

NOTES ON CAPITAL THEORY: A COMMENT ON 'THE ECONOMIC BASIS OF INTEREST RATES'

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1. In a recent article in this *Journal*, Professor Wilkie⁽²¹⁾ gave an account of the determination of the rate of interest in a Fisher-Hirshleifer model. In his analysis of a production economy (§§ 3-7), he appeared to take as intuitively reasonable, not to say axiomatic, the existence of an inverse monotonic relation between the value of investment and the rate of interest. 'A reduction in the auctioneer's rate of interest will stimulate capital investment.' (p. 290) Now one of the conclusions to emerge from the capital theory debates in economics of the 1950s and 1960s is that the value of investment is not in general an inversely monotonic function of the rate of interest (see, for example, Harcourt⁽⁸⁾). The purpose of these *Notes* is to alert actuaries to the existence of this conclusion and discuss some of its implications. To achieve the first objective, I shall need to construct a simple model—this is done in § 2. I devote § 3 to the analysis of the relation between the value of investment and the rate of interest. I discuss wider implications and draw some conclusions in § 4.

2.1 Consider a simple two-sector model, where the first industry produces iron, the second coal. Both commodities are used as means of production (or capital goods) in each industry, though they are completely used up in any production process. Thus, I deal with a single-product industries, circulating capital model, as in Part I of Sraffa⁽¹⁶⁾. (I abstract from such non-produced means of production as land.) I assume that both industries have the same period of production and, initially, each has only one process, subject to constant returns to scale. To describe production processes, I require the following notation:

X_{ij} = quantity of commodity i advanced as means of production
(and used up) in industry j

L_j = input of direct labour into industry j

X_j = gross output of commodity j

$i, j = 1, 2$, where 1 refers to iron, 2 to coal. I have implicitly assumed that the economy is in a stationary state—time suffixes are absent from inputs and outputs. This means that any net output (i.e. gross output minus inter-industry advances used up in the production process) is consumed. Gross investment consists of replacement only, net investment to increase productive capacity being zero. The process in the iron industry is described by the quadruple $\{X_{11}, X_{21}; L_1; X_1\}$, that in the coal industry by $\{X_{12}, X_{22}; L_2; X_2\}$. By the constant returns

hypothesis, all information for the first process can be carried by the triple $\{a_{11}, a_{21}, l_1\} = (I)$, that for the second process by $\{a_{12}, a_{22}, l_2\} = (II)$, where:

$a_{ij} = X_{ij}/X_j =$ input of commodity i per unit of output of commodity j

$l_j = L_j/X_j =$ input of direct labour per unit of output of commodity j

My model resembles that in §§4–7 of Wilkie⁽²¹⁾ in having a period of production of one year but differs from it in the explicit recognition of heterogeneous capital inputs.

2.2 Having described production processes, it is now time to derive the price equations. Note, first, that the price of one commodity cannot be determined in isolation: the price of coal depends on that of iron and vice versa. I use the following notation:

$p_j =$ price of commodity j

$w =$ wage rate, assumed uniform

$r =$ rate of profit, assumed uniform

Consider the first industry. The price of iron, p_1 , must be set to cover wage costs, wl_1 , depreciation charges, $p_1a_{11} + p_2a_{21}$, and pay profits at a uniform rate on the value of capital advanced, $r(p_1a_{11} + p_2a_{21})$. That is:

$$p_1 = wl_1 + (1 + r)(p_1a_{11} + p_2a_{21}) \quad (1a)$$

In this model, where there is only circulating capital, the value of capital advanced is equal to the value of capital used up. A similar equation holds for the second industry:

$$p_2 = wl_2 + (1 + r)(p_1a_{12} + p_2a_{22}) \quad (1b)$$

The choice of a uniform rate of profit in (1) is not at all arbitrary; rather, it reflects the operation of competitive forces in a capitalist economy. If an intersectoral profit rate differential arises, there is a tendency for entrepreneurs to move their capital from that industry with the lower rate of profit to that with the higher rate. In this way, the price system described in (1) is established. Adam Smith referred to it as a centre of gravitation, Ricardo as the natural system of prices. It is reasonable to assume that in equilibrium the rate of return on physical capital will be equal to the rate of return on financial capital; hence, the rate of interest can be identified with the rate of profit.

(1a) and (1b) constitute a system of two equations in four unknowns. If one of the commodities, say the first, is chosen as standard of value or numeraire, (1a) and (1b) can be rewritten as:

$$1 = wl_1 + (1 + r)(a_{11} + pa_{21}) \quad (2a)$$

$$p = wl_2 + (1 + r)(a_{12} + pa_{22}) \quad (2b)$$

where p is the relative price of coal and w is the real wage, both in terms of iron.

Selection of a numeraire reduces the number of unknowns by one; there is one degree of freedom in (2). Eliminate p from (2) to obtain:

$$w = \frac{(1 - (1 + r)a_{11})(1 - (1 + r)a_{22}) - (1 + r)^2 a_{12}a_{21}}{l_1(1 - (1 + r)a_{22}) + (1 + r)l_2a_{21}} = f(r)/g(r) \quad (3a)$$

Eliminate w from (2) to obtain:

$$p = \frac{l_2(1 - (1 + r)a_{11}) + (1 + r)l_1a_{12}}{l_1(1 - (1 + r)a_{22}) + (1 + r)l_2a_{21}} = h(r)/g(r) \quad (3b)$$

I am interested only in a model which supports a positive price system, i.e. $p \geq 0$, $w \geq 0$, $r \geq 0$. It can be shown from (3) that, if the economy is capable of producing a positive net output of both commodities, there is a positive value of r , denoted by R , such that:

- (i) $p > 0$, $w > 0$ for $0 \leq r < R$;
- (ii) $p > 0$, $w = 0$ for $r = R$;
- (iii) $dw/dr < 0$ for $0 \leq r \leq R$.

dp/dr is either positive, negative or zero throughout the interval $0 \leq r \leq R$; likewise, d^2w/dr^2 .

