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NOTES ON BONUS AND SOLVENCY VALUATIONS

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I. THE PROBLEM

My intention is to give some simple illustrations of the difficulties which arise when the market rate of interest has sunk below the limit of solvency for parts of a company's stock of life contracts. In fact, the "partial liquidation" which at such a contingency would appear most natural to the unbiased mind of an actuary, does not seem to be seriously contemplated in insurance legislation. As regards Sweden, this mathematically correct reduction of the liabilities of some of the contracts could perhaps be realized within the legal frame in Mutual Offices. But it is impossible for other companies. It seems therefore necessary to attack the problem from the purely actuarial point of view. If we can find a technically satisfactory solution, it might then be possible to go farther with the question of giving the procedure legal shape.

In the numerical illustrations which will follow, I shall use a simple valuation technique. The loading to meet costs of administration, risk fluctuations, etc., will be supposed to consist of

(*a*) 7% of the continuous premium,

(*b*) an annual addition of 2·8‰ of the sum insured and bonus,

treated in such a way that generally the force of mortality is increased and the force of interest simultaneously decreased by that amount, ·0028.

These additions should suffice to cover all costs, including initial expenses. The unadjusted force of mortality is taken in close touch with observed data in Sweden as

$$1000\mu_x = 3 + \cdot 06 . 10^{\cdot 042 . x}$$

with a slight modification for low ages where it is put equal to a constant.

The profit is assumed to be distributed according to a bonus method which I have characterized as "floating bonus". All benefit is transformed to a bonus addition which is, however, only declared for one year and is not, as British bonuses, supposed to increase during the life of the insured, but to remain constant *with the valuation rate*. It can thus both rise and fall, and it can even disappear. This very simple bonus distribution which is used by some Scandinavian insurance companies will be easy to handle in our calculations, but the *form* of bonus payment does not seriously affect the results which we have in view and might be modified to suit British practice.

For sake of simplicity, we shall assume a discontinuous production of business. Every fifth year, £10 millions of endowment assurances are acquired, all with an age at entry of 35 years and an age at maturity of 55 years. The development is supposed to follow the table of mortality exactly, and we have accordingly the following stock of contracts at different valuations:

Total sum assured, in millions of pounds

Valuation year	Years of entry							Total stock
	Low premiums			High premiums*				
	1920	1925	1930	1935	1940	1945	1950 etc.	
1940	8·450	9·005	9·419	9·740	10·000	—	—	46·614
1945	—	8·450	9·005	9·419	9·740	10·000	—	46·614
1950	—	—	8·450	9·005	9·419	9·740	10·000	46·614
1955	—	—	—	8·450	9·005	9·419	9·740 etc.	46·614
1960	—	—	—	—	8·450	9·005	9·419 etc.	46·614
1965	—	—	—	—	—	8·450	9·005 etc.	46·614
1970	—	—	—	—	—	—	8·450 etc.	46·614

(The first year of entry is supposed to be left in the stock till after the valuation. The reduction factors are simple l_x-quotients.)

We shall further suppose that the market rate of interest has been 4% since 1920 and that the bonus addition corresponding to the premium paid has been declared at each previous valuation, but only for the time till the next valuation. The continuous premium p'' is supposed to be 47·22 ‰ for the years of entry 1920, 1925 and 1930. The corresponding bonus addition for 4% will be denoted by k_4, and can be computed from the following equation:

$$93 \cdot p'' \cdot \bar{a} = \mathrm{P}'' \cdot \bar{a} = (1 + k_4)\{1 - (\delta_4 - \cdot 0028) \cdot \bar{a}\},$$

* See p. 121.

if $\bar{a}$ denotes the 4% annuity value $\bar{a}_{35\,\overline{20|}}$, and $\delta_4 = \log_e 1{\cdot}04$. We will assume that the premiums have been raised from the year of entry 1935 inclusive (owing to uncertainty as regards future interest) to $51{\cdot}27\,^{\circ}/_{\circ\circ}$. Calculating the bonus additions for both premiums at 4%, we obtain 100 and 195 per mille, respectively.

These additions are assumed to have been declared at the 1935 valuation, but in 1940 the situation seems so uncertain that it is resolved to make valuations at 2%, 3% and 4%. We shall then have to compute the assets available at that date and the liabilities according to the pure contracts (solvency valuation), or with the inclusion of a reasonable bonus for different rates of interest, in accordance with the adopted method of bonus distribution (bonus valuation).

In order to fix our ideas, we shall denote by (w) a *bonus basis* involving any assumed rates of mortality, withdrawal, expense, and rate of interest, but in our simplified scheme expressed in terms of a modified rate of interest i_w coupled always with those fixed costs of administration which we have cited above. To each bonus basis (w) there corresponds naturally a bonus rate k_w, computed in our hypothetical case by the formula:

$$P''.\bar{a}^{(w)} = (1 + k_w).\bar{A}^{(w)},$$

where

$$\bar{A}^{(w)} = 1 - (\delta_w - {\cdot}0028)\,\bar{a}^{(w)},$$

$$\delta_w = \log_e (1 + i_w),$$

and $\bar{a}^{(w)}$ is calculated at rate of interest i_w. Denoting by $\bar{a}^{(w)}(t)$ the corresponding annuity value after the expiration of t years, we can define a special *bonus reserve* belonging to the basis (w) by the prospective formula:

$$U^{(w)}(t) = (1 + k_w)\,\bar{A}^{(w)}(t) - P''.\bar{a}^{(w)}(t)$$

$$= (1 + k_w)\,\{1 - (\delta_w - {\cdot}0028).\bar{a}^{(w)}(t)\} - P''.\bar{a}^{(w)}(t).$$

Observing that (w), owing to the corresponding bonus additions k_w, is what has been called a "pure basis", it is evident that the retrospective reserve is identical with the prospective reserve $U^{(w)}(t)$. It is to be observed that k_w can also take negative values. We have accordingly to denote the reserve function U as a *natural reserve* or an *unrestricted reserve*. If it is decided that we must not without some special arrangement reduce the liabilities of the first order,* the reserve has to be changed. We shall denote by $W^{(w)}(t)$ the same reserve as $U^{(w)}(t)$ so long as k_w is positive, and where k_w is negative the same reserve with k_w put equal to zero. In that case the value (prospective only) is

$$W^{(w)}(t) = 1 - (P'' + \delta_w - {\cdot}0028)\,\bar{a}^{(w)}(t).$$

If we consider this reserve as a function of i_w, the point for which we shall have $k_w = 0$ can be denoted as the *liquidation point* of the contract. Below this point, the function $W^{(w)}(t)$ is represented by the formula we have just given, and for higher values of i_w it becomes equal to the expression $U^{(w)}(t)$ defined above. The graph of the function W will thus present a minimum cusp or vertex at the liquidation point.

The fact is illustrated by Diagram I. It need hardly be said that the ideas with which we are dealing might also be extended to any form of life insurance (other than whole-life or endowment assurance), although their elementary character does not seem to make this necessary.

II. VALUATIONS (TECHNICAL BALANCE SHEETS)

We will now pass to the question of a general *valuation basis* (u), which for sake of simplicity may be considered as containing as sole variable the rate of interest i_u. In the general case all valuation factors must of course be included. Whereas the functions U and W studied above concern nothing but the liabilities, the valuation basis (u) will also have to take account of the valuation of the *assets* of the company. It must accordingly contain also an assumption concerning the mode of investment. We shall confine ourselves to the study of two alternatives, viz. (1) "short" investments, i.e. very short-dated investments, day-to-day money, etc., where there will be no loss from depreciation, and (2) investment in perpetuities, implying the risk of capital losses for rising rates of interest, but not vice-versa, capital gains being neutralized altogether by the existence of the right of converting the bonds to a lower nominal rate. We are thus placing ourselves in a rather unfavourable situation, but it is not very far from reality.

* *i.e.* the pure liabilities according to the contract *without any bonus additions*.

If the bonus k_w is determined according to a basis (w) and the valuation is made on the basis (u), the reserve functions will be

$$U^{(u,w)}(t) = (1 + k_w) . \bar{A}^{(u)}(t) - P'' . \bar{a}^{(u)}(t)$$

and $W^{(u,w)}(t)$, which is the same function as long as $k_w > 0$, but if $k_w < 0$ is the corresponding function with $k_w = 0$.

