

NOTES ON THE PAPERS WRITTEN FOR THE TWELFTH INTERNATIONAL CONGRESS OF ACTUARIES

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THE Congress was to have been held at Lucerne in June 1940. When war broke out, most of the papers had been sent in, and the remainder were received not long afterwards. Dr Marchand, in his 1941 report to the Swiss Actuarial Congress, stated that it had been decided to publish these papers. Effect was given to this decision by the production of four volumes, excellently printed and bound in spite of war-time handicaps.

The ninety-eight papers sent in are classified according to subject-matter as follows:

Subject	No. of papers
The theory of probability in Insurance	19
Cover for special hazards in Life Assurance	4
Investment of Life Assurance funds	11
Terms to be granted on withdrawal	24
Sickness Insurance	11
Treatment of withdrawals in Life Assurance	5
War risk in Life Assurance	9
Variations in groups of persons	8
Indemnity for loss or diminution of professional income	1
Unclassified	6

Note. In the following description of the papers, the letter E, F, G, or I after an author's name indicates the language in which the paper has been written. An asterisk indicates that the paper will be found in Vol. iv.

THE THEORY OF PROBABILITY IN INSURANCE

The reporter was Jecklin. The notion of probability itself is discussed in the papers of Göring, Boschan. The part that the theory of probability plays in insurance is regarded as important by Baptist, de Finetti, Berger, doubtful by Hammon and Clarke, Lah, and of little consequence by Hagstroem, Shannon, ten Pas. Hagstroem in particular stresses the point that fundamental changes must outweigh random fluctuations. The papers by Vajda and Lukacs develop the theme of the so-called 'extended mortality table' and make it clear that the ordinary standard formulae are only admissible if special assumptions are made. The paper by Redington and Michaelson discusses numerical deviations in death-rates based on the A 1924-29 experience. Broadly speaking, much of the matter in these papers turns on the questions: (1) to what extent can the death-rate be regarded as an *a priori* probability? (2) of what nature are the deviations from the assumed value? The applicability of the theory of probability to Insurance other than Life Assurance is considered practicable by Riebesell but not by Jecklin. Special aspects of the theory form the subjects of papers by Jacob, Kolodziejczyk, Haferl, Nolfi.

- A. BERGER* (G). *What hypotheses lie at the base of actuarial science and to what degree can the use of the theory of probability and of the theory of risk be justified?* The writer describes three ways of envisaging the question of reserves: (1) the normal way, based on frequencies, (2) that based on Hattendorff's theorem, (3) the notion here and elsewhere set out by the author. In the summary this is described as the reserve connected with the 'smallest amount of mean loss'. What is meant is a reserve which results in smallest mean square deviation, and in its most generalized form this would cover variations in the rates of expense also. It is not claimed that any workable system of valuation could be based on this theory.
- P. RIEBESELL* (G). *The actuarial bases of Non-Life Insurance.* The writer claims that, although it has hitherto only been possible to administer Indemnity Insurance on empirical lines, the advances which statistical research has made in the last thirty years now permit a more scientific handling of this branch of Insurance business. The formulae which he derives are based on an average loss and Poisson's exponential for the Law of Small Numbers: they include formulae for premiums and reserves. Reference is made to previous papers by the writer and by his pupils.
- J. BAPTIST (F). *The theory of probability in Insurance.* Plain statistical frequencies are the starting-point of our actuarial tables, but if these are to be further developed the theory of probability cannot be ignored. An elementary application is the derivation of compound functions such as $A^1_{x:n|}$ from the life table and the interest table. The frequencies which are the basis of the life table are compounded of a number of parameters, of the relative values of which we have at present insufficient knowledge, but it is in the light of this fact, and the fact that the force of each parameter is liable to vary during the experience on which our table is based, that further improvements are to be expected.
- S. SHANNON (E). *Some assumptions and hypotheses underlying actuarial calculations.* Emphasizes the known fact that mortality risk is of all Life Assurance risks the one least liable to be influenced by human volition, variations in interest and expenses being practically impossible to forecast. In the second part of the paper the extent to which the normal curve of error represents distributions is discussed; its applicability is modified by the inequality between p and q and the fact that the numbers concerned are not sufficiently great.
- P. BOSCHAN (E). *Some considerations concerning probability in actuarial science and the foundation of the Extended Life Table.* The writer refers to a paper by Schlick and holds that the question whether we construct our tables on *a priori* or *a posteriori* principles is simply a matter of expediency. There is no room for the *calculus* of probability though the estimates of the actuary now belong to the more extended form of the *theory* of probability. We cannot, however, arrive at anything approximating to an aleatory distribution round each year of age until we have first separated the secular trend.
- E. LUKACS (G). *The theory of mean risk founded on the theory of probability.* Gives an analysis based on the idea that the probabilities, obtained *a posteriori*, of $0, 1, \dots, a, \dots, s$ deaths among s persons aged x are separately considered, and that the probabilities pertaining to each corresponding batch of survivors are similarly dealt with, and so on. Each age is thus regarded as

being independent of the next one, but no assumption is made regarding the standard deviations of the deaths; i.e. the distribution of the deaths need not necessarily follow the Gaussian or Bernoullian law. If the distribution follows the Gaussian law we get the formulae of the classical theory of risk.

