OUTSTANDING CLAIMS RESERVES

Report by G.I.S.G. Working Party

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Introduction

Definitions To reflect accurately the true position of his business at any instant, an insurer must make allowance for two types of liability incurred in respect of claim outstanding:

1. An estimate of the amount which will be paid in the future on claims of which the insurer has received notification. This is called the provision for reported claims. Alternatively it may be referred to as claims I.B.N.E.R. (incurred but not enough reported) or as the outstanding claims reserve.

2. An allowance for payments on claims arising from events which have occurred but of which no notification has yet been received. This is called the provision for claims incurred but not reported (I.B.N.R.)

Deciding on a Method of Valuation

There is no one method which is recognised as a standard procedure for estimating such reserves. There are a large number of methods which have been proposed and/or used ranging in complexity from individual case estimates to methods using advanced statistical techniques. A very large number of papers on the subject have been written and printed in diverse publications. The person charged with estimating the reserves for his office should be aware of the methods which are available, but to wade through all the literature on the subject would be an immense task.

A major purpose of this report is to make an easier job of finding suitable methods which might be used for an individual office's own particular portfolio of business. To this end the working party considered a large number of the papers which might be relevant to the subject of outstanding claims reserving. These papers are listed in Appendix 1, in alphabetical order of author, under the following headings:

Author(s)
Title of paper
Code

The code has the following meaning:

Code

0. No particular method is given: this would normally be an overview or general paper or one only touching on the subject of outstanding claims reserving.

1. One particular method is presented and described. This is normally a method derived by the author(s).

2. More than one method is presented. These methods are either ones using a similar approach or the paper is a survey of different methods.

3. Not considered relevant. (There were a very small number of such papers).

4. Not yet investigated. (Usually because of time problems)

All the papers (apart obviously from those coded 3 or 4) have been summarised in Appendix 2, in alphabetical order by author. The headings used are:

Author(s)
Title
Source
Summary of paper

Appendix 3 consists of an indexed summary of most of the methods found in the papers summarised in Appendix 2. This is divided into four sections:

1. Direct business : general methods

2. Direct business : specific applications

3. Reinsurance business : general methods

4. Reinsurance business : specific applications.

Within each of these sections the methods have been sorted into order by increasing complexity of the calculations required. This of course is a value judgement and therefore may not coincide with the views of other people. The headings used are:

Method : a brief summary

Author(s)

Paper : title

Source : of paper

Applications : o/s claims reserves and/or I.B.N.R. etc.

Data required

Main assumptions

Comments

Author-paper-source shows, where known, the original source of the method. Any other papers which describe the method are not usually identified apart from two particular cases i.e. the B3 General Insurance Monograph on Estimating Outstanding Claims (Ackman, Green and Young) and Surveys of Actuarial Studies No. 1 on Loss Reserving Methods (J.van Eeghen).

Usage of Report

There are various ways in which the summaries in this Report can be utilised. Two such ways are presented here.

a) Algorithm for finding a suitable method to use for a particular portfolio of business.

   1. Determine whether the business is direct or reinsurance business. It is suggested that a general method would probably be preferred to a specific application.

2. Find the appropriate section in Appendix 3.

3. Find a summarised method which looks as though it might be applicable (based on the application required, data available, assumptions holding etc. and either on the degree of complexity which might be thought necessary or starting with the least complex method). If all the methods in the general method section have been eliminated, try the specific application section and if all these have been eliminated you have problems'.

4. For the method found, for each source listed read the appropriate summary in Appendix 2.

5. If the method is not considered to be suitable go back to step 3, otherwise obtain and read the original paper.

6. If the method is now considered not to be suitable go back to step 3, otherwise use the method on the portfolio to be valued.

7. If the method is still not what is required go back to step 3.

8. Other methods should also be investigated before deciding on a valuation basis.

b) Algorithm for deciding whether a particular paper should be obtained as the source of a method of valuation.

1. Find the author and paper title in Appendix 1. If it is not listed or code 4 then the actual paper will have to be read, otherwise:

2. If the paper is code 0 or code 3 then it will not be of use, otherwise:

3. Find summary of paper in Appendix 2 and read.

4. If a method, as presented by the summary of the paper, appears to be suitable then either obtain the relevant paper <u>or</u> find out whether there is an alternative source for the method using algorithm (a).

<u>Particular Papers</u>

Of the papers summarised in Appendix 2, the following three are of particular note.

1. Ackman, Green and Young: Estimating Outstanding Claims. This is of particular interest as it is the proposed B3 General Insurance Monograph on Outstanding Claims.

2. J.van Eeghen: Loss Reserving Methods. This is a survey of loss reserving methods and a full description of thirteen methods is given. This publication is often a better source for understanding the methods than the original papers. The relevant methods are listed in the summary of the paper.

3. E. Straub: I.B.N.R. - A Difficult Marriage between Practice and Theory.

This paper gives a practical eight-step I.B.N.R. loss reserving process and this is shown in full in the summary of the paper.

Sources of Papers

The following four publications each consists of a set of papers most of which are relevant to the subject of claims reserving, and have hence been summarised in this Report. The "source" in Appendix 2 and Appendix 3 has been given as A.S., I.A.A., I.M.A. or N.R.G. respectively. The full titles are as follows:

1. A.S. "I.B.N.R.": Proceedings of the first meeting of the Contact Group "Actuarial Sciences". Edited by F.de Vylder and M.J. Goovaerts (1978). Instituut Voor Actuariele Wetenschappen.

2. I.A.A. Institute of Actuaries of Australia. General Insurance Seminar (1978). Working Section 2 - Outstanding Claims.

3. I.M.A. "Claims Provisions for Non-life Insurance Business". Proceedings of a Symposium organised by the Institute of Mathematics and its Applications (1974).

4. N.R.G. "I.B.N.R." The Prize-winning Papers in the Boleslaw Monic Fund Competition (1971). Nederlande Reassurantie Group N.V. (What scientific and quantitative methods can be used for determining the I.B.N.R. for reserve and underwriting purposes?)

Other abbreviations used for "source" (apart from ASTIN, GIRO, GISG, JIA, JIAss and OECD) are

M.V.S.V.: Mitteilungen der Vereinigung Schweizerischer Versicherungsmathematiker.

P.C.A.S.: Proceedings of the Casualty Actuarial Society

S.A.S. : Surveys of Actuarial Studies No. 1. Loss Reserving Methods. J.van Eeghen. Nationale Nederlanden N.V.

T.I.C.A.: Transactions of the International Congress of Actuaries.

Degree of Uncertainty

Apart from reviewing and summarising papers and classifying methods, our other term of reference is to discuss the problem of estimating the degree of uncertainty associated with the valuation of outstanding claims.

"Degree of uncertainty" is a probabilistic concept. In theory if the whole claim process was probabilistic, and if its structure and parameters were known, it would be possible to use simulation to calculate confidence limits on the value of outstanding claims as predicted by a particular model.

However even if the claims process were a realisation of a set of probability distributions it would never be possible to discover the structure and parameters of all these distributions and hence impossible to construct a "true" model of the claim process. Unfortunately the confidence limits placed on a particular estimation method will be highly dependent on the assumed model; if model and estimation method assume the same structure for the claim process the confidence limits will be fairly narrow, if they assume different structures the limits will be wider. In reality therefore any accuracy implied by the confidence limits would be totally spurious.

Estimation of the degree of uncertainty of a valuation must therefore be far rougher. It could involve a number of stages:

1. List the properties of the claim process that must be stable over time for the proposed statistical estimation method to work.

2. Run down the checklist of factors that may disturb the stability of the various properties of the claim process.

3. Decide whether the method can be used without adjustment.

4. Adjust the past data if the lack of stability lies in the past.

5. Adjust the future projection if the lack of stability lies in the future.

6. Apply the method to obtain a best estimate of the necessary reserves.

7. Estimate reasonable upper and lower bounds for the adjustments in 4 and 5.

8. Apply the method again.

A user of statistical estimation methods probably already goes through the process, even if he doesn't think explicitly of the various stages. The Working Party does not therefore claim originality apart from the drawing up of the checklist.

The difficulty, obviously, is that stage 3 will almost certainly show that the method cannot be used without adjustment. Sometimes a special investigation will enable the effect of a past change to be quantified fairly accurately, and hence step 4 may be possible. Step 5 will, however, usually be impossible as most of the factors which will affect the future stability of the claim process will be unknown at the valuation date. Assumptions will therefore usually be conservative, i.e. whether or not a past trend is assumed to increase in the future will depend on whether or not it will increase the necessary reserve.

Checklist of Factors

A. Claim Settlement Pattern

1. Timing of Claim Occurrences
2. Allocated Loss Adjustment Expenses
3. Changes in Settlement Patterns
4. Severity of Claims
5. Claim Frequency
6. Increasing Use of Partial Settlements
7. Special Settlements
8. Nil Claims - Precautionary Advices
9. Judicial Awards
10. Large Claims

B. Nature/Mix of Business

1. Changes in Portfolio Volumes
2. Change in Mix of Business
3. Change in Policy Conditions

C. Data Constraints

1. Computer Systems
2. Availability of Data
3. Reliability/Credibility of Data
4. Processing Backlogs
5. Heterogeneity of Data

D. Exogenous Influences

1. Changes in Legislation
2. Social Environment
3. Weather Conditions
4. Currency Movements
5. Miscellaneous

E. Reinsurance Arrangements

1. Inwards Reinsurance

   i) Data
   ii) Claim Payment Delays
   iii) Valuation Methods

2. Outwards Reinsurance

   i) Net Liability Calculation
   ii) Catastrophe Covers/Large Claims

The above factors are amplified in the following section. The Reinsurance part is presented in a different format to the other parts as it incorporates many features from them.

It should be noted that the effect of nearly all of these factors is unquantifiable for various reasons. Therefore any system for calculating the standard deviation or confidence limits for the outstanding claims reserve is necessarily suspect.

UNCERTAINTY ASSOCIATED WITH THE VALUATION OF OUTSTANDING CLAIMS

This section covers the wide range of factors some or all of which should be considered when using any method to estimate the value of outstanding claims based on past data.

The parameters which give rise to uncertainty when estimating outstanding claims can be split into five broad groups:-

A. Claim Settlement Pattern
B. Nature/Mix of Business
C. Data Constraints
D. Exogenous Influences
E. Reinsurance Arrangements

For many of the items which fall into each group little amplification is necessary. For others practical examples are given.

A. CLAIM SETTLEMENT PATTERN

Stability of the claim payment or settlement pattern is an assumption which is widely used. In practice this ideal is rarely realised and the following is a checklist of the factors which can lead to uncertainty.

1. Timing of Claim Occurrences

Unusually adverse weather in December will increase the proportion of payments and settlements made in the first development year after the year of origin. On a household account where settlement delays are short the distortion could be severe.

2. Allocated Loss Adjustment Expenses

If any specific allocated expenses are included in payment data, changes in the method of allocation (e.g. between classes of business) or timing of allocation (e.g. date of payments, date of settlement) will distort the payment data.

3. Changes in Settlement Patterns

These could arise in a number of ways e.g.:-

Policy decisions to press for early settlements
Changes in claims handling efficiency, staffing levels etc.
Closing off exercises on outstanding claims

4. Severity of Claims

If large claims have a longer settlement pattern than small ones any change in the mix of severity of claims will change the overall settlement pattern.

5. Claim Frequency

A change in frequency without a change in the mix of type of claims will not affect most methods, but a change resulting from increased claim awareness, introduction of bonus protected motor policies etc. may affect the settlement pattern (see effects 4 and 8).

6. Increasing Use of Partial Settlements

7. Special Settlements

Changes in policies towards ex gratia payments or the attitude to borderline cases may affect settlement patterns, particularly in the later development years.

8. Nil Claims - Precautionary Advices

The effect on any method using the average claim size is obvious. The volume of precautionary advices can be changed by publicity about the need for such advices.

9. Judicial Awards

The level of settlements for injury etc. may not follow any price or earnings index, but have sharp upward movements after particular judicial awards followed by a period of stability.

10. Large Claims

Settlement patterns can be seriously affected by random variations in the number, amount and date of payment of large claims.

B. NATURE/MIX OF BUSINESS

1. Changes in Portfolio Volumes

With a significant change in portfolio volume, it would be questionable whether the nature/mix of the business would remain similar over time. Invariably, there would be a change in underwriting standards/type of risk brought about either explicitly by extending cover to certain policyholders previously declined cover or implicitly by starting to charge below average premium rates. This fundamental change in the underlying nature of the "average" risk and the difficulty in quantifying the effect on the claims process would immediately introduce an air of caution into the statistical calculation of the outstanding claims reserve.

Even in the unlikely event of a similar before and after portfolio, the effect of random fluctuations (which are more prevalent with a low volume portfolio) must be allowed for when projecting the outstanding reserves for a larger portfolio using historical low-volume statistics. Similarly, when going from a high to a low volume situation, the stability which might have been experienced in the past due to the minimal effect of such random fluctuations, could well disappear in the low-volume portfolio thus leading to uncertainty about the correctness of the statistical reserve.

On an even more practical side, it would be necessary to be aware of the effect on the claims staff of a large increase in portfolio volume. Invariably the numbers/experience of such staff would not keep pace with the changing portfolio (at least initially) and hence the claims handling procedures would change. This could lead to delays in computer notifications, less thorough investigations and delays in settlements with potentially higher ultimate costs. Hence the inherent nature of the claims could have changed and certainly the statistics of the emerging cohort of business (associated with the increased volume) would be different from previous low volume cohorts.

2. Change in Mix of Business

If a class of business is very narrowly defined with all risks which fall under this classification being similar in claim characteristics, then there is little problem. On the other hand consider a company with a single motor private classification. If it experiences a swing away from predominantly non-comprehensive business to comprehensive policies than not only will the claim size distribution be significantly different but also the reporting and settlement patterns will have changed.

3. Change in Policy Conditions

Some changes can, in practice, have little effect - for example increasing a policy excess by £50 should cause the average net claim to subsequently decrease by a somewhat similar figure. In reality the before and after situation might well not be too dis-similar thus making projection less uncertain.

Other changes can have a significant effect - for example extending private motor cover to broken windscreens without application of an excess and without any effect on the NCD or granting protected NCD's for a minimal arbitrary premium could cause the incidence of claims to increase sharply and have a significant effect on the claim size distribution. In this case a projection of the current outstanding claims based on past averages etc. is likely to give an over-inflated reserve.

An even more significant change would be where a NCD scale was changed and/or where the rules for determining position on the scale were amended. Such a change could have a significant effect on the type of claim subsequently reported.

C. DATA CONSTRAINTS

Data can be inadequate or erroneous in content for a number of reasons and these are examined below:-

1. Computer Systems

A company's computer system is usually the main, if not the only, source of information open to the claims reserver. Computer systems capable of handling large volumes of diversified policy and revenue information are necessarily complex. In order to interpret computer generated statistics, it is essential to have a sound knowledge of the computer's database, together with a good understanding of the system's inter-related structure. Sometimes, data can be processed incorrectly owing to a misunderstanding of how the computer system works. The statistician must always be aware of current processing policy if data processing errors of this nature are to be avoided. Accurate statistics are essential. A close association with the database engenders a sense of awareness to processing errors. The sooner errors are highlighted and corrected, the cleaner the data becomes.

Inception to date statistics show revenue development by contract or group of contracts and should tie in exactly with revenue details. Extraneous factors can sometimes hinder this relationship. Statistics must always be reconciled with revenue accounts. Otherwise, distortions will manifest in the projection of outstanding reserves.

The importance of knowing one's computer system may be illustrated by the following questions, the answers to which must be known:-

a) are premiums gross or net of commissions?

b) does the term commission include acquisition costs such as premium taxes, fire brigade taxes, profit commission?

c) do paid losses incorporate provision for settlement costs such as legal fees, court costs?

d) does the term outstanding losses represent only the amount advised by the broker or are there additional elements to cover outstanding court costs, legal fees, additional reserves assessed by the claims manager?

2. Availability of Data

A computer system is only as good as the information it contains and of the accessibility of that information. It is pointless storing information which cannot be accessed or reported in the format required. A statistician can find himself in a situation where certain information is required, but no programs exist to extract it in the format required.

Consider, for example, the situation where foreign business statistics are reported on an all to sterling basis and a large claims development requires investigation. The only claims activity report available reports movements in original currency. Imagine the difficulty of converting every claim into sterling in order to establish claim size significance for the purpose of identifying the large claims. Clearly the information exists but is not available in the format required.

Sometimes, information essential to claims reserving is absent from the computer system. Certain reinsurances, recorded manually, may never have been processed. In such cases, allowance must be made when projecting net statistics to compensate for the known deficiency. Availability of all data, be they manual or computer records,is essential for proper reserving.

3. Reliability/Credibility of Data

The 'rubbish in - rubbish out' scenario is very apt in claims reserving. The statistician must always watch out for processing errors (e.g. from punching in errors, incorrect currency codes or exchange rates, non-processing of reinsurances etc.).

Sometimes data can be processed wrongly owing to lack of training or understanding on behalf of a processing technician.

The following example shows how a lack of understanding about the processing of proportional business can lead to under-reserving. Proportional treaties are settled on an accounting balance basis, that is, premiums less paid claims less funds withheld constitute the balance due to or from the reinsurer. It is common to have a clause written in the treaty whereby the cedant retains a set percentage of premiums and/or loss reserves. These are retained and released in accordance with the policy conditions. Consider now those funds in relation to outstanding loss reserves. Loss funds withheld will represent an agreed percentage of outstanding loss reserves. The percentage is usually set at 100 but can be as high as 150. It is normal for proportional treaty accounts to be rendered on a quarterly basis, with the outstanding loss reserves advised to the reinsurer at least once a year, but not necessarily every quarter. It can happen that for those quarters where no outstanding losses are advised but loss funds withheld exist, the technician processes a zero outstanding loss. Of course, the correct action would have been to extract the policy details, establish the relation between reserves and funds withheld, and calculate the outstanding reserves commensurate with the funds withheld reported on the quarterly account.

In contrast, statistics net of reinsurance may be overstated if certain reinsurance amounts have not been processed owing to omission or timing lags. Unless allowances are made, projection of such data will generate an overstatement in reserves.

Clean, credible data is all important when establishing the best estimate of the reserves to be carried in the company accounts.

4. Processing Backlogs

These can arise, for example, from staff shortages, an increasing portfolio or holidays.

If the backlog is seasonal (e.g. due to holidays) then this should have occurred in the past and statistical developments may not be distorted. If however, the backlog has arisen for reasons which have not happened in the past and should never have arisen in the first place, then subjective allowance must be made to accommodate any statistical distortion which results.

Processing backlogs restrict the normal flow of statistical development. To avoid an over or under statement in projected claims reserve, the backlog must be understood and its impact assessed.

5. Heterogeneity of Data

A commonly used technique in claims reserving is to divide the database up into homogeneous sub-sets. Taken to the extreme, we would end up with hundreds of data sets having little if any statistical stability. Besides being impractical, this is also undesirable. The statistician must therefore restrict himself to broader sub-sets such as,for example, by DOT class. Depending on the size of a reserving class, a further sub-division by business type may be appropriate.

If any characteristics change with time different developments will result. Unless these are known and allowed for, wrongful projections of claims reserves will be made. In practice, however,it may be very difficult to identify changes in business profile from computer data alone.