2.3 Now, I generalize the model by permitting the second industry to have alternative processes, say $(II\alpha)$ and $(II\beta)$: $(II\alpha) = \{a_{12}^\alpha, a_{22}^\alpha; l_2^\alpha\}$. I can construct two techniques of production, $\{\alpha\} = \{I, II\alpha\}$ and $\{\beta\} = \{I, II\beta\}$. It will be clear that this simple extension introduces the question of choice of technique, hitherto absent. The analysis of §2.2 can be applied to (α) and (β) separately. To deal with the question of choice of technique, suppose that (α) is initially employed at a rate of profit r lying between 0 and R^α , the maximum rate of profit supported by (α) . The corresponding wage rate is $w^\alpha = f^\alpha(r)/g^\alpha(r)$ and the relative price is $p^\alpha = h^\alpha(r)/g^\alpha(r)$. Now suppose that process $(II\beta)$ becomes available: should a capitalist in the second industry switch his process of production? To answer this, the capitalist evaluates $(II\beta)$ at the prevailing (i.e. (α) 's) prices; that is, he calculates:

$$s_2(\beta:\alpha) = p^\alpha - w^\alpha l_2^\beta - (1 + r)(a_{12}^\beta + p^\alpha a_{22}^\beta) \quad (4)$$

The right-hand side of (4) consists of the receipts, p^α , minus the costs, $w^\alpha l_2^\beta + (1 + r)(a_{12}^\beta + p^\alpha a_{22}^\beta)$. If $s_2(\beta:\alpha) > 0$, the capitalist is justified in switching because he can achieve a higher rate of profit, at least in the short run. If $s_2(\beta:\alpha) < 0$, $(II\beta)$ is more expensive than $(II\alpha)$. Finally, if $s_2(\beta:\alpha) = 0$, it is a matter of indifference whether $(II\alpha)$ or $(II\beta)$ is used.

To proceed with this analysis, substitute from (3a) and (3b) into (4) to obtain after straightforward but lengthy manipulations:

$$s_2(\beta:\alpha) = [l_2^\beta f^\alpha(r) - l_2^\beta f^\alpha(r) + (1 + r)l_1 k(r)]/g^\alpha(r) \quad (5a)$$

where

$$k^\alpha(r) = a_{12}^\alpha(1 - (1 + r)a_{22}^\alpha) - a_{12}^\beta(1 - (1 + r)a_{22}^\alpha) \quad (5b)$$

As remarked above, $s_2(\beta:\alpha)$ can be of any sign. Suppose for the sake of argument that $s_2(\beta:\alpha) > 0$: a capitalist in the second sector is justified in switching to process (II β). Suppose that the switch is made and (β)'s price system is established at the same rate of profit (i.e. solve (2) with (II β) replacing (II α) in (2b)). Would it ever be profitable for the capitalist to switch back to (II α)? To answer this question, it is necessary to calculate:

$$s_2(\alpha:\beta) = p^\beta - w^\beta l_2^\alpha - (1+r)(a_{12}^\alpha + p^\beta a_{22}^\alpha) \quad (6)$$

i.e. to evaluate (II α) at (β)'s prices. Substituting from (3) into (6) yields:

$$s_2(\alpha:\beta) = [-l_2^\alpha f^\beta(r) + l_2^\beta f^\alpha(r) - (1+r)l_1 k(r)]/g^\beta(r) \quad (7)$$

If $s_2(\beta:\alpha) > 0$, it follows from (7) that $s_2(\alpha:\beta) < 0$, because $g^\alpha(r) > 0$, $g^\beta(r) > 0$ at feasible values of r . This means that choice of technique is determinate for, using (5a) and (7), I have:

$$\text{sign } s_2(\beta:\alpha) = (-1) \text{sign } s_2(\alpha:\beta) \quad (8)$$

It is possible that $s_2(\alpha:\beta) = s_2(\beta:\alpha) = 0$, in which case the given value of r is called a switch-point.

Inferences on choice of technique can be drawn by using the diagram of w - r curves. To see this, calculate $w^\alpha - w^\beta$. From (3a), for (α) and (β), I obtain:

$$w^\alpha - w^\beta = (1+r)a_{12}[-l_2^\alpha f^\beta(r) + l_2^\beta f^\alpha(r) - (1+r)l_1 k(r)]/g^\alpha(r)g^\beta(r) \quad (9)$$

so that:

$$\text{sign } (w^\alpha - w^\beta) = \text{sign } s_2(\alpha:\beta) \quad (10)$$

If $s_2(\alpha:\beta) > 0$, (α) is the cost-minimizing technique because it is cheaper than (β) at both (α)'s and (β)'s prices: from (10), (α) supports the higher real wage. In Figure 1, (α) is chosen for $0 \leq r < \bar{r}$, (β) for $\bar{r} < r \leq R^\beta$. At $\bar{r}$, either (α) or (β) can be chosen. (This model is analysed further in Woods⁽²³⁾.)

2.4 Now consider the following numerical example, due to Garegnani⁽⁵⁾:

$$(I) = (1/12, 1/3; 1)$$

$$(II\alpha) = (1/6, 1/6; 1)$$

$$(II\beta) = (137/546, 19/273; 92/91)$$

This is illustrated in Figure 2. w^α and w^β intersect at $r_1 = 1/3$ and $r_2 = 1/2$. (α) is cost-minimizing for $0 \leq r < 1/3$ and $1/2 < r \leq R^\alpha$, while (β) is cost-minimizing for $1/3 < r < 1/2$. This is called reswitching of techniques.

3.1 The significance of reswitching lies in the fact that it is sufficient to imply that the value of investment is not a monotonically decreasing function of the rate of profit. To see this, consider Figure 3. As the economy is in a stationary state, net output consists only of consumption. If the rate of profit is zero, net income is equal to wages. As the values of net income and output are identical, it follows that in a stationary state both are measured by the vertical intercept $0W$.

