CAMRADATA Analytical Services

# Financial Modelling Needs to Change in Light of the Credit Crunch

Pensions Conference, Leeds

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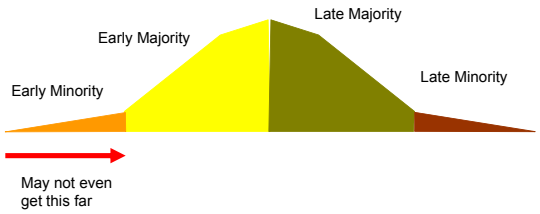
## Set expectations up front

- This is about modelling VAR
- How do ideas propagate?




## New experiences for mathematicians:

- Natural Numbers, How Many People?
- Integers, Differences between natural numbers
- Rational Numbers, fractions
  - Hippasus of Metapontum 5th Cen. BC: $\sqrt{2}$
  - Number & Geometry inseparable
- Real Numbers
  - Zero,
  - Negative numbers Middle Ages in the East

## Imagine a number like 1.5

- Natural Numbers, How Many People?
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- In what context is there a number between 1&2?

## Imagine a number like 1.5

- Natural Numbers, How Many People? (No)
- Integers, Differences between natural numbers (No)
- Rational Numbers, fractions (Yes)
  - Hippasus of Metapontum 5th Cen. BC:
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- In what context is there a number between 1&2?

## Imagine a number like √-1

- Natural Numbers, How Many Objects? (No)
- Integers, Differences between natural numbers (No)
- Rational Numbers, fractions (Yes)
  - Hippasus of Metapontum 5th Cen. BC:
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- Real Numbers (Yes)
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- In what context is there a number between 1&2?
- In what context is there a number √-1?

## Imagine a number like √-1

- Natural Numbers, How Many Objects? (No) No
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  - Hippasus of Metapontum 5th Cen. BC:
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- Real Numbers (Yes) No
  - Zero, Negative numbers Middle Ages in the East
- In what context is there a number between 1&2?
- In what context is there a number √-1? None

## Imagine a number like √-1

- Hamilton ditched the imaginary notion by expanding definitions of numbers
- His Complex number system, is an ordered number pairs (a,b) approach.
- He redefined arithmetic operators to accommodate a(b+c) = a*b +a*c.
- Risk and economic systems may well need to be redefined as more general paired number systems too.
- Borrowing the free thinking ideas of Hamilton

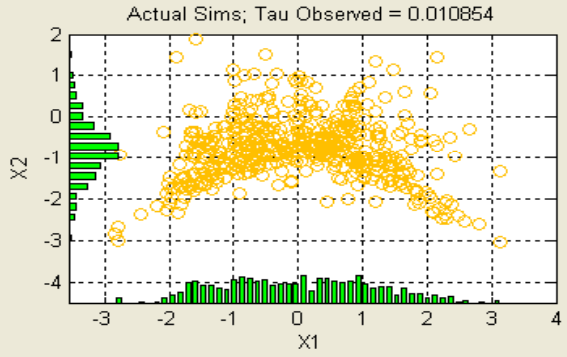
## Example of systems

| Number system | Portfolio Variance system |
|---|---|
| -inf +inf | >=0 |
| Rules: Associative, Distributive a(b+c) = ab+ac, etc etc.... | Rules: Commutative, VAR(A+B),VAR(A*B), VAR(a(b+c)) ≠ VAR(ab+ac).... ? |
| Hidden dimension: Though complex number system | Correlation captures linkages between dimensions<br>We can have hidden dimensions |

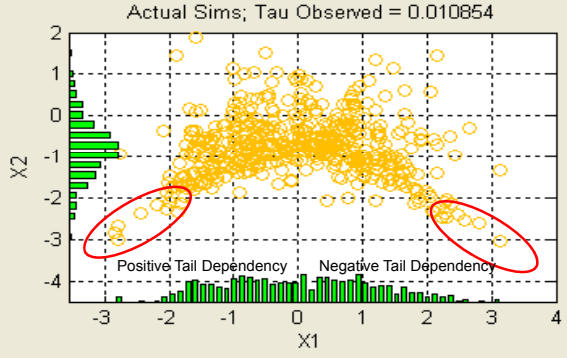
## The point

- Anything is possible
- Imagine/expand a system
- Use it to show something
- There are things we want to show that we can't currently do.

