

NOTES ON OTHER ACTUARIAL JOURNALS

GERMANY

Blätter der Deutschen Gesellschaft für Versicherungsmathematik, 19
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- REISS, R.-D. *Statistical Inference Based on Large Claims via Poisson Approximation. Part I: Poisson Random Variables*. The author assumes that a certain Pareto assumption is justified for the modelling of large claims. The estimator for the unknown parameter is based on claim frequencies. The statistical inference is carried out via a Poisson approximation to the actual model of claim size distributions and the application of the maximum likelihood method. In connection with practical aspects the bootstrap method is briefly mentioned.
- KREMER, E. A *(New) Burning Cost Method in Case of Trend*. The author reconsiders the problem of predicting or calculating the premium of a reinsurance cover with the so-called burning cost method. The classical assumption of constant mean yearly burning costs is replaced by the assumption of a (linear) trend. A practicable burning cost method is derived by applying results of the econometrics.
- PATER, R. *The Run-off-Triangle: Least Squares—Against Chainladder Estimations*. To estimate the lower, unknown, part of a run-off triangle is one of the many problems an actuary has to face. Often the chain ladder method is used. The predictions can be improved by using a least squares model. Maximum likelihood estimators can be obtained. An inequality coefficient is introduced to discriminate between models.
- KREMER, E. *On the Numerical Evaluation of the Ultimate Ruin Probability*. A new method for computing the ultimate ruin probability is presented. The accuracy of the method is demonstrated in some examples, where the exact results are known. Finally the practicability of the method is shown.
- LÜTHY, H. *Die Entwicklung von Rentnerbeständen als stochastischer Prozeß*. This work has as content the examination of the development of the number of annuitants with time, where this number is built up of given, time-dependent numbers of active persons. The development of such numbers of annuitants is to be understood as a stochastic process $R(t)$.
The moment generating function for $R(t)$ is deduced, which unequivocally determines the distribution. The proof of different limit value theorems can then be demonstrated. One interesting point makes itself apparent—the distribution of the number of annuitants over longer periods of time in general follows the Poisson distribution very closely. Furthermore, functions of $R(t)$ are so defined that the resulting processes are either Martingales or Submartingales. A Kolmogorov inequality for positive Submartingales permits, for example, assertions concerning the probability of ruin in different models of financing.
Finally, the calculation of a concrete example is presented. With the help of the utility principle (in this specific case: the exponential principle) the stop-loss premium is calculated for the reinsurance of inflation compensation for current old age annuities.
- ETTL, W. *Optimal Dividend Policy*. The optimal dividend policy is derived under general conditions which allow variable risk parameters and discounting. For the compound Poisson distribution claim model as well as for the Wiener process claim model higher moments of the sum of the discounted dividend payments are derived and the optimal dividend policy is derived. For models with barriers for dividends the higher moments of the sum of the discounted dividend payments are derived. The combination of the time of ruin and higher moments are also considered as criteria for dividend policy. An outlook on the application of the distribution function of the discounted dividend payments is also given.
- NETZEL, C. *Optionen, Wahrscheinlichkeit, Zeit und Information oder ist das Theorem von Cantelli irreführend?* This paper deals with the question whether some policy options can be offered

priceless according to the theorem of Cantelli. These options can be a valuable asset for the policyholder because of his growing information during the lifetime of the policy. For the valuation the formula of Black and Scholes for capital market options is used. A capital option for an annuity plan is considered to illustrate the basic idea.

- STICKER, A. *Die Mindestrückvergütung in der Kapitallebensversicherung*. In conjunction with the new mortality tables '86 M/F, the Federal Insurance Supervisory Authority requires the life insurance offices to calculate certain minimum surrender values. The paper introduces two constant parameters (A and B) which reflect the individual contract, as a function of the age of entry (x) and the duration of payment of premiums (t). It can be proven that ${}_mR_x \cdot A + B$ produces a good approximation of the required minimum surrender value even after changes of the underlying insurance contract, whereas ${}_mR_x$ is the actually justified surrender value after m years of contract duration.