In addition to the valuation basis $W^{(u,w)}(t)$, there is also a better known reserve function, which we will denote by $\overline{W}^{(u)}(t)$, viz. the prospective reserve for *the liabilities of the first order*:

$$\overline{W}^{(u)}(t) = \bar{A}^{(u)}(t) - P'' . \bar{a}^{(u)}(t)$$
$$= 1 - (P'' + \delta_u - \cdot 0028) . \bar{a}^{(u)}(t).$$

This function is identical with $W^{(u)}(t)$ below the liquidation point, but continues above this point on a lower level than $U^{(u)}(t)$. It is steadily decreasing with increasing rate of interest i_u. The valuation premium P'' is treated as a constant. The simple technical reserve

$$V^{(u)}(t) = 1 - \bar{a}^{(u)}(t) / \bar{a}^{(u)}$$

has no direct bearing upon the practical problem because it takes for granted that it is always possible to make the valuation with a "natural reserve premium"

$$P^{(u)} = \bar{A}^{(u)} / \bar{a}^{(u)},$$

whereas the premium income in reality must be taken as given by the contract. Of course there is no use in making a valuation with such an arbitrary premium as the natural reserve premium unless it is definitely smaller than the real premium and the frame of administration expenses is constructed according to the corresponding differences—a combined hypothesis which seldom happens to be true.

According to the kind of reserve functions entering in the valuation, we may speak of an U-valuation, a W-valuation or a $\overline{W}$-valuation (the simple V-valuation being for reasons given above omitted altogether). The general term would be of the form: *a W-valuation at the basis* (u), *including bonus at the basis* (w). We shall also introduce the term *L-valuation*, denoting a valuation of the type U, but where the negative k_w will be transformed in a way to be explained later.

As an example of bonus valuations according to these ideas, we may take the following set:

W-valuations in 1940; $u=w$ (if $k_w > 0$)

Floating-bonus method; "Short" investment

Valuation basis (u) ...		$i_u=2\%$		$i_u=3\%$		$i_u=4\%$	
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £
Year of entry							
1920	9·295	8·450	+·845	8·535	+·760	9·295	—
1925	6·569	6·468	+·101	6·210	+·359	6·569	—
1930	4·100	4·447	−·347	3·982	+·118	4·100	—
1935	2·078	2·051	+·027	2·070	+·008	2·078	—
1940	—	—	—	—	—	—	—
Total stock	22·042	21·416	+·626	20·797	+1·245	22·042	—

As we have assumed the rate of interest to be 4 % until 1940 and the corresponding k_w (equal to ·100 for the premium ·04722, or ·195 for the premium ·05127) to have been paid on all claims up to that time, it is evident that the assets must be equal to the sum of the calculated reserves W (or U) at 4 %. It is then also evident that the valuation result for $i_u=4\%$ must be 0, denoting that the continuation of the payments k_4 will only be possible if the rate of interest continues to be at least 4 %. If it falls below 4 %, a reduction of the bonus below k_4 must be made. The surplus in the valuation at 3 % indicates that bonus at the rate k_3 may be paid in future, if the rate of interest continues to be at least 3 %. For if the surplus of £1·245 millions be set aside, the remaining assets, yielding 3 %, will just suffice—according to the definition of the reserve U or W—to meet the total liabilities $1+k_3$. The bonus additions k_w are as follows:

Entries	Premium (continuous)	Bonus addition k_w corresponding to		
		$i_w=2\%$	$i_w=3\%$	$i_w=4\%$
1920–30	·04722	−·073	+·010	+·100
1935–	·05127	+·006	+·097	+·195

The valuation at 3 % represents a normal case, all bonus additions k_3 being still positive. The real difficulties appear only at 2 %, part of the contracts being then insolvent, whereas the total valuation shows a surplus. We shall try to examine this case from different points of view.

First suppose that we cut down the profits immediately to those corresponding to 2 %, which we at once assume will be stable for the future. As the W-valuation shows a surplus of £·626 millions, we can set this amount aside as a general reserve, and yet we have money enough to carry through our business. This means that the surplus already collected for the year of entry 1920 is used to cover the deficiency for the year of entry 1930 and equalizes a heavy adjustment of interest rates between different "generations". The period of insurance from 1920 to 1940 and the period from 1930 to 1950 will give rise to enormous differences in the effective rate of interest yielded by the sum of reserves collected for a contract of the kind in question. How far does it seem advisable to level the fluctuations of interest and equalize the treatment of different generations of policyholders?

It would evidently be unfair, if, in our hypothetical case, we reduce the surplus of 10 % for the year of entry 1920 to zero. From the point of view of actuarial equity, it would even be unfair to make *any* reduction for that year. Such a bonus method is clearly rather too effective against the policyholders—even though in the interest of real security—and it follows that it must be manipulated with a delicate sense of fairness to the holders of old contracts. If it be thought necessary, the method may be completed by a successive transformation of bonus liabilities to *unconditional* liabilities ("of the first order") provided this be made sufficiently slowly, and in any case not faster than would be the case under the well-known British methods of bonus distribution.

Supposing then that the 10 % addition for the year of entry 1920 is left untouched, the surplus of the W-valuation at 2 % is changed into a deficit of £·219 millions. We shall try to follow the development of this deficit under different hypotheses. As the result is given directly by the valuation if the rate of interest always remains at 2 %, we shall study only the case of a temporary depression of the interest. We shall put the interest at 2 % during 10 years, and then let it suddenly rise again to its former level of 4 %.

The effect of this disturbance will be the main object of our examination.

It is worth while observing that the W-valuation is a solvency valuation only with regard to those years of entry which are characterized by premiums which are insufficient at the rates i_u. If we make, instead of a W-valuation, a $\overline{W}$-valuation, the result will be as follows:

$\overline{W}$-valuations in 1940
"Short" investment

Valuation basis (u) ...		$i_u=2\%$		$i_u=3\%$		$i_u=4\%$	
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £
Year of entry							
1920	9·295	8·450	+·845	8·450	+ ·845	8·450	+ ·845
1925	6·569	6·468	+·101	6·131	+ ·438	5·813	+ ·756
1930	4·100	4·447	−·347	3·909	+ ·191	3·430	+ ·670
1935	2·078	2·005	+·073	1·416	+ ·662	·923	+1·155
1940	—	− ·046	+·046	− ·598	+ ·598	−1·017	+1·017
Total stock	22·042	21·324	+·718	19·308	+2·734	17·599	+4·443

As a comparison, we also write down the result of a pure U-valuation, negative bonus additions being throughout included:

U-valuations in 1940; $u=w$
Floating-bonus method; "Short" investment

Valuation basis (u) ...		$i_u=2\%$		$i_u=3\%$		$i_u=4\%$	
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £
Year of entry							
1920	9·295	7·833	+1·462	8·535	+ ·760	9·295	—
1925	6·569	5·863	+ ·706	6·210	+ ·359	6·569	—
1930	4·100	3·860	+ ·240	3·982	+ ·118	4·100	—
1935	2·078	2·051	+ ·027	2·070	+ ·008	2·078	—
1940	—	—	—	—	—	—	—
Total stock	22·042	19·607	+2·435	20·797	+1·245	22·042	—

The opposite trends of the functions U and $\overline{W}$ as seen in our Diagram I are reflected in the corresponding U- and $\overline{W}$-valuations, whereas the most important valuation from the practical point of view, the W-valuation, will, on account of the changing trend of the