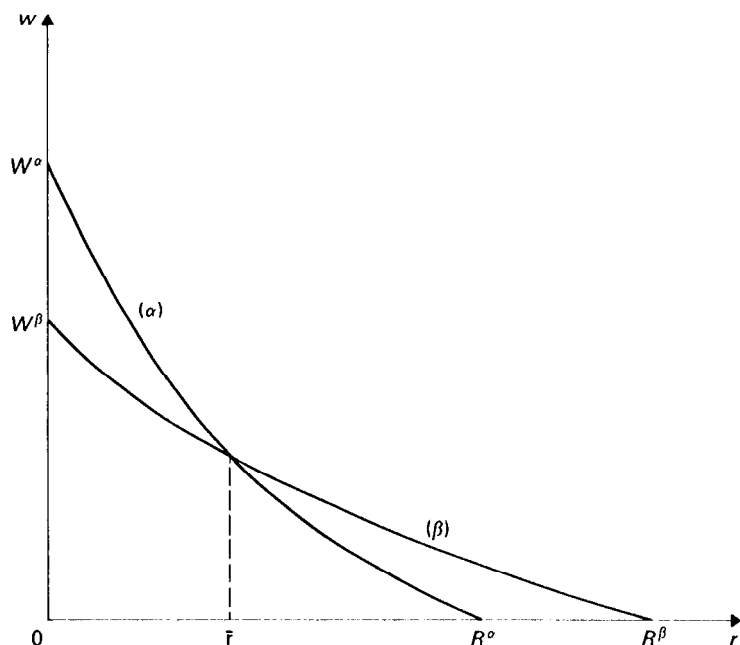


Figure 1. Two techniques: one switch-point.

As $0\bar{w}$ is the wage rate, $0W - 0\bar{w} = \bar{w}W$ is the value of profits per head. In triangle $AW\bar{w}$, $\bar{w}A = 0\bar{r}$ is the rate of profit. So, $\tan \bar{w}AW = \bar{w}W/\bar{w}A$ is the value of capital per head, being the value of profits per head divided by the rate of profit. However, in this model, the value of capital is equal to the value of investment.

Now consider Figure 2 for increasing values of r from 0. At r_1 , there is a switch from (α) to (β) , from the technique with the higher value of capital per man to that with the lower value of capital per man: at this value of r , investment and the rate of profit are inversely related. However, at r_2 , there is a switch from (β) to (α) , from the technique with the lower value of capital per man to that with the higher value of capital per man: in other words, at r_2 , investment and the rate of profit are positively related.

Garegnani's numerical example of reswitching conclusively demonstrates, *inter alia*, that there is not necessarily an inverse monotonic relation between the value of investment per head and the rate of profit.

3.2 It is important to realize that reswitching is sufficient, not necessary, for the 'perverse' phenomenon described above. Consider Figure 4, in which technique (γ) has been superimposed on Figure 2. The second sector has a third process, $(II\gamma)$. The rule for choice of technique discussed in § 2.3—at a given value

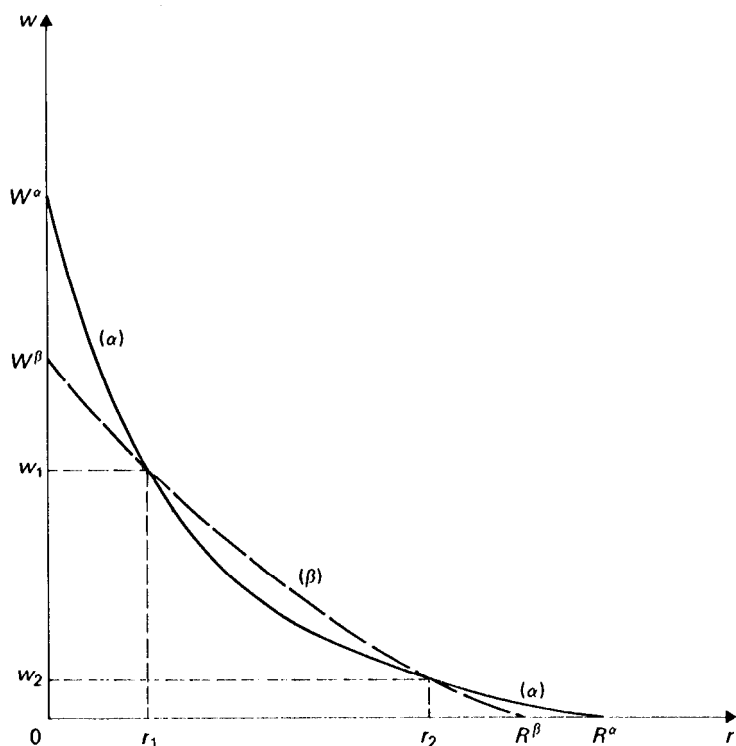
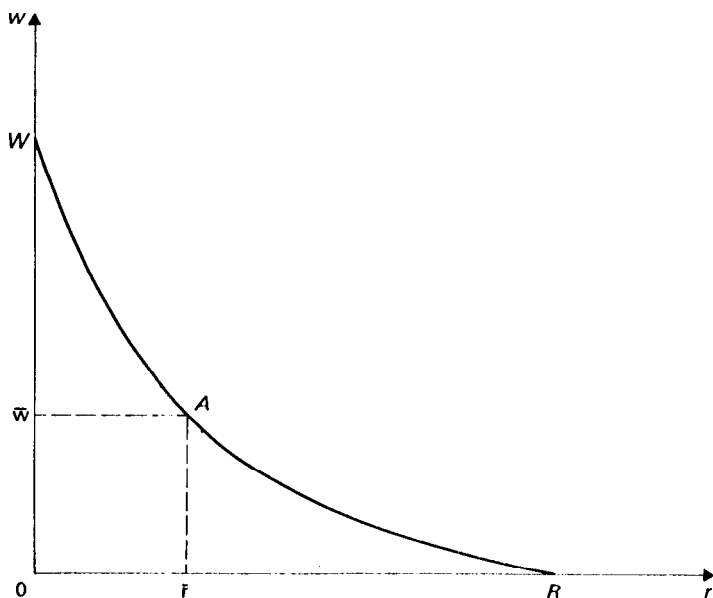