- P. H. HAMMON and R. D. CLARKE (E). *Some effects upon Insurance problems of modern criticisms of the frequency theory of probability.* Sets out the smallness of the part which the theory of probability plays in the practical handling of Life Assurance risks. The body of lives on which a life table is based is necessarily heterogeneous; all we can hope for is that its composition is in the main the same as that of the assured of the company which uses the table; moreover, such a table does not usually take into account secular trends. Each individual must be considered on his merits and actuaries must make the best compromise which they can between *a priori* judgment and frequency statistics.
- F. M. REDINGTON and R. L. MICHAELSON (E). *An aspect of the 'a priori' probability theory of mortality.* A test, using the A 1924-29 tables, founded on the assumption that at each age there exists an *a priori* probability of death, and that the experience on which the table is based is homogeneous: we should then expect to find the function $\frac{\Delta^2 q_x}{\sigma(\Delta^3 q_x)}$ represented by a frequency distribution with a mean value of approximately zero and a standard deviation of approximately unity. This is found to be the case for the select, but not for the ultimate, table. The data are obviously not homogeneous, and it is possible that the disruption caused by this state of affairs becomes more marked as selection wears off.
- S. VAJDA (G). *The Extended Mortality Table and its application in the theory of probabilities.* This is another version of the idea expounded by Lukacs.
- B. DE FINETTI (I). *The theory of probability in Insurance.* Like Baptist, this writer considers that the theory of probability constitutes the only suitable and adequate instrument for dealing with the problems of Insurance, although it is not possible definitely to classify the distributions involved as analogous to arrangements of a schematic type. Reference is made to many writings on the subject, and in particular to a recent one by the author himself: *Ai margini del dominio della matematica nei problemi dell' assicurazione.*
- W. G. J. TEN PAS (G). *Probability and statistics in actuarial science.* The writer maintains that Life Assurance cannot properly be said to be founded on the theory of probability but that life assurers must, like traders in other spheres, adopt, in the main, prices and methods which have served them well in the past. He admits that to assume linear secular diminution of mortality is empirical, but asserts that no precision could be claimed for any other assumption, such as that embodied in a quadratic or logarithmic formula. Figures based on the vital statistics of Holland, Norway, and Sweden, for the years 1921-34, are given.
- M. JACOB (I). *On a method of approximation for the calculation of the mean risk.* Expounds a method, based on Steffensen's inequalities, of obtaining approximate maximum and minimum values of the mean square of the risk,

according to Hattendorff's formula: it can be extended to decreasing premium risks, and to impaired risks with increasing extra mortality. Numerical examples founded on the H^M table are given.

- S. KOŁODZIEJCZYK (F). *On events of high probability*. Events of high probability confer greater stability than those of low probability. An analysis is given, and numerical examples taken from the Fire Insurance branch of the writer's office.
- K. G. HAGSTROEM (F). *Remarks on the function of the theory of probabilities in Insurance practice*. The essential conclusion is that variations from year to year are due to real changes in risk intensities and that estimates of probabilities of certain deviations based on the pure chance theory afford no practical guide.
- E. GÖRING (G). *An extension of Mises's collective and the corresponding development of the theory of probability*. Deals with von Mises's definition which not only is not general enough to cover collectives which include impaired lives, but is inadequate for large sections which are inevitably not homogeneous. An attempt is made to express the heterogeneity of a distribution in a mathematical way.
- E. HAFERL (G). *The determination of the self-retention in Life Assurance*. The point is stressed that there should be a lower as well as an upper limit to an office's self-retention, the former being conditioned by the principle of a minimum profit to the reinsurer, and independence of the size of the portfolio. It is then shewn how, on the assumption that the variations of a group with a given death rate may be represented by a normal distribution, they may be combined with those of a group having another death rate, and the probabilities of all possible combinations of variations would then be represented by a surface (similar to a correlation surface) with elliptical cross-sections parallel to the XY plane. The probability of joint deviations beyond a certain limit would then be represented by the relative size of that part of the total volume cut off by an elliptical cylinder cutting the surface along the line of a cross-section parallel to the XY plane.
- H. JECKLIN (G). *Can the theory of probability be better applied in Life Insurance than in Non-Life Insurance?* The writer points out that in Indemnity Insurance we have not, as we have in Life Assurance, data sufficient to enable us to say that to a limited extent the theory of probability is applicable; but he is of opinion that the difference between the two main types of Insurance is not so great in this respect as is commonly supposed. We have the analogy of a small Life Assurance company where numbers are not sufficient to allow the law of great numbers to manifest itself in each age group; it may nevertheless be applicable in some degree to the portfolio as a whole.
- P. NOLFI (G). *The yearly mortality fluctuations and their signification for the theory of probability*. The influence of causes which affect the so-called basic value of q , such as wars and epidemics, affects all companies more or less equally; the writer surmises that wars might double the mortality at young ages twice in a century. A formula is arrived at, which assumes that both the normal accidental fluctuations and the periodical fluctuations affecting the basic value of q follow a Gaussian law.

IVO LAH (G). *The theory of probability and Insurance*. The difficulty of applying the theory of probability, entirely suited to games of chance, urn problems, etc., to social and economic entities, is again emphasized, but it is considered that it still has its value if suitable allowance is made for the periodical influences which seriously affect the assumed *a priori* probabilities. The writer suggests that the Permanent Committee should endeavour to make arrangements for an international exchange of statistical observations.

COVER FOR SPECIAL HAZARDS IN LIFE ASSURANCE

The report on this section was made by Burlet. Of the four papers submitted only that by Franckx dealt with the analytical side of the question. The other three, by Boehm, Derrien, Meertens, described and discussed the practices of the writers' respective countries.

C. BOEHM* (G). *Coverage of special risks in Life Insurance*. Extra risks are here divided into three categories. The first category embraces those which are calculable in advance and includes of course climatic and occupational extra risks for which German statistics are given; statistics are also given for aviation risks.

E. FRANCKX (F). *Coverage of special risks and the corresponding usual forms of Life Insurance*. An investigation of the conditions under which the problem of extra risk in endowment assurances may be met by varying the amount payable at death, making it a function of the reserve, instead of charging extra premiums.