To illustrate this, let us consider long tail business where business written is gradually changing from a claims occurrence to a claims made basis (i.e. instead of the insured being covered for claims occuring during the policy period, he is protected against claims being filed over the policy period). Business written on a claims made basis has a much shorter tail. Unless the computer contains an indicator to differentiate between the two types of business, it would be impossible to segregate them for the purpose of reserving. We would tend to over-reserve if there has been a gradual shift from a claims occurred to a claims made basis; the converse also being true. In this example, we have a data constraint which prevents us sub-dividing the business into homogeneous sub-sets. The underwriter is probably the best person to approach for a guide as to the change in business type by year.

Taking this into account, we can subjectively recognise and make allowances for the heterogeneity of the data when projecting claims reserves. To be more scientific is impossible.

Data constraints often prevent the statistician from dividing his database into the homogeneous sub-sets. If heterogeneity cannot be avoided, the statistician must apply subjective analysis when establishing outstanding claims reserves.

D. EXOGENOUS INFLUENCES

Such influences are to a large extent outside the control of the insurer. Two of the most important of these influences are inflation in the general level of prices and earnings (to the extent that these affect claim settlements) and uncertainty of investment yields. As these factors are specifically excluded from the terms of reference of the Working Party, no further reference is made to them in this report.

Other exogenous influences affecting the claims experience may be revealed by a gradual trend in the figures being analysed, but in many cases the exogenous influences will cause a sharp discontinuity in the experience. Where a discontinuity is known to have occurred and its effects can be reasonably quantified, observed experience should, of course, be adjusted to eliminate the effects of the discontinuity before making an assessment of the provision necessary for outstanding claims. Since most types of exogenous influence are not amenable to statistical measurement, it is not possible to include in the provision for outstanding claims a scientific assessment of reserves for exogenous influences which may or may not arise in future. All that can be done is to include an arbitrary margin in the provision made. Even for those exogenous influences where some statistical assessment could in theory be made (e.g. the risk of catastrophes), the limited nature of the data available to the individual insurer may render a statistical approach inappropriate. These features are illustrated in the examples given in the following paragraphs.

1. Changes in Legislation

Changes in legislation, whether fiscal or otherwise, are clearly factors beyond the control of the individual insurer. For example, an increase in the rate of value-added tax could result in increased claim costs for (say) motor car repairs effected after the relevant date, whether or not the damage was inflicted before the relevant date. Provided the proportion of claim costs subject to VAT is known, the effect of a change in the rate of VAT can be quantified and past experience can be adjusted to produce consistent figures for assessment of the outstanding claims provision. It is not possible to make allowance on a statistical basis for any future changes in the rate of VAT. Another legislative change which might be expected to produce a sharp discontinuity in the experience is the recent introduction in the UK of a law requiring the wearing of seat belts by the driver and front-seat passengers. The increased use of seat belts should result in a decrease in claims costs. However, as it may be difficult to isolate this reduction in claims cost from other features in the experience, any attempt to quantify the reduction so as to produce consistent past experience figures may not be worthwhile. Not all legislative changes need produce a sharp discontinuity in experience. For example, if the legislation, perhaps in conjunction with other factors, encourages people to switch from public transport to private transport, the increase in traffic density may result in a gradual increase in claim costs which would be revealed as a trend over some years.

2. Social Environment

Apart from legislative changes, there may be changes in the social environment which lead to uncertainty in the valuation of outstanding claims. For example, a more sympathetic attitude towards disabled claimants may be reflected in higher compensation payments awarded by courts, particularly where the payment is determined by a jury. This feature would normally be revealed as a gradual trend in settlement costs, although not necessarily on a steady basis, and may be indistinguishable from other features affecting the claim settlement process discussed in A above. However, occasionally, this type of change may produce a sharp discontinuity in the experience. The recent "Barrell" case which caused such an upheaval in the Australian liability market is an illustration of the significant effect that court awards can have on the assessment of outstanding claims provisions previously thought to be adequate.

3. Weather Conditions

Where the outstanding claims provision is assessed at the end of each calendar year, the vagaries of the UK weather can result in a fluctuating incidence of claims between different calendar years, and this may give rise to different settlement patterns (see Section A.1). The problem is greatest when the most recent calendar year has experienced abnormal weather conditions. It is unlikely that the data available on past experience would be sufficiently credible to allow any adjustment to the assumed future run-off pattern for the latest year other than on the basis of informed subjective judgement. Infrequent climatic events, such as typhoons, hurricanes and other catastrophes, can also complicate the analysis of past experience. The preferred course of action is likely to be to eliminate catastrophe claims from past experience, to use these adjusted claims figures to assess the outstanding claims provisions, and then to incorporate a further provision for any outstanding catastrophe claims known at the valuation date. The difficulties arise in deciding which claims (or which part of a claim payment) are attributable to a catastrophe. Although much of the catastrophe risk may be covered by reinsurance, the accumulation of payments net of reinsurance is still likely to distort the claims pattern. The provision to be made for catastrophes occurring after the valuation date is a problem for the unexpired risk reserve rather than the outstanding claims reserve. (It is unlikely that there would be any catastrophes incurred but notknown at the valuation date).

4. Currency Movements

It is clearly desirable for the sake of homogeneity that experience in different territories should be examined separately, provided there is a sufficient volume of data. Although the figures in the UK supervisory returns are to be shown in sterling, the analysis of experience should be made in original currencies to obviate the distortion caused by fluctuating exchange rates. However, it is not always possible to segregate the data completely on the basis of currency. For some risk groups, particularly those risks that are of an international nature, the currency in which a claim is made may not be known in advance. Even where the policy conditions prescribe payment in a particular currency, the amount of claim ultimately paid may be effectively linked to some other currency, e.g. depending on the location of the claim event or on the country in which court action was pursued. If records are available of the proportion of claim payments made (or effectively made) in each currency within the particular risk group being analysed, then adjustments to the experience figures can be made to compensate for past changes in exchange rates. However, it is likely that only approximate adjustments would be possible in practice.

5. Miscellaneous

This paragraph considers a further set of distorting influences which are not outside the control of the insurer, but which are extraneous to the claims experience. For example, some methods of estimating outstanding claims involve the calculation of ratios of claim payments (however defined) to earned premiums. In such cases, it is important to be aware of any changes in the general level of premium rates and to adjust the ratios for such changes so that the figures examined are on a consistent basis. Similarly, previous changes in reserving techniques may make earlier figures not directly comparable with later figures. It may also happen that pressure from management in their desire either to maximise or to stabilise company profits could have had a variable effect in the assessment of reserves in previous years. The specialist within the insurance company who is charged with responsibility for assessing outstanding claims ought to be aware of all these factors, but the specialist outside the company, relying only on published information, may not be. In that case, a greater degree of uncertainty may be expected in estimating the outstanding claims provision.

E. REINSURANCE ARRANGEMENTS

Many of the factors affecting direct business are also relevant when considering reinsurance arrangements. The two aspects of reinsurance must be considered, i.e. the reinsurance that an insurance company accepts from other insurance companies and the reinsurance which it cedes to its reinsurers.

1. Inwards Reinsurance

There are a number of reasons why there is greater uncertainty associated with the valuation of liabilities for reinsurance than for direct insurance. These are:-

i) **Data**

The reinsurer's data can be unreliable, sparse, unobtainable and difficult to use. Considering the different sections of reinsurance business:-

a) For fire and accident proportional treaty business only minimal information will be available, i.e. for each treaty for each quarterly account the reinsurer will be told the overall premiums, claims, commission, premium reserve and loss reserve (plus possibly information on large claims).

b) For accident and fire non-proportional business he will be provided with individual claims information, i.e. for each claim instalment he will be informed of the claim instalment amount and the revised outstanding claim, both the cedant's gross ("from the ground up") and the reinsurer's share. However, although the reinsurer supposedly has full information on the outstanding claims he still has to determine the IBNR position and what he does not know is whether the reinsured's outstanding claims procedures have changed for any reason, e.g. due to a management directive on future inflation or employment of a more (or less) conservative claims manager. If the reinsurer does not have a claims manager whose brief is to investigate and assess outstanding claims then he has to place complete reliance on the insurers' claims advisers.

c) For marine and aviation business outstanding claims information is not always supplied by the reinsured and when it is supplied is not necessarily reliable, for various reasons. This business is worldwide (and multi-currency) and information from some territories is particularly difficult to obtain. Also the procedures are not available to channel the information required.

Information on numbers of claims is not available for proportional business and is not of any particular use for estimating liabilities for non-proportional business as only claims above retentions are known about and numbers of claims above retentions are particularly volatile.

There are also problems with variations in the data. That is, business received from different reinsureds will be completely different: with different underwriting characteristics, retention levels etc. and therefore likely to be much more heterogeneous than direct business.

ii) Claim Payment Delays

For reinsurance the delays in payment of claims (and, to a lesser extent, premiums) are much longer than for direct business. The following table shows the flow of reinsurance premiums and claims for a typical underwriting year (based on the "Mercantile and General Company's" experience).

The flow of reinsurance premiums and claims for a typical year:-

Development Year	Fire Non-Proportional NPI %	Fire Non-Proportional Claims %	Accident Non-Proportional NPI %	Accident Non-Proportional Claims %	Marine NPI %	Marine Claims %	Aviation NPI %	Aviation Claims %
1	51	19	53	1	20	5	13	6
2	43	52	34	5	59	36	59	26
3	6	18	9	8	17	30	20	21
4		6	2	10	4	14	7	18
5		3	1	11		7	1	5*
6		2	1	12		3		16
7				11		2		3
8				9		1		2
9				6		1		1
10				4		1		1
11				3				1
12 and over				20**				
Total	100	100	100	100	100	100	100	100

Notes on Table:-

* The development for each account has been smoothed, except for the low figure for Aviation in development year 5 which is due to underlying portfolio movements.

** Accident non-proportional could take up to 25 years (or more) to fully develop: the 20% for development years 12 and over is an approximate estimate only.

Any method of calculation of liabilities based on estimating results so far in the future will be prone to error for the following reasons:-

a) Inflation rates, particularly as they affect claims payments will change unpredictably.

b) If future payments are based on historical past payments then a number of past year's data will be required to determine that there is a pattern and then to provide a basis using this pattern. Patterns of payment may well change due to changing circumstances (see Section A) and changing mixes of business (see Section B).

c) If grossing up factors such as development ratios are used they will be very large for the most recent underwriting years.

d) There may well be insufficient data available, particularly for a new company or one writing a new class of business, to provide a valuation basis.

For different categories of business:-

a) For Accident non-proportional business the fact that claims take such a long time to develop necessitates using valuation methods which are based on development of incurred claims (i.e. accumulated claims paid plus known outstanding claims). Note that the incurred claims may take seven years to develop fully if treaties are subject to stability clauses and even longer for liability business with no stability clause.

b) Marine and Aviation business, is not as long-tailed as Accident non-proportional but still takes 10 or 11 years for claims to fully develop. However, here incurred claims cannot be used to derive the I.B.N.R. for the reasons given in (i)(c) above.

iii) Valuation Methods

There is no consensus of opinion as to what methods of valuation should be used to determine the liabilities for reinsurance business. However, the number of methods which can be used are more limited for reinsurance than for direct business because of the problems with limited data and the long-tailed nature of the business. The reinsurer has therefore to select from the limited number of methods which he can use rather than one which may be less prone to error but requires more information than is available.

The problem is most apparent for fire and accident proportional business. This is accounted for on an "open" and "closed" year basis, i.e. "open" for those treaties where further quarterly returns are expected in regard to premium; and for a treaty where the fourth quarterly account has already been received, the accounting is "closed" for that particular treaty by means of portfolio transfers. It was the custom until recently to, in effect, take the liability for the open year as break-even, i.e. as premiums less commission, management expenses and losses paid and for the closed year as the premium reserves plus claims reserves notified by the cedants, i.e. with no allowance for I.B.N.R. It has become increasingly apparent that reserves should be set up for I.B.N.R. However, we are here very short of methods which could be used as data is very limited and in particular underwriting year information is not available. About the only method which can be applied is to look at the past history of loss ratios for the account, consider from experience and market considerations whether there is an improvement or deterioration in the underwriting performance, hence estimate what loss ratio is expected for the "open" or "closed" year under consideration and apply this loss ratio. It can be seen that this method will be prone to considerable error!

2. Outwards Reinsurance

i) Net liability calculation

An insurer, whether it writes reinsurance or not, will need to ascertain its run-off liabilities net of outwards reinsurance as well as its gross liabilities before retrocession outwards. There are in effect two ways in which the net liabilities may be calculated.

a) By calculating the gross liability and the reinsurance outwards liability separately and hence calculating the net liability as the difference between these two figures.

b) By using data which is net of retrocession and so calculating the net liability directly.

Both of these methods can be considered although different results are likely as the valuation methods are not usually additive models. If method (b) is used then the gross liability still needs to be calculated and problems may be caused by using this order of calculation, i.e. in obtaining results which are consistent. In general it would appear to be more logical to consider gross and reinsurance outwards as two separate entities, particularly where the proportion reinsured is substantial. Where method (b) would seem to be advantageous is in the treatment of large claims covered by catastrophe reinsurance whereby calculating the figures net could be done with less problems. The gross, reinsurance outwards approach in this case would produce distorted results unless special allowance is made for large claims.

ii) Catastrophe Covers/Large Claims

The treatment of large claims and catastrophe covers can produce distortions in the results as there are a number of ways in which allowance is made for these. Also, problems are caused by whole account and other forms of catastrophe cover which are used to protect more than one account as it is then very difficult to obtain the liability figures for each account separately.

APPENDIX 1

Papers on Outstanding Claims Reserves

The following is a list, in alphabetical order of author, of those papers which the working party considered might be of relevance to the subject of outstanding claims reserves. The "code" assigned to each paper has the following meaning:

Code	Meaning
0	No particular method given: Normally an overview or general paper.
1	One method in particular is presented and described.
2	More than one method is presented: either of a particular type or a survey of methods.
3	Not considered relevant to the subject so no summary given.
4	Not yet investigated.

Summaries of papers are therefore given for codes 0, 1 and 2, but not for codes 3 or 4,in Appendix 2.

Author(s)	Title	Code
Abbott, W.M. et al	Some thoughts on technical reserves and statutory returns in general insurance.	2
Ackman, R.C., Green, P.A.G., and Young, A.G.	Estimating outstanding claims: general insurance monograph.	2
Albrecht, P.	Parametric multiple regression risk models: Some connections with I.B.N.R.	2
Allen, T.W.	An accountant's viewpoint	0
Beard, R.E.	Some historical, theoretical and practical aspects.	2
Beard, R.E.	Some statistical aspects of non-life insurance.	3
Beard, R.E.	Verification of outstanding claim provisions - separation technique.	1
Beck, J. H.	General insurance.	1
Benktander, G.	An approach to credibility in calculating I.B.N.R. for casualty excess reinsurance.	4
Bennett, M.C. and Taylor, J.M.	Motor outstanding claims	2
Bennett, M.C.	Models in motor insurance	3
Berquist, J.R. and Sherman, R.E.	Loss reserve adequacy testing: a comprehensive, systematic approach.	4
Bornhuetter, R.L. and Ferguson, R.E.	The actuary and I.B.N.R.	2
Brown, A.	Stability of claim payment patterns	0
Bühlmann, H. Schnieper, R. and Straub, E.	Claims reserves in casualty insurance based on a probabilistic model.	2
Buoro, G, Ferrara, G. and Quario, G	Statistical approach to the outstanding claims reserve in motor insurance.	1
Caffin, S. W.	The design of output tables for the analysis of general insurance statistics.	3
Clarke, H.E. and Eagles, L.M.	Mathematical density functions applied to a liability insurance portfolio.	1
Clarke, T.G.	An Actuary looks at claims provisions in general insurance.	2
Clarke, T.G. and Harland, N.	A practical statistical method of estimating claims liability and claims cash flow.	1

Author(s)	Title	Code
Clarke,T.G. et al	Technical reserves working party report No. 1: I.B.N.R. reserves	2
Clarke,T.G. et al	Technical reserves working party report No. 2: outstanding claims reserves	2
Committee on Loss Reserves.	Statement of principles regarding property and casualty loss and loss adjustment expenses.	0
Cork,C.F.	Trend and Loss development factors	4
Craighead,D.H.	Some aspects of the London reinsurance market in world-wide short-term business.	1
Eeghen,J. van	Loss reserving methods	2
Ferrara,G. and Quario,G.	Distribution of the number of claims in motor insurance according to the lag of settlement.	1
Finger,R.J.	Modelling loss reserve developments.	1
Fisher,W. and Lange,J.	Loss reserve testing: a report year approach.	4
Fisher,W. and Lester, E.P.	Loss reserve testing in a changing environment.	2
Forbes,S.W.	Loss reserve valuations and financial results in non-life insurance.	0
Foster,G.T.	The determination of I.B.N.R. reserves and the effect of I.B.N.R. on the underwriting of excess of loss business.	1
Fowler,T.W.	Liability I.B.N.R. reserves	1
Franckx,T.W.	La théorie de la crédibilité et son utilité.	0
Guaschi,F.E.	Delay problems in reinsurance	2
Hachemeister,C.A.	A stochastic model for loss reserving.	1
Hachemeister,C.A.	A structural model for the analysis of loss reserves	0
Hachemeister,C.A.	Breaking down the loss reserving process	4
Harborne, E.S.	The setting of reserves in an inflationary world.	2
Harding,V.	Treatment of incurred but not reported claims.	1
Homewood,C.J.	An administrator's definition of the problem.	0

Author(s)	Title	Code
Homewood,C.J.	Verification of technical reserves with particular reference to motor insurance.	0
Johnson,P.D.	Translation of a memorandum describing the German reserve clock method of arriving at reserves for outstanding claims.	1
Jong,P. and Zehnwirth,B.	A random coefficients approach to claims reserving.	4
Jong,P. and Zehnwirth,B.	Claims reserving, state-space models and the Kalman filter.	1
Karlsson,J.E.	The expected value of I.B.N.R. claims	1
Khury,C.K.	I.B.N.R. methods for the liability excess of loss reinsurer	1
Kramreiter,H and Straub,E.	On the calculation of I.B.N.R. reserves	1
Landin,D.	Determining the I.B.N.R.	0
Linnemann,P.	A multiplicative model of loss reserves: a stochastic process approach.	1
Masterson,N.E.	Problems in motor insurance - claims reserves	1
Matthews,T.J.	The valuation of general insurance claim reserves.	1
McClenahan,C.L.	A mathematical model for loss reserve analysis.	1
Pentikainen,T.	Reserves of motor-vehicle insurance in Finland	1
Philipson,C.	A generalised model for the risk process and its applications to a tentative evaluation of outstanding liabilities.	1
Pollard,J. H	Outstanding claim provisions: a distribution-free statistical approach.	1
Reid,D. H.	Claim reserves in general insurance.	1
Ryder,J.M.	Claim estimation models.	0
Sawkins,R.W.	Analyses of claims run-off data - a broad view.	2
Sawkins,R.W.	Some problems of long-term claims in general insurance.	1
Simon,L.J.	Distortion in I.B.N.R. factors	1
Skurnick,D.	A survey of loss reserving methods.	4
Straub,E.	I.B.N.R. - a difficult marriage between practice and theory.	1
Straub,E	On the calculation of I.B.N.R. reserves	1

Author(s)	Title	Code
Tarbell,T.F.	Incurred but not reported claim reserves.	1
Taylor,G.C. and Ashe,F.R.	Second moments of estimates of outstanding claims.	4
Taylor,G.C. and Matthews,F.J.	Experimentation with the estimation of the provision for outstanding claims in non-life insurance.	4
Taylor, G.C.	An investigation of the use of weighted averages in the estimation of the mean of a long-tailed claim size distribution.	4
Taylor, G.C.	General insurance database and methods of claim reserving	0
Taylor, G.C.	Separation of inflation and other effects from the distribution of non-life insurance claims delays.	1
Taylor, G.C.	Some practical variations of the separation method.	1
Taylor, G.C.	Statistical testing of a non-life insurance run-off model.	2
Taylor, G.C.	Testing goodness of fit of an estimated run-off triangle.	0
Verbeck, H.G.	An approach to the analysis of claims experience in motor liability excess of loss reinsurance.	1
Vylder, F. de	Estimation of I.B.N.R. claims by least squares.	1
Wit,G.W. and Kastelijn,W.M.	I.B.N.R.	2
Wit,G.W. and Kastelijn,W.M.	The solvency margin in non-life insurance companies.	3

APPENDIX 2

Summaries of Papers

In alphabetical order of the author(s).

