INVESTMENT STRATEGY WORKING PARTY - PAPER ON WITH-PROFITS INVESTMENT STRATEGY

Life Convention 2004

AUTHORS & ACKNOWLEDGEMENTS

This paper has been produced by a working party of the Institute of Actuaries. The members of the working party are:

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Responsibility for the paper obviously lies with the authors. We would, however, like to acknowledge the contribution from Trevor Maynard who was an original member of the party until his transfer to General Insurance. Also we acknowledge the modelling work carried out by past and present members of Norwich Union's ALM team, namely Rachid Bouchaib, Helen Evans, David Walsh and Phil Eyley.

1. INTRODUCTION

This paper proposes a dynamic investment strategy that can be used by Life offices to control the risks in their With-Profits Fund. The strategy can be used to manage the funds' Equity Backing Ratio (EBR) and is based on the concept of a Theoretical EBR. The Theoretical EBR indicates how onerous the guarantees are and the strategy involves rebalancing the assets of the fund so that the actual EBR is close to the theoretical EBR.

The dynamic strategy tends to give favourable outcomes in the 'tails' of a distribution (compared to floating or fixed investment strategies). This is because the Theoretical EBR falls following poor equity returns and rises following good equity returns. Therefore, the dynamic strategy protects investors from sustained poor equity returns and enables investors to benefit from sustained good equity returns.

We investigate the impact of the dynamic strategy on controlling the risks faced by a model office by investigating its position at the end of 2002 assuming it followed a dynamic strategy and comparing the results to alternative strategies. Work has also been carried out using a stochastic model to project a future position, with example results included in the appendices.

Applying the results of our investigation help us to forecast:

- Probabilities of ruin
- Future levels of EBR
- Size and timing of future cashflows
- Affordability and sustainability of future bonus strategies

2. THE MODEL OFFICE

2.1 Generation of the Model Office

A model office was set up on which to carry out the investigations of the working party. This was done in such a way that it was sufficiently flexible to allow changes to be made to the overall model by changing just a few parameters. The office was kept simple whilst reflecting some of the potentially significant guarantees that might have been issued by a typical UK Life Office over the last 25 years.

The model office generates guarantees and asset shares for:

- Conventional Life endowments

- Unitised With Profits Life single premium bonds

- Unitised With Profits Pension single and regular premium contracts with guaranteed growth rates

The following sections describe the theoretical office, "Trio Life", that has been produced and on which the modelling has been based.

2.2 The Development of Trio Life

Trio Life is a mutual life office based in the UK. It started writing conventional endowment business in 1978 and this remained its only product line until the late 1980's. Bonus rates of 4% on sum assured and 7% on previously allocated bonus were declared throughout this period. Business volumes increased steadily but unspectacularly over this period.

In 1988, Trio Life introduced a unitised with profits pensions contract on which it offered a guaranteed annual bonus rate of 4% p.a for units allocated in that year. An additional 6% bonus was offered on this contract in the first few years, giving a total bonus allocation of 10%.

In 1992, a new line of business was introduced in the form of a unitised with profits investment bond. The initial rate of bonus was 8%. The bond offered a guarantee at the 10^{th} anniversary that a policyholder cashing in the policy would receive the full face value of their units, i.e. that no market value reduction would be applied.

From 1991, the company recognised a need to gradually reduce the bonus rates offered as accumulated guarantees increased. Bonus rates were reduced steadily over the next 7 years so that by 1998 the bonus rates offered were as follows:

Table 2.1 Declared Bonus 1998

	Conventional Life		Unitised	
	On Sum Assured	On Bonus	Life	Pensions
1998 Bonus	2.50%	4.50%	6.00%	7.00%

In 1996, the company had also withdrawn the 4% guarantee offered on pensions business.

During this period, sales of endowment policies increased substantially, as the market for mortgage endowments increased. Volumes of investment bonds had also grown substantially since the product was launched and pensions sales had been steadily increasing.

From 1997, there was a change in the mix of business being sold, as sales of endowments declined to almost zero in three years, as confidence in the suitability of the product was eroded. Trio Life ceased writing endowment business in 2001. Over this time, however, there were further strong rises in investment bond and pensions sales, boosted by a strong stock market. Whilst pension sales remained strong from 2000 onwards, sales of investment bonds declined significantly as the stock market yielded negative returns, impacting adversely on consumer confidence in investment products.

Over this period, Trio Life also recognised a need to further reduce bonus rates and the reductions seen over the previous few years were accelerated. By 2002, the bonus rates offered were:

Table 2.2 Declared Bonus 2002				
	Conventional Life		Unitised	
	On Sum Assured	On Bonus	Life	Pensions
1998 Bonus	1.50%	2.50%	3.75%	4.75%

Since outset, the company had operated a policy of managing its investments through setting a target equity backing ratio which was the same across all policies and product types. Initially this was 60% through until 1996, after which it was steadily increased to 75% by 1999. In later years, it was reduced in response to the declining stock market so that by 2002, the equity backing ratio was 55%.

2.3 Trio Life Today

At the end of its first 25 years, Trio Life had become an office writing only unitised with profits business through an investment bond and single and regular premium pensions products. The latter generated roughly 75% of its new business in 2002. The company had, however, built up a significant block of in force business which included conventional endowments.

Total asset shares across the product lines had accumulated to £10.9bn from a total value of assets of £12.2bn, so that the company had an available estate of £1.3bn.

The table below summarises the high-level position of the office at end 2002.

Table 2.3 Summary at 31/12/2002	
Product	Asset Shares £bn
UWP Bond	2.8
UWP Pension Regular	2.3
UWP Pension Single	2.3
Conventional Endowment	3.4
Total Asset Shares	10.9
Market Value of Assets	12.2
Available Estate	1.3

At end 2002, the start point of the model office projections, the equity backing ratio (including property investment) was 55% across all business. Its investment strategy was as follows:

Table 2.4: Investment Strategy	
Equities	Invested in a diversified mix of UK and overseas stocks. Approximate split 70/30 UK/Overseas
Fixed Interest	Benchmark UK Government Fixed Interest

3. MODEL OFFICE PROJECTIONS

3.1 Methodology

In investigating the benefits of a dynamic investment strategy we have adopted two approaches:

- We have investigated the position of Trio Life at the end of 2002 assuming that it used a range of different investment strategies in the past.

- Using stochastic simulation techniques we have projected the position of Trio Life until the end of 2025 when the last policies mature. This has enabled us to assess the risks that the guaranteed maturity benefits pose to the office and to investigate the impact that its future investment strategy has on the risk profile.

3.2 Theoretical Equity Backing Ratio

The dynamic investment strategy is used to manage the funds' Equity Backing Ratio (EBR) and is based on the concept of a Theoretical EBR. This section introduces the Theoretical EBR and discusses factors which influence it. The approach is similar to that adopted by Hibbert & Turnbull (2003) and Appendix F contains a comparison with their approach.

We can construct a dynamic hedge portfolio to meet the guaranteed maturity values of a fund. The hedge portfolio involves investing in zero coupon bonds to match the guarantees and using the remainder of the assets to replicate equity call options. The replication of the equity call options involves rebalancing between risky assets and risk-free bonds depending on equity price movements.

