

NOTES ON OTHER ACTUARIAL JOURNALS

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FRANCE

Bulletin Trimestriel de L'Institut des Actuaire Français

No. 202, March 1953

BAUDEZ, G. *Étude de distributions dans le cas des causes rythmées*, pp. 23-37.
 Argues that observed frequency distributions are better explained by 'rhythmic causes' than by chance effects.

GAUTHIER, M. *Influence de la mise à la retraite sur la mortalité du personnel des sociétés d'assurances*, pp. 39-60. Examines the 1919-44 mortality experience of 'normal' male pensioners of the French life-office staffs (1024 deaths). There is no evidence that, for a given retirement age, mortality decreases as the period since retirement increases. However, the mortality rate at age x is lower, the nearer x is to retirement—indicating that 'requests for retirement are influenced by the employee's state of health'. An aggregate table of pensioners' mortality (P.S.A.R.M.) is successfully graduated by Makeham. We note that, at 3 %, $a_{65} = 8.866$.

COMBES, B. *Le problème de la généralisation de l'inégalité de Bienaymé*, pp. 61-69.
 A geometrical argument throws new light on the problem of generalizing the Bienaymé (Tchebyshev) inequality.

Bulletin de l'association des actuaire diplômés de l'institut de science financière et d'assurances, April 1953

EYRAUD, H. *Du monopole privé au monopole d'Etat*, pp. 1-5.

SOUSSELIER, J. *L'assurance des risques catastrophiques*, pp. 6-21. In general, catastrophic risks (flood, hail, forest fire, avalanche, etc.) are characterized by a high frequency of small claims. The author discusses insurance, reinsurance, state participation and cover through a consortium.

FRANÇOIS, J. L. *Essai de calcul rationnel de la prime de rendement*, pp. 22-60.
 Attempts to assess scientifically the proper incentive bonus to pay for piece work, such as that done by punched-card operators.

RAGE, G. *Généralités sur l'Assurance Maritime*, pp. 61-68.

GERMANY

Blätter der Deutschen Gesellschaft für Versicherungsmathematik,

Vol. 1, Part 4, 1953

BOEHM, C. *Vergleich der verschiedenen Methoden zur Berechnung der Prämienreserve eines Versicherungsbestandes*, pp. 3-18. Reviews Karup's, Lidstone's Z , Jecklin's t (Akt. Vedy, 1936), Pöttker's Scalar (Blätter, 1950) and

Jecklin's F (*J.S.S.* x, 119) methods of valuation from the viewpoints of their (i) accuracy (reserves and expected sums at risk falling-in), and (ii) simplicity of application. Pörtker's method is very accurate but requires four valuation constants (see *J.I.A.* LXXVII, 128).

WÜNSCHE, G. *Sequential-Testverfahren in der Versicherungstechnik*, pp. 19-37. Illustrates Wald's sequential method of sampling (i) to detect whether a file of cards can be considered to contain less than 1% or more than 4% in error, (ii) to decide whether the mean reserve of a portfolio is less than 175 or greater than 225, (iii) to choose between 5 and 6 as the mean number of policies per insured.

AMMETER, H. *Der doppelseitige und die einseitigen ($I\chi^2$ -) Tests und ihre Leistungsfähigkeit für die wahrscheinlichkeitstheoretische Überprüfung von Sterbetafeln*, pp. 39-60. The mathematical development of the tests described by the author in the 1951 Swiss Bulletin (see *J.I.A.* LXXVIII, 138).

