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Opening Address by the President.

The Nature and History of Actuarial Work as exemplifying the Mode of Development and the Methods of Science. A Presidential Address delivered before the Institute of Actuaries on the 30th of November 1896, *by* T. E. YOUNG, B.A., PRESIDENT.

I. INTRODUCTION.

IN pursuing the traditional practice of introducing myself officially by an Inaugural Address, I desire unaffectedly to state that any conventionality, which this course may have acquired from custom, is merged and lost in a deep feeling of grateful recognition of your generous confidence and goodwill.

With the historic associations which cluster around our famous Institute,—associations admirably blending the prosecution of Scientific Method with Practical Research; both the Experimenta Lucifera and Fructifera of Bacon,—to belong to this Fraternity of Experts as a student should now prove an inspiration; to reach official position and ampler scope of devotion to its interests is a signal distinction; while the attainment of its Headship, by the kindly suffrages of Colleagues, not merely constitutes the "consummate flower" of a genuine ambition, but is also suggestive, at the moment of realisation, of humbling reflections that personal service in the past, however earnestly

intended, had not assumed a more valid proportion to the crowning honour of so high a trust.

Although the limits of a Presidential Address are generously wide, and, indeed, somewhat nebulous, certain obvious considerations assign a reasonable range. The introduction of questions involving a distinctly controversial character should wisely be avoided; and a disquisition upon a definite Professional subject would be reft of its stimulating and instructive form by reason of the conventional absence of subsequent discussion. An Introductory Address, moreover, should, if possible, appeal to a larger and more general sphere of thoughts and sympathies than can usually be discovered within the narrower confines of a technical thesis.

In this embarrassed suspense of motive, the conviction finally dawned that the dignified position of the Institute; the commanding status which the Profession has attained after vigorous years of arduous and fruitful toil; the hopes and interests especially of the younger Members, which must ever exercise a pathetic attractiveness for their senior colleagues, who watch with sympathetic thoughtfulness their nascent career: all these reflections concurrently suggested that I might fitly engage your attention in a rapid survey of the Nature and History of Actuarial Work as exemplifying the Mode of Development and the Methods of Science.

And in explicitly claiming at the outset for our Actuarial studies the title and dignity of Science in the ample meaning of that term, I am misled by no unsubstantial ambition or delusive hope, nor do I employ mere language of generality expressive rather of indefinite assumption than assertive of a formal and completed fact. For I propose to ascertain the conception of the Nature and Methods of Science elaborated by acknowledged Masters* who have deduced the Principles of *Science* from a survey of the rational evolution of the *Sciences* themselves, and to exhibit the essential congruity, with whatever secondary limitations may be requisite, which exists between this established Body of philosophical truths and the growth and form of the investigations with which we ourselves are proud to be associated.

This course will also afford me the opportunity of discussing

* Herschel: Discourse on the Study of Natural Philosophy; Herbert Spencer: Essays, Scientific, Political, and Speculative, Vols. 1 and 3; Whewell: Novum Organon Renovatum; Jevons: The Principles of Science; Mill: A System of Logic, Vols. 1 and 2; and Bain: Logic: Part II, on Induction.

the aptitudes which constitute the Actuarial character; the interchange of influence which unceasingly mediates between the Science created and the creating Mind, as the area of dominion widens; and the appropriate educational training and equipment which the survey may suggest.

II. Common Knowledge and Science.

The Criterion of Scientific Knowledge as distinguished from Common or Uncultivated Knowledge cannot be assigned as the possession of an Organised Body of classified facts, deduced inferences, and anticipated results; nor does it consist in Prevision of consequences based upon observed relations among phenomena; for these features are essentially discernible in each period of Knowledge as the tentative Prelude and the rigorous Sequel of a common Progress. But the culmination of Ordinary Knowledge into Science is attained when its previsions are not merely *qualitative*, or assertive solely of the *kind* of phenomena to be expected under stated conditions, but reach the completing stage of *quantitative* expression, where anticipated consequences are capable of *measurement* in precise numerical form. In this advance, the Modifications in phenomena produced by operating forces,—for all physical Sciences are exclusively concerned with movements in materials effected by incident Force,—may be too irregular and indeterminate to proceed beyond the sphere of empirical and fragmentary deduction, or may finally, by successive essays of approximation, emerge so clear and definite that, in a felicitous moment as Herschel termed it, the detected relations may even suggest the class of mathematical functions which symbolise their bond of connexion, and thus signalise the advent of the epoch of exactness.

And, without anticipating at this point any detailed exposition, I may summarily and justly appeal to our investigations into phenomena,—Mortality Statistics,—under the operation of the Force of Dissolution*, and to our technical results, as adequate evidence that the history of Actuarial study, from its elementary notions derived from games of chance, has developed gradually into preciser modes which, in later days, have received a vast accession of scope and power by application of the Instrumental Principles of the Differential and Integral Calculus.

* Employing this term as distinctive from the technical phase of the "Force of Mortality."

III. THE CLASSIFICATION OF THE SCIENCES.

With this conception of the Nature of Science, we may proceed to a general Scheme of Classification of the Sciences, which will prove sufficient for our purpose without discussion of the theoretical differences which still separate the two philosophical scientists, Mr. Herbert Spencer and Professor Bain.*

The Sciences, then, may be summarily partitioned into—

(i) ABSTRACT SCIENCES,—such as Logic and Mathematics,—which treat of Time and Space or the *Forms* in which phenomena are cognised, whatever be the Metaphysics we profess regarding the nature of these Notions as Forms of Thought or Forms of Things :

(ii) ABSTRACT-CONCRETE SCIENCES, like Mechanics, Physics, and Chemistry, which are concerned with *phenomena* in their *elements*, as they are presented under these Universal Forms, and

(iii) CONCRETE SCIENCES, such as Geology, Biology, and Sociology, whose subject-matter consists of *Aggregates* or Totalities of *phenomena*.

Our own Science is clearly included within the final Department, and we observe that the Principles evolved in the Abstract section, when applied to the phenomena we study, are instrumental in attaining that exactness of expression and mathematical relationship which provide the Scientific form.

Moreover, all Concrete Sciences are concerned both with phenomena and with the manifold Forces which their varying movements embody. Astronomy—to select the exactest Science in this Division—is dominated by the Force of Gravity, whose permanent uniform action is termed a Law. The connotation of this expression is unhappily capable of metaphorical misuse, but the Scientific definition must be remembered that Laws of Nature are not Rules† which phenomena *must* obey, but consist simply of uniformities of co-existence and succession which they have been discovered to display.

In our wildest ambition we cannot range our Science in approximate relationship with a finished system like Astronomy,

* Spencer: The Classification of the Sciences: Essays, Vol. 3.
Bain: Logic: Part I: Deduction: Appendix A.

† Whewell: "Bridgewater" Treatise on Astronomy and General Physics: Introduction, cap. 2.
Spencer: The Study of Sociology, cap. 2.

but I have selected this illustration with a view to showing, by juxtaposition, that the factors in the two regions of research are similar—materials to observe and classify, and a Force whose Law or uniform operation is to be deciphered and interpreted, since every phenomenon is a manifestation of Force under changing modes.* The conception of Force, too, in these compared provinces is precisely identical, for the only possible knowledge of Force is that of Movement in Objective Facts.

The phenomena and forces, again, in the more rigorous systems of Truth, are of a stable character, or subject only to periodic changes which contain implicitly within themselves the re-establishment of prior relations, while our data are modified continually by a combination of Causes, multitudinous, complex, and fluctuating. Consider for a moment how the medical and surgical discoveries of the present generation alone have fundamentally affected our materials. And this example is illustrative further of that interaction of development and expression which is continuously mediating between the different spheres of Scientific Truth, in evidence of which may be cited the significant mode in which Optical discoveries have proved ancillary to Astronomy. The Antiseptic Method of Lister is undoubtedly one of the most momentous discoveries of any age. Many of the accessories which accompanied its introduction have naturally disappeared, but its essential character is permanent in its recognition of the external origination of disease and in the creation of an artificial atmosphere or environment which these exterior pathogenic organisms are incompetent to penetrate. So memorable and distinctive a revolution has thus been effected that it is difficult to express a statistical comparison with the prior state of Surgery with numerical precision. Amputations alone are available as the term of contrast; and here, previous to 1875, a mortality of 22 per-cent was considered to be extremely favourable, while, since the adoption of Antiseptic precautions, the percentage in similar cases is only 2·8. For the signalising character of that method consists in the circumstance that operations of most vital and capital nature; thoracic and abdominal explorations; thrombosis of the cerebral sinuses; forms of intestinal obstruction; and direct treatment of the kidneys, were enterprises which, anterior to Lister's epoch, no professional daring, skill, or

* If indeed Science should not hereafter regard the Universe as simply Matter under changing forms or purely Force in varying modes.

sagacity even attempted to perform. And, parenthetically interposing a remark in relation to Annuitants, the result of my enquiries definitely shows that, independent of the improvement of general conditions affecting longevity, the successful practice of Ovariotomy on Antiseptic principles is becoming largely accountable for the prolonged existence of Annuitants who are chiefly recruited from the class of Females. In this operation, the death-rate, which, prior to 1878, varied from 65 to 87 per-cent, has now been reduced within a range of 7 to 10 per-cent.*

But in what direction are our data thus affected as compared with the practically changeless nature of the materials of the completed Science I have chosen as a distinctive superior? Not in the Methods employed for determining quantitative ratios; but in the modes of incidence of the associated Force; and the nicer discrimination of the degree of its intensity of action at various stages, with the resulting practical application to systems of graduation, rates of premium, and valuations.