M. DERRIEN (F). *Life Assurance and enlarged risk due to profession*. Deals specially with occupational risks. French companies ignore them to an extent which is perhaps not quite fair to the good risks. Certain dangerous occupations are classified in groups and the charging of extra premiums or reductions in benefit are advocated.

C. MEERTENS (G). *Cover for special hazards in Life Assurance*. Describes the general practice of Dutch companies. Climatic risks are, owing to the improvement in living conditions in the tropics, leniently dealt with, and policies are usually world-wide after three years' home residence.

INVESTMENT OF LIFE ASSURANCE FUNDS

The report was made by Röthlisberger. Guillaume's paper gives mathematical form to the financial transactions of the country as a whole, and their bearing on Life Assurance. MacCharles, de Crefft, Bernus, Moosbrugger, ten Pas, Ullrich, write mainly about the investment practices of their own countries, and so does Poznański, who, however, concentrates chiefly on the subject of policy loans. The subject is more generally discussed by Bjerreskov, Hagstroem, Altenburger, the last of whom advocates international safeguarding. The reporter expresses a regret that no paper was received on the subject from England, Italy, or the U.S.A., but as regards Great Britain it may be suggested that there is little that could be said that would be of practical guidance to countries hampered in this respect as are those of the Continent; it is clear that the degree to which they are thus hampered varies considerably.

- H. ULLRICH* (G). *Investments of German Life Insurance companies.* Following a paper submitted to the Rome Congress, the writer gives particulars of German Life Assurance investment law and policy. Mortgages still constitute by far the most important item, but the way in which the situation was saved in the inflationary post-war period by the modest percentage of funds invested in real estate has led to a larger proportional purchase of properties. The item 'securities' has increased.
- F. DE CREEFT (F). *Proposals for a new policy of investing in property.* Investment in houses is considered unsatisfactory, owing to the rapid depreciation of small dwellings. It may be noted that some of the arguments used in this pre-war paper no longer apply.
- F. D. MACCHARLES (E). *Investment practices of Canadian Life Insurance companies.* The legal restrictions to which Canadian Life Assurance investments are subject, and the general policy followed, are described and compared with those of American companies.
- S. BJERRESKOV (F). *The financial policy of Life Assurance companies.* The somewhat timid outlook on investment policy shown here would seem to be the natural outcome of the restrictive conditions to which, in varying degree, Continental companies are subjected. In this connexion the comparative table giving the classification of investments in various European countries is interesting. It is only in Great Britain that an appreciable proportion of the funds is placed in stocks and shares.
- P. BERNUS (F). *The investments of Life Insurance companies.* Reference is made to the French law of 30 December 1938 on the working of which it is not yet possible to comment. There are new rules for dealing with redeemable securities; if bought under redemption price they must be kept at that figure in the balance-sheet. French companies invest largely in real property.
- E. MOOSBRUGGER (F). *The investments of Life Insurance companies in France and their valuation in the balance-sheet.* This paper also deals with French practice and the probable effects of the decree of 1938. The author considers that the rules for dealing with redeemable securities are unnecessarily complicated.
- J. ALTENBURGER (G). *On the question of investments in Life Assurance.* The writer's observations on the effects of war apply to-day with far greater force than when they were written. He has in view a kind of international financial Red Cross, a worthy ideal to which however it would obviously be difficult to give practical effect.
- W. G. J. TEN PAS (E). *Retrospective investigations into the investments of Life Assurance funds in the Netherlands.* An analysis of investment trends in Holland from 1925 to 1938. The law of 1922 is closer than most Continental Insurance laws to that of Great Britain and imposes no restrictions on the choice of investments or their valuation in the balance-sheet. Mortgages are still the most important item but the percentage of these has decreased and those of real estate and of Government loans have increased, the latter markedly.
- T. POZNAŃSKI (F). *On the character of investments consisting of policy loans.* Deals in particular with the course to be adopted in the event of the insolvency of the assurer, or in the event of a change in the currency of the

policy (assumed to be the same as that of the policy loan). In the case of the insolvency of the assurer, it might be admissible to scale down the debt if the surrender value were to become less than the amount of the loan.

- K. G. HAGSTROEM (F). *On the formation of reserves and the necessity of making the funds yield interest.* The first part of the paper emphasizes the need for reserves, the levy system being one which can only be adopted by the State; it involves, moreover, injustice to the non-insured. In the second part of the paper it is pointed out that the assets should include real property and equities as a safeguard against inflation; the difficulty of distributing profits in the event of marked inflation is mentioned. The suggestion that an Insurance company might work through a commercial bank would hardly find favour here, as the Insurance companies themselves are adequately equipped and the staffs trained to deal with all suitable investments. Incidentally this part of the paper includes a well-grounded protest against the Continental attitude of State and private Insurance institutions towards the profession.
- E. GUILLAUME (F). *Constitution of premium reserves in connexion with issues of securities in a given country and currency.* The writer sets out in the form of a differential equation the working of a country's borrowing and saving transactions. The conclusion that a nation should increase its new indebtedness more or less according to the investment needs of banks, Insurance companies, etc., seems, however, curious.

TERMS TO BE GRANTED ON WITHDRAWAL FROM THE CONTRACT OF LIFE ASSURANCE

More papers were submitted on this subject than on any other but they were for the most part short. The reporter was Wolfer. Fifteen papers dealt with the practices of the writers' respective countries: these were by Dolezel Löer and Parthier, Maurice, Stephenson, Nielsen, Neumann, Junnila, Loisel, Coe and Owen, Hickox and Puckridge, Horsley and Murray, Biró and Jacob, Schärf, Ploeg, Palmquist, Štastný. The other nine papers by Bjerreskov, Johansen, Altenburger, Bodoni, Crosato, Manfredi, Masciotti, Thesen, Zwinggi, were of a general nature.