Using this approach does not require an actual investment in options or zero coupon bonds because it follows that the dynamic hedge portfolio can be decomposed into three components using Black Scholes option pricing techniques:

- Risk-free bond held to match guarantees
- Risky portion of call option
- Risk-free bond portion of call option

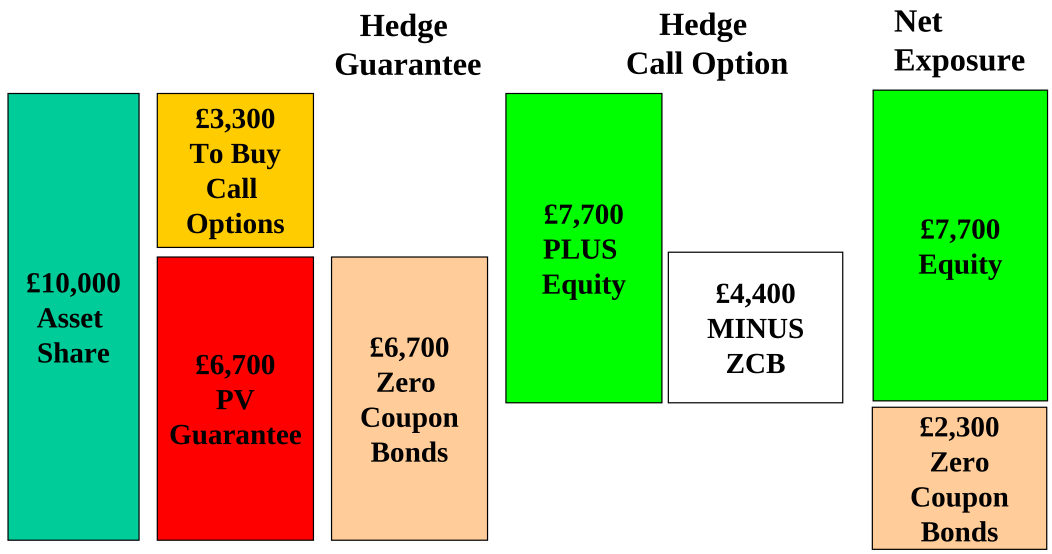
This enables us to calculate the Theoretical EBR - the proportion of fund that can be invested in risky assets such as equities and property - and to manage our investment strategy accordingly.

We illustrate this with a simple example of a single premium UWP bond.

A bond written 3 years ago with 7 years left to run has Asset Share (net of future charges) of £10,000. Projected Guaranteed Benefit (including an allowance for future bonus) is £9,500, with a present value of £6,700.

From this we can calculate the amount available to spend on the call option is £3,300 (being £10,000 - £6,700). A replicating portfolio for this might be £7,700 of risky asset and £4,400 of risk free. It follows that the total of risk free assets (ZCBs) in the portfolio is £2,300; being the present value of the guaranteed benefits backed by notional ZCBs (£6,700) less the risk free element of the replicating portfolio (£4,400). The geared equity exposure arising from the call option gives the equity content of the theoretical equity-backing ratio.

We can present this as:



In our example the Theoretical Equity-Backing Ratio is 77%; being 7,700 / (7,700+2,300). As a comparison it's worth noting that investing the surplus £3,300 (between asset share and PV of guaranteed benefits) wholly and directly in equity would give an EBR of only 33%.

Factors influencing the level of the theoretical EBR are:

- *The fund value relative to the guarantees.* This will change when asset values move, or when changes to the guarantee are made that were not anticipated e.g. as a result of new business. The higher the fund value relative to the guarantees, the higher the theoretical EBR.

- *The term to maturity.* Time provides the horizon over which investment risk is taken. This heavily influences the trade-off between risks and rewards.

- *Equity Volatility.* Equity volatilities can change over time and must reflect the riskiness of the equity-type assets held by the fund. The higher the equity volatility, the lower the theoretical EBR.

- *Gilt yields.* A fall in the gilt yield causes a fall in the theoretical EBR as the cost of matching the guarantee rises.

In certain cases the Theoretical EBR may be undesirably high and to control this we apply an upper limit of 70% in our model.

3.3 Introducing the Matching Parameter

The matching parameter is the proportion of the guarantees to be matched by zero coupon bonds when calculating theoretical EBR. Our example above assumed a matching factor of 100%, but introducing a multiplier to the PV of guarantees gives control over the level of risk. A higher value of the matching parameter leads to a lower theoretical EBR and corresponds to a less risky strategy.

In a With-Profits fund the matching parameter would be set with regard to the size of inherited estate and should correspond to an acceptable Value-at-Risk calculated from a stochastic projection of the future state of the fund.

In our investigation we have used the matching parameter to align initial EBRs and give comparability between different investment strategies.

3.4 Key Modelling Techniques and Assumptions

Some of the key modelling techniques and assumptions used are described below:

- To calculate the return on fixed interest stocks we have assumed that a matching fixed interest portfolio is held. There are two possible approaches to calculating asset share:

  Group Method: The return on the whole portfolio is calculated and the portfolio return is credited to all asset shares.

  Individual Method: The gilt return credited to a policy's asset share reflects the change in value of the bonds backing the policy i.e. bonds with maturity equal to the maturity of the policy (where known).

  We have used the individual method, but further analysis of the approaches is given in Appendix B.

- Asset shares have been accumulated allowing for investment returns, expenses, tax and premiums. Investment returns (on equities and fixed interest) and expenses were based on actual historic returns for different asset classes used in the model office. We have assumed that life business was taxed at 20%.

- We have developed a dynamic regular bonus rule which is linked to the Theoretical EBR. Regular bonuses are calculated for conventional life, unitised life and unitised pension business separately as:

  Theoretical EBR * Gilt Yield * (1 – tax rate) / 1.125

  The rule provides a link between future bonuses given and the strength of the fund and gilt returns. A change of no more than 0.5% a year is allowed, in order to give some smoothing of bonuses from year to year. Historic bonuses are shown in Graph 4.5, but we have also examined the effect of using the dynamic rule historically.

- When a policy matures the payout is restricted to a proportion of asset share. The payout is calculated as:

  Max(Guarantee, Payout Factor * Asset Share) where the payout factor is less than 1

  When asset shares are above the payout the surplus is transferred to the estate. When asset shares are below the guarantee there is a claim on the estate. The payout factor is chosen so that the total positive and negative cashflows over time are the same on average and the expected net claim on the estate is zero.

- To align initial EBRs and give comparability between different investment strategies we introduce a 'matching parameter'. The matching parameter is the proportion of the guarantees to be matched by zero coupon bonds when calculating the theoretical EBR. The matching parameter gives control over the level of risk. A lower value of the matching parameter leads to a higher initial EBR and corresponds to a riskier strategy.

- Ruin occurs when the estate becomes negative.

Appendix A gives more detail on the model projections and the mathematics involved.

4. RESULTS

4.1 Investigations Carried Out

We have investigated the position of Trio Life at the end of 2002 assuming that it used the following investment strategies in the past:

- Fixed Strategy: 55% EBR every year since 1978
- Floating Strategy: The office starts with an initial EBR of 55% in 1978. At later times, the EBR only changes due to market movements. The office does not perform any switches between equity-type and fixed interest-type assets, and new business investment and disinvestment for claims are made in proportion to the current asset mix.
- Dynamic Strategy: EBR equal to the calculated theoretical EBR of the office every year since 1978 with a matching parameter of 100%. (The EBR using a 100% matching parameter would have been around 55% in 1978.)
- Model Office Strategy: EBRs as used by Trio Life (see Graph 4.2).

We have done this firstly by assuming the office paid the bonuses paid by Trio Life and then using the dynamic bonus rule given in section 3.3.

In addition we have carried out stochastic projections of Trio Life's position following different strategies starting from the actual position at the end of 2002.

4.2 Historic Investigation with Actual Bonuses

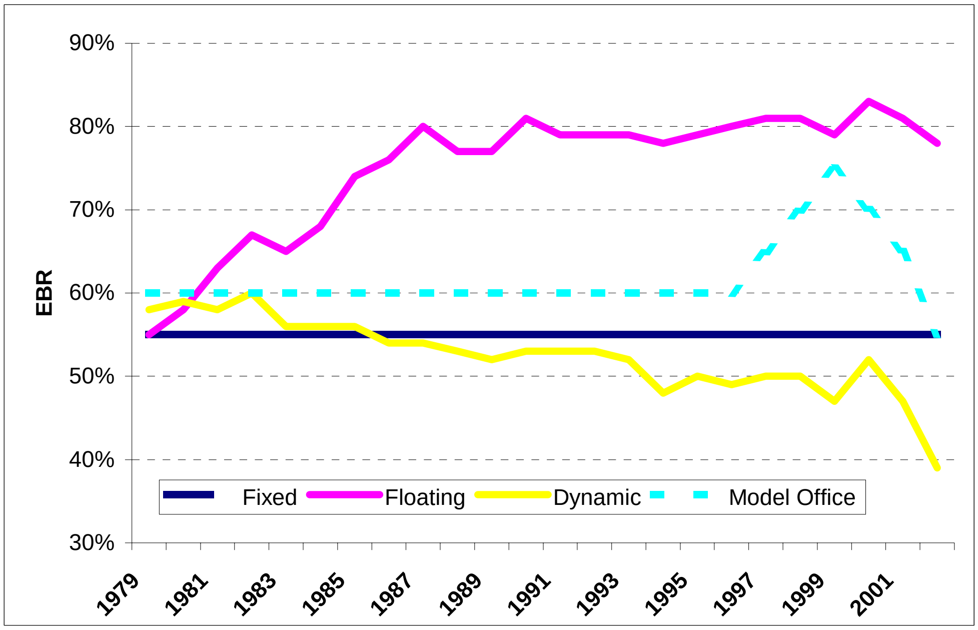
The table below shows the asset shares, guarantees and calculated EBRs for each part of the business and the whole office at the end of 2002 for each investment strategy. Note that the guarantees apply at future dates and so are not comparable with the asset shares. They are included for comparison with the lower level of guarantees built up under the dynamic bonus strategy shown in 4.3.

