Institute and Faculty of Actuaries

Multi population mortality models

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Motivation

- The mortality at older ages is still under considerable research
- Limited research on the multi population models
- Vast space for improvement

Aims

- We aim to derive better multi population mortality models
  - Consistent and robust fitting method
  - Identifiability problems correction
  - Forecast joint mortality rates based on the models

The models

- $ln[m_{(x,t,i)}] = \alpha_{(x,i)} + B_{(x)}K_{(t)} + \beta_{(x,i)}\kappa_{(t,i)} + \varepsilon_{(x,t,i)}$
- $ln[m_{(x,t,i)}]$
- $ln[m_{(x,t,i)}]$ $\alpha_{(x,i)}$
- $ln[m_{(x,t,i)}] = \alpha_{(x,i)} + \beta^1_{(x)}\kappa^1_{(t,i)} + \beta^2_{(x)}\kappa^2_{(t,i)} + \varepsilon_{(x,t,i)}$

$$m_{(x,t,i)} = \frac{Death}{Exposure}$$

$\beta_{(x,i)}$

$(\beta_{(x,i)} \equiv B_{(x)})$

age

$\kappa^1_{(t,i)}$ and $\kappa^2_{(t,i)}$
time/country dependent
parameters

The Li and Lee model

The Li and Lee variation I

The Li and Lee variation II

The Kleinow model

Li and Lee – fit of the model

$$\hookrightarrow ln[m_{(x,t,i)}] = \alpha_{(x,i)} + B_{(x)}K_{(t)} + \beta_{(x,i)}\kappa_{(t,i)} + \varepsilon_{(x,t,i)}$$

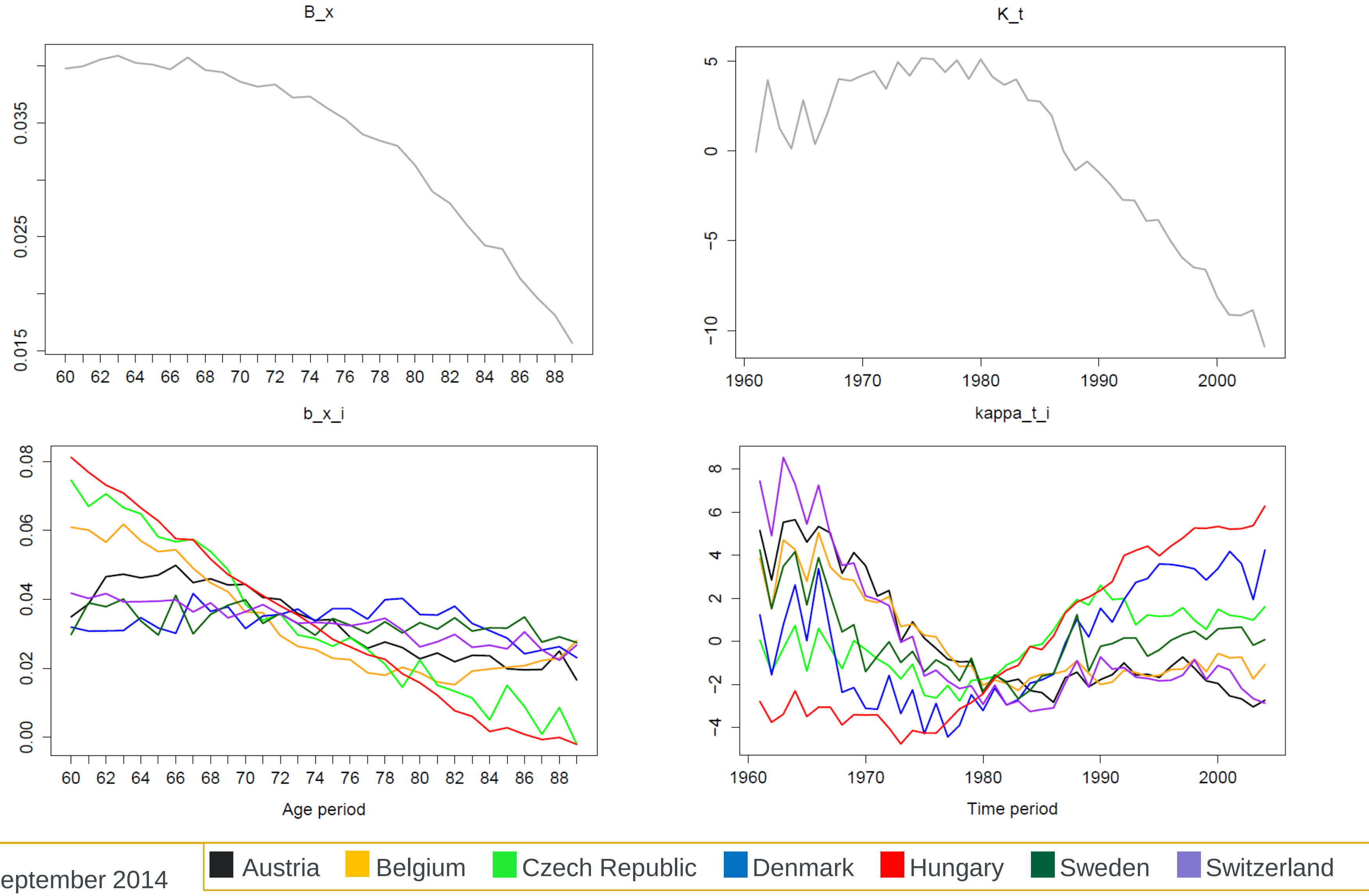
- Maximum likelihood estimation
  - One step - single likelihood function
  - Newton-Raphson iterative process
- Identifiability problems
  - Standard constraints
- Model specifics
  - Very heavy model
  - A lot parameters to estimate – total of 802
  - Flat or almost flat log-likelihoot function at some dimmentions