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- REICHEL, G. *Reine Erlebensfallversicherungen im Spannungsfeld zwischen Risiko- und Nutzentheorie*. This paper is based on a similar article in issue No. 21 in *Angewandte Versicherungsmathematik* which proposed to analyse life insurance contracts having regard to the different judgement principles of both contractual parties. The present paper applies these considerations on pure endowment policies. Under the chosen assumptions as to insurance duration and age of entry, the sample projections of dividends produce different priorities if the insurer applies risk theory for the determination of his minimum premiums and if the policyholder values his possible maximum contribution according to a profit theory. A comparison of mixed endowment policies with pure endowment policies indicates that the latter offer more opportunities in the market.
- REISS, R.-D. *Statistical Inference Based on Large Claims via Poisson Approximation. Part II: Poisson Process Approach*. The author assumes that the tail of the claim size distribution is of Pareto type. The estimator of the unknown parameters is based on the numbers exceeding a non-random threshold. The estimator is related to the maximum likelihood estimator in a parametric model of Poisson processes. For this purpose, we compare the model of empirical point processes and the model of Poisson processes. Moreover, we prove the asymptotic normality of the estimator.
- SCHMIDT, K. D. *A Note on Positive Supermartingales in Ruin Theory*. In this note, we present an elementary proof of Kolmogorov's inequality for positive supermartingales and an application of this inequality to the ruin problem for a class of surplus processes for which an adjustment coefficient needs to exist.
- KREMER, E. *Remarks on Calculating the Premium for an Excess-of-Loss Cover in Earthquake Insurance, According to De Saran's Approach*. The problem of calculating the premium for an excess-of-loss cover in earthquake insurance is reconsidered. A well known rating method is discussed and conjectures for improving are given. A special modification for rating treaties with larger priorities is presented.
- HEILMANN, W.-R. *Versicherungsmathematische Methoden des Risk Management*. Risk Management is a term that has come into widespread use in Europe only during the last two decades, and it still lacks precise meaning. It is not even possible to provide a definition of the word risk which is generally accepted. So we start with defining the notions of risk and risk management in the two sections following the introduction.
- In the fourth section we consider insurance and reinsurance as two prominent tools of risk management, and stress the connections between engineering, risk management and insurance. In the following sections we describe risks by a mathematical model, and represent attitudes towards risk by utility functions. Subsequently, these models are used to characterise optimal forms of insurance and reinsurance. In the final section we list a number of practical risk management techniques.

ENGBROKS, H. *Diskussionsbeitrag zur Bewertung von Pensionsverpflichtungen*. The paper suggests a modification to the Teilwert method, being the valuation basis for pension liabilities, in order to pay account of the date-of-valuation principle also with respect to the biometric risk. From a business point of view this modification allows for a more satisfactory valuation of pension liabilities.

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HIPP, C. *Feller-Ross-Approximation für Summenverteilungen*. Feller's approximate inversion of the Laplace transform is used to derive approximations for compound distributions. If the compound distribution has decreasing density, then the approximation is monotone. Theoretical error bounds and two numerical examples are given.

KREMER, E. *On a Generalized Total Claims Amount*. In the paper the author defines a generalised total claims amount and reformulates it with help of the so-called spacings. The corresponding distribution function is evaluated for different claims size distributions. The application of the results in reinsurance is demonstrated.

AMSLER, M.-H. *Lundbergs obere Schranke für die Ruinwahrscheinlichkeit. Ein elementarer Beweis*. A proof of the famous Lundberg formula for the probability of ruin of an insurance portfolio is given by means of elementary mathematical tools.

KREMER, E. *On the Probable Maximum Loss*. In some non-life insurance branches the probable maximum loss is of great importance for judging a given risk or a given collective of risks. In the present paper, the author gives a general mathematical definition and a new practicable calculation method for the probable maximum loss.

ZIGENHORN, U. *Zur Modellierung des Ausgleichs im Kollektiv*. Albrecht's definition of Equalisation in Collectives is generalised for application to inhomogeneous collectives.

It is further discussed to analyse the probability of ruin and the premium loadings as examples.

A new definition introduces two parameters to weight both targets and to steer the degree of approximation to the chosen results.

This definition is also used for finite collectives with the help of a non-uniform error barrier to determine the conditions that guarantee a given maximum loss probability. The resulting critical values are much better than those derived from the uniform Berry-Esseen barrier.

SCHMIDBAUER, H. *Demographische Mortalitätsanalyse: Ein Punktprozeß-Modell*. In the present paper, a point process model for a population is constructed. We calculate the distribution of random variables which are of relevance for demographic mortality analysis. With these results, classical life table methods can be analysed with respect to random fluctuations. Using a given life table, a numerical analysis is carried out that permits an evaluation of the quality of the life table methods.