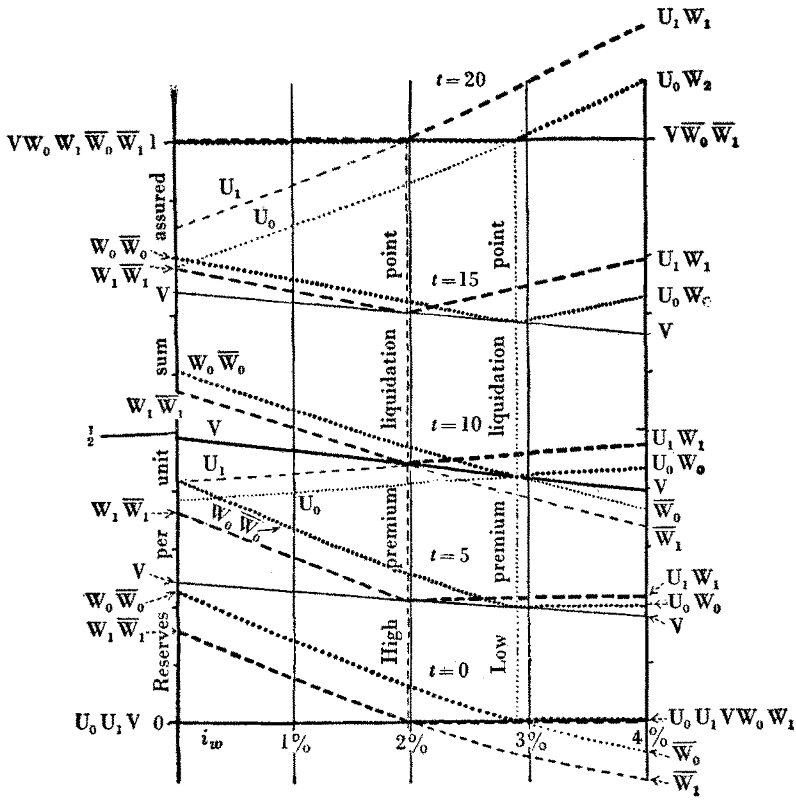

Diagram I. Showing reserve functions per unit sum assured for different rates of interest. *Index* o and *dotted curves*: *low* premiums, point of liquidation high. *Index* 1 and *broken curves*: *high* premiums, point of liquidation low. V (t), technical reserve function. W (t), valuation reserve function, if no reduction is admitted; consisting of two branches: U (t) = unrestricted reserve, above liquidation point, $\overline{W}$ (t) = prospective reserve for liabilities of the first order, below liquidation point.

function W, be more difficult to master, as soon as the liquidation points intervene.

III. THE PARTIAL LIQUIDATION

We have stated that in 1940, after the payment of k_4 to the policies of the year of entry 1920, a deficiency has appeared in the W-valuation at 2 %. This is the indication for introducing a partial liquidation. Supposing that all conditions which seem adequate for the application of such a drastic remedy have been fulfilled, we cut down all liabilities for the years of issue 1925 and 1930 by the same amount, ·073 per unit of insured capital. After this reduction,

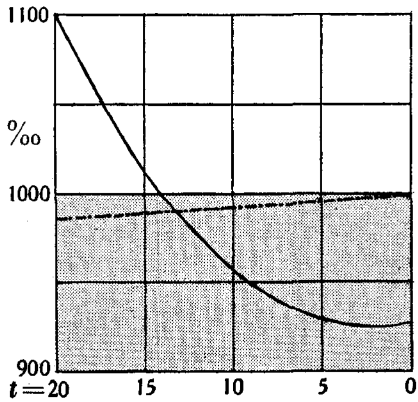

Diagram II. Comparison between legal reduction (Sections 139 and 140 of Swedish Law) and no-adjustment reduction. Legal reduction: —·—·—·—·— No-adjustment reduction: ———.

the deficits of these contracts are changed into surpluses, represented by the differences

$$U^{(4)}(t) - U^{(2)}(t).$$

In the general case, the reserve $U^{(w)}(t)$ is, by definition, the amount necessary for the payment of the sums of insurance $1 + k_w$ during the remaining time of the contract. It follows that a supplementary reserve of

$$U^{(w^1)}(t) - U^{(w)}(t)$$

will be necessary and sufficient for the constant payment of another sum $1 + k_{w^1}$, if the rate of interest has increased from i_w to i_{w^1}. In a stationary stock of insurance, where all durations are represented in the same way at any moment, these supplementary reserves will always have the same average size, and it follows that a period of low interest, appearing as an isolated interval, would have no disturbing effect whatever, provided the liability can be adjusted exactly to the amount $1+k_w$ corresponding to the low rate. After the expiration

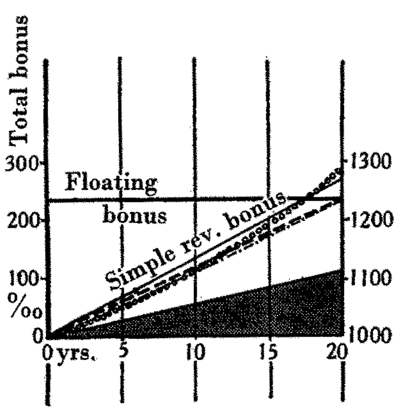

Diagram III

20-year endowment assurance.
Age at entry 35 years.

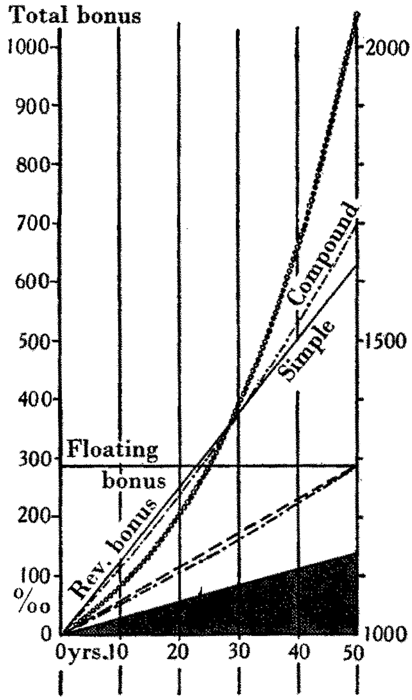

Diagram IV

Whole life assurance.
Age at entry 40 years.

Comparison of total capital assured according to different bonus methods. Floating bonus: $1+k_4$. Shaded: $d_w(n)=\frac{1}{2}k_w$. Capitalized amount according to the constant-premium-reduction scheme: ooooooooooo.

of the interval, the sum of supplementary reserves would suffice to bring up the reserves to those needed according to the high rate i_{w1}, and it should even have been possible also to pay interest on the supplements in the meantime. This demonstration, however, is based on the assumption that no capital losses are experienced (i.e. that investment has been made in short money) and that the premiums remain unaltered.

We have already stated that it would be unfair to deprive the policy-year 1920 of its bonus at rate k_4 (or even to include that year in the reduction), and we have thus rejected the possibility of a strict adjustment according to the new level of the interest. It seems then natural to make a similar compromise also as regards later claims, if this be possible. Suppose for instance that we try to avoid the actual reductions as long as possible. The surplus we have found in the U-valuation at 2 % should then be used as long as possible to neutralize the reductions on claims arising after 1940. The value required to avoid all reductions is equal to the single premium for an assurance of the amount ·073 for the remaining part of the contract.

Excluding the 1920 entrants the surplus in the U-valuation at 2 % in 1940 is stated to be £·973 millions. For the entry-year 1925 the total cost in 1940 of an assurance of ·073 payable on death or at maturity in 1945 is found to be £·606 millions. For the entry-year 1930 the total cost in 1940 of a temporary assurance of ·073 till 1950* is £·080 millions, and the total cost of the pure endowment of a like amount is £·506 millions. The surplus in 1940 will therefore suffice to cover the full claims during the total period of low interest except the last mentioned £·506 millions, out of which only £·287 millions will be covered. Postponing the bringing into practice of the reduction as long as possible, we find that the only remaining reduction will be an amount of ·032 per unit of capital insured in respect of the survivors at the end of 1950 of those entering in 1930. Our surplus is then completely exhausted, but the partial liquidation has not done much harm to the policy-holders. It may be of interest to compare the result with what would have happened if we had decided also to abolish all payments of bonus. This would have saved an assurance of ·006 for contract years 1935 and later, and it is easily found that the final cover of £·287 millions for the pure endowment relating to the year 1930 would have risen to £·299 millions—a relatively insignificant difference. But in this case, it must be admitted that the office is making the new contracts pay for the losses caused by old ones. That is why it seems from a purely actuarial point of view preferable to continue the payments of surplus also during the period of (actual or postponed) liquidation.

* *i.e.* the year when interest is assumed to revert to 4% p.a.

