multiplicative expression with one factor for each premium argument which is taken into consideration. For simplicity of notation, we assume the number of premium arguments to be three. The premium arguments may take on m_1, m_2 and m_3 different values, respectively.

The number of cells is then $m = m_1\ m_2\ m_2$ and the general cell index c is replaced by

$$ijk\ (i = 1\cdots m_1,\ j = 1\cdots m_2,\ k = 1\cdots m_3)$$

Risk premiums π_c are required to be of the form $\pi_{ijk} = \bar{p}\, u_i v_j w_k$, i.e. the likelihood function

$$(2)\qquad L(\lambda_{ijk},\ \mu_{ijk},\ \sigma_{ijk}) = L_1(\lambda_{ijk})\cdot L_2(\mu_{ijk},\sigma_{ijk})$$

should be maximized under the constraints

$$(3)\qquad \lambda_{ijk}\mu_{ijk} = \bar{p}\ u_i v_j w_k$$

Here, $\bar{p}$ is the observed overall riskpremium, while u_i, v_j, w_k are nonnegative parameters. These are free parameters apart from a proportionality normalization. The latter can e.g. be of the form $u_1 = v_1 = w_1$ from which it is seen that there are $1+(m_1-1)+(m_2-1)+(m_3-1) = d$ free parameters of this kind.

In addition we have the λ_{ijk} as free parameters. When they are also determined we get the μ_{ijk} from

(4) $$\mu_{ijk} = \bar{p}\, u_i v_j w_k / \lambda_{ijk}$$

We now have (d+m) free parameters. Finally, the standard deviations σ_{ijk} are also assumed to be unknown. To be able to estimate them from the observed data we have to make simplyfying assumptions, e.g.

(5) $$\sigma_{ijk} \text{ all equal, say } \sigma_{ijk} = \sigma$$

or

(6) $$\sigma_{ijk} \text{ proportional to } \mu_{ijk},\ \sigma_{ijk} = \sigma\mu_{ijk}$$

or, as an intermediate case,

(7) $$\sigma_{ijk} \text{ proportional to } \sqrt{\mu_{ijk}},\ \sigma_{ijk} = \sigma\sqrt{\mu_{ijk}}$$

In all three cases, the total number of free parameters, i.e. the number of degress of freedom, will be d+m+1.

In order to estimate the parameters we should maximize (2) taking into consideration the constraint (4) and one of the alternative constraints (5), (6) and (7).

We adopt the following approximate procedure. In (2) the first factor $L_1(\lambda_{ijk})$ is maximized by putting theoretical frequencies equal to the observed ones, i.e.

$$\hat{\lambda}_{ijk} = s_{ijk}$$

Then there remains the maximization of

$L_2(\mu_{ijk}, \sigma_{ijk})$. From (4)

(8) $$\mu_{ijk} = \bar{p}\, u_i v_j w_k / s_{ijk}$$

and from (1)

(9) $$L_2(\mu_{ijk},\sigma_{ijk}) = \text{const} \cdot \prod_{ijk} \sigma_{ijk}^{-1} \cdot \exp(-Q/2)$$

with

(10) $$Q = \sum_{ijk} n_{ijk}(P_{ijk} - \bar{p}\, u_i v_j w_k)^2 / s_{ijk}\sigma_{ijk}^2$$

Under the constraint (5), $\sigma_{ijk} = \sigma$, the maximization of L_2 is equivalent to minimizing, with respect to u_i, v_j, w_k

$$Q_a = \sum_{ijk} n_{ijk}(P_{ijk} - \bar{p}\, u_i v_j w_k)^2 / s_{ijk}$$

i.e Least squares with n_{ijk}/s_{ijk} as weights.

Then $\hat{\sigma}$ is obtained by

$$\hat{\sigma}^2 = \hat{Q}_a/m$$

Of the two constraints (6) and (7) we choose to treat (7) in some more detail. Under this constraint, (8)-(10) yield

(11) $$L_2(u_i, v_j, w_k, \sigma) = \text{Const}\cdot\sigma^{-m} \cdot \prod_{ijk} (u_i v_j w_k)^{-1/2} \cdot \exp(-Q/2)$$

with

$$Q = \sigma^{-2} \sum_{ijk} n_{ijk}\, (p_{ijk} - \bar{p}\, u_i v_j w_k)^2 / \bar{p}\, u_i v_j w_k = \sigma^{-2} Q_A$$

To maximize L_2 or, equivalently, Log L_2 we put the partial derivatives of Log L_2 w.r. to σ, $u_i (i = 1 \cdots m_1)$, $v_j (j=1 \cdots m_2)$, $w_k (k=1 \cdots m_3)$ equal to zero. Observing that

(12) $$Q_A = \left[\Sigma\, n_{ijk} \frac{p_{ijk}^2}{\bar{p}\, u_i v_j w_k} - 2\, \Sigma\, n_{ijk} p_{ijk} + \Sigma n_{ijk} \bar{p}\, u_i v_j w_k\right]$$

we get

$$(d/d\sigma=0) \qquad -m/\sigma + \sigma^{-3} Q_A = 0$$

$$(u_i d/du_i=0) \qquad -m_2 m_3 + \sigma^{-2}\left[\sum_{jk} n_{ijk} \frac{p_{ijk}^2}{\bar{p}\, u_i v_j w_k} - \sum_{jk} n_{ijk} \bar{p}\, u_i v_j w_k \right] = 0$$

$$(i = 1 \cdots m_1)$$

and the analogous equations $v_j d/dv_j=0$, $w_k d/dw_k = 0$.

Rearranging, the system becomes

$$(13a) \qquad Q_A = m\sigma^2$$

$$(13b) \quad \begin{cases} \sum_{jk} n_{ijk} \dfrac{p_{ijk}^2}{\bar{p} u_i v_j w_k} - \sum_{jk} n_{ijk} \bar{p}\, u_i v_j w_k = m_2 m_3 \sigma^2 & (i = 1 \cdots m_1) \\ \sum_{ik} \quad -''- \quad - \sum_{ik} \quad -''- = m_1 m_3 \sigma^2 & (j = 1 \cdots m_2) \\ \sum_{ij} \quad -''- \quad - \sum_{ij} \quad -''- = m_1 m_2 \sigma^2 & (k = 1 \cdots m_3) \end{cases}$$

Summing e.g. the first set of m_1 equations w.r. to i, we find

$$\sum_{ijk} n_{ijk} \frac{p_{ijk}^2}{\bar{p} u_i v_j w_k} - \sum_{ijk} n_{ijk} \bar{p} u_i v_j w_k = m_1 m_2 m_3 \sigma^2 =$$

$$= m\sigma^2 = Q_A$$

and thus, from (12)

$$\sum_{ijk} n_{ijk} \bar{p} u_i v_j w_k = \sum_{ijk} n_{ijk} p_{ijk}$$

i.e. the graduated total claims cost will be equal to the observed one.

To solve the system (13), we may choose an initial value for σ and then solve equations (13b) for u_i, v_j, w_k (it can be proved that there is one, essentially unique, solution). Putting the values obtained for u_i, v_j, w_k into (13a), we get a new value for σ and repeat the procedure. The process is stopped when two successive σ-values are equal.

From (11) and (13a) we then have

(14) $$\hat{Q} = \hat{\sigma}^{-2}\,\hat{Q}_A = m = \text{Const}$$

and

(15) $$2\text{Log}\hat{L}_2 = \text{Const} - m\text{Log}\hat{Q}_A - m_2m_3\sum_i \text{Log}\hat{u}_i - m_1m_3\sum_j \text{Log}\hat{v}_j - m_1m_2\sum_k \text{Log}\,\hat{w}_k$$

As earlier pointed out, the number of degrees of freedom is $(m_1+m_2+m_3-2)+m+1$. The three terms in this sum correspond, respectively, to the factors u, v and w, to the estimated frequencies and to σ.