Figure 2. Two techniques of production: reswitching.

of r , the cost-minimizing technique supports the highest real wage—remains valid. So, (γ) is chosen for $0 \leq r < r_3$, (β) for $r_3 < r < r_2$ and (α) for $r_2 < r \leq R^2$. At r_3 , the switch is to the technique with the lower value of investment but, at r_2 , it is to that with the higher value of investment. Non-monotonicity of investment as a function of the rate of profit obtains in the absence of reswitching. In Figures 2 and 4, the problem arises (if there is one) at r_2 . What occurs there is called capital-reversing or a negative real Wicksell effect (so called because it was first discussed by Ricardo).

4.1 The existence of the reswitching and capital reversing phenomena has quite wide-ranging implications for economic theory. An early casualty was the traditional neo-classical aggregate production function. (Indeed, the first skirmishes in the capital theory debates were fought over this concept—see Robinson⁽¹⁵⁾, Champernowne⁽²⁾ and Harcourt⁽⁸⁾.) To perceive the effect of the two phenomena on the aggregate production function, reconsider the example of

Figure 3. w - r curve for one technique.

§ 2.4, illustrated in Figures 2 and 3. Applying the theory developed in § 3, I can construct Figure 5, relating net output per head, y , to the value of capital per head, k . This can be interpreted as a production function, though it does not exhibit the usual concavity. From Figure 2, it can be seen that (α) is employed at rate of profit $r=0$, with net output per head given by $y^\alpha(0)=W^\alpha$ and value of capital per head by $k^\alpha(0)$. The economy is at point $A=(k^\alpha(0), W^\alpha)$. As r increases towards r_1 , the value of capital per head in (α) decreases, as can be seen using Figure 3, though net output is constant. Roughly speaking, as r increases from 0 to r_1 , the economy moves from A to $B=(k^\alpha(r_1), W^\alpha)$. At r_1 , either (α) or (β) could be employed. If (β) were employed, the economy would be at $C=(k^\beta(r_1), W^\beta)$. Now, at a switch-point between (α) and (β) , either (α) or (β) or a convex combination of the two is cost-minimizing: BC represents the locus of such combinations. For values of r between r_1 and r_2 , (β) is cost-minimizing, with corresponding k and y given by an appropriate point on CD . Consider next the switch-point r_2 , where (α) and (β) are again cost-minimizing. If (α) were employed, the economy would be at $E=(k^\alpha(r_2), W^\alpha)$. The line DE represents convex combinations of (α) and (β) that are cost-minimizing at r_2 . For values of r between r_2 and R , the corresponding values of y and k are given by a point on EF .

It is clear that the line $ABCDEF$ does not represent the graph of a concave

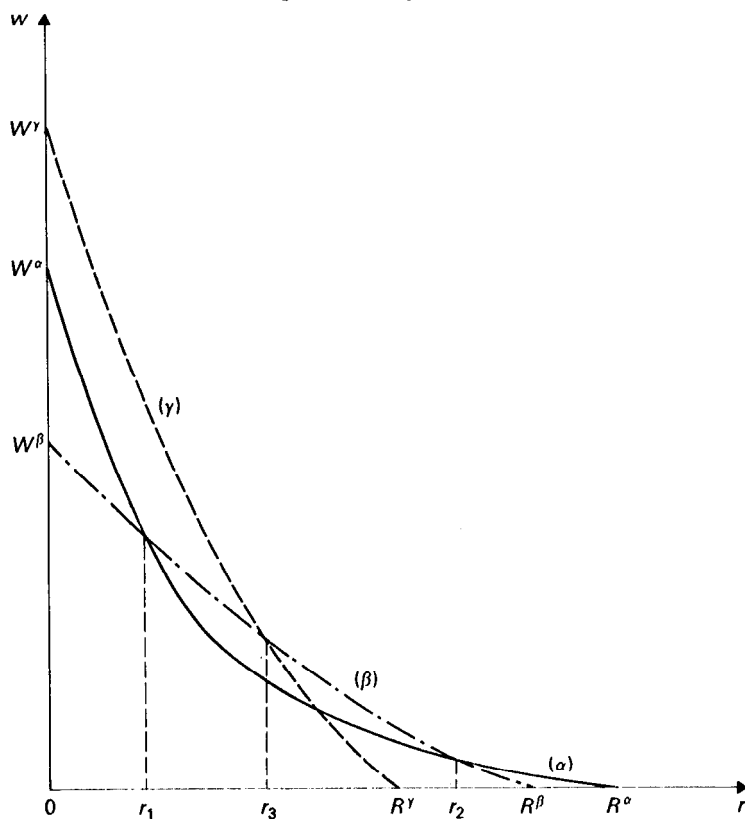


Figure 4. Three techniques of production: capital reversing without reswitching.

function. Note that GCH is not dominated by EF , though it lies vertically beneath it. Each point on $ABCDEF$ is efficient, meaning that at some rate of profit it is cost-minimizing. So, $H = (k^\beta(r), W^\beta)$ represents a cost-minimizing technique for some value r between r_1 and r_2 , while $F = (k^\alpha(R^\alpha), W^\alpha)$ represents a cost-minimizing technique at R^α : in the former, it is (β) that is cheapest while, in the latter, it is (α) . Evidently, the calculations are being performed at different rates of profit: that one technique has a higher net output per head than the other, for the same amount of capital per head, is not surprising once this simple point is realized.

The interpretation of Figure 5 as a production function could be challenged on the following grounds: normally, a production function is regarded as a physical concept yet, in Figure 5, there are both price and quantity changes occurring.