Equivalent fitted mortality rates with different set of estimated parameters

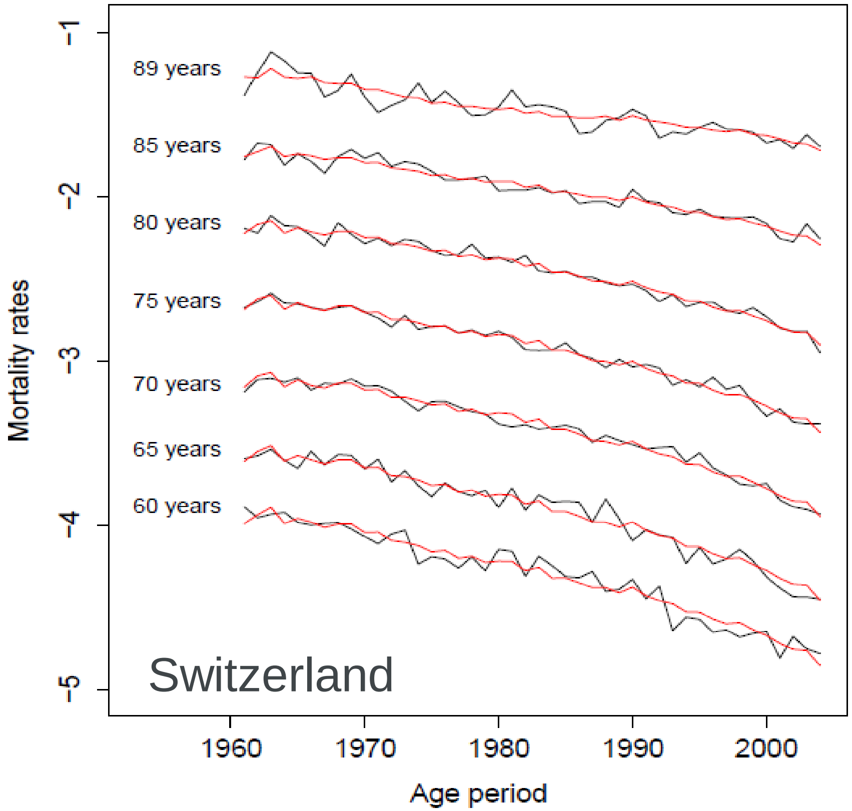
$$\sum_x B_{(x)} = 1 \qquad \sum_t K_{(t)} = 0$$

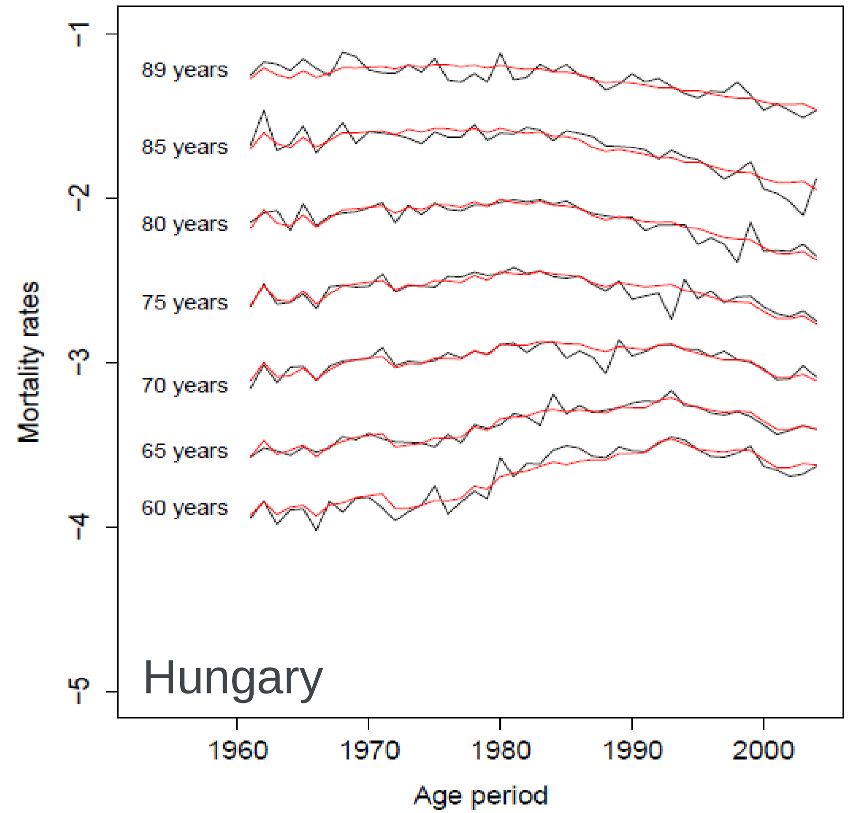
$$\underbrace{\sum_x \beta_{(x,i)} = 1 \qquad \sum_t \kappa_{(t,i)} = 0}_{for\ every\ i=1,2,\dots,7}$$