- R. DOLEZEL, K. LÖER, and H. PARTHIER* (G). *Benefits in case of withdrawal from a Life Assurance contract.* The subject is envisaged from the assured's point of view and a formula is given which indicates a measure of the inclination to withdraw. A distinction is made between assurances where this inducement is strongest in the early years (the commonest case), and the converse instance. With regard to collective assurances at a level premium it is held that refunds on withdrawal should be based on the actual reserve multiplied by $\frac{\bar{P}}{P_x}$, where \bar{P} is the level premium and $\bar{P} < P_x$.

- H. MAURICE (F). *The value of a Life Insurance contract in case of withdrawal.* Surrender values in Belgium are conditioned by the laws of 1930 and 1931, which limit the allowance for initial expenses to 3 % of the sum assured or 90 % of the annual premium. An analysis is given of claims, surrenders, and withdrawals, according to duration, and another is given by calendar years, reflecting the effect of the crisis of 1935. The writer considers that the laws are in the main adequate.

- H. R. STEPHENSON (E). *Surrender values of Canadian Life Insurance companies.* An account of the law and practice governing surrenders in Canada. A comparison is made with the arrangements in the U.S.A.
- S. BJERRESKOV (F). *On the principles of granting a right of surrender and those regarding the calculation of surrender values of life policies.* Suggests surrender values based on the formula ${}_tR = {}_tV - \alpha' \frac{a_{x+t:\overline{n-t}|}}{a_{x:n}|}$, where α' takes account of initial expenses and losses and profits due to early lapses.
- P. JOHANSEN (E). *Retrospective calculation of surrender values.* Tries to find a solution of the difficulty arising from the disappointment generally caused by the smallness of the surrender values granted during the early years of an assurance. The assured naturally resents being mulcted owing to the payment of a heavy initial commission to an agent. The suggestion that agents might be made to refund some of their commission seems, however, hardly practicable.
- A. K. NIELSEN (E). *Calculation of surrender values in Danish Life Assurance.* Deals particularly with conditions governing surrenders in Denmark, where the law enjoins no special restriction on the companies. Like Johansen the writer considers the question retrospectively and alludes to the almost insoluble problem arising from the payment of high procuration commissions.
- E. M. NEUMANN (E). *Values granted upon withdrawal or cessation of premium payments.* Describes surrender and non-forfeiture practices in the U.S.A. These are largely conditioned by State legislation, and consequently to some extent frozen. A table comparing guaranteed surrender values and asset shares is given, showing that the surrender values are in the main adequate. The law is not considered to provide sufficient protection to the companies if they are called upon to pay surrender values on a large scale owing to financial panic.
- A. JUNNILA (E). *Terms to be granted on withdrawal from the contract of Life Assurance.* Conditions governing surrenders in Finland are described. For policies not paid-up, these are calculated according to the Fredholm formula, allowing for the deduction from the reserve of the unliquidated portion of the initial expenses, taken as 35.3% of the sum assured. It is considered that some modification in favour of the assured might be made in the early years of a policy.
- J. LOISEL (F). *The intrinsic life policy value in case of withdrawal.* The withdrawal conditions which are a result of statutory limitations result in certain cases in unfairness or inconsistency. For example, surrender values for very long-term endowment assurances may become less than those for corresponding whole-life policies. Suggestions are made which involve smaller reduction values and larger surrender values.
- N. E. COE and R. H. OWEN (E). *Terms to be granted on withdrawal from the contract of Life Assurance.* The practices in Great Britain of British and of Dominion offices in case of non-renewal are described. The automatic loan non-forfeiture scheme is universally adopted by the Dominion offices but only by a minority of the British offices considered.

- W. E. H. HICKOX and C. E. PUCKRIDGE (E). *The terms to be granted on withdrawal from a contract of Life Assurance*. The paper in *J.I.A.* Vol. LXVII by Hickox is referred to and the following are the main points made. Guaranteed values may be a danger to a company in times of crisis, though an elastic guarantee, dependent on some such criterion as the value of Consols, might be reasonably safe. Unzillmerized reserves are unsuitable as a basis. The prospective method of calculation is the right one and a distinction is made between maximum and minimum values, the former including an item representing the value of future bonuses.
- N. A. HORSLEY and D. L. MURRAY (E). *Terms to be granted on withdrawal from the contract of Life Assurance*. Some of the aspects of the question referred to in the previous paper are again discussed and the results of a questionnaire addressed to 54 offices established in Great Britain are given. The value of the paid-up policy is shown to be the basis for the surrender values of whole-life policies in 14 cases and for those of endowment assurances in 37 cases. A table is given showing the numbers of cases in which offices require the payment of one, two and three years' premiums respectively before admitting liability to pay a surrender value.
- J. ALTENBURGER (G). *Indemnity in case of withdrawal from a Life Insurance contract*. Lays stress on the idea that persons surrendering exercise an anti-selection against the insuring company, in support of which the writer quotes his own early experience, and gives the opinion that such people have no claim on the reserve for future profits.
- E. BIRÓ and M. JACOB (I). *Paid-up policy and surrender values as problems of actuarial practice*. The authors refer to a paper of 1932 and contend that surrender values should be linked with paid-up policy values in such a manner that the interest on a loan on the paid-up policy should not appreciably exceed the premium on a new policy for the difference between the amount of the paid-up policy and the loan. A formula is given embodying this idea and there are examples from the pre-war experience of Poland and Hungary, whose practices conformed with the above principle. It may incidentally be noted that this criterion would have little weight if applied to with-profit policies.
- G. BODONI (I). *Paid-up policy and surrender values and the transformation of a Life Insurance contract*. Treats of the difficulties which may arise if a policy is transformed (as distinct from being converted into a paid-up policy or surrendered). If the calculation is based on the reserve, ignoring the surrender value or the paid-up policy value, there may be inconsistency. The opinion is given that a transformation should be based on the zillmerized reserve.
- P. CROSATO (I). *Influence of additional risks on the theoretical value of a Life Insurance contract*. Here attention is drawn to the fact that an assurance which involves the payment of a constant annual extra premium for a diminishing extra risk may result in a negative extra reserve in the earlier years of the policy. It would obviously be difficult to give practical effect to this phenomenon.
- M. MANFREDI (I). *The value of a Life Insurance in case of transformation*. On the same lines as Bodoni's paper. Transformation should in theory be made on

the basis of full reserves but the zillmerized reserves may have, wholly or partially, to be taken into account in order to avoid inconsistencies. Numerical examples are given.