And having, for the purpose of vivid comparison, adduced the elaborated Science of Astronomy, it is agreeable to notice, on the other hand, that in relation to other branches included within the Concrete circle, our own investigations present an admirable superiority. In Biology, Political Economy, and Sociology, many valid uniformities have been traced, but it is evident, on examination, that those departments of specialised knowledge do not display that general character of exactness which distinguishes our Science. Dr. Whewell† and Professor Jevons‡ have ingeniously striven,—the former, to illustrate, the latter,

* The statistical information above presented, expresses the experience of one of our principal London Hospitals, and may be accepted, I am assured, as typical of that prevailing in the Metropolitan Hospitals generally.

The subject is so impressive in relation to our data, that a brief reference to the disease of Pulmonary Consumption should not be omitted. The view formerly adopted regarded Phthisis Pulmonalis as essentially originating within the system, though various elements contributed a predisposing force; but the discovery by Dr. Koch and others of the existence of vegetable organisms or bacilli in tubercular growths has materially modified the ancient teaching. With the doctrine of the dependence of Tuberculosis upon vital organisms, which must of necessity be introduced within the system from external sources, attention to exterior agencies may now be deemed to be the effective instrument for protection or mitigation. We are thus confronted with the two factors of (i) a possible constitutional predisposition, or (to speak figuratively) a possible provision of appropriate pabulum for these pathogenic organisms, and (ii) the possible prevention, even where such a diathesis exists, of the intrusion of bacilli by suitable conditions of isolation and physical environment.

† Whewell: Cambridge Philosophical Transactions for 1829, 1831, and 1850.

‡ Jevons: The Theory of Political Economy.

systematically to prove,—that Political Economy, for example, is capable of deduction and extension by the processes of Algebra and the Calculus, but the more temperate view of Professor Cairns * in opposition to this doctrine seems to be conclusive. The essential materials are analogous in each of the two ranges of observation and thought, but while in our work the adoption of mathematical analysis appears to be naturally congruent, the mathematical relations so employed in Political Economy exhibit merely a graphic form of symbolic illustration of truths elsewhere elicited, void of the capacity of expansion into an organon of inference and research.

In proceeding to more detailed exposition, I merely deal, statistically and historically, with the common elements of our Professional study; and the only novelty I claim is that of so massing and presenting the materials in connexion with the fundamental processes which the development of Science involves as to exhibit, by their grouping and succession, the conformity of our procedure with genuine Scientific Method. With a view to forming an independent judgment, I have found it necessary to peruse once more the original Treatises of our great predecessors,—a laborious enterprise, it is true, but one which has been enriched with ample compensation in the freshness of thought and ingenuity of resource which distinguish the pages of De Moivre, Simpson, Milne, Gompertz, and Davies.

IV. The Basis of Science.

In the multitudinous variety of the phenomena which Nature presents, with the diverse uniformities of the embodied forces, a primary process is termed the Decomposition or Analysis of Facts with a view to their Classification into Elementary Data which shall possess the characteristics of reality, limitation, clearness of apprehension, and definiteness of ascertainment. This initial step has necessarily preceded the discovery of the Laws of Nature, hitherto revealed. A fundamental distinction, as I have observed, is at once apparent in the circumstance that, in the physical Sciences, the substances and forces constitute stable combinations and fixed and determinate relations, while, in our materials of mortality, we possess the entanglement of these external factors with the phenomena of human beings exhibiting determining features of their own: a complex intermixture of the distinguishing

* Cairns: The Character and Logical Method of Political Economy.

facts of Human Nature with the elements of General Nature itself. Even were the complete materials and their uniformities fully displayed to our gaze, the definite goal of exact expression would yet be infinitely distant, since the complicated problem would still appeal to a restricted intellect. For, unhappily, the development of knowledge, and the consequent specialisation which it compels, has not been accompanied by a concurrent enlargement of our powers of insight and causal combination.

Observations of facts, again, as every Science reveals, are indissolubly interwoven with the investigator's personal judgments, presumptions, and almost unconscious interpretations, which require, as far as possible, to be detected and eliminated. This natural confounding of inference with observation is conspicuously evident, to employ a passing example, in the common causal induction between superior longevity and married life. Milne* discussed the problem briefly and inadequately in 1815, but it was reserved to Mr. Herbert Spencer† to prove decisively that the deduction drawn from mortality observations of the Married and Single was capable, on the ground of this psychological limitation, of quite a different interpretation.

These Elementary Facts, again, besides being true and definite, must be *appropriate* for the purposes of the specific enquiries which the Science pursues.

But in the Collection and Classification of facts, as well as their *Colligation*, to adopt Whewell's‡ term, the observer, if a stable foundation is to be secured, must be guided by certain appropriate Mental Conceptions, derived, as John Stuart Mill§ has remarked, either from the groups of Intellectual Notions collected in other Sciences, or obtained by the more arduous and delicate process of eduction from the very observations which they are intended subsequently to connect. It would be futile in Chemical problems to apply merely the conception of Mechanical Force, just as research in Physiology would be nugatory were the notions of Mechanics and Chemistry alone to be employed. In our own Mortality investigations, to anticipate for a moment, it is clear that the conception of Vital Powers is the fitting basis of union; and though, at first sight, this proposition presents the appearance of the ancient fallacy of Circulus

* A Treatise on the Valuation of Annuities and Assurances; Vol. 2; Appendix No. 5.

† The Study of Sociology, cap. 5.

‡ Whewell: Novum Organon Renovatum, cap. 4.

§ Mill: A System of Logic, Vol. I, Lib. 3, cap. 2.

in Definiendo, it serves, at all events, the purpose of *defining* the region of Intellectual Notions to which we must appeal, and has been utilised,—baselessly, I deeply regret to feel,—in the hypothesis of Gompertz.

These appropriate scientific conceptions of Analysis and Colligation are generally due, in the first instance, to the native sagacity and tested skill of the investigator, and become clearer and more capable of definite service with the concurrent advance of specialised Intellectual Education, and the clarifying and corrective discipline of Scientific Discussion.

With all these difficulties confronting us, the history of Actuarial work has exhibited a congenial assimilation with established science.

Our British investigator, Dr. Edmund Halley,* expounded for the first time in 1693 the necessity of the Conception of Age, in the preparation of the Breslau Table as a classification of facts. In the compilation of the Bills of Mortality in this Country,—commenced as a late sequel to the General Visitation of Religious Establishments in 1538, and fitfully continued until their formal organisation into the General Register Office of 1836,—the ages were omitted, and it was not until John Smart,† with similar insight, enlarged upon the defect in 1726‡ that, in 1727-1728, this element was included.

The illustrious Grand Pensionary, Johan de Witt,§ again, presented his Report on Annuity calculations to the States-General of Holland and West Friesland, at the earlier date of 1671, in which the results were based upon the ages recorded. Thus, at the earliest period of our systematic history, this governing conception of Age in the tabulation and employment of Statistics was distinctly recognised in consonance with approved Scientific Method.

The second indispensable scientific conception,—that of the *Appropriateness* of the data for the specific investigation,—was also possessed by Halley, though in a confused form. He justly condemned the London records as an unsuitable index of Metropolitan Mortality in consequence of the deranging effect produced

* Philosophical Transactions of the Royal Society: reprinted in the Journal of the Institute of Actuaries, Vol. xviii, pp. 251 and 262.

† Smart: Tables of Interest and Annuities.