PÖTTKER, W. *Eine Methode zur summarischen Berechnung der Prämienreserve ohne Hilfssahlen*, pp. 61-69. If $P_{x:\overline{n}|}$ could be written as $\alpha_x/(\beta_x + n)$ the expression $\sum_j S_j {}_tV_{x:\overline{n}|} = {}_tV_{x:\overline{n}|} \sum_j S_j$ is exactly satisfied when

$$\bar{n} = \sum_j n_j P_{x:\overline{n_j}|} S_j / \sum_j P_{x:\overline{n_j}|} S_j.$$

However, $P_{x:\overline{n}|}$ is well approximated by $\alpha_x/(\beta_x + n) - \gamma_x$ for 15-year ranges of n , and this means that $\bar{n}$ is in defect or in excess according as $\gamma_x \gtrless 0$. Since ${}_tV_{x:\overline{n}|}$ varies slowly with x , a simple average x may be used over wide entry-age ranges. The result is an easy method of approximate valuation.

DIENST, H. R. *Untersuchungen zum Eindeutigkeitsproblem der Nomographie*, pp. 71-96. Investigates the conditions under which two or more straight-line nomograms of $F(u, v, w) = 0$ are topologically different.

HOLLAND

Het Verzeerings-Archief (Actuarieel Bijvoegsel), Vol. xxx, 1953

DUSSART, R. L. G. *Sur le complexe invalidité-décès*, pp. 23-31. Application of the multiplication rule of probabilities to certain probabilities encountered in disability insurance. A much-generalized version of these ideas is to be found in Dubourdieu, *Les principes fondamentaux du calcul des probabilités et la théorie de l'assurance-maladie*, Paris, 1939.

YNTEMA, L. *Some notes on the principle of discrete equivalence*, pp. 32-40. This principle is defined by the identity

$$F(t+T) \equiv e^{\delta T} F(t) \quad \text{all } t.$$

A good graduation of the curve of the mean rate of interest realized on the investments of the Dutch life offices from 1926 ($t=0$) to 1951, inclusive, is provided by the assumption that the value of a unit of capital at time t , $F(t)$, may be determined from

$$F(t+40) = (1.043)^{40} F(t).$$

ROOIJEN, J. P. VAN. *On numerical integration*, pp. 41-53. Derives formulae of the type

$$\int_a^{a+h} f(x)dx = h \sum_{k=0}^n c_k f\left(a + k \frac{h}{n}\right) + R_1 = h \sum_{k=1}^n \bar{c}_k f\left(a + 2k-1 \frac{h}{2n}\right) + R_2,$$

and provides expressions for c_k , $\bar{c}_k$, R_1 and R_2 .

JONGH, B. H. DE. *An extension of the theory of games and economic behaviour*, pp. 54-61. A discussion, in the terminology of the two-person zero-sum game (see Longley-Cook, *J.S.S.* x, 142), of the observed fact that a course of action preferred economically may not be a strategy of maximum expectation.

ITALY

Giornale dell' Istituto Italiano degli Attuari, Vol. xv, 1952

SANSONE, G. *Su di una equazione integrale di F. P. Cantelli suggerita da un problema di statistica matematica*, pp. 201-18. A number of conditions are stated under which the solution of the integral equation

$$\int_{-\infty}^{\infty} e^{\alpha^2 \phi(t)/2 + \alpha t - t^2} dt = \sqrt{\pi} e^{\alpha^2/4} \quad (0 < \alpha < \infty)$$

is $\phi(t) \equiv 0$.

OTTAVIANI, G. *Sulla convergenza uniforme delle successioni di funzioni*, pp. 219-34.

CHIARO, A. DEL. *Sulla determinazione delle probabilità di eliminazione*, pp. 235-49. Considers various formulae for deriving, from observations, an estimate of the probability that an individual aged x will be a decrement of a specified type before age t . Attention is paid to the logical consistency of the approximating assumptions made.

JECKLIN, H. *Sull' interpolazione iperbolica*, pp. 250-60. Shows numerically that the assumption $y = (ax + b)/(cx + d)$ is an accurate interpolation formula for $(1+i)^n$, $a_{\overline{n}|i}$ (i or n variable) and $1 - a_{\overline{n-1}|i}/a_{\overline{n}|i}$ (t variable).