- R. MASCIOTTI (I). *A fundamental theorem and its applications for the determination of surrender values*. The writer gives a formula according to which, starting with an average age at entry x for a whole portfolio, he arrives at the conclusion that the probability of survival on the books for the collective may be found from the probability of such survival of an individual aged x by multiplying that probability by a constant; the reasoning involves certain assumptions as regards Makeham's constants. This postulate leads to the formula

$$(\beta\pi_{x+t:\overline{n-t}} - \alpha\pi_{x:\overline{n}}) a_{x+t:\overline{n-t}}$$

for the surrender value. Figures in justification of the original thesis are given from the experience of the Industrial Branch of the *Istituto Nazionale delle Assicurazioni*, with $\alpha = .89$ and $\beta = .82$. It is claimed that the formula keeps closer to the true values than the commonly adopted proportional one, except for very short durations. It may be noted that the actuarial expression ${}_tP_x$ is here given a significance which differs from the one usually accepted.

- G. THESEN (G). *Calculation of surrender values in Life Assurance*. Sets out the practical considerations which should govern the determination of surrender values in order so far as possible to avoid leaving the policyholder with a grievance. The proportional rule adopted for endowment assurances and whole-life limited-payment policies is easy to explain but it is considered that a return of a certain proportion of the premiums paid with interest constitutes an even better arrangement.
- A. G. PLOEG (G). *The problem of surrender in Dutch Life Insurance*. Describes the difficulties with which Dutch companies have had to cope. Competition has tended to raise surrender values so high as to cause undue encroachment on profit funds, and to risk an undesirably large number of surrenders in times of financial crisis.
- H. SCHÄRF (G). *Reflections on surrender values of non-participating Life Insurances with special application to the Polish portfolio*. The writer mentions that Insurance law varies in Poland according to whether the region involved was originally Russian, German, or Austrian, the rules for the computation of surrender values being, however, the same in all three districts. He then gives in some detail his own views on the theoretical requirements to which such computations should conform and gives numerical examples on 'model office' lines.
- R. PALMQUIST (F). *Terms granted by Swedish Life Insurance companies on withdrawal from the contract*. Describes the difficulties of Swedish companies in connexion with this question. Initial costs are taken into consideration to the extent of about 3.2% (formerly about 4%). The well-known difficulty of giving satisfaction to the policyholder is mentioned, but the writer, while in favour of liberal surrender values, considers that those who take a paid-up policy should receive treatment which is relatively more favourable than that accorded to those who surrender.
- E. ZWINGGI (G). *The determination of withdrawal values in Life Assurance when the profit reserve is taken into account*. Discusses analytically the question

of allotting a portion of the profit reserve to persons surrendering. This reserve may be considered as the excess of the second order reserve (computed on bonus reserve valuation lines) over the technical valuation reserve.

- K. ŠTASTNÝ* (G). *Terms to be granted in case of withdrawal from a Life Assurance contract.* Two schools of thought prevail, which advocate maximum (even to the point of loss) and minimum surrender values respectively. Conditions in former Czechoslovakia are referred to and a formula is given, the object of which is to bring surrender values into line with paid-up policy values, the latter being, generally speaking, calculated on a more liberal basis than the former.

BASIS AND TECHNIQUE OF SICKNESS INSURANCE

Grütten, the reporter, points out that contributors did not in the main confine themselves to the question submitted for discussion. The theme is, however, a new one, not having been considered at previous Congresses except as a concomitant to Life Assurance. The papers of Tosberg, Rusam, Grünewald and Wogan, deal in the main with questions of definition and classification. Heath, Clarke, Williamson, Graham, Katz, write mainly of the practices of their respective countries. Bassford, Valentinuzzi, give statistics derived from the experiences of their own offices. Lundberg emphasizes the difficulty of dealing with heterogeneous material and illustrates his ideas in mathematical form.

- A. GRÜNEWALD and R. WOGAN* (G). *Bases and structure of Social Sickness Insurance in Germany.* The writers refer to the German experience and class benefits according to five main groups. Statistics are given showing, by five-year groups, the number of cases of sickness involving benefit and the duration of the benefit (in days). Statistics are also given of the relative losses due to certain main groups of diseases. It is considered that statistics by professional groups would be valuable, but no figures on these lines are given.
- F. RUSAM* (G). *Actuarial principles relating to private Sickness Insurance.* The tariffs for private Sickness Insurance in Germany were arrived at by the rough method of ascertaining the average loss per head. Some of the cost of maternity benefit is thrown on the male section. Tables based on the experience of the *Handwerk, Handel, und Gewerbe* company of Dortmund are given. Select tables are advocated.
- A. TOSBERG* (G). *Basis and technique of private Sickness Insurance.* This writer also stresses the difficulties under which private Sickness Insurance labours in Germany. Each company has to make estimates based on an aggregation of the various sickness indemnities at each age, and the lack of tables showing separately the cost of medical attendance, hospital treatment, and surgical fees, etc., renders the application of uniform statutory provisions impracticable. That this difficulty could be overcome, given sufficient data, is unquestionable, and this fact is illustrated by figures drawn from the experience of the *Deutschen Kranken-Versicherungs A.G.*
- H. R. BASSFORD (E). *The basis and technique of Personal Accident and Sickness Insurance.* Statistics based on the experience of the Metropolitan Life Assurance Company from 1921 to 1938 are given. The tables set out particulars of the occupational classification; of the relation of the claims

rate to age and to duration; of the experience by calendar years, reflecting the effects of economic crisis; and of the influence of selection, which in Accident Insurance and in Sickness Insurance shows a diminution in the claims rate as policy duration increases.