‡ In connexion with the employment of these observations for Mortality deductions.

§ Journal of the Institute of Actuaries: Vol. ii, pp. 121, 222: Vol. iii, p. 93. [Vide Note on p. 121.]

by incessant immigration from the Provinces under the pressure of prevailing social and economic conditions of industrial life.* The Breslau observations appeared to him, for specific reasons he assigned, to constitute a more adequate standard, and though his conception of appropriateness was thus vague and loose,—necessarily limited, as it was, by the materials at his command,—we must admire the scientific insight and disciplined ingenuity which he displayed at this great historical epoch of our Science.

A similar recognition of the necessity of Appropriateness was exhibited by de Witt, since it is clear that, for the purpose of determining the values of Life Annuities, he collected his statistics, not from the experience of the general population but, from the Registers of Annuities granted by the States of Holland and West Friesland.

It has proved a most happy stimulus in the rapid and stable progress of our work,—thus avoiding merely tentative principles of which subsequent experience would require the abandonment,—that these two distinguished precursors,—Halley and de Witt,—were possessed,—by mental constitution and facile grasp of mathematical discrimination and scientific analogies,—of the primary conceptions of Age and Appropriateness of data in constructing the foundation of Actuarial knowledge.

A partial illustration of the conception of Suitability is evident also in the formation of the Northampton and Carlisle Tables.

The genuine course was more fully pursued by Dr. Farr in 1843, 1853, and 1864, in the preparation of Tables appropriately expressive of the general mortality of this country; while, fitly consummating the scientific process, the splendid investigation of the Institute of Actuaries now in progress, worthily sequent to its similar enterprise in 1863, should be honoured as culminating the search for suitability of Experience as a basis of deduction in our sphere of work,—the finest example in our annals of the Conception of Appropriateness of data for specific prevision, and competent in this respect to rank with the most notable achievements of corresponding Scientific procedure.

I might also briefly specify the Decomposition of facts in the distinction between the mortality experience of Male and Female lives,—apparently first attempted by De Parcieux in 1746,—nor should the important resulting detection be omitted of the

* This consideration was especially significant in Halley's time, when the population of London, as we may infer from Macaulay's History of England, only approximated to about 500,000 persons.

exceptional rate of mortality at early adult ages in the case of Males,* a valuable addition to our knowledge which composite Tables could not have revealed; and finally I allude to the most significant results,—destined, I conceive, to affect appreciably our practical judgments,—deduced from the analysis and construction of materials under the guiding conception of *Selection*.†

V. Hypotheses, and the Laws of Phenomena.

The object of all researches into Nature, and the power that stimulates and directs the enlarging scope of mental conception,—the more rigorous and competent colligation of phenomena,—is the discovery of those Laws or Uniformities of operation which the forces inherent (so to speak) in the materials actually present.

Every scientific advance has been signalised and completed by the ascertainment of the permanent relations latent in phenomena; by the successive subsumption of narrower generalisations under inductions of deeper and wider import; and by the consummate stage of expressing these uniformities within the compact compass of mathematical functions as the advent of the quantitative era of research. For although all physical inductions and deductions involve a purely empirical and approximate character, the phenomena of Nature and the laws they display are exact and minutely regular; the mathematics we have discovered and generalised are implicitly and punctually embodied in the varied Universe; the phenomena of the Heavens and the Earth are but objective symbols of mathematical truth; and the discrepancy between these august exhibitions of mental power and symmetry, and the feeble picture we alone are competent to pourtray, expresses simply the presentation of an infinitely complex problem to the finitely constituted mind.

In the extraction of Laws from phenomena, we may either *tentatively and directly assume some apparent order which the* facts ostensibly present, and by successive simplifications and modifications endeavour to decipher the Law, or this anticipation

* Journal of the Institute of Actuaries, Vol. xiv, p. 247.

† Journal of the Institute, Vol. i, p. 179; Vol. xv, p. 328; Vol. xx, p. 95; Vol. xxi, p. 229; Vol. xxii, p. 391.

In the Tabulation of the numbers collected, and the deduction of the ratios involved, I might include the Conception of the appropriate mode of operation. The Tables of Halley, Simpson, and Price were based upon the deaths, and the accurate method was first fully pursued in Milne's investigations where the deaths occurring at each age are brought into relation with the corresponding numbers living.

of the uniformity may be guided by some prior Hypothesis* or supposition,—some general conception derived from the accumulations of cognate branches of Science. In this operation, the slightest suggestion of symmetry, the faintest indication of order, the most fugitive hint of definite progression, may prove the acceptable supposition by which to colligate the facts. Indeed, as John Stuart Mill† has observed, the limits of hypothetical assumption in pursuing the quest into natural regularities are simply coincident with the limits of the human imagination itself, provided the hypothesis is harmonious with ascertained Laws of Nature, and admits of affirmation or negation by subsequent appeal to Experience. The dictum of Lord Bacon‡ against the Anticipation of Nature has long been exploded as an authentic Rule in Science; and the history of every Science forms a perpetual and consistent demonstration that this process of Anticipation, with its sequent Verification, has proved the impressive and opulent source whence the large generalisations and imposing accumulations of exact knowledge have been added to the intellectual heritage of the Race. Newton's "Hypotheses non fingo",§ again, only applied to suppositions that rested on no probable basis and violated the system of established truths, for the Law of Gravitation was itself the grandest hypothesis which the genius of man has yet devised, although transcending the grasp of human conception through the absolute indifference of its Force to interposing bodies. The Undulatory Theory of Light, again, shows the deepest perplexities in the way of harmony of conception: the conjunction of an infinite mobility or elasticity in the Ether with a vaster solidity than Steel! Yet it possesses the property of a Vera Causa, for not only are the phenomena of Light, Heat, and Radiant Energy explicable on its assumption in analogous congruence with the phenomena of Sound, but Clerk

* Hypotheses or Suppositions are the mental conceptions which, under the tendency of the mind towards generalisation and unity, are provisionally "placed beneath" (as the term etymologically signifies), the apparently disconnected facts as their rational support and explanation. And although scientific language is not precisely determinate upon the point, we may legitimately affirm that an hypothesis, which has received adequate confirmation by recognised scientific tests, may, at that supreme moment, be designated a *Theory*, or an original fact of Nature which the Speculator or Ideal Spectator (for this etymological implication the term "Theory" involves) would, if gifted with commanding power of mental vision, directly observe as an integral constituent of the Physical Scheme.

† Mill: A System of Logic, Vol. 2, Lib. 3, cap. 14.

‡ Bacon: Novum Organum: Lib. 1, Aph. 26.

§ Newton: Principia, Lib. 3.

Maxwell has adduced valid reason for surmise that the complicated manifestations of Electricity and Magnetism constitute but stresses and motions in this marvellous adamantine medium.

An essential condition of a serviceable hypothesis as a probable representation of Nature consists in the possibility of comparison between its calculated consequences and the phenomena actually observed. It was on this ground that the hypothesis of Vortices of Descartes as an explanatory exposition of Celestial Mechanics failed; the conditions of verification could not be rigidly applied: and, in Optics, the rival pretensions of the Undulatory and Corpuscular Hypotheses were definitely decided by the Experimentum Crucis of the relative velocity of light in denser and rarer media.

And a final attribute of a genuine hypothesis resides in the circumstance that, as it reaches its ultimate shape, a deeper and wider simplicity of form is presented, with a consequent facility and extension of application.

In our own sphere a vigorous and persistent search has been instituted to discover the Law of Mortality supposed to be decipherable from our observations. This adventure appears to have started in what Comte described as the theological stage, or that primitive mode of conception, which, in his judgment, constituted the origination of Science. The mystic number 7, derived from Chaldean usage, was invoked in explanation, and septennial periods of life were assumed to be naturally expressive. In an Act of Parliament passed in 1540, it was accepted on that basis, that a single life was equivalent to a lease of 7 years, two lives to a lease of 14 years, and three to a lease of 21 years, thus creating the stereotyped restriction of leases to 7, 14, and 21 years. Even after the publication of Halley's researches in 1693, Acts were passed in 1694 and in 1703, in which the ancient numerical relationship (somewhat modified) was adopted in the assessment of pecuniary values.

We reach, in 1671, the important epoch of de Witt, who assumed that the probability of death continued uniform during various consecutive stages of life, but in each period presented a definite numerical proportion of increase, varying with the interval, to the initial ratio. From researches which have recently been made into the primitive history of Life Contingencies in Holland, I should imagine that these results were found to be approximately presented by the series of numbers living without the introduction of any directing anticipation. It is interesting

to notice that, in the correspondence between de Witt and the scientific Burgomaster, Hudde, in 1671, a provisional supposition is mentioned that, from an early age, the decrements of life are so constituted that, out of 80 young persons alive, about one dies annually until the whole body is extinct.