ZWINGGI, E. *Un procedimento per determinare il saggio di interesse di rendite vitalizie e rendite certe*, pp. 261-68. Uses the methods of an earlier article in the *Blätter* (see *J.I.A.* LXXIX, 100) to find the rate of interest involved in an annuity value, $\ddot{a}_{x:\overline{n}|}$ or $\ddot{a}_{\overline{n}|}$. It is assumed that commutation columns or values of v^t , respectively, are available at a standard rate of interest. Numerical results show that the method is accurate even when the standard rate differs from that sought by as much as 1 %.

NICOSIA, F. M. *Sulla teorica dei capitali accumulati*, pp. 269-75. Shows that a result of Wyss (see *J.I.A.* LXXV, 115) is a particular case of Cantelli's theory of 'mutuality' (see *J.I.A.* LXIX, 308).

HAGSTROEM, K. G. *Considerazioni intorno al simbolismo matematico e alla capitalizzazione*, pp. 276-85. Discusses (1) an alternative notation for $\ddot{a}_{x:\overline{65-x}|}$, (2) the pair of symbols $s_{\overline{n}|}$ and $\bar{s}_{\overline{n}|}$, (3) nominal and effective rates of interest, (4) the rate of discount, and (5) a shorthand notation for common forms of insurance.

TEDESCHI, B. *Teorica dei tre usuali sistemi di sconto e critiche relative (Nota II)*, pp. 286-304. In continuation of his earlier note (see *J.I.A.* LXXIX, 227) considers transactions involving successive loans and sets of repayments.

CANTELLI, F. P. *Ulteriori considerazioni che si riferiscono alla conferenza su 'Lo sconto commerciale nella matematica finanziaria'*, pp. 305-12.

Risposta ad una critica su 'Lo sconto commerciale nella matematica finanziaria', pp. 313-23.

In his 1951 'conference' (see *J.I.A.* LXXVIII, 370) the author showed that banker's discount was really more favourable to the client than simple interest. In these articles he reviews the opinions of a number of actuaries from various countries.

This number also contains a summary of a conference by d'Addario on premiums for 'premier feu' insurances.

SCANDINAVIA

Skandinavisk Aktuarietidskrift, 1952, Parts 3 and 4

SVERDRUP, E. *Basic concepts in life assurance mathematics*, pp. 115-31. A rigorous proof is given of the theorem that, if premiums are loaded for contingencies, the probability of a loss tends to zero as the portfolio tends to infinity. A similar theorem is proved for reserves. Further, a novel theoretical solution of the graduation problem is provided by finding the maximum-likelihood estimators of q_x and a_j in the relation

$$q_x = \sum_{j=-b}^b a_j q_{x+j}.$$

WOLFOWITZ, J. *Consistent estimators of the parameters of a linear structural relation*, pp. 132-51. In econometric analysis, observation yields the simultaneous values of two random variables X and Y connected with ξ and η , two other random variables, by the equations

$$X = \xi + U \quad \text{and} \quad Y = \eta + V,$$

U and V being interpreted as errors. The variables ξ and η are known to be connected structurally by the relation

$$\xi \cos \theta + \eta \sin \theta = p.$$

The author constructs two sequences of functions, θ_n and p_n , of n pairs of random observations, such that $\theta_n \rightarrow \theta$ and $p_n \rightarrow p$ stochastically as $n \rightarrow \infty$.

OTTESTAD, P. *On the analysis of variance of percentage fractions*, pp. 152-59. A practical discussion of the interpretation of the analysis of variance of k observed relative frequencies.

LEFÈVRE, J. *Application de la théorie collective du risque à la réassurance 'Excess-Loss'*, pp. 160-87. Using Lundberg's 'collective' assumptions it is possible to determine the probability distribution of an insurance company's total loss after P units of risk premium have been received. The 'excess loss' reinsurance premium may thus be determined. It is shown,

inter alia, that simple asymptotic results are quite accurate, whereas the normal approximation based on 'individual' theory may be considerably in error.

CHUNG, K. L. *On the renewal theorem in higher dimensions*, pp. 188-94.

