NOTES ON FOREIGN ACTUARIAL JOURNALS

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HOLLAND

Het Verzekerings-Archief, Vol. XXIII, 1942

W. Meyer. *Overzicht van de theorie der winstuitkeering aan verzekerden*, pp. 1–14. A discussion of the various 'mechanical' and 'natural' systems of distribution of surplus used in Britain, America, and on the Continent.

J. B. D. Derksen. *De berekening der sterftekansen bij de samenstelling van sterftetafels; een mathematisch-statistische studie*, pp. 15–39. It has been customary in the Dutch population tables to estimate the mortality during ten inter-censal years by using the unweighted mean of the ten individual values of q_x involved. This procedure is here justified on the assumption of a linear secular trend.

K. Goldziher. *Logistische Bearbeitung der säkularen Änderungen in der niederländischen Volkssterblichkeit*, pp. 40–72. Fits the logistic curve

$$p_x(t) = p_x(\infty)\,(1 + e^{\alpha - rt})^{1/\rho},$$

where $p_x(\infty)$ is taken as unity, to Dutch population data since 1870. The idea is to illustrate the method of fitting with a view to its application to annuity data in order to facilitate 'forecasting'.

M. van Haaften. *Combinaties van verzekeringen bij leven op* 0, 1 *en* 2 *levens*, pp. 73–74.

M. van Haaften. *La connexion de l'assurance épargne, l'assurance à terme fixe et l'assurance mixte*, pp. 75–79. If $\mathrm{P}^{t}_{x\overline{n}|} = \dfrac{\mathrm{A}_{\overline{n}|}}{\mathrm{a}_{x\overline{n}|}}$ and $\mathrm{P}^{r}_{x\overline{n}|} = \dfrac{\mathrm{a}_{\overline{n}|} - \mathrm{a}_{x\overline{n}|}}{\mathrm{a}_{x\overline{n}|}}$, then

$$\mathrm{P}_{x\overline{n}|} = \mathrm{P}^{t}_{x\overline{n}|} + \mathrm{P}^{r}_{x\overline{n}|}\, d = \mathrm{P}_{\overline{n}|} + \mathrm{P}^{r}_{x\overline{n}|}\, \mathrm{a}^{-1}_{\overline{n}|}\,.$$

C. Campagne. *De sommatie-formule van Euler*, pp. 81–94. A proof of the Euler-Maclaurin formula with remainder term, valid for certain holomorphic functions.

J. Brands. 15*e Verzekeringsdag. De kapitalisatie bij verzekeringmaatschappijen*, pp. 95–101. Resumé of an address on the financing of Dutch insurance companies delivered on the fifteenth anniversary of the (Dutch) Insurance Society.

S. Vajda. *On some approximations from valuation statistics*, pp. 102–103. It is proved mathematically that, whatever the distribution by age of sums assured, the mean ages derived from the ratios V.S.A. ÷ S.A. and V.N.P. ÷ N.P. increase with the valuation rate of interest.

W. VENEMA. *Enkele beschouwingen over herverzekering*, pp. 104–124. An interesting review of the articles of the Eighth and Eleventh Congresses on reassurance. Meidell's formula for the desirable maximum retention of a life office, viz. $\frac{2}{3}m\,(1+N^{\frac{1}{2}}+N^{-\frac{1}{2}})$, where m = mean sum assured, N = total sum assured, is derived and discussed.

G. SCHELTEMA. *Besmetting met den tuberkelbacil*, pp. 125–129. Comparisons of the percentages of 12-year-old children with tubercular infection in different places and at different times.

C. VAN EBBENHORST TENGBERGEN. *Leeftijdsopbouw en uitdijing van een bevolking bij gelijkblijvende relatieve geboorte- en sterfte-frequenties*, pp. 143–145. Mathematical notes on the stable community, namely that in which birth- and death-rates are invariant whilst the number of births increases or decreases at a constant rate.

B. H. DE JONGH. *Eigen-behoud-theorie bij brandverzekering*, pp. 177–188. The calculation of the maximum sum assured which a fire office should retain on its own account. The criterion of stability used is the 'relative risk', $\mu_2/\mu_1'^2$.

W. DE GEUS. *Over koers en koersformules van leeningen met continue betaling*, pp. 189–204. It is shown by solving a basic differential equation that, for instance, Makeham's formula for the value at rate i of a loan subject to interest at rate g and repayable by instalments during n years holds also in the continuous case. Although this was to be foreseen it provides a useful confirmation that mathematics and 'good sense' do not conflict.