## The point

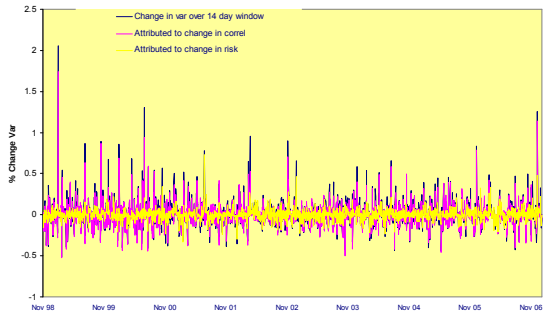



## The point



## The Point

**Variance attribution**
**Equally weighted index of 9 broad sectors**




## Weird Covariance

- The rho thingy ?

$$\sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y\rho_{(x,y)}$$

- The square root of the covariance thingy?

$$\mu + \sqrt{H} \times z$$

Where $\mu$ = mean

$\sqrt{H}$ = Square root of covariance

$z$ = Uncorrelated random numbers

## Weird Covariance

- The rho thingy ?

$$\sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y\rho_{(x,y)}$$

- The square root of the covariance thingy?

Infinite ways to do this

$$\mu + \sqrt{H} \times z$$

Where $\mu$ = mean

$\sqrt{H}$ = Square root of covariance

$z$ = Uncorrelated random numbers

## Imaginary Correlation

$$\sigma_x^2\rho_{(x)}^2+\sigma_y^2\rho_{(y)}^2+2\sigma_x\sigma_y\rho_{(x)}\rho_{(y)}$$

- Looks right, could be a paired number, or something tied to a hidden dimension
- Operators on imaginary correlation need to be defined to tie up with a 'Variance System'

## Why?

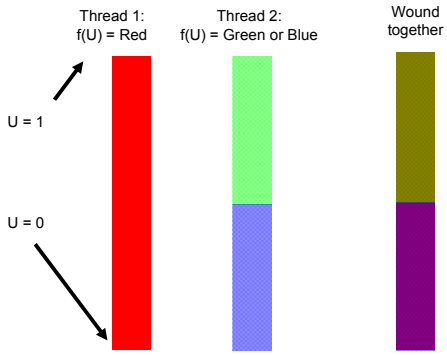
- I want to define 'a priori' different dependency structures across n assets.
- A copula would then fall out.
  - Can't currently do this….
- My guess at a process to do this uses a hidden dimension.
- The D distribution. D for dependency driven

## D distribution

$$X=\mu_{f(u)}+\sigma_{f(u)}Z$$

- There is a hidden dimension controlled by U(0,1)
- Each asset has a relationship to the hidden dimension f(U).
- All other dependencies are ignored.
- **Things happen in the world which effect Returns and Risks**
- **Manifests itself as correlation**

## Think about it as wound thread

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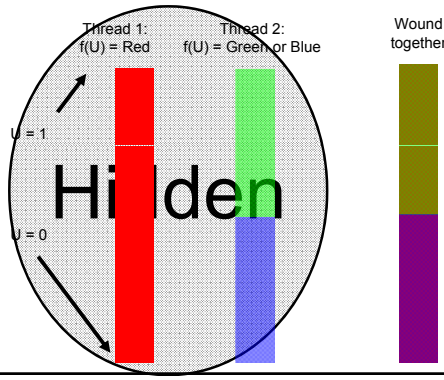
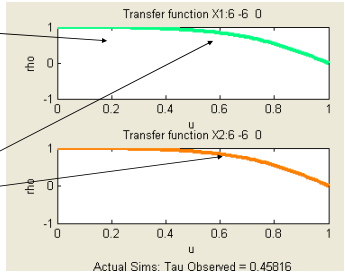
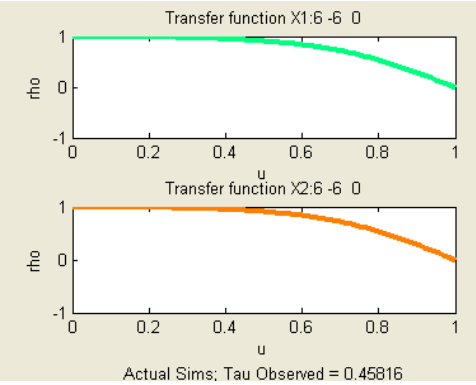