If we want to test e.g. the hypothesis that all w_k can be put equal to one, i.e. the third factor variable has no influence, we have to solve the system (13) putting all $w_k=1$ (the third line of (13b) should be omitted, of course). If the estimates obtained are denoted by two "hats", the likelihood test statistic becomes

$$2\text{Log}\hat{L}_2 - 2\text{Log}\hat{\hat{L}}_2 = \left(-m\text{Log}\hat{Q}_A - m_2m_3\sum_i \text{Log}\hat{u}_i - m_1m_3\sum_j \text{Log}\hat{v}_j - m_1m_2\sum_k \text{Log}\hat{w}_k\right) - \left(-m\text{Log}\hat{\hat{Q}}_A - m_2m_3\sum_i \text{Log}\hat{\hat{u}}_i - m_1m_3\sum_j \text{Log}\hat{\hat{v}}_j\right)$$

It is approximatively chi-square distributed with degrees of freedom equal to

$$(m_1m_2+m_3-2) - (m_1+m_2-1) = m_3-1$$

For comparisons of one factor model with a more specialized one, the important part of $2\text{Log}L_2$ is, therefore, minus m times the sum of

$\text{Log}Q_A$

and, for each factor variable,

the arithmetic mean of the logarithms of the factors

The relevant part of the number of degrees of freedom equals the total number of factors minus the number of factor variables plus one.

3. An example of multiplicative rate making practise

In practical work the theory above has been used in the following simplified, and somewhat rough, way.

To solve the system (13), we put the initial value for σ equal to zero. Equations (13b) are then solved with their right-hand members equal to zero. This means putting the partial derivatives of Q_A w.r. to u_i, v_j, w_k equal to zero. The estimates obtained are thus those which minimize Q_A.

We then compute σ^2 from (13a) and <u>assume</u> that if this σ^2 was inserted in (13b), and (13b) was solved for u_i, v_j, w_k, we would get roughly the same σ^2 back from (13a). In reality, we would, of course, obtain a larger σ^2 as Q_A would increase somewhat from its minimum value.

Thus we assume that the iteration would (approximately) stop after one further step, and so we do not have to carry it through to obtain σ^2:

$$m\hat{\sigma}^2 = \hat{Q}_A = \text{minimum } Q_A \text{ w.r. to } u,v,w.$$

From (12) and (13b) with right hand members equal to zero, we get

$$\min Q_A = \hat{Q}_A = 2\Sigma n_{ijk}\bar{p}u_u v_j w_k - 2\ \Sigma n_{ijk}p_{ijk}$$

i.e.

$$\hat{Q}_A = 2 \text{ (graduated - observed claims cost)}$$

where the graduation is done by those factors u,v,w which minimize Q_A. This coincides with the original Boehm-Mehring program [ ] for fitting a multiplicative model.

As $\hat{Q}_A$ is positive, the factors u,v and w thus far obtained overestimate the observed claims cost. Instead of carrying through the iterations to the end, we now adopt Jung's [ ] approach to estimate the factors. This means that we put not only the total graduated claims cost equal to the observed one, but do so also for each marginal sum.

$$(14)\quad \begin{cases} \sum_{jk} n_{ijk}\, p_{ijk} - \sum_{jk} n_{ijk}\, \bar{p} u_i v_j w_k = 0 \quad (i = 1 \cdots m_1) \\ \sum_{ik} \text{ -"- } \sum_{ik} \text{ -"- } = 0 \quad (j = 1 \cdots m_2) \\ \sum_{ik} \text{ -"- } \sum_{ij} \text{ -"- } = 0 \quad (k = 1 \cdots m_3) \end{cases}$$

These equations have one, assentially unique, solution [ ].

This gives us our estimates $\hat{u}$,$\hat{v}$ and $\hat{w}$.

The quantities $\hat{Q}_A$, $\hat{u}$, $\hat{v}$ and $\hat{w}$ thus obtained are then used e.g. to construct likelihood ratio tests according to the previous section.

4. The hypothesis of no risk premium differentiation

The simplest tarif model is to assign the same premium to all cells. This may be considered as a multiplicative model with all factors equal,

$$\pi_{ijk} = \bar{p}u \quad \text{for all } i,j,k$$

where u is the only factor to be estimated. The exact likelihood estimate according to section 2 will make graduated and observed claims costs equal, and so

$$\hat{u} = 1$$

$$m\hat{\sigma}^2 = \hat{Q}_A = \Sigma n_{ijk} (p_{ijk} - \bar{p})^2/\bar{p}$$

The important part of Log L_2 is $-m\mathrm{Log}\hat{Q}_A$ and the corresponding number of degrees of freedom is equal to one.

If we use, instead, the practical approach of section 3, $\hat{Q}_A$ is obtained as the minimum of

$$Q_A = \Sigma\, n_{ijk}(p_{ijk} - \overline{p}u\,)^2 / \overline{p}u$$

w.r.to u. This yields

$$\overline{p}u = (\Sigma n_{ijk} p_{ijk}{}^2 / \Sigma n_{ijk}\,)^{1/2} > \overline{p}$$

and

$$\hat{Q}_A = 2\Sigma n_{ijk}\,(\overline{p}u - \overline{p}\,)$$

As estimate of the unknown factor u we still use $\hat{u} = 1$.

The simple hypothesis of no premium differentiation between cells can then be tested against a more complicated model by taking the difference between the (important part of) Log L_2 for the latter and the $-m\mathrm{Log}\hat{Q}_A$ of this section. Also, one minus the quotient between the two $\hat{Q}_A$'s may be taken as a descriptive measure of the variance reduction that is accomplished by the more complicated model.

INSTITUTE OF LIBRARY ACTUARIES

Description of the multiplicative ratemaking model.
(Free translation from a description by G Andreasson, Oct 1968, augumented by B Ajne and G Green, Dec 1972)

1. The model

Assume that, within a certain branch of insurance, we have tarif arguments U, V, W, ... In motor they could be e.g. make of car, age of car, geographical district etc. In fire we could have e.g. building material (stone, wood,...), year of erection, geographical location etc.

We also have statistical experience consisting of

(1) exposures $n_{ijk....}$

e.g. number of policy years or total sum insured for risks in "cell" i, j, k, ...

(2) (relative) risk measures $p_{ijk....}$

e.g. claims' cost per policy year or as a proportion of sum insured.

The model assumption is now

$P_{ijk...} = \bar{p}\, u_i v_j w_k ...$, where p is defined by (3) on p.2

i.e. the influence of the tarif arguments U, V, W,.... is assumed to be multiplicative. If the tarif arguments U, V, W,..... have I, J, K, ... levels, this means that we have replaced the estimation of the $I \cdot J \cdot K \cdot ....$ cell risk premiums by the estimation of $I + J + K + ...$ risk factors $u_1 ... u_I$, $v_1 ... v_J$, $w_1 ... w_K$,...

(I, J, K are denoted by m_1, m_2 and m_3 in the main paper)

Thus the model assumption implies a considerable reduction of the number of parameters to be estimated from the data. It also supplies a convenient logic for the construction of tarifs, perhaps to a larger extent than an additive model would do as in insurance one is used to express differences in percentages rather than in absolute amounts. Finally, one also gets a possibility to study, via the risk factors, the influence of one tarif argument isolated from those of the others.

