NOTES ON FOREIGN ACTUARIAL JOURNALS

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AMERICA

Transactions of the Actuarial Society of America, Vol. XL, Part II

R. D. MURPHY. Some Observations on Governmental Regulation of Life Insurance in the United States. Interesting Presidential Address.

W. G. BOWERMAN. Centenarians. Discussion of cases and evidence. A note of warning about birth certificates or baptismal entries may be added: there was almost a custom in some districts and families even well into the nineteenth century that when a child died in early infancy a subsequent child should bear the same Christian name. No birth or baptismal record should be accepted as conclusive evidence till the doubt indicated has been eliminated.

J. E. HOSKINS. Asset Shares and their Relation to Non-Forfeiture Values. "Asset Share" may be either with Profit deducted (Natural Reserve) or without Profit deducted. Some interesting points are brought out.

A. HUNTER. Cancer. An Analysis of Life Insurance Experience. The experience relates to 2716 insurances on men and 523 on women effected in 1913–22 and 1925–35 with the New York Life and was checked against an equally sized control group of existing policies. The conclusions reached are:

    1. Site of Cancer—Of deaths from cancer, 65% of those among men and 41% of those among women were of the digestive tract or peritoneum; among women 24% of the deaths were due to cancer of the breast. The site of the disease is somewhat less common in the genito-urinary organs among men than among women, when uterine cancer is included. Cancer of the mouth and pharynx covers 5% of the deaths among men and less than 1% among women.

    2. Occupation—For men, the proportion of cancer deaths was higher than average among skilled and semi-skilled workers and

lower than average among professional men, farmers and clerks. For women, the proportion of cancer deaths was higher among teachers, nurses, librarians and other professional women than among housewives, but it should be noted that there is probably a movement from the former group to the latter group between the issue of the policy and the date of death.

Cancer of the mouth or pharynx was more prominent among professional men and clerical workers than among other occupational groups, while cancer of the digestive tract and of the respiratory system was more important among skilled and semi-skilled workmen. Breast cancer was more prevalent among professional women than among other groups of women.

3. Predisposition—The present investigation showed no indication of an hereditary predisposition to cancer.

4. Influence of Build—Overweight in this and other investigations has been an unfavourable factor affecting cancer incidence, especially of the digestive and urogenital tracts.

5. Birthplace and Residence—Our statistics indicated a higher than average proportion of deaths from cancer among the foreign born, particularly from Russia (Jews), and among residents of the New England and Middle Atlantic Seaboard States.

6. Blood Pressure—The blood pressure readings at issue did not seem to have much relationship to subsequent death from cancer.

7. Habits—A study of the statements of the applicants with reference to their use of alcohol indicated that of men who died of cancer there was a somewhat greater proportion who drank alcoholic beverages than among the control group. This was especially noticeable in the case of men who died of cancer of the digestive tract, the mouth or pharynx. But other material, previously published, indicates that inferences on this matter should be accepted with caution.

J. M. LAIRD and E. J. MᶜA. LENNEY. The effect of Misrepresentation. A study of the legal position in the United States of America.

K. CAMP. Practical Interpolation Methods with Second-Order Curves. The methods are adapted to mechanical technique.

The Record. American Institute of Actuaries, Vol. XXVIII, Part 1

V. R. SMITH. Presidential address containing suggestions for improving relations with the public and the part played by the profession therein. Also discusses briefly the training and qualifications of an actuary.

J. A. BUDINGER. Variations in withdrawal rates in relation to non-participating premiums. Answers the question "how many years will probably elapse before the asset share equals the legal reserve on the policy?" by taking three series of withdrawal rates and applying them to an ordinary life assurance effected at age 35.

R. J. MYERS. Cost estimates for the old age insurance system of the Social Security Act. Presents the detailed cost estimate developed by the Committee on Economic Security.

L. L. STEVENS. Attained-age method of valuation applied to Annuities. Factors are used to avoid segregating annuity policies of various types (cp. methods used by Karup, Altenburger and others).

L. L. STEVENS. A practical method for obtaining the amount of guaranteed interest in instalment payments on Annuities-Certain.

HOLLAND

Het Verzekerings-Archief, Vol. XX, Nos. 1-4

D. BIJL. Onkosten Analyse. Remarks on expenses with particular reference to the constituents of the annual expenses incurred whilst a policy is on the books.

S. VAJDA. Die Durchschnittsprämie und das Durchschnittsalter. Provided $q_{x+1} = a + bq_x$, where a and b are independent of x, then, for several life contingency functions, the mean age of a group decreases with an increase in the term.

E. T. STELLER. Kritische beschouwingen over enkele hoofdstukken uit de verzekeringswiskunde. On weighted means. A few errors which have been made in Dutch publications are mentioned and the author concludes with a consideration of the equation for the equated time.