Table 4.1: Trio Life's position at end 2002 given actual bonuses.

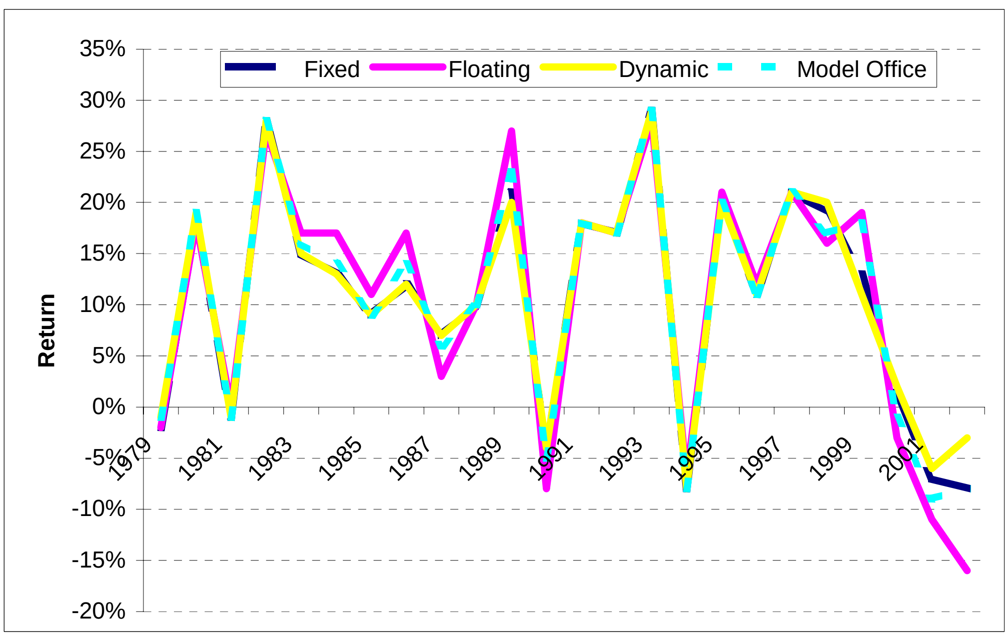
Product	Gtees (£bn)	Fixed		Floating		Dynamic		Model Office	
		Asset Share	EBR	Asset Share	EBR	Asset Share	EBR	Asset Share	EBR
Conventional Life	6.5	3.4	34%	3.2	29%	3.6	36%	3.4	34%
Unitised Pensions	5.1	4.6	18%	4.2	0%	4.9	26%	4.6	17%
Unitised Life	3.3	2.8	1%	2.6	0%	2.9	10%	2.8	0%
Overall	**14.9**	**10.9**	**24%**	**9.9**	**14%**	**11.4**	**29%**	**10.9**	**24%**

The graphs below show the EBRs and annual investment returns experienced by the office following each investment strategy. The results on which the graphs were based are shown in Appendix C.

Graph 4.2: Historic Equity Backing Ratios



Graph 4.3: Historic investment returns



4.3 Historic Investigation with Dynamic Bonuses

An alternative projection was carried out which looked at the bonuses that would have resulted from following the bonus strategy described by the dynamic rules in 3.3 above i.e. with bonuses linked to the theoretical EBR.

The table below shows the asset shares, guarantees and calculated theoretical EBRs for each part of the business and the whole model office at the end of 2002 for each investment strategy:

Table 4.4: Trio Life's position at end 2002 given dynamic bonuses.

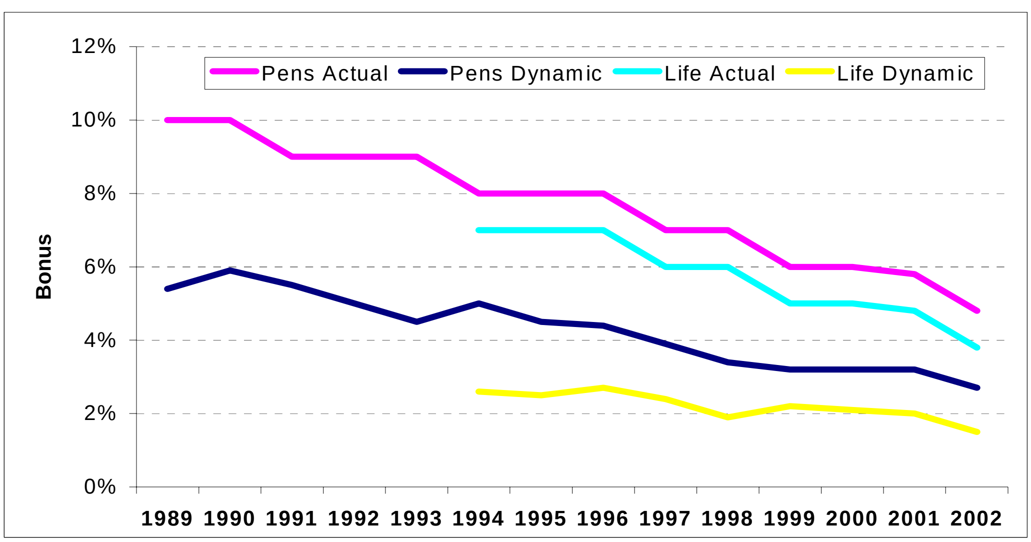
Product	Gtees (£bn)	Fixed		Floating		Dynamic		Model Office	
		Asset Share	EBR	Asset Share	EBR	Asset Share	EBR	Asset Share	EBR
Conventional Life	6.1	3.4	41%	3.2	37%	3.5	42%	3.4	41%
Unitised Pensions	4.5	4.6	40%	4.2	24%	4.7	41%	4.6	39%
Unitised Life	2.9	2.8	24%	2.6	6%	2.8	26%	2.8	23%
Overall	**13.5**	**10.9**	**38%**	**9.9**	**28%**	**11.0**	**39%**	**10.9**	**38%**

A comparison of the results above with Table 4.2 shows that following the dynamic bonus rules would have led to a lower level of guarantees and a higher Theoretical EBR.

The graph below shows the bonuses that would have been declared assuming a fixed investment strategy. Using any of the other three investment strategies would have resulted in similar bonuses. The results are shown for unitised life and pensions business. A similar pattern emerges for conventional business. For comparison, the bonuses actually declared by the model office are also shown.

It can be seen that over the period, bonuses declared were consistently higher than those produced by the dynamic bonus methodology and consequently guarantees were built up more quickly. Appendix C shows a full comparison of bonuses under the two approaches for the different lines of business.

Graph 4.5: Actual and dynamic bonus rates for unitised business.



4.4 Projecting a Dynamic Investment Strategy

We have used stochastic simulation techniques to project the position of the office and of individual product lines over the next 10 years. This helps us to assess the risks that the guaranteed maturity benefits pose to the office and to investigate the impact that investment strategy has on the risk profile. We have not considered the risks posed by smoothing.

Appendix D shows some of the results produced using this approach, for a portfolio of single premium unitised with profits bonds. We investigated two possible investment strategies:

- Floating investment strategy with an initial EBR of 55%.

- Dynamic investment strategy with an initial EBR of 55%. The office rebalances its assets so that the EBR is equal to the calculated EBR at the start of each year. The matching parameter (as described in 3.3) is 76% in order to align the starting EBRs.