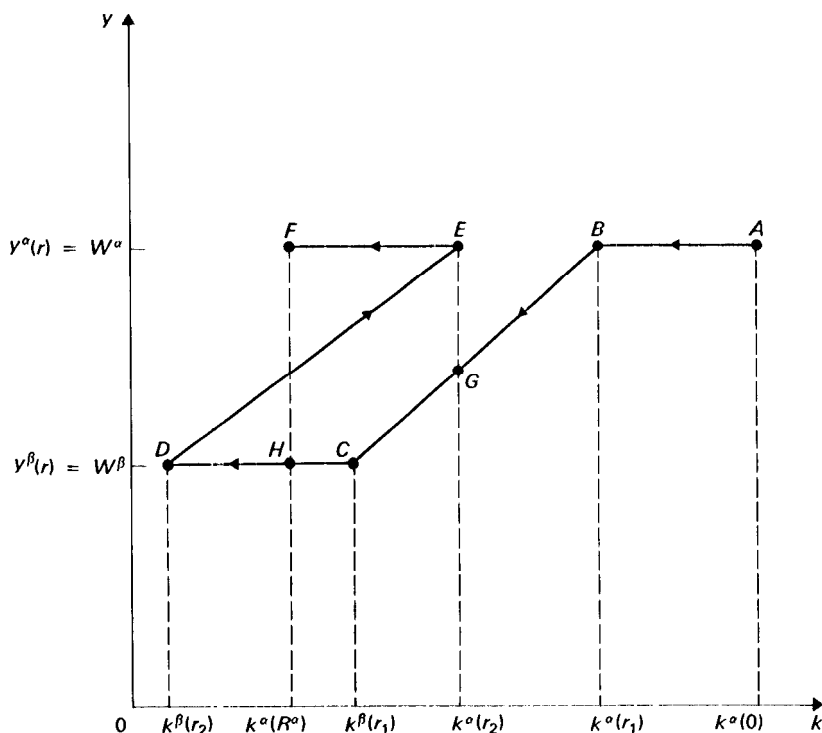


Figure 5. A production function corresponding to Figure 2.

Along AB , CD and EF it is only prices that are changing because, in each instance, the technique of production is fixed. On the other hand, only quantities change along BC and DE because the price systems of the two techniques are identical at a switch-point. It is natural to enquire if these price changes can be excluded so that a purely physical production function is obtained. Suppose, following Champernowne⁽²⁾, that a chain-index of capital is constructed which changes in value only when there is a change in the physical composition of the capital stock, i.e. only when the technique of production changes. The chain-index of capital would then have one value, say K^α , for the whole interval between 0 and r_1 . At r_1 , the chain-index value would be altered to, say, $K^{\alpha\beta}$, reflecting the fact that there had been a switch to (β) . The chain-index would be fixed at this new value between r_1 and r_2 . At the second switch-point, the chain-index would increase to, say $K^{\beta\alpha}$, as there is a switch back to (α) . So, I obtain Figure 6, which again does not represent a concave function. Comparing Figures

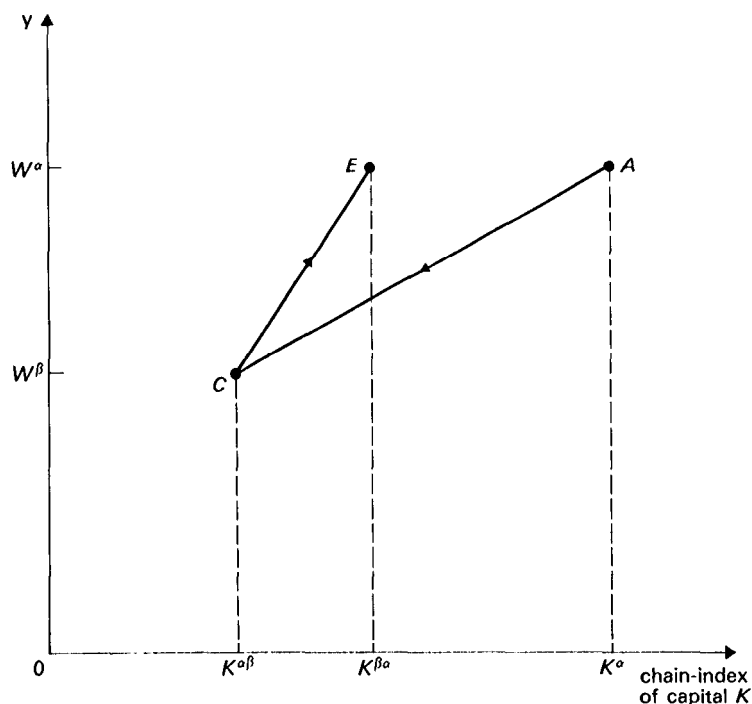


Figure 6. A production function corresponding to Figure 2 using a chain index of capital.

5 and 6, it is *as if* AB , from the former, had been collapsed onto A , in the latter, CD onto C and EF onto E . (Note that the horizontal axes in the two Figures are expressed in terms of different units.) A represents technique (α) between rates of interest 0 and r_1 , with chain-index value of capital K^α . E represents exactly the same set of physical capital goods in the same proportions as A but has a different chain-index value of capital $K^{\beta\alpha}$, simply because it is valued over a different range of values of r . Note, however, that if (β) did not exist the chain-index value of capital in (α) over the interval between r_2 and R^2 would be K^α , not $K^{\beta\alpha}$, because it would be equal to K^α for all values of r between 0 and R^2 . The chain-index approach does not provide a means of calculating capital values absolutely: the same technique can have two chain-index values, as Figure 6 demonstrates.

