

NOTES ON FOREIGN ACTUARIAL JOURNALS

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AMERICA

Transactions of the Actuarial Society, Vol. L, Part I, May 1949

- H. R. BASSFORD. *Presidential Address*, pp. 1-8. Actuaries and Insurance 1889-1949.
- W. D. LAIRD. *The revenue of the period of account and its relation to premiums, valuation and dividends*, pp. 9-47.
- H. F. BLAIR and L. W. HAINES. *Mortality experience according to build on standard insurance in the Provident Mutual*, pp. 48-58. Most favourable among under-weights and improving with duration. Over-weights showed consistently unfavourable experience becoming worse with duration. There is some evidence that short individuals require more careful selection or stricter weight standards than had been applied.
- J. R. HERMAN. *Actuaries—past, present and future*, pp. 59-70. Statistics from the Actuarial Society and American Institute. Table 4 shows the average number of years required to become a Fellow by examination and the average age at completion of last examination and indicates the effect of the war.
- J. M. BOERMEESTER. *Certain implications which arise when the assumption is made that premiums are paid continuously and death benefits are paid at the moment of death*, pp. 71-75.
- A. E. ARCHIBALD. *Policy expiry date under automatic premium loan clause*, pp. 76-78.
- W. W. STEFFEN. *Attained age valuation constants for policies with varying premiums*, pp. 79-81.
- Aviation Statistics*, pp. 97-101. The favourable record of U.S. airlines in 1948 approximately offsets the unfavourable record of 1947.
- A. HUNTER. *Remarks on final meeting*, pp. 141-144. These remarks were made at the final meeting of the Actuarial Society by one who had been a member for fifty years.

CZECHOSLOVAKIA

Aktuárské Vědy, Vol. VIII (1948-49), Part 2

The articles in this number were contributed by pupils and friends of Professor E. Schoenbaum to commemorate his sixtieth birthday, 10 June 1942.

- E. BUNICKY. *Remarque sur le critère de l'indépendance des caractères*, pp. 53-60. The well-known criterion of independence for two attributes A and B is that

$$\frac{(AB)}{(B)} = \frac{(A\bar{B})}{(\bar{B})},$$

where brackets enclosing a letter denote the number in the class designated by that letter, and a superimposed bar denotes negation (Kendall's *Advanced Statistics*, ch. 13). It is here shown that whereas, for example, the relation

$$N^2 (AB) (\bar{A}\bar{B}) = (A) (B) (\bar{A}) (\bar{B}),$$

with $N = (A) + (\bar{A}) = (B) + (\bar{B})$, is necessary and sufficient for independence, the relation

$$N^2 (AB) (\bar{A}B) = (A) (B)^2 (\bar{A})$$

is a necessary but not a sufficient condition.

- B. HOSTINSKÝ. *O výpočtu pravděpodobností, které se vztahují k časovému vývoji soustav*, pp. 61-66. A French résumé is provided, p. 67, under the title *Sur le calcul des probabilités relatives à l'évolution d'un système*.
- J. JANKO. *Difference equation of the policy reserve*, pp. 68-75. The difference equation formed by the recurrence relation connecting policy values of the generalized endowment and annuity assurance (viz. an assurance for a variable sum assured on death or annuity payment on survival) is solved in general terms and various particular cases are shown to reduce to familiar forms.
- A. ZELEŇKA. *Rentrée en validité dans l'assurance-invalidité*, pp. 76-87. Derivation and consideration of the differential equations connecting l_x^a , l_x^i and the forces of mortality, invalidity and recuperation. The complications that arise when the latter is not identically zero are clearly indicated.