This anticipation of De Moivre* naturally introduces the hypothesis of the latter, published in 1825. It may be preliminarily remarked that, however incompetent the attempt, De Moivre possessed the merit of treating in reasoned detail of a general uniformity in the natural sequences of the numbers recorded from age to age; while, at the same time, he displayed a sound appreciation of the necessity of verification from various sources, and of the need of congruity between any Mortality law and the character of Natural uniformities already established. His supposition was avowedly designed as the tentative basis of a facile method for computing the values of Life Annuities. With the tact of a mathematician, he at once differenced the numbers contained in Halley's Table, and noting the approximate uniformity of decrements, he assumed, on general and special considerations, that the limiting age might be fixed at 86, with the involved hypothesis which enshrines his name. But the hypothesis was a mere verbal enunciation of the disclosed series of differences; it was extracted from a limited and defectively constituted Table; it possessed, therefore, no natural generality of expression; and its sole recommendation, beyond its aid to calculation, appears to have consisted in its specious aspect of simplicity in consonance with crude notions of physical truth.

The genuinely ambitious attempt to obtain a Law of phenomena which our annals record, and the one which is precisely in harmony with scientific method, is the hypothesis of Gompertz. And had the principle which guided him in his selection of a formula been an authentic induction from physiological data, or countenanced by physiological research, that hypothesis would have justly ranked among the most admirable achievements of Science, having regard to the intrinsic importance of the problem and the manifold complexity of the facts.

Dealing generally, in his preliminary Memoir of 1820,† with the development of functions which decrease with increments of time, he made an impressive advance in his famous Memoir of 1825† with the enunciation, conceived in the true philosophic

* De Moivre: Annuities upon Lives.

† Gompertz: Philosophical Transactions of the Royal Society.

spirit, of the possible co-existence of fortuitous and continuous forces in determining the reduction of vital power. Proceeding at first on the basis of the latter supposition alone, he obtained his celebrated formula expressive of the intensity of mortality, with the deductive formula for the number living at any given age.

Mr. Jellicoe,* as far as I can ascertain, was the first writer to approve the formula on the distinctive ground of its philosophic character, and subsequent authors,† as Gray, De Morgan, Makeham, and Woolhouse, have assigned to it a validity and natural supremacy by reason of the physiological principle on which it was professedly founded. For Gompertz proceeded in the rigorous manner which all eminent discoveries of physical laws disclose. In almost every supreme instance in the Sciences, the resulting mathematical formula, which expressed the bond of connexion between the observed facts, or their representative numbers, has not been derived empirically from the observations themselves, but has been suggested primarily by some appropriate hypothesis respecting the probable nature of the connexion itself. A remarkable example is afforded by the explanation of the Fringes of Shadows in Optics, which defied even the marvellous sagacity and intuitive vision of Newton, who confined himself to direct examination of the phenomena alone, and it was only when the problem was approached under the guidance of the Principle of the Interference of Undulations, by Fresnel, Young, and others, that the hidden law became revealed. It will be remembered, too, that Darwin was directed to Natural Selection and the Survival of the Fittest in Natural History by means of the Theory of Population of Malthus. In a similar scientific spirit, Gompertz pursued the legitimate course of examining the series of numbers living in the light of his assumed principle of the mode and measure of reduction of power in the physical frame. But the principle of selection must be a genuine physical induction as well as appropriate in its nature to the observations; and unless that condition be satisfied its introduction is futile and delusive so far as any natural authority is thereby conferred upon the dependent formula. The mathematical function will only bear the authentic symbol of physical

* Journal of the Institute of Actuaries: Vol. iv, p. 199.

† Journal of the Institute: Vol. vii, p. 121; Vol. viii, p. 181; Vol. xiii, p. 325; Vol. xv, p. 389.

truth when the hypothesis by which it is imprinted constitutes a valid fact of Nature. Now Gompertz's physical assumption is clearly appropriate to the enquiry; but, unhappily for any natural distinction with which it might endow the formula, it is a purely unfounded speculation which no physiological generalisation, no record of physiological investigation, no acknowledged scientist, past or living, either indicates or confirms. To express a principle in the language of physiology is essentially different from the announcement of physiological truth.

The formula of Gompertz further failed to satisfy the requisite conditions of scientific verification: the disruption of continuity through change of constants marked its want of harmony with Nature; and his subsequent introduction of additional constants* merely burdened the function with cumbrous elements, like the successive accumulation of appendages to the hypothetical mechanism of Epicycles and Eccentrics, with consequent remoteness from the severe simplicity of scientific truth.

I need only refer to Makeham's admirable generalisation† by the inclusion of a function representing the operation of the accidental causes, which expanded the range of the formula while preserving its symmetric form, since the inherent defect in the primal assumption obviously attaches to all extensions of the expression.

Gompertz, though postulating accidental causes of disease, clearly did not anticipate consciously the recent teachings of Bacteriology; but it is nevertheless a striking circumstance that his happy and sagacious divination is coincident with the modern discovery of extraneous forces or vital organisms with their character of fortuitous incidence.

A definite Law of Mortality then would seem to be as shadowy as a dream; and the advent of the happy discoverer,—combining the powers of the Physiologist, the Physicist, and Mathematician,—to form but a baseless hope.

I fear I have lingered out of due proportion upon this fascinating aspect of scientific work, more especially as our Science has unfortunately failed to realise this object of its quest. The high intellectual charm of the subject must win forgiveness, and to this claim I add my genuine admiration of the scientific

* Journal of the Institute of Actuaries: Vol. xvi, p. 329.

† Journal of the Institute: Vol. viii, p. 301; Vol. xxviii, pp. 152, 185, and 316.

power and philosophic spirit displayed by Gompertz in his supremely able attempt.*

Comte justly insisted that power of Prediction constituted an essential attribute of a valid hypothesis; and the history of Science is richly emblazoned with these felicitous and skilful scientific guesses. Astronomy is laden with memorable examples, of which the subtle mathematical vision of Neptune is not the least; Physical Optics includes the marvellous predictions of Fresnel and Sir W. R. Hamilton in the complicated phenomena of double and conical refraction; in General Physics, Professor James Thomson's prevision of the depression of the melting point of ice under pressure has been finely confirmed; and finally we learn that Clerk Maxwell's famous prediction of the constitution of Saturn's rings has recently been spectroscopically verified.† Had Gompertz's formula rightly expressed the facts of mortality, this scientific requirement would have been excellently exemplified by inclusion of the rule which De Morgan‡ termed the Law of Uniform Seniority.

Beyond the general prediction which a Table of Mortality involves, I can only recall at the moment a prevision uttered by Milne§ in 1815, that a heavier rate of Mortality would be found prevailing in the Upper Classes as compared with the middle Classes of Society, with which he united the hope that specific investigations would be pursued. The memorable researches into the Mortality of Peerage Families in 1861‖ fulfilled that hope, but at the same stroke finally shattered the prophetic anticipation.

VI.—The Instruments of Science.

The distinctive epochs in Science have generally been marked, and frequently inaugurated, by the invention of some Instrument of Research and Deduction,—the introduction of Instantiæ Radii,

* Reference might also be made to an investigation (Journal of the Institute of Actuaries: Vol. xvii, p. 56) instituted into the law supposed to be exhibited in the ages at which Assurances are effected. Even if this uniformity were established, it would prove to be simply of an empirical character implying no adequate ground for extension beyond the individual or composite experience in which it might be observed.

† By Professor Keeler.

‡ Journal of the Institute of Actuaries: Vol. viii, p. 181.

§ A Treatise on the Valuation of Annuities and Assurances: Introduction, p. 51.

‖ Journal of the Institute: Vol. ix, p. 305.

to employ the picturesque phraseology of Bacon.* In the language of Sir Humphrey Davy: "Nothing tends so much "to the advancement of knowledge as the application of a new "Instrument."

The establishment of the Science of Thermotics may almost be said to have been effected by the invention of the Thermometer; in Chemistry, the nice and delicate adaptation of the Balance inaugurated an era of fruitful discovery; the application of the Microscope has supplied the key to many Biological secrets by its revelation of primitive life-histories in Embryology; while, in more modern times, the significant results obtained from the perfection and skilful use of the Spectroscope have immeasurably widened our knowledge of cosmical spaces, and gathered into an imposing unity of structure the myriad spheres and systems of the Skies. I have selected in our own Science four illustrative Instruments which appear to be deserving of special distinction.