## Model of assets using this principle

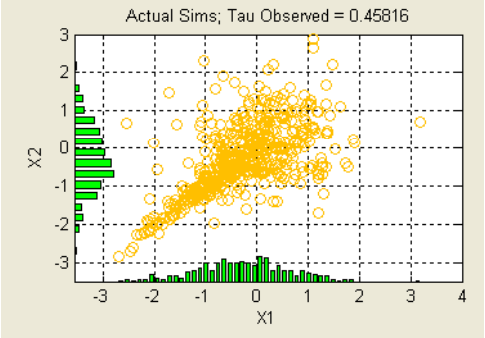
- Shows correlation to hidden dimension over U(0,1)
- Individual correlations, given U, interact as "imaginary correlations"



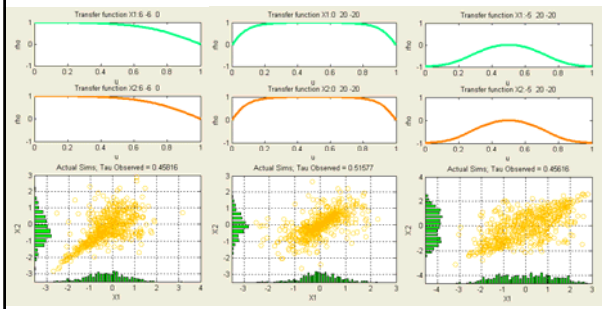
## Model of assets using this principle



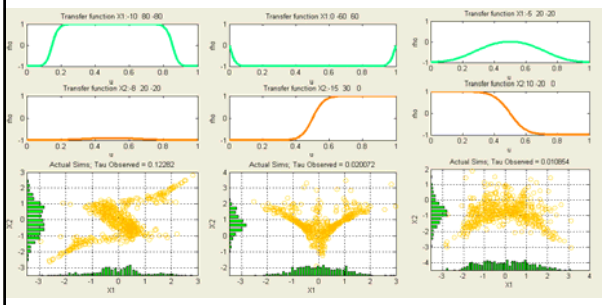

## An oops distribution




## Central and tail dependency



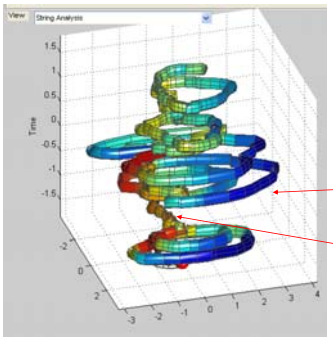
## Zoro, Stag, Banana



## The passage of Time

- Are investment returns related to calendar years or calendar months?
- They are closely related to the pendulum of the economic clock & economic policy
- The clock may go backwards or skip
- How do you measure the clock?

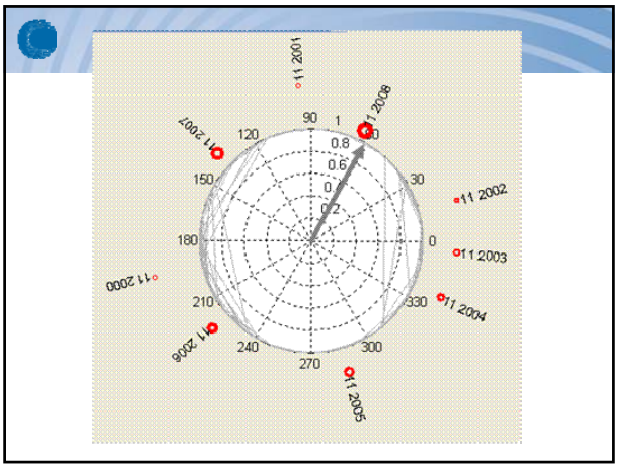
## Could U be the economic cycle?