Reswitching is sufficient, not necessary, for a non-concave production function. If the line of analysis above were applied to the techniques in Figure 4, non-concavities would arise. Having recognized the problems that would be caused by reswitching and capital reversing, Champernowne⁽²⁾ excluded them by

the simple expedient of assuming that they would not occur, though he admitted that "there is no logical justification for the assumption(s)". (p. 119)

It is clear from the discussion in § 3 and above that the value of capital is not an inversely monotonic function of the rate of interest; neither, for that matter, is the capital-output ratio, nor the level of consumption per head. In a stationary state, the level of consumption per head is equal to net income per head, which is given by the maximum real wage for the technique in question. Referring to Figure 2, consumption per head is equal to W^α for low values of r , W^β for intermediate values of r and W^γ again for high values of r . Inverse monotonicity of capital per head, the capital-output ratio and consumption per head as functions of the rate of interest were among what Harcourt⁽⁸⁾ referred to as the 'neo-classical parables'. These parables are not generally valid.

One advantage of dealing with a circulating capital model is that any conclusion drawn about capital applies immediately to investment: for, when all capital is used up during the production period, the value of capital is necessarily equal to the value of investment. From the discussion above, the value of investment is not an inverse monotonic function of the rate of interest. In other words, the traditional investment function, or the marginal efficiency of capital as Keynes called it, is shown to be without theoretical foundation. This may seem to be a strong conclusion to be drawn from the analysis of a special case. Yet, if it were well founded, the investment function would be robust in the analysis of particular examples. Nonetheless, it might be useful to consider models where the values of capital and investment are not identical (i.e. where there is fixed capital). Two options present themselves: first, where there are *a priori* specified depreciation coefficients, as in Samuelson⁽¹⁶⁾, Champernowne⁽²⁾ or Garegnani⁽⁴⁾; second, where fixed capital is treated within a joint production framework, as in Part II of Sraffa⁽¹⁸⁾. Though unsatisfactory from a theoretical viewpoint, the first option was the one chosen by most contributors to the capital theory debates. It suffices to record here the fact that the possibilities discussed above for the circulating capital model also arise when the first option is selected (see Garegnani⁽⁴⁾, for example). Reswitching and capital reversing can occur in two ways in proper fixed capital models, that is, when the second option is chosen. First, there can be reswitching between two processes, where each is defined in terms of the use of a particular durable instrument of production. Second, when there is only one machine-using process, the same optimal economic lifetime can occur at both low and high values of the rate of interest, with a different optimal lifetime at intermediate values. The implication is that the possibilities illustrated for the simplest capital model can occur in more general models. Should residual doubt remain on this, I have analysed in the Appendix two of Samuelson's⁽¹⁷⁾ numerical examples—one of a maturing-labour inputs model, the other of a durable capital model. (The spectrum of production models is treated in Woods⁽²⁴⁾, to which the reader is referred for a complete account of the theory of choice of technique.)

At this juncture, I can refer to the observation made in § 1 that Professor Wilkie

assumes the value of investment to be an inverse monotonic function of the rate of interest. From §§ 2–3 and the discussion above it is clear that, at worst, the hypothesized relation is not generally valid and, at best, requires justification. A corollary of the first (pessimistic) interpretation could well be the abandonment of the investment function altogether, as advocated by Eatwell and Milgate⁽³⁾. If the second (more optimistic) interpretation were adopted, the next stage could well be a sensitivity analysis of the underlying assumptions. For example, if each sector has a differentiable concave production function, reswitching of techniques cannot arise. However, as Burmeister and Dobell⁽¹⁾ have demonstrated, capital reversing can still occur (see their Chapter 9). This means that, even in differentiable neo-classical economies, the value of investment is not necessarily an inverse monotonic function of the rate of interest. The so-called ‘Paradoxes in Capital Theory’⁽¹³⁾ cannot be eliminated by choosing a continuous rather than a discrete technology. (See Woods⁽²²⁾ for further discussion of the relation between these two representations of technical possibilities.) It is difficult to avoid the general conclusion that the value of investment is not an inversely monotonic function of the rate of interest. In turn this has serious implications for the convergence process described in § 7 of Wilkie⁽²¹⁾. The (imaginary Walrasian) auctioneer will experience considerable, not to say insuperable, problems: a reduction in the rate of interest does not necessarily stimulate capital investment. This point has been discussed forcefully by Garegnani^{(4),(6),(7)}; in particular, the demand-and-supply mechanism may be unable to function as normally supposed, thereby calling into question the whole basis of economic equilibrium.

An inference to be drawn from this discussion is that two pillars of traditional macro-economic theory—the aggregate production function and the investment function—do not have firm foundations. The implications of reswitching and capital reversing do not rest there. Reconsider the numerical example in § 2.4. As the wage rate falls from W^a through w_1 , the second sector’s demand for labour per unit of output rises from 1 to 92/91. Further falls in the wage rate through r_2 result in a *decrease* in this sector’s demand for labour from 92/91 to 1. In other words, the demand function for labour is not an inverse monotonic function of the real wage rate. Steedman⁽¹⁹⁾ has demonstrated that this conclusion can be extended to the aggregate demand for labour. So, the traditional demand function for labour, yet another component of macro-economic models, is shown to be without solid foundations.

The discussion so far has concentrated on the macro-economic implications of reswitching and capital reversing. However, in the previous paragraph, I have alluded to their micro-economic implications. Further consideration of the numerical example in § 2.4 reveals another line of argument. As demonstrated in Woods⁽²³⁾, the relative price is either a positive or an inverse monotonic function of the rate of interest for both techniques when there is reswitching. Immediately, this implies that the second sector’s demand for a particular type of capital good cannot be an inverse monotonic function of its price (or rental). Thus, the principle of substitution does not seem to apply to either produced or non-

produced inputs, for I have shown that, as the price of an input changes in one direction, the quantity employed does not necessarily change in the opposite direction. The earlier emphasis in the reswitching debates established the macro-economic analogue of this, namely that a fall in the rate of interest does not necessarily induce a switch to more capital-intensive techniques. (See Pasinetti⁽¹¹⁾,⁽¹²⁾ for further discussion of the notion of substitution in production models.)