Headings:

Author(s)

Title

Source

Summary of paper

Abbott, W.M. et al.

Some Thoughts on Technical Reserves and Statutory Returns in General Insurance

JIA 101 (1974) p.217

This paper contains five sections which are defined as follows:

(i) The function, contents and scope of the statistical returns required under the Insurance Companies (Accounts and Forms) Regulations 1968 (S.I. No.1408).

(ii) Observations on the completion of the Returns.

(iii) Data extracted from the Returns.

(iv) Estimation of Technical Reserves.

(v) Some theoretical considerations.

The first three sections are not directly relevant to the problem of outstanding claims reserves (also the 1968 Regulations have now been superceded) As regards the other two sections:

Section (iv) Attention is focused on outstanding claims reserves, IBNR reserves, unearned premium reserves and unexpired risk reserves. The authors state that they are not putting forward a procedure for determining the adequacy of the reserves for outstanding claims based on the statutory statistical returns. However, they do give a list of factors which they consider are relevant when examining outstanding claims reserves.

(a) Size of Account - the smaller the account, the more variation and hence the need to refer to industry data.

(b) Company Practice.

(c) Type of Portfolio.

Four possible approaches to the estimation of outstanding claims reserves from the DTI returns are proposed.

1. Use of the ratio of total incurred cost for a cohort of claims to the total payments already made at that time.

2. By calculating the average costs by duration to settlement and combining these with an estimated decrement table.

3. By comparing the total incurred cost and total payments on settled claims with the percentage of claims settled. This is an extension of 1.

4. By developing an estimate for the Average Incurred Amount.

The remainder of this section deals with the other reserves mentioned above.

Section (v) This section refers to two fundamental matters, namely, heterogeneity and the variances of reserves.

Ackman, R.C.; Green, P.A.G. and Young, A.G.

Estimating Outstanding Claims

B3: General Insurance Monograph

This draft monograph contains substantial amounts of general basic information on the subject of outstanding claims.

On the question of the statistical methods available and in particular when such methods require external assumptions such as future inflation rates, the comment is made that it is usual to employ a range of assumptions to indicate the sensitivity of the result to differing bases. The paper analyses which classes lend themselves to statistical methods before proceeding to describe the common statistical approaches currently in use namely:-

(i) Simple Ratio

(ii) Run-Off Triangle

(iii) Basic Chain Ladder

(iv) Inflation Adjusted Chain Ladder

(v) Separation Method

(vi) Average Cost per Claim Method

(vii) Reid's Method

There follows a quite detailed section on why statistical methods 'Fail' with the main reasons being stated as due to:-

(a) External Influences - Uncertainty as regards future claims inflation and frequency.

(b) Changing Claims Process - Reporting of Nil claims, settling procedures etc.

(c) Changing Business Mix.

(d) Small Portfolios, Large Claims and Random Fluctuations.

The next section of the paper concerns itself with statistical methods for estimating IBNRs which in practice is a description of the construction of a delay table.

The paper then considers the contentious issue of whether or not provisions should be discounted and the point is made that in general for published or statutory accounts, undiscounted provisions are used. However they go on to say that where provisions are to be used for management accounts or premium rating, then there is a strong argument for the use of discounted provisions at suitably conservative rates of interest.

Finally, attention is turned to the monitoring of results which would normally consist of identifying the difference between actual and expected claim payments. Far more difficult is an explanation of the differences. For this, something similar to a life analysis of surplus has to be carried out identifying the sources of suuplus and deficit.

Albrecht, P

Parametric Multiple Regression Risk Models: (3 papers)

ASTIN Colloquium (1982)

A standard compound Poisson distribution is used for the total claim amount of an individual risk, the parameters being functionally dependent on a set of exogenous variables. A detailed theoretical model is then formed for the whole portfolio of risks.

The main paper then gives examples of possible models (multiple linear Poisson regression, log-linear Poisson regression etc). The author uses maximum likelihood estimation to obtain the parameters and a number of theoretical theorems are given. Various chi-square and F-distribution tests of the goodness of fit are given and there are sections on the choice of exogenous variables and transformations on them. The actual algebra is complex and cannot easily be summarised. The reader is referred to the original papers for further details.

The second paper shows how a number of IBNR models (by Verbeek, de Vylder, van Eeghen, Clarke/Eagles etc.) can be represented within the framework of the basic model.

The third paper shows that the model could be applied to premium rating and compares its flexibility with a number of existing methods.

Allen, T.W.

An Accountant's Viewpoint

I.M.A. p.156

The subject of claims provisions for non-life insurance is considered from the viewpoint of an accountant and auditor. It does not purport therefore to show any ways of calculating reserves. It is interesting from its stated viewpoint, e.g. in expanding the four fundamental accounting aspects in S.S.A.P.2: "going concern", "accruals", "consistency" and "prudence" and hence to explain why a profit is only allowed to come in as it is earned but a loss has to be provided immediately.

Beard, R.E.

Some Historical, Theoretical and Practical Aspects

I.M.A. p.26

The author considers the estimation of the provisions for liability in respect of non-life claims. He outlines some of the historical background, provides a guide to the mathematical and statistical contributions to both the structural patterns of the claims distributions and the related risk theoretical models and links the theoretical aspects with practical conditions.

After a brief historical introduction the operation of an insurance company is considered from a risk theory viewpoint and hence the effect on the ruin probability of under-estimating the outstanding claims liability. The distributions of claims by amount and over time are discussed. The problems in determining the result and reasons for inaccuracies are considered. The two methods whereby a company can calculate its reserves are given as case by case estimation and the simple average claims method (or a combination of both), although further analysis, particularly on data presented in year of account within year of origin format, would be needed for longer tailed portfolios.

Assessment of a company's results by outside observers and the use here of claim distributions and of chain ladder techniques are considered. An analysis is given of statutory returns based on limited data and, in the Addendum, of the mathematical structure of long tail settlements based on a longer period of data.

The paper is worth reading for the outline it gives of the problems and some practical solutions to calculating outstanding claims provisions.

Beard, R.E.

Verification of Outstanding Claim Provisions - Separation Technique

ASTIN Bulletin Volume 9 (1977) Parts 1 & 2 p.26

The paper shows a method applying the separation technique where there is no information on the number of claims.

If Pij is the total amount paid in development year j for year of origin i the standard separation method expresses this as the product of three factors:-

$\hat{P}_{ij} = n_i \, r_j \, \lambda_{i+j}$ where n_i is a measure of number of claims.

The paper estimates the values of $P_{i,j+1} / P_{ij}$ as $R_j L_{i+j}$ where $R_j = r_{j+1}/r_j$ and $L_{i+j} = \lambda_{i+j+1} / \lambda_{i+j}$. In the standard method the sum of the r's is unity. The paper expresses the sum of the R's as a scaling factor Z. This can be set to any value (e.g. unity) and the R's and L's estimated by the usual separation method. There are two problems: bias can be introduced by a distortion of claim payments in a particular calendar year and projection of the L's involves estimating future changes in the rate of inflation. If the underlying r's and λ's are wanted the paper shows a possible method of estimating Z.

The author admits that the method can be difficult to handle when conditions are changing rapidly, but says that in practice the results would be compared with those of the chain ladder as the differences can provide information about the claim settlement structure.

Beck, J.H.

General Insurance

Magazine of the Faculty of Actuaries' Students Society Vol.V, No.4,p.29 (1979)

This paper is not specifically concerned with outstanding claim reserves, being more a wide ranging overview of the work of an Actuary in General Insurance.

However, the author does suggest that 'one of the Actuary's most important tasks is to make an overall estimate of the reserve as a check on the adequacy of the total of individual estimates' and an example is then given based on the author's own Company's U.K. private motor position.

The method adopted is to calculate the inflation adjusted average payments in each year of development, identify and exclude any out-of-line figures and plot the resultant points on log graph paper. With his example he finds that all the points after the first two lie on a straight line and he then produces his 'smoothed average payment development pattern using his first two actual points followed by his newly ascertained geometrically progressing series'.

The normal methods of making allowance for future different levels of inflation are used and his conclusions are that his results are very sensitive to the assumed rate of future inflation. He suggests that if his model is at all realistic, the conclusion has important consequences.

By comparing the actual reserves held with his statistical method, he ascertains the future rate of inflation which would cause these two reserves to equate. He then goes on to conclude that if all other sources of error could be ignored, the inference is that the company believes that a sustained inflation figure equal to that found is highly unlikely.

Another consequence is that an increase in the rate of inflation will require a more than corresponding increase in the reserve held for existing claims. The author then proceeds to reject the suggestion that higher claim payments in respect of existing claims would be automatically offset by an increase in investment income. He suggests that the yield from the existing investments will not increase as inflation increases.

This was considered to be an interesting and useful paper.

Bennett, M.C. and Taylor, J.M.

Motor Outstanding Claims

JIAss Vol.23 (1978) p.75

The main part of this paper is devoted to examining methods of estimating motor outstanding claims based on past Payments. Traditionally this involves using an inflation adjusted chain ladder method or the Separation method (which are subsequently described) but the authors suggest an alternative approach based on the inflation adjusted average payments for each development year. Thus the future payments are not obtained by grossing up the payments to date but by ignoring such payments and taking simply the average payments in each development year.

These three methods are then applied to two different sets of data and the authors' conclusions are that their suggested method (modified if necessary to cope with particular circumstances) is likely to be as satisfactory as any other method for motor insurance outstanding claims.

The paper concludes with brief descriptions of several other methods of minimal interest.

Bornhuetter, R.L and Ferguson, R.E.

The Actuary and IBNR

P.C.A.S. 59 (1972) p.181

The first part of the paper discusses a basic IBNR formula developed by T.F. Tarbell in 1934.

Tarbell's formula was:

$$\frac{N^{y}{}_{10-11-12} \times C^{y}{}_{10-11-12}}{N^{y-1}{}_{10-11-12} \times C^{y-1}{}_{10-11-12}} \times I^{y-1}$$

where:
N = number of notices
C = average incurred per notice
I = amount of IBNR
y = designates current year
y-1 = designates previous year
Subscripts denote month

In other words, the actual IBNR as realised in a given period of time was related to some base, and the resulting factor was then applied to the current base. Any number of bases can be used including earned premiums, case incurred, outstanding case reserves, or premiums in force.

When selecting an exposure base, one must consider the potential distortion in the IBNR reserve which can occur if the book of business is growing rapidly (especially a new book of business). This distortion can be a very real problem and is partly a function of the base used, the term of business, and the length of the tail. The formula is best suited to stable volumes of business and short-tail classes.

A one year run-off method, such as Tarbell's, will lead to a woefully inadequate reserve structure if loss development patterns are deteriorating and especially if the volume of business is increasing.

The paper then goes on to describe the classical chain ladder approach and the necessary pre-requisite of homogeneous sub-sets of data.

Brown, A.

Stability of Claim Payment Patterns

I.A.A. p.61

Most statistical methods for estimating outstanding claims depend on the assumption that the claim payment pattern by development year remains stable over time.

The Kolmogorov-Smirnov test is used with the null hypothesis that two samples of observed values (in this case, r_j, the cumulative proportion of ultimate payments made after time j in Taylor's method) come from populations with the same distribution function. The basic test is applied to claim numbers and to payments provided each claim payment is of equal size. A modification is made to allow for the variability in claim size.

This test was applied to a company's data. The first sample used data for the years 1970 to 1974 in calculating a set of r_j's. The second sample used the r_j's derived from the accident year 1975. The null hypothesis (that the claim payment is stable) was rejected at 99.9% significance level.

Bühlmann, H., Schnieper, R. and Straub, E.

Claims Reserves in Casualty Insurance based on a Probabilistic Model

M.V.S.V. 80,1, p.21

The probabilistic model assumed in this paper treats the individual claim amounts, including partial payments, as a random process over time. Thus, ${}^{m}Z_{ij}^{(n)}$ denotes claim amounts paid up to the i th development year for individual claim number (n) occurring in the j th underwriting year, where m is the year in which the claim was first reported. The final claim amount for the individual claim is ${}^{m}Z_{j}^{(n)} = \lim_{i \to \infty} {}^{m}Z_{ij}^{(n)}$. If the cumulative total of claims paid up to i th development year and the final total of claims are denoted by Xij and Xj respectively for the j th underwriting year, we have:

$$X_{ij} = \sum_{m=1}^{i} {}^{m}X_{ij} \qquad \text{where } {}^{m}X_{ij} = \sum {}^{m}Z_{ij}^{(n)}$$

summed over all claims with reporting year m.

$$X_j = \sum_{m=1}^{\infty} {}^{m}X_j \qquad \text{where } {}^{m}X_j = \sum {}^{m}Z_j^{(n)}$$

The reserve required is the difference between the final total and the known total, and this difference is considered in two parts:

IBNER reserve (incurred but not enough reported) = $\sum_{m=1}^{i} \left({}^{m}X_j - {}^{m}X_{ij}\right)$

IBNR reserve = $\sum_{m=i+1}^{\infty} {}^{m}X_j$

The probabilistic assumptions made for the model are:

1. The distribution of number of claims for each underwriting year j is independent, and is Poisson with parameter $V_j\gamma$, where Vj is a measure of volume and γ is unknown.

2. Reporting year and claim amounts are each independent of claim frequency.

3. Events in different underwriting years are independent.

4. Amounts of claim are independent, but have the same distribution. Simular assumption for reporting year.

5. Stationarity of growth rates of individual claim amounts, and so growth rate ${}^{m}\lambda_i$ does not depend on underwriting year.

The data assumed to be available are the usual run-off triangle for cumulative claims paid, but broken down into "m" subsidiary triangles, one for each reporting year. A run-off triangle of cumulative numbers of claims is also assumed to be available.

On the assumption that the distributions of the individual claim amount ${}^{m}Z_{ij}$ and its subsequent development are log-normal, the following expressions are derived for the reserves required at end of year k:

IBNER reserve = $\sum_{m=1}^{k} \left(\prod_{i \geq k} {}^{m}\lambda_i - 1\right) . {}^{m}X_{kj}$

IBNR reserve = $\sum_{m=k+1}^{\infty} p(m) . E\left[{}^{m}Z_j\right] . V_j\gamma$

where p(m) is an unknown variable representing the probability of a claim being reported in year m.

A numerical example is given for a distribution with known parameter values, as this enables the "true" reserve to be calculated using the above formulae. As a comparison of the "true" reserves with the results from a standard chain ladder calculation do not seem very satisfactory, the authors consider three alternative methods of estimating the parameters. Under the main alternative (the M-method) ${}^{m}\lambda_{i-1}$ is estimated as a weighted average of ${}^{m}X_{ij} / {}^{m}X_{i-1,j}$ using

$$ {}^{m}\hat{\lambda}_{i-1} = \sum_{j=1}^{k+1-i} \frac{{}^{m}X_{ij} \cdot {}^{m}X_{i-1,j}}{U_{mj}} \Bigg/ \sum_{j=1}^{k+1-i} \frac{({}^{m}X_{i-1})^{2}}{U_{mj}} $$

where Umj is the number of new claims reported in the m th development year. Similarly, $\rho(\hat{m}) = \sum_{j=1}^{k+1-m} U_{mj} / \sum_{j=1}^{k+1-m} V_j, \ldots$ and the estimate for $E[{}^{m}Z_j]$ is taken as $E[{}^{m}Z_{mj}] . \prod {}^{m}\lambda_i , \ i \geq m,$ where it is assumed that $E[{}^{m}Z_{mj}] = C_m^{-} (1+\delta)^{j-1}$. Then Cm and δ can be determined by postulating certain conditions, which in the M-method are Cm = Cm (δ) and a "least squares" assumption in relation to the average claim amount for new claims. Two modifications to the M-method were considered to overcome the problem of the later reporting years where the number of new claims is few, but these modifications produced a bias to over-reserving.

The four different methods discussed in the paper were then applied to 50 simulations of the distribution with known parameter values, and the conclusions reached were that the standard chain ladder method was as satisfactory as any of the others (except possibly for the most recent underwriting year) and that the standard deviation is relatively high for all methods.

Buoro, G., Ferrara, G., and Quario, G.

Statistical Approach to the Outstanding Claim Reserve in Motor Insurance

T.I.C.A. (170),3, p.455

The paper presents a simple method of assessing the reserves for a portfolio where small claims prevail. It is deemed to ensure accuracy without the administrative work involved in individual estimating. The claims are subdivided into two categories when the claims are first recorded. For the large claims a continuous inventory is envisaged while for small claims the automatic procedure is proposed. This is based on a classification in ranges of size of first evaluation by the adjuster and an analysis of the cost of settled claims classified by first evaluation, amount and age.

If t is a given delay of payment and S_t the amount paid during the last 12 months for claims with duration t and n_t the relevant number of claims, the average cost to be attached to claims with age equal to t_0 is:

$$E\left[S_t : t > t_0\right] = \frac{\sum_{t=t_0}^{\infty} S_{t+1}}{\sum_{t=t_0}^{\infty} n_{t+1}}$$

This average is multiplied by the number of outstanding claims, which is determined by using a Pareto type formula distributed according to duration, to give the reserve at the prices of the year of occurrence. This is then adjusted for inflation to give the final figures.

Clarke, H.E. & Eagles, L. M.

Mathematical Density Functions Applied to a Liability Insurance Portfolio

T.I.C.A. 1980, 2, p. 59

For liability claims the paper studies how the claim distribution depends on the delay of claims settlement. The frequency of claim events are assumed to follow a Poisson distribution and the claim amounts lognormally distributed with moments depending on the delay of settlement.

Various estimators for the moments are discussed and the paper leads the reader through a complete analysis of a portfolio of liability business, calculation of risk premium for future years, the cost of reinsurance protection, and calculation of reserves.

The method used would be useful for classes of business which are stable over time, where the claims events are independent, the delay and claim size are positively correlated and the claim amounts can be fitted by a statistical distribution depending on the delay of settlement. These criteria would rarely apply.

Clarke, T.G. and Harland, N.

A Practical Statistical Method of Estimating Claims Liability and Claims Cash Flow

ASTIN Bulletin Vol. 8 Part 1 (1974) p.26

Most methods involving claim payment triangles fail to take account of different patterns of claim occurrences within different years of origin (e.g. unusually adverse weather causing a large claims volume in January or December one year). The authors accept that a full analysis by month of origin/month of development would be preferable, but say that the volume of data needed would be excessive. Instead they analyse data by year of origin/month of development.