WALDMANN, K.-H. *Versicherungsmathematische Behandlung erhöhter Risiken am Beispiel der terminalen Niereninsuffizienz*. Terminal renal insufficiency is considered as a heightened risk, which cannot be insured by standard methods. This is due to the high mortality during the first years to be observed independent of the therapy (dialysis, transplantation). Based on the empirical data, published by the European Dialysis and Transplant Association, a differential treatment (with respect to the age of the patient and the duration of the therapy) of the mortality is given, and, on this basis, the premiums and reserves are calculated. Moreover, resulting from contracts with negative reserves, some possibilities are discussed to reduce the total loss of the portfolio in size and duration.

NEUBURGER, E. *Unabhängigkeit von Rentenanwartschaftsbarwerten von der Zahlungsweise*. Under most recent developments, actuarial valuations aim to reflect situations where disability and survivors' pensions are not paid immediately as of the date of the claim, but only as of the next following due date of the pension. The paper shows that the present values of such deferred titles do not depend on the frequency of pension payments provided that throughout the year there is an equal distribution of the dates of the claim and of the dates of the termination of pension payments. Under the given conditions, an interesting by-product is proven, conclusively and without recourse to approximations, namely the existence and the formula of $k^{(t)}e$ independent

of x and only depending on t the number of pension instalments throughout the year which produces $\ddot{a}^{(t)} = \ddot{a}_x - k^{(t)}$ for the present values of immediate annuities:

- \ddot{a}_x due immediately with annual payments, and
- $\ddot{a}_x^{(t)}$ due immediately with t instalments per year.

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REICHEL, G. *Notwendige und hinreichende Bedingungen für das Theorem von Cantelli*. Cantelli's theorem is often used in practice if one cause for withdrawals is to be left out of consideration. A special form of this theorem is possible if the claim at such withdrawal is restricted to the mathematical reserves.

Henryk Schärf has given a justification of this special theorem, which is based on general observations of invariances. This basic result offers necessary and sufficient conditions. Interestingly enough, the literature (including Schärf's paper) offers only sufficient conditions for the general form of Cantelli's theorem.

The present paper deduces necessary and sufficient conditions for Cantelli's theorem in a generalised observation of insurance forms which comprises the continuous and the discontinuous method as special cases.

After a description of the general models, Thiele's equation and Cantelli's theorem are derived and the latter is then discussed for the cases of special interest.

KREMER, E. *Practical Exponential Smoothing Credibility*. Credibility estimators with geometric weights have proved to be of some practical importance. An alternative is to use exponentially smoothed credibility estimators. In the present paper the latter are prepared for practical application.

MAIER, H. *Autoregressive Modelle in der privaten Krankenversicherung*. The article describes statistical methods which are usable for a private health insurance company. With these methods forecasting the claims experience in case of illness is possible without any iteration. These methods are based on the theory of auto-regressive models. All methods are proved exactly and their use in practice is discussed.

LAUX, H. and LISSNER, J. *Modellbetrachtungen zum Verfahren der vereinfachten Abwicklung im Bausparen*. In the present paper the history of the process of the 'vereinfachte Abwicklung' (simplified liquidation of a building-society) is told and the supposition to make it work is described. After this a model of the 'vereinfachte Abwicklung' is presented and the effects of some influences on the process are shown.

GOHDES, A. E. *Versicherungsmathematische Finanzierungsmethoden für betriebliche Versorgungsverpflichtungen*. The diverse valuation methods available for funding company pensions in the U.S.A. and U.K. are briefly described and their respective main characteristics commented upon. In addition, methods for valuing pension plan assets and amortising any surplus or deficit are touched upon too. No single approach is regarded as correct. The consulting actuary is constrained in his choice of approach both by legal and accounting rules and by specific information in respect of the relevant pension fund being valued. The paper is essentially a synthesis of existing literature published in the U.S.A., U.K. and Germany.

RUPPRECHT, G. *Neue Rechnungsgrundlagen für die Berufsunfähigkeitsversicherung*. The last detailed survey on the probability of the occurrence of disability among actively employed persons, the mortality rates and the rehabilitation behaviour of disabled persons was conducted in 1964. At that time, figures were backed up mainly by foreign observation results. In the meantime, the joint statistics of the Association of German Life Insurers provides a data base from which can be assessed modern calculation bases for private disability insurance which are tailored for the German market. The present paper reports on the results of this assessment.