FRANCE

Bulletin Trimestriel de l'Institut des Actuaire Français, No. 185, 1948

- R. DULAC. *Ajustement analytique des tables de mortalité*, pp. 383-454. Effects a number of graduations of the French population tables of 1928-33 (males, females, persons) by means of formulae for μ_x like

$$A + Bc^x \text{ (Makeham),}$$

$$a + bc^x + df^x \text{ (Quiquet),}$$

$$\alpha_1 b_1^x + \alpha_2 b_2^{(x-c)^3} + \alpha_3 b_3^x \text{ (Thiele), and}$$

$$(A + Bc^x)/(Kc^{-x} + 1 + Dc^x) \text{ (Perks).}$$

The basic data are the official graphically graduated values of l_x at quinquennial ages and Quiquet's law is found to be the most suitable. This thesis for graduate membership of the French Institute was written whilst the author was a prisoner of war and without access to works of reference or aids to calculation. One cannot but admire the assiduity of the author.

No. 186, 1949

In addition to a brief reference to the Centenary of the Institute of Actuaries this number contains an account of and radio speeches by HENRI MARAÏ (pp. 22-26) who died in November 1940 as a member of the Free French Forces in London and whose body was brought back to Paris in 1949; and (pp. 34-42) some personal memories of ALBERT QUIQUET written by P. Richard in 1944 or the occasion of the tenth anniversary of Quiquet's death.

- J. POUDÉVIGNE. *Physique actuarielle: détermination électrique des annuités*, pp. 46-54.

Journal de la Société de Statistique de Paris, Vol. LXXXVIII, 1947.

The July-August 1947 number of this periodical contains an article by the French mathematician M. FRECHET entitled *Sur les expressions analytiques de la mortalité valables pour la vie entière* together with a discussion by members of the Society (pp. 261-285). The author advocates fitting a polynomial of the sixth degree in $\log x$ to $\log q_x$ and argues that a reasonable fit throughout life results. There is some discussion on what precisely constitutes a good fit and the χ^2 criterion is considered too stringent.

GERMANY

Blätter für Versicherungsmathematik

Six of the ten parts of Vol. v of the *Blätter* were reviewed in *J.I.A.* Vol. LXXIV, p. 356. The remaining four parts of that volume have now become available to the reviewers and together with Vol. VI, Part I are reviewed below. With the exception of Vol. IV, Part 10, notices of the German *Blätter* are now complete.

Vol. v, Part I

- J. ZAPFF. *Die Ausgleichung von Sterbetafeln unter besonderer Berücksichtigung der Gewichte der Einzelbeobachtungen*, pp. 1-21 (and pp. 49-68 of Part 2). Argues that it is unjustified to introduce the ungraduated or graduated value of q_x into the weight to be used at age x in graduating mortality data. Instead E_x is chosen as the weight and illustrations are given of its application in graduations by linear compounding formulae and in various methods of fitting a Makeham law.

- H. RICHTER. *Die Konvergenz der Erneuerungsfunktion*, pp. 21-35. The integral equation of renewal theory is

$$1 = l(x) + \int_0^x l(x-t) \phi(t) dt,$$

where $l(x)$ represents the probability of surviving x years from entry and $\phi(t) dt$ is the probability of entry into the community at time t . On the hypotheses that $l(x) \equiv 0$, $x > \omega$, and that $l'(x)$ exists and is continuous for successive sections of the curve of $l(x)$, it is proved that $\phi(x)$ tends to a limit as $x \rightarrow \infty$ (cf. Feller, *Ann. Math. Statist.*, 1941).

- H. JECKLIN. *Zur Tarifanalyse*, pp. 35-39. If an office premium $\pi_{x|\overline{n}|}$ is based on a relation of the type

$$\pi_{x|\overline{n}|} = (1 - \beta)^{-1} \left(P_{x|\overline{n}|} + \frac{\alpha}{a_{x|\overline{n}|}} + \gamma \right),$$

a knowledge of the basic mortality and interest does not enable the calculation of α , β and γ to be made uniquely from three values of $\pi_{x|\overline{n}|}$. On the other hand, if likely limits can be set on α a reasonable set of values of the three loading constants can be obtained.