We have assumed that the estate is invested in the same way as asset shares, and that all the asset shares have the same EBR.

The stochastic results show again how the dynamic approach produces more favourable results. The estate ruin probability with the floating investment strategy is 9.6%. The estate ruin probability with the dynamic investment strategy is 9.0%.

Although the concept behind the dynamic investment strategy involves hedging guarantees, shortfalls can arise using the dynamic strategy. Reasons for this include:

-The fund is not rebalanced continuously in practice
-We apply the same EBR to all the asset shares
-Transaction costs and tax must be paid

The modelling ignores the risks implicit in holding different fixed interest investments for varying duration from the matching profile. We have been addressing this by changing our fixed interest investments so that the cashflows are close to the liabilities.

Consideration can be given to the investment strategy for the Estate. If the probability of ruin is too high when the Estate is invested in the same way as Asset Shares, action can be taken to reduce this to an acceptable level by altering the strategy to one that is more appropriate. For example, using the model of Trio Life we can reduce the estate ruin probability to close to 4% by either

- Investing the estate in fixed interest; or

- Increase the matching parameter such that the initial EBR reduces from 55% to 46%

Investing the estate in fixed interest while also withdrawing the entitlement to future bonuses would give a reduction in matching parameter equivalent to an initial EBR of 80% and a projected ruin of 4%.

5. ACTIONS TO CONTROL RISK & CONCLUSIONS

5.1 Introduction

The results presented in this paper give a clear indication of a number of risk areas for the model office and hence point to ways to control and manage risk. It is interesting to note that since we started this work one large life office has publicly altered its approach to managing risk along some of the lines suggested here.

5.2 Historical Risk Control

The results in sections 4.2 and 4.3 give some insight into how successful the office has been in controlling its risks. In outline they show that:

- Following the floating investment strategy historically, where the asset mix is simply allowed to drift in line with market movements, would have been a disaster.
- The asset strategy followed has in practice controlled risk fairly effectively relative to the floating strategy.
- Following a dynamic investment strategy, where the asset mix is altered each year in line with the theoretical EBR, would have worked better than the actual strategy followed, but not by a great deal.
- The bonus strategy has arguably been too generous. It has certainly increased the level of guarantees relative to the funding position such that the theoretical EBR has fallen significantly in recent years (the time path of the EBRs is shown in Appendix C).

5.3 Prospective Risk Control – Investment Strategy

Looking forwards the office has a number of options for controlling risk:

- Reduce the overall EBR to more in line with the theoretical level, perhaps using different EBRs for different product lines.
- Charge for guarantees. Note that the dynamic investment strategy implicitly charges for guarantees since the cost of hedging will materialise by impacting the investment return achieved, resulting in a lower expected return.
- Rely on the estate to meet all guarantees.

To date the model office has used a single equity backing ratio for all with profits business. It is noticeable from the different theoretical EBRs for different products (see Table 4.4) and different cohorts of each product (see Appendix E) that there is a considerable amount of risk cross-subsidisation. The office may feel that PRE requires a single EBR for all cohorts of a single product, with the resulting cross-subsidisation being an inherent feature of with-profits business. However, this may or may not extend to using the same EBR for different product lines, perhaps depending on what policyholders have historically been told about the investment strategy for their policies. It is certainly expected that a greater level of risk control could be achieved by matching the EBR for each product more closely with the theoretical EBR for those policies and perhaps by choosing a lower EBR for the estate. This is something we intend to investigate next with the model office.

Whilst the discussion above has focused on the equity / bond split, the bond strategy is also important. The model office has assumed that the term of the bonds held is matched to the aggregate term of the guarantees embedded in the liabilities. If this is not done then the risk management will clearly be less effective. Further control could be achieved by matching at the individual policy level, as has been assumed here, but as the results in Appendix B show this has relatively small impact in the context of the equity risk.

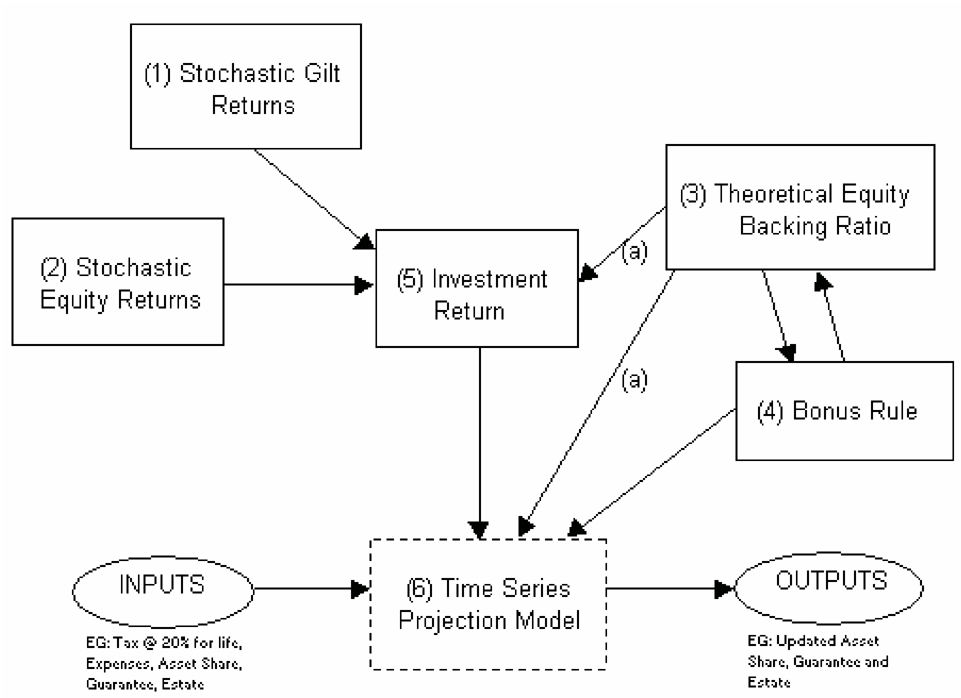
5.4 Prospective Risk Control – Bonus Strategy

The model office has assumed that some risk control will be exercised through bonus strategy, with the level of bonuses dependent on the overall solvency of the fund. This is clearly important – the lower the level of guarantees that is built up the higher the theoretical EBR and hence the lower the level of risk run for a given EBR.

This is perhaps an example of a general principle for managing with-profits business, namely the value of maintaining management flexibility. If actions are chosen to retain as much flexibility as possible, for example by keeping bonuses and guarantees down or by avoiding committing to rigid rules for allocating investment returns to asset shares, then there is less need to rely solely on the equity backing ratio for risk control purposes. This may increase expected returns to policyholders, but must obviously be weighed up against PRE in other areas.

Appendix A: Overview of the Model and Mathematical Statement of Problem

The diagram outlines the key components of the methodology used to project outcomes for the model office:



(a) Assumes a dynamic investment strategy is being modelled.

A1: Stochastic Gilt Returns

Gilt returns are calculated stochastically from the Hull-White model[1]. The model is based on the following process for the short rate *r:*

$$dr = [\theta(t) - ar]dt + \sigma dz$$

The vector theta is defined as:

$$\theta(t) = \frac{f(0,t) - f(0,t-dt)}{dt} + af(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})$$

Where $f(t,T)$ = Forward rate for date T as at time t.
a = Reversion rate
σ = Volatility
dz = Standard Normal variable

In the model, the short rate follows the slope of the forward rate curve. If it deviates from the curve then it reverts back to it at rate a. We have used the parameters a = 6.1% and σ = 1%.

[1] See J Hull and A White's paper 'Pricing Interest Rate Derivative Securities' in *Review of Financial Studies,* 3, no.4 (1990) for more details.

A2: Stochastic Equity Returns

Equity returns are calculated for each time step t using the lognormal model:

$$\log(1+ER) = \mu - \frac{1}{2}\sigma^2 + \sigma.dz$$

Where: *ER* = Equity Return
μ = Equity Drift
σ = Equity Volatility
dz = Standard Normal variable

Drift and volatility are assumed to be constant throughout the projection. We have used the parameters μ = 7.5% and σ = 17%.

Note that there is no mean reversion in the equity model and equity and bond returns are independent.