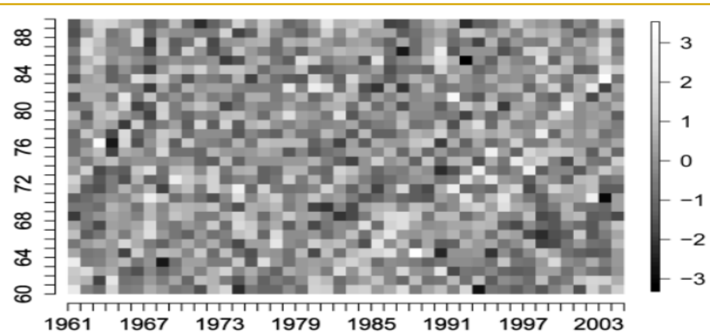
Li and Lee – estimated parameters

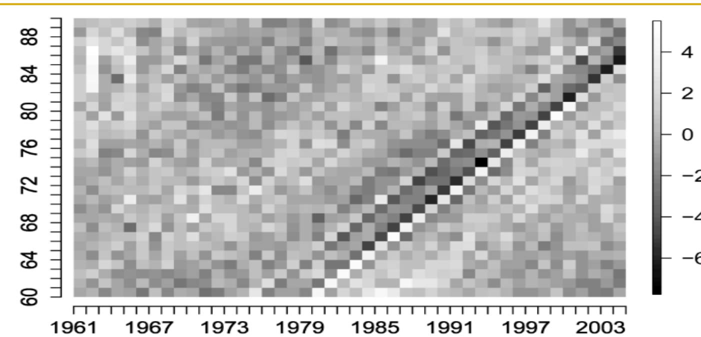


Li and Lee – fitted mortality rates









Real mortality rates   Fitted mortality rates

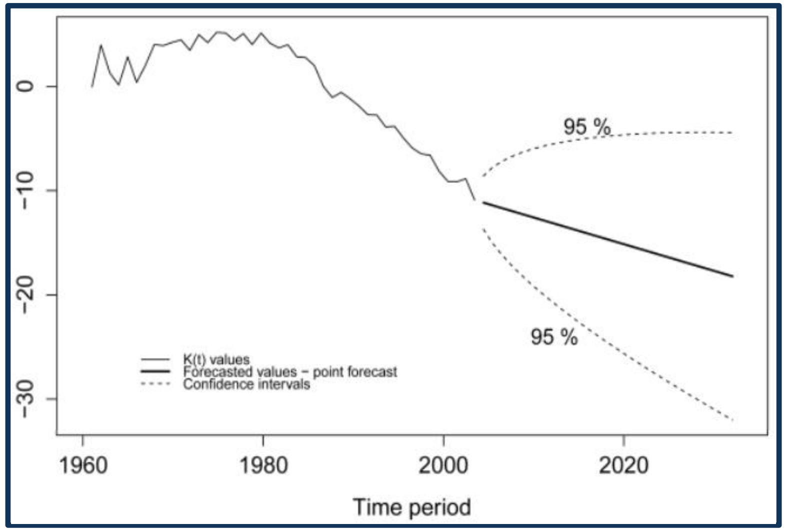
Li and Lee – forecasting the parameters

$\hookrightarrow \ln[m_{(x,t,i)}] = \alpha_{(x,i)} + B_{(x)}K_{(t)} + \beta_{(x,i)}\kappa_{(t,i)} + \varepsilon_{(x,t,i)}$

- Two time dependent parameters to forecast
  - A global time dependent $K_{(t)}$ paramerer

$$K_{(t)} = K_{(t-1)} + d + \varepsilon$$

Random walk with drift