W. KATZ. Theorie en praktijk der ziektekostenverzekering. Interesting remarks on the insurance of medical expenses. Contrast is drawn between this type of insurance and the usual (in England) sickness insurance.

K. GOLDZIHER. Beitrag zur Bibliographie der politischen Arithmetik seit 1930. Additions to previous bibliographies in this journal of books on compound interest and investments.

C. v. E. TENGBERGEN. Pensioenprincipes en pensioenfinanciering. A note on an aspect of pension fund interest.

F. H. RUSTING. Een eigenschap betreffende de berekening van koopsommen voor abnormale risico's. The derivation, and some consequences, of the theorem

$$\sum_{t=0}^{n} f_t \, {}_{t|}q'_x = \sum_{t=0}^{n} c_t A_t,$$

where

$$c_t = \begin{cases} \dfrac{{}_t p'_x}{{}_t p_x} - \dfrac{{}_{t+1} p'_x}{{}_{t+1} p_x}, & (0 \leqslant t \leqslant n-1) \\ {}_t p'_x / {}_t p_x, & (t = n) \end{cases}$$

and

$$A_t = \sum_{t=0}^{s-1} f_t \, {}_{t|}q_x + f_s \, {}_s p_x. \quad (0 \leqslant s \leqslant n).$$

E. T. STELLER. Kritische beschouwingen over enkele hoofdstukken uit de verzekeringswiskunde. The conclusion of a paper commenced in the previous number. This part contains, *i.a.*, critical discussions of Steffensen's, Palmqvist's, and Poukka's methods of obtaining an isolated annuity value from life contingency functions at another rate of interest, and of ten Pas' linear representation of the progression of q_x (x fixed) with the passage of years (see *J.I.A.* Vol. LXIX, p. 308).

E. STERN. Kursformeln für Anleihen mit gleichmässig gestaffeltem Zins. A full algebraic treatment of the purchase price of a loan subject to a rate of dividend increasing in A.P. and repayable by equal instalments of capital.

J. B. D. DERKSEN. Berekening van sterftekansen. A reply to one of Steller's "Critical Remarks" in the previous issue. In a postscript Steller maintains that his original contention was misunderstood.

ITALY

Giornale dell' Istituto Italiano degli Attuari, Vol. X, Parts 3–4

G. OTTAVIANI. Sulla probabilità che una prova su due variabili casuali X e Y verifichi la disuguaglianza X < Y e sul corrispondente scarto quadratico medio. Transvariation exists where a typical value (e.g. the mean) of X is greater than the same typical value of Y. Survivorship assurances provide a practical aspect of the problem.

K.-G. HAGSTROEM. Rilievi sulla teoria del deprezzamento. An interesting essay in the application of the principles of life assurance to the insurance of landed property, based on the concept of a constant "force of destruction" *during*, and the worthlessness of the buildings *after*, a certain period. Specimen tables of premiums for "endowment assurances" to cover these risks are given.

E. FRANCKX. I capitali di sopravvivenza. This follows and generalizes an earlier paper in *G.I.I.A.* Vol. X, parts 1-2. On the assumption that mortality follows a "Quiquet" law of the pth order, any survivorship assurance can be expressed as a linear function of $p + 1$ definite integrals of the form

$$\int_0^\omega v^t \phi_i(t) \frac{l_{(x+t)}}{l_{(x)}} \dots \frac{l_{(u+t)}}{l_{(u)}} \dots dt,$$

one of the coefficients of the definite integrals being arbitrarily chosen.

A connexion is shown to exist between the generalized formula and (1) the formula obtained in the previous paper, (2) the formulas set out by Lidstone in *J.I.A.* Vol. LXVIII, p. 535.

Lidstone's notes were a means to an end—the use of the formula in connexion with a table of equivalent lives of equal age—but the paper in *G.I.I.A.* is, presumably, the end itself. And this rather illustrates the general difference between the contents of *G.I.I.A.* and *J.I.A.*

C. E. DIEULEFAIT. Sui momenti delle distribuzioni ipergeometriche. A new method of determining these moments is given. The typical "Pólya" scheme (which includes the hypergeometrical) is here considered. It is that of an urn containing black and white balls, a specified number of balls of the same colour as that of the one extracted being replaced in the urn after each extraction.