Vol. v, Part 2

- P. RIEBESELL. *Neue deutsche Forschungen über das Gesetz der grossen Zahl*, pp. 68-75. A review of five German articles written between 1936 and 1939

on interval estimation. The earlier works are based on the Bayes approach with a uniform distribution of prior probabilities, but the later publications are concerned with confidence intervals (Mutungsbereichen), without, however, reference to their originator (Neyman, 1934).

- R. SCHÖNWIESE. *Witwenrentenversicherung nach direkter Methode*, pp. 75-87. This author's articles on the valuation of widows' pensions by the reversionary method in the 1917 *Zeits. gesamte Versich.-Wissens.* and the 1928 *Blätter* are now classic. Indications are given here for the practical application of the formulae derived. No numerical examples are given.

Vol. v, Part 3

- K.-G. HAGSTROEM. *Über Versorgungsversicherung*, pp. 97-120. A theoretical and practical study of the family income benefit and the uniformly decreasing term assurance. Concludes, *inter alia*, that extra mortality of tuberculosis type has a considerable effect on the incidence of negative reserves, and that, if a family income benefit is to be added to a whole-life assurance subject to payment of a uniform premium throughout life, the annual payment under the former should not exceed between about 15 % and 35 % (depending on the term of the income benefit) of the sum assured under the latter if the total policy value is never to be negative.
- H. PARTHIER. *Über Änderungen an den Grundgrößen einer Zerfallordnung der Aktiven*, pp. 121-140. A compendium of the numerical effects on the remaining functions of percentage alterations in the dependent functions q , i , q^t and q^{aa} , singly and jointly.

Vol. v, Part 10

- F. HUMBERT. *Verrechnung von Vermögen in Sterbegeld-Gruppenversicherungsverträgen*, pp. 427-443. A method is proposed for the allocation of an initial single premium payment in respect of the members of a group entering on a whole-life group insurance subject to an annual premium independent of age (cf. p. 151 of Vajda's review in *J.S.S.* Vol. vi).
- A. ANDRAE. *Jährliche und unterjährige Beitragszahlung zweckmässiger eingerichtet*, pp. 443-449. Points out that there would be no practical distinction between true and instalment mthly premiums if, when a claim arises, it were the practice to add to the sum assured that portion of the current premium which relates to the fractional period beyond the date of death of the life assured.
- E. MEYER. *Die Senkung des Rechnungszinsfusses vom Gesichtspunkte der gerechten Belastung der Versicherungsnehmer aus*, pp. 449-455. Maintains that, if the reserve basis has been strengthened because of probable lower interest earnings, surrender values should not be calculated on the higher reserve values.
- J. WISSING. *Kleiner Beitrag zur Risikozwischenversicherung*, pp. 456-460. Puts Jecklin's formula for extended term insurance (cf. *J.I.A.* Vol. LXV, p. 256) in a conveniently practical form.
- G. HUSZÁR. *Die technisch richtige Bewertung der zur Deckung von Prämienreserven dienenden Immobilien*, pp. 460-468. Proposes that the value of real estate in an insurance company's assets should be the commuted value of the net income for k years plus the discounted value of the re-sale value at the end of k years. The problem considered is to choose an appropriate k .

Vol. VI, Part I

- E. SCHRÖPFER. *Rechnungsgrundlagen für die Ruhegeldversicherung*, pp. 1-29. Decremental tables and commutation values at 3%, 3½% and 4% for disability income and widows' pensions based mainly on the 1928-32 data from three German pension funds for bank employees.
- H. BRAUN. *Der relative Wert der Vergleichung der wirklichen Sterblichkeit oder Invalidität mit der wahrscheinlichen*, pp. 30-34. Claims that the periodic component in mortality (see *J.I.A.* Vol. LXXIV, p. 357) invalidates the comparison of actual and expected deaths in respect of any one calendar year. Warns against published data based only on 'peak' or 'trough' calendar years.
- B. MEIDELL. *Der Effektivzins*, pp. 34-43. The culmination of previous work on the calculation of the yield in a financial transaction (see *J.I.A.* Vol. LXX, p. 217 and Vol. LXXI, p. 139) is the simple formula