W. F. LIEUWEN. *Subjectieve balanswaardeering bij levensverzekeringmaatschappijen en particuliere pensioenfondsen*, pp. 205–219. Principles of the valuation of assets for a life office's balance-sheet. Shares, property holdings, redeemable securities, and mortgages are dealt with in turn.

J. A. T. M. BRANS. *Veranderingen in den rentevoet en haar invloed op de private en sociale verzekering*, pp. 220–252. Review of the papers given to the Eleventh Congress about the influence of a changing rate of interest on private and social insurance.

F. H. RUSTING. *Sluitende berekenings- en omzettingsformules*, pp. 257–304. The axiomatics of a life office's contracts. Six axioms (for example, the third is 'If a contract of form T is changed to a new form T′ and then again to a form T″ the effect is the same as if there had been one change from T to T″') are proposed and discussed mathematically.

Het Verzekerings-Archief, Vol. XXIV, 1943

W. J. C. DE HEER. *Correspondeerende jaarpremiën en koopsommen voor uitgesteld ouderdomspensioen*, pp. 1–7. Explains why a pension assured by successive level annual 'single premiums' is not the same as that obtained for an annual premium based on a specified age at entry. The explanation lies in the method of loading usually adopted.

TJ. S. VISSER, H. G. HAGELEN, W. MEYER, C. VAN EBBENHORST TENGBERGEN, TH. NIEUWENHUIS, JR. *Welke premiebedragen zijn gestort voor een bepaalde jaartermijn van een erfrente?*, pp. 8–12 and pp. 188–206. A lively exchange of views on the effect of tax on educational annuities.

C. VAN EBBENHORST TENGBERGEN. *Sterfte onder Europeesche ambtenaren in Nederlandsch-Indischen dienst en onder hun weduwen*, pp. 13–14. Supplementary figures to extend the observations in this author's previous article in this journal (1940) from 1935 to 1938.

W. DE GEUS. *Koersbepaling m jaar na het sluiten eener leening*, pp. 15–20.

J. VAN DE WEG. *Algemeene koersformule m jaar na het sluiten eener leening met eindigen duur*, pp. 238–244. The price per unit of that portion of a loan repayable by instalments which remains outstanding after m of the original n years have elapsed is usually obtained by writing $n-m$ for n in the formula for the issue price. This rule is here shown not to be universally valid. The second author shows, however, that providing an appropriate formula is used for the issue price the substitution is permissible.

J. B. D. DERKSEN. *Afronding der sterftetafel G.B.M. 1931–1940 (II) volgens Makeham*, pp. 81–95. A Makeham graduation of the Dutch population data of 1931–1940 (males, war deaths excluded) for ages 20–79. Although perhaps satisfactory as a broad approximation, the result strikingly illustrates the systematic deviations of modern data from Makeham's law.

TH. NIEUWENHUIS, JR. *Het analyseren van de bedrijfswinst van een levensverzekeringmaatschappij naar de bronnen*, pp. 96–120 and pp. 227–231 (correction: 1944, p. 52). Investigates the modifications necessary to the relation connecting ${}_{m}\mathrm{V}$ and ${}_{m+1}\mathrm{V}$ to render it suitable for analysing surplus based on reserves calculated by formulae like

$$\bar{\mathrm{A}}_{x+\frac{1}{2}} - \mathrm{P}_{\cdot\frac{1}{2}|}\mathrm{a}_{x+\frac{1}{2}}.$$

C. CAMPAGNE. *De stelling van Hattendorf en haar algemeene geldigheid door de stelling van Cantelli*, pp. 121–144. Hattendorf's theorem states that the variance of the random variable expressing the (mortality) profit (or loss) of an insurance over its whole term is equal to the sum of the variances of the profits of the individual years, although these profits are not independent. A new and direct proof is given and linked up with earlier Italian work. Strangely, Steffensen's three proofs in *Skandinavisk Aktuarietidskrift*, 1929, are not mentioned.

J. ENGELFRIET. *Quelques formules actuarielles relatives à l'invalidité totale ou partielle par suite d'accident*, pp. 145–161, and 1944, pp. 33–46. The Dutch workmen's compensation is an annuity for an amount varying with the degree of incapacity for work. Until now the collective method has been used to value this liability, but an attempt is made here to provide formulae in which passage between the varying degrees of invalidity is regarded from a multiple decrement point of view. The theory is illustrated by an example from practice.