U-valuations in 1950; $u=w$

"Short" investment

Valuation basis (u) ...	...	$i_u=2$ %		$i_u=3$ %		$i_u=4$ %	
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £	Liabilities mill. £	Diff. mill. £
Year of entry: 1930	—	—	—	—	—	—	—
1935	6·361	6·361	—	6·746	−·385	7·137	− ·776
1940	4·187	4·187	—	4·327	−·140	4·456	− ·269
1945	2·051	2·051	—	2·070	−·019	2·078	− ·027
1950	—	—	—	—	—	—	—
Total stock	12·599	12·599	—	13·143	−·544	13·671	−1·072

U-valuations in 1950; $u=w$

Perpetual investment

Valuation basis (u)...	...	$i_u=2$ %		$i_u=3$ %			$i_u=4$ %		
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Assets mill. £	Liabilities mill. £	Diff. mill. £	Assets mill. £	Liabilities mill. £	Diff. mill. £
Year of entry: 1930	—	—	—	—	—	—	—	—	—
1935	6·361	6·361	—	4·241	6·746	−2·505	3·180	7·137	−3·957
1940	4·187	4·187	—	2·791	4·327	−1·536	2·094	4·456	−2·362
1945	2·051	2·051	—	1·367	2·070	− ·703	1·026	2·078	−1·052
1950	—	—	—	—	—	—	—	—	—
Total stock	12·599	12·599	—	8·399	13·143	−4·744	6·300	13·671	−7·371

We have supposed that the surplus disclosed by a U-valuation at the moment of introducing in principle a partial liquidation will be utilized in trying to postpone the practical consequences of the liquidation. This cannot be done indefinitely, because a long period of low interest will make the reductions unavoidable. The surplus consisting of the differences between U-reserves having been exhausted, the return to a high level of interest will find the company with a loss as regards the U-reserves for a higher rate. The U-valuations in 1950 will be as shown in the table on p. 131, the year of issue 1930 being now omitted.

We may now see how long it will be before the company will be able *again to grant* k_4 bonuses. This delay will be found if, starting with the $U^{(2)}(t)$ as single premiums, we undertake a valuation at 4% of these premiums together with the continuous P'' falling due after 1950, diminishing the result with the values of successive costs of claims $1+k_2$. At the moment when the total value of these amounts is equal to the sum of U-reserves at 4%, the sum assured can be raised to $1+k_4$.

The results of the successive valuations at 4% will be seen from the following tables:

U-valuations at 4%

Floating-bonus method; "Short" investment

Year of entry	1955			1960		
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Assets mill. £	Liabilities mill. £	Diff. mill. £
1935	9·296	10·098	−·802	·967	—	+ ·967
1940	6·923	7·137	−·214	9·978	10·098	− ·120
1945	4·518	4·456	+·062	7·326	7·137	+ ·189
1950	2·160	2·078	+·082	4·650	4·456	+ ·194
1955				2·160	2·078	+ ·082
1960						
Total stock	22·897	23·769	−·872	25·081	23·769	+1·312

It is seen that the level of 4% profits is reached again after 7 or 8 years for "short" investment and after 43 years for perpetual investment. In practice, the result will lie between those extremes. We have thus got the general impression that *the adaptation to a*

U-valuations at 4%

Floating-bonus method; Perpetual investment

Year of entry	1990			1995		
	Assets mill. £	Liabilities mill. £	Diff. mill. £	Assets mill. £	Liabilities mill. £	Diff. mill. £
1935	−12·137	—	−12·137	−14·767	—	−14·767
1940	− 5·261	—	− 5·261	− 6·401	—	− 6·401
1945	+ ·315	—	+ ·315	+ ·383	—	+ ·383
1950	+ 4·749	—	+ 4·749	+ 5·778	—	+ 5·778
1955	+ 3·903	—	+ 3·903	+ 4·749	—	+ 4·749
1960	+ 3·208	—	+ 3·208	+ 3·903	—	+ 3·903
1965	+ 2·637	—	+ 2·637	+ 3·208	—	+ 3·208
1970	+10·668	+10·098	+ ·570	+ 2·637	—	+ 2·637
1975	+ 7·486	+ 7·137	+ ·349	+10·668	+10·098	+ ·570
1980	+ 4·649	+ 4·456	+ ·193	+ 7·486	+ 7·137	+ ·349
1985	+ 2·159	+ 2·078	+ ·081	+ 4·649	+ 4·456	+ ·193
1990				+ 2·159	+ 2·078	+ ·081
1995						
Total stock	+22·376	+23·769	− 1·393	+24·452	+23·769	+ ·683

new level of interest will perhaps require more than half the life-time of a single insurance. By adaptation is then understood the procedure by which the definite consequences of the level of interest are postponed, if the floating-bonus method is applied with due regard to the equity for single contracts. A strict application of its principles would always be possible without delay up to the limit given by existing extra reserves, but this will hardly become a practical alternative. The interval of adaptation will depend upon the real composition of the insurance stock, represented in our hypothetical case by 20 years, and also upon the span between the levels of interest. If the market rate were oscillating incessantly between, say, 4% and 2%, the possibility of taking account of the "real" effective rate for the investment of the funds will evidently depend upon the period of the market fluctuations. Long-term variations of the interest will exert a marked influence, short-term will be less noticeable. Perhaps a company would best meet the interests of policyholders if it could organize its bonus method in such a way that on a very long-term average the mean result of valuations should be zero. But this makes it necessary also to admit periods of negative results without any consequent liquidation.

IV. PARTIAL LIQUIDATION FOR SURVIVAL LIABILITIES ONLY?

We have found that, in our special case, the practical consequences of liquidation could be avoided during a long period, and that they could be restricted to a diminution of the pure endowment sum. This makes it desirable to discuss the possibilities of constructing the technique of partial liquidation in such a way as to give an absolute priority to the payments for death claims. This idea was brought up in a memoir to the Paris Congress in 1937.* The difficulties of realizing it seem only to be insurmountable for whole life policies and such endowment assurances as mature at very high ages. Denoting the death liabilities of a contract with index 1, the survival liabilities with 2, and the "bonus" basis with the mark (w), we can write the equation

$$P''.\bar{a}^{(w)}=(1+k_{w.1}).\bar{A}_1{}^{(w)}+(1+k_{w.2}).\bar{A}_2{}^{(w)},$$

where $k_{w.1}=k_{w.2}>0$ for values of i_w above the liquidation point. Below this point, we can try to determine $k_{w.1}$ and $k_{w.2}$ independently. First let it be possible to put $k_{w.1}=0$ and $-1\leqq k_{w.2}\leqq 0$: in this case the death liabilities will be unaffected and the survival liabilities reduced. If this is impossible, we can put $k_{w.2}=-1$ and $-1\leqq k_{w.1}\leqq 0$. The liabilities for the survival or pure endowment part will then disappear and those for the death case be reduced. For practical reasons, it may be advisable, in all cases with a higher age of maturity (e.g. above 65 years), to put

$$P''.\bar{a}^{(w)}=(1+k_w).(\bar{A}_1{}^{(w)}+\bar{A}_2{}^{(w)})$$

with negative k_w.

The results of this calculation for different assurances are exemplified (the premiums being purely nominal) in the table on p. 135.

It is easily seen that a valuation according to the reductions thus computed—which may be called an *L-valuation*—does not materially differ from the corresponding U-valuation. Consequently, the main results hold also for this case.

* Hagstroem-Palmqvist: *Les fluctuations de l'intérêt. XI^e Congrès International d'Actuaires*, Paris, 1937, Vol. 1, p. 189.

Age at entry	Age at maturity	Annual premium adopted (continuously) °/°° (p'')	Profit addition at 4 % k_4 °/°°	Profits (reductions) at 2 %: Death case $k_{2.1}$ °/°°	Profits (reductions) at 2 %: Pure endowment case $k_{2.2}$ °/°°
		Endowment assurances			
20	35	63·89	+ 87	0	− 61
30	45	64·70	+ 97	0	− 50
50	65	72·56	+100	0	− 32
20	40	45·15	+ 83	0	−113
30	50	46·19	+ 97	0	−100
20	50	27·49	+ 75	0	−218
30	60	29·16	+ 95	0	−207
		Whole life assurances (*a*) Different; (*b*) Equal reductions for death and survival cases			
20	90	14·95	+ 59	(*a*) −195	−1000
				(*b*) −202	− 202
30	90	19·79	+ 94	(*a*) −143	−1000
				(*b*) −152	− 152
50	90	42·41	+100	(*a*) − 31	−1000
				(*b*) − 45	− 45
		Temporary death assurance			
30	40	8·97	+ 68	+ 60	—

V. LEGISLATION IN SOME COUNTRIES

We have said that partial liquidation is not seriously contemplated in insurance law, but in certain clauses the idea seems to exist. We shall cite a few examples but shall see that the legislation has left the details undetermined.

In Swedish Law a reduction of liabilities is only contemplated for mutual offices, and the law* provides that if

the assets of a mutual insurance company do not correspond with its liabilities—insurance fund, security fund and bonus fund included—then the deficiency shall, save in a case where only the assets of the company shall be liable for its obligations, and in so far as the deficiency will not be covered by diminution of the security fund and bonus fund, be allotted among those persons who have been members of the company....