JENSEN, A. *A short remark on the theory of random sampling and the theory of variance*, pp. 195-200. Ingenious derivation of confidence intervals for the mean $\bar{Y}$ of a normal population of N individuals on the basis of a random sample of n joint observations (x, y) , the population value of x being assumed known.

BLOMQUIST, N. *On an exhaustion process*, pp. 201-10. m persons in succession traverse a path which contains n mines. Each person has a probability p of exploding a previously unexploded mine. How many explosions will occur?

WHITTLE, P. *Certain non-linear models of population and epidemic theory*, pp. 211-22. Shows how to calculate the successive moments of the probability distribution of the number in a population after exposure to several forces of entry and exit for a specified period of time.

WHITTLE, P. *On principal components and least square methods of factor analysis*, pp. 223-39.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker,
Vol. LIII, Part 1, 1953

WYSS, H. *Die Risikotheorie und ihre Bedeutung für die Versicherungsmathematik*, pp. 23-45. An expository account of the individual and collective theories of risk.

KREIS, H. *Über die Orthogonalpolynome*, pp. 46-56. A development of the orthogonal polynomials of least squares. The English reader may prefer to consult Aitken, *Proc. Roy. Soc. Edinb.* LIII (1933).

JECKLIN, H. *Beitrag zur technischen Behandlung anormaler Risiken in der Lebensversicherung*, pp. 57-77. If the mortality of a standard table can be represented approximately by $q_x = a + bc^x$ and if that of a substandard life can be written $q'_x = a' + b'c^x$, then

$$P'_{x:\overline{m}|} \div P_{x+m:\overline{n}|} + \left(\frac{1}{\ddot{a}_{\overline{n}|i'}} - \frac{1}{\ddot{a}_{\overline{n}|i}} \right) (1 + zi),$$

where $m = 25 \log(b'/b)$ and $i' = i + a' - a$. Numerical illustrations are provided.

LAH, I. *Die Taylorsche Reihe der generalisierten Poukkaschen Funktion und ihre Anwendung*, pp. 78-91. The author returns to the problem of calculating $\ddot{a}_{x:\overline{n}|}$ at a rate of interest i' , given a set of basic tables at i (see *J.I.A.* LXXIII, 444 and LXXVIII, 140). The new formulae have lost the sheer simplicity of the Poukka approximation but are very accurate. For example, by using calculated commutation columns at 3% the value of $\ddot{a}_{39}$ at 6% was obtained with an error of only .001.

NOLFI, P. *Gedanken und Grundlagen zur Invaliditätsversicherung*, pp. 92-105. 'An increase in the mortality of ill-health pensioners results in an increase

in the mortality of active employees.' Observational results of the Zurich municipal employees' pension fund are used to illustrate how the mortality of ill-health pensioners increases as the rate of ill-health retirement decreases.

SAXER, W. *Über die Variation der Invaliditätswahrscheinlichkeiten bei der Berechnung des Barwertes anwartschaftlicher Invalidenrenten und ihrer Prämien*, pp. 106-15. If $P_{\overline{an}}^i$ is written for the annual premium for a unit annuity to age s on disability within n years, a change in disability rates, i_x , may be allowed for by writing

$$P'_{\overline{an}} \doteq \frac{1}{n} \left(\frac{i'_x}{i_x} + \frac{i'_{x+1}}{i_{x+1}} + \dots + \frac{i'_{x+n-1}}{i_{x+n-1}} \right) P_{\overline{an}}^i.$$

Numerical illustrations are provided.

VOGEL, W. *Eine Invarianzeigenschaft von Standard-Absterbeordnungen und deren praktische Anwendung*, pp. 116-28. The pure endowment ${}_nE_x$ is invariant under a linear transformation of the age x to $m(x+p)$ simultaneously with the homogeneous transformation of $\mu_x + \delta$ to $\frac{1}{m}(\mu_x + \delta)$; and conversely.