J. P. VAN ROOYEN and J. B. D. DERKSEN. *De berekening der periodieke sterftekansen*, pp. 176–187 and pp. 232–237. A criticism of Derksen's paper in Vol. XXIII, 1942, and a reply.

J. DE GANS. *De transformatiemethode van Wilhelm Göhring*, pp. 209–213. A criticism of a German method of utilizing a tabulator to calculate reserves.

W. MEYER. *De actuarieele behandeling van subnormale risico's*, pp. 214–226. Suggests that, if extra mortality is measured by a factor α where

$$q'_x = (1+\alpha)\, q_x, \quad 0 < \alpha < 1,$$

an equivalent age rating-up is given by h where

$$a'_{x\overline{n}|} = a_{x+h:\overline{n}|} = (1+\alpha)\, a_{x\overline{n}|} - \alpha a_{\overline{n}|}.$$

Discusses the problem of the reserve to be held in such cases.

M. VAN HAAFTEN. *De tontineleening voor de Nieuwe of Ronde Luthersche Kerk te Amsterdam uit* 1671, pp. 245–254. Points out that the standard references to the so-called Amsterdam State Tontine of 1671 derive from Struyck (cf. the presentation volume to the Seventh Actuarial Congress) and that close reading of his work reveals that, on the contrary, the Tontine was raised by the Lutheran community in Amsterdam to build a church.

J. N. VAN WOERKOM. *De levensverzekering en de vermogensbelasting*, pp. 261–280. The effect of Dutch income tax on life assurance.

W. J. VAN DER MAËN. *Het berekenen van sterftekansen*, pp. 281–300. Discusses (1) the graduation by linear compounding of values of $q_{x+\frac{1}{2}}$ obtained from population data, and (2) the representation of mortality at old ages by formulae like

$$\mu_x = A + Bc^x + \frac{k}{\omega - x},$$

or

$$\mu_x = a + bx + cx^2 + \frac{d}{\omega - x}.$$

J. J. SCHOKKING. *De verklaring van niet-beleenbaarheid ten aanzien van kapitaalverzekeringen met lijfrenteclausule en de inkomstenbelasting*, pp. 301–308.

J. M. VAN DER HEYDEN. *Selectie bij het sluiten en afbreken van lijfrenteverzekeringen*, pp. 309–313. An investigation of the 'selection' exercised by deferred annuity policyholders who have taken paid-up policies. The experience is based on the optional old-age pension scheme provided by the State.

M. VAN HAAFTEN. *Tontines uit* 1671 *op aandeelen van de Oost-Indische Compagnie*, pp. 314–326. Tonti's suggested method to the French Government for raising a loan dates from 1650. It was not adopted in France until 1689 but was used by the Dutch town of Kampen in 1670. This paper describes a private tontine floated by the Dutch East Indian Company in 1671.

Het Verzekerings-Archief, Vol. XXV, 1944

J. P. VAN ROOIJEN. *De Logistische Functie in de Nederlandsche Sterftestatistiek*, pp. 1–32. The logistic function

$$\eta=\frac{e^{\xi}}{1+e^{\xi}} \quad \left(\eta=\frac{y-\alpha}{\beta},\ \xi=\frac{\gamma-x}{\delta}\right)$$

has been used to describe the development of p_x with time. The alternative form

$$\eta=\frac{\xi}{\sqrt{(1+\xi^2)}}$$

is here considered and is fitted by moments to Dutch population statistics of 1870–1940. The application to 'generation' mortality tables is discussed.

M. VAN HAAFTEN. *De Spaansche lijfrente-negociatie uit* 1775 *geen tontine*, pp. 47–51. Correction of a historical note in Vol. VII, 1926, relating to a Spanish life-annuity loan of 1775.