If the deficiency will not be covered through allotment, as aforesaid, or if only the assets of the company are liable for its obligations...the de-

* Lag den 25 Maj 1917 om försäkringsrörelse, ss. 139 and 140. The quotation is a translation from the Act.

ficiency shall, in so far as it will not be covered by diminution of the security fund and bonus fund, be neutralized...by reduction of the premium reserve and by a consequent reduction of the sums assured....

Allotment...shall be effected in proportion to the capital insured to each member. In case of reduction of members' shares in the premium reserve and of the sums to be paid...all shares and all sums shall be reduced in the same proportion....

If the statutes of the company contain regulations differing from those in the Act then the company's regulations shall be valid.

This legislation is a relic from ancient times when mutuality in its proper form still existed in life assurance. A modification introduced to deal with the new form of mutuality, where there is a guarantee capital, did not go further than to consider the deficiency as a capital loss which must be distributed among the owners of the total capital of the company according to the amount of each member's share. It has always been considered that the regulations have been insufficiently elaborated and probably the general meaning is that when the danger of insolvency approaches, it should suffice to introduce into the statutes of the company regulations that may be deemed to be appropriate to the particular circumstances. Before actual insolvency, the actuary would have time to work out an equitable distribution of profits and losses over successive generations of policyholders. Although we may be unable to set out principles for the general case of insolvency, we can probably in a special case suggest terms that are at least as equitable as any other terms, or, at any rate, introduce no obvious inequity.

Taking a particular case we will assume as before that a company's insurances are composed of 20-year endowment assurances maturing at age 55 divided into equal numbers of contracts of duration 0, 5, 10, 15 and 20 years and that all the premiums are $47{\cdot}22\,^{\circ}/_{\circ\circ}$. Ignoring in this instance the reductions by the l_x factors the sums assured are given by line (*a*) of the table on p. 137.

Let us now suppose that 4% has been the rate of interest and that bonus has been distributed continuously according to the floating-bonus system (4), i.e. £100 per £1000 assured. The remaining reserve is therefore what we have called $U^{(4)}(t)$, and the assets in hand are given in line (*b*) of our table. If the rate of interest at this moment suddenly drops to 2% the W-valuation at 2%,

$$W^{(w)}(t) = 1 - (P'' + \delta_w - {\cdot}0028)\,\bar{a}^{(w)}(t),$$

Millions of pounds

		$t=0$	$t=5$	$t=10$	$t=15$	$t=20$	Total stock
Sums assured	(*a*)	10·000	10·000	10·000	10·000	10·000	50·000
Assets	(*b*)	·000	1·961	4·353	7·295	11·000	24·609
Liabilities	(*c*)	·539	2·521	4·721	7·183	10·000	24·964
Liabilities reduced according to Swedish law	(*d*)	·531	2·485	4·654	7·081	9·858	24·609
Sums reduced according to Swedish law	(*e*)	9·990	9·955	9·921	9·889	9·858	—
No-adjustment reduction	(*f*)	9·270	9·293	9·569	10·121	11·000	—
Full-adjustment reduction	(*g*)	9·270	9·270	9·270	9·270	11·000	—

will give the value of the liabilities "of the first order". The results are given in line (*c*). If the assets are £24·609 millions there is a deficiency of £·355 millions.

Now we may ask how the law cited above deals with the loss. The first step is shown in line (*d*); the liabilities are reduced proportionately so as to correspond with the assets existing. After that, the reserve of each policy is taken as a single premium, and the sum assured is computed corresponding to this for single premiums and the contract premium, P″, payable during the remaining $n-t$ years. All the calculations are made according to the new rate of 2%. This must be what the law considers as a "consequent reduction". The reduced sums assured are given in line (*e*). The amounts do not give the impression of being reasonable. One reduction seems to be *a priori* evident; for when $t=0$ the reduced sum assured should be $1+k_2$ or 927 ‰ because the bonus k_2 is negative, i.e. $-·073$. In this case however the legal reduction is a minimum.

If we try to express in figures the opinion (rather doubtful in itself) that no adjustment whatever should be allowed between periods of high interest and periods of low interest we shall have to credit each policyholder with "his own reserve" $U^{(4)}(t)$ and calculate the reduction on this assumption. The results are given in line (*f*). The opposite view, that the new level of interest should be reflected in all contracts (except those where $t=20$) may be denoted as the "full-adjustment reduction" and is illustrated in line (*g*). Between these extremes, a prudent quasi-automatic application of the floating-bonus method might be evolved.

Under Dutch Law* partial liquidation of existing business is possible but the administration of the company would pass to the Verzekeringskamer. The German Law† is similar but does not go quite so far. In both cases it seems to be thought that insolvency can be due only to faulty management and could not arise through extremely adverse financial conditions beyond the control of any insurance management. It is doubtful if in either case a company would be allowed to do new business.

Under English Law‡, however, in the event of a company's insolvency contracts may be reduced on such terms and subject to such conditions as the Court considers to be just. Presumably the Court would not, necessarily, prohibit new business, and it appears therefore that what I have called partial liquidation can take place.

VI. MODIFICATION OF FLOATING-BONUS METHOD

We have seen that the floating-bonus method implies rather too much freedom to reduce the bonus in the event of decreasing interest. It was shown that this freedom could be used to postpone a partial liquidation even when this would ultimately become necessary. We have stated that this freedom must be handled with great care if it is to avoid inequity against old contracts. The method can however be amended to give the company less freedom to reduce the bonus rate for old policies. This can, for instance, be done by splitting up the rate k_w declared at a bonus valuation, which has shown that the bonus system (w) is applicable, into two parts, namely

(*a*) a *definite* bonus, $d_w(t)$, which is formed in much the same way as the British reversionary bonus (simple or compound), i.e. by declaring a constant rate per annum which cannot be reduced at later valuations, and

(*b*) an *interim*§ bonus, $k_w - d_w(t)$, which will bring the total bonus, in the case of death only, up to the rate given by basis (w), but which can be reduced at any later valuations.

* Wet op het Levensverzekeringbedrijf (Stbl 1922, no. 716). See especially ss. 40 and 49.

† Gesetz über die Beaufsichtigung der privaten Versicherungsunternehmungen und Bausparkassen, 6 June 1931. See ss. 81 and 89.

‡ Assurance Companies Act, 1909, s. 18.

§ I have borrowed this term from actual British practice, but given it another meaning.

This arrangement will diminish the safety of the method because a smaller amount of the bonus reserve is available as a free fund in case of emergency, but it will make the method more attractive to the public. The definite part of the total bonus will ultimately rise to the neighbourhood of k_w but it should perhaps be sufficient to put

$$d_w\,(n)=\tfrac{1}{2}k_w.$$

I have given in Diagrams III and IV two specimens of bonus rates according to the floating-bonus method compared with the corresponding reversionary bonus schemes. The formulae used and the numerical work are appended.

I. 20-*Year Endowment Assurance*

Age at entry: 35 years. Continuous premium $p''=\cdot 05301$ (average of British participating premiums).

Floating bonus.

$$93\cdot p''\,.\,\bar{a}^{(4)}=\mathrm{P}''\,.\,\bar{a}^{(4)}=(1+k_4)\,.\,\{1-(\delta_4-\cdot 0028)\,.\,\bar{a}^{(4)}\};\quad k_4=\cdot 23531.$$

Simple reversionary bonus†: total rate c, annual rate $c/20$.

$$\mathrm{P}''\,.\,\bar{a}^{(4)}=c/20\,.\,(\overline{\mathrm{R}}'_{35}-\overline{\mathrm{R}}'_{55}-20\,.\,\overline{\mathrm{M}}'_{55})/\mathrm{D}_{35}$$
$$+1-(\delta_4-\cdot 0028)\,.\,\bar{a}^{(4)}+c\,.\,\mathrm{D}_{55}/\mathrm{D}_{35};\qquad c=\cdot 2714.$$

Compound reversionary bonus: annual rate p.

$$\mathrm{P}''\,.\,\bar{a}^{(4)}=\int_0^n\frac{l_{x+t}}{l_x}\cdot\mu_{x+t}\cdot(1+p)^t\,.\,(1+j)^{-t}\,dt+\frac{l_{x+n}}{l_x}\,.\,(1+p)^n\,.\,(1+j)^{-n}$$
$$=1-(\delta^*-\cdot 0028)\,.\,\bar{a}^*;\quad 1+j=e^{\delta_4-\cdot 0028};\quad \frac{1+j}{1+p}=e^{\delta^*-\cdot 0028};$$
$$1+p=e^{\delta_4}/e^{\delta^*}=1\cdot 04/(1+i^*);\quad i^*=\cdot 0276;\quad p=\cdot 0121.$$

† The functions $\overline{\mathrm{M}}'_x$ and $\overline{\mathrm{R}}'_x$ used in the formulae for the simple bonus cases are defined as follows:

$$\overline{\mathrm{M}}'_x=\mathrm{D}_x-(\delta_4-\cdot 0028)\,\overline{\mathrm{N}}_x,$$
$$\overline{\mathrm{R}}'_x=\int_x^\infty\overline{\mathrm{M}}'_t\,dt.$$

The formulae thus make allowance for an expense loading of $2\cdot 8\,^0\!/_{00}$ of the bonuses as well as of the sum assured.