$$i = g \left[1 + \left(\frac{A - S_0}{1 - A} + \frac{gS_1}{1 - S_0} \right)^{-1} \right],$$

where $S_0 = \sum C_t v_0^t$ and $S_1 = \sum t C_t v_0^{t+1}$ (see below). In sixty-six numerical examples of loans in which the capital is repaid (i) as part of a level annuity payment, (ii) in equal amounts each year for n years, and (iii) in one sum at the end of n years, the yield according to the above formula is correct to five significant figures in fifteen cases, and in only ten cases is the fourth significant figure 1 or 2 units in error.

- P. GOTAAS. *Eine Bemerkung zu der letzten Meidellschen Zinsformel*, pp. 43-45. Shows that the convergency criterion for the validity of Meidell's formula (above) is that the difference between i and g must be less than half of g , that is to say, $|g - i| < g/2$.
- A. KLOSE. *Neuere Untersuchungen zur Korrelationsrechnung*, pp. 45-54. A review of some of the more recent suggestions for measuring the intensity of correlation (see *J.I.A.* Vol. LXXII, p. 525, and Vol. LXXIII, p. 151).

HOLLAND

Het Verzekeerings-Archief, Vol. XXVIII, Nos. 1 and 2, 1949

- M. VAN HAAFTEN. *L'âge limite ω* , pp. 1-11 (with reply by E. ZWINGGI, pp. 11-14). Discusses the effect of the alternative definitions (i) $l_\omega = d_\omega \neq 0$, (ii) $l_\omega = 0$, $l_{\omega-1} > 0$, (iii) $\omega \rightarrow \infty$. Concludes that the divergence of opinion among textbook writers makes a standardization of notation undesirable.
- W. MEYER. *Over het verband tussen verlaging van de gemiddelde rekenrentevoet der premiereserve, verhoging van de premiereserve en verhoging van de interestwinst*, pp. 15-18. An expression is derived for the increased interest surplus due to a decrease in the valuation rate of interest.
- G. W. DE WIT. *Stochastische problemen in de verzekeeringswiskunde*, pp. 19-45. Mainly a discussion of the collective theory of risk as applicable to life assurance: also deals with sickness and accident, etc. Refers to most of the modern development but some useful Swiss and Italian work is not mentioned.

- B. GROOTENBOER. *Enige resultaten van groepsgewijze reserveberekeningen voor verzekeringen van weduwepensioen*, pp. 46–50. Further discussion of a Makeham approximation for use in the valuation of reversionary annuities purchased by annual premiums (cp. J. Hage in same Journal [1948], *J.I.A.* Vol. LXXIV, p. 359). A numerical example from practice illustrates the points made.
- J. DE GANS. *Differentierekening, interpolatie- en benaderingsmethoden*, pp. 51–135. Notes on finite differences intended for students preparing for the second (and final) part of the Dutch actuarial examinations. An English student without knowledge of the language could follow it.

PORTUGAL

Instituto dos Actuários Portugueses—Boletim, No. 3, December 1948

The number includes a note on the centenary of the Institute of Actuaries and the following which may be of interest to members of the Institute.

- H. JECKLIN. *Considérations élémentaires sur les combinaisons d'assurance-vie apparentées aux contrats d'épargne*, pp. 15–30. If the sum assured at death by an endowment is the premiums paid accumulated at interest there is no mortality involved. The author makes various assumptions with linear increases in the sums assured: some of these might be useful for sub-standard lives.
- O. W. SPRING. *L'Assurance Sociale en Suisse*, pp. 41–53. A short account with bases, etc.
- E. HARTA and A. LEÃO. *Ensaio de uma tábua de morbidex portuguesa*, pp. 55–69. A sickness table based on a small experience of male beneficiaries in 1944 and 1945 of the C.S.P. da Pancificação of Lisbon. The actual experience is not given.

