NOTES ON OTHER ACTUARIAL JOURNALS

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BRAZIL

Boletim do Instituto Brasileiro de Atuária

7, 1951

OLIFIERS, E. *Uma simbologia racional das fórmulas seletas dos expostos ao risco de invalidez e sua representação gráfica*, pp. 3–28. Discusses the derivation of independent rates of exit by disability from censuses taken at 31 December, in select form. The treatment is elementary.

MONTELLO, J. *Nota sôbre o regimen financeiro de capitalização*, pp. 29–50. The mathematical theory of a fund continuously recruited by new entrants, and supported by contributions related to salary. The benefits are invalidity pensions equal to salary on becoming disabled, pensions to dependants of the disabled who have died, and lump sum death benefits. The author investigates the conditions under which the existing participants and future entrants may be separated in the equations expressing financial equilibrium.

NOGUEIRA, R. *Contribuição a teoria de capitalização*, pp. 51–175. A very generalized formulation of the mathematical theory of premiums and policy values for assurances payable on the occurrence of certain life contingencies or, partially or wholly, in the event of successful participation in one or more, possibly interdependent, lotteries. The numerical examples relate to a case without life contingencies where there is a monthly lottery and 'certain' payment of a unit at the end of 25 or 30 years. The sum assured at time u in months is $C(u)=C_1(u)+C_2(u)$ with

$$C_1(u)=1+[\{\max(u, 276)-1\}/275]^2$$

and
$$C_2(u)=\begin{cases}1-\{(u-1)/275\}^2 & u=12, 24, 36, \ldots, 276,\\ 0 & \text{all other } u.\end{cases}$$

MADEIRA, J. L. *Tarifação de riscos subnormais*, pp. 176–84. Well-known methods of obtaining the value of $P_{x\overline{n}|}$ when the mortality basis is changed from μ_x to $\alpha\mu_x+\beta$.

8 and 9, 1952 and 1953

OLIFIERS, E. *Uma simbologia racional das fórmulas seletas dos expostos ao risco de invalidez e sua representação gráfica*, pp. 3–17. If Makeham holds it has been shown by Reed and Merrell that

$$_nq_x \simeq 1-\exp\left[-\left\{m_x^{(1/n)}+\frac{\log_e c}{12}\,n\left(m_x^{(1/n)}\right)^2\right\}\right].$$

The author derives several more terms of the series in the exponent.

YUDENITCH, A. *Sôbre a axiomática do Cálculo das Probabilidades*, pp. 19–60. A discussion of the philosophical difficulties of constructing a sound theory of probability on assumptions which are not self-contradictory. Numerous, if somewhat brief, references to the literature of the subject indicate the author's preference for von Mises's approach. Over 50 works are mentioned in the Bibliography.

GREVILLE, T. N. E. *Interação infinita de um processo de graduação de uma sucessão finita de dados*, pp. 61–7. Let $\mathbf{u}$ be a column matrix whose components are n ungraduated values of a certain function. $\mathbf{A}$ is a $n \times n$ matrix and the product $\mathbf{Au}$ produces the graduated values of the given function. The problem is to determine the conditions under which $\mathbf{A}^r\mathbf{u}$, the result of repeated application of the graduation formula, tends to a limit as r tends to infinity. The solution is provided by a $p \times p$ $(p<n)$ matrix $\mathbf{D}$ whose characteristic roots are all absolutely less than unity. The author also provides an explicit relation for $\mathbf{A}^\infty$.

GERMANY

Blätter der Deutschen Gesellschaft für Versicherungsmathematik, Vol. 2, Part 2, 1955

KNÖRLEIN, F. *Zur Mathematik der Gruppenversicherung gegen technische Durchschnittsprämie*, pp. 153–76.