2. Fitting the model to the data.

Assume for simplicity that we have three tarif arguments U, V, W. Denote by $\bar{p}$ the overall risk measure defined by

$$(3) \qquad \bar{p} = \sum_{i,j,k} n_{ijk} p_{ijk} / \sum_{i,j,k} n_{ijk}$$

Introduce the normalized risk measures r_{ijk} defined by

$$(4) \qquad r_{ijk} = p_{ijk}/\bar{p}$$

Thus

$$(5) \qquad \bar{r} = \Sigma\, n_{ijk}\, r_{ijk} \,/\, \Sigma\, n_{ijk} = 1$$

The computation starts out from the normalized risk measures, i.e. we start with the estimation problem

$$(6) \qquad r_{ijk} \approx u_i v_j w_k$$

Two ways have been used to estimate the u's, v's an w's from the r's. The first is the chi square minimum method according to which the following expression should be minimized w.r.to u_i, v_j, w_k:

$$(7) \qquad \chi^2 = \text{Nons} \cdot \sum_{i,j,k} n_{ijk} \frac{(u_i v_j w_k - r_{ijk})^2}{u_i v_j w_k}$$

((7) corresponds to Q_A in the main paper)

Equating the partial derivatives to zero yields a system of $I + J + K$ equations for the $I + J + K$ unknown quantities u_i, v_j, w_k:

$$(8)\quad \begin{aligned} u_i &= \sqrt{\sum_{j,k} \frac{n_{ijk}\, r^2_{ijk}}{v_j w_k} \Big/ \sum_{j,k} n_{ijk} v_j w_k} \\ v_j &= \sqrt{\sum_{i,k} \frac{n_{ijk}\, r^2_{ijk}}{u_i w_k} \Big/ \sum_{i,k} n_{ijk} u_i w_k} \\ w_k &= \sqrt{\sum_{i,j} \frac{n_{ijk}\, r^2_{ijk}}{u_i v_j} \Big/ \sum_{i,j} n_{ijk} u_i v_j} \end{aligned}$$

(This is system (13b) of the main paper with $\sigma = 0$)

The second way could be called the method of unbiased estimation of marginal riskmeasures, which means putting the graduated marginal risk measures equal to the observed ones:

$$(7')\quad \sum_{j,k} n_{ijk}\, \bar{p} u_i v_j w_k = \sum_{j,k} n_{ijk}\, p_{ijk}$$

(Eq.(14) of main paper)

for $i = 1 \ldots I$, and corresponding equations for each fixed j and each fixed k. These equations can be put into the following form, which differs from (8) in that the quantities r^2_{ijk} have been replaced throughout by $r_{ijk} u_i v_j w_k$:

$$(8')\quad \begin{aligned} u_i &= \sum_{j,k} n_{ijk}\, r_{ijk} \Big/ \sum_{j,k} n_{ijk} v_j w_k \\ v_j &= \sum_{i,k} n_{ijk}\, r_{ijk} \Big/ \sum_{i,k} n_{ijk} u_i w_k \\ w_k &= \sum_{i,j} n_{ijk}\, r_{ijk} \Big/ \sum_{i,j} n_{ijk} u_i v_j \end{aligned}$$

Both (8) and (8') are well apt to solution by iteration. Initial values may conveniently be chosen to be the observed marginal risk measures, viz.

$$(9) \qquad u_i = \sum_{j,k} n_{ijk}\, r_{ijk} \,/\, \sum n_{ijk}$$

$$v_j = \sum_{i,k} n_{ijk}\, r_{ijk} \,/\, \sum n_{ijk}$$

$$w_k = \sum_{i,k} n_{ijk}\, r_{ijk} \,/\, \sum n_{ijk}$$

According to experience the iteration process converges rapidly, ten steps usually being quite sufficient. In principle the iteration is stopped when the m:th and (m+1):th iteration yields the same result to a certain number of significant figures,

$$(10) \qquad {}^{m+1}u_i = {}^{m}u_i = u_i^*$$

$$ {}^{m+1}v_j = {}^{m}v_j = v_j^*$$

$$ {}^{m+1}w_k = {}^{m}w_k = w_k^*$$

Hence we get the estimates

$$(11) \qquad r_{ijk}^* = u_i^*\, v_j^*\, w_k^*$$

and finally

$$(12) \qquad p_{ijk}^* = \bar{p}\, r_{ijk}^*$$

3. Testing the goodness of fit.

If the graduation is done according to formulas (7)-(8), chi square minimum, the fit of the model to the data can be tested by computing the quantity S defined by

$$(13) \qquad S = \frac{\sum n_{ijk}\, u_i^* v_j^* w_k^*}{\sum n_{ijk}\, r_{ijk}}$$

According to main paper, section 3, min χ^2 = 2 obs claims x (S-1)

i.e. graduated total claims cost divided by observed total claims cost. From (7) it can be proved that S is always greater than one, and the deviation (S-1) can be used as a measure of the "badness" of fit (S=1 means perfect fit).

Also the following marginal quotients are computed

$$(14) \quad S_{i..} = \sum_{j,k} n_{ijk} u_i v_j w_k \Big/ \sum_{j,k} n_{ijk} r_{ijk} \qquad i = 1,2,\ldots\ldots,\nu_i$$

$$S_{.j.} = \sum_{i,k} n_{ijk} u_i v_j w_k \Big/ \sum_{i,k} n_{ijk} r_{ijk} \qquad j = 1,2,\ldots\ldots,\nu_j$$

$$S_{..k} = \sum_{i,j} n_{ijk} u_i v_j w_k \Big/ \sum_{i,j} n_{ijk} r_{ijk} \qquad k = 1,2,\ldots\ldots,\nu_k$$

and their deviations from one may be used to judge the balance of the graduation.

For the method of graduation described by formulas (7')-(8') all S-values will naturally be equal to one, but for rounding off errors. However, they are still computed as a check.

4. Experience with the multiplicative model.

The multiplicative rate making model has, from the late sixties and onwards, found a fairly extensive use in the graduation of swedish claims' statistics within Motor Insurance.

It is also used, at least in one swedish company, to revise tarifs within Home-owner's Comprehensive, House-owner's Comprehensive, Combined Shop Insurance, Pleasure Boat Insurance etc.

The swedish fire tarif has a multiplicative structure and, thus, is well suited for multiplicative graduation. However, some care has to be taken as to the large fire claims, the influence of which has to be smoothed before the graduation.

Often the graduation according to (7)-(8) is used to test the model, while if the fit is considered satisfactory, formulas (7')-(8') are used to actually estimate the risk factors. The later method pays a little more attention to the smaller risk groups and also fulfills the condition of equity for the tarif arguments taken one at a time.

Also, in practical tarif work the estimated risk factors are often rounded off to get a nice-looking tarif, or are smoothed by a straight line or otherwise in order to avoid too great discontinuities between closely related riskgroups.

If there are conditions a priori on the structure of the tarif these should if possible be introduced as side conditions in the computations leading to the estimated risk factors. In especial if a priori conditions are so restrictive as to leave free only the influence of one tarif argument, multiplicative graduation is no longer necessary.