4.2 The issues of reswitching and capital reversing have to a large extent been discussed in relation to the question of capital measurement. This is, perhaps, not surprising, given that the influential contributions of Robinson⁽¹⁵⁾ and Champernowne⁽²⁾ were addressed to this question. The dependence of capital values on income distribution (the rate of interest and the real wage) was suggested by Ricardo⁽¹⁴⁾ in Chapter I of *'The Principles of Political Economy and Taxation'*. The Classical/Ricardian approach was subsequently abandoned, having to await Sraffa⁽¹⁸⁾, Robinson⁽¹⁵⁾ and others for its revival. In the intervening period of a century or so, it was the Swedish economist Wickcell who drew attention to the problems inherent in the neo-classical approach to capital theory. Wickcell⁽²⁰⁾ was able to prove that, even under perfect competition, the rate of interest is not equal to the marginal product of capital (cf. condition (iv) in § 7 of Wilkie⁽²¹⁾, p. 290). Wickcell's approach was repeated by Metzler⁽¹⁰⁾, another neo-classical economist, who argued that the divergence between the marginal product of capital and the rate of interest "is attributable to the fact that it is impossible to find an invariant unit in which to measure the social quantity of capital. To put the matter another way, we may say that a change in the supply of capital—arising, for example, from new voluntary savings—alters the units in which all the previously existing capital is measured; and it is therefore incorrect to say that the supply of capital as a whole has increased by the amount of the voluntary saving. It is important to emphasize that this problem of measuring the quantity of capital is not an index-number problem. There are, to be sure, numerous index-number problems of the greatest complexity in the theory of capital. But the problem to which I now refer would exist even in the simplest economy in which all output consisted of a single type of consumer's good and all firms were exactly alike." As Wickcell and Metzler demonstrated and as I have discussed above, the problem with capital is all-pervasive, having serious implications for micro- as well as macro-economics.

4.3 As a final remark, let me say that, although I have been critical of Wilkie's account of the rate of interest, no criticism is intended of the use to which the concept has been put, as in McCutcheon and Scott⁽⁹⁾, for example. Nonetheless, actuaries should resist the temptation to associate reswitching of techniques with multiple internal rates of return: the former arises in a general equilibrium framework, the latter in a partial equilibrium framework.

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APPENDIX

A.1 In this Appendix, I analyse two examples due to Samuelson⁽¹⁷⁾. The first is of a maturing-labour inputs model, the second of a proper fixed capital model.

A.2 Let (α) be described by these processes:

$$7 \text{ units of labour} \oplus 1 \text{ litre of brandy} \quad (1)$$

$$1 \text{ litre of brandy} \oplus 1 \text{ litre of champagne}$$

7 units of labour produce 1 litre of brandy which, when left to stand for one period, ferments into 1 litre of champagne. Assuming wages are paid at the end of the production period, the price equations are:

$$7w^\alpha =: p_1^\alpha \quad (2)$$

$$(1+r)p_1^\alpha =: p^\alpha = 1$$

where p_1^α is the price of brandy in terms of champagne, the numeraire. From (2), I obtain:

$$w^\alpha = 1/7(1+r)$$

$$dw^\alpha/dr < 0, d^2w^\alpha/dr^2 > 0. \quad (3)$$

Let (β) be described by these three processes:

$$2 \text{ units of labour} \oplus 1 \text{ litre of grape-juice}$$

$$1 \text{ litre of grape-juice} \oplus 1 \text{ litre of wine} \quad (4)$$

$$1 \text{ litre of wine} \oplus 6 \text{ units of labour} \oplus 1 \text{ litre of champagne}$$

2 units of labour produce 1 litre of grape-juice which ripens by itself into wine. When the wine is shaken by 6 units of labour, 1 litre of champagne is obtained. The corresponding price equations are:

$$2w^\beta = p_2^\beta$$

$$(1+r)p_2^\beta = p_3^\beta \quad (5)$$

$$(1+r)p_3^\beta + 6w^\beta = p^\beta = 1$$

where p_2^β is the price of grape-juice, p_3^β that of wine. From (5), I obtain:

$$w^\beta = 1/[2(1+r)^2 + 6] \quad (6)$$

$$dw^\beta/dr < 0, d^2w^\beta/dr^2 > 0$$

(3) and (6) intersect at the solutions of:

$$1/7(1+r) = 1/[2(1+r)^2 + 6] \quad \text{or}$$

$$\begin{aligned}
 2(1+r)^2 - 7(1+r) + 6 &= 0 & \text{or} \\
 [2(1+r) - 3][(1+r) - 2] &= 0 \\
 r_1 = 0.5, r_2 = 1. & \quad (\text{See Figure 7.})
 \end{aligned}
 \tag{7}$$

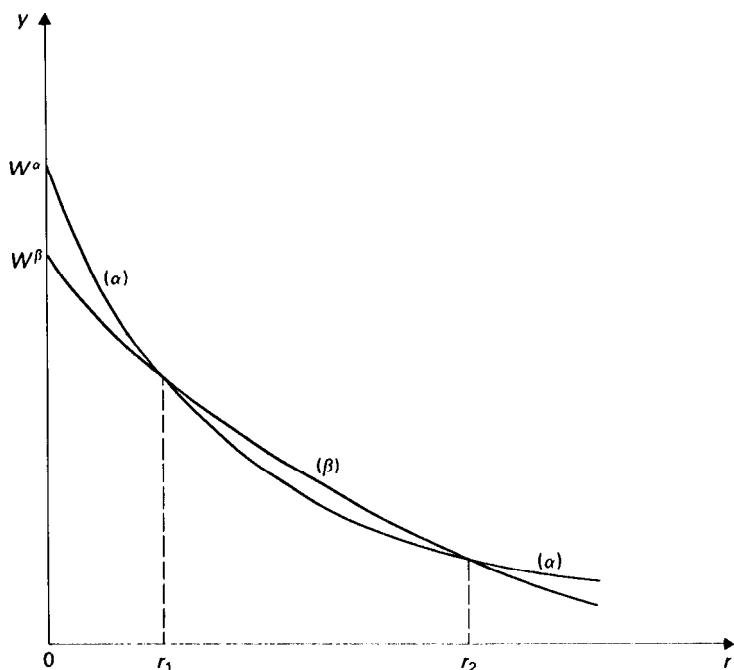


Figure 7. w - r curves for the maturing-labour inputs model.