DIENST, H. R. *Einige Kriterien über das Auftreten negativer Prämienreserven bei Gruppenversicherungen gegen technische Durchschnittsprämie*, pp. 177–92. During the nineteen-thirties German actuaries engaged in lively discussions about the theoretical soundness of group insurance written on the basis of so-called 'technical mean premiums'. Suppose, for example, that group coverage for a unit sum assured extends from age a to age 65, the 'technical mean premium' for a group consisting of n_x lives aged x at entry $(x=a, a+1, \ldots, 64)$ would be

$$\bar{P} = \sum_{x=a}^{64} n_x A^{1}_{x\,\overline{65-x|}} \Big/ \sum_{x=a}^{64} n_x \ddot{a}_{x\,\overline{65-x|}}.$$

This is to be compared with the arithmetic mean premium

$$P = \sum_{x=a}^{64} n_x P^{1}_{x\,\overline{65-x|}} \Big/ \sum_{x=a}^{64} n_x,$$

and, in fact,

$$P^{1}_{a\,\overline{65-a|}} < \bar{P} < P < P^{1}_{64\,\overline{1|}}.$$

The use of $\bar{P}$ with a 'closed' group of lives leads to surprising consequences. Even when the actual mortality coincides with that expected the total policy value of the group can be negative, and when mortality diverges from expectation prospective and retrospective policy values differ from one another. When the group is 'open' and withdrawals can take place the problems multiply. One of these concerns the year-to-year adjustments to be made to $\bar{P}$, another the surrender value to be granted to an individual who withdraws.

The reader coming fresh to the subject is advised to read first Knörlein's paper in the *Blätter*, **4**, 357 (see *J.I.A.* **70**, 213) and then Sachs's article in *Blätter*, **2**, 217 (see *J.I.A.* **63**, 247). Zwinggi's textbook (*Versicherungsmathematik*, Basle, 1945) contains a chapter on the subject but it is highly compressed.

The practical development of this type of group insurance was prevented in Germany by a government ruling issued in 1934. However, recent rulings have withdrawn this ban and Knörlein reviews the theory developed pre-war and lays down a number of practical principles for the successful conduct of the business. Dienst derives criteria to determine whether the total policy value of a closed group will ever become negative and illustrates them with numerical examples.

GUMBEL, E. J. *Die Bedeutung der Parameter in der Gompertz-Makehamschen Formel*, pp. 193–8. Expresses A, B and c of the Makeham law in terms of the μ's and the ages (x_1 and x_2) at the minimum and maximum points of the 'curve of deaths'. These results appear in Fraser's classic paper (*J.I.A.* **45**, 476). However, two useful inequalities developed by the author are

$$B<A<\frac{\mu_{x_1}+\mu_{x_2}}{4} \quad \text{and} \quad \mu_{x_2}<\log_e c<2\mu_{x_2}.$$

BARTSCH, R. *Über ein Verfahren zur näherungsweisen Ermittlung von Kommutations- und Rentenbarwerten*, pp. 199–202. By expressing first-order differential coefficients in terms of the three end-values of u_x, Lubbock's formula (to first derivatives) is rewritten as a weighted sum of equidistant values of u_x. A numerical example applies the result to calculate N_{40}.

BIERLEIN, D. *Sterbetafeln lassen sich nicht so ausgleichen, dass die Reservekurven generell hyperbolisch sind*, pp. 203–8. Shows that the relation

$${}_tV=\frac{A+Bt}{C+Dt} \quad \text{with} \quad AD-BC\neq 0 \text{ and } t \text{ integral,}$$

is contradictory when ${}_tV={}_tV_x$ or ${}_tV_{x\overline{n}|}$. Note that Jecklin's F-method (see, e.g. *J.S.S.* **10**, 119) uses this relation as an *interpolation formula*.

BOEHM, C. *Über den Charakter von Verbindungen zwischen Todes- und Erlebensfallversicherungen*, pp. 209–23. If the 'risk premium'

$$vq_{x+t-1}(S_t-V_t)+(PE)_t$$

is positive for all t the assurance is said to have the character of a life assurance; conversely it has the character of a pure endowment. It is shown that this definition is better than others that have been proposed, and that when the sign of the risk premium *changes* an analysis of the assurance into its constituent (sign invariant) parts may be of practical value in the choice of a mortality basis.