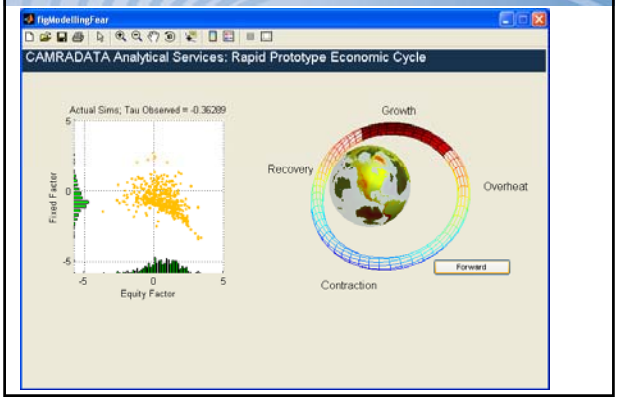
Colour:
US GDP, 1951 to present

Cooling

Warming




## Condition economic model on U

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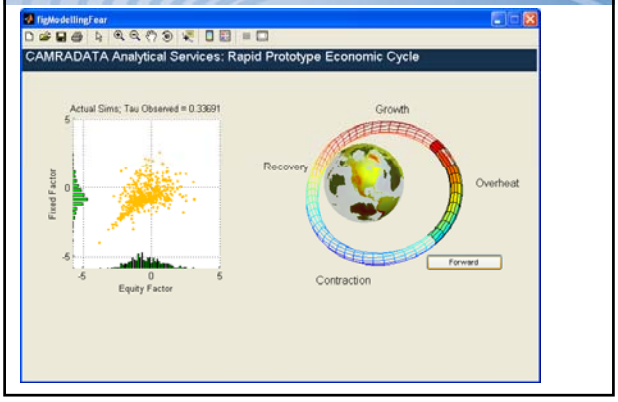

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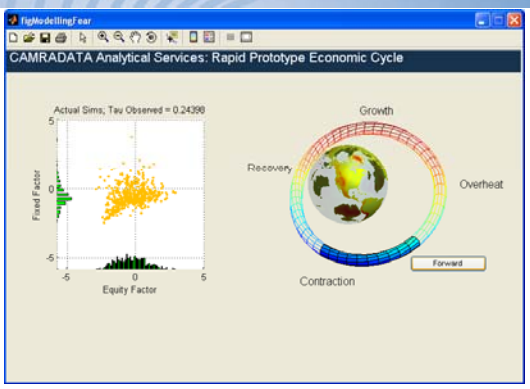


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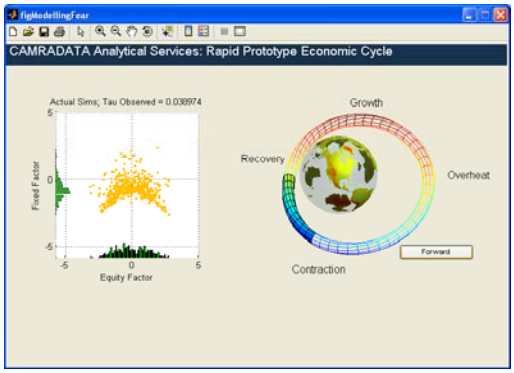


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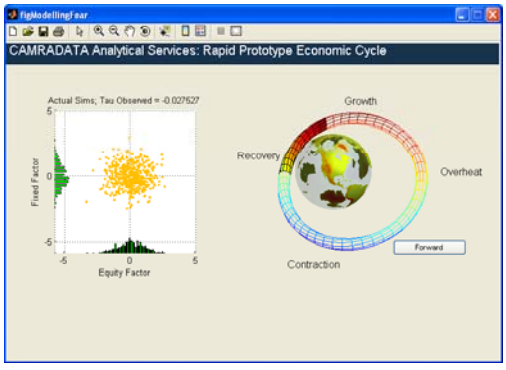

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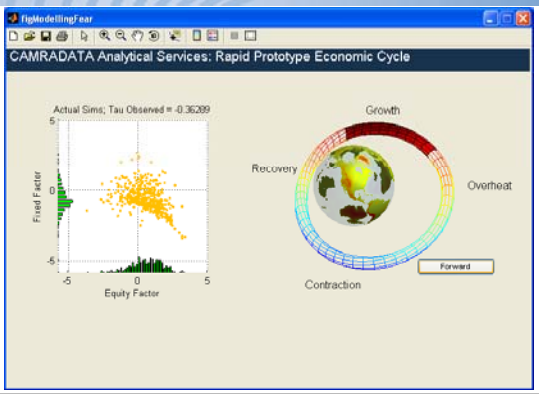


## Contact

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