(i) The first to be recorded is the Life Table itself; and it has proved of happiest augury in our history that, from the commencement of research, this instrument was devised in so masterly a form as to facilitate investigation and deduction in the completest mode. Halley's Table† has since been modified only,—so adequate was its construction,—in the separate expression of the decrements of life, first furnished explicitly, but in an inconvenient manner, by Thomas Simpson‡ in 1742.

De Witt did not produce a Table of Mortality, but his calculation of the Value of a Life Annuity as a type of procedure presumes a similar basis.

The honour of origination is distinctly to be accorded to Halley.

(ii) The Columnar or Commutation Method may be honourably specified as a second Instrument. A careful comparison of authorities, and actual computations, definitely assign priority of conception to William Dale,§ who, in 1772, clearly explained the

* Novum Organum: Lib. ii: Aph. 39.

† It is curious to notice that the interpretation of this Table has not proved generally evident. Montucla conceived that the number (1000) placed against "age curt. 1" expressed the births: Daniel Bernouilli understood that the number of infants born was not furnished, and that the 1000 were supposed to attain the age of 1: he accordingly estimated the radix of the Table to be 1300; while Farren considered that the 1000 represented the number of children who were aged one year.

‡ The Doctrine of Annuities and Reversions: containing a Table deduced from the London Bills of Mortality.

§ Journal of the Institute of Actuaries; Vol. i, p. 15*: "Calculations deduced from First Principles."

principle of the method and furnished correct examples of its use. Had he applied his process from the earliest age instead of age 50 (for his calculations were intended for employment by Pension Societies whose grants commenced at the latter epoch), or had he multiplied his actual figures by a constant factor of interest determined by the initial age he selected, the contents of his columns would have been obtained in the precise form to which we are now accustomed.

Professor Johan Nicolaus Tetens,* of Kiel and Copenhagen, independently devised the same conception in 1786, for, recalculating his results, it is obvious that his column C_x is exactly the same as our column D_x, while his column E_x is precisely similar to the modern N_x, but expressed in the form of N_{x-1}.

Barrett's arrangement,† absolutely independent of that of Tetens, as the process of Tetens was unsuggested by Dale's—was published in 1813, although conceived in 1786; and the method received its final extension by Griffith Davies‡ in 1825. It is constantly occurring in Science that when any Preludial stage of knowledge is verging upon its Historic Epoch, to employ the happy language of Whewell, the magic word of discovery is ready to be uttered by many a well-trained voice; and the history of Science acquires much of its human interest and attractiveness from the simultaneous or promptly successive unison of these disciplined inspirations of mind.

(iii) The third instrument I select is the Continuous Method of Annuities and Assurances.§

Surveying a Table of Mortality in its customary form of mere numerical expression, we gain no clear conception of the full meaning it implies; but project the numbers successively living in the form of a Curve, and at once the suggestion arises of a gradual progression in time. Under the guidance of this conception, we contemplate the race of Man, or an Aggregate of Men, as a Quantity or Fluent, to adopt the language of Newton's Calculus, flowing continuously through time, of which the Fluxion or Differential Co-efficient is the Rate of Flow. Hence, naturally and spontaneously, we perceive, as in an objective spectacle, how minutely the Higher Calculus is adapted to the interpretation;

* Journal of the Institute of Actuaries: Vol. i, p. 2.
† Journal of the Institute, Vol. i, p. 1; Vol. iv, p. 185.
‡ Tables of Life Contingencies.
§ Journal of the Institute: Vol. xv, p. 96.

and when, as employed in our calculations, the factor of interest exhibits the same continuous character, the perfect congruity of this method is not merely observed as a felicitous union of Principle with appropriate Materials, but its very natural assimilation to our work, if I may so express it, affords a sure prophecy of its extended and fruitful use.

(iv) I confine myself to the selection of one other Method or Instrument, which, I think, I may justly include. This is presented under the form of what has been variously described (in somewhat unhappy language) as the Model, or Average, or Representative Office for the exhibition of Valuation results.* I conceive that a complete change was produced in our reasonings, or at least in their general and effective shape, when, from isolated comparison of individual Policy-Values, at different ages and for various durations, we passed to the assemblage of values which the Representative Table displayed, and which alone are practically serviceable in forming an adequate conception of the aggregate effect of various Mortality Tables and Rates of Interest.

VII. The Methods Employed in the Formation of Science.

All methods adopted in the construction of Science are allied in the essential feature of facilitating the detection of the Order of Progression or the Modes of Relation, of which phenomena are the material signs, and their completed aim consists in the discovery of a mathematical function from which results in agreement with the facts of observation may be deduced.

Adapting the technical processes of physical Science, the successive procedure in our investigations, if we pursue a mathematical law, comprises the selection of the Independent Variable, the Construction of a Formula of Expression, and the Determination of the Co-efficients it involves. And in the whole compass of scientific enquiries with which my very restricted knowledge is cognisant, the palmary researches in 1817 of the French physicists, Dulong and Petit,† into the relation between a series of temperatures and the velocity of the cooling of bodies, present, in principle, very curious analogies with the processes we must

* Journal of the Institute of Actuaries: Vol. xiv, p. 249.

† Whewell: Nov. Org. Renov.: Lib. 3, cap. vi.
Balfour Stewart: A Treatise on Heat: Lib. ii, cap. 4.

adopt, and appear to furnish an instructive type of operation if its object be the ascertainment of a Law. The Independent Variable of the temperature was made to increase in arithmetical progression while, for each change, the quantity of the variant,—the rate of cooling—was observed. Sagacity and experiment, aided by mathematical artifices, then tentatively investigated an algebraic formula of an exponential character, by which the observed relations became congruously expressible. These elements correspond in our case to the Independent Variable of the Age, and the varying numbers of the living or dying. It is true that, in accordance with the established course in Science, Dulong and Petit were guided in their provisional selection of a formula by the physical hypothesis of Prévost respecting the mutual Exchanges of Radiant Heat between contiguous bodies, and hence the mathematical equation they obtained was not merely indicative of a uniformity of Nature, based upon a valid prior induction, but served the ancillary purpose of all true Theories in further establishing and expanding the principle on which it proceeded. In our labours we must be content, so far as we pursue this method, with adopting these mathematical operations upon numbers, since we are devoid of the assistance of any physiological principle of direction which, had it been genuine, would have proved the counterpart of Prévost's Theory of Exchanges.

Without describing the entire series of methods, I select the two which, in Science, have been termed the method of Means and the method of Curves.

The method of Means has been extensively employed in Science, and in the great Report in 1829 of our first President,*—whose honoured name is happily and worthily still continued in our ranks,—we possess an instructive example. In the various modifications which have since been attempted, an important advance consists in the introduction of a completer scheme for utilising the "Weights" of observations.† And certainly, to my mind, no more admirable adaptation of this method has ever been essayed than the system which Woolhouse‡ devised as an instrument of graduation. Notwithstanding its limitation of application to our work, it ranks in our history as a remarkable specimen of mathematical grasp and ingenuity, worthy of being

* J. Finlaison: On the Evidence and Elementary Facts on which the Tables of Life Annuities are founded.

† Journal of the Institute of Actuaries: Vol. xxii, p. 24.

‡ Journal of the Institute: Vol. xv, p. 389.

included in the impressive collection of serviceable processes which the rich accumulations of Science contain.

A further scheme appears likely to prove the most prophetic of success in our approximate efforts. For the relations between the age and the numbers living or dying may be projected into a Curve,* when, by this aid of space-representation, the uniformities seem to spring at once into evident relief, and reveal or suggest, as in a picture, the ideal and general regularity, of which the numbers individually are the fragmentary signs, however interrupted by minor breaks and tortuosities which the errors of observation and collection may have introduced. The eye,—our most intellectual sensual organ, as Bain has remarked,—trained by experience and fashioned by acquired skill into a fineness of perception of symmetry, deciphers, from the flow of the line, the uniformity of which it is in search; and, not infrequently, the disciplined organ of mind, following, in this united venture, the perception of the eye, detects, as the flexibilities of the Curve are pursued, the class of mathematical functions to which the regularity appertains, and thus compresses the pictured scene within the framework of symbolic representation. At all events, if a mathematical formula be not secured, the adjusted flow of the Curve, appropriately modified and adequately tested, will exhibit, with sufficient approximation, the natural progression which the facts embody. This method has been conspicuously productive of result in the investigations of Astronomy, Meteorology, and the Tides.† Admirable examples of the employment of the Method also enrich the literature of our special subject; and I venture to express the general view that our investigations will be most successfully aided and recompensed,—not by any assumed Law of Mortality which we expect the phenomena to disclose; not by the introduction of any physiological principle which possesses no foundation in Biology; not even by the adoption of any method of Means, however elaborate and ingenious, but,—by the judicious and skilful application of the Graphic Method. Some writers‡ have expressed the apprehension, and even the conviction, that the employment of this method by different investigators would produce seriously divergent conclusions on account of its supposed

* Journal of the Institute of Actuaries: Vol. xxvi, p. 77.