SCANDINAVIA

Nordisk Försäkringstidskrift for July 1949 contains a lecture by F. LUNDBERG on research work in life assurance (pp. 233–242) and a contribution to the debate on the subject by R. BRUNANDER (pp. 271–285). They are in Swedish but an abstract in English is available.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker, Vol. XLIX, 1949, Part I

- E. MARCHAND. *Le Centenaire de l'Institut des Actuaires de Londres*, pp. 23–29. A sympathetic description of the Centenary celebrations by the President of the Swiss Association.
- C. JÉQUIER. *Quelques remarques sur les réserves mathématiques de l'assurance à terme fixe et de l'assurance d'annuités*, pp. 30–34. Writes the policy value of a family income benefit in the form $a_{\overline{n}|} \cdot V_{\overline{am}|} - (a_{\overline{n}|} - a_{\overline{n-1}|})$ and discusses the effect of changing the mortality basis. A similar procedure is adopted for *terme fixe* policy values.

- H. AMMETER. *Die Elemente der kollektiven Risikotheorie von festen und zufallsartig schwankenden Grundwahrscheinlichkeiten*, pp. 35-95. A restatement and continuation of a paper in *Skand. Aktuar.-tidskr.* 1948 (cf. *J.I.A.* Vol. LXXV, p. 110). Numerical examples show that the maximum retention should be decreased, perhaps considerably, if the probability of a claim is itself subject to a probability distribution law.
- O. W. SPRING. *Lebensversicherung und Geldwert*, pp. 96-119. Detailed development of the mathematics of insurance during a period of changing cost of living. The effect on the gross premium of increasing management expenses is also discussed.
- H. KREIS. *Über eine Erweiterung des Ausgleichsverfahrens von Karup*, pp. 120-130. An explicit extension of Karup's linear compounding graduation formula from nineteen terms to any arbitrary odd number. (See *Trans. Second Int. Cong. of Actuaries*, p. 31.)
- H. RUCH. *Zum Zinsfussproblem*, pp. 131-138. The usual requirement for an endowment assurance net premium to increase with a decrease in i is that μ_x should always increase with x . It is here proved that if n does not exceed the integer k , where $D_{x+k} < \frac{1+i}{2} D_x \leq D_{x+k-1}$, then $P_{\overline{a}|x}$ always increases as i decreases. The reasoning is extended to produce an explicit sufficient criterion for n .
- G. ALBERS and W. PÖTTKER. *Der Einfluss der Parameter in der Formel von Gompertz-Makeham auf die Prämienreserve*, pp. 139-147. By writing ${}_tV_{\overline{a}|x}$ as ${}_tV_{\overline{a}|x}$ plus a remainder which is called the risk portion of the policy value, it is shown that, with close approximation, an increase in the Makeham constant g tends to decrease numerically the risk portion and that an increase in c increases or decreases ${}_tV_{\overline{a}|x}$ according as t is smaller or greater than a critical duration t_0 which is nearly independent of age. The latter result should have useful applications in connexion with Lidstone's Z method.
- F. HUMBERT. *Zusammengefasste Berechnung der Verwaltungskostenreserve für einen aus verschiedenen Versicherungsarten gemischten Bestand*, pp. 148-151. Derives a formula to calculate the reserve for management expenses from a knowledge of the total net premium and 'Zillmerized' reserves for the business as a whole.
- W. RUCHTI. *Betrachtungen über die Praxis der gestaffelten Rückversicherungskommission*, pp. 152-157. A problem of commissions occurring in the practice of fire reinsurance.
- E. ZWINGGI. *Variation der Rechnungsgrundlagen in der Invalidenversicherung*, pp. 158-164. On the assumption that the rate of disablement can be expressed in Makeham form it is shown how to estimate rapidly and accurately the effect of changes in the rate of disablement on the annual premium for an annuity during disability.