The variables used are:-

n_i = number of claims occurring in month i of a calendar year

c_i = average cost of claim occurring in month i

s_r = proportion of total liability for a monthly cohort paid within r months of beginning of month of claim

k_i = proportion of total liability for a year's cohort paid within i months of beginning of year of claim (i can be greater than 12)

The k's are expressed in terms of the n, c and s, and these formulae are inverted to give s in terms of n, c and k and solved using data on old, well developed cohorts. If the number of claims still IBNR is small, the n's are known. It is not necessary to know the absolute value of the c's, only the ratio between them. These are expressed in terms of seasonal and inflationary factors which may be largely subjective. Calculating the k's involves estimating the outstanding liability for the old yearly cohorts, but the use of well developed cohorts means that any error will be small. From the s's the k's for the most recent yearly cohorts are calculated, although since the model uses number of claims occurred it is necessary to estimate the numbers of IBNR claims for the latest months. Cash paid to date on these annual cohorts is then divided by the relevant k to give the cohort liability.

Although the method is sensitive to changes in speed of payment for the most recent cohort the authors say it has given acceptable answers.

Clarke, T.G.

An Actuary Looks at Claims Provisions in General Insurance

I.M.A. p. 84

This paper highlights some of the methods of estimating the technical reserves in general insurance. It will be found that much of the data currently available and the nature of certain classes of business do not allow sophisticated statistical methods to be used. In estimating the reserve three steps should be taken:

(a) the detailed administrative and contractual requirements of the class of business should be ascertained.

(b) the effect of future changes in economic conditions should be projected.

(c) more than one method of estimating the reserves should be used.

Technical reserves can be divided into four main groups:

(i) outstanding claims, (ii) I.B.N.R. claims, (iii) unexpired premium reserve; and (iv) fluctuation reserve. The paper examines the calculation of each of these groups in some depth. Under (i), five methods are explained.

1. Individual estimates by claims staff (with a projection of the probable underestimation based on past patterns).

2. Average cost per claim method (with due allowance for inflation).

3. Standard table method (based on an underlying mathematical model, usually additive or multiplicative on various parameters, and a large amount of computerised data: therefore, probably only applicable to a large private car account).

4. Average outstanding claims cost (based on the progression of the average cost of outstanding claims by development period within each year of claim).

5. Grossing up method (based on a study of the pattern of claim payments at each stage of the run-off, say each month, to give an average and range of values ascertained from historic data).

Two approaches to the estimation of the IBNR reserve are given:

1. Estimate the number of late reported claims and apply an average cost per claim, adjusted if necessary.

2. Gross up the estimated liability for the claims already notified.

Clarke, T.G. et al

REPORT No. 1 : IBNR Reserves

Technical Reserves Working Party Report : GIRO Seminar (1975)

This paper was the first of a series of papers to be produced by the technical reserves working party, although it was considered that it might have been more logical to produce the outstanding claims reserves paper (Report No. 2) first. The paper states that:

> "It is pointless to strive for greater precision in the estimation of IBNR reserves than can be achieved for the reserves for reported claims. In practice both these reserves should be considered as one when deciding on the overall adequacy of the reserves for an individual class of business".

The paper is divided into two distinct sections:

1. Direct Business
2. Reinsurance Business

These two sections are further sub-divided into four parts:

i) the definitions
ii) a description of theoretical foundations underlying the practical methods
iii) considerations in the selection of practical methods
iv) a discussion of practical applications.

For direct business the IBNR reserve may be considered as the product of

(a) the projected number of IBNR claims (using delay factors based on monthly development of numbers of claims)

(b) a suitable average cost per claim, e.g. using a ratio:

$$\frac{\text{average cost of IBNR claims}}{\text{average cost of reported claims}}$$

Non-proportional reinsurance is considered under four categories:

1. Short Tail

   No IBNR provision is needed.

2. Long Tail (Accident): with cumulative incurred claims being fully developed over about 10 years.

   Four methods are considered:

   (i) In effect the basic chain ladder (on cumulative incurred claims)

   (ii) Calculation of a settlement pattern of total claims paid for a given development year from the chain ladder projection to project cumulative claims paid.

(iii) Equating final expected claims for each underwriting year with the break-even loss ratio based on the development settlement pattern and an assumed rate of interest.

(iv) Projecting the number of claims and multipying ultimate number of claims by an average claim figure (current average claim times a developed modification factor) for each underwriting year.

3. Marine and Aviation

The settlement pattern extends to around 10 years and information is available only for claim and premium amounts and then only at settlement. Two methods of estimation are the appropriate modifications of methods (ii) and (iii) for Long Tail business.

Proportional reinsurance is not normally considered to have an IBNR problem although it may be considered desirable to build up the Fund carried forward from the open year if the relevant treaty year is expected to make a loss.

Clarke, T.G. et al

REPORT No. 2 : Outstanding Claims Reserves

Technical Reserves Working Party Report : GIRO Seminar (1976)

This paper is the second in the series of papers to be produced by the technical reserves working party and is restricted to reserves for reported outstanding claims. The first report was on IBNR reserves. Some of the methods described in this paper, however, could be used in certain circumstances to predict the total reserve for outstanding reported claims plus IBNR and even plus unexpired risk reserves.

The purpose of the report "is to highlight some of the considerations which must be taken into account by the statistical investigator in General Insurance and to indicate possible practical approaches".

The paper describes in quite simplified terms the various common methods which can be used:

1. Case estimates

2. Average cost per claim estimates

3. Methods employing the aggregate of claim payments

    (i) Basic chain ladder
    (ii) Chain ladder with inflation adjustment
    (iii) Chain ladder with separation technique
    (iv) Geometric progression
    (v) Delay table related to premiums

The paper makes the specific point that while long tailed business is not usually amenable to chain ladder methods, it has been observed to exhibit the feature that after an initial period claim payments tend to form a proportion of payments of the previous year i.e. they run into geometrical progression (Method 3(iv)).

One of the concluding suggestions is that it may be useful to obtain information on claim size distribution and to develop a suitable statistical method using such data.

Committee on Loss Reserves

Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expenses

ASTIN Bulletin 10, 3 (1979) p.305-317 or P.C.A.S. 65 (1978)p.74

An excellent paper for students. It introduces and defines many terms in common use within the general insurance industry. It explains, for example, the difference between allocated and unallocated loss adjustment expenses.

Very good definitions are given of accounting date, valuation date and total loss reserve. The paper sets out the various ingredients of total loss reserves, which are:

(i) Case reserves

(ii) Provision for future development on known claims

(iii) Re-opened claims reserve

(iv) IBNR

(v) Provision for claims in transit

Terms (i) to (iii) are usually referred to as reserves for known claims, and terms (iv) to (v) are collectively referred to as IBNR.

Mention is made of the many considerations an actuary must make when establishing reserves. These fall under the headings: homogeneity of data, credibility, date availability, emergence patterns, settlement patterns, development patterns, frequency and severity of claims, re-opened claims potential, aggregate limits, collateral sources, reinsurances, pools and associations, operational changes, change in loss distribution, external influences and reasonableness.

This paper gives the reader a sound introduction to the subject of loss reserving.

Craighead, D.H.

Some Aspects of the London Reinsurance Market in World-wide Short-term Business

JIA 106 (1979) Part 3 p.227

This paper covers what is usually known as the 'Lloyds System' and opens with a description of Lloyd's and its main features.

Lloyd's lays down strict Audit requirements relating to the reserves involved in closing the account. Among the requirements, there is a test for adequacy of the fund, applicable more particularly to the 'reserve at close' at the end of the third year and the run-off of closed years but also required to be applied to the two 'open' years. The test requires that the higher of two amounts be used:-

(a) A stated percentage (depending on class, audit code and development year) applied to the net absolute premium advised

or

(b) The total of the estimated outstanding liabilities - how this is to be quantified, is not set out.

As regards the need to allow for IBNR when closing the account, the author makes the point that, although theory would suggest that there should only be few IBNRs a couple of years after the underwriting year has ended, in practice there can be an appreciable volume. In fact the author suggests that delaying factors are such an important part of reinsurance that IBNR become far more prevalent in the case of reinsurance than in direct insurance.

An essential feature of reinsurance business is that, for the most part, the number of claims is unknown. Hence the statistical approach used in claim analysis on direct insurance business based on numbers reported and average amount of claim is no longer applicable in the case of reinsurance. Other means have to be used.

Since reinsurance business is grouped by Underwriting Year, the author suggests that as a statistical basis for the calculation of IBNR, the only starting point effectively available is the comparative experience table of loss ratios set up in the form of a 'triangulation'. The essential feature of this approach is to provide a comparable incurred loss ratio as a yardstick based on the Company's own experience. It is suggested that separate triangulations be used for each 'type' of reinsurance, namely:-

Direct
Facultative Reinsurance
Proportional Treaties
Catastrophe Excess of Loss Treaties

and in each case differentiating between 'short-tail' and 'all others'. The validity of the concept rests on the premise that, at any elapsed point in time for any Underwriting Year, IBNR is a constant percentage of claims already paid or notified. A problem arises however in the allowance to be made for inflation particularly if a worldwide inflation measure is relevant.

To test the method suggested, data derived from a particular Reinsurance Company was used in an attempt to fit suitable curves by computer analysis, with the curve fitted being of the form:-

Incurred Loss Ratio at time $t = L_{*}\left(1 - e^{-(t/b)^{c}}\right)$
where L_{*}= Ultimate Loss Ratio

The remainder of the paper tends to be of a more academic nature, being concerned mostly with the best method of curve fitting.

Eeghen, J. van

Loss Reserving Methods

Surveys of Actuarial Studies No. 1;
Nationale Nederlanden N.V. (1981)

This is an excellent publication and should be required reading for anyone interested in the subject of loss reserving. The number of actuarial papers published is ever increasing so that it is a complicated and time-consuming task to assemble and arrange all contributions to a given subject. The purpose of the series "Surveys of Actuarial Studies" is to lighten the task by summarising and grouping all important papers on a given subject. This publication is the first in a series .

The introduction talks generally about loss reserving, defines I.B.N.E.R. (Incurred But Not Enough Reserved) and I.B.N.R. (Incurred But Not Reported), and describes some of the problems involved. The data have to be premanipulated before they can be used and to this end the I.B.N.R. loss reserving process described by Straub ("I.B.N.R. - a difficult marriage between practice and theory") is described together with a selection (to serve as an example) of the questions for department executives proposed by Berquist and Sherman ("Loss reserve adequacy testing: a comprehensive, systematic approach"). Indexed summaries of the methods are given with the following column headings:

Method
Main field of application
Required data
Main assumptions
Remarks

Thirteen methods are presented and the suggested grouping is:

1. Simple methods where statistical estimation problems are not involved.

2-6. The entries in the triangle are influenced only by a column effect and a row effect.

7. A fixed pattern for the development years is supposed.

8-11 The entries in the triangle are influenced only by a column effect and a diagonal effect. The diagonal effect represents inflation-like influences - the methods produce estimates of past inflation rates and require prediction of future inflation rates.

12. A very general model, more a way of thinking than an algorithm.

13. A very general model, able to deal with a great variety of external influences. An explicit estimator, including confidence bounds, is derived for the loss reserve.

In general presentation of the methods obeys the following pattern:

a) Model and assumptions.

b) Comments on the assumptions: in which the (abstract) assumptions are translated into situations from practice and circumstances are enumerated which can break down the model.

c) Comments on the data.

d) Computations: the procedure for going from the data to the required answer is given. (Proofs are omitted).

e) Numerical example: this clarifies part d) and serves as a check for those who wish to write a computer program for the method.

In the computations as much uniformity as possible has been maintained, e.g. in indexation of the run-off triangles.

The methods presented are as follows (with the papers from which they are derived):

1. Examples of Simple Methods. Skurnick, D: "A Survey of Loss Reserving Methods".

2. Chain-Ladder Method. (Various Sources).

3. Variations on the Chain-Ladder Method. Berquist & Sherman: "Loss Reserve Adequacy Testing : a Comprehensive, Systematic Approach".

4. De Vylder's Method by Least Squares. Vylder, F. de: "Estimation of I.B.N.R. Claims by Least Squares".

5. Straub's Method. Straub, E: "On the Calculation of I.B.N.R. Reserves".

6. Probabilistic Model. Bühlmann, Schnieper and Straub: "Claims Reserves in Casualty Insurance Based on a Probabilistic Model".

7. McClenahan's Method. McClenahan, C.L.: "A Mathematical Model for Loss Reserve Analysis".

8. Verbeek's Separation Method. Verbeek, H.G.: "An Approach to the Analysis of Claims Experience in Motor Liability Excess of Loss Insurance".

9. Taylor's Arithmetic Separation Method. Taylor, G.C.: "Separation of Inflation and Other Effects from the Distribution of Non-Life Insurance Claim Delays".

10. Taylor's Geometric Separation Method. Taylor, G.C.: "Statistical Testing of a Non-Life Insurance Run-Off Model".

11. Taylor's Regression Method. (Same Paper as Method 10).

12. Hachemeister's Model. Hachemeister, C.L.: "A Stochastic Model for Loss Reserving".

13. Reid's Model. Reid, D.H.: "Claim Reserves in General Insurance".

Finally, a bibliography of 52 relevant papers (including those mentioned above), is given.

Ferrara, G & Quario, G.

Distribution of the Number of Claims in Motor Insurance According to the Lag of Settlement

ASTIN Bulletin, Volume 9 (1977) Parts 1 & 2 p. 119

The paper uses conventional actuarial life table notation to model the number of claims outstanding as a function of the time since they occurred.

If x is the duration since occurrence then $l(x)$ is the number of claims still outstanding and $\mu(x)$ is the instantaneous "force of settlement". The paper assumes that $\mu(x)$ is an increasing function of the number outstanding (i.e. the force of settlement decreases as increases). Assuming a simple function:-

$$\mu(x) = \mu(l(x)) = k.l^{\beta}, \beta > 0$$

a solution $l = (x_0/(x + x_0))$ is obtained for $x > 0$ where $x_0 = 1/\beta k$ & $\alpha = 1/\beta$ (i.e. $\alpha > 0$ & $x_0 > 0$)

The formula is therefore purely ad hoc, but based on a feasible assumption about the rate at which claims are settled. The authors tested the formula by applying it to a portfolio of motor material damage claims. The claims were divided into 6 groups according to the claims handlers' first assessment of their size. Claims over an upper limit were not analysed. The parameters were calculated by a least squares method and gave good fits in 5 out of the 6 groups and it was concluded that the model could usefully be used to calculate future settlement patterns, Finger, R.J.

Modelling Loss Reserve Developments

P.C.A.S. 63 (1976) p.90

An interesting paper for the calculus specialist. Partial differentiation and iterative methods abound. Basically, the method minimises the sum of squares of the differences between observations and estimates.

The paper is considered to be too theoretical at this stage.

Fisher, W. and Lester, E.P.

Loss Reserve Testing in a Changing Environment

P.C.A.S. 62 (1975) p.154

The purpose of the paper is to demonstrate the importance of determining any underlying changes in the claim environment in selecting a reserve test. The paper examines and compares how several different reserve methodologies react to changes in two variables: the calendar/accident year loss ratio and the adequacy of the reserves for reported claims. All other factors which may normally change over time are assumed to remain constant.

Three reserving methods are analysed. They are:

(i) Loss Development - ultimate loss development factors are determined for each accident year based on recent emergence patterns of incurred losses. These factors are used to estimate the ultimate loss liability for each accident year.

(ii) Expected Loss Approach - the IBNR needed for each accident year is the product of the expected losses (based on the expected loss ratio) and 1-1/ULD where ULD is the appropriate ultimate loss development factor.

(iii) Percentage of Premium Method - IBNR factors are computed from historical emerged losses as a percentage of premium.

The paper shows that for a particular line of insurance where results are static over a period of several accident years, both as to ultimate loss ratio and loss emergence patterns, the choice of method is unimportant. All yield the same result.

The paper demonstrates the gearing of each method. This may be summarised as follows:

(i)	Loss Development Method	-	highly geared and produced the highest estimates in the light of loss ratio deterioration
(ii)	Expected Loss Approach	-	the least sensitive of change
(iii)	Percentage of Premium Approach	-	reflected change gradually, the middle of the road approach.

Forbes, S.W.

Loss reserve valuations and financial results in non-life insurance

Journal of Risk and Insurance - 1972/3,p.369

The purpose of this paper is to demonstrate that incorrect estimates of claims reserves can have a significant effect on a company's reported financial results. The paper suggests a method for monitoring the mis-statements of claim reserves at successive year ends, with a view to reconstructing correct earnings statements and improving the accuracy of future estimates of outstanding claims.

Foster, G.T.

The Determination of IBNR Reserves and the Effect of IBNR on the Underwriting of Excess of Loss Business

N.R.G. p.7

This paper examines the problem of estimating IBNR claims for a reinsurer transacting excess of loss business. It is suggested that the direct application of the chain ladder method may well produce unreliable results because of disturbing factors in the experience and that, although a method could be devised based on the frequency of claims and the average amount of claim in relation to the gross premium income of the ceding office for each class of business reinsured, it could be necessary to aggregate experience under dissimilar portfolios in order to obtain sufficient data for such a method. The paper proposes that IBNR reserves could be assessed by an empirical method using the following data recorded for each development year in respect of each underwriting year: (a) amount of new claims advised, (b) savings on expected settlements, (c) adjustments in the value of outstanding claims. The IBNR experience can then be examined on the basis of new claims and on the basis of new claims less savings, i.e. item (a) less (b) plus (c), and trends in the experience can be detected more readily if these two amounts are expressed as percentages of the premiums written. Having determined the assumed percentage factors for these items, the IBNR reserve can be calculated for each underwriting year using the sum of the relevant percentage factors for each outstanding development year. It is suggested that interest should not be taken into account in the IBNR assessment, but used as a provision towards expenses and as a buffer against possible hyper-inflation. A check on the adequacy of IBNR reserves should be made by analysing the revenue account separately for each underwriting year.

Fowler, T.W.

Liability IBNR Reserves

N.R.G. p.30

This paper is concerned mainly with the determination of IBNR reserves for a reinsurer transacting excess of loss liability business. On the basis of the normal run-off triangle of claims paid up to the end of each development year in respect of each underwriting year, cumulative development factors are derived by the standard chain ladder method. The reciprocals of these development factors are referred to as 'lag factors' which can be considered as the ratio of claims paid up to the end of a particular development year to the ultimate amount of claims paid. The lag factors derived from the observed data are then smoothed by an exponential curve-fitting process, and IBNR reserves are determined by applying the complements of the lag factors to the expected ultimate amount of claims. The paper suggests that the ultimate level of claims could be taken as the 'risk earned premium' provided this reasonably reflects the exposure to loss. The need to assess the predictive value of the data used in the construction of the lag factors is considered, and a numerical illustration is given of the effect on IBNR reserves of excluding the earlier underwriting years for one particular set of data. Factors affecting the obsolescence of the data include changes in retention limits or in rates of inflation or in the nature of the business in force.

The level of IBNR reserves should be tested each year by comparing the expected run-off pattern with actual experience, analysing the experience of each underwriting year separately. This analysis may point to reasons for any deviations from the expected pattern and indicate how the proposed level of IBNR reserves should be modified. The analysis should also be made for each individual treaty and for all treaties effected with each ceding company in order to assess possible changes in the experience particular to one ceding company. This analysis may be difficult because of the greater susceptibility to chance fluctuations in small amounts of business.