W. VENEMA. *De integraalvergelijkingen voor de premiereserve van de spaarverzekering*, pp. 53–61. A clear and simple derivation of the solution of Volterra's integral equation of the second kind, viz.

$$\phi(x)=f(x)+\lambda\int_a^x k(x,s)\,\phi(s)\,ds$$

($f(x)$ and $k(x,s)$ given), is followed by an application to the policy value of a capital redemption assurance, viz.

$${}_t\overline{V}_{\overline{n}|}=t\overline{P}_{\overline{n}|}+\delta\int_0^t {}_s\overline{V}_{\overline{n}|}\,ds.$$

W. DE GEUS. *Invloed van rente-uitstel op de slotwaarde van een kapitaal*, pp. 73–96. When a waiting-period of $k<1$ occurs between the receipt of interest and its reinvestment, the accumulation factor $e^{\delta t}$ becomes

$$\sum_{h=0}^{p+1}\frac{\delta^h}{h!}\overline{(t-\overline{h-1}.k)}^h, \quad pk\leqslant t\leqslant(p+1)\,k.$$

The corresponding expression for the discrete case, $\left(1+\frac{j}{m}\right)^{mt}$, is also considered and numerical examples given.

J. ENGELFRIET. *Invaliditeit na ongeval*, pp. 97–127 and pp. 197–224. Consideration of invalidity insurance in relation to the progress of the Dutch national accident fund. A detailed discussion follows the paper.

J. DE GANS. *Lijfrentetafels, afgeleid uit de gegevens, die door de levensverzekeringmaatschappijen jaarlijks aan de Verzekeringskamer verstrekt worden*, pp. 128–142. Aggregate mortality tables (male and female) based on the annual returns of Dutch life offices are derived for the period 1931–40. An attempted Makeham graduation for ages $50\frac{1}{2}$–$85\frac{1}{2}$ reveals for males a value of α of $-\cdot00003$, so that Gompertz is applicable. Whilst the 'female' values of a_x are roughly equivalent to $a_{x-2\frac{1}{2}}$ according to the Dutch population tables 1931–40, the 'male' values do not reveal a mortality uniformly better than that of Holland as a whole.

J. P. VERGOUWEN. *De makelaardij in assurantiën in Italie, Spanje en Vlaanderen voor* 1575, pp. 157–176. The history of insurance brokerage in Italy, Spain and Flanders prior to 1575.

N. E. H. VAN ESVELD. *Op de grens tusschen sociale en particuliere verzekering*, pp. 177–196. The relations between, and limits of, social and private insurance.

C. VAN EBBENHORST TENGBERGEN. *Een doeltreffend financieringsstelsel voor pensioenfondsen*, pp. 241–253. Draws attention to the desirability of ensuring that payments out of a pension fund develop proportionately to the earnings of the fund. Some illustrative mathematical examples are given.

W. J. C. DE HEER. *De premiereserve van de levenslange verzekering bij overlijden met tijdelijke premiebetaling*, pp. 254–256. Provided valuation loadings are made the same, the reserve of a limited-payment whole-life assurance during the term of payment is equal to that of an endowment assurance effected at the same age for a term determined by equating the two valuation premiums concerned.

Het Verzekerings-Archief, Vol. XXVI, 1945–46

K. DE RINGH. *Schets eener algemeene formeele verzekeringswiskunde*, pp. 1–36. Develops relations for the value of a very general type of assurance and shows that they hold for the particular cases of life, invalidity, sickness, fire and 'lottery' insurance. Also considers net premiums and policy values for the general case.

W. MEYER. *De practijk der risico-herverzekering*, pp. 37–58. Considers, for various types of assurance, the decrease in the sum reassured when this takes the form $K(1-{}_mV)-M$, where K is the sum assured under the principal policy (or policies) and M is the office's retention. The effect of replacing this natural decrease by a straight line is discussed.

C. VAN EBBENHORST TENGBERGEN. *Sterfte bij de Indische pensioenfondsen onder de in Europa aanwezige gepensionneerden*, pp. 59–68, 186–189. In continuation of her researches in the 1940 and 1943 volumes of this journal, the author first considers only the mortality of officials who served in the Indies and returned home on pension. It is concluded that at every age their expectation of life is about 2 years less than that of the average Dutchman. Widows' mortality is also considered. Officials who remained in the Indies after pension suffered heavier mortality, but there was a distinct class selection at work.

K. FREUDENBERG. *Zur Ausgleichung doppelt abgestufter Sterbetafeln*, pp. 69–100. A detailed description of the graduation of select rates of mortality by means of the relation

$$q_{[x-t]+t}=q_x\left[1-\alpha_x\left(1-\frac{t}{T}\right)^2\right],$$

where T is the select period. The method of weighted least squares (approximate weights) is used and illustrated on the German 1924–26 table where $T=10$. An interesting result in this case is that α_x can be regarded as independent of x.