ITERATIVE SOLUTION OF EQ.(8) OF THE "DESCRIPTION" (APP 1)

Appendix 2A

A = 1.000 B = 0.0 C = 0.0 D = 0.0 E = 0.0 F = 1.000

N = 8966800. P = 2.89642 Q = 2.89642 R = 1.000061

(Total exposure, Σn_{ijk} sum inserted in KIEK) ($\bar{P}$ expressed as ‰)

STUFE 0 S = 1.089530

0.24773	0.31909	0.96677	0.26837	0.12945	0.91049	2.88682	2.06416	0.51300	0.18590
0.91603	4.69370	0.31014	0.15614	0.89537	0.36547	3.19380	1.39424	0.87958	5.87692
3.87528	1.43274	4.51325	3.73742	2.54897	0.52864	0.46248	0.54022	0.52573	0.70464
0.70457	0.14911	0.46988	0.59776	3.51533					
1.72333	0.97252	0.58015							
2.08495	1.26193	2.93292	1.91525	1.01186	0.58407	0.52834	0.34897	0.37784	0.25422

STUFE 1 S = 1.332819

0.23376	0.29427	1.15750	0.44045	0.48768	1.33834	3.60193	3.09827	0.38589	0.18284
0.74361	3.93192	0.55831	0.50300	1.15923	0.58614	3.59212	2.76643	1.63764	3.37875
4.80874	2.06454	5.02996	5.36216	2.24989	1.36288	0.72400	1.00730	0.78241	1.20848
0.97154	0.20310	0.58081	1.36692	3.08728					
1.76578	0.91664	0.69338							
1.69709	1.18710	3.05781	2.69275	0.93707	0.60855	0.67872	0.56841	0.49736	0.44078

INSTITUTE OF ACTUARIES LIBRARY

STUFE 10 S = 1.331231

U	0.24757	0.31540	1.15144	0.42460	0.40300	1.41398	3.71393	3.09015	0.39984	0.20605
	0.82918	4.02180	0.52927	0.44648	1.22819	0.56814	3.82730	2.70260	1.47793	3.33218
	5.00973	2.03639	5.09457	5.28410	2.34680	1.22511	0.70272	0.92573	0.77262	1.21403
	0.99323	0.21844	0.61425	1.11243	2.75229					
V	1.82199	0.90928	0.68084							
W	1.68007	1.16664	3.06471	2.74110	0.93731	0.60786	0.69684	0.57696	0.50430	0.45321

I	J	K	L	M	N	FAKTOR F	LOG F	S-WERTE	N-WERTE	P-WERTE	S*P-WERTE
02						0.24757	-1.396058	1.420887	204259.	0.71752	1.01951
04						0.31540	-1.153919	1.699754	107557.	0.92421	1.57092
11						1.15144	0.141017	1.276405	1481350.	2.80019	3.57417
12						0.42460	-0.856613	1.435064	429943.	0.77731	1.11549
14						0.40300	-0.908827	3.749654	315714.	0.37494	1.40590
15						1.41398	0.346407	1.333318	842067.	2.63715	3.51616
16						3.71393	1.312090	1.425457	141758.	8.36144	11.91888
18						3.09015	1.128221	1.277611	324732.	5.97867	7.63841
21						0.39984	-0.916702	1.309945	42552.	1.48587	1.94641
22						0.20605	-1.579621	1.579278	29681.	0.53846	0.85038
23						0.82918	-0.187321	1.400345	20759.	2.65321	3.71541
24						4.02180	1.391729	1.248915	40951.	13.59493	16.97891
26						0.52927	-0.636249	1.559153	46679.	0.89830	1.40059
27						0.44648	-0.806360	2.153018	579006.	0.45224	0.97368
28						1.22819	0.205540	1.357776	479516.	2.59336	3.52120
31						0.56814	-0.565392	1.419295	262705.	1.05854	1.50238
33						3.82730	1.342158	1.601200	114557.	9.25059	14.81204
34						2.70268	0.994245	1.819105	100276.	4.03831	7.34612
36						1.47793	0.390641	2.334092	50980.	2.54764	5.94643
38						3.33218	1.203627	1.100823	78076.	17.02205	18.73827
42						5.00973	1.611382	1.203177	82478.	11.22446	13.50501
45						2.03639	0.711178	1.183638	481095.	4.14983	4.91189
46						5.09457	1.628174	1.212720	83965.	13.07227	15.85301
48						5.28410	1.664702	1.324579	138700.	10.82514	14.33876
51						2.34690	0.853052	1.429221	68853.	7.38290	10.55179
52						1.22511	0.203031	2.371166	72621.	1.53115	3.63062
53						0.70272	-0.352801	1.252741	738192.	1.33954	1.67810
62						0.92573	-0.077177	1.511609	98667.	1.56471	2.36522
63						0.77262	-0.257965	1.185754	766822.	1.52273	1.80558
64						1.21403	0.193942	1.518761	130995.	2.04094	3.09970
65						0.99323	-0.006792	1.782528	79485.	2.04072	3.63764
72						0.21844	-1.521263	1.288191	152338.	0.43189	0.55636
93						0.61425	-0.487354	1.363911	227952.	1.36097	1.85624
94						1.11243	0.106544	1.638132	54460.	1.73138	2.83623
96						2.75229	1.012433	1.070160	97059.	10.18187	10.89623
						1.82199	0.599928	1.360283	1261703.	4.99150	6.78986

I	J	K	L	M	N	FAKTOR F	LOG F	S-WERTE	N-WERTE	P-WERTE	S*P-WERTE
	2					0.90928	-0.095100	1.231086	5920226.	2.81684	3.46777
	3					0.68094	-0.384422	1.827032	1784871.	1.68037	3.07009
		01				1.68007	0.518835	1.122615	1168392.	6.03889	6.77934
		02				1.16654	0.154130	1.364999	1108457.	3.65508	4.98917
		03				3.06471	1.119952	1.600656	342039.	8.49497	13.59752
		04				2.74110	1.008359	2.035530	37804.	5.54736	11.29181
		05				0.93731	-0.064746	1.218653	1772757.	2.93077	3.57158
		06				0.60786	-0.497808	1.374595	1337800.	1.69170	2.32540
		07				0.69684	-0.361202	1.409671	2169039.	1.53030	2.15722
		08				0.57696	-0.549975	2.028332	253717.	1.01077	2.05017
		09				0.50430	-0.684582	1.861185	404139.	1.09438	2.03684
		10				0.45321	-0.791398	2.125382	372656.	0.73632	1.56495

ITERATIVE SOLUTION OF EQ. (8') OF THE "DESCRIPTION" (APP 1)

Appendix 2B

A = 1.000 B = 0.0 C = 0.0 D = 0.0 E = 0.0 F = 1.000

N = 8966800. P = 2.89642 Q = 2.89642 R = 1.000061

STUFE 0 S = 1.089530

U	0.24773	0.31909	0.96677	0.26837	0.12945	0.91049	2.88682	2.06416	0.51300	0.18590
	0.91603	4.69370	0.31014	0.15614	0.89537	0.36547	3.19380	1.39424	0.87958	5.87692
	3.87528	1.43274	4.51325	3.73742	2.54897	0.52864	0.46248	0.54022	0.52573	0.70464
	0.70457	0.14911	0.46988	0.59776	3.51533					
V	1.72333	0.97252	0.58015							
W	2.08495	1.26193	2.93292	1.91525	1.01186	0.58407	0.52834	0.34897	0.37784	0.25422

STUFE 1 S = 0.999999

U	0.16719	0.18107	0.85562	0.28010	0.13930	1.04270	2.57094	2.42023	0.30387	0.12454
	0.54058	3.16707	0.37092	0.23454	0.89947	0.38076	2.14791	1.42913	0.60704	2.97137
	3.99325	1.68628	4.06824	3.85344	1.47689	0.49156	0.56223	0.60010	0.66448	0.78254
	0.57544	0.16798	0.43314	0.81294	2.73944					
V	1.74006	1.00121	0.49192							
	2.05763	1.15586	2.62650	1.82698	1.03336	0.57463	0.67736	0.38132	0.35859	0.26891