Franckx, E.

La Théorie de la Crédibilité et son Utilité

A.S. p.1

This general introduction to the above Proceedings explains the need for a sound scientific basis to justify the practical methods adopted for determining outstanding claims reserves, and comments on the different approach required for non-life business compared to life business.

Guaschi, F.E.

Delay Problems in Reinsurance

I.M.A. p.124

This paper considers the subject of claims provision from the point of view of a reinsurance company. The basic problem is that of delay, in the notification to the reinsurer that a claim has been paid and in the settlement of a claim, and also in the payment of the premium. A short discussion is given of the characteristics of reinsurance which give rise to these delays. In order to study the pattern of delay reinsurers find it convenient to relate claims to the calendar year in which the event occurs : the underwriting year.

No one method of determining the size of the provision for claims is so reliable as to preclude the necessity for any other to be used. On the contrary, it is essential to attack the problem from different angles.

Two main methods of calculating the claim reserve are explained:

1. Basic chain ladder method. A number of years of data are required (up to 11, or more, development years). The development ratios used may be based on the maximum available number of years in the development triangle, or, if it is considered that using the whole column of each development period gives undue weight to data which are out of date, the factors can be based on, say, the five most recent revenue years. Claims paid and outstanding may be projected instead of claims paid as a shorter development period is then required.

2. An average claims method. The percentage of claims by number reported each year is determined. Using the number of claims reported to date for each underwriting year the expected ultimate number of claims is determined, or, prudently, the upper, say, 1 per thousand limit to the number of claims (by assuming that the above percentages are binomial variables). Using development triangles of claims paid and outstanding, and numbers of reported claims, the average claim paid and outstanding for each development year is calculated, and hence the ultimate average claim determined by extrapolation.

Then for underwriting year u which has d development years, claims liability = $N^1{}_u . \dfrac{C_u}{N_u} . \dfrac{M^1}{M_d}$

where $N^1{}_u$ = ultimate number of claims for underwriting year u

C_u = claims paid and outstanding, development year d for underwriting year u

N_u = number of claims reported to development year d, for underwriting ye ar u

M^1 = ultimate average claim

M_d = average claim paid and outstanding for development year d

In calculating outstanding loss reserves allowance should be made for the fact that premium income is due in respect of the most recent underwriting years.

Hachemeister, C.A.

A Stochastic Model for Loss Reserving

T.I.C.A. 1980, 1, p.1 85 or A.S. p.6

The author attempts to deal with the problems of analysing loss reserves under changing conditions and with a limited number of actual losses by considering the loss size of individual claims as a stochastic process and further by identifying discrete sub-categories of conditions within which claims can be analysed. Such categories of claims might include claims for which the current reserve is less than some fixed amount, claims involving bodily injury, claims of doubtful liability, etc. By specifically including size ranges as part of the identification of the claim categories the value of claims falling within such categories will be subject to limited variance. The analysis of claims therefore concentrates on the analysis of the average claim within each category and the probability of claims moving from one category to another. By properly choosing claim categories it is hopeful that only a small part of the variation in the claim process will come from variation within these claim categories.

A structure for incorporating these conditions into the loss reserving model is developed but this is based on the notation and concepts introduced in a previous paper by the author 'A Structural Model for the Analysis of Loss Reserving'. The notation introduced there (apparently) considered the use of partial information through some point in time to condition the random variables associated with an arbitrary insurance related stochastic process. The present paper develops the statistical relationships between means and variances of the loss reserves and explores the implications towards bias and the accuracy of the loss reserve estimates with different definitions of the categories.

The mathematics presented looks rather daunting and no examples are given. The paper is considered to be of theoretical interest only.

Hachemeister, C.A.

A Structural Model for the Analysis of Loss Reserves

Bulletin d'Association Royale des Actuaries Belges No. 73 (1978) p.17

This paper develops from first principles the framework of a model for loss reserves, and explains the relevant insurance concepts for the benefit of a reader with no previous knowledge of insurance matters. Standard types of notation, frequently used in statistical work, are introduced for random variables, realizations, probability distributions, moments and estimators. Actuaries not immersed in theoretical statistics might find the paper useful for this reason, but otherwise there seems no advantage in summarising the paper.

Harborne, E.S.

The Setting of Reserves in an Inflationary World

I.M.A. p.139

The period over which claim payments become due is spread over a number of years and the amount of the liability is often determined at the prices ruling many years after the premium rate was determined, particularly for third party injury claims. The traditional methods of calculating liabilities are considered to be inadequate and it is important to make explicit forecasts of the rates of inflation and other factors used to arrive at these liabilities. Two methods are put forward (outline only):

1. Calculation of liabilities using averages

Large claims are considered separately on a portfolio basis. The rest of the data is analysed by rating factors to produce period by period average levels of claim frequency and claim amount, and these are then plotted against time. Projections are made using a combination of statistical techniques and outside information and judgement, and formal estimates of the rate of inflation are determined. The reserves are then:

(a) Unsettled claims: Up to 6 months : number of claims reported in each risk class x average settled claim amount for appropriate period.

Over 6 months : case estimates after specific review.

(b) IBNR: Number of policies in force x delay factor x estimated average settled claim amount.

(c) Unexpired risks: Number of policies in force x days to run x estimated claims frequency x estimated average settled claim amount.

2. Estimation of Liabilities from run-off statements

Various methods have been put forward which multiply up the amount paid at the end of usually the first year of an account by reference to the pattern established in previous years. Some drawbacks to this approach are set out in the paper. The most important factor at present (1974) that can cause insurers to lose money is inflation. The proposed method for taking account of the best estimates of the rates of inflation is to 'de-inflate' the historic data to base year prices, project the results and use the forecast inflation rates to produce new figures, i.e. the inflation - linked chain ladder method.

Harding, V.

Treatment of Incurred But Not Reported Claims

N.R.G. p.55

The author explains that the assessment of IBNR reserves is a more difficult problem for the excess of loss reinsurer than for a direct insurer. The reinsurer is involved in a much smaller number of claims, with a much higher average amount and a slower settlement pattern, and the likely amount may be difficult to estimate at the time of notification. For the direct insurer, past experience should be sufficiently credible for ratios derived from that experience to be used in the calculation of IBNR reserves. The ratios examined could be either the ratio of IBNR claims subsequently paid to the claims paid to date (or to earned premium income) or alternatively the ratio of the number of claims paid to date (or to number of policy years exposed) with a separate assessment of the average amount of claim.

For the reinsurer, chance fluctuations in the small number of IBNR claims for one underwriting year under one treaty are so significant that past experience under one treaty does not provide a credible basis. Amalgamation of different treaties is complicated by variations in retention limits and in the nature of the risk. The paper proposes that IBNR reserves should be determined as the expected ultimate claims cost for the whole portfolio, less claims already paid and known claims outstanding assessed on the basis of individual estimates. The ultimate claims cost is determined by a statistical model on the assumption that:

(a) the number of reinsurance claims follows a Poisson distribution;.

(b) amount of claim in excess of retention limit (E) follows a log-normal distribution, truncated below E.

(c) reinsurance claims are stochastically independent.

The ultimate claims cost as a proportion of gross premium income written by the direct insurer can be expressed as $m_E \cdot \int_E^{\infty} (z - E) \frac{dF(z)}{1 - F(E)}$

where m_E denotes expected number of reinsurance claims per unit of premium income and $F(z)$ is the log-normal distribution for original claim amount z. After adjusting the claims data for inflation and the premium income for any significant changes in rating, the original claims are amalgamated, to the extent possible on the basis of a common E, to estimate the parameters of the claims distributions. The ultimate claims cost can then be estimated using a simplified form of the above integral.

Homewood, C.J.

An Administrator's Definition of the Problem

I.M.A. p.3

This paper sets out in non-mathematical terms the nature of theproblem of testing the adequacy of non-life reserves and explains how it had reached its then present (1974) level of importance. The chronology of relevant legislation is traced and related to the evolution of the Department of Trade's thinking on the subject in the context of supervision.

The paper is of no relevance to the calculation of claims reserves, and is out of date as a chronology of events.

Homewood, C.J.

Verification of Technical Reserves with Particular Reference to Motor Insurance

O.E.C.D. Report by the Working Party on the Verification of Technical Reserves No. AS(68) 1

This is one of a series of papers prepared by a Working Party of the Insurance Committee of the Organisation for Economic Co-operation and Development. Although the Working Party concentrated on motor insurance initially, subsequent papers in the series covered other classes of general business. As the aim of the Working Party was to determine the best method(s) that could be employed by supervisory authorities in the verification of technical reserves, they were not directly concerned with methods of estimating reserves that might be appropriate for insurers to adopt. After considering various tests, including simple ratio tests, the Working Party concluded that a statistical approach was to be preferred and they recommended that the chain ladder method applied to the run off triangle of data would normally be a satisfactory approach. The Working Party had hoped to investigate the limits within which random fluctuations in the experience would be likely to occur, and proposed that claims data should be collected and analysed on the basis of a classification of claims into various cost bands. However, little cost band analysis was in fact completed. The series of papers is of historical interest in explaining the background to the introduction of run-off data in supervisory returns and for the recognition given to the chain ladder method, but is not a convenient source of reference for studying the chain ladder method.

Johnson, P.D.

Translation of a memorandum describing the German "Reserve Clock" method of arriving at reserves for outstanding claims

GIRO Bulletin No. 3

This GIRO Bulletin consists of the translation Peter Johnson carried out in 1975 of a memorandum describing the German "Reserve Clock" method of arriving at reserves for outstanding claims.

Each recorded motor liability claim (open or closed) is included in the reserve clock for the first time after 6 months and for the second time after 12 months.

All claims occurring in a particular month are grouped together and the total incurred cost (payments + reserves) are calculated as well as the average incurred cost. These costs are then combined in a rolling 12 month format to produce what is called a moving year clock.

To estimate the year end reserve for the claims of a particular year of account, the following procedure is used:-

(a) Let the number of claims recorded in the year of account be x

(b) Let the average incurred cost based on the latest results of the moving year clock be y

(c) Multiply x by y to determine the estimated incurred cost for the year of account.

(d) From this sum, subtract the payments already made in the year of account on the claims recorded in that year.

The rest of the paper then goes into the pro's and con's of whether there is any need to employ any of the more traditional formula type calculations based on recorded payments for closed claims.

Jong, P. & Zehnwirth, B.

Claims Reserving, State-Space Models and the Kalman Filter

J.I.A. 110 (1983)

The paper shows how the State-Space approach can be applied to claims reserving. A State-Space model consists of two sets of equations.

The first set embodies the assumption that each component of a vector (of observations) is made up of a known, time varying linear combination of unknown parameters, and a zero mean error.

Let $y(t)$ be the observed vector
$\beta(t)$ be the vector of unknown parameters
$\upsilon(t)$ be the vector of errors.

Then

$$y(t) = x(t).\beta(t) + \upsilon(t)$$

where $x(t)$ is a known 'design' matrix specifying the manner in which the observations are related to the unknown parameters.

There is also a 'state' equation describing the evolution over time of the 'state' vector (t).

$$\beta(t) = H(t).\beta(t-1) + G(t).V(t)$$

where $H(t)$ and $G(t)$ are known matrices and
$V(t)$ is a vector of zero mean errors.

The paper then goes through the completed matrix algebra to show the vector β (t) can be estimated. It notes that the solution presents computational problems even for microcomputers. There is a major problem in initiating the recursion with an initial estimate of $\beta(0)$, the parameter vector.

The main advantages are the ability to fit any of the common methods to the State-Space framework and the ability to update existing estimates with latest observations.

However the complexity and computing time necessary are big drawbacks.

Karlsson, J. E.

The expected value of IBNR claims

Scandinavian Actuarial Journal - 1976, p. 108

This is a short mathematical note giving the theoretical derivation of an expression for the mean value of IBNR claims on certain assumptions about the independence and the distribution function of the variables involved.

Khury, C. K.

IBNR Methods for the Liability Excess of Loss Reinsurer

N.R.G. p. 65

The paper describes a method of determining IBNR reserves for a reinsurer transacting excess of loss business. It is assumed that each cedant may have a different retention limit, but that changes in a cedant's retention can occur only at the beginning of each underwriting year. The data required by the reinsurer from each cedant at the end of each underwriting year is a listing of individual claim amounts for all reported claims, broken down by underwriting year of occurence.

The notation used for the data is:

r(i,j) = retention for i^{th} cedant (i = 1,2,...n) in j^{th} underwriting year (j 1,...10say); N(i,j,t) no. of claims incurred by i^{th} cedant in the j^{th} underwriting year, as reported up to the end of underwriting year t;
Im(i,j,t,k) = gross amount of k^{th} claim (counted at year t) incurred by i^{th} cedant in j^{th} underwriting year, as assessed at end of development year m.
Corresponding to Im, CIm and RIm denote the amount of the total loss borne by the cedant and the reinsurer respectively. Thus, Im = CIm + RIm, and CIM $\leqslant$ r (i,j).

The reinsurer recasts the data, for its whole portfolio, to obtain the following run off triangles for claims paid plus known claims outstanding:

(a) gross loss experience for the cedant's business in which the reinsurer has an interest (Im);

(b) net loss experience for business in (a) on the assumption that the retention limit r(i,j) applied to the whole business — this is repeated for every i and j to give a set of 10n run-off triangles for the different retention limits (although there may well be some duplicates, since not all r(i,j) may be different;

(c) loss experience suffered by reinsurer on same assumption as in (b) - this gives another set of 10n run-off triangles representing the difference between (a) and (b).

To project ultimate loss levels, development factors are determined by the standard chain ladder method with the incorporation of weighting factors to give greater credibility to more recent date. The ultimate level of claims is determined for the gross experience (a) denoted by U(o,j) and for each set of (b), where U(p,j) can denote the ultimate claims for the set p (for p + 1,2,.....10n) for underwriting year j. The reinsurer's ultimate liability on the assumptions made is then

RU (p,j) = U(o,j) - U(p,j) for the 10n sets.

The reinsurer's liability for IBNR claims can be theoretically determined as IBNR (p,j) = RU(p,j) - RI(p,j), where RI(p,j) is the total of known paid and outstanding claims for the set p in the j^{th} underwriting year, determined from (c) above.

IBNR (p,j) is therefore an estimate of the reserve that would be required if the retention for the p^{th} set had applied to all cedants in the j^{th} underwriting year. Actual IBNR reserves required are determined using the factor $F(p,j) = IBNR/I(j)$ where $I(j)$ denotes the gross claims paid and outstanding for the j^{th} year, as determined from (a) above. The factor $F(p,j)$ for the appropriate retention limit applying to a particular cedant in the j^{th} year is then applied to the gross known loss for that cedant. The method described utilises the whole experience available to the reinsurer to determine the F factors for different retention levels; the credibility of the factors is therefore much greater than would be obtained from a separate analysis of each cedant's business.

Kramreiter, H and Straub, E

On The Calculation of IBNR Reserves

M.V.S.V. 1973 p. 177

The general result of the minimum quadratic error method of estimating outstanding claims was given in an earlier paper with the same title by E. Straub. This extension of that paper considers some particular mathematical models for the claims run-off pattern, and shows that less complex expressions can be derived for estimating reserves for these models. Three models are considered, viz:

(1) a miltiplicative model, where $X_i^{h+1} = k_{h+1} \cdot X_i^h$

(2) an additive model, where $X_i^{h+1} = X_i^h + Y_{h+1}$

(3) a combination of an additive and multiplicative model.

The analysis of the multiplicative model shows that the estimate of ultimate claims by this method gives the same result as the orthodox chain ladder calculation, viz. the product of claims paid up to the latest development year (q say) and $\prod_{h=q}^{\infty} k_{h+1}$

Landin, D

Determining the IBNR

N.R.G. p.95

The content of this paper is more concerned with correct rating of excess of loss reinsurance business than with the assessment of adequate reserves. The author examines a specific example of the rate-making process in relation to a tranche of motor business which an insurance company was seeking to reinsure. Although data were available for 8 previous years, the number of large claims incurred was relatively few. In examining past experience, the paper suggests that claim amounts and retention levels should be adjusted in line with earnings inflation to put the statistics on a comparable basis and that, since the rate was to be quoted as a percentage of premium income, changes in the company's rating structure should be taken into account. The run-off of cumulative claims frequency was examined to assess the probable ultimate level of claims. However, as the experience was not fully developed and as the small number of claims could be subject to chance fluctuations, it was difficult to assess the full extent of IBNR claims. On the assumption that the distribution of claims followed a Pareto distribution, and with some subjective adjustments, calculations were made to show that an adequate rate was much higher than might have been supposed from the initial data provided.

Linnemann, P.

A multiplicative model of loss reserves; a stochastic process approach.

Laboratory of Actuarial Mathematics, University of Copenhagen
Working Paper No.32(1980)

The model used in this approach takes into account the waiting time until the claim is reported, partial payments of claim amounts, seasonal variation in claims frequencies, inflation, and discounting. The average partial payments of claims are assumed to be represented by a multiplicative model, and the parameters are estimated by a method of analytic graduation suggested by Hoem* (see below).

The starting point is a Lexis diagram recording the time interval in which a claim is incurred, plotted at a 45 degree angle against the payment interval in which the claim is reported or a payment is made. (The time intervals were taken as quarters of a calendar year).

This diagram gives a series of run-off parallelograms I(t,j) for underwriting periods t + 0, 1, ...T and payment intervals j + 0, 1,.....w. The end of the last underwriting period, i.e. time T + 1, is the date at which reserves are being estimated. For claim no. s out of N(t) incurred in the t^{th} underwriting period, let the waiting time until the claim is reported be oVs (which was assumed to fall within four quarters of incurrence), and assume there are Ns partial payments of amount iYs made after waiting time iVs, for i + 1, 2,.....Ns. The number of partial payments made for the s th claim in the (t,j) interval, and the amount of these payments are given by:

$$N_s(t,j) = \sum_{i}^{N_s} I_n\{{}_iV_s \in I(t,j)\} \quad \text{and} \quad Y_s(t,j) = \sum_{1}^{N_s} {}_iY_s \,.\, I_n \,.\{{}_iV_s \in I(t,j)\}$$

where I_n denotes the indicator function. Special action is required for the run-off parallelograms I (t, T-t) for t+0, 1.....T, to segregate events occurring after time T+1.

No. of claims incurred in t^{th} underwriting period and reported in k^{th} payment interval before T+1 is

$$N(t,k) = \sum_{s=1}^{N(t)} I_n.\{{}_0V_s \in I(t,k)\} \quad \text{for} \quad k = 0, 1, \dots, (T-t)$$

Hence no. of claims reported before time T+1 is $N_t =$ N(t,k) for the t^{th} underwriting period and $N = \sum_{t=0}^{T} N_t$ in total.

With the model assumed, the conditional expectation of the amount of claim for the i^{th} payment of claim no.s is represented by:

$$E\{{}_iY_s : {}_0V_s \in I(t,k) \text{ and } {}_iV_s \in I(t,k)\} = a_j \,.\, b_{t+j+1} \,.\, c_k \,.\, d_t$$

where a_j is an effect of the payment period j.
b_{t+j+1} is a measure of inflation.
c_k is an effect of reporting time k
d_t allows for seasonal fluctuation factor

Assumptions made are:

(1) No. of payments for a claim in each payment interval is independent of time incurred, viz

$$E\{N_s(t,j) : {}_0V_s \in I(t,k)\} = \Pi(k,j)$$

(2) Distribution of reporting times for incurred claims is independent of underwriting period.