F. H. RUSTING. *Over keuzecontracten zonder sterfterisico*, pp. 113–159. A further contribution to the axiomatics of an insurance contract (cf. previous article in 1942 volume). In this article a purely 'savings' contract is considered which gives rise to a succession of options at fixed points of time (premium payment is, of course, such an option).

W. F. LIEUWEN. *Het pensioen en het laatstgenoten salaris. Een 'nettosalaris'-pensioenregeling*, pp. 161–185. Compares the contributions and pensions of the employee who has enjoyed exceptional promotion, on the basis of 'final salary', 'average salary' and 'money value' plans. Proposes a pension based on the net salary after payment of contributions as fairer to the exceptional individual and to the mass of employees.

J. VAN HOORN. *Invloed van sterfteverbetering op de kosten van gecombineerde verzekering van eigen weduwepensioen*, pp. 190–202. On the assumption that recent improvements in Dutch population mortality continue, calculations are made of their effect on the single and annual premiums for old-age pensions, widows' pensions, and 'combined' (2:1) pensions. A summary of the discussion is included.

JHR. F. DE CREEFT. *Levensverzekering op privaat en sociaal gebied in België sinds de laatste tien jaren*, pp. 203–222. An address reviewing the developments of life assurance in Belgium during the last ten years, including the effects of war.

SCANDINAVIA

Skandinavisk Aktuarietidskrift, Vol. XXVIII, 1945.

J. F. STEFFENSEN. *On certain formulas of mechanical quadrature*, pp. 1–19. An instructive discussion of the effect of moving slightly the ordinates of the Tchebycheff and Gauss formulae of approximate integration so that rational abscissae may be employed. The point is made that, if the sign of $f^{(m)}(x)$ remains unchanged throughout the interval of integration, the result of using a formula with remainder term $|k_1| f^{(m)}(x)$ and one with remainder $-|k_2| f^{(m)}(x)$ is to ensure two values one on either side of the truth.

J. F. STEFFENSEN. *Further remarks on iteration*, pp. 44–55. In a previous paper in the same journal (1933) the author considered the numerical solution of equations of the form $x=f(x)$. If $f(x)$ is decreasing in the interval under consideration then the root will always lie between x_ν and $x_{\nu+1}$, but if $f(x)$ is increasing then all x_ν are on the same side and to obtain numerical approximations we must iterate from both sides. This is avoided by means of the transformation $x=z^\alpha$. Applications to compound interest problems are given.

ÅKE KJELLSTRÖM. *Über eine Abrundungsmethode bei Berechnung von Pensionsraten*, pp. 56–67. A method of 'rounding off' monthly pension rates used in connexion with the Swedish old-age and incapacity pension regulations.

S. TÄCKLIND. *Fourieranalytische Behandlung vom Erneuerungsproblem*, pp. 68–105. A theoretical discussion of the application of Fourier integrals to the problem of renewals (a sequel to the paper in same journal, 1944).

H. BERSGTRÖM. *On the central limit theorem in the space* R_k, $k>1$, pp. 106–127.

C.-E. QUENSEL. *Studies of the logarithmic normal curve*, pp. 141–153. Discusses criteria for fitting the logarithmic normal curve (viz. the normal curve with log x instead of x as variable), and its Charlier Type A extension, to observations. The truncated curve is also given attention.

W. SIMONSEN. *On the numerical solution of systems of equations by means of iteration*, pp. 154–170. Newton's method of iteration applied to the solution of the system of equations

$$\phi_i(x_1, x_2, \ldots, x_n) = 0, \quad i = 1, 2, \ldots, n,$$

is lengthy and successive results do not indicate the rapidity of convergence to the true solution. A method is here developed to overcome these objections.

P. JOHANSEN. *Approximate valuation of joint-life policies*, pp. 171–180. Investigates the use of a single life to replace joint lives and to permit inclusion of such policies in the main valuation group. The simple method proposed produces reserves about 1% to 2% on the 'safe side'.

H. WOLD. *A theorem on regression coefficients obtained from successively extended sets of variables*, pp. 181–200. A theorem is proved which expresses mathematically the experience in multiple regression analysis that the introduction of a further variable increases about as often as it decreases any specified coefficient $b_{01.23\ldots n}$.

O. REIERSÖL. *Residual variables in regression and confluence analysis*, pp. 201–217. A generalization to several variables of a theorem which states that if residuals from a linear (time) trend are used as variables in a regression analysis the effect is exactly the same as if the regression is carried out on the original variables with 'time' as an additional variable.