STUFE 10 S = 0.999999

U	0.16505	0.17817	0.85144	0.27575	0.10991	1.04966	2.77068	2.56827	0.29653	0.12319
	0.53074	3.19966	0.34714	0.19046	0.91781	0.38489	2.20162	1.46075	0.61963	2.95766
	3.85499	1.65635	3.97172	3.78068	1.49779	0.48322	0.57870	0.59761	0.66180	0.76099
	0.60224	0.16065	0.43725	0.74405	2.60518					
V	1.76382	0.99794	0.48743							
W	2.04733	1.14659	2.61293	1.80207	1.03119	0.57723	0.69433	0.38283	0.35900	0.27052

I	J	K	L	M	N	FAKTOR F	LOG F	S-WERTE	N-WERTE	P-WERTE	S*P-WERTE
02						0.16505	-1.801480	1.000016	204259.	0.71752	0.71753
04						0.17817	-1.724989	1.000017	107557.	0.92421	0.92422
11						0.85144	-0.160822	1.000015	1481350.	2.80019	2.80023
12						0.27575	-1.288278	1.000013	429943.	0.77731	0.77732
14						0.10991	-2.208115	1.000016	315714.	0.37494	0.37495
15						1.04966	0.048471	1.000014	842067.	2.63715	2.63719
16						2.77068	1.019092	1.000016	141758.	8.36144	8.36158
18						2.56827	0.943231	1.000016	324732.	5.97867	5.97877
21						0.29653	-1.215598	1.000016	42552.	1.48587	1.48590
22						0.12319	-2.094033	1.000013	29681.	0.53846	0.53847
23						0.53074	-0.633482	1.000013	20759.	2.65321	2.65324
24						3.19966	1.163045	1.000012	40951.	13.59493	13.59510
26						0.34714	-1.058040	1.000014	46679.	0.89830	0.89832
27						0.19046	-1.658327	1.000014	579006.	0.45224	0.45225
28						0.91781	-0.085770	1.000015	479516.	2.59336	2.59340
31						0.38489	-0.954707	1.000013	262705.	1.05854	1.05855
33						2.20162	0.789194	1.000016	114557.	9.25059	9.25074
34						1.46075	0.378948	1.000015	100276.	4.03831	4.03837
36						0.61963	-0.478628	1.000015	50980.	2.54764	2.54768
38						2.95766	1.084399	1.000017	78076.	17.02205	17.02234
42						3.85489	1.349342	1.000017	82478.	11.22446	11.22465
45						1.65636	0.504623	1.000016	481095.	4.14983	4.14990
46						3.97172	1.379199	1.000017	83965.	13.07227	13.07250
48						3.78068	1.329903	1.000017	138700.	10.82514	10.82533
51						1.49779	0.403990	1.000014	68853.	7.38290	7.38300
52						0.48322	-0.727278	1.000016	72621.	1.53115	1.53118
53						0.57870	-0.546975	1.000015	738192.	1.33954	1.33956
62						0.59741	-0.514823	1.000018	98667.	1.56471	1.56473
63						0.66190	-0.412791	1.000015	766822.	1.52273	1.52275
64						0.76099	-0.273141	1.000016	130995.	2.04094	2.04098
65						0.60224	-0.507105	1.000017	79485.	2.04072	2.04076
72						0.16066	-1.828467	1.000017	152338.	0.43189	0.43190
93						0.43725	-0.827259	1.000014	227952.	1.36097	1.36099
94						0.74405	-0.295648	1.000015	54460.	1.73138	1.73140
96						2.60518	0.957502	1.000016	97059.	10.18187	10.18204
						1.76392	0.567484	0.999998	1261703.	4.99150	4.99149

I	J	K	L	M	N	FAKTOR F	LOG F	S-WERTE	N-WERTE	P-WERTE	S*P-WERTE
	2					0.99704	-0.002062	1.000000	5920226.	2.81684	2.81684
	3					0.48743	-0.718609	1.000024	1784871.	1.68037	1.68041
		01				2.04723	0.716534	1.000014	1168392.	6.03889	6.03897
		02				1.14659	0.136789	1.000008	1108457.	3.65508	3.65510
		03				2.61293	0.960474	1.000015	342039.	8.49497	8.49510
		04				1.80207	0.588935	1.000010	37804.	5.54736	5.54742
		05				1.03119	0.030711	1.000011	1772757.	2.93077	2.93080
		06				0.57723	-0.549513	1.000020	1337800.	1.69170	1.69174
		07				0.69433	-0.364802	1.000019	2169039.	1.53030	1.53033
		08				0.38283	-0.960175	1.000011	253717.	1.01077	1.01078
		09				0.35900	-1.024429	1.000008	404139.	1.09438	1.09438
		10				0.27052	-1.307426	1.000011	372656.	0.73632	0.73632

From appendices 2A and 2B the following can be computed

- m = number of cells = $35 \times 3 \times 10 = 1{,}050$ Degrees of freedom (important part) = $35+3+10-3+1 = 46$
- $\min \chi^2 = \hat{Q}_A$ of section 3 of main paper = 2 obs claims cost $\times (S-1)$, S taken from 2A, =
  $= 2 \times NP \times (S-1) = 2 \times 8{,}966{,}800 \times 2.89462 \times 0.331231 = 17{,}205{,}210$
- $\hat{\sigma}^2$ of section 3 = $\hat{Q}_A / m = 1{,}624.0$ $\hat{\sigma}^2_{unbiased} = \hat{Q}_A/(m-46) = 1{,}698.4$ $\hat{\sigma} = 41.2$
- $2 \operatorname{Log} L = -m\left(\log \hat{Q}_A + \frac{1}{36}\Sigma \log u_i + \frac{1}{3}\Sigma \operatorname{Log} v_j + \frac{1}{10}\Sigma \log w_k\right) = -1{,}050 \log \hat{Q}_A + 521.1$
- E.g. the hypothesis of no premium differentiation has $2 \operatorname{Log} L_1 = -1{,}050\, \hat{Q}_A^1$ where $Q_A^1 \geq \hat{Q}_A$ i.e. $-1{,}050 \operatorname{Log} \hat{Q}_A^1 \leq -1{,}050\, \hat{Q}_A$ so

  $2 \operatorname{Log} L - 2 \operatorname{Log} L_1 \geq 521.1$ to be compared with χ^2 with $46-1 = 45$ degrees of freedom, a highly significant deviation.

INSTITUTE OF ACTUARIES LIBRARY

Home-owner´s comprehensive as an illustration of multiplicative rate making.

1. Short description of home-owner s comprehensive

Home-owner´s comprehensive in my company consists of two parts, viz. insurance for the building and a combined householder´s insurance. The latter usually covers private property within the house (carpets, furniture, TV-sets etc) against fire, water damage and burglary. It also includes cover for personal liability (as a private person, not as a house-owner) and some other things, depending on which one of three alternatives the policyholder has chosen.

In the following we will only consider the insurance for the building. It covers fire, storm, damage to windows, water damage and machinery breakdown, damage to the house in connection with burglary, and liability as a house-owner (e.g. snow from the roof causing personal damage). The two dominating risks are fire and water damage, as seen from the following table. Storm may now and then, every tenth year or so, pop up and show noteworthy figures.

Table 1. Fire and water damage claims as a percentage of total claims cost. Building insurance within home-owner´s comprehensive.

Year of incurrence	73	74	75	76	77	78	79	80
Fire, %	25	31	29	29	23	28	25	25
Water damage, %	52	58	57	57	54	58	64	62
Fire + water, %	77	89	86	86	77	86	89	89

2. Tarif structure of building insurance

In 1979, realizing the increasing impact of water damage claims as a conequence of the increased standard in the households, we revised the tarif structure. Thus the premium was split into two parts and a new premium argument (the number of "water units", see below) was introduced.