- W. J. GRAHAM (E). *The development of sickness and hospitalization plans, private and social, in the United States*. The writer is of opinion that injuries due to accident, the costs of serious illness, and loss of income, can be insured against on a reasonably sound basis, but small illnesses cannot easily be dealt with. For group hospital benefits it is essential to contract on the collective principle; in connexion with this a table of maximum surgical fees is given.
- W. R. WILLIAMSON (E). *Disability Insurance, individual, group and social*. In the main a setting-out of the various types of Sickness, Disability, and Accident Insurance in the U.S.A. It is stated that there may be from ten to fifteen million lives insured under individual disability contracts, and some three million under group disability contracts, but that there is no social insurance protection except what is furnished by Workmen's Compensation laws and some other (small scale) categories.
- A. S. CLARKE (E). *Sickness Insurance practice in Great Britain*. Details are in the first place given of the National Health Insurance scheme. In spite of the fact that the scheme provided benefits formerly offered by the friendly societies, these societies provide supplementary benefits, and have increased their membership. Insurance offices provide both immediate and deferred sickness benefits, generally ceasing at age 60; the policy is on an annual renewal basis terminable by the office. Sickness benefits coupled with Life Insurance have never had the same vogue as they have had in the U.S.A. (whose unfavourable experience was noted) and are generally confined to waiver of premiums in the event of permanent disability.
- E. A. J. HEATH (E). *Basis and technique of Sickness Insurance*. It is pointed out that insurance against sickness and permanent disability has not been considered by previous Congresses, except as a concomitant to Life Assurance. The two main types of benefit, immediate and deferred, are described, as are also methods of valuation and a variety of practical matters. The moral hazard is very great, and makes the profitable transaction of this class of business difficult.
- A. VALENTINUZZI (I). *Illness due to accidents among Italian seamen and fishers insured under the compulsory scheme*. Certain particulars are given of the experience of the Cassa Maritima di Trieste. Foreign sources (it is not mentioned which) had to be used to estimate the frequency of occurrence of sickness or invalidity, and the Italian statistics only deal with the durations of these when once incurred.
- W. W. KATZ (G). *Sickness Insurance*. Deals with the insurance in Holland of medical benefits, the premiums for which are obviously more difficult to assess than those for loss of earnings compensation (little practised in that country). No tables or formulae exist and the premiums have been built up empirically. An analysis of medical costs by age groups is given. It is pointed out that, owing to differences in practice and scales of fees, the experience of one country provides no sure basis for the premium rates of another.

- O. LUNDBERG (E). *On the importance of regrading risk premiums in voluntary Sickness and Accident Insurance*. The author points out the heterogeneity of an average group of persons insured against sickness or accident, and in particular the difficulty of eliminating the bad moral risks (aggravators). A plan providing for the periodical adjustment of premiums, in the light of practical experience, is advocated.

TREATMENT OF WITHDRAWALS IN LIFE ASSURANCE

The subject-matter of this section, which both Germans and Italians designate by the compact word 'Storno', relates to the subject generally, and in particular to its bearing on premiums and reserves: the earlier section in which the word 'withdrawals' appears is confined to the terms offered to persons withdrawing. The question of premiums and reserves is dealt with in two papers only, those of Taucer and Gisi. Stowe gives tables of frequencies derived from Canadian experience and Bonifacio and Sala both give statistics, based on the experiences of various countries.

- H. J. STOWE (E). *Withdrawals in Life Insurance*. Gives statistics for Canada from 1900 to 1937. Withdrawals generally are high and for 1932-35 exceeded new business. Figures showing trends by year of insurance and by district are also given and schemes for encouraging the upkeep of policies described.
- G. BONIFACIO (I). *A statistical enquiry regarding lapses in Life Insurance*. A study of the rate of withdrawal during the first three years of assurance based on an experience of the years 1932-34, arranged by three-monthly periods. The statistics are drawn from different countries, and graphs are given.
- F. SALA (I). *On the preparation of the material serving for the determination of the frequency of lapses in Life Assurance*. An investigation on lines similar to those adopted by Bonifacio. The plan of a table is adumbrated, graded both according to age at entry and according to duration, at three-monthly intervals.
- R. TAUCER (I). *On the question of lapses in Life Insurance*. The theoretical continuous equation postulating the return of a certain proportion of the reserve is given. As withdrawals in Life Assurance depend on the will of the assured, they cannot be allowed for in the calculation of premiums, etc., but may in certain cases be taken into account in calculations connected with pension funds and the like.
- E. GISI (G). *The influence of withdrawals upon tariff premiums, premium reserves and the participating rates of endowment assurances with a share in the profits*. The effect of withdrawals is examined in relation to the bonus system. In Switzerland the method of bonus distribution is not taken into account when computing surrender values. If bonuses are on an increasing basis, a considerable profit may thus accrue to the assurer.

COVER FOR WAR RISK IN LIFE ASSURANCE

The reporter for this subject was Wyss, and what was related or suggested by the contributors is interesting mainly in the light of what has actually happened. Each writer gave his particular views; in the papers of Mattfeld and

Brix, Jagt, Pale, von Geusau, Moser, Wyss, these were accompanied in each case by a reference to the practices of the writer's country; even in Germany these practices were remarkably divergent. Crowe, Hesselberg, wrote from a more general standpoint. Citteri propounded a technical approach to the question: his paper and Crowe's were in fact the only ones containing actuarial formulae.