M. P. GEPPERT. Su una classe di distribuzioni in due variabili casuali. Shows that, when x and y are correlated variables, linearity of regression is maintained for any rotation of the axes of X and Y only in cases where the correlation surface can be represented by an equation of the form

$$z = f(ax^2 + 2bxy + cy^2).$$

E. FELDHEIM. Nuova dimostrazione e generalizzazione di un teorema di calcolo della probabilità. Let p, less than $\frac{1}{2}$, be the probability of a given event. Let r be the number of times the event takes place in n trials. Assume $m = np$ (integral). Then if

$$P = \text{probability}, \quad (r < m)$$
$$Q = \text{probability}, \quad (r > m)$$

$P - Q$ is positive and, as n increases, tends to the value

$$\frac{q - p}{3\sqrt{(2\pi npq)}}.$$

The case where np is not integral is also investigated.

This double number also contains a review of recent insurance developments in various countries and a short summary of the following lectures at the Seminario Attuarile:

M. CASTELLANI. Vantaggi e svantaggi dell' assicurazione per gruppi.

R. OTTAVIANI. L' utile di mortalità nei bilanci delle imprese di assicurazioni sulla vita.

JUGOSLAVIA

Udruženja aktuara Kraljevine Yugoslavije, Vol. IV, Parts 1-4

D. MARKOVIČ. Način računanja bruto dodakta kod osiguravajučih preduzeča u Yugoslaviji. Particulars of the construction of life assurance premiums in Yugoslav companies.

A. URBANC. Slovenska bratovščina sv. Hieronima v Vidmu iz leta 1452. A description of the Slovene confraternity of St Jerome at Udine in the year 1452. This society provided sickness, orphan, burial and dowry benefits. A rough adherence to the insurance principle is shown by the fact that merchants and house owners paid 12 soldi a year and the poorer classes 24 soldi a year. A facsimile of the statutes is given, drawn up in a medieval form of Italian. It was laid down that the chamerari who administered the Society must be persons of substance and position lest owing to misery and poverty they should be suspected of fraud. Fines for dereliction of duty on their part were payable in oil or wax.

K. VORONJEC. O porodičnom saosiguranju. The Family Insurance described includes an arrangement whereby on payment of an extra of $2\,^o/_{oo}$ of the sum assured, the wife is insured, without further payment of premiums, for the remainder of the endowment period, in the event of the death of her husband during that period.

H. KOEPPLER. Sumarne enačhe in njihova rešitev. Defines summation equations, shows their connexion with the Volterra integral equation of the second order, and gives examples relating to life assurance.

The volume also gives particulars of the law relating to private insurance.

SCANDINAVIA

Skandinavisk Aktuarietidskrift, 1939, Parts III and IV

P. JOHANSEN. Iteration von Funktionen zweier reellen Variabeln und einer komplexen Variabeln. Theorems relating to the convergency of the series $\{x_r\}$ obtained by iteration performed on the relation $x = f(x)$ are here extended to the cases (i) $x = \phi\,(x, y)$, $y = \psi\,(x, y)$ and (ii) $z = f\,(z)$, where $z = x + iy$.

F. BORCH. Über die technischen Grundlage der Invalidenversicherung. The author's previous work on select tables (see *J.I.A.* Vol. LXVIII, p. 593) is here extended to disability tables. With the aid of auxiliary functions, $a^i_{[x]:\overline{n}|}$, $P^{ai}_{x:\overline{n}|}$, etc. may be found from the ultimate mortality table applicable to disabled lives.

B. MEIDELL. Zur Theorie und Praxis der Berechnung des effektiven Zinsfusses bei Anleihen. Among the methods of approximating to the rate of interest yielded by a security there is no accurate general formula stating the yield explicitly. It is suggested that the formula

$$i = [g + \sqrt{\{(A - S_0)^2 + 4(1 - A)gS_1\lambda\}} - (A - S_0)]/2S_1\lambda,$$

where

$$S_0 = \sum_{t=1}^{n} C_t v_g^t, \quad S_1 = \sum_{t=1}^{n} tC_t v_g^{t+1}, \quad S_2 = \sum_{t=1}^{n} (t+1)^{(2)} C_t v_g^{t+2}$$

and

$$\lambda = 1 - \frac{g}{2} \frac{A-1}{A-S_0} \frac{S_2}{S_1},$$

satisfies these desiderata. Detailed numerical examples are supplied.

J. THURMANN-MOE. Die neuen norwegischen Rechnungsgrundlagen für Rentenversicherung, R 1939, und für Lebensversicherung, N 1939. Discusses the new Norwegian official mortality and disability bases for assured lives. An extrapolation of previous results has been made and advantage taken of Makeham's formula.

C. E. QUENSEL. Changes in Fertility following Birth Restriction. The mathematics of fertility taking account of variations in the previous number of births of each female and the divorce rate, and referring particularly to the effect of an increase in the practice of birth control. There is a warning against attaching much importance to large and sudden changes in the undifferentiated fertility rate at any age.