$P = P_1 + P_2$

P_1 covers water damage and liability

P_2 covers remaining risks

Both premiums are loaded for expenses. P_1 is split into a fixed and a variable part

$$P_1 = F_1 + V_1$$

F_1 covers the liability risk premium and fixed expenses. V_1 covers water damage and variable expenses. It is of the form

$$V_1 = B_1 \, 10^{0.01 p_1}$$

where p_1 is the total number of points of type one in our points system, see appendix 1. An increase of p_1 by 30 points thus means a doubling of V_1.

We have eight premium arguments, cf appendix 1, viz.

1. Byggnadsyta (area covered by the building, length times width) in m^2. The corresponding number of points of type 1, p_{11}, is proportional to the logarithm of the area. This means that V_1 is proportional to the area itself, i.e. we take out a premium per square metre of building area.This is thus our measure of exposure.

2. Hustyp, type of house. Corresponding points p_{12} (and p_{22} for P_2, see below) are given in a séparate table not shown here.

3. Material (in outer walls), stone or wood, and

4. Byggnadsår, the year when the building was finished (eight classes). These two arguments are given combined points, p_{134} and p_{234}, shown in separate tables.

5. Antal våtenheter, number of water units. You have to count the total number of units connected to the water supply system of the house such as sinks, dishing machines, wash basins, washing machines, WC´s, baths, separate shower baths and - if indoors - swimming pool (two units). Points, p_{51} and p_{52}, as functions of the number of water units are in separate tables.

6. Uppvärmningssätt, method for warming up the house (circulating water, electrical radiators, other, none - the form is used also for summer houses), and

7. Komm vatten, if water supply from a central water plant of the district or not. Points, p_{167} and p_{267}, in separate tables.

8. Belägenhet, the geographical location of the house (eight choices for permanent resedence). Points, p_{18} and p_{28} in separate tables.

Thus

$$p_1 = p_{11} + p_{12} + p_{134} + p_{15} + p_{167} + p_{18}$$

so that the variable premium V_1 is a product with one factor for each premium argument - arguments 3 and 4 and also 6 and 7 being combined, though (multiplicative tarif model).

The premium P_2 has no fixed part (this is customary for fire dominated premiums) and so is of the form

$$P_2 = B_2 \, 10^{0.01 \, p_2}$$

where p_2 is the sum of points of type two, p_{2x}.

All premium arguments, with their values, are reprinted on the policy. The policy holder can thus check that the premium is founded on correct information.

3. Statistical analysis of the tarif structure.

The introduction of a new tarif structure in 1979 was preceded by statistical analyses, with some guessing involved as to the appearance in our portfolio ot the new premium argument water unit.

Analyses were again carried out in 1980 and 1981. We will use part of the latter as an illustration of multiplicative rate making. I will mainly refer to the paper "Description of the multiplicative ratemaking model" which is included as appendix 1 in the material previously sent out.

Appendix 2 shows a multiplicative graduation of observed water damage risk premiums. These are the base for the variable premium V_1.

Claims are censored at 50 KSEK, i.e. for each claim only the amount up to this point is taken into account.

The column N-WERTE shows exposures, marginal for each premium argument value and total, in hundreds of square metres of building area. The total exposure amounts to 424605, corresponding to some 400000 policy years (100000 policies during the four years of incurrence 1976-79).

P-WERTE are observed marginal and total risk premiums in KSEK per 100 square metres building area. Thus the total observed claims cost is

$$424605 \times 0.19759 = 83\ 898 \text{ KSEK}$$

S-WERTE are the marginal and total quotients between graduated and observed claims costs referred to in the Description. As they are greater than one it is evident that the graduation has been done according to formulas (7)-(8) there. The total S, the deviation of which from one is our primary measure of goodness-of-fit, amounts to 1.552328.

FAKTOR F gives the factors for the different premium argument values. As explained in section 3 of the theoretical paper (main paper) we do not actually use these factors, but

the factors obtained from graduation according to formulas (7') and (8') in the Description. These are shown in appendix 3. The N-WERTE and P-WERTE are the same as before, but the S-WERTE are now all equal to one as they should.

The following figures may be shown

No of premium argument values						No of cells
m_1	m_2	m_3	m_4	m_5	m_6	m
8	7	6	3	2	7	14112

Degrees of freedom $\sum m_i - 6+1 = 28$

$$\hat{Q}_A = \min \chi^2 = 2 \times \text{obs claims} \times (S-1) =$$

$$= 2 \times 83898 \times 0.552328 = 92678$$

(S taken from appendix 2)

$$\hat{\sigma}^2 = \hat{Q}_A/m = 6.5673$$

$$\hat{\sigma} = 2.5627$$

In the model underlying the analysis this means that for each cell the standard deviation of the distribution of individual claims sizes equals 2.5627 times the square root of the average claims size.

Twice the log likelihood of the graduation, according to section 3 of the theoretical paper, is computed as

$$\log \hat{Q}_A = \text{constant} + \log (S-1) = C-0.593613$$

and adding, for each premium argument, the arithmetic mean of the logarithms of its factors,

which gives, in addition -0.053594

finally multiplication by $-m = -14112$ should be carried out which gives twice the likelihood (apart from general additive constants) as $-14112\ (C - 0.647207)$.

4. Comparisons between tarif structures

In the analysis described above, material and age of building were combined while the remaining premium arguments were supposed to have independent multiplicative factors in the risk premium.

Several analyses were carried out, in which different groups of arguments were combined. As an illustration the loglikelihood for one additional such analysis is given below together with the loglikelihood above and the loglikelihood for the structure with no premium differentiation at all between cells, i.e. all factors equal to one.

	(i) No premium differentiation	(ii) Material and Age combined	(iii) Material and Age and No. of water units combined
d.f.	1	28	70
S	1.789470	1.552328	1.505716
log (S-1)	-0.236393	-0.593613	-0.681780
Sum of means of logfactors	0	-0.053594	0.013516
2xLoglikelihood /-14.112	C-0.236393	C-0.647207	C-0.668264

To test (i) against (ii) the difference between twice the loglikelihoods is taken and compared to chi-square with 28-1 = 27 d.f.

14112 (0.647207-0.236393) = 5797

which is highly significant.

The corresponding difference for testing (ii) against (iii), with 42 d.f., is

14112 (0.668264-0.647207) = 297.2

which is highly significant (above the 99.9 percentile), too, though to a lesser extent.

The reduction of (S-1), compared to no premium differentiation, is 30 % for structure (ii) and 36 % for structure (iii).

5. What happened to the tarif?

In spite of the foregoing result, Water units were not combined with Material and Age in tarif 82. Reasons of complexity and statistical instability could be invoked against a structure with many parameters.

Material and Age are combined, however, and so are the arguments for Warming up method and Central water plant.

In appendix 4, the tarif factors for no. of water units are compared to those in the statistical analysis of appendix 3. As seen, the tarif does not incorporate the full effect of the observed differientation. Confidence intervals may be pretty wide, though.