G. MATTFELD and H. BRIX* (G). *Cover for war risk in Life Assurance*. This paper, written shortly before the outbreak of the present war, starts appropriately with a text from Schiller: *Der Krieg ist schrecklich wie des Himmels Plagen*. German companies had adopted divergent practices in regard to the coverage of war risk and the most interesting part of the paper is a postscript of which the following is a translation. 'The above paper was already completed when the war of the Allies (Poland, France, and England) against Germany broke out in September 1939. Soon after the commencement of hostilities the Government, by circular R 44 of 7th October 1939, annulled the whole of the existing divergent policy war conditions and decreed that all Life Assurance offices working in Germany should provide uniform conditions for the coverage of war risk. The most important decisions are as follows:

'The coverage of war risk will, without any period of deferment, be undertaken for combatants as well as non-combatants. For new business there shall be charged a single extra premium at the inception of the policy, based on the amount assured; war cover is limited to a maximum of R.M. 100,000. For the covering of war risk† a special levy is prescribed, which will involve special retrospective assessments, the details of which will be determined later on. In order to prevent evasion of this liability by policyholders through premature withdrawals, special deductions will be made in the event of surrender.'

A résumé of the practices of twenty other European countries is given.

E. W. CROWE (E). *War risk*. War risks cannot be dealt with on ordinary statistical lines. The main difficulties are set out and various methods of dealing with these risks are described, such as the charging of extra premiums or reductions in benefit.

J. A. JAGT (G). *War risk cover for Life Insurances in Denmark*. Deals with conditions in Denmark and in particular with the recommendations made by a Commission in 1936, which followed closely the lines of the German 'Musterbestimmungen'. These recommendations, which are set out in detail, were put into force in July 1940.

E. PALE (E). *War risk coverage for Life Insurances in Finland*. Gives the recommendations of a Committee appointed in 1938 in Finland. The main interest lies in the suggested policy conditions set out in the appendix. The recommendation on p. 161 regarding secondary losses, by which are meant first and foremost investment losses, runs as follows: 'Secondary losses may during wartime run up to sums impossible to estimate. They may even force the whole company into insolvency. For this reason it is not sufficient to devise a system confined to the cover of war death claims but it should be conceived in such manner as to offer security also against the secondary losses of the company.'

† By implication, on existing policies.

- U. CITTERI (I). *Coverage of war risk in Life Insurance*. War risk is defined as the risk run by all insured; only that arising from extra mortality is discussed. The lines on which the extra mortality might be investigated are adumbrated. Alternative methods of dealing, *a posteriori*, with the loss incurred, are mentioned.
- I. HESSELBERG (G). *Life Assurance and war*. Here, as in the previous paper, the three sources of war loss mentioned in Elderton's *Review* paper of November 1937 are referred to, and investment losses are considered as well as those due to mortality. Companies with large bonus reserves have of course a better chance than those not so well off in this respect to leave contracts unmodified after the war. It is interesting to note that (reasoning from the experience of the 1914-18 war) the writer anticipates considerable compensation for war losses from a rise in the interest rate.
- G. M. M. A. VON GEUSAU (F). *War risk coverage in Life Assurance*. It is considered here that losses due to extra mortality are the only ones which can be dealt with. The definition of war risk coverage, and the various plans for meeting the extra risk, are set out almost as fully in the summary as in the paper itself.
- H. F. MOSER (G). *The transformation of Life Insurance companies into societies of mutual aid in case of war*. The provisions regarding war risk made by the twelve leading Swiss companies prior to the outbreak of the present conflict represented considerable divergencies of practice. In consultation these companies have since agreed upon certain uniform principles. In the event of Switzerland being herself drawn into the war, it is anticipated that Federal sanction would be given to an arrangement which would modify fundamentally the relations of insurer and insured, giving the companies the character of mutual aid societies.
- H. WYSS (G). *The coverage of war risk in Swiss Life Insurance*. This author also writes about the conclusions of the special Committee appointed by the leading Swiss offices to make recommendations for a uniform treatment of war risks. These conclusions, given in full, were confirmed by the offices and the Federal Insurance Ministry. It is presumed here also that in the event of Switzerland being drawn into the war legal effect would be given to the resolutions by the Bundesrat. An important feature is the elimination of distinctions between combatants and non-combatants.

STUDY OF INTERNAL VARIATION IN GROUPS OF PERSONS

This interesting and difficult subject, about which so much has been written in recent years, both in the public press and in technical journals, may be discussed either in relation to a whole population or a more limited collectivity such as that represented by a large Life Assurance portfolio. The comprehensive report by Zwinggi shows that there are three methods of approach to the problem: (1) the purely mathematical method, the basis of which is the function of renewal, and which has not so far progressed beyond the domain of theory; (2) the extrapolation method; (3) the statistical method, based on assumptions as to the future courses of fertility and mortality, the practical application of

which is still hampered by the scantiness of fertility statistics. Method (1) is used by Hadwiger and Wegmüller, Maret, Presburger, Zwinggi; also by Tarjan in connexion with the capital requirements of a new Life Assurance company. Here there is a considerable diversity of notation and nomenclature: the renewal function may be either a number or a rate and may include new entrants only or new entrants less exits other than those by death. Method (3) is exemplified in the papers of Dobbernack and Tietz, Townsend, Keinänen. The latter gives Finnish population statistics.