A3: Theoretical Equity Backing Ratio (EBR)

We use Black-Scholes[2] option pricing techniques to calculate the cost of a European Call and EBR:

Where:

$$EuropeanCallCost = S.\Phi(d_1) - K.\Phi(d_2)$$

And:

$$EBR = \frac{Max(0, S-K).\Phi(d_1)}{EuropeanCallCost}$$

Where: K = Present value of the guarantee plus bonuses (strike price)
S = Adjusted Asset Share
r = Net interest rate
t = Time to maturity (years)
σ = Equity Volatility

The EBR is resolved through iteration because the maturity value depends on the bonus allowance, which itself depends on the current EBR.

For Black-Scholes, the adjusted asset share, S, is the asset share discounted for future expenses plus the present value of all future premiums (less the present value of the shareholder transfer from future bonuses for a proprietary office).

The strike price, K, is the maturity value discounted at the net rate for t years; where the maturity value is the sum assured plus bonuses to date rolled up to maturity assuming a rate of future bonus in each year derived from the bonus rule [given below in (4)].

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[2] Where $d_1 = \frac{\log(S/K) + rt + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$

A4: Bonus Rule

Regular bonuses were calculated for conventional life, unitised life and unitised pension business as:

*Theoretical EBR * Gilt Yield * (1 – tax rate) / 1.125*

A5: Investment Return

The investment return is calculated as:

*Equity Return * EBR + Gilt Return * (1 – EBR)*

A6: Further details of model assumptions

The following points set out some of the detail of the modelling approach and assumptions used.

- The model uses annual time steps.
- We have assumed that there are two asset classes; equity and fixed interest.
- To project asset shares, we have made an allowance for expenses, policy lapses, tax on life policies and for future regular premiums on conventional policies.
- We assume that conventional endowment regular premiums are received at the end of the year. We assume that charges of 0.8% of asset share are taken at the end of the year. We assume that the rate of tax on investment returns is 20% for life business.
- We have assumed that 4% of policies lapse each year.
- The theoretical EBR is capped at 70%. There is no minimum.
- We have assumed that all the UWP bond policyholders withdraw on the no-MVR guarantee date (if their policy has not lapsed by that time).
- We have not allowed for the impact of new business or future regular premiums for unitised business.
- We have not considered the impact of statutory solvency.
- Mortality has been ignored.

Appendix B: Asset Share Methodology

This section gives the expectation and volatility of asset shares at maturity using the individual and group methods of calculating asset shares (see Section 3.3).

In our model we have assumed an individual method where the gilt return credited to a policy's asset share reflects the change in value of the bonds backing the policy. Section 3.3 explained how this differs from a group method.

The table below shows the expectation and standard deviation of asset shares of maturing policies (using the floating EBR investment strategy) in the following situations:

- Gilt Yields do not vary stochastically (σ =0%) and the group method of calculating asset shares is used
- Gilt Yields vary stochastically and the group method of calculating asset shares is used
- Gilt Yields vary stochastically and the individual method of calculating asset shares is used

Table B.1: Expectations and Standard Deviations of Maturing Asset Shares (£m)

Year	Non Stochastic		Stochastic			
			Group Method		Individual Method	
	Expectation	SD	Expectation	SD	Expectation	SD
2003	851	72	851	74	851	72
2004	838	102	839	103	839	102
2005	884	127	884	128	885	127
2006	933	155	932	156	933	155
2007	1,011	191	1,010	191	1,012	191
2008	1,118	230	1,116	232	1,118	230
2009	1,057	239	1,055	240	1,057	239
2010	1,186	296	1,185	298	1,187	296
2011	961	256	961	257	961	256
2012	941	262	942	263	942	262
2013	683	206	685	207	683	206
2014	622	194	624	196	623	194
2015	648	206	651	208	649	206
2016	710	240	715	242	712	240
2017	691	245	697	248	694	245
2018	353	102	356	103	355	102
2019	428	128	433	130	431	129
2020	420	125	425	127	423	125
2021	410	126	416	128	414	126
2022	384	117	391	119	389	117
2023	203	61	207	63	206	61
2024	102	31	104	32	103	31
2025	41	12	42	13	41	13

The standard deviation results in this table indicate that volatility of gilt yields poses an additional risk to offices where asset shares of maturing policies may drop below the guarantees or where payouts are smoothed from year to year. The need to buy or sell bonds before they mature means it is difficult to eliminate the risk from movements in gilt yields. However, the increased risk is small relative to the equity risk. Using the individual method of calculating asset shares reduces the risk of increased shortfalls.

Appendix C: Projection of Trio Life up to 2002

The table below shows the EBRs and Returns underlying the graphs in Section 4.2

Table C.1: Simulated Historical EBRs and Returns

Year	Fixed		Floating		Dynamic		Model Office	
	EBR	Return	EBR	Return	EBR	Return	EBR	Return
1979	55%	-2%	55%	-2%	58%	-1%	60%	-1%
1980	55%	18%	58%	18%	59%	19%	60%	19%
1981	55%	-1%	63%	0%	58%	-1%	60%	-1%
1982	55%	28%	67%	27%	60%	28%	60%	28%
1983	55%	15%	65%	17%	56%	15%	60%	16%
1984	55%	13%	68%	17%	56%	13%	60%	14%
1985	55%	9%	74%	11%	56%	9%	60%	9%
1986	55%	12%	76%	17%	54%	12%	60%	14%
1987	55%	7%	80%	3%	54%	7%	60%	6%
1988	55%	10%	77%	10%	53%	10%	60%	10%
1989	55%	21%	77%	27%	52%	20%	60%	23%
1990	55%	-4%	81%	-8%	53%	-4%	60%	-5%
1991	55%	18%	79%	18%	53%	18%	60%	18%
1992	55%	17%	79%	17%	53%	17%	60%	17%
1993	55%	29%	79%	28%	52%	29%	60%	29%
1994	55%	-8%	78%	-7%	48%	-8%	60%	-8%
1995	55%	20%	79%	21%	50%	20%	60%	20%
1996	55%	11%	80%	12%	49%	11%	60%	11%
1997	55%	21%	81%	21%	50%	21%	65%	21%
1998	55%	19%	81%	16%	50%	20%	70%	17%
1999	55%	13%	79%	19%	47%	11%	75%	18%
2000	55%	1%	83%	-3%	52%	2%	70%	-1%
2001	55%	-7%	81%	-11%	47%	-6%	65%	-9%
2002	55%	-8%	78%	-16%	39%	-3%	55%	-8%

The following table shows the bonuses that would have been declared under the dynamic approach described in Section 4.3 and illustrated in Graph 4.5

Table C.2: Simulated Dynamic Historical Bonuses

Year	Conventional Life	Unitised Pensions	Unitised Life
1979	6.1%		
1980	5.8%		
1981	6.3%		
1982	5.8%		
1983	5.3%		
1984	4.8%		
1985	4.3%		
1986	3.9%		
1987	3.6%		
1988	3.4%		
1989	3.6%	5.4%	
1990	4.0%	5.9%	
1991	3.6%	5.5%	
1992	3.2%	5.0%	
1993	2.7%	4.5%	

1994	3.1%	5.0%	2.6%
1995	2.7%	4.5%	2.5%
1996	2.8%	4.4%	2.7%
1997	2.3%	3.9%	2.4%
1998	1.8%	3.4%	1.9%
1999	1.9%	3.2%	2.2%
2000	1.8%	3.2%	2.1%
2001	1.8%	3.2%	2.0%
2002	1.5%	2.7%	1.5%

The regular bonuses paid by Trio Life in the model office are shown below for comparison:

Table C.3: Actual Historical Bonuses

Year	Conventional Life		Unitised Pensions	Unitised Life
	Sum Assured	Regular Bonus		
1979	4.0%	7.0%		
1980	4.0%	7.0%		
1981	4.0%	7.0%		
1982	4.0%	7.0%		
1983	4.0%	7.0%		
1984	4.0%	7.0%		
1985	4.0%	7.0%		
1986	4.0%	7.0%		
1987	4.0%	7.0%		
1988	4.0%	7.0%		
1989	4.0%	7.0%	10.0%	
1990	4.0%	7.0%	10.0%	
1991	4.0%	7.0%	9.0%	
1992	4.0%	7.0%	9.0%	
1993	3.5%	6.5%	9.0%	
1994	3.5%	6.5%	8.0%	7.0%
1995	3.0%	6.0%	8.0%	7.0%
1996	3.0%	5.5%	8.0%	7.0%
1997	3.0%	5.0%	7.0%	6.0%
1998	2.5%	4.5%	7.0%	6.0%
1999	2.0%	4.0%	6.0%	5.0%
2000	1.5%	3.5%	6.0%	5.0%
2001	1.5%	3.0%	5.8%	4.8%
2002	1.5%	2.5%	4.8%	3.8%

Appendix D: Comparing Floating and Dynamic Investment Strategies

We have used stochastic simulation techniques to project the position of the office and of individual product lines. In this Appendix we show the results for the unitised life bond product on a standalone basis, increasing the volume of business by a factor of 3.