SKANDIA

Ansökan – Villa Fritidshus Separat lösöreförsäkring

APPENDIX 1

Personnummer (10 siffror): c 3 5 0 3 0 4 0 4 9 9 Ordn nr 02

Kunduppgifter

Fyll inte i rastrade fält = Skandias noteringar

☒ Villaförsäkring ☐ Fritidshusförsäkring ☐ Separat lösöreförsäkring

☐ Nyteckning ☐ Omskrivning

Begynnelsedatum (år-mån-dag)

Fältkod, försäljning | AT ZK risk ☐ Ja

Namn: Björn Ajne

Fältkod, bestånd | Kundkategori

Utdelningsadress: Kritvägen 3

Specialavisering ☐ 05 ICA ☐ 02 Skandia | Kontonr

Postnummer och ortnamn: 141 34 Huddinge

Annan förfallodag

Fastighetsbeteckning och kommun: Flyttblocket 21 Huddinge

Ersatt försäkring nr

Kommunkod | Förskattsberoln. kr | Distrib bevakning

Personlig lösegendom

Omfattning: ☐ Stor ☐ Liten ☐ Mini ☐ Fritidshus | Försäkr. bel kkr*

Omf | Förhöjd självrisk kr | Typ | Medförsäkrads namn

* 1 000 kr förkortas i kkr (kilokronor)

Påslag/Rabatt Lösegendom % Typ | Premie Lösegendom

Points of type 1 2 = → | Självriskfaktor | Lösegendom

Huvudbyggnad *Se anvisningar på blankettens baksida*

(1) Byggnadsyta (utvändiga mått i markplan) längd × bredd = 130 m²

(2) Hustyp — Till bostad inredda delar är skuggade på skisserna

1 En våning utan källare och utan inredd vind ☐
2 En våning utan källare men med inredd vind ☐
3 En våning med källare men utan inredd vind ☒
4 En våning med källare och inredd vind ☐
5 Två våningar utan källare och utan inredd vind ☐
6 Två våningar med källare men utan inredd vind ☐
7 Två våningar utan källare men med inredd vind ☐
8 Två våningar med källare och inredd vind ☐
9 Tre eller flera våningar ☐

(3) Material i ytterväggar: ☒ Sten ☐ Trä

(4) Byggnadsår: 1965

(5) Antal våtenheter — Var god fyll i blankettens baksida = 12 stycken

(6) Uppvärmningssätt: ☒ Cirkulerande varmvatten ☐ Elektriska radiatorer ☐ Annan uppvärmning ☐ Ingen uppvärmning

(7) Komm vatten: ☒ Ja ☐ Nej

(8) Belägenhet: ☐ 1 ☐ 2 ☐ 3 ☒ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9

Omf | Förhöjd självrisk 1,000 kr | Typ | Förstariskbelopp kkr* | Brand/Allt framtid kkr*

↑ raised deductible (500 kr normal)

Poäng 1	Poäng 2		
53	53	Premie Huvudbyggnad	
		Premie 1 + 515	← P_1
		Premie 2 + 184	← P_2
		Avdrag all framtid −	
4	15	= 699	
Fri = 0: 20	Fri = 0: 0	Självriskfaktor 0,90	← Rabatt factor deduct.
60	10	= 629 →	Huvudbyggnad + 629
19	8	Påslag/Rabatt Huvudbyggnad % Typ	P
16	16		
Summa 1: 172	Summa 2: 102		

Övriga byggnader

Nr	Benämning	Byggnadsyta m²	Vinterbonat	Omf	Förhöjd självrisk Kronor	Typ	Förstariskbelopp Röjningskostnad kkr*	Typ
		m²	☐ Ja ☐ Nej				kkr*	
		m²	☐ Ja ☐ Nej				kkr*	
		m²	☐ Ja ☐ Nej				kkr*	

Påslag/Rabatt Övriga byggnader % Typ | Övrig byggnad +

P_2 P_1

Övriga uppgifter

Husbocksförsäkring: ☐ Ja ☐ Nej | Ersätter husbocksförsäkring i (bolagets namn)

Husbock +

Skandiakonto: ☐ önskas ☐ finns | ☐ Bank-giro ☐ Post-giro | Kontonr

Total premie kronor

Har Ni tidigare haft hem-, villa- eller fritidshusförsäkring i annat bolag? ☐ Ja ☐ Nej

Noteringar

Tidigare försäkringsbolag

Blev försäkringen uppsagd eller endast förnyad mot tilläggspremie? ☐ Ja ☐ Nej

Har Ni haft mer än en skada under de senaste 12 månaderna? ☐ Ja ☐ Nej

Handläggare

SH 8003-4

Vand!

ANDR VI BY 75-79. RPR YYT.IUR 1. RKLALD.VATENH.HTYP.HSPV.PPGRP. JRR. VATTEN 50TKR

(INRCG CGVBV)

APPENDIX 2

I	J	K	L	M	N	FAKTOR F	Q-WERTE	S-WERTE	N-WERTE	P-WERTE	MEDELQUSUM
S02						1.69832	1.193927	1.958937	24469.	0.15047	0.46272E+01
S09					02 = 0-2 y.	1.59483	1.151731	1.404454	76139.	0.19575	0.18633E+01
S19						1.83364	1.245886	1.366957	69090.	0.21606	0.20319E+01
S99	Material x Age of				09 = 3-9 y.	1.99249	1.213166	1.668935	44389.	0.19482	0.25988E+01
T02	building (4 classes)				19 = 10-19 y.	1.41731	1.202765	2.372066	33089.	0.12235	0.41618E+01
T09						1.51137	1.185425	1.514817	69060.	0.19370	0.24082E+01
T19					99 = 20- y.	1.79239	1.229287	1.609576	38510.	0.22339	0.39280E+01
T99						1.65467	1.364299	1.501180	71041.	0.22417	0.25683E+01
	3	0-3				0.89644	1.651943	2.906545	23280.	0.11704	0.48428E+01
	5	4-5				0.76953	1.245355	1.847799	52040.	0.14327	0.21058E+01
	7	6-7	water			0.83295	1.208089	1.484864	71546.	0.18564	0.16685E+01
	9	8-9	units			0.92130	1.122092	1.444588	102676.	0.18687	0.14255E+01
	11	10-11				0.94487	1.128857	1.354022	114556.	0.20191	0.13481E+01
	15	12-15				1.39555	1.084204	1.462454	53796.	0.28936	0.40494E+01
	99	16-				3.07204	1.349604	2.727624	6858.	0.37481	0.41672E+02
		1	Type of			0.85209	1.347766	1.708292	95105.	0.14712	0.18828E+01
		2	house (six			1.01270	1.325772	1.529068	74443.	0.19234	0.21686E+01
		3	most frequent			0.88915	1.163685	1.514752	100803.	0.16923	0.26634E+01
		4				1.11356	1.189601	1.521857	52664.	0.22713	0.31628E+01
		5				1.11559	1.273140	1.505052	58654.	0.23842	0.32231E+01
		6	types)			1.24453	1.121955	1.392954	43072.	0.29259	0.42136E+01
			1 Water	Warming		0.94537	1.159883	1.411240	265123.	0.21660	0.21716E+01
			2 Electrical	up		0.99177	1.228332	1.739982	149732.	0.16611	0.26093E+01
			3 Other	system		1.82524	1.491286	3.662278	9846.	0.16377	0.18240E+02
			E No	central		1.41583	1.367153	2.714273	50755.	0.15749	0.79558E+01
			V Yes	water plant		0.91310	1.160514	1.429486	373919.	0.20303	0.19851E+01
					1	1.19609	1.181590	1.321895	21878.	0.18735	0.37996E+01
					2	0.67035	1.137278	1.428855	142159.	0.14539	0.10226E+01
			Geographical		3	1.04210	1.153585	1.438247	79795.	0.22287	0.25616E+01
			location. There		4	0.85125	1.383226	1.565212	98161.	0.17446	0.16848E+01
			were seven in the		5	1.34691	1.180557	1.527461	50655.	0.27978	0.46387E+01
			previous tarif,		6	1.81403	1.211382	1.741374	25790.	0.31402	0.82528E+01
			tarif 80.		7	1.76803	1.520453	2.141401	15324.	0.22214	0.70934E+01
							1.22[illegible]31	1.552328	424605.	0.19759	0.269953878D+01