II. *Whole Life Assurance*

Age at entry: 40 years. Continuous premium $p'' = \cdot 03270$ (average of British participating premiums).

Floating bonus.

$$P''.\bar{a}^{(4)} = (1 + k_4).\{1 - (\delta_4 - \cdot 0028).\bar{a}^{(4)}\}; \quad k_4 = \cdot 28682.$$

Simple reversionary bonus†: total rate c, annual rate $c/50$.

$$P''.\bar{a}^{(4)} = c/50.(\overline{R}'_{40} - \overline{R}'_{90} - 50.\overline{M}'_{90})/D_{40}$$
$$+ 1 - (\delta_4 - \cdot 0028).\bar{a}^{(4)} + c.D_{90}/D_{40}; \qquad c = \cdot 6348.$$

Compound reversionary bonus: annual rate p.

$$P''.\bar{a}^{(4)} = 1 - (\delta^* - \cdot 0028).\bar{a}^*;$$
$$1 + p = e^{\delta_4}/e^{\delta^*} = 1 \cdot 04/(1 + i^*); \quad i^* = \cdot 029; \quad p = \cdot 0107.$$

† The functions $\overline{M}'_x$ and $\overline{R}'_x$ used in the formulae for the simple bonus cases are defined as follows:

$$\overline{M}'_x = D_x - (\delta_4 - \cdot 0028)\,\overline{N}_x,$$
$$\overline{R}'_x = \int_x^\infty \overline{M}'_t\,dt.$$

The formulae thus make allowance for an expense loading of $2\cdot 8\,^0/_{00}$ of the bonuses as well as of the sum assured.

ABSTRACT OF THE DISCUSSION

Dr K.-G. Hagstroem, in introducing his paper, said that the securities of the Swedish State now yielded about 2·3 % (as compared with the yield on Consols of 3·6 %); Swedish municipal and industrial securities at fixed interest rates yielded about 2·4 % and 2·9 %; and, what was most astonishing, first-class mortgages up to two-thirds of the total value of the estate yielded only 3 %, or even a little less. The causes of that strange situation were not easily grasped but he was inclined to ascribe it to the influence of internal politics.

In order not to be misunderstood, he would like to emphasize that the heavy fall of interest rates in Sweden had not, so far as he knew, touched the solvency of any Swedish life assurance company. He would also like to point out that the premiums which he had cited were hypothetical and did not correspond to actual Swedish conditions, and also that the floating-bonus system referred to in the paper was not a Swedish speciality; it was only his own company which was practising it in Sweden, and some Danish and Norwegian companies had a somewhat similar though not an identical method. For his conclusions and points of view he was, of course, alone responsible.

Mr K. A. Usherwood, in opening the discussion, said that the subject of the paper was one which seemed to have received rather less than its due meed of attention by the Institute, perhaps because failures of companies had been happily rare and those few which had taken place had arisen from causes other than those which the author had investigated. The only recent references which he had been able to trace were a paper by F. L. Collins (*J.I.A.* Vol. LVI, p. 152) on the difficulties arising in an actual insolvency, and some passages in E. H. Lever's paper on long-term investments (*J.I.A.* Vol. LXIX, p. 10) which touched on the difficulties caused by serious fluctuations in the rate of interest.

In order to exhibit the problem on abstract lines, free of all extraneous issues, the author had made certain postulates. In the first place, it was assumed that the element of competition was entirely absent, and that if other offices existed they were equally affected by the same fluctuations, so that the flow of new business after the fluctuations or the catastrophe had occurred was not disturbed. Equally it was assumed that there was no need to consider the flotation of new offices, because it was implicit in the paper that it was possible to recover past losses out of present and future profits, and that would no longer be possible if it were feasible for other offices to be floated which would not suffer from the burden of past deficiencies. He felt that even with those assumptions the question of the flow of new business was of importance, because the volume of life assurance transacted was a function not only of need but also of price, so that if the price were at any time uneconomic then the volume would be diminished, and the period which the author's model office took to reach solvency would be extended.

The author had also assumed that the office was mutual, and that all business was participating. The floating-bonus system dealt with in the paper was equivalent to a mortuary bonus declared for one year only but reserved for by the office on the basis of its continuation. In the United Kingdom such bonuses had arisen in the past in Industrial branch practice, though not so far as he knew in Ordinary branch practice, and the nearest equivalent would seem to be a reversionary bonus declared annually by an office making a bonus reserve valuation. Using for convenience the A 1924–29 mortality table and employing two rates of premium throwing up approximately the same amounts of floating bonus at 2% and 4% respectively as given in the paper, he found that the equivalent simple reversionary bonuses were 11*s.* % per annum for the lower scale of premiums, and 23*s.* % per annum for the higher scale. Those might seem low by present-day standards, but if taken as a mean of with-profit and without-profit business were not an unreasonable approximation to the facts.

On the author's assumptions of a reduction, more or less catastrophic, in the rate of interest, followed after an interval by an increase, the simple reversionary bonus system might be expected to throw up a time-lag compared with the floating-bonus system, since bonuses, once declared, were treated as payable equally with sums assured and could not be withdrawn. On the assumption of "short" investments—i.e. no depreciation—the author's model office reached equilibrium again—i.e. the full rate of bonus—somewhere about 1957–58, whereas he found that a model office under the reversionary bonus system would not reach equilibrium until 1972; there was thus a time-lag of about 14 years due to the difference in bonus method. That was on the assumption which the author had made that no bonus was declared until it was possible to declare the full rate which the premiums would theoretically support. If, on the other hand, bonuses were to be declared from time to time at such rates as were found possible on a bonus reserve basis, as would probably take place in practice, he thought that the full rate of bonus would never be reached again but that the tendency of the bonus rate would be asymptotic to the original rate. With the author's severe assumptions of investment in perpetual securities, of full depreciation in 1950 though no appreciation in 1940 because of conversion options, the effect of the reversionary bonus system was naturally very acute, and while the author's model office reached solvency apparently about 1978 and equilibrium on the old rates of bonus about 1993, with his own model office he arrived at the position where the accumulated deficiency was so large that the profit on new business was insufficient even to meet the interest on the deficiency, and therefore the office went steadily downhill, the last pound vanishing out of the coffers about 1985.

It was self-evident that a reversionary bonus system was much less flexible than a floating-bonus system, and the ability to remove bonuses already declared was a great advantage from the point of view of the office. He felt, however, that once policyholders had had a reversionary bonus system, it would be extremely difficult to convince them of the advantages of a mortuary bonus. The tendency in this country had actually been in the other direction, and certain offices which formerly had declared mortuary bonuses in Industrial branch business had found it advisable to change over to reversionary bonuses by slow degrees.

He had found some difficulty in applying the author's suggestions for dealing with the situation in 1940. Firstly, the author had suggested that it would be inequitable not to pay in full the maturities due on the date of valuation. If the position were clarified by assuming a continuous flow of new business instead of an issue every five years, it would be necessary to differentiate between maturities due on the valuation date, those due on the next day, those due two days later, and so on. He could see no grounds for drawing a distinction, other than that of degree, between liabilities which had actually matured and liabilities which would mature in the near or distant future. Secondly, there was what might perhaps be described as a recurring motif in the paper suggesting the preferential treatment of death claims at the expense of maturities. Whatever views might be held on the need for life assurance as such, he felt that an insurance contract was indivisible, and that maturity claims were entitled to treatment *pari passu* with death claims. Thirdly—the most fundamental point—it seemed to be implicit in the treatment suggested that, when the circumstances envisaged had arisen in 1940, the office should carry on and hope for better times. To his mind that seemed unjustifiable, for if in 1940 it was a matter of knowledge that the rate of interest would in 10 years increase again, then he thought that it was the duty of the actuary to value on the basis of 2% for 10 years and 4% thereafter, and the insolvency would not arise. If, on the other hand, as seemed more probable, the future course of the rate of interest was on the lap of the gods, he saw no option but to make a valuation at the immediate rate of interest of 2% and according to the result to treat the office as either solvent or insolvent, after making due allowance for bonuses which might have vested.