Suppose (β) is initially employed at r . The surplus/loss relative to the prevailing rate of profit by employing (α) is obtained from:

$$\begin{aligned}
 s(\alpha:\beta) &= 1 - 7(1+r)w^\beta \\
 &= 1 - 7(1+r)/[6 + 2(1+r)^2] \\
 &= [2(1+r)^2 - 7(1+r) + 6]/[6 + 2(1+r)^2] \\
 &= F(r)/[6 + 2(1+r)^2]
 \end{aligned}
 \tag{8}$$

On the other hand, if (α) is initially employed, the extra profit to be obtained by switching to (β) is given by:

$$\begin{aligned}
 s(\beta:\alpha) &= 1 - w[6 + 2(1+r)^2] \\
 &= 1 - [6 + 2(1+r)^2]/7(1+r) \\
 &= [-2(1+r)^2 + 7(1+r) - 6]/7(1+r) \\
 &= -F(r)/7(1+r)
 \end{aligned} \tag{9}$$

From (8) and (9):

$$\text{sign } s(\alpha:\beta) = -\text{sign } s(\beta:\alpha) \tag{10}$$

Clearly, from (8) to (10), choice of technique is determinate, the cost-minimizing technique being the one to support the higher real wage. From (7) to (9) and Figure 7, there are two switch-points, with (α) employed for low and high values of r and (β) for intermediate values. Thus, reswitching and capital reversing can occur in maturing-labour input models.

A.3 I now consider a fixed capital model. Let technique (γ) consist of the following processes:

$$\begin{aligned}
 &1 \text{ labour} \oplus 1 \text{ new machine} \\
 &1 \text{ new machine} \oplus 18 \text{ units output} \oplus 1 \text{ one-year old machine} \\
 &1 \text{ one-year old machine} \oplus 1 \text{ two-year old machine} \\
 &1 \text{ two-year old machine} \oplus 54 \text{ units output}
 \end{aligned} \tag{11}$$

1 unit of labour produces a new machine, which produces 18 units of output one period later and 54 units of output three periods later. The price equations are:

$$\begin{aligned}
 w^\gamma &= p^\gamma(0) \\
 (1+r)p^\gamma(0) &= 18p^\gamma + p^\gamma(1) = 18 + p^\gamma(1) \\
 (1+r)p^\gamma(1) &= p^\gamma(2) \\
 (1+r)p^\gamma(2) &= 54
 \end{aligned} \tag{12}$$

where $p^\gamma = 1$ is the price of output and $p^\gamma(t)$ is the price of a t -year old machine. From (12), I obtain:

$$\begin{aligned}
 w^\gamma &= [54 + 18(1+r)^2]/(1+r)^3 \\
 dw^\gamma/dr &< 0, \quad d^2w^\gamma/dr^2 > 0.
 \end{aligned} \tag{13}$$

Let (δ) have processes:

$$\begin{aligned}
 &1 \text{ unit labour} \oplus 1 \text{ new machine} \\
 &1 \text{ new machine} \oplus 1 \text{ one-year old machine} \\
 &1 \text{ one-year old machine} \oplus 63 \text{ units of output}
 \end{aligned} \tag{14}$$

1 unit of labour produces a new machine (different from that in (γ)), which in turn produces 63 units of output two periods later. The price equations are:

$$\begin{aligned}w^\delta &= p^\delta(0) \\(1+r)p^\delta(0) &= p^\delta(1) \\(1+r)p^\delta(1) &= 63p^\delta = 63\end{aligned}\tag{15}$$

From (15), I have:

$$\begin{aligned}w^\delta &= 63/(1+r)^2 \\dw^\delta/dr &< 0, d^2w^\delta/dr^2 > 0.\end{aligned}\tag{16}$$

Equating (13) and (16), I obtain:

$$2(1+r)^2 - 7(1+r) + 6 = 0\tag{17}$$

which is identical to (7). As $W^\gamma > W^\delta$, it is clear that the diagram of $w-r$ curves is qualitatively identical to Figure 7, with (γ) replacing (α) , (δ) replacing (β) . To determine choice of technique, evaluate (γ) at (δ) 's prices:

$$\begin{aligned}s(\gamma:\delta) &= 54 - [(1+r)^3w^\delta - 18(1+r)^2] \\&= 54 - (1+r)^3 \cdot 63/(1+r)^2 + 18(1+r)^2 \\&= 9[2(1+r)^2 - 7(1+r) + 6]/(1+r) \\&= 9F(r)/(1+r)\end{aligned}\tag{18}$$

Now evaluate (δ) at (γ) 's prices:

$$\begin{aligned}s(\delta:\gamma) &= 63 - (1+r)^2 \cdot w^\gamma \\&= 63 - (1+r)^2[54 + 18(1+r)^2]/(1+r)^3 \\&= -9F(r)/(1+r)\end{aligned}\tag{19}$$

From (18) and (19):

$$\text{sign } s(\gamma:\delta) = -\text{sign } s(\delta:\gamma)\tag{20}$$

(20) means that choice of technique is determinate, with the cost-minimizing technique supporting the higher real wage. So, (γ) is optimal over two disjoint intervals of the rate of interest. In other words, reswitching can occur in a proper fixed capital model. (Strictly speaking, this conclusion has not been established: I should first have demonstrated that it is never optimal to scrap the machine in (γ) after the first year. In fact, it is always optimal to employ the machine in (γ) for three years.)