† Herschel: Investigation of the Orbits of Double Stars;
Herschel: Meteorology, § 29.
Whewell: Researches on the Tides.

‡ Farren, *e.g.*: The Chances of Premature Death and the Value of Selection.

largely subjective character. This objection appears to me to be futile. We do not entrust the processes of the Differential Calculus to him who has merely mastered the Binomial Theorem; and the use of this method by competent students, disciplined in the practice of our work, may confidently be expected to exhibit a practically uniform result.

The necessity may be urged, in passing, of presenting in every case, the *original* facts, purged only by approved methods from errors incidental to observation and classification, and placing in juxtaposition the graduated figures determined by the mode of adjustment which appears to be appropriate. We should then observe how, in many older graduations, important and capital features, were deliberately (through ignorance or pre-conception) obscured * or lost, especially the significant change in the rate of mortality at early adult male ages. It is obvious that, confirmed as this fact has been by authentic experience, no formula or method which suppresses it can be in conformity with Nature. Just as the mathematical formula for a Curve presents the curious phenomena of Isolated or Conjugate Points and a Continuous Line, each element being equally and essentially involved in the equation itself, so any mathematical expression of uniformities deduced from observations on Mortality should include, as an integral constituent of its construction, the striking phenomenon I have mentioned. This consideration once again pronounces that the expectation of a mathematical statement of a Law of Mortality must remain within the sphere of hope, and directs our anticipations of practical success more conclusively to the resources of the Graphic Method.

VIII. Modes of Computation, and their Simplification and Extension.

Science inevitably—and as a guarantee of a genuine progress—tends to evolve simpler, and therefore more general, modes of investigation and calculation from scattered, cumbrous, and isolated forms. We can appeal to both the mathematical and physical departments of knowledge for apt and suggestive illustrations of this process.

One of the most admirable simplifications in Pure Mathematics—important in its consequences, and simple in its form—was the

* Dr. Price, *e.g.* under a preconception, arbitrarily altered the decrements of the Northampton table between the ages of 20 and 30. (Journal of the Institute of Actuaries: Vol. v, p. 284.)

suggestion of Harriott * in 1631 of the transposition of the members of an equation to one side and equating to zero.

In the development of Functions, we perceive the Binomial Theorem, which Newton confessedly obtained by induction, comprehended as a particular case in Taylor's Theorem in the Differential Calculus, which again has become merged in the higher mathematical generalisations of Laplace and Lagrange. A wider and successively wider formula in order of simplicity and comprehensiveness emerges as the Science proceeds.

In this region, I need only, again, refer to the foundation and development of the Higher Calculus: the Scheme of Exhaustions of the Greek Geometricians, with its necessary confirmatory supplement of the Reductio ad Absurdum; the Method of Indivisibles of Cavalerius, which reduced the ancient process into a compacter compass, and relieved it of the verifying encumbrance; and the absorption, with added rigour of demonstration, of all these symbolic systems in the Fluxions of Newton and the Infinitesimal Calculus of Leibniz.†

In the physical Sciences, the relevant process consists in the introduction of a simplification of the problem presented, and its graduated extension into wider generality of expression by successive approximation. Newton's solution of the problem of planetary motions entirely depended at the outset on a conspicuous simplification by proving that homogeneous material spheres acted as though their masses were condensed into the centres, and thus comprising them within the range of the Law of Gravitation as applicable directly to infinitely small particles.

Dr. Halley pursued, in 1693, the correct method of determining the values of Life Annuities, which had previously been assessed by guess-work, by combining the successive probabilities of survivance with the factor of discount. So masterly was the command of mathematical resources, and so keen and ample the gift of scientific insight, of that remarkable man, that at one felicitous stroke, he laid the permanent fortunes of our Science in the construction equally of the basis and mode of deduction of Annuity computation. But in each series of calculations, the summation of which constituted the value at any age, he was compelled to proceed independently, so that the values of one series failed to be utilised in the construction of the rest. Hence

* Ball: A History of Mathematics.

† Carnot: Reflections on the Metaphysical Principles of the Infinitesimal Analysis.

the laborious character of the process necessitated the limitation of his results to a fragmentary Table. And he confessed that he perceived no method of generalisation which could diminish the vast extent of toil.

But earlier, in 1671, the illustrious de Witt had pursued a similar, though apparently an equally discontinuous, plan, in the Report presented to the States-General, for though he computed his results by combining the successive values of Annuities-certain with the Annual probabilities of Death, his formula is obviously transformable into the established system devised by Halley.

In 1654, the subtle intellect of Pascal, in his correspondence with the mathematician, Fermat, had securely based the foundation and principles of the Calculus of Probabilities, and, with that instrumental Science thus newly created, gifted minds were alert to prolong its power into practical applications, exhibited in our domain by the absolutely independent investigations of these two distinguished men. To whom should the priority of originality be conceded? The famous Canon, termed Waring's Rule, which has frequently been invoked in deciding controversies of this character, determined priority on the basis of Publication, and the scope of Publication has usually been accepted as defined by public announcement through the press. Judged by this standard, Halley's claim would possess precedence, but seeing that de Witt's Report was actually printed in 1671, though its circulation was limited to the Members of the States-General, it must undoubtedly be conceded that the originator of the principle of the true method is de Witt.* We hold them, however, in equal honour, without invidious appraisement of merit; and the discovery ranks, in its degree, within the category which has been rendered illustrious by the independent discovery of Newton and Leibniz in the Calculus; the suggestion of the Undulatory Theory of Light by Fresnel and Young; the announcement of Natural

* Recent investigations into the early history of Life Contingencies in Holland have been fertile of interesting discoveries. We now learn (i) that when de Witt's Treatise was presented to the Assembly of the States-General of Holland and West Friesland in 1671, it was at once ordered to be officially printed. This was done; and the document was circulated among the Members of the States; probably about 100 in number; (ii) Extracts from the Resolutions of the States of Holland and West Friesland were regularly printed officially shortly after the Resolutions had been passed. In the Volume containing Extracts, dated December 1670, de Witt's Report again appears under date of the 30th of July 1671. This Volume I have had an opportunity of inspecting, and copies are exceedingly rare; and (iii) the manuscript of the Treatise still remains in the Archives of State at the Hague.

Selection as part of the mechanism of Evolution by Darwin and Wallace; and the marvellous intellectual reach of power of Adams and Le Verrier.

In 1725, a striking simplification was effected when these isolated Methods were merged into the wider formula by which De Moivre* proceeded, from the value at any age, to deduce the corresponding value at the next younger age.

De Moivre pursued his investigation in the mode which the historians† of Mathematics describe as Rhetorical Reasoning, where the process is conducted in language without the intervention of Symbols. In 1742, however, Thomas Simpson,‡ though adopting the same procedure, concluded with a mathematical demonstration in which the modern formula is precisely expressed under the more general form of a combination of joint lives. It is not difficult to pronounce, with authoritative certainty, that the old dispute between the claims of De Moivre and Simpson on the question of priority of conception must be decided definitely in favour of the former.

The Summation-formula of Lubbock,§ involving Finite Differences; that of Woolhouse,‖ including Differential Coefficients; and the modern methods of approximate Valuation of Annuity and Assurance problems by means of Definite Integrals¶ are interesting examples of wider and more simple methods, in gradual succession of extension.

The history, already furnished, of the Commutation Method is an additional illustration of this development in simplicity and capacity of form, both in respect of its substitution for more cumbrous and isolated modes of computation,** and the progressive improvement of the Method itself.

And regarding the subject generally, Thomas Simpson,†† in 1742, appears to have been the first writer who avowedly

* Annuities on lives: $a_x = vp_x(1 + a_{x+1})$.