(3) Independence of individual claims, but each has identical distribution.

(4) Independence is assumed between the number of claims and the amounts paid and the number of payments on the claims.

The present value of the outstanding payments for claim no. s, reported before T + 1, can be derived from $X_S(t) = \sum v^{j-(T-t)} . Y_S(t,j)$ for j = T-t+1 to w where v is a discount factor. The IBNER reserve for the time interval (t,k) is then $X(t,k) = \sum X_s(t)$ for ${}_0V_s \in I(t,k)$ and a summation of X(t,k) over all t and k will give the total IBNER reserve. The estimated reserve is taken as the expected value of the IBNER reserve, and with the above assumptions, this can be represented for interval (t,k) as:

$$N(t,k) \cdot \sum_{j=T-t+1}^{w} v^{j-(T-t)} a_j \cdot b_{t+j+1} \cdot c_K \cdot d_t \cdot E_k \{ N_s(t,j) \}$$

where the conditional expectation $E_k \{N_s(t,j)\} = \Pi(k,j)$ for $j > T-t$, The parameters a_j for j=0, ...T, b_t for t = 1,....T+1, C_k for k = 0, ...3 and d_t for t=0,.....T are estimated from the data, and inflation and interest are evaluated by judgement.

The IBNR reserve can be derived by a similar process, using

$$\bar{E} \{ N_s(t,j) \, I_n [{}_0V_s \in I(t,k)] \} = \pi(k,j) \, . \, \bar{P} \; {}_0V_s \in I(t,k) \}$$

where $\bar{E}$ and $\bar{P}$ denote the conditional expectation and the conditional probability of a claim being reported after time T +1. The probabilities $p_k = p\{ {}_0V_s \in I(t,k) \}$ can be estimated by standard methods, if N(t) and N(t,k) are multinominally distributed.

The main parameters a, b, c, d, and π are then estimated by Hoem's method of analytical graduation. Using a log transformation to produce a linear structure, it can be shown that analytic graduation with the inverse covariance matrix is asymptotically similar to linear regression. It then follows that the estimators of the parameters are asymp totically multidimensionally normally distributed, and this enables the variance of the estimator of the reserve to be calculated.

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*Hoem J M (1972): "On the statistical theory of analytical graduation: Proceedings of 6th Berkeley Symposium.

Masterson, N.E.

Problems in Motor Insurance - Claims Reserves

ASTIN Bulletin. Vol 2 Part 1 (1962) p.152

The author draws up triangles (by year of origin and development year) of

(i) number of claims reported to date
(ii) number of claims closed to date
(iii) mean cost of closed claims

The numbers of claims closed is used to monitor any changes in the speed of claim settlement.

The method assumes that there is stability, over the various years of origin of claims, of the mean claim size development ratios:-

(Ultimate mean claim): (mean closed claim after 1, 2, 3,.....years.)

A chain ladder type calculation is performed on the triangle of mean costs of closed claims to get average increases in mean costs of closed claims between development years 1 and 2, 2 and 3 etc. For the earliest year of origin the ratio of ultimate mean claim to mean claim closed to date appears to be calculated by using a figure for reserves on claims still open, derived from individual estimation of the claims concerned. This is not stated explicitly, but is consistent with the numerical example given.

The ultimate mean claim size is multiplied by the number of reported claims (the method does not therefore estimate IBNR) to arrive at a projected final cost. Payments to date are subtracted to give an estimate of outstanding claims.

The author acknowledges that distortions can be caused by inflation, changes in excess etc. He also says that subjective adjustments can be made in the mean claim development ratios if the speed of settlement changes.

The author says that long experience shows that it is better to calculate reserves for outstanding claims by taking the difference between total cost and total payments to date on all claims than to use numbers and average sizes of open claims.

The method is applied to U.S. motor insurances, treating third party personal injury, third party property damage and insureds' own damage separately. Its application to other classes is not covered.

Matthews, T.J.

The Valuation of General Insurance Claim Reserves

I.A.A. p.69

Having discussed the need for statistical methods in assessing the adequacy of claims reserves, the paper promotes the concept of a claims reserve valuation basis.

This is the set of assumptions on which a projection of claim payments is based. The basis should include some or all of the following components:-

a) the claim payment pattern.

b) the tail of the claim payment distribution

c) claims inflation

d) interest earnings

e) taxation

f) expenses

g) contingency loading

The valuation process should be as follows:-

i) Analyse historical claim payments data using several statistical methods.

ii) Select a claim payment distribution

iii) Incorporate assumptions about b) to g) above.

iv) Calculate reserves

Components a) and b) above are then considered in some detail

Claim payment pattern

Claim payments per Taylors model are:-

$$c_{ij} = n_i \, r_j \, \lambda_{i+j} + e_{ij}$$

where e_{ij} is an error term and the other symbols take the normal meaning.

The CL method can be represented in this form if all the λ_{i+j} are held constant. The inflation adjusted CL method results from choosing past values of λ_{i+j} and the separation method produces estimates of both r_j and λ_{i+j}.

The sets of r_j are important and these sets are compared for the three methods. These exhibit certain irregularity as they are derived from portfolios which cannot be described as large. The crude r_j's obtained should, therefore, be smoothed.

The CL set of r_j can be regarded as a basic set that results from choosing zero past inflation. Using higher past inflation produces a shorter distribution. Instead of specifying the past inflation and deriving the claim payment pattern we could specify the claim pattern and derive a set of past balues. This method could be called the "Distribution Adjusted CL method". Deviations from this standard set of r_j can be tested.

Problem of the Tail

It is a convenient practice to accept the company's estimate for the oldest cohort of outstanding claims and using this estimate as a base for estimates of the tail for all later years. Inaccuracies here can have a dramatic effect on long tail business. It is suggested that a set of consistent assumptions should be used e.g.

a) The use of a grossing factor based on longer term industry-wide data.

b) An "average" (over say 4 years) tail.

c) Determine the underlying real run-off ratio implied by various estimates of the tail.

McLenahan, C.L.

A Mathematical Model for Loss Reserve Analysis

P.C.A.S. 62(1975), p.134

The paper describes a method of reserving based solely on loss payment development. The model assumes that claim severity, that is, pure loss cost resulting from the average claim, is constant over time.

The basic relationship is:

$$nPm = Cpq^{n-d} (1+x)^{m+n} (1+y)^{m} \text{ ; if } n<d \text{ then } nPm = 0$$

where: nPm represents the amount paid during month m+n ($n \geq 0$) on losses incurred during accident month m.

C represents the constant value losses

x represents the uniform monthly rate of change in severity

y represents the uniform monthly rate of change in accident month incurred losses due to claim frequency and exposure volume changes

p represents the probability of payment

q = 1-p

d represents the average delay in months between loss occurence and loss reporting

The paper then continues to develop this basic relationship, and with a lot of algebra, produces a usable reserving model.

Pentikainen, T

Reserves of Motor-Vehicle Insurance in Finland

ASTIN Bulletin Vol 2 Part 1 (1962) p.161

The author describes the simple practical methods in use in Finland twenty years ago. IBNR was calculated by monitoring late notifications in each development year of previous cohorts and expressing the amounts as percentages of the premium in the relevant year. These percentages were then averaged over the different cohorts, and significant explicit margins were added. Reserves for known outstanding claims could be calculated in a similar way using amounts paid in the various development years (on claims known and outstanding at the end of the year of origin) or by multiplying a full year's incurred claims amount by a mean time to settle. This second method involves analysing the times to settlement of a sample of closed claims.

Some companies were too small to produce credible factors, and also lacked the mathematical expertise. The supervisory authority therefore recommended bases, and the author shows the third party motor basis as an example:-

Known claims	Personal injury	- case estimates
	Large property	- case estimates
	Other property damage	- $0.18P + 0.07P_{-1} + 0.07P_{-}$
IBNR	$0.31P + 0.06P_{-1}$	

where large property claims are ones greated than 5% of the last year's premium, and P, P_{-1}, P_{-2} are premiums in the last three years.

Since 1952 Finnish Law has required an explicit solvency margin "calculated in accordance with the principles of risk theory". The last section of the paper uses a number of the basic risk theory formulae to produce practical, and partly empirical, working formulae for a solvency margin.

Philipson, C.

A Generalised Model for the Risk Process and its Applications to a Tentative Evaluation of Outstanding Liabilities.

ASTIN Bulletin Vol 3 Part 3 (1965) p.215

The claims process is considered as a compound Poisson distribution with a variable (over time) "intensity function" rather than a constant Poisson parameter. The model is essentially for premium rating, but by extending it to cover the distribution over time of the various payments on each claim it can be used to value outstanding liabilites.

The paper is very theoretical and fails either to state clearly the assumptions it makes or to show how to derive the necessary parameters and distributions from available data. It is unlikely to be of interest for practical use.

Pollard, J.H.

Outstanding Claims Provisions : a Distribution-Free Statistical Approach

JIA 109, 3, p.417

The author suggests that in the more normal methods for estimating provisions for outstanding claims, the lack of a theoretical statistical basis prevents information about the reliability of the resulting provisions from being calculated. With the method he is putting forward, it is possible to calculate confidence intervals for the outstanding claims provisions.

In this paper, a theory is developed in which the complications of the claim process are overcome by considering the joint distribution of payments for a single claim in successive development periods. The model does not assume specific distribution for claim size, time to settlement or payments prior to settlement by amount and time. Adjustments are made for past inflation so that all amounts are in up-to-date money terms.

The paper makes extensive use of vector analysis and the following are the steps followed:-

(i) From past experience determine the expected payments in each year of development and their variance all expressed in up-to-date money terms.

(ii) Produce a claims payment triangle showing for each year of origin and development year, the payments made in that development year adjusted for past inflation.

(iii) For each year of origin, compare the development pattern obtained to date with that expected from (i) by using a chi-squared goodness-of-fit test. If it appears that the past development appears inappropriate, the author suggests a method for making suitable adjustments to the future provision calculation for each year of origin.

(iv) The author makes specific allowance for both future claims inflation and future investment earnings in arriving at the expected value of discounted future claims payments for each year of origin.

(v) The end result is a figure for the expected value of the outstanding claim reserve as well as a figure for the corresponding variance. Since the author then suggests that for a reasonably large portfolio with a reasonably large number of claims the Central Limit Theorem will be applicable, he concludes that the distribution of the outstanding claim payments will be approximated by a multivariate normal variable. This then allows him to determine the 95% confidence interval for the discounted value of all future claim payments as

$$\text{expected value} \pm 1.96 \sqrt{\text{variance}}$$

Reid, D.H.

Claim Reserves in General Insurance

JIA 105 (1978), 3, p.211

This paper is concerned with the development of a mathematical model for claims experience in General Business and can be regarded as being comparable to that used in Life Business with its probabilistic and discounting components. The paper then proceeds to develop the model into the area of reserving for outstanding or IBNR claims.

Reid cites as a major limitation of case estimation that the resulting estimates are devoid of any indication of their reliability or of the extent of the margin for error built into them. As regards the normal statistical approaches, he regards them as too rigid to reflect varying future conditions and too susceptible to distortions by large claims.

The author begins his approach by extracting base information from an event year some distance in the past. This data consists of numbers of claims both by periods of settlement and by ranges of ultimate cost. The underlying assumptions used are that this base year is complete, that more recent years can be used to update this base year information and essentially each years claim settlements follow a similar pattern to the base year. An important aspect of this approach is that to put together the base year table it is necessary to know the ultimate value of all claims.

Attention is then focused on later years which introduces an immediate problem in that there will be progressively more claims outstanding. Instead of using the base year experience in a simple manner to complete the outstanding claims, it is "stretched and contracted" to reflect the current situation for the later year under consideration. The extent of the stretching is determined in two ways namely -

1. For past years, it is arranged so as to reflect patterns of available data.

2. For future years, it is done on the basis of future assumptions/expectations.

The process of stretching and fitting is carried out separately for each of the origin years between the base year and the accounting date. Eventually it is possible to extract the required statistics relating to outstanding claims.

This is a long and very complicated paper. Although it is technically impressive there are reservations about its practical usefulness.

Ryder, J.M.

Claim Estimation Models

I.A.A. p. 98

This paper is not very well written.

It makes comments on three problems encountered in the use of Taylor's generalised model (of which the separation method is a specific case).

The model is:

$$E\ (C_{ij}) = \mu_i\, r_j\, \lambda_{i+j}$$

a) There are difficulties in estimating tail values of r_j. Both the shape and the length of the tail have to be 'guessed'. Two methods are suggested.

b) There may be unreliability in recent information e.g. in ℓ_{ij} and $n_i = \mu_i$. It would be more reliable to use graduated values/forecasts rather than unadjusted observations.

c) "Claims expected" are adjusted to equal "Actual claims paid to date" for each accident year. This means that μ_i which we started off with as being equal to n_i has to be re-estimated. The use of "<u>total</u> expected payments to date" being made equal to "actual payments" is acceptable.

Sawkins, R.W.

Analyses of Claims Run-off Data - A Broad View

I.A.A. p. 30

The paper develops some general thoughts on analysing claims run-off data. The data consists of, for each year of origin, for each development year:

No. of claims reported in the development year
No. of claims outstanding at the end of the development year
Payments made in the development year

The analysis can be carried out in three stages:

A Set up a model
B Determine past inflation rates
C Choice of method for future projection

A Setting up a Model

It must try and deal with the following four aspects:

a) The amount of business (and hence the amount of claims) will differ in respect of different years of origin.

b) There is assumed to be an underlying claim settlement (or payment) pattern.

c) There is a factor affecting payments which is dependent on the year in which the payment is made.

d) Data may be subject to random fluctuations, changes in company policy and administration, lack of homogeneity, long term trends and data errors.

The following methods are considered. All of them use data adjusted for inflation based on increases in earnings. All methods can use either discrete or cumulative data.

1. Ratio of payments -

$$\frac{\text{Payments in development year } t+1}{\text{Payments in development year } t} = \text{Constant for a given } t$$

Example: Inflation adjusted chain ladder

2. Payments per Unit of Exposed to Risk -

Three measures of exposed to risk for different years of origin are:

i) Claim payments in development year 0.

ii) No. of claims reported in development year 0.

iii) No. of claims occurring in the year of origin.

Such methods make a specific allowance for changes in the amount of risk from one year of origin to another.

Example: Separation method

3. Number of Claims -

The run-off pattern of numbers of claims is combined with the development of payment per claim. The no. of claims can be expressed as those:

i) Settled in each development year.

ii) Outstanding at the beginning (or end) of each development year.

iii) Handled in each development year.

B Determining Past Inflation Rates

In general inflation is taken as a single factor which affects all payments equally in a payment year - it is likely that claim inflation in a year is not uniform for all development years. The methods described in A used the Average Weekly Earnings (AWE) index to adjust for inflation. Assumed past rates can be based on:

1) General considerations (like the AWE assumption)

2) External information (e.g. industry averages) about the class of business concerned

3) An analysis of the actual data.

The use of actual data will not produce the same answers as 1 or 2 because it will include other trends and influences occurring in the data, such as the speeding up of settlement, changes in type of claim, random fluctuations, etc.

C Methods of Projection

For a given past year of origin certain payments will already have been made. Using a given claim payment distribution there are three ways in which the future payments can be determined:

1) Any variation in past payments from the assumed distribution will continue proportionately in the future.

2) Future payments will be in accordance with the distribution, regardless of past payments.

3) Total payments will be in accordance with the distribution, so that future payments are the total payments less the past payments.

Summary

The description above shows the very large number of estimates that can be made.

The paper is worth reading.

Sawkins, R.W.

Some problems of long-term Claims in General Insurance

Transactions of the Institute of Actuaries of Australia and New Zealand, 1975, p. 336

The paper prepares sets of accounts using claims data supplied to the NSW Workers Compensation Commission and premiums based on tariff rates. The traditional financial year revenue accounts are split into amounts attributable to a claim year. Amongst other things this process reveals the inaccuracy in the provision for outstanding claims and the following method is suggested for a better estimate of the provision.

Data available. For each of the last 6 (1968 - 1973) financial years, for each year of claim the amount paid in the year and the amount outstanding at the end of the year. From this can be derived:

$C(n,t)$ = the payment made in elapsed year (or duration) t on claims which occurred in year n.

Payments made after 1968 were deflated (the percentage used being increase in Average Weekly Earnings plus 1½%). Payments in prior years were not deflated because complete run-off statistics were not available for the model company.

$C'(n,t)$ = Deflated value of $C(n,t)$

$$p(t) = \frac{C'(n,t)}{C'(n,t-1)} \quad \text{with } p(1) = 1$$

= Relative amount paid in elapsed year t.

$$P(t) = \sum p(t)$$

= Relative total amount paid up to the end of elapsed year t.

Knowing the amounts paid on claims in the recent past the future payments, excluding inflation, can be projected. By making assumptions about future rates of inflation on claims, payments allowing for inflation can be calculated. These are summed over all claim years to obtain the provision for outstanding claims.

Data was available for 6 financial years and various values of $p(t)$ were calculated:

a) the arithmetic average

b) an average weighted according to the amount of claims paid (in real terms) in each year

$$\frac{\sum C'(n,t)}{\sum C'(n,t-1)}$$

c) a geometric average

d) a credibility type average.

There was very little variation between them and for the calculations a smoothed value of p(t) was used.

This method differs from the CL method because:

a) it uses discrete payments

b) it uses the data from the last 10-20 accident years instead of the 5-6 years used by the CL method.

However, there are similarities with the CL:

a) No use is made of claim numbers

b) Only payments are considered

Simon, L.J.

Distortion in IBNR Factors

P.C.A.S. 57 (1970) p. 64

The paper discusses the distortions in reserve analysis which can occur when a portfolio is increasing or decreasing or when a new line of business is introduced.

The paper introduces an IBNR factor, B, which is :

$$\text{IBNR Factor B} = \frac{\text{Incurred Losses at some subsequent date}}{\text{Incurred Losses at the accounting date}}$$

The objective is to convert the Factor B into an equivalent time period representing the portion of time that is missing from the losses. For example, a Factor B = 1.14 will represent, under certain conditions, the loss of 1.5 months of incurred losses from the data. We will then assume that during some subsequent accounting period the same conditions will maintain and 1.5 months of losses will be missing from that accounting period also.

Formulae of algebraic nicety are developed to demonstrate the distortions caused by a growing book of three year risks as well as one year risks.

The paper was worth reading and the reader gains a valuable insight of the nature of distortions caused by changing volumes in portfolio. At times however, the algebra became a little arduous.

Straub, E

IBNR - a difficult marriage between practice and theory

A.S.p.29

This is a short descriptive paper emphasising a pragmatic approach to the assessment of IBNR reserves. The author's view, despite the elaborate statistical techniques he suggested in earlier papers, is that the practical factors are much more important than statistical methods.

An eight-step IBNR loss-reserving process is proposed:

1st step

Find out importance and priority of a realistic assessment of IBNR's within the context of the performance analysis of the portfolio under consideration. This means, among other things, visualising quantitatively the many different consequences of an unrealistic IBNR estimate.
Continue with the following steps, if IBNR's are found to be crucial, otherwise not or only partly.

2nd step

Make up complete run-off statistics for the three types of figures: paid, paid plus outstanding and paid plus outstanding plus IBNR.

3rd step

Give detailed description and run-off of the largest individual claims (e.g. the 10 largest ones) from date of occurrence til settlement, also for those claims which later on are superseded by even larger ones.