D1. Estate Distribution

The following tables show the distribution of the estate starting with £1.25bn using the dynamic strategy and the floating strategy. The dynamic strategy results have been colour-coded. A green cell indicates that the result is higher for the dynamic strategy. A red cell indicates that the result is lower for the dynamic strategy.

The estate ruin probability with the floating investment strategy is 10.1%. The estate ruin probability with the dynamic investment strategy is 9.2%.

Dynamic Strategy:

Percentile	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
1%	1.1	1.0	1.0	0.9	0.8	0.5	0.2	-0.2	-0.5	-0.5	-0.5
5%	1.2	1.1	1.1	1.0	1.0	0.7	0.4	0.0	-0.2	-0.2	-0.2
10%	1.2	1.1	1.2	1.2	1.1	0.9	0.6	0.3	0.1	0.0	0.0
25%	1.3	1.3	1.4	1.4	1.4	1.3	1.1	0.8	0.6	0.7	0.7
50%	1.4	1.4	1.5	1.7	1.8	1.8	1.7	1.6	1.6	1.7	1.7
75%	1.4	1.6	1.8	1.9	2.2	2.4	2.6	2.7	2.8	3.0	3.1
90%	1.5	1.7	2.0	2.3	2.5	2.9	3.2	3.5	3.8	4.1	4.2
95%	1.6	1.8	2.1	2.4	2.7	3.1	3.5	3.9	4.3	4.7	5.0
99%	1.7	2.0	2.4	2.8	3.1	3.6	4.2	4.7	5.3	5.9	6.1

Floating Strategy:

Percentile	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
1%	1.1	0.9	0.9	0.8	0.6	0.3	-0.2	-0.7	-0.9	-1.0	-1.0
5%	1.2	1.1	1.1	1.0	0.9	0.6	0.3	-0.2	-0.5	-0.5	-0.5
10%	1.2	1.1	1.2	1.2	1.1	0.9	0.6	0.2	0.0	0.0	0.0
25%	1.3	1.3	1.4	1.4	1.5	1.4	1.2	0.9	0.8	0.9	0.9
50%	1.4	1.4	1.6	1.7	1.8	1.9	1.9	1.8	1.8	2.0	2.0
75%	1.4	1.6	1.8	1.9	2.1	2.4	2.6	2.7	2.8	3.0	3.1
90%	1.5	1.7	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.1
95%	1.6	1.8	2.0	2.3	2.7	3.0	3.4	3.8	4.2	4.6	5.0
99%	1.7	2.0	2.3	2.7	3.0	3.5	4.1	4.5	5.1	5.7	6.2

D2. EBR Distribution

The following tables show the distribution of the EBR using the dynamic strategy and the floating strategy. The dynamic strategy results have been colour-coded as above.

Dynamic Strategy:

Percentile	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1%	55%	31%	21%	6%	0%	0%	0%	0%	4%	50%
5%	55%	40%	32%	17%	6%	0%	0%	0%	21%	59%
10%	55%	44%	38%	27%	18%	5%	0%	0%	33%	67%
25%	55%	52%	48%	40%	35%	28%	21%	22%	52%	70%
50%	55%	58%	60%	56%	57%	55%	55%	61%	70%	70%
75%	55%	66%	70%	70%	70%	70%	70%	70%	70%	70%
90%	55%	70%	70%	70%	70%	70%	70%	70%	70%	70%
95%	55%	70%	70%	70%	70%	70%	70%	70%	70%	70%
99%	55%	70%	70%	70%	70%	70%	70%	70%	70%	70%

Floating Strategy:

Percentile	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1%	55%	48%	45%	42%	41%	40%	39%	38%	37%	35%
5%	55%	50%	48%	46%	46%	45%	44%	43%	43%	42%
10%	55%	51%	50%	49%	48%	47%	47%	47%	46%	45%
25%	55%	53%	53%	52%	52%	52%	52%	52%	52%	51%
50%	55%	55%	56%	56%	57%	57%	57%	58%	58%	58%
75%	55%	58%	59%	60%	61%	62%	63%	64%	64%	64%
90%	55%	60%	62%	63%	65%	66%	67%	68%	68%	69%
95%	55%	61%	64%	65%	67%	68%	70%	70%	70%	70%
99%	55%	63%	67%	69%	70%	70%	70%	70%	70%	70%

D3. Bonus Distribution

The following tables show the distribution of the regular bonuses using the dynamic strategy and the floating strategy. The dynamic strategy results have been colour-coded as above. Note that in these results, in early years, the bonuses are constrained by the smoothing rules.

Dynamic Strategy:

Percentile	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
5%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
10%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
25%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
50%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.5%
75%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.8%	0.8%	1.2%	1.5%
90%	3.3%	2.8%	2.3%	1.8%	1.7%	1.8%	1.9%	2.0%	2.3%	2.8%
95%	3.3%	2.8%	2.3%	2.0%	2.3%	2.3%	2.4%	2.6%	2.9%	3.3%
99%	3.3%	2.8%	2.7%	2.8%	3.0%	3.2%	3.4%	3.8%	4.1%	4.4%

Floating Strategy:

Percentile	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
5%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
10%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
25%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.0%	0.0%
50%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.3%	0.0%	0.3%	0.6%
75%	3.3%	2.8%	2.3%	1.8%	1.3%	0.8%	0.8%	0.8%	1.2%	1.6%
90%	3.3%	2.8%	2.3%	1.8%	1.6%	1.8%	1.7%	1.8%	2.3%	2.8%
95%	3.3%	2.8%	2.3%	1.9%	2.3%	2.1%	2.3%	2.4%	2.8%	3.2%
99%	3.3%	2.8%	2.5%	2.8%	2.9%	3.0%	3.4%	3.7%	4.0%	4.4%

Appendix E: Theoretical EBRs

This Appendix shows what the theoretical EBRs would be for each year's business considered separately.

Colour code key:

Colour	Theoretical EBR Range
Green	55%-100%
Amber	30%-55%
Red	0%-30%

Table E.1: Theoretical EBRs for UWP Pensions

	Year of Maturity														
Year units purchased	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
1988	45%														
1989	52%	39%													
1990	0%	0%	0%												
1991	74%	60%	52%	46%											
1992	48%	36%	30%	26%	22%										
1993	12%	7%	5%	3%	1%	0%									
1994	0%	0%	0%	0%	0%	0%	0%								
1995	23%	15%	12%	9%	7%	6%	4%	3%							
1996	0%	4%	14%	20%	24%	27%	30%	32%	35%						
1997	0%	0%	7%	14%	20%	24%	27%	30%	32%	34%					
1998	0%	0%	0%	0%	0%	3%	9%	13%	17%	21%	23%				
1999	0%	0%	0%	0%	0%	0%	0%	2%	6%	11%	14%	17%			
2000	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	3%	7%	11%		
2001	0%	0%	0%	0%	0%	0%	0%	0%	4%	8%	12%	15%	18%	21%	
2002	0%	0%	0%	0%	8%	14%	18%	21%	25%	27%	30%	32%	33%	35%	36%

Table E.2: Theoretical EBRs for UWP Bond

	Year of Maturity									
Year of Entry	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1993	64%									
1994		1%								
1995			49%							
1996				27%						
1997					22%					
1998						3%				
1999							0%			
2000								0%		
2001									0%	
2002										18%

Table E.3: Theoretical EBRs for Conventional Endowments

Year of Entry	of Maturity	
1978	2003	**94%**
1979	2004	**83%**
1980	2005	**72%**
1981	2006	**62%**
1982	2007	**55%**
1983	2008	**47%**
1984	2009	**42%**
1985	2010	**39%**
1986	2011	**37%**
1987	2012	**35%**
1988	2013	**35%**
1989	2014	**34%**
1990	2015	**33%**
1991	2016	**33%**
1992	2017	**33%**
1993	2018	**33%**
1994	2019	**34%**
1995	2020	**35%**
1996	2021	**36%**
1997	2022	**37%**
1998	2023	**38%**
1999	2024	**39%**
2000	2025	**41%**

Appendix F: Comparison with Hibbert & Turnbull Approach

Hibbert & Turnbull (2003) propose an approach that involves investing asset shares in risky assets and then purchasing put options to provide downside protection. We can use put call parity to determine the conditions under which this approach is equivalent to the approach used in this paper.