- W. DOBBERNACK and G. TIETZ* (G). *The development of a totality of persons considered from the point of view of Social Insurance*. The writers mention the population laws of Euler and Verhulst and give a short account of the biological method, the one most likely to achieve results of some value. The applicability of this method to Social Insurance is then discussed. By applying mortality and invalidity rates only and comparing actual with expected exits, an estimate may be made of the lapse rate.
- E. KEINÄNEN (G). *Distribution of the population according to age*. The known difficulty of estimating population trends is referred to. Finnish statistics from 1865 to 1935 are given, excluding, on account of their unreliability, those for ages over 65, and constants are derived according to the method set out by Cramér in the *Skandinavisk Aktuarietidskrift* for 1935; this method involves the use of a hypothetical interest rate and the writer has recomputed this and the constant K according to the Finnish data.
- S. S. TOWNSEND (E). *Some observations on the internal variations in groups of lives assured in Industrial Assurance and in the general population of England and Wales*. Allusion is made to the line of approach which takes the net reproduction rate into account and the analogy between general population age distribution and that of Life Assurance institutions is shown to be closest in the case of Industrial Assurance. The population maximum should be reached sooner than that of Industrial Assurance business. It may be that the conclusions have been somewhat vitiated since the writing of the paper.
- R. TARJAN (G). *Investigations concerning the capital needed to run a Life Insurance business*. On the assumption that the 'Zillmer' allowance for initial expenses is in a new company insufficient, capital must be found to make good the deficiency. The process of capital redemption is symbolized, and a criterion is indicated for the estimation of the length of the 'critical period'. Integral equations are involved.
- M. PRESBURGER (F). *General study of the internal variation in groups of persons*. Equations illustrating changes in the structure of a group are given, and it is shown how a comparison can be made between the number of years spent in the group by 'exits' who have since attained a given age and those spent by the persons of that age remaining in the group. This may be of value in estimating the adequacy of benefits given on withdrawal from pension funds.
- H. HADWIGER and W. WEGMÜLLER (G). *Development and structure of groups of persons*. Here there is an investigation of the special case of a collectivity of which the law of exits is known, and where new entrants all come in at age zero and follow the same law of exits as that of the initial group. A

criterion is given for stability (corresponding to an asymptotic course of the structure function), and also a method for determining the renewal function.

- A. MARET (G). *Direct calculation of the 'functions of process' of an 'open aggregate'*. Allusion is made to a paper submitted to the Ninth International Congress by Moser, who showed that any process function (e.g. rate of invalidity) of an open collectivity kept up to strength by renewal from external sources could be found from the process function of the corresponding closed collectivity and the renewal function of the open one. The writer proceeds to show that for this purpose a knowledge of the renewal and the process functions of the closed collectivity is sufficient. The renewal function is of course a particular type of process function and it is shown that in the case of the closed collectivity the renewal function (necessarily negative if births are not a factor), is the first differential with regard to t of the collectivity, a fact which is incidentally obvious from first principles.
- E. ZWINGGI (G). *Connexions between the technical stability of a Social Insurance fund and the formula of development for the number of insured persons*. Here certain considerations relating to the effect which future entries may have on a Social Insurance fund are elaborated. The normal actuarial procedure is of course to take account only of existing membership but if future entries have a certain trend a latent deficit may result. Different types of development formulae are investigated and it is shown in particular that if the formula is of the so-called increasing logistical type the latent deficit tends to increase.

BASIS OF INDEMNITY TO BE ALLOWED FOR LOSS OR DIMINUTION OF PROFESSIONAL INCOME

One paper only was submitted, reported on by Marchand.

- W. THALMANN (G). *The determination of accident-disability annuities for Social Insurance in Switzerland*. The author is the actuary of the Swiss National Accident Insurance Fund, and gives figures based on the experience of that fund. The criterion is the percentage reduction of earning power. Possibility of recovery is taken into account as well as mortality, and annuities are graded in relation to duration as well as age.

MEMOIRS OUTSIDE THE PROGRAMME

- N. E. ANDERSEN (F). *On the mean duration of a number of arbitrary sums as a function of interest intensity and half invariants of the sum distribution*. The problem is to find by a quick approximate method a mean duration n for a series of discounted sums. The usual linear interpolation is shown to be justified on the assumption that the sums referred to are fairly represented by a Gaussian distribution.
- A. HUNTER (E). *Recent experience on persons with medical impairments*. The paper embodies the main results of the 'Impairment Study—1938' of the American Joint Committee. The pathological conditions dealt with are those involving a history of: (1) syphilis, (2) intermittent pulse, (3) irregular pulse, (4) asthma, (5) appendicitis, (6) albuminuria, (7) casts, (8) nephrectomy, (9) non-toxic goitre.

- R. RISSER and L. MAZOUÉ (F). *Two methods of complementary Disability Insurance*. This is apparently a condensed version of a report made in connexion with the initiation of a pension and disability fund for mining and engineering industries in France. Two types of contract are recommended.
- C. D. RICH (E). *Mortality and variation in health*. A condensed version of the paper by the same author in *J.I.A.* Vol. LXX. There is an appendix embodying suggestions made by Fraser and Joseph.
- G. HUSZÁR (E). *On the result of Baily's formula*. The writer refutes the assertion hitherto made to the effect that Baily's formula for ascertaining the rate of interest pertaining to a temporary annuity, the value and duration of which are known, always gives approximate values in excess of the true rate.
- U. CAROSONE (I). *Observations on the approximate valuation of premiums for insurances with a variable benefit in case of death, and some practical applications*. It is pointed out that in most standard tables C_x does not vary much from ages 25 to 75. This fact may be used to obtain approximate values of increasing or decreasing benefits, such as an assurance covering the balance of a debt which is being reduced by an annuity, guaranteed bonuses (so-called), etc. The basic formula is

$$(v\bar{A})_{\overline{x}|i_0} \approx \phi [a_{\overline{n}|i_0} - a_{\overline{xn}|i_0}],$$

where ϕ is a financial factor depending on the duration of the assurance and the rates of interest used for calculating the premium tariff and the annuity certain respectively.