The question of the insolvency of a life assurance company must (in contrast to that of a trading company) be very largely a matter of opinion; but there must come a point when the actuary could not conscientiously say that the office was solvent, and if that point were reached, or if steps such as the legislature contemplated were taken from outside, then there arose, under English law, a totally different problem. If the office were insolvent under those conditions, mercantile practice, and he believed law, suggested a proportionate reduction in all liabilities, due immediately or at a future date, all policies being treated alike except those which, as a result of the forethought of policyholders, had been partially secured by loans. He mentioned the latter because he had known outside Europe a case where, for reasons not reflecting on the companies but of an international character, general doubt had arisen as to the ability of a particular group of offices to meet their liabilities, and those policyholders who had given thought to the situation had met it by taking immediate loans for the maximum amounts possible. It was probable, therefore, that in an insolvency the amount of loans on policies would be found to have greatly increased.

When the point of insolvency was reached, it seemed that the problem could not be discussed in an atmosphere of pure equity; other considerations stepped in. For example, in England under the 1909 Act a net premium valuation must, he believed, be made, and it would not appear possible, therefore, to distinguish between differences in rates of premium for the same class of business, even if the differences were as extreme as in the paper. Under the Insurance Undertakings Bill it seemed that an office premium valuation was required, and the position might arise, therefore, as in the paper, of a portion of the business being solvent while the balance, written at lower rates of premium, was insolvent. Whatever equity might suggest in those circumstances he thought it would be difficult to convince the Court that some policyholders should receive a surplus while others should go without part of their sums assured; and, speaking as a layman, he felt that the Court would take the view that all contractual liabilities were entitled to treatment on the same lines. He had used the term "contractual liabilities" because under the author's conditions that would cover the sums assured only, and in United Kingdom current practice it would cover bonuses already declared. The reference in the Undertakings Bill to "actual premiums" suggested that it might be possible to lay before the Court the consideration that high premium policies should be less severely treated than low premium policies, even to the point of distinguishing between the reductions to be applied, but he felt that that was the limit to which a Court brought up in English mercantile practice could be expected to go.

Mr H. E. Raynes remarked that the author had dealt with a rate of interest falling from 4% to 2%, but on long-dated British Government securities the net rate which could be secured at the present time was about £2. 11*s*. %. Possibly the worst had been passed but it was found that the net yield on life office funds was still falling as investments were turned over. There had been depreciation in capital values since 1932, but even if capital values had moved in inverse ratio to the fall in the rate of interest difficulties would still arise from the necessity of investing 4% premiums in a $2\frac{1}{2}$% market.

With a growing life fund, depreciation of capital values, providing it were associated with a corresponding rise in the net yield, was better than appreciation with a fall in the net yield. Unfortunately, full advantage could never be taken in investment policy of such changes in the market. The dice were loaded against the lender, as the author seemed to realize when he gave the alternatives of investment either in discount bills and deposit money, or in securities where the borrower always had the right to take advantage of the market.

In his own country the author had a remedy available in the floating bonus; in British offices, bonuses, once allotted, were a definite liability, and the author's second alternative of a moderate definite rate of bonus with a fluctuating interim rate, though tempting, would not conform to British practice.

British practice had been to make a stronger valuation than the current market conditions justified, thus holding back some surplus for future bonuses. A net premium valuation at a low rate of interest had been adopted by some actuaries and others had valued on the bonus reserve basis. The first method gave additional reserves of an indefinite amount and produced the uncomfortable feeling that net premiums had been valued which fell little short of the gross premiums. In his opinion the most practical method was to value with-profit policies on a bonus reserve basis and non-profit business on a gross premium basis, reserving an adequate sum for future expenses, at a rate of interest based on current market conditions. In addition there should be a reserve against the possibility of a further fall in the rate of interest and that other ever-present possibility, European war. Of the necessity for such an additional reserve he was convinced, and it should be built up from current profits so that policyholders would have to forgo immediate benefits for future safety.

Mr M. E. Ogborn said that, in his opinion, the paper had demonstrated that the terms for which the assets were invested should be closely linked with the terms of the liabilities. In the paper it was assumed that there were 20-year endowment assurances and that in the one case the assets were invested in "short" money, and in the other in investments which were perpetual although the borrower had an option to repay. The author would, of course, admit that neither investment method was the ideal. The effects of departing from the ideal were shown on pp. 132 and 133, and, particularly in the second case, were very severe.

The paper had also shown that for the sake of stability of life funds, it was desirable to consider the average experience over a long period. It was not right to take a point of time at which the experience was at one level without considering what was likely to be the experience in the future. If rates of interest fell to 2%, it did not necessarily follow that they would then increase again, but in general experience it was at least very probable, and some effect would generally be given to that in practice. Any distribution of surplus should be subordinated to the principle of stability, which came first. To obtain a true average experience, it seemed that the unit of time should be the policy generation and not the valuation period, as had commonly been assumed. It was implicit in the author's work that he considered the problem from the point of view of the yield over the unexpired term of the policy.

Mr K. J. Britt said that the industrial offices which granted mortuary bonuses usually did so only as a stop-gap and because they wanted to give something to holders of policies of long durations immediately if a claim arose instead of waiting until they had accumulated sufficient surplus to be able to grant reversionary bonuses. They did not intend to continue the method of mortuary bonuses indefinitely, and usually they set aside each year an amount considerably more than the sum required to grant the mortuary bonus for the following year, with the purpose of accumulating a reserve to transform that bonus gradually into a reversionary bonus. It was true that those offices which granted mortuary bonuses had had to suspend them during the last war, but nevertheless the policy-holders had been better off than if the offices had waited till reversionary bonuses could have been granted. Most of the offices which had previously practised the mortuary bonus method had since adopted the reversionary bonus system, and he believed that others might be expected to follow. Another point to be remembered in connexion with the mortuary bonus scheme as generally practised in the United Kingdom was that the mortuary bonuses were usually granted only to those policies which had been a number of years in force.

He thought that the main object of a proposer for a 20-year endowment assurance was investment, and he agreed with the opener that it would create dissatisfaction if a reduction were made in the bonus payable at maturity rather than in that payable at death.

On the question of dealing with insolvency, he agreed with the opener in thinking that the problem was quite different if all offices were in a similar position than if one or two only were affected. If they were all affected, he thought it very probable that legislation would be enacted to deal with the problem. If one or two only were affected, then in any practical solution it would be undesirable to encourage the holders of recently effected policies (where surpluses existed) into discontinuing their policies and so making the position worse. There was much to be said for dealing with the problem by calculating for each policy the amount of the sum assured that the future office premiums would buy on the basis of current mortality, interest and expenses, the balance of the sum assured being reduced to accord with the proportion of assets available to valuation liabilities.

Mr Henry Brown remarked that he had read the paper with great interest. The author deserved congratulation for his courage in struggling with the idioms of the English language, though it was also noteworthy how much he had been able to convey in a comparatively small space by means of his formulae and tables. Everyone must admire the author's brevity and concentration of thought, and the rigorous development of his ideas.

There was one sentence in which he thought that Dr Hagstroem had not quite maintained his high standard of rigorous deduction from his hypotheses. The sentence in question occurred immediately below the table on p. 132, and read: "It is seen that the level of 4% profits is reached again after 7 or 8 years for 'short' investment and after 43 years for perpetual investment." Looking at the table it would be seen that in 1955, which was 5 years after the rate of interest was assumed to have gone back to 4%, there was a deficiency in a 4% valuation on the "short" investment basis, while in 1960, which was 10 years after the rate of interest had gone back to 4%, there was a surplus, and at first sight it seemed reasonable to say that the level of 4% profits would be reached about half-way between those dates, *viz.* after 7 or 8 years; but if a little more thought were given to the matter, the conclusion would be reached that that was not quite correct. The 1955 valuation was on a 4% basis with what the author described as k_4 bonuses, *i.e.* the bonuses which the premiums would provide on a 4% basis, and the liabilities which were shown against year of entry 1935, of 10·098, were for sums assured and k_4 bonuses; but the hypothesis was that the office went on paying only k_2 bonuses until it was in a position to pay k_4 bonuses on everything, and the actual payment which would be made on 1 January 1956 in respect of the 1935 policies then maturing was not 10·098 but 8·450 for sums assured and ·051 for k_2 bonuses, making a total of 8·501, which was less than the assets in respect of those policies. There was thus a surplus of ·795, which, as the author showed, grew to ·967 by 1960. On 1 January 1956 that surplus would be more than enough to meet the rest of the deficiency, and the real point of time when the level of 4% profits was reached seemed to be the night of 1 January 1956 when the policies then maturing had been paid. He thought, therefore, that that sentence should be amended to say that the level of 4% profits would be reached immediately after the end of 5 years. The same point arose with regard to the table on p. 133, where on the basis of "perpetual investment" the level of 4% profits would be reached immediately after the end of 40 years.