To take a simple example, consider a single premium with-profit T year bond, with guaranteed maturity proceeds of G_T. The initial Policy Value (assumed to be the gross initial premium) is G_0 and expense deductions are Q. Suppose that the T year risk free discount factor is D_T. The with-profit fund invests in a combination of matching T year zero coupon bonds and accumulation units in a single risky asset S with current unit price S_0. For simplicity we will ignore interest rate volatility.

Under our approach, we determine the number of call options on units of S, with a strike of ;

$$X = \frac{S_0 G_T}{G_0}$$

that can be purchased with the initial premium net of expenses Q and net of the cost of zero coupon bonds securing the guarantee . In other words if n is the number of call options purchased we have (using the notation $c(X)$ and $p(X)$ to denote call and put options on S at strike X);

$$n = \frac{G_0 - Q - D_T G_T}{c(X)},$$

We can then work out the effective exposure to the risky asset under our approach using the delta of the call option given by:

$$\Delta_{call} = \Phi(d_1)$$

where $\Phi(.)$ denotes the cumulative normal distribution,

$$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

and σ is the volatility of S per unit time. The effective equity exposure, or equity backing ratio is thus:

$$Ebr = \frac{n\Delta_{call} S_0}{G_0} = \frac{S_0\left(1 - \frac{Q}{G_0} - \frac{D_T G_T}{G_0}\right)}{c(X)} \Phi(d_1)$$

If we choose our contract size for options on S so that $S_0 = G_0$ and $X = G_T$ then this simplifies to:

$$Ebr = \frac{S_0 - Q - D_T X}{c(X)} \Phi(d_1)$$

This is basically the single premium equivalent of the formula given in appendix A3 except that expenses are defined explicitly rather than implicitly, along with allowance for future premiums and shareholder transfers in the definition of asset shares.

We will now show how in this simple example our approach can be reconciled with Hibbert & Turnbull. Under their approach, suppose we hold *m* units of *S* in the with profit fund. We then need to buy (charged *to* the policy *by* the estate) *m* put options to support this allocation. However, since these combined holdings guarantee that we have at least *mX* at maturity, we only need to allocate $D_T(G_T - mX)$ to zero coupon bonds in order to ensure that the combined bond, risky asset and put option positions meet the guaranteed liability. We thus find that, using put-call parity, if we allow for the charge of *mp(X)* for the cost of the put options, *m* must satisfy the budget equation:

$$G_0 - Q = m(S_0 + p(X)) + D_T(G_T - mX) = mc(X) + D_T G_T$$

hence

$$m = \frac{G_0 - Q - D_T G_T}{c(X)} = n$$

In other words, provided that we charge the policy for the cost of the put options we hold, the same number of put options (combined with holdings of the risky asset) under the Hibbert & Turnbull approach as we hold call options (combined with the risk free bond) under our approach. Again invoking put call parity, the equity backing ratios must again be identical *if* we include the equity allocation of the put option backing the policy that is implicitly embedded in the estate.

The main advantages of our approach, in comparison to the Hibbert and Turnbull method, arise from the fact that it can be applied irrespective of the size of inherited estate. For example using the Theoretical EBR approach,

- The estate is free to follow an independent investment strategy. (This is illustrated in Section 4.4, while Section 3.3. described how the approach can be modified to take account of Estate).

- A straightforward comparison of relative strengths and weaknesses in different cohorts of business is possible. (We demonstrated this in the heatmaps presented in Appendix E).

Volatile Interest Rates

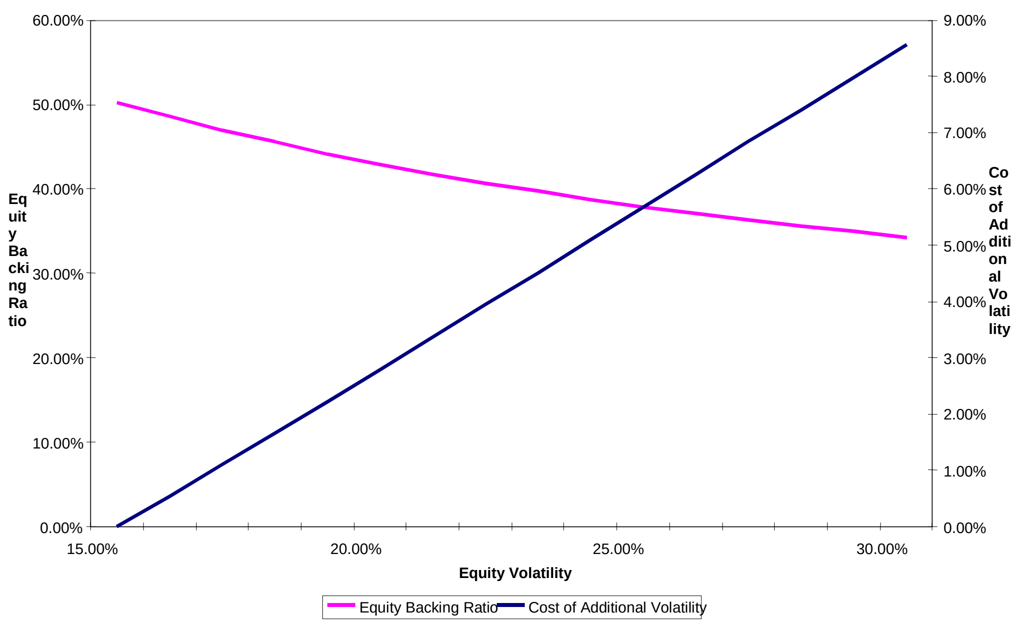
If interest rates are allowed to be volatile then the above derivations become slightly more complex but the basic principles of put call parity still apply. Where guarantees apply only at a single terminal date it is relatively easy to deal with volatile interest rates in our simple example of a *T* term bond, by working in forward risk neutral terms with respect to the *T* term discount bond. However, being of relatively minor importance relative to the other uncertainties in the model (see below) we do not pursue this aspect further in this paper.

Assumed Volatility of Risky Asset

A key assumption in the approach is the volatility of the risky asset. The Black Scholes hedging algorithm, and the associated equity backing ratio, will only work if the actual underlying volatility of the risky asset is in line with the assumption.

The graph below shows how the implied volatility in 5 year FTSE 100 equity options has varied over the past four years (source: Lehman Brothers). It can be seen that the market implied equity volatility in this contract has varied between less than 15%pa and just under 25%pa over the period (the term of option required in calculating theoretical EBR is likely to be longer but the volatilities here are indicative).

The graph below shows how the equity backing ratio would vary according to the assumed volatility of the equity market for our simple 10 year single premium policy (guaranteed return assumed to be 3%pa below the 10 year interest rate). It will be seen that if the hedging policy assumes an equity volatility of 15%pa, but the actual experienced volatility is 30% then the implemented equity backing ratio will be around 15% higher than is actually supported by the premium charged (left hand axis). This can be restated (right hand axis) as a cost to the estate, calculated as the difference between the cost of the market cost of call or put option positions at policy inception (based on actual market implied volatility shown on the horizontal axis) compared with the assumed cost (based on 15% pa volatility in this example).