R = 1.000105

JUNG VI BY 76-79. RPR BYYTNORM. BKLÅLD,VATENH,HTYP,UPPV,PRGRP,OMR. VATTEN 50TKR

APPENDIX 3

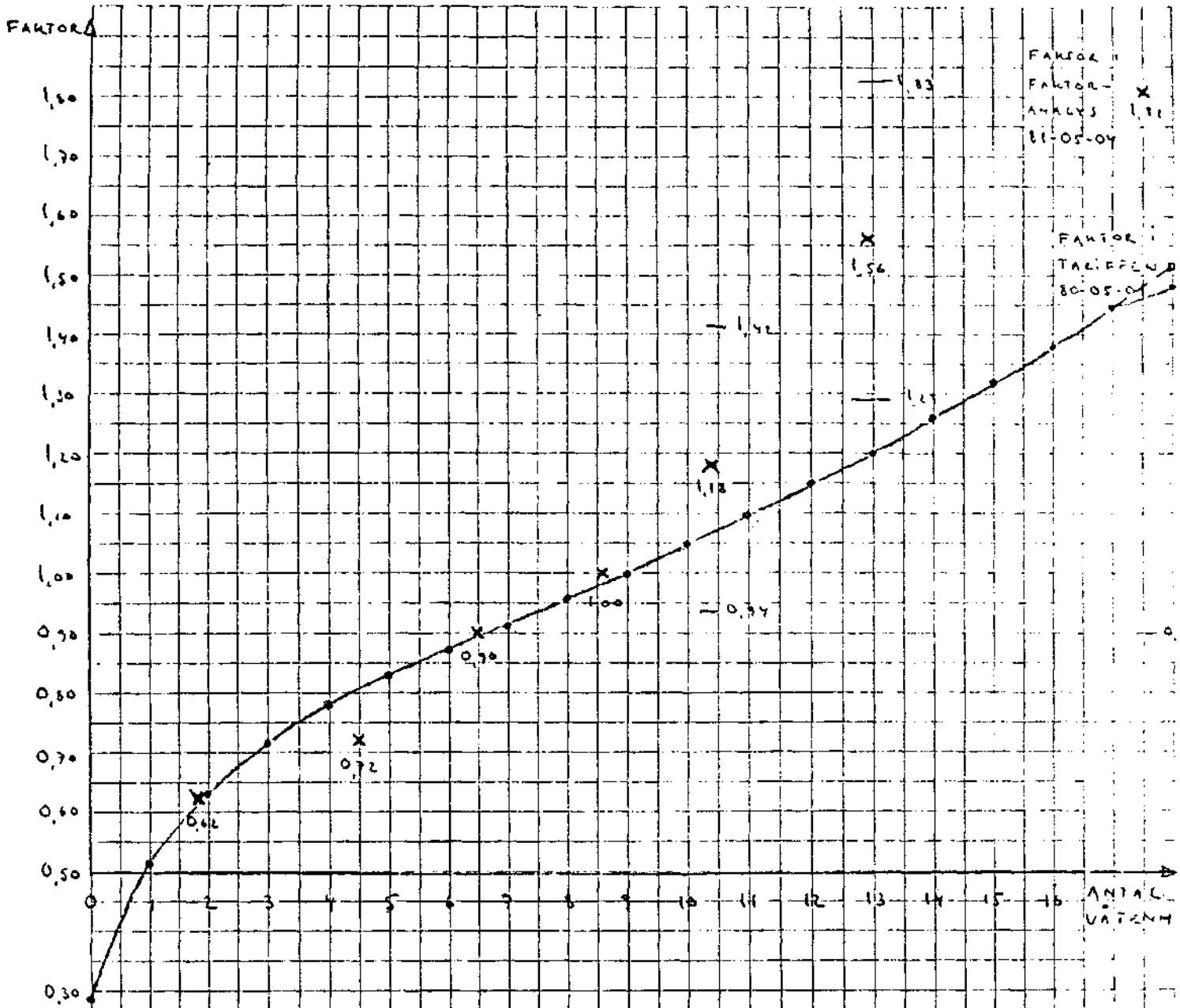
≈(1) (inkl. CGVBV)

I	J	K	L	M	N	FAKTOR F	Q-WERTE	S-WERTE	N-WERTE	P-WERTE	MEDELQUSUM
S02						0.74672	0.954017	1.000128	24469.	0.15047	0.36820E+01
S09						0.95290	0.922842	1.000088	76139.	0.19575	0.14930E+01
S19						1.09329	0.906093	0.999964	69080.	0.21606	0.14777E+01
S99						1.07074	0.880023	0.999998	44389.	0.19482	0.18852E+01
T02						0.56916	0.932285	1.000095	33089.	0.12235	0.32259E+01
T09						0.92555	0.913214	1.000136	68060.	0.19370	0.18552E+01
T19						1.07561	0.915604	0.999964	38510.	0.22339	0.29257E+01
T99						1.12532	0.794533	0.999956	71041.	0.22417	0.14968E+01
	3					0.58868	0.929913	0.999982	23280.	0.11704	0.27261E+01
	5					0.68263	0.888369	0.999976	52040.	0.14327	0.15022E+01
	7					0.85392	0.908794	0.999993	71546.	0.18564	0.12551E+01
	9					0.94798	0.857472	1.000000	102676.	0.18687	0.10893E+01
	11					1.11682	0.872954	0.999985	114556.	0.20191	0.10425E+01
	15					1.47671	0.862109	0.999975	53796.	0.28936	0.32199E+01
	99					1.71513	0.970659	1.000015	6858.	0.37481	0.29977E+02
		1				0.82743	0.891835	1.000073	95105.	0.14712	0.12458E+01
		2				1.11554	0.911252	0.999984	74443.	0.19234	0.14894E+01
		3				0.83307	0.952972	0.999936	100803.	0.16923	0.21811E+01
		4				1.11883	0.864045	0.999926	52664.	0.22713	0.22971E+01
		5				1.16326	0.906013	0.999901	58654.	0.23842	0.22847E+01
		6				1.28979	0.848468	0.999919	43072.	0.29259	0.31865E+01
			1			1.04939	0.866995	0.999970	265123.	0.21660	0.16233E+01
			2			0.92397	0.925918	0.999965	149732.	0.16611	0.19669E+01
			3			0.96430	0.967681	0.999966	9846.	0.16377	0.11836E+02
				E		0.88790	0.957169	0.999893	50755.	0.15749	0.55700E+01
				V		1.01715	0.873615	0.999906	373919.	0.20303	0.14942E+01
					1	0.91666	0.931929	0.999996	21878.	0.18735	0.29968E+01
					2	0.75742	0.831916	0.999968	142169.	0.14539	0.74799E+00
					3	1.15210	0.913150	0.999966	79785.	0.22287	0.20191E+01
					4	0.81404	0.938330	1.000072	88161.	0.17446	0.11429E+01
					5	1.38000	0.874949	0.999984	50655.	0.27978	0.34379E+01
					6	1.63500	0.896071	1.000018	26780.	0.31402	0.61047E+01
					7	1.33018	0.980356	1.000004	15324.	0.22214	0.45737E+01
							0.900029	1.000254	424605.	0.19759	0.198205411D+01

R = 1.000105

Avsende	Datum	Ev klassificering	Sida
VILLA BY VATTEN	81-05-12		
	Utfärdare/ref PC		

Appendix 4



Relative sizes of factors for no. of water units in tarif 80 (V_1)

Deviation in tarif 81 (only for the highest no. exhibited in this figure, viz. 18)

X Observed relative sizes in the analysis described in the text (placed over the portfolio points of gravity for the seven intervals studied)