The distinguished mathematician, Euler,—as we learn from Todhunter's History of the Mathematical Theory of Probability,—investigated the problem in 1760 (published in 1767), and showed that the value at any age provided a mode of immediately determining the value at the *succeeding* age. This conclusion was apparently deduced quite independently of the enquiries of his predecessors, and the precise form of the modern expression is, of course, at once obtained by a simple algebraical transformation.

† Ball: A History of Mathematics: Cap. v.

‡ Simpson: The Doctrine of Annuities and Reversions.

§ Journal of the Institute of Actuaries: Vol. xviii, p. 305.

‖ Journal of the Institute: Vol. xi, p. 301; Vol. xv, p. 95.

¶ Journal of the Institute: Vol. xxiv, p. 95: Vol. xxvi, p. 276 Vol. xxvii, p. 122.

** Journal of the Institute: Vol. i, p. 96*.

†† The Doctrine of Annuities and Reversions.

attempted to devise a universal system of procedure, accompanied by a general Notation, without restriction to the incidents and peculiarities of any specialised class of observations.

IX.—The Approximative Character of Science.

No teaching is more impressive and valuable in the historical evolution of every physical Science than its frank confession of the approximate character of its processes and results.

The infinite variety of Nature; the complex combination of Causes; the consequent intermixture of Effects; and the bewildering interactions of diverse laws, present, even in comparatively simple phenomena, a tangled maze to the baffled intellect, while, ever existent behind the seeming chaos, shine serene simplicity and symmetric precision.

In almost every mathematical expression in physical research, apparently so complete and definitely inclusive, a vaster number of terms are rejected through mental limitation, than the sparse and larger terms that are retained. In the finest development of Science,—Astronomy,—the problem of the Three Bodies is a purely approximate solution, while the more complicated scheme of attractions existing between the members of a larger system require to be decomposed, approximation upon approximation, into separate sections involving three planets only.* No planet, again, exactly conforms to Kepler's laws, which only hold precisely true of infinitely minute bodies, and the processes applied under the Law of Gravitation assume that each planet is a perfect ellipsoid of homogeneous structure. Professors Thomson and Tait† have furnished a most impressive lesson of pure approximation in the consideration of apparently the simplest problem in Statics,—the employment of a crowbar in raising a heavy body; and, without wearying you with additional illustrations, the stamp of distinct empiricism, limitation, and conditionality is impressed upon practically every physico-mathematical result which the genius of the race has contributed to the opulent treasure-house of Science. It has often occurred to me that an Actuarial student would commence his enquiries into our special subject with a mind more efficiently trained and accustomed to practical considerations, and less liable to be misled by the

* Cheyne: Treatise on the Planetary Theory. Godfray: Treatise on the Lunar Theory.

† Treatise on Natural Philosophy. Vol. ii, cap. 5.

decimal fallacy, as I may term it, of affected and baseless precision, if he rendered himself familiar in some degree with this universal and memorable lesson of Scientific truth.

Sir John Leslie,* in 1804, raised a serious warning to physical investigators against pretended and delusive exactness: I repeat the needed caution to my younger colleagues, and point out to them that this decimal precision is not merely illusory and impracticable, but is absolutely misleading in presenting an ostentatious appearance of exactitude to which no valid pretension can be raised. And to mention a small, perhaps, but significant illustration, I yet hope to survive the time when the idle parade of decimals in our Valuation results will be discarded as an error. But let me not be misunderstood. A sound mathematical equipment is essential to a successful prosecution of our labours in any useful form, but the teaching which I urge with earnestness upon our younger members is that mathematical processes are simply instruments to be employed, over which judgment, experience, and a practical acquaintance with the affairs of life and with the necessary limitations of our work should preside in autocratic and unquestioned power. Even in the purer realms of Geometry, we are attended by the spirit of approximation, for the ideas to which our reasonings apply possess no reality in Nature, but are simply, so to speak, the intellectual limits of sensible quantities and qualities. The character of rigorous exactness is secured because the elements of calculation are ideal.

X. THE LANGUAGE OF SCIENCE.

The exposition of the evolution of a Science usually includes a reference to the Scheme of Language which embodies and perpetuates its discoveries; and this custom I venture to pursue in relation to our Professional Speech.

The advance of knowledge requires to be expressed in clear and definite Propositions, compacting into manageable masses individual and isolated facts; but, in addition, is demanded a system of Nomenclature and Terminology, which constitutes the current coin of mental gains, and provides a simple and expressive medium of intellectual intercourse.

In the Symbolic Notation which now dignifies the Science of Life Contingencies, we possess, I conceive, a scheme of abbreviated language which is at once simple, lucid, definite, and

* An Experimental Enquiry into the Nature of Heat: Note iv.

connected. The visual relief and mental gain are almost startling when we pass from the bizarre pages of Milne with their bewildering maze of eleven different forms of type to the adequate and congruent system in which we are now competent to present our processes and results. The advantage I have mentioned is not merely the exhibition of symmetry to the eye, but that distinctive aid in investigation which accompanies every wise and organic reform of language, in mental relief, intellectual clearness, and enlarged power of concentration. But when we turn from this admirable lexicon of symbols to our current Nomenclature, we descend from the cultured pages of the philosopher to the cumbrous dialect of the peasant. What consistent or adequate significance is conveyed in our "net", "mathematical", or "risk" premium? Where is the inevitable appropriateness or precision of meaning in our "office" or "gross" premium? And we attain the acme of linguistic barbarity in that abhorrent term, the "loading." Our scientific language should fulfil the requisites of precision and consistency in the work of specialists, and of freedom from alien connotations when our words are uttered by the laymen whom we teach.

The Criteria of a Scientific language have been adequately, and almost finally, expounded by Whewell* and John Stuart Mill,† to whose authority all cultured scientists appeal; and I shall not despair of the advent of the time when the sagacious and constructive skill which has elaborated our refined scheme of analytical speech refurbishes these unpresentable expressions, or rather substitutes a more consonant triad of technical terms.

XI. The Benefit of Science to the Community.

In no portion of Bacon's Novum Organum does his language assume a more sonorous and dignified expression or his predictions a more vivid and impressive tone than in those inspiring passages which expound the social benefits conferred by Science and expatiate upon the widening area over which her future successes should beneficently extend.

And following this custom, which has been observed by historians of Science down to Herschel and Whewell, I conclude this portion of my Address with a few brief reflections in adaptation of such a survey to our special work.

* Aphorisms on the Language of Science: Nov. Org. Renov.
† A System of Logic: Vol. ii., Lib. iv., cap. 4 to 6.

I need only summarily refer to the vast practical scheme of Assurance which our principles have founded, directed, and expanded, with its countless social, economic, and even political consequences; and addressing myself to a more specialised subject, I point with admiration to the commercial and financial advantages to the community which the introduction of Market Values, by the appropriate adaptation of Actuarial principles, has permanently created. The history of this feature is deserving of a momentary chronicle. In the earlier writers, the purchase and sale of Life Interests were based exclusively upon pure Annuity-Values.* In the Treatises of De Moivre† in 1752 and Dodson‡ in 1753, we discover for the first time a solution of the problem in which Life Assurance was involved as an illustrative element; but the purchaser became his own Assurer. The absence or imperfect conception of a general Assurance system must naturally be remembered when considering these attempts. It was only in 1825 that Griffith Davies§ first practically solved the problem as the commencing stage of a varied and exhaustive application. Davies deduced his result from first principles, and it is interesting to observe, as was subsequently pointed out,‖ that, by means of the Conception of Analogy,—so fertile an instrument of discovery in the advance of Science,—he might have detected the process by comparison with the "Sinking-fund" factor in the valuation of an Annuity-Certain.

I recall, too, the memorable monetary advantage to the Public Funds which our first President¶ established, when, in 1819,—guided by the Conception of Appropriateness of data,—he denounced the employment of the Northampton Table in the grant of Government Annuities.

And another illustration may be adduced of the practical efficiency of our methods for commercial purposes, in the admirable and conclusive researches, commenced in 1859, into the Probabilities of Issue deduced from the records of Peerage Families,** which have rendered numberless interests marketable to which no definite value had previously attached.

* Journal of the Institute of Actuaries: Vol. vii, p. 136.
† Treatise on Annuities: 4th edition.
‡ Mathematical Repository.
§ $\frac{1}{d+p} - 1$: Treatise on Annuities: cap. iv., § 5.
‖ Journal of the Institute: Vol. xx, p. 435.
¶ Journal of the Institute: Vol. x, p. 147.
** Journal of the Institute: Vol. viii, p. 127; Vol. x, p. 181; Vol. xii, p. 185; and Vol. xxi, p. 406.