This potential additional cost (or profit, if volatility is lower than anticipated) would of course be hedged if the insurance company actually bought the options from a bank at policy inception. By effectively running its own dynamic hedge of the option positions the insurance company exposes itself to the same "P&L" risks that a bank would face in writing such options. This profit or loss is ultimately borne by the estate whether the put options are implicitly bought from the estate under the Hibbert & Turnbull approach or whether the asset allocation of the asset share hedge the call option under our approach, based on a mis-specified volatility –

since the payout from the hedge will not match the assumed policy payout obligations in either case.

In addition to uncertainty in market volatility, banks (and thus the insurers hedging options themselves) do of course also run the risks associated with “fat tails” or “jumps” in the actual equity return distribution. Some of these risks could in principle be reduced by the with profit fund hedging its exposure to implied volatility by purchasing exchange traded equity options (so called “vega” hedging), netting off any associated equity market “delta” against the required overall equity backing ratio for the fund.

Other Approaches

It needs to be stressed that although our approach and the Hibbert & Turnbull approach can be reconciled, there remains some arbitrariness at the heart of both. The approaches formalise the with profit investment process by defining the payout as being essentially the same as a guaranteed equity fund – and then working backwards to establish how much of the fund can be invested in equity, given the cost of the guarantee. However, it is not clear that the pay out profile of a guaranteed equity fund necessarily represents the reasonable expectations of with profit policyholders. In particular the approach implies unbounded exposures to equities (rising over 100% of the policy value as equities rise and falling as close as you like to zero if equities fall). We have addressed this in our model by imposing limits on the equity allocation, which make the model more realistic but at the expense of breaking some of the assumptions behind the theoretical analysis above.

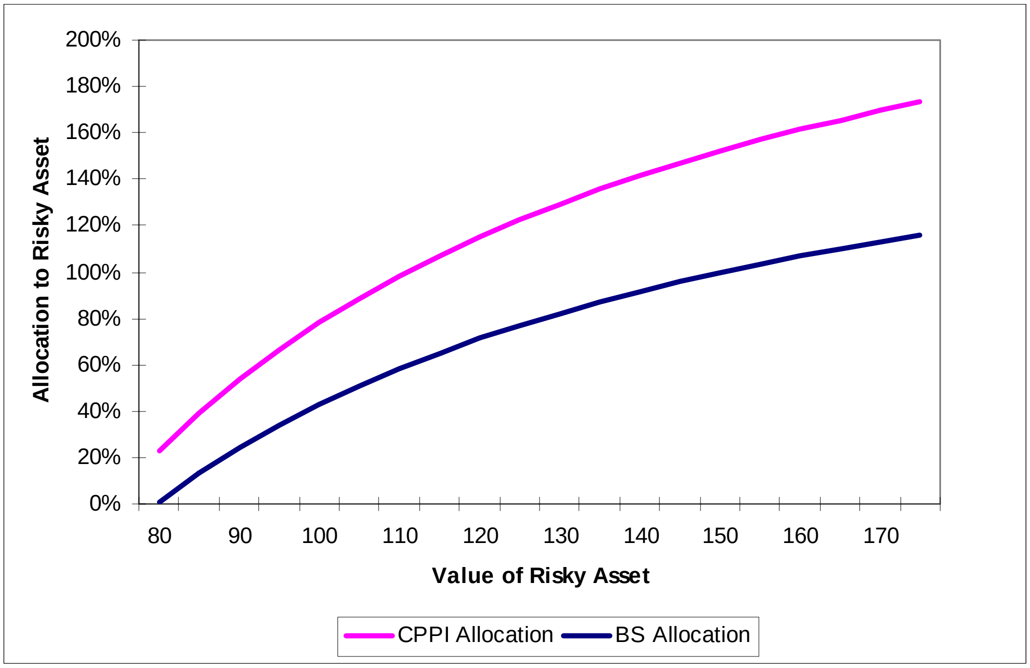
This suggests the possibility of a number of alternative processes. For example instead of letting the characterisation of a with profit contract as a guaranteed equity fund drive the asset allocation algorithm, the contract could instead be characterised in terms of the asset allocation policy itself. An obvious example of this would be constant proportion portfolio insurance. Under this approach the payout function is defined by the asset allocation algorithm (as opposed to an option, where the asset allocation algorithm is defined by the payout function).

Black & Perold (1992) show that if we define a “cushion” C_T as the difference between the asset portfolio and some guaranteed fund (most easily approached by defining the matching zero coupon bond for the guaranteed liability as our numeraire) and if the allocation to the risky asset is simply m times the cushion, then C_T satisfies:

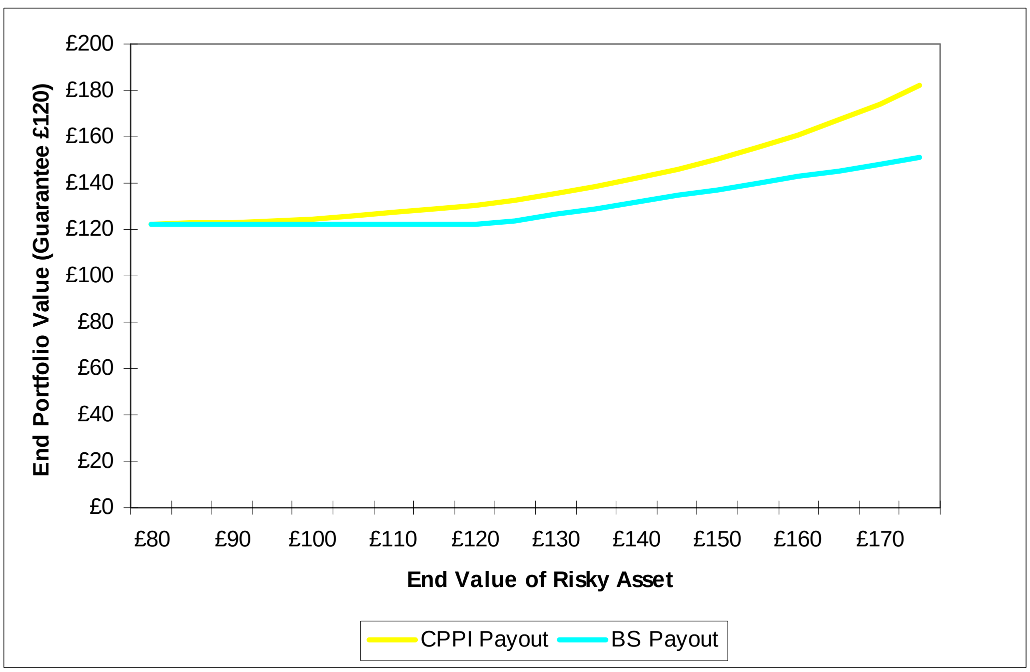
$$C_T = C_0 e^{\frac{1}{2}(m-m^2)\sigma^2 T}\left(\frac{S_T}{S_0}\right)^m$$

Of course the unbounded (indeed, potentially highly geared) nature of the asset allocation under this approach also makes it unsuitable, unadjusted as a representation of policyholders' expectations. Indeed, by imposing a maximum equity allocation we switch into what is essentially the reverse of CPPI, namely a rebalancing policy at higher asset levels. However, in comparing the payouts and asset allocations of the option hedging versus CPPI approaches it is difficult to say categorically that one or other better represents policyholder expectations over the central region.

For example the graph below compares the equity allocation under our hedging approach with a CPPI (with a cushion multiple m = 3) approach for the same 10 year with profit bond characteristics as discussed above.



The end payoffs from our approach versus the CPPI approach then compare as follows:



Thus it would appear to be perfectly plausible to start from the opposite end and define the pay-out in terms of the assumed asset allocation – rather than defining the asset allocation in terms of an assumed pay out. For reasonable choices of parameter within the central range of outcomes the answers may not be dissimilar – and under both approaches we need to impose a subjective limit on the allocation at the extremities (e.g. prohibit gearing).

However, accepting that the approach described here is an approximation, it does nevertheless reveal a number of useful insights. In particular, we argue that it is an improvement on approaches that do not explicitly hedge guarantees. It must also be stressed that the basic link between dynamic asset allocation of a with profit fund and buying or writing portfolio protection that underlies this paper has universal application and would remain relevant if the actual strategy were to be decomposed more accurately into an amalgam of option hedging, CPPI, rebalancing and other approaches.

References

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