4th step

Quantitative (!) analysis of main reasons for the hitherto observed IBNR pattern: points of gravity, e.g. individual ceding companies, type of claim, unforeseen developments in court practice, legislation, consumer's consciousness or influence of index clause.

5th step

Break up portfolio into parts according to above points of gravity (i.e. some sort of non-mathematical cluster analysis) and estimate the I.B.N.R. of each part separately and by different methods:

- largest claims primarily based on the opinion of well-informed claims experts,
- some critical types of claims by taking into account most recent developments and probable future behaviour of courts and public,
- ceding companies showing an exceptional IBNR pattern by taking into account any known reasons for being exceptional, as well as any information on changes of the companies' claims handling,

- for remaining 'normal' claims by using some of the statistical procedures described in the literature.

<u>6th step</u>

Cautious but still realistic forecast of future investment income on claims reserves.

<u>7th step</u>

Build together estimated final claim costs, amounts and frequency of claims payments and interest rates to a realistic overall IBNR reserve (which, by the way, need not necessarily be identical with the published one).

<u>8th step</u>

Analyse every year the accuracy of last year's and earlier IBNR estimates, give reasons why things went right or wrong and go back to step no 1.

Straub, E.

On the calculation of IBNR reserves

N.R.G. p. 123

The method of assessing reserves described in this paper avoids any assumption about the nature of the distribution of claims. The expected final "burning cost" for an underwriting year (i.e. the ultimate loss ratio) is estimated on the assumptions that it is a linear function of the observed loss ratios and that it is unbiased. The estimate is made on the basis that the expected quadratic error between the observed values and the linear estimator is minimised. If $X_i^{(h)}$ denotes the ratio of claims observed up to the end of the h th. development year to the premium income P' for the i^{th} underwriting year, it is assumed that $X_i^{(h)}$ is stochastically independent of $X_{i'}^{(h')}$ for $i \neq i'$. (It is also assumed that the ratios $X_i^{(h)}$ are adjusted for claim information and tariff changes, if necessary). The paper shows that, on the assumptions made, an estimate of the final burning cost for an underwriting year can be expressed in terms of the premium income and the mean and covariance of the observed values, i.e. $e^{(h)}$ and $c_{hh'}$, which are independent of i, viz:

$$e^{(h)} = E\left[X_i^{(h)}\right] \quad \text{for } i = 1, 2, \ldots\ldots\ldots\ldots$$

$$c_{hh'} = P_i \times \mathrm{Cov}\left[X_i^{(h)}, X_i^{(h')}\right] \quad \text{for } i = 1,2, \ldots\ldots\ldots$$

Normally the true means and covariance would not be known, and estimates of the true values would have to be used. The estimate could be derived either from the observed $X_i^{(h)}$ only, or by taking into account other experience. Although the numerical work required for this method is in practice rather cumbersome, the paper claims it is the simplest of all distribution-free methods as it involves only first and second order moments.

Tarbell, T.F.

Incurred but not Reported Claim Reserves

P.C.A.S. 58 (1971) p. 84

A short paper discussing Tarbell's approach to IBNR reserves.

Reference is made to certain factors which affect IBNR reserving. They are:

i) Volume of exposure

ii) Accident frequency

iii) Average claim costs

The paper then goes on to describe the general method. In formula form it may be represented as:

$$\text{Reserve} = \frac{N^{y}\ 10\text{-}11\text{-}12 \ \times\ C^{y}\ 10\text{-}11\text{-}12}{N^{y-1}\ 10\text{-}11\text{-}12 \ \times\ C^{y-1}\ 10\text{-}11\text{-}12} \times I^{y-1}$$

where - N = number of notices

C = average incurred per notice

I = amount of IBNR

y = designates the current calendar year

y-1 = designates the previous calendar year

Subscripts designate calendar months

The formula is not applicable to lines having a low accident frequency and a large factor of variation in average claim cost. The paper suggests that the formula produces good estimates for short-tail lines such as property, motor damage, accident and health indemnity.

Taylor, G.C.

General Insurance Database & Methods of Claim Reserving

I.A.A. p. 24

All methods of analysis are based on a model and this can be of two types:

Simple - this may be unrealistic but is clear and easily comprehended

Elaborate - may be realistic but there is a greater potential for ill-conditioning in the fitting process and for consequent instability of results.

Moving towards elaborate methods is an improvement and using them would make substantial demands on data. The use of Reid's method, requiring the following data for each claim, may not require major modifications to claim files:

Date of occurrence

Date of notification

Date of settlement

Payments made to date (with dates)

Taylor, G.C.

Separation of Inflation and Other Effects from the Distribution of Non-Life Insurance Claim Delays

ASTIN Bulletin - Volume 9 Parts 1 & 2 (1977) p.217

This is the basic paper describing the separation method. (After discussing how chaining rates of inflation and other exogeneous factors can distort projections made by chain ladder and similar methods).

The paper assumes that if the conditions affecting individual claim size (i.e. price index and other exogeneous effects) remained constant the ratios of average claim amount paid in development year j to the average amount paid up to the end of development year k (the highest development year in the run off triangle) for the same year of origin i would have an expected value r_j independent of i. It further assumes that all exogenous factors can be expressed in terms of an index λ_{i+j} dependent on year of payment only.

Defining c_{ij} = cumulative payments to end of development year j for year of origin i

n_i = estimate of number of claims occurring in year i made at end of that year

$s_{ij} = (c_{ij} - c_{i,j-1}) / n_i$

Then $\hat{s}_{ij} = r_j \lambda_{i+j}$

By summing rows and columns of the Sij and $r_j \lambda_{i+j}$ matrices and using the fact that the sum of the r's is unity, the r's and λ's can be estimated. The paper refers to Verbeek's paper, which was earlier and applied the method to claim numbers in excess of loss reinsurance.

The paper mentions a possible extension to allow for changes in claim size depending on the year of origin i.e. $\hat{s}_{ij} = \varepsilon_i r_j \lambda_{i+j}$ but this introduces computational difficulties, reduces the number of degree of freedom and was not pursued.

Estimating payments in development years beyond K involves some other means of estimating outstanding claims, e.g. case estimates. The paper recommends using only the estimate for the earliest year of origin defined as s_{oK+}. Future values of λ are projected using either fixed or variable inflation allowing all necessary values of $\hat{s}_{ij}$ to be calculated (j running up to development year K). The $\hat{s}_{iK+}$ are set to $\hat{s}_{oK+} (\lambda_{K+i+1} / \lambda_{K+1})$. Use of case estimates is rejected because of differences between the λ's used explicitly for pay ments up to development year K and those used implicitly by the claims estimator for subsequent years.

The author gives numerical results for two 4 by 4 development triangles : one for a motor account and one for a pecuniary loss account. A good fit to the actual past payments was achieved in the former case but the author points out that just as important as a good fit is that the r's and λ's should only change by a small amount when a 3 by 3 development triangle is used instead of a 4 by 4. In the pecuniary loss case the fit was very bad as it was obvious that the r's were not independent of the year of origin.

Taylor, G.C.

Some Practical variations of the Separation Method

General Insurance Bulletin, 11 (1977) p. 9

This paper considers the possibility of using the separation method when the runoff data are not available in a triangular form.

Two possible combinations of missing data are considered namely missing top left entries and missing top left as well as top right entries.

The conclusions reached are that if the missing entries are zero filled, the normal separation method can then be applied to obtain the estimated provisions for the outstanding claims.

Taylor, G.C.

Statistical Testing of a non-life Insurance run-off Model

A.S. p. 37

One way of estimating outstanding claims is to devise a statistical model to represent the claims data in a run-off triangle, and to estimate the unknown parameters in the model from the data. To assess whether the model selected is a good fit to the data, a test based on some "chi-square" statistics was suggested by Taylor & Matthews in an earlier paper (reference (45), q.v.). Subsequent experience has shown this test to be inadequate, and an alternative approach is discussed in this paper. The approach is considered mainly in relation to two methods of parameter estimation, viz:

1) the arithmetic separation method, which was discussed in an earlier paper (reviewed), with either unknown or known claims inflation index, and

2) the geometric separation method, which is similar to the arithmetic method except that parameters are estimated by taking column and diagonal products rather than sums.

Working from the result that the most efficient unbiased estimator of a parameter is a function of a sufficient statistic for that parameter, the author shows that the arithmetic separation method is generall better in fitting a model to the data than the geometric separation method, except where there is a relatively small amount of data. Nevertheless, as the geometric separation method is more tractable statistically, it is used to develop the proposed test for goodness of fit.

The paper then considers a regression approach to estimating the parameters, and demonstrates that the regression method is identical in principle to the geometric separation method. Variation of the observed data about the values predicted by the model chosen can therefore be measured by calculating the multiple correlation coefficient or, more helpfully, by examining the distribution of the residuals. By means of a simple transformation on the residuals, it is possible to obtain a statistic which has a Student's t-distribution with (n-p) degrees of freedom, where n denotes the no. of observations in the run-off triangle and p is the no. of parameters to be estimated. This enables a judgement to be made on whether the model provides a good fit to the data. The regression approach can also be used, with modifications, to test models fitted by other methods, e.g. the inflation-adjusted chain ladder method. Examples show that a model fitted by the arithmetic separation method can give very similar results to the regression approach.

Taylor, G.C.

Testing Goodness of Fit of an Estimated Run-Off Triangle

ASTIN Bulletin - Vol 10 Part 1 (1978) p. 78

A standard chi-square statistic calculated from the difference between an actual claim payment run-off triangle and the expected (i.e. fitted) triangle derived from a model would only actually be distributed as chi-square if the marginal distributions of the payments for each year of origin/year of development are binomial.

The author acknowledges that this is not true. He makes assumptions leading to each distribution being a compound Poisson and derives a statistic which will be distributed as chi-square. His main assumptions are:

1) Payments C_{ij} (= total paid for year of origin i, development year j) are known.

2) "Disturbing influences" (e.g. inflation, changing rates of growth) can be removed giving C'_{ij}

3) Expected proportions of total cost paid in each development year are independent of year of origin, i.e. $\mu'_{ij} = C'_i r_j$

4) Number of claims in each i, j cell is a stationary Poisson variable.

5) Individual claim amounts in i, j cell are independent identically distributed random variables with 1st and 2nd moments depending on development year, but not year of origin.

The test statistic is the sum over all the cells of:

$$(1-\hat{r}_j).(\alpha_{1j}/\alpha_{2j})(C'_{ij}-\hat{\mu}'_{ij})^2/\hat{\mu}'_{ij}$$

where the α's are the 1st and 2nd moments.

An individual company will be able to estimate these moments but a supervisory authority will not have access to the necessary data. The author says that the ratio of 1st to 2nd moment varies considerably from company to company, but that the ratio of the square of the 1st moment to the 2nd moment does not. He shows how to derive this ratio from data on a number of companies, but also uses an empirical value as an upper limit. The latter approach makes the test over strict. These approximations using the ratio of the square of the 1st moment to the second moment require the number of claims in each i, j cell to be known.

Although payments can definitely be allocated to an i, j cell the paper assumes implicitly that claims can be allocated in the same way. Since partial payments are made before final settlement this is not so. It then becomes unclear exactly what the α's are 1st and 2nd moments of.

Although a simple chi-square test makes assumptions that are not true, the improvements developed in the paper run into significant problems and the final result may not be an improvement.

Verbeek, H.G.

An approach to the Analysis of Claims Experience in Motor Liability Excess of Loss Reinsurance

ASTIN Bulletin Vol 6 Part 3 (1972) p. 195

The paper concentrates on the number of claims reported to an excess of loss reinsurer. Total claims amount is calculated by multiplying by a conservative estimate of mean excess claim since they "...... empirically show a rather stable pattern for fixed excess points".

The main assumptions are:

1) Number of claims occurring in the direct insurers business in a particular year of origin is a Poisson process

2) A claim occurring in a particular year of origin has a probability of being reported in the same year of r1, of being reported the next year of r2 etc. All claims are assumed to be reported by the end of k years. "reported" appears to mean reported to the direct writer and notification to the reinsurer appears to be assumed to be instantaneous. The r's are independent of claim.

3) The excess level and the claim size distribution are dependent on the year of notification, not the year of origin.

4) The data has been pre-manipulated to allow it to be treated as coming from the same basic business i.e. all years' of origin can be treated as having the same Poisson parameter

Defining i = calendar year of occurrence

j = development year of reporting (i.e. 1, 2 .....k giving a calendar year of reporting of i + j - 1)

Then the number of claims occurring in year i, reported in development year j has Poisson parameter αr_j . The proportion of these that affect the reinsurer is, by assumption 3 dependent on the calendar year of reporting only. Writing λ (i + j - 1) as the product of α and this proportion the number of claims notified to the reinsurer has a Poisson parameter $\lambda_{i+j-1} \cdot r_j$. This is then fitted to the triangle of numbers of actual notifications to the reinsurer (adjusted in line with assumption 4) using the standard separation technique.

Projection of the λ's into the future is not dealt with in detail, but "Empirical data suggest that extrapolation by means of exponential curves may be realistic".

The paper makes very restrictive assumptions. In particular assumption 3 will not usually be true as excess levels are more likely to be dependent on the year of origin of claim (with an allowance for inflation to the date of notification or settlement), and the assumed stable pattern of average excess claims will be disturbed by a change in excess level.

Vylder, F. de

Estimation of IBNR Claims by Least Squares

A.S. p. 22 or MVSV (1978) p. 249

This paper is concerned with the "classical model" for estimating outstanding claims by least squares. If c_{ij} denotes the amount of claims paid in the jth development year for the ith underwriting year, then c_{ij} can be represented by $x_i \cdot p_j \cdot u^{i+j}$, where:

x_i = total amount of claims for i^{th} underwriting year in inflation-free units,

p_j = proportion of claims paid in j^{th} development year for each underwriting year,

u^{i+j} = claim inflation index

The unknowns x_i and p_j can be obtained by minimizing

$$\sum w_{ij} \left(x_i \cdot p_j \cdot u^{i+j} - c_{ij} \right)^2$$ summed over all known cells c_{ij}.

In the classical model, $u^{i+j} = 1$ for all i and j, and it is usually assumed that $w_{ij} = 1$ for all i, j, and $\sum p_j = 1$. If it is only the values of the unknown C_{ij} which are required, the last two assumptions are unnecessary since the values of x_i and p_j can still be determined, e.g. by using an iterative process. By noting that $x'_i \cdot p'_j$ is also a solution of the least squares equation (where $x'_i = x_i \cdot k \cdot u^{-i}$ and $p'_j = p_j \cdot k^{-1} \cdot u^{-j}$ the paper shows that the least squares method of estimating outstanding claims for the "exponential model" with an implicit inflation assumption for u^{i+j} is the same in practice as for the classical model with $u^{i+j} = 1$. The least squares method does not enable an estimate of the inflation index to be made.

Wit, G.W. and Kastelijn, W.M.

I.B.N.R.

A.S.p.65

The paper is a reaction to the first meeting of the contact group "Actuarial Sciences". When considering the calculation of I.B.N.R. the following aspects are considered to be important:

1. Different statistical methods lead to different I.B.N.R. results.

2. Should future rates of interest be included in the reserve calculation?

3. What are the reasons for the systematic under-estimation of claims?

4. What is the influence of the largest claim on the run-off triangle?

Straub covers these points in his paper "I.B.N.R. - a difficult marriage between practice and theory".

To illustrate point 1 an example is worked out in this paper using four different methods, i.e. basic chain ladder, two of Taylor's separation methods and Verbeek's separation method.

APPENDIX 3

Indexed Summaries of Methods

Sections:

1. Direct business, general methods
2. Direct business, specific applications
3. Reinsurance business, general methods
4. Reinsurance business, specific applications.

Within each section methods are in increasing order of complexity

Column headings:

Method

Author(s)

Paper

Source

Applications

Data required

Main assumptions

Comments

Direct Business: General Methods

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Individual Estimates	Clarke, T.G.	An Actuary Looks at Claims Provisions in General Insurance	I.M.A.	O/s claims reserves	Individual claim Estimates	Not too much data; preferably not liability	A simple method: time-consuming
Simple ratio methods (of items normally found in company accounts)	Ackman, Green and Young	GIRO Monograph on Estimating Outstanding Claims	G.I.S.G.	O/s claims reserves and I.B.N.R.	Various items found in company accounts	Stability of account	Extremely limited applicability
Average cost per claim (times number of claims)	Clarke, T.G.	An Actuary Looks at Claims Provisions in General Insurance	I.M.A.	O/s claims reserves	History of numbers and costs of claims	Portfolio relatively stable or of a reasonable size	Relatively crude method: easy and quick to apply
Average cost per claim: projecting numbers of claims handled and estimating average future payments	Ackman, Green and Young	GIRO Monograph on Estimating Outstanding Claims	G.I.S.G.	O/s claims reserves and I.B.N.R	Development data of claims paid and numbers of claims, paid and outstanding	Usual stability assumptions	Makes it possible to detect and allow for changes in number of claims settled by development period
Average cost per claim for I.B.N.R. (after estimating the number of late reported claims)	Clarke, T.G.	An Actuary Looks at Claims Provisions in General Insurance	I.M.A.	I.B.N.R.	History of numbers of claims ("late reported" and "already reported") and of average late reported claims	Stable account	Must consider whether "late reported" claims are of different value to already reported claims
Average outstanding claims cost	Clarke, T.G.	An Actuary Looks at Claims Provisions in General Insurance	I.M.A.	O/s claims reserves	Historic information on outstanding claims costs by year of origin	Average outstanding claim amount can be extrapolated	Method lends itself to consideration of fluctuation reserves as projected figures will tend to fluctuate around a mean.
Grossing-up based on a study of claim payments at each stage of the run-off (say each month)	Clarke, T.G. Clarke, T.G. & Harland, N.	An Actuary Looks at Claims Provisions in General Insurance A Practical Statistical Method of Estimating Claims Liability and Claims Cash Flow	I.M.A. ASTIN Bulletin Volume 8 Part 1	O/s claims reserves	A study of the pattern of claim payments at each stage of the run-off	The run-off claim payments for any "year of claim" follows a reasonably stable pattern	Claims payment information can be used as it passes through the accounting system: readily adaptable so a computer can produce monthly updated liabilities. But a subjective adjustment is used for the rate of settlement