N. E. ANDERSEN. *The valuation of the temporary annuity* $\bar{a}_{\overline{n}|} - \bar{a}_{x\overline{n}|}$, pp. 218–227. A neat proof that a level premium $(\bar{a}_{\overline{2n}|} - \bar{a}_{x\overline{2n}|})/\bar{a}_{x\overline{n}|}$ produces positive policy values if μ_x increases with x and ${}_np_x > v^n$.

L. TÖRNQVIST. *On the economic theory of lottery-gambles*, pp. 228–246. The paper concludes with the remark that the 'formulas can be used in a general theory of economic activity. The classic price theory cannot be used for a theory of lottery gambles.'

O. ERICSON. *Un problème de la répartition des revenus*, pp. 247–257. If each of n individuals has an equal chance of earning any one unit of a total of S units representing the national income, the probability that a specified man earns x units is

$$\binom{S-x+n-2}{S-x} \div \binom{S+n-1}{S} \to \frac{n}{S} e^{-nx/S}.$$

This simple law is in surprisingly good general agreement with the distributions of incomes in Sweden in 1931 and 1945.

S. VAJDA. *Calculation of policy-values for different rates of interest*, pp. 258–271. If policy values, at a given age at entry and a given duration, are calculated at several equidistant rates of interest, the ratio between two successive values is nearly constant. Constancy being assumed, a table of factors enables us to pass from one rate of interest to another. The results are close for H^M, A1924–29, and other tables.

Skandinavisk Aktuarietidskrift, Vol. XXIX, 1946

S. N. MEYER. *Some theoretical views on the shape of the tubercular infection curve and its dependence on infection risk*, pp. 1–11. The observed curves of the proportion, y, of individuals with tubercular infection rise from zero at birth to a high percentage at age 30. If $\phi(x)$ denotes the force of infection at age x among the uninfected,

$$\frac{dy}{dx} = (1-y)\,\phi(x).$$

By solving for y it is shown that the theory of a decreasing risk of infection during school years is not the only interpretation possible of the slowing-up of the rate of increase of y at these ages.

S. G. LINDBLOM. *On the connection between tests of significance for correlation coefficients and for differences between means*, pp. 12–29. A novel and instructive discussion of the relationships between random variables like

$$u = \frac{(\bar{x}_1 - \bar{x}_2)\sqrt{[n_1 n_2/(n_1+n_2)]}}{s_{1+2}} \quad \text{and} \quad t = \frac{(\bar{x}_1 - \bar{x}_2)\sqrt{[n_1 n_2/(n_1+n_2)]}}{\sqrt{\left\{\frac{1}{n_1+n_2-2}\left[(n_1-1)\,s_1^2 + (n_2-1)\,s_2^2\right]\right\}}},$$

where the x's are normal variates and s is the sample S.D. with $n-1$ as denominator; k-dimensional generalizations are also given.

T. K. ORE. *Withdrawal among Norwegian life assured* 1910–1935, pp. 30–56. The withdrawals included lapses, surrenders and conversions to paid-up policies. The following points are discussed: (1) effect of economic conditions on new business and withdrawals; (2) variation in withdrawal rates according to age attained and duration; (3) higher withdrawal rates in the cases accepted without medical examination; (4) variations exhibited when sum assured is used as the unit instead of policy; (5) the lower withdrawal rates among women than among men; (6) the total withdrawals during the first five and first ten years of assurance. A valuable statistical investigation.

H. HOLME. *On some practical working formulae for determining the effective rate of interest of loans*, pp. 57–79. The formulae given depend ultimately on a linear inverse interpolation applied to a properly chosen function depending on the type of loan in question. A high degree of accuracy is obtainable without much labour.

E. MICHALUP. *Beitrag zur Amortisationsrechnung*, pp. 80–84. An example of the Lindelöf method of speeding up the convergence of a slowly convergent series. The series chosen is $a_{\overline{1/p}|}$ (p an integer).

H. CRAMÉR. *A contribution to the theory of statistical estimation*, pp. 85–94. Completion of the proof of a theorem in the author's recent book on mathematical statistics which related to 'joint efficient estimators' of the k parameters of a probability law.