It will summarily be observed that the examples and applications of Actuarial principles which I have described throughout this Address, amply and impressively exemplify a general criterion of Science in the progressive advance from the Isolated to the Combined; from the Complex to the Simple; from the Homogeneous to Heterogeneity both of structure and method.

XII.—The Character and Aims of the Actuarial Student.

The advance of Science, and the specialised Education of the enquirer, proceed in mutual interaction. Enhanced resourcefulness of investigation ensues as the accumulated facts and laws become added to the armoury of research. Defective processes assume a keener precision and pliability; each happy interpretation and generalisation of Nature preludes a wider amplitude of survey and more assured confidence of insight; the intellectual obedience of man to the teachings of Nature becomes transformed into a supremacy of power; and thus, besides conferring an exacter and more refined character upon the Science, Man himself, as the Minister et Interpres Naturæ, in Bacon's phrase, gradually becomes possessed of a finer sagacity in search, a developed skill in method, and a more vigilant vision of Ends.

And the historians of Science naturally deduce, from their survey of research, the assemblage of attributes, which distinguish the genuine cultivators of this systematised form of knowledge. In pursuance of this customary course, I venture to address a few reflections, specially and earnestly, to my younger colleagues; and though most of us cannot hope to emulate the commanding qualities of the Masters who have contributed to the development of our work, we may yet gain courage from the circumstance that the characteristics they displayed must exist, though only in potential germ, in every student who has rightly discovered his vocation or calling in our sphere. And those higher qualities which win our unstinted admiration, constitute the Ideal form towards which, though we may only dare to cast a wistful regard, we yet sedulously and hopefully direct our way. And despair is banished by the thought that progress in all Sciences presents a corresponding aspect, enlarging in vivid contrast with each advancing stage, and that not simply in Ethics, but also in Intellectual and Practical work, every movement of memorable achievement in History and Life has become the more purified, stimulated, and

enriched as the Ideal End assumed a minuter distinctness and glowed with a steadier and serener light. In the History of our Profession we may, I think, distinguish three definite stages of development of character and equipment: the Mathematical; the Administrative; and the Financial.

(i) In the primitive period, the Actuary was chiefly a Mathematician,* from the necessity of the case and in the absence of that practical exhibition of Actuarial principles which the system of Life Assurance embodies. His pursuit was mainly that of calculation, based upon the application of established mathematical processes to the novel materials of mortality statistics which had then assumed a collected form.

(ii) More practical considerations gradually emerged, as I have shown in the solutions of De Moivre and Dodson, in 1752 and 1753, which introduced within our sphere commercial elements, like market values, and,—with the elaboration of the system of Assurance, in which his principles discovered their fullest sway, involving subtle Actuarial problems of rational premiums,† Valuations, and Distribution of Profits,—the Actuary, while retaining his special character, gradually added the attribute of an Administrator, and thus united to the mathematical aptitude, a practical knowledge of large business affairs and contact with financial and economic problems.

(iii) At the present day, we perceive a further and very definite transformation where general skill in Finance and practical mastery of financial questions have assumed an imperative supremacy. But an acquaintance with problems of finance will be absolutely impotent if confined to theoretic study. The student of our Profession must now, more sedulously than ever, mix with men in business enterprise, and learn from mastered experience in the world itself, the modes of dealing with financial subjects and the methods and meaning of financial work. *Seclusion* in the study implies *exclusion* from the bracing rivalries and the highest usefulness of Professional life; a strict devotion to the region of pure mathematics paralyzes the tact and practical skill which are essential to the estimation of

* As the Science of Numbers is the basis of every mode of Actuarial work, early speculations and applications constituted a rich armoury of weapons with which, as observations became more complete and specialised, succeeding enquirers could successfully attack the increasing complexity of questions which the progress of our science produced. The investigations of Daniel Bernouilli in 1760, of D'Alembert in 1761, and of Laplace in 1812, into the mortality due to smallpox, are instructive examples of mathematical skill applied to limited data.

† Gompertz: Memoir of 1820.

probable evidence; and in obedience to the great Law of Distribution of Force, the specialised cultivation of the Mind, without the concurrent development of Feeling and of Interest in Humanity, produces but a fragmentary and unfinished man.

I counsel the young members, with sincere and earnest hopefulness, to acquire this direct and practical financial training, and I am confident that the resources of the Institute, which should be ever vigilant of the appropriate education of its youthful Alumni, will speedily devise a plan of wisely guiding them in this specialised direction, where their prosperous and honourable usefulness distinctly and permanently lies.

The essential aim of our general and professional life is the quest of a genuine system of Education; the appropriate and laborious educing of the faculties we possess; the provision of an organised method of training enquirers to collect and elaborate knowledge for themselves; so that we may become, to adopt a figurative expression, not mere mental sponges capable of absorbing a vast amount of knowledge, but unhappily simply competent of discharging it minutely as it was acquired, somewhat the muddier perchance from the process; but rather that our minds should assume the functions of a Chemical Laboratory where supplies of information constitute merely the crude materials which the subtle Alchemy of the Intellect and the Practical Power shall convert into fresher and more finished forms.

We possess admirable Text-Books of our Science, worthy of the honoured position which the Institute fulfils, but the mere mental absorption of a Text-Book, however exhaustive be its treatment and extensive its scope, will leave the student a commonplace storehouse of garnered knowledge which remains in the precise condition in which it was imbibed, void of stimulating energy, impotent as an active instrument of mind. Search your Text-Books for hints and directions, but sedulously pursue the real and sovereign mode of education in studying the original works of the great Masters themselves; in following the researches which enrich our *Journal;* and seeking to employ all these acquisitions both as stable elements of learning but essentially as means of sharpening, invigorating, and guiding the activity and vivid perceptiveness of your individual powers. A minute fragment of original thoughtful effort outweighs, in the true scale of educative purpose, the vastest accumulation of unassimilated

and mechanical facts. An Echo, though precisely articulate, is yet but a repeated Voice.

There is, unhappily, no example so common as that of men who fancy that, by introducing into their speech the terms of Philosophy and Science, they are conversing in a philosophic and scientific mode, and the sole corrective of this mischief is the study of original and fruitful works with alert and teachable intelligence. Even the moral qualities of patience and humility which such a study involves constitute an ennobling education of permanent value and hope. We thus learn, again, the impotency of that glib assumption, fatal both in Theology and Life, and too prevalent in this thoughtless age, that the deepest and widest problems are capable of expression within the narrow boundaries of a summary dogmatic statement. We gain the helpful and steadying conviction that social, professional, and scientific questions are usually of too profound an import to become adequately compressible into a portable formula, however specious and ingenious; and I can never hear, in the discussion of comprehensive and complex problems, whether of life or professional work, the frequent assertion, to employ the current phrase, that the solution lies within the compass of a "nutshell", without humbly thinking that the nutshell measures, not the dimensions of the question but, rather the character and capacity of the speaker's head.

I finally urge the importance of the student attending our debates, and modestly taking a part in our discussions. He will never fail of a kindly and cordial reception; even a single question, derived from the speaker's own freshness of enquiry and reflection, is often of material service to us all,—the "prudens interrogatio" which is not infrequently the "dimidium scientiæ";* and, moreover, an early participation in debate, conducted solely in the pursuit of Truth, affords steadiness of conception, readiness of resource, flexibility of utterance, and that modest self-reliance, which so largely determine the complete command of all our powers.

XIII.—Conclusion.

In thus concluding the task which I assigned to myself in this Address, the reflection naturally arises, as I survey the course I have traversed, that, with fragmentary knowledge and imperfect

* Bacon: De Augmentis Scientiarum: Lib. v, cap. 3.

enquiry, I have attempted too vast and intricate an enterprise,—a local pilot steering his vessel amid perilous and unnavigated seas,—but a generous interpretation of a feeble effort will, I am confident, ensue in the remembrance that motives of surpassing interest have impelled me in this quest: a sincere affection for our great Institute, in whose honour, devotion and labour can recognise no bounds; a deepening pride in our traditions of memorable public usefulness, and of strenuous and unresting toil in fashioning the structure of Science which we now inherit as an honourable Trust; a profound personal concern in the fortunes of the younger Members of our Body, destined to receive, and worthily uplift, the torch of knowledge from our passing hands; and an ambition to kindle within their minds a vivid sympathy with the high vocation of an Institute whose signal services in the past are authentically prophetic of an even ampler history in the future, and a yet loftier station in the hierarchy of Institutions devoted to the Common Weal.