SLATMANN, W. *Der XIV. internationale Kongress der Versicherungsmathematiker*, pp. 225–35.

Vol. 2, Part 3, 1955

AMMETER, H. *Über die risikotheoretischen Grenzen der Versicherbarkeit*, pp. 261–77. Uses the collective theory of risk to show that whereas independent risks are insurable without limit—though the 'risk loading' or the 'contingency reserve' may be commercially unattainable—the reverse holds for dependent risks.

BEHR, I. VON. *Gemischte Kapitalversicherungen mit erhöhter Erlebensfallsumme*, pp. 279–84. Considers an assurance under which the endowment at maturity is $K > 1$ and the death benefit is unity or the policy value, whichever is the greater (see Jordan, *Life Contingencies*, pp. 150–3). Writing m for the number of years during which the sum assured is unity the author provides an interpretation of $\ddot{a}_{x\overline{m}|}$ and $\ddot{a}_{\overline{m}|}$ which satisfies the relation $d+P=1/\ddot{a}$, and develops a geometrical method for finding m.

JECKLIN, H. *Über die Möglichkeit, Sterbetafeln so auszugleichen, dass die Reservekurven generell hyperbolisch sind*, pp. 285–9. In connexion with a paper by Bierlein (see preceding page), shows that mortality laws exist for which the reserves are hyperbolic in form, but with the axes of the hyperbolae not parallel with the axes of coordinates. De Moivre's law is one such example.

BIERLEIN, D. *Optimalmethoden für die Summenapproximation in Jecklins F-Methode*, pp. 291–352. An 'optimal method' of approximate valuation is defined by means of game theory and the conception applied to Jecklin's method of approximating to reserve-values by means of hyperbolae. A numerical example is given.

DÖRING, H. *Die Statistische Zentralstelle des Verbandes der Lebensversicherungsunternehmen und ihre Aufgaben*, pp. 353–62. An account of the work of the Statistical Central Office of the Life Offices' Association in Germany.

Vol. 2, Part 4, 1956

MÜNZNER, H. *Zur Frage: Binomialverteilung oder Poissongesetz?*, pp. 405–12. An examination of the conditions under which the Poisson or Binomial, respectively, characterizes the distribution of insurance claims at a given point of time assuming that no new insurances are effected. The essential distinction is whether the probability of a *further* claim is directly proportional to the number of insurances left in force or can be considered independent of this number (see *J.S.S.* **8**, 204). The author's generalizations are unconvincing.

DIENST, H. R. *Witwenrentenanwartschaft und Ehestandshäufigkeit*, pp. 413–30. Expresses h^t_{x+n}, the probability of a man being married on his death at (exact) age $x+n$, in terms of wives' survivorship probabilities and the marriage and divorce rates of males between ages x and $x+n$. It is thus possible to write the 'collective' value of a widow's pension in a form equivalent to the 'individual' value for a male with a wife of average age. Numerical calculation of h_{20+n} reveals that certain survivorship and marriage rates are incompatible.

TOSBERG, A. *Beitrag zur Entwicklung einer Mathematik der Krankheitskostenversicherung*, pp. 431–68. Further developments in the theory of hospital-expense insurance (see *J.I.A.* 80, 114).

SCHOBE, W. *Rationale Approximationen der Potenzfunktion*, pp. 469–84. Shows that $x^y \sim H_k(x,y)/H_k(x,-y)$, where $H_k(x,y)=\sum_{m=0}^{k}\binom{k+y}{m}\binom{k-y}{k-m}x^m$ with a relative error of the order of $2\sin\pi y\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)^{2k+1}$. A similar approximation is also derived for $\log x$.

BOEHM, C. and RÖPER, G. *Elektronische Rechnenmaschinen und Informationsverarbeitung*, pp. 485–510. An excellent review of the literature of electronic data-processing machines.

HOLLAND

Het Verzekerings-Archief: Actuarieel Bijvoegsel, 33, 1956

YNTEMA, L. *An elementary proof of the central limit theorem*, pp. 19–40. The proof uses characteristic functions, the Dirichlet integral and the Montel–Helly theorem. Although 'elementary' it is not 'simple'.

PORTUGAL

Boletim do Instituto dos Actuários Portugueses, 11, 1955

LAH, I. *Analytical graduation of fertility rates*, pp. 1–7. Following Mortara, the fertility rate at age y is defined as $F(y+1)-F(y)$, where $F(y)$ is the ratio of the total number of live children to which women aged $(y-1/2)$ to $(y+1/2)$ have given birth to the number of women living between those ages. The curve fitted to observed values of $F(y)$ between ages α and ω is of the form

$$\log_e \Phi(y)=a_1\left[\frac{\omega-y}{\omega-\alpha}+\log_e\frac{y-\alpha}{\omega-\alpha}\right]+\sum_{j=2}^{n}a_j(\omega-y)^j.$$

Brazilian census data are employed in a numerical example. The same article appears in the September 1956 number of *J. Amer. Statist. Ass.*

ROCHA, E. *Grandezas actuariais em múltiplas cadeias abertas*, pp. 9–18. Consider n groups of distinguishable elements, the force of transfer from group i to group j $(i<j)$ being $\mu_{ij}(t)$ at time t. Two formal solutions are developed for the problem of determining $F_i(t)$, the number of elements in the ith group at time t. The results are applied to a familiar problem in invalidity insurance.

CASTRO, G. DE. *O comportamento indutivo e a Estatística Matemática*, pp. 19–45. An expository treatment of inverse probability, induction and decision theory.

CASTRO, G. DE. *Uma pequena bibliografia sobre a teoria matemática da decisão*, pp. 49–60.

SCANDINAVIA

Skandinavisk Aktuarietidskrift, **38**, 1955

CONOLLY, B. W. *Unbiased premiums for stop-loss reinsurance*, pp. 127–34. If $p(x, \theta)$ is the probability density of annual claims x, θ being a parameter, the average amount payable by a reinsuring office with a stop-loss contract to pay claims in excess of c is

$$\Pi = \int_c^\infty (x-c)\, p(x, \theta)\, dx.$$

Vajda (see *J.I.A.* **78**, 371) has suggested the estimate

$$P = \int_c^\infty (x-c)\, p(x, t)\, dx,$$

where t is an unbiased estimate of θ, but has admitted that in general P is not unbiased, i.e. $\mathscr{E}(P) \neq \Pi$. The author suggests that for certain values of c Vajda's estimate may be unbiased, and illustrates the procedure for determining such c values without, however, proving his point.

RIDDERSTRÖM, S. *On ratio estimates in simple random sampling with some practical applications*, pp. 135–62. A summary of known results is followed by illustrations drawn from U.S. life insurance material (reserves, deferred premiums, due and unpaid premiums).

MICHALUP, E. *Some approximation formulae of the effective rate and force of interest*, pp. 163–4. i in terms of δ and vice versa with errors not exceeding 10^{-11} when $i = \cdot 1$.

LAH, I. *Das Restglied der Taylorschen Reihe des Rentenbarwertes und einige Formeln des Zinsfussproblems für grösse Zinsspannungen*, pp. 165–79. The remainder term is derived in multiple integral form and is used to provide approximate formulae for a_x at rate i_1, given a_x and the related commutation columns of higher order (S_x, etc.) at rate i.

VAJDA, S. *Analytical studies in stop-loss reinsurance. II*, pp. 180–91. A very neat determination of an unbiased P (see above) of minimum variance when $p(x, \theta)$ is Normal.

HESELDEN, G. P. M. *Some inequalities satisfied by incomplete beta functions*, pp. 192–200.

PHILIPSON, C. *A tentative application of the collective risk theory to crop insurance*, pp. 201–53. An obscure (to the reviewer) treatment of a multivariate stochastic process. No numerical examples.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker, **56**, 1956

ROBERT, J.-P. *Problèmes techniques soulevés par le projet de révision de la loi fédérale sur l'assurance en cas de maladie et d'accidents*, pp. 25–40. Criticizes the low reserve requirements (a minimum of one year's claims) from the standpoint of the increasing average age of the insured population.

HALDY, M. and TAILLENS, E. *Limites dans l'évolution de la mortalité*, pp. 41–8. Shows the effect on $2\frac{1}{2}\%$ life contingency functions of a decrease in over-all mortality to a level determined by current mortality due to violence, senility and the diseases of the circulatory system (only).

JECKLIN, H. *Varia zur hyperbolischen Interpolation von Reservekurven*, pp. 49–63. Some further developments, with numerical examples, of the ϕ method of reserve approximation, viz. that based on the assumption

$$ {}_tV = Ct/n + At/(1-\phi t). $$

See *J.I.A.* **81**, 308 and **82**, 146.

KREIS, H. *Über das Renditenproblem festverzinslicher Titel*, pp. 64–70. The equation $A = 1 + (g-i)\, a_{\overline{n}|}$ is rewritten as $\frac{g-i}{A-1} = \frac{1}{a_{\overline{n}|}}$, and the yield i is then given by the intersection of the straight line $y = \frac{g-i}{A-1}$ and the curve $y = (a_{\overline{n}|})^{-1}$. The latter is replaced by a hyperbola and an explicit relation for i results. Numerical illustrations.

LUKACS, E. *Über Auslosungsversicherungen*, pp. 71–6. Consider a lottery endowment assurance under which the sum assured is payable on death or on the drawing of the policy in a lottery. If the chance of success in the lottery remains constant the effect on the usual premium is (obviously) the same as an increase in the force of interest.

WÜNSCHE, G. *Ein Sequenz-Test zur Kontrolle von Ausscheidehäufigkeiten*, pp. 77–89. An expository account of the application of sequential methods to test whether the mean of a Poisson process is $\mu_0 t$ (t being time) rather than $\mu_1 t$. The successive deaths of a group of coeval annuitants provide a numerical illustration ($\mu_0 = \cdot 029$, $\mu_1 = \cdot 042$).

GUBLER, H. *Über eine allgemeine Methode der Lösung des Zinsfussproblems für verschiedene Versicherungsformen und die Darstellung der darin auftretenden Momente*, pp. 91–144. The calculation of functions at non-tabulated rates of interest is based on the approximate formula

$$ f(i) \approx f(i_0)\, e^{ah + \frac{1}{2}bh^2}, $$

where $1+h = (1+i_0)/(1+i)$ and a and b are functions of the moments of the tabulated commutation functions at interest rate i. Numerical examples are given, and tables of moments of D_x and C_x are included for the Swiss tables SM and SF 1939/44 at 3% interest.

UNITED STATES AND CANADA

Transactions of the Society of Actuaries, 7, 1955

KLEM, W. *Professional ethics*, pp. 333–41. The Board of Governors of the Society recently appointed a standing committee on professional conduct. The question is discussed whether or not an official code of ethics should be adopted.

HOLCOMBE, S. M. *Adding or increasing substandard extras on policy changes* (and discussion), pp. 342–48. Derives simple formulas for the uniform annual extra premium payable on a sub-standard life after a change from a standard low risk policy to an assurance with a higher amount at risk.

KELTON, W. H. *A valuation study of disability benefits included in life insurance policies* (and discussion), pp. 349–65. Comparison of a 'conservative' and a 'liberal' valuation (both at $3\frac{1}{2}\%$ interest) of the Travellers' disability income policies issued between 1918 and 1931.

ARNOLD, E. A. *Analysis of approximate valuation methods* (and discussion), pp. 366–81. The methods described apply to small or difficult items (e.g. the valuation of policies providing for a life income to the beneficiary after the sum assured has been paid in instalments) rather than to the whole portfolio.

BAILLIE, D. C. *Term versus whole life—Actuarial note* (and discussion), pp. 382–90. Suppose (x) effects a unit n-year temporary assurance (with interest computed at force δ) and simultaneously invests the difference, $\bar{P}_{x(\delta)}-\bar{P}^{1}_{x\,\overline{n}|(\delta)}$, at force of interest δ_1. On death shortly thereafter his estate exceeds a unit by the accumulation of the investment. If another individual aged x effected a unit whole life assurance he would need to increase his premium continuously to secure a reversion to an estate of the same size as that of (x). But this would mean that the second individual would be paying out more than (x), and to secure equality of outgo (x) must invest, at age $x+t$, the difference, $\Pi_t-\bar{P}^{1}_{x\,\overline{n}|(\delta)}$, between the amount of the second individual's cumulated whole life premium and (x)'s temporary assurance premium. If F_t is the amount of (x)'s invested fund at age $x+t$ the assumed equality of outgo at every point of time after age x results in the following differential equations:

$$\Pi'_t=\bar{P}_{x+t}F'_t \quad \text{and} \quad F'_t=\delta_1 F_t+(\Pi_t-\bar{P}^{1}_{x\,\overline{n}|(\delta)}).$$

The author obtains explicit relations for F_t and Π_t and extends the arguments to include gross premiums.

MORTON, A. P. *Family history of cardiovascular-renal disease* (and discussion), pp. 391–403. An analysis of the Prudential's 35% share of the data included in the *1951 Impairment Study*.

This number also includes a digest of informal discussions of: Under-writing, Group Insurance, Valuation with Electronic Data Processing Machines, Annuities, and Society Meetings.

The 1955 Reports of Mortality and Morbidity Experience (published as a Supplement to the *Transactions*) contains the experience of 'standard ordinary' issues in the policy year 1953–54, certain aviation statistics, group life data for 1951–54, group weekly indemnity and hospital and surgical expense insurance material for 1952–54, and group annuity mortality data for 1954 and for 1951–54.

8, 1956

BOERMEESTER, J. M. *Frequency distribution of mortality costs*, pp. 1–9. Describes the application of an electronic data processing machine to the production of random numbers by the (faulty) mid-square method (*Monte Carlo Method*, Nat. Bur. Standards, 1951) and the subsequent computation of sampling distributions of the liability to pay immediate annuities to n individuals. In a numerical example with $n=10$ the author provides five quantiles (viz. the least variate, the mean, the 90% variate, the 95% variate and the greatest variate) of a 100-sample distribution of which the true mean is

$$\sum_{j=1}^{10} S_j a_{65} \quad \text{with} \quad S_j = 1, j = 1, 2, \ldots, 9,$$

and S_{10} equal to one of the alternate values 1, 2, 5, 10, 25 or 50. The last of these distributions indicates, for example, what could happen when one of 10 pensioners receives a pension 50 times as large as each of the rest.

NOWLIN, P. W. and GREVILLE, T. N. E. *Payment of reserve in addition to face amount*, pp. 10–11. A simpler derivation than that of Jordan's text-book.

TOMPA, P. M. *Life agents' retirement plans under New York State expense limitations*, pp. 12–43.

SAFFEIR, H. J. *Mortality differences between payee and nonpayee elections arising from insurance death claims*, pp. 44–8. What is the amount of self-selection involved when the female beneficiary under a life insurance elects a life income in lieu of a lump-sum settlement? This may be measured by comparing the mortality of such annuitants with that of others who were nominated by the insured prior to his death. Using the 1951 Settlement Option Study the author reaches an age set-back of 1·3 years.

SUTTON, H. L., JR. *Effect of family history on longevity after age 45*, pp. 49–52. Prudential's experience between 1911 and 1931, and between 1931 and 1951 anniversaries.

DUFFIELD, D. C. *Group conversion charges—accounting for annual statement*, pp. 53–6.

This number also contains a digest of informal discussions on Social Security, Group Insurance, Accident and Sickness Insurance, Underwriting and General topics.