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Grossing up the estimated liability for the claims already notified	Clarke, T.G.	An Actuary Looks at Claims Provisions in General Insurance	I.M.A.	I.B.N.R.	Past relationships of late reported liability to alreading reported liability	Reported liability correctly assessed; stable account	Difficulty may be experienced in obtaining relationships for most recent years: particularly for liability classes
German Reserve Clock: rolling 12 month average incurred claims costs	Johnson, P.D. (translated by)	A Memorandum describing the German Reserve Clock Method of Arriving at Reserves for Outstanding Claims	GIRO Bulletin No. 3	O/s claims reserves	Incurred costs of past claims grouped by month of occurrence	Reasonable Stability	Monthly fluctuations are eliminated.
Basic Chain Ladder: (projection of claims run-off triangle using derived development ratios)	Ackman, Green & Young	GIRO Monograph on Estimating Outstanding claims.	G.I.S.G.	O/s claims reserves and I.B.N.R. for direct business and reinsurance	Run-off triangle of cumulative claim payments	The triangle columns are proportional: exogenous factors can be ignored (e.g. inflation)	Simple method : does not make explicit allowance for inflation
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Inflation adjusted Chain Ladder: the run-off triangle of cumulative claims is expressed in constant money values before projecting and future inflation rates are allowed for explicitly	Ackman, Green and Young	GIRO Monograph on Estimating Outstanding Claims	G.I.S.G.	O/s claims reserves and I.B.N.R. for direct business and reinsurance	Run-off triangle of claim payments; past claims inflation indices and forecasts of future inflation rates.	Inflation adjusted triangle columns are proportional; index chosen is proxy for claims inflation and future inflation rates can be projected.	Explicit allowance is made for inflation. This may not be possible for reinsurance (particularly for world-wide account)
Chain Ladder type calculation applied to the triangle of mean closed claims	Masterson, N.E.	Problems in Motor Insurance - Claims Reserves	ASTIN Bulletin Vol. 2	O/s claims reserves	Development triangles of numbers of claims reported and numbers and mean costs of claims closed to date	Stability (over years of origin) of the development of mean cost of closed claims	Found to have been a good practical method for U.S. motor business. Subjective adjustments to data may be needed
Calculate average inflation adjusted payments for each year of development (uses a geometric progression)	Beck, J.H.	General Insurance	Magazine of the Faculty of Actuaries' Students Society: Vol. 5, No. 4 (1979)	O/s claims reserves and I.B.N.R.	Historical payments data	Usual stability assumptions	Interesting treatment of future inflation

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Lag factors, the reciprocals of cumulative development factors from the basic chain ladder method are smoothed by an exponential curve-fitting process	Fowler, T.W.	Liability I.B.N.R. Reserves	N.R.G.	O/s claims Reserves and I.B.N.R.	Run-off triangle of claims paid	Basic chain ladder assumptions	Straightforward variation of basic chain ladder method
Taylor's Arithmetic separation method. Average payment in a development year of a cohort of claims is expressed as the product of two factors: one r_j is dependent on the development year and the other on the date of payment & $\Sigma r_j = 1$	Taylor, G.C.	Separation of Inflation and Other Effects from the Distribution of Non-Life Insurance Claims Delays	ASTIN Bulletin Vol. 9	O/s claims reserves and (maybe) I.B.N.R.	Development triangles of total claim payments. Estimates of total numbers of claims occurring for each year of origin (estimates made at end of 1st development year)	Inflation and other exogenous factors dependent on year of payment only. Stability of proportion of total claims amount paid in various development years.	Standard separation method
	Ackman, Green and Young	GIRO Monograph on Estimating Outstanding Claims	G.I.S.G.				
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Taylor's Geometric Separation method. As per Arithmetic Method but $\Pi r_j = 1$ where the r's are the development year factors	Taylor, G.C.	Statistical Testing of a Non-Life Insurance Run-off model	A.S.	O/s claims reserves and (maybe) I.B.N.R.	c.f. Taylor's Arithmetic Separation Method	c.f. Taylor's Arithmetic Separation Method	The computations are similar to those for the arithmetic separation method but produce different results which are not very stable when there are many observations
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Taylor's Regression Method. As per Arithmetic Method but no constraint on Σr_j or Πr_j and uses linear regression techniques	Taylor, G.C.	Statistical Testing of a Non-Life Insurance Run-Off Model	A.S.	O/s claims reserves and (maybe) I.B.N.R.	c.f. Taylor's Arithmetic Separation Method	c.f. Taylor's Arithmetic Separation Method	Yields estimates which are identical to those obtained by the Geometric Separation Method but does provide additional information e.g. correlation coefficient & confidence intervals (for what they are worth); and can be adapted for use with incomplete triangles
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
De Vylder's Least Squares Method. Claim in j th dev. year for i th. u/wtg year is of form $x_i p_j u^{i+j}$ (u constant rate of inflation) find and to minimise $\sum (c_{ij} - x_i p_j)^2$ by an iterative process	Vylder, F. de	Estimation of I.B.N.R. by Least Squares	MVSV (1978) or A.S.	O/s claims reserves and I.B.N.R. for both direct insurance and reinsurance	Run-off triangle of non-cumulative claims payments. The triangle may be incomplete	Triangle columns proportional; constant inflation	"Suspected" data can be omitted from the triangle
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Hachemeister's Model: Data sub-divided by discrete sub-categories reducing estimation of reserves to estimation for small homogeneous groups with due allowance for movement between groups	Hachemeister, C.A.	A Stochastic Model for Loss Reserving	T.I.C.A. (1980) Vol. 1 or A.S.	O/s claims reserves and I.B.N.R.	Registration of numbers of claims, paid amounts and the history of each individual case estimate	Very general assumptions	The method describes a way to formalise the loss process. Algorithms for estimation are not given
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Separation technique: for each year of origin ratios of payments in successive development years are taken to remove effect of numbers of claims and separation method then performed on triangle of ratios	Beard, R.E.	Verification of Outstanding Claims Provision - Separation Technique	ASTIN Bulletin Vol. 9	O/s claims reserves and I.B.N.R.	Development triangle of total claim payments. Data on claims numbers <u>not</u> required.	Stability (over years of origin) of ratios of payments made in development year $n+1$ to development year n	More difficult and volatile than standard separation method.
McClenahan's Formula: $nPm = Cpq^{n-d}(1+x)^{m+n}(1+y)^m$ to give the amount paid in year m+n on losses incurred during accident month m.	McClenahan, C.L.	A Mathematical Model for Loss Reserve Analysis	P.C.A.S. 62 (1975) P.134	O/s claims reserves and I.B.N.R.	Loss payment development statistics. These include claim frequency, severity and average values. (severity = mean average pure loss cost from claims which occurred in the month under consideration)	The run-off pattern has a "geometrical" form and constant rates of change in severity and number of incurred losses	A very involved method with lots of algebra. It is hard to see why reality would follow the described pattern although the model seemed to be usable in McClenahan's example of motor & bodily injury insurance
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Straub's method - modified (to use multiplicative or additive model, or combination thereof, for the claims run-off pattern	Kramreiter and Straub	On the Calculation of I.B.N.R. Reserves	M.V.S.V. (1973)	O/s claim reserves and I.B.N.R. (or final expected burning cost) for both direct insurance and reinsurance	Run-off triangles of cumultative loss ratios (or burning costs)	As per Straub's original method plus a multiplicative or additive claims run-off model (or combination thereof)	Less complex expressions are derived than for Straub's original method
Straub's Method: Based on past experience the unknown final result is estimated in a distribution free and unbiased manner such that the mean quadratic error is minimised.	Straub, E.	On the Calculation of I.B.N.R. Reserves	N.R.G.	O/s claims reserves and I.B.N.R. (or final expected burning cost) for both direct insurance and reinsurance	Cumulative loss ratios (or burning costs) run-off triangles	1. The claim experiences of different years of origin are stochastically independent 2. The mean of the observed values is independent of the underwriting year 3. The product of the premium for an u/wtg year and the covariance of the observed values is independent of the underwriting year	The method avoids an explicit assumption about the nature of the claims distribution and involves only 1st and 2nd order moments. In practice the true means and co-variances would not be known and estimates thereof would be required. A cumbersome method
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Reid's Model: A mathematical model is used to simulate the claims experience incorporating probabilistic and discounting components	Reid, D.H.	Claims Reserves in General Insurance	J.I.A. Vol.105 (1978)	O/s claims reserves and (maybe) I.B.N.R.	Number of claims by year of risk, jointly categorised by claims size and settlement time	The base year is complete, the more recent years can be used to update this base year information; each year's claim settlements follow a similar pattern to the base year	The method is complicated. A confidence interval can be obtained
	Ackman, Green and Young	GIRO Monograph on Estimating Outstanding Claims	G.I.S.G.				
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Vector analysis approach using amounts in up-to-date money values and considering the joint distribution of payments for a single claim in successive development periods	Pollard, J.H.	Outstanding Claims Provisions: a Distribution-free Statistical Approach	J.I.A. Vol.109 Part 3	O/s claims reserves and I.B.N.R.	Claims payment run-off triangles in up-to-date money values	Applicability of central limit theorem	Only applicable where there are a large number of claims. Allows determination of a confidence interval

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
State-Space model framework and the Kalman Filter. Algorithm to embody most common methods	Jong and Zehnwirth	Claims Reserving, State-Space Models and the Kalman Filter	J.I.A. Vol.110	O/s claims reserves and I.B.N.R.	Dependent on underlying methods used	2 sets of equations 1st set: each vector (of observations) is made up of a known, time varying linear combination of unknown parameters, and a zero mean error. 2nd set: state equation describing evolution over time of the "state" vector	Complicated matrix algebra presenting computational problems even for computers. Advantages: Ability to fit any common method to State-Space framework and ability to update existing estimates with new observations.
Finger's Method: Basically, minimizes the sum of squares of the differences between observations and estimates	Finger, R.J.	Modelling Loss Reserve Developments	P.C.A.S. 63(1976) P.90	I.B.N.R.	Aggregate claim amounts, numbers or average claim amounts	Independence between different parameter sets	Very complex and considered too theoretical
Probabilistic model: using Poisson and log-normal distributions. Three slightly different variations are given.	Bühlmann, Schnieper and Straub	Claims Reserves in Casualty Insurance Based on a Probalistic Model	M.V.S.V. (1980)	O/s claims reserves and I.B.N.R.	Run-off triangles of cumulative claims paid, for each reporting year. Run-off triangles of cumulative numbers of claims	A fair number of assumptions about distributions applying and independence of variables	The complexity of calculations and volume of data required seems to render method impracticable especially as no improvement in accuracy over simpler methods
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
Multiplicative model using Hoem's method of analytical graduation: taking a partial claims payment as being a function of 4 factors: reporting time, payment period, inflationary factor and seasonal factor	Linnemann, P.	A multiplicative Model of Loss Reserves: a Stochastic Process Approach	Laboratory of Actuarial Mathematics, University of Copenhagen (1980), Working Paper No. 32	O/s claims reserves and I.B.N.R.	History of Numbers and amounts of partial claim payments in each payment interval for each underwriting year. (Payment interval normally a quarter of a year)	A number of assumptions about independence of variables	The complexity of the method and the volume of data required would seem to render the method unsuitable for practical use.

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Claims process considered as a compound Poisson distribution with a variable (over time) "intensity function" rather than a constant Poisson parameter	Philipson, C.P.	A Generalised Model for the Risk Process and its Application to a Tentative Evaluation of Out-standing Claims.	ASTIN Bulletin Vol. 3	O/S claims reserves and I.B.N.R.	Not obvious	Not clearly stated	Very theoretical and fails to show clearly how to derive parameters and distributions. Unlikely to be of practical use

Direct Business: Specific Applications

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Simple Method based on average claim amounts with claims data sub-divided by size of first evaluation ranges and delay of payment ranges	Buoro, Ferrara and Quario	Statistical Approach to the Outstanding Claims Reserve in Motor Insurance	T.I.C.A. (1976)	O/s claims reserves for portfolios composed mainly of small claims	Settlements in previous 12 months by delay and size of first evaluation; outstanding claims by size of first evaluation ranges	The first evaluation represents a way by which the adjuster graduates his judgement on the claim pattern	A useful simple method
Grossing-up factors for estimating the "tail" for long-tail business	Matthews, T.J.	The valuation of General Insurance Claims Reserves	I.A.A.	O/s claims reserves and I.B.N.R.: "tail" of long-tailed distributions (for both direct business and reinsurance)	Industry-wide data: run-off triangles of cumulative claims paid	The portfolio matches industry-wide data	Useful where the run-off triangles for a portfolio are of insufficient size to use to estimate the "tail"
Tarbell's Formula: Reserve = $\frac{N^y}{N^{y-1}} \frac{C^y}{C^{y-1}} I^{y-1}$ where N = number of claims, C = average claim size, I = I.B.N.R.	Tarbell, T.F. Bornhuetter, R.L. and Ferugson, R.E.	Incurred But Not Reported Claim Reserves The Actuary and I.B.N.R.	P.C.A.S. 58(1971) P.84 P.C.A.S. 59(1972) P.181	I.B.N.R. reserves for direct short-tail classes such as property, motor damage with high claims frequency and stable claims costs	Number and average size of claims together with the I.B.N.R. booked at the same time in the previous year	Stable average claims costs, loss development patterns and volume of business	A very simplistic model. Different bases can be used to estimate I.B.N.R. e.g. earned premiums, incurred losses, premiums in force
Tests goodness of fit of an estimated run-off triangle by developing a chi-square test statistic	Taylor, G.C.	Testing Goodness of Fit of an Estimated Run-Off Triangle	ASTIN Bulletin Vol.10	Tests goodness of fit of estimated run-off triangle	Input payment triangle used by method under test and the fit to that data is produced by the method. Also 1st & 2nd moments of claim amount distribution for each development year	1. Distribution of expected claim delays constant over varying years of origin. 2. Number of claims in each cell is a stationary Poisson variable 3. Sizes of individual claims in each cell are independent random variables	Assumptions and approximations become increasingly complex if claim amount moments are not known.
Standard table method: additive or multiplicative model using multivariate analysis	Clarke, T.G.	An Actuary Looks at Claims Provisions in General Insurance	I.M.A.	O/s claims reserves for a large private car account	Computerized information on required parameters (e.g. age of policyholder, number of years of NCD, vehicle size) for each claim	Additive or multiplacative model applies to given parameters	Considerable amount of data required with detailed coding of claims. This has not been carried out to any great degree for classes other than motor

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
A formula for the number of claims outstanding as a function of duration from occurrence is derived and fitted to the data	Ferrara and Quario	Distribution of the Number of Claims in Motor Insurance According to the Lag of Settlement	ASTIN Bulletin No. 9 (1977)	Future settlement patterns for numbers of claims (for motor business)	Number of claims in a cohort (possibly several years of origin) outstanding at various durations (in months) from date of occurrence	Ad hoc assumptions about the rate of settlement of claims for increasing times since occurrence	Found to give a reasonable fit on motor data
A mathematical model using a Poisson distribution for claims frequency and a distribution with moments depending on delay of settlement for claims amounts	Clarke and Eagles	Mathematical Density Functions Applied to a Liability Insurance Portfolio	T.I.C.A. (1980)	O/s claims reserves and I.B.N.R. for liability claims	Full records of individual claims for a number of years.	Business stable over time, claims events independent, delay and claim size positively correlated, claim distribution can be fitted by a statistical distribution (e.g. log-normal) depending on the delay of settlement	The assumptions required would rarely apply in practice.

Reinsurance: General Methods

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Basic chain ladder with modifications e.g. 1. to use only, say, 5 most recent years in development ratios 2. to use cumulative incurred claims run-off triangles	Guaschi, F.E.	Delay Problems in Reinsurance	I.M.A.	O/s claims reserves and I.B.N.R.	Run-off triangles of cumulative claims paid and/or incurred claims (and of premiums) for up to 11 (or more) development years	Triangle columns are proportional: exogenous factors can be ignored e.g. future inflation rates approximately equal to past inflation rates	Usually world-wide account so that knowledge about inflation rates (past and future) very limited so that number of methods is limited.
Triangulation of incurred Loss Ratio and Curve Fitting. A golden section search is suggested to obtain the best fit. Formulae of form: $l_{x:r} = L_x \{ 1 - e^{-(t/B)^c} \}$	Craighead, D.G.	Some aspects of the London Reinsurance Market in World-Wide Short-term Business.	J.I.A. Vol.106 (1979)	O/s claims reserves and I.B.N.R.	Run-off triangles of incurred loss ratios	At any elapsed point in time for any underwriting year I.B.N.R. is proportional to claims incurred to date	Designed to overcome problem that current loss ratio subject to considerable random fluctuation and to allow for known changes in underwriting practice.

Other methods which have been listed in the "direct business: general methods" section which are also particularly suited for reinsurance are:

a) Basic Chain Ladder

b) Inflation-linked Chain Ladder

c) De Vylder's Least Squares

d) Straub (original and modified)

Reinsurance: Specific Applications

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Empirical method for I.B.N.R. reserves, based on an examination of trends in new claims notified and adjustments made to previous year's estimates	Foster, G.T.	The Determination of I.B.N.R. Reserves and the Effect of I.B.N.R. on the Underwriting of Excess of Loss Business	N.R.G.	I.B.N.R. reserves for excess of loss reinsurance business assuming an individual claims system	For each development year in respect of each underwriting year: a) amounts of new claims notified b) savings on expected settlements c) adjustments in the value of outstanding claims	Trends in past experience can be extrapolated into the future	The proposed empirical approach would probably fit in with the monitoring approach applied by a reinsurer to each treaty written
Average claims methods projecting numbers of claims by u/wtg year and extrapolating the average claim by dev. year to give the ultimate average claim amount	Guaschi, F.E.	Delay Problems in Reinsurance	I.M.A.	O/s claims reserves and I.B.N.R. for reinsurance business on an underwriting year basis, where the numbers of claims are known	Run-off triangles of claims paid and known outstanding claims and numbers of reported claims	Reasonable stability of development patterns for numbers of claims and average claim amount	The method should be modified to allow for inflation
Multi-triangle method, basic chain ladder (on incurred claims for a) gross loss experience; b) net loss experience for a) assuming the retention limit in each u/wtg year for each cedant applied; c) theoretical experience of reinsurance as a) - b).) to obtain reserves to use for various retention levels to apply to each cedant's figures	Khury, C.K.	I.B.N.R. Methods for the Liability Excess of Loss Reinsurer	N.R.G.	I.B.N.R. reserves for excess of loss reinsurance business	A listing from each cedant at the end of each revenue year of the individual claim amounts (paid and outstanding) by u/wtg year for all reported claims	Usual basic chain ladder assumptions. Also that retention limits change only at the beginning of a revenue year	The whole experience available to the reinsurer is utilised to determine I.B.N.R. reserves for different retention levels so the credibility of the assessment should be much greater than otherwise. However, the work involved is formidable.

METHOD	AUTHOR	PAPER	SOURCE	APPLICATIONS	DATA REQUIRED	MAIN ASSUMPTIONS	COMMENTS
Verbeek's separation method: used on numbers of claims reported, the two parameters being calendar year of notification and years from origin to notification Multiply by conservative estimate of mean excess claim which "empirically show a rather stable pattern for fixed excess points"	Verbeek, H.G.	An Approach to the Analysis of Claims Experience in Motor Liability Excess of Loss Reinsurance	ASTIN Bulletin Vol. 6 (1972)	O/s claims reserves and I.B.N.R. for excess of loss reinsurance (where numbers of claims are known)	Number of claims notified to reinsurer, split by year of origin and year of notification	Number of claims to direct insurer for particular year of origin are Poisson. Claims size distribution dependent on year of notification only. Average size of excess claim known and stable	Assumptions are very restrictive and unlikely to be true. (No details given of how method has worked in practice)
	Eeghen, J.van	Loss Reserving Methods	S.A.S.				
An integral expression is evaluated to determine ultimate claims cost assuming numbers of claims are Poisson distributed and amounts are log-normally distributed	Harding, V.	Treatment of Incurred But Not Reported Claims	N.R.G.	O/s claims reserves and I.B.N.R. for excess of loss reinsurance (with an individual claims system)	Gross premium income (or earned premium less expenses loadings) for each accounting year, incurred claims cost for each accounting year for each individual claim exceeding the reinsured's retention limit	1. The number of reinsurance claims follows a Poisson distribution. 2. Amounts of claims follow a log-normal distribution (truncated below the retention limit) 3. Reinsurance claims are stochastically independent	The experience for different treaties, with different retention limits, can be aggregated to improve the credibility of the result.