E. FRANCKX. La Théorie des chaînes de Markoff. By means of an elementary method depending on inequalities, necessary and sufficient conditions are obtained for a Markoff chain to be "regular". These conditions are less restrictive than those due to Markoff himself.

W. DOEBLIN. Sur certains mouvements aléatoires discontinus. A general form is found for the probability that a discontinuously moving point situated at x at an instant s should be in a given set of points E at time t. A highly theoretical treatment.

E. LUKACS. Zur Theorie der Selekttafeln. A paper on the theory of damaged lives. If it is assumed that (i) with increasing y, $\mu_{[y]+x-y}$ converges uniformly towards a limit μ_x, and (ii) (in the Hamza notation) $\lim\limits_{x\to\infty} \dfrac{l_x^i}{l_x^{aa}}=0$, then $\mu_x^{aa}=\mu_{[x]}$, $\mu_x^{ai}=-\dfrac{d}{dx}\log l_{[x]}-\mu_{[x]}$ and $\mu_x^i=(l_x\mu_x-l_{[x]}\mu_{[x]})/(l_x-l_{[x]})$.

The author promises a paper on the practical aspects of these relations.

1940, Parts I and II

A. J. LOTKA. The Theory of Industrial Replacement. Reply to Prein-reich's recent criticism.

H. CHRISTEN and A. LINDER. Une application de la nomographie au système complet de rentes viagères de Blaschke-Gram. Nomographs for annuities at various rates of interest and mortality provided mortality follows Makeham's law. The underlying formulae are those of Blaschke and Gram.

K.-G. HAGSTROEM. Sur la notion de prime d'épargne. The idea of a marginal premium.

A. TAUBER. Das Sinken der Sterblichkeit im Zeitverlauf. Study of the decrease in mortality from 1876 to 1933 using $_{15}q_x^t$, where t signifies the time and expressing it in the form $a+b\theta^{t^3}$.

W. ANDERSSON. A General Formula for the Normal Mean Errors of the Coefficients in Parabolic Least Squares Graduation. Reaches the formula

$$\epsilon^2\,(a_p^{(k)})=\frac{L_p^{(k)}}{N-k-1}\frac{1}{p!}\frac{\sigma_y^2}{\sigma_x^{2p}}\,(1-r^2),$$

where $a_p^{(k)}$ is the coefficient of x^p in a parabolic curve involving terms to x^k and

$$L_p^{(k)}=1+\frac{(p+1)\,(p+2)}{2^2\,(1!)^2}+\frac{(p+1)\,(p+2)\,(p+3)\,(p+4)}{2^4\,(2!)^2}+\cdots.$$

Gives table of $L_p^{(k)}/p!$

K.-G. HAGSTROEM. Stochastik ein neues—und doch ein altes Wort. An interesting etymological note.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker,
Part xxxix

H. Wyss. Beobachtungen über die Sterblichkeit bei Gruppenversiche-rungen. A description of a non-medical insurance mortality in-vestigation, during the years 1925–37, covering about 57% of all Swiss group life insurances. The data related to pure endowments, whole life assurances and deferred annuities, separately or in con-junction. A standard "safe side" table is constructed for males and females separately and comparisons are made with seven well-known mortality bases. The common occurrence, in group in-surances, of annuities with long periods of deferment is commented upon and their dangers are noted.

C. Jéquier. L'assurance d'annuités, cas particulier de l'assurance temporaire. It is proved that in the case of decreasing temporary assurances (the sum assured for the tth year being B_t and $B_1 = 1/q_0$) the policy values are always positive, zero, or negative according as

$$B_t q_{t-1} \lessgtr B_{t+1} q_t,$$

and this result is applied to the annuity portion of the family income policy (cp. W. F. Marples in *J.I.A.* Vol. LXIII, p. 82).

W. Ruchti. Amonotonie der Sterblichkeitsabnahme in ersten Lebens-jahr. A comparison of the Swiss population monthly rates of mortality during the first year of life analysed by calendar month of birth.

J. Meier. Zur Theorie der unabhängigen Wahrscheinlichkeiten. In a multiple decrement table the relation connecting "rates" and "probabilities" is

$$1 - \sum_{t=1}^{k} q_t = \prod_{t=1}^{k} (1 - r_t),$$

either member representing the probability of "remaining in service" at the end of a specified period (e.g. one year). Having established three axioms which any solution of this equation must satisfy, the author considers several such solutions which are shown numerically to differ only slightly. The mathematics are readily followed.

E. Dasen. Recherches sur la détermination approximative du taux de rendement des emprunts à taux d'intérêt nominal variable. A method whereby Bond Value Tables may be applied directly to find the yield on a security with a rate of dividend which changes before the date of redemption.