R. BENTZEL and H. WOLD. *On statistical demand analysis from the viewpoint of simultaneous equations*, pp. 95–114.

P. QVALE. *Policy-valuation by means of risk-premiums*, pp. 137–142. An interesting use of the punched-card multiplying punch in connexion with the annual valuation.

G. TRIER. *Technical basis for collective (group) pensions insurance in Norway*—K1946, pp. 143–174. A new mortality, disability, marriage and issue basis developed for use by the four companies providing collective pension benefits additional to those granted under national insurance.

G. ELFVING. *Contributions to the theory of integer-valued Markoff processes*, pp. 175–205. Miscellaneous theorems concerning elementary discrete random processes.

L. TÖRNQVIST. *On the distribution function for a function of n statistic variables and the central limit theorem in the mathematical theory of probability*, pp. 206–229. Extension to non-independent variables of investigations by von Mises and Cramér.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker, Vol. XLVII, 1947, Part I

ERNST KAISER. *Demographische und wirtschaftliche Rechnungsgrundlagen der eidgenössischen Alters- und Hinterlassenenversicherung*, pp. 31–50. A description of the technical bases used in the new Swiss compulsory old-age, widows' and orphans' pensions insurance.

HANS STREIT. *Die massgebenden Gesichtspunkte bei der Wahl der Rechnungsgrundlagen im privaten Versicherungsbetrieb*, pp. 51–122. A prize-winning essay on the influences governing the choice of mortality, interest, loading, etc., bases for premiums, reserves, surrender values, and bonus calculations. A useful guide to Swiss practice.

H. JECKLIN and M. EISENRING. *Die elementaren Mittelwerte*, pp. 123–165. Any mean value of n positive numbers $a_1 \leqslant a_2 \leqslant \ldots \leqslant a_n$ must satisfy the requirements of (*a*) continuity and uniqueness, (*b*) symmetricity, and (*c*) 'betweenness' (i.e. $a_1 \leqslant \mathrm{M} \leqslant a_n$). Various general methods of formation and comparisons *inter se* are propounded.

IVO LAH. *Das Zinsproblem*, pp. 167–247. The differential equation

$$\sum_{k=1}^{n+1} \sigma_{n+1}^{k} \frac{d^k a_x}{di^k} \sum_{k=1}^{n-1} \sigma_{n-1}^{k} \frac{d^k a_x}{di^k} = \frac{n+1}{n} \frac{\mathrm{S}_{x+1}^{(n+1)} \mathrm{S}_{x+1}^{(n-1)}}{(\mathrm{S}_{x+1}^{(n)})^2} \left(\sum_{k=1}^{n} \sigma_n^k \frac{d^k a_x}{di^k} \right)^2,$$

where the σ_n^k are Stirling's numbers defined by

$$x^{(n)} = \sum_{k=1}^{n} \sigma_n^k x^k,$$

and

$$\mathrm{S}_x^{(n)} = \frac{1}{n!} \sum_{t=0}^{\omega-x} (n+t)^{(n)} \mathrm{D}_{x+t},$$

plays a fundamental role in problems involving changes in the rate of interest i. In particular, the formulae bearing the names of Steffensen, Poukka, Frucht, Evans, etc., all appear when suitable approximations and boundary conditions are employed. This is an ingenious synthesis of the various aspects of the interest problem in life contingencies.

B. ROMER. *Die Bestimmung von durchschnittlichen Krankenkosten an Stichproben*, pp. 249–271. An illustration of elementary (finite population) sample theory in its application to the frequency distribution of the cost of sickness claims due to heart disease, bronchitis, and rheumatism. The conclusions as to the criteria for choosing the minimum useful sample size need critical examination.

K. STAUBER. *Begriff und Verwendung des Reduktionsfaktors in der Krankenversicherung*, pp. 273–280. If $1-R_x^w$ represents the proportion surviving sick after w days' sickness from a unit becoming sick at age x, the net premium for a unit temporary annuity during sickness after a waiting-period of w days is

$$\frac{1}{D_x}\sum_{t=0}^{n-1} v^{\frac{1}{2}} D_{x+t} k_{x+t} (1-R_{x+t}^w),$$

where k_x is the mean annual number of days' sickness at age x. It is argued that the use of the approximation

$$(1-R_{x+\frac{1}{2}n}^w)\frac{1}{D_x}\sum_{t=0}^{n-1} v^{\frac{1}{2}} D_{x+t} k_{x+t}$$

gives reasonably close results both for premiums and reserves.