He had come to the conclusion, with regret, that the author's suggestions as to practice, such as the floating-bonus method and giving a higher bonus to death claims than to maturities by survival did not seem likely to be of value in England, but the way in which the author had developed his "pocket" valuations might give English actuaries useful tools in making preliminary investigations of their own, and in any case it had been a great pleasure to study the paper.

Mr G. D. Stockman referred to the question raised by the author whether under English legislation it would be possible to have what amounted to a partial winding-up. He believed that there had been such an occurrence in the case of one company, where there was a reduction in the contracts very much on the lines suggested by Mr Britt, the amounts of sums assured which would be provided under the new conditions by the actual premiums payable being determined and the assets being apportioned in order to increase such sums assured. Unfortunately, the method did not have the desired result, because at a later stage the company had to be completely wound up. On the other hand, some friendly societies had been partially liquidated, and their subsequent careers had been quite successful.

When reading the paper he had been struck by what was perhaps an elementary truth, which some people might say was quite obvious, but which he did not think had always been brought out. Whereas with depreciation it was necessary to bear the full brunt of the loss, in the opposite case of appreciation the full benefit could very rarely be realized. The dice, as Mr Raynes had said, were loaded against the lender.

Sir William Elderton, in closing the discussion, said that the problem of the paper as he saw it was the problem of the partial liquidation of a life assurance fund. There was one form of partial liquidation which he thought every actuary would regard as being desirable in English law, and that was the partial liquidation of a composite company in respect of its life assurance fund, where the life assurance fund was solvent and failure had occurred owing to what might be called the failure of the underwriting policy. At present in English law such a partial liquidation was impossible, and it was to be sincerely hoped that new legislation would be introduced before any other unfortunate event happened in which a life assurance fund was robbed for the benefit of people who did not deserve it.

In the old days there was at least one English life assurance company which granted a bonus that could subsequently be reduced in altered conditions, the bonus lasting only till the next valuation. In spite of the custom at meetings of the Institute not to mention the names of companies, he thought that in the particular circumstances he might be forgiven for saying that it was the old Mutual. He believed that that method of bonus was described and perhaps invented by Manly. As the opener had said, there were great advantages in being able to wipe out bonuses which had been given too heavily in the past, but unfortunately that was usually impossible without going into liquidation. Possibly on the Continent the insurance offices were better off than in the United Kingdom as to a certain extent their premiums were fixed by a tariff, and consequently all the offices could by arrangement make alterations simultaneously; competition thus being presumably cut out to a large degree.

To those who thought that a sudden decrease in the rate of interest was merely an academic proposition, he suggested that they should consider how much increase in income tax was required to make a life assurance fund in England insolvent. It would be found that with a sudden increase in income tax the very problem which the author had put forward would arise, and he wondered whether, if that increase in income tax were to happen, the offices would ever get back to the old condition again.

He thought that in English Law it was contemplated that winding-up could be done in the form of partial liquidation, and he believed that the opener had confused the cash division of assets in the case of a complete winding-up with the condition of affairs when the Court decided to reduce contracts. Section 18 of the Assurance Companies Act, 1909, stated that "The Court, in the case of an assurance company which has been proved to be unable to pay its debts, may, if it thinks fit, reduce the amount of the contracts of the company upon such terms and subject to such conditions as the Court thinks just, in place of making a winding-up order". It was therefore open to the Court to reduce the contracts in any form it liked, and if it were proved to the Court that it would be unfair to reduce contracts within two years of maturity of an endowment assurance—if that could be proved—he saw no reason for saying that that would be wrong in law as the law now stood, although, of course, a good deal of evidence would be necessary.

In order to indicate a way in which it would be unfair for endowment assurances near maturity not to be paid the full sum assured although in other cases a reduction might be fair, he would take an example of an office rather like the author's model office which had made its investments so as to match its maturities. It was not a ridiculous idea from the point of view of investment; a certain number of offices thought it was advisable if they had large sinking-fund policies maturing at a particular time. If an office had wilfully done that, then surely if it had enough to pay its sums assured and bonuses in respect of those maturities, it might be argued that it would be hard on those policies to have the amounts which they would receive cut down. Another example would be the case of endowment assurances, some of which were going to mature very shortly, there having been a heavy decrease in the rate of interest not related to income tax, so that there was an appreciation in assets. It might then be considered that it was fair for those policies which were soon maturing to be paid, if anything, more than their sums assured rather than less!

The author's suggestion that in a partial liquidation it might be permissible to reduce the bonus for maturities and not to reduce it for death claims had been criticized that evening, but while he did not wholeheartedly agree with the suggestion, he thought that there was more to be said for it from one point of view than perhaps appeared at first sight. Surely if insolvency took place, the objection to a cash distribution was that a man might be a bad life and could not replace his life assurance. He could replace his investments if he could save money in the future, but he could not replace his assurance unless he happened to be assurable at ordinary rates. That being the objection to a cash distribution in the insolvency of a company, surely in a sense it ought to bear some weight in the case of partial insolvency. While he appreciated that there were difficulties in putting it into practice, he did not see that it should necessarily be ruled out altogether. He pointed out one difficulty in applying the author's suggestion, namely, that if it were done for endowment assurances generally it would presumably have to be done for all endowment assurances. If, therefore, an office had a large block of endowment assurances which had been effected mainly for ordinary purposes and another large block effected primarily to provide pensions, it would be difficult to deal with them differently, and yet to meet the requirements of the two classes they ought to be dealt with in different ways.

He added that it had been a great pleasure to him that the author was present, and to members generally that someone from Scandinavia had come to read a paper to them. It was a good thing to hear what people in other countries had to say; those who heard only the point of view of their own country were apt to become stereotyped, and the wider the variation of view which could be obtained the better it was for everyone.

The President (Col. H. J. P. Oakley), in proposing a vote of thanks to the author, said he was quite sure that all the members would endorse the last remarks of Sir William Elderton; they were indeed glad to have the views of someone outside their own country. The author had dealt with a pessimistic subject in a most delightful way, and he would probably agree that, pessimistic though the subject was, it had been discussed that evening with optimism. The author was really fulfilling the first function of an actuary in looking into the future and in devising means to deal with that future, whatever it might hold in store, and the paper which he had written and the warning which he had given would not be forgotten.

There was a very interesting gleam of light in the remarks of the opener when he spoke of the possibility of all policyholders taking policy loans. Such action would be advantageous to the office, because, of course, if all the policyholders would take policy loans to the limits of their surrender values any investment difficulties on the part of the office would be solved forthwith, and insolvency would be turned into solvency.

Perhaps the brightest feature in the discussion was the intervention in the debate by his predecessor and he felt sure that all the members had been delighted to hear Mr Henry Brown take part in the discussion in his old-time manner and with all his argumentative power.

Dr K.-G. Hagstroem, in reply, said it was generally agreed that there were differences between their practices, between their aims and between their opinions, but there were also great resemblances, and it was the resemblances which were the more important. The resemblances between life assurance in different countries were fundamental; the differences rested on the surface. The problem which had been discussed might seem more or less academic, but at some time such questions would no doubt arise, and even if opinions differed widely as to how they were best treated, it was gratifying to find so much unanimity as had been expressed in the discussion.

Fluctuations of interest, of course, grew less and less embarrassing the more it was possible to raise the importance of the risk element as against the purely endowment part of the stock of policies, and in that connexion he expressed his great admiration for the reform which had been carried out in England by the introduction of the family income policy. Going further on those lines, it would no doubt be possible to increase materially the immunity of life assurance to unfavourable changes in the rate of interest. He was convinced that under the guidance of actuarial forecasts of the future, imperfect and doubtful though they might appear, life assurance would be able to get rid of the difficulties. Once established, the idea of life assurance would always subsist, even if it must assume an aspect and undergo a metamorphosis which could not be foreseen. The strength of the idea was such that even universal stupidity—one of its greatest foes—could not hurt it in the long run.