UNITED STATES

Transactions of the Society of Actuaries,
Vol. IV, November 1952

HOLMES, B. T. *The actuarial profession within the free communities*, pp. 423-29. Presidential address.

ECKLER, S. *State hospital insurance in Canada*, pp. 430-47 (with discussion). Hospital insurance is compulsory in British Columbia and Saskatchewan. In the former province the utilization rate is about $1\frac{1}{2}$ days a year, in the latter $2\frac{1}{4}$, the difference being due to the restrictions on chronic coverage in British Columbia.

ORMSBY, C. A. *The cost to reinsure individual life insurance policies*, pp. 448-84 (with discussion). Points to be considered by a life office entering the reinsurance field on a co-insurance or yearly renewable term basis, respectively.

PEDOE, A. *The trend of life insurance company expenses*, pp. 485-510 (with discussion). Traces the trend of expenses in twenty Canadian non-industrial companies since 1939.

MUSHER, J. *The actuarial implications of the 1951 Railroad Retirement amendments*, pp. 511-45. The effect of increased benefits is to raise the cost from 12.60 to 14.43 % of payroll computed by ignoring individual compensation in excess of \$3600 p.a. The tax rate permitted by law is 12.5 %.

HOSKINS, J. E. *A convenient method of providing for mortality improvement based on the a-1949 table*, pp. 546-73 (with discussion). Approximates Projection B of the a-1949 Table (see *J.I.A.* LXXVI, 168) by using the basic a-1949 Table and imposing an age set-back of .075 (male) and .06 (female) for each year by which the year of birth post-dated 1875.

RASOR, E. A. and GREVILLE, T. N. E. *Complete annuities*, pp. 574-84 (with discussion). Examines the consequences of assuming that the fractional payment to 'complete' an m thly annuity t years after the last full payment is due, is

$$\{(1+i)^t - 1\}/i^{(m)} \quad \left(0 < t < \frac{1}{m}\right).$$

In particular,

$$\bar{A}_{x:\overline{m}|} = 1 - i^{(m)} \bar{a}_{x:\overline{m}|}^{(m)}.$$

SMITH, T. C. and GRIFFIN, F. L., JR. *Work-life expectancy as a measure of damages*, pp. 585-620 (with discussion). Applies the rates of disability and voluntary retirement derived in the 1946 Railroad Retirement Board Report to fix the monetary value of a railroad employee killed or injured as a result of its negligence. In the discussion Fenichel summarizes the actuarial evidence offered in about 30 actions brought as a result of the two Long Island Railroad crashes a few years ago.

GELLES, M. *Actuarial considerations in cost analysis*, pp. 621-48 (with discussion). Three methods are mentioned: (1) Allocation of a fair proportion of expenses to each branch of the business; (2) Formulation of a standard cost for each branch on the assumption it is efficient; (3) Calculation of the additional cost incurred as a result of a new branch.

This number also contains a digest of informal discussion on substandard insurance, war hazards, and general points.

1953, No. 2

This number contains the 1952 reports of the mortality and morbidity committees.

The analysis of 'standard' mortality in each calendar year since 1930 is made for medical and non-medical issues separately for each of the first fifteen policy years, and for both types of issue together for the aggregate of policy years thereafter.

Group accident and sickness data, including hospital and surgical expense claims, are examined for each of the years 1947-51.

Group annuity deaths among 'normal' male pensioners over age 65 during 1951 were 106.5 % of those expected by the 1937 Standard Annuity table.

The passenger death rate per 1000 domestic passenger hours in 1951 continued at the same level as in the previous three years, namely at slightly above .002.

The disability experience under supplementary contracts was examined in four separate quinary calendar year groups ending in 1950. This represents the first publication of a joint experience since the 1926 investigation—which was based on limited data. Although five different classes of benefits are studied, only one of them (viz. waiver of premium after a six months waiting period) is currently used with new issues. Under this type of clause, claims during 1946-50 were running at 41 % of those expected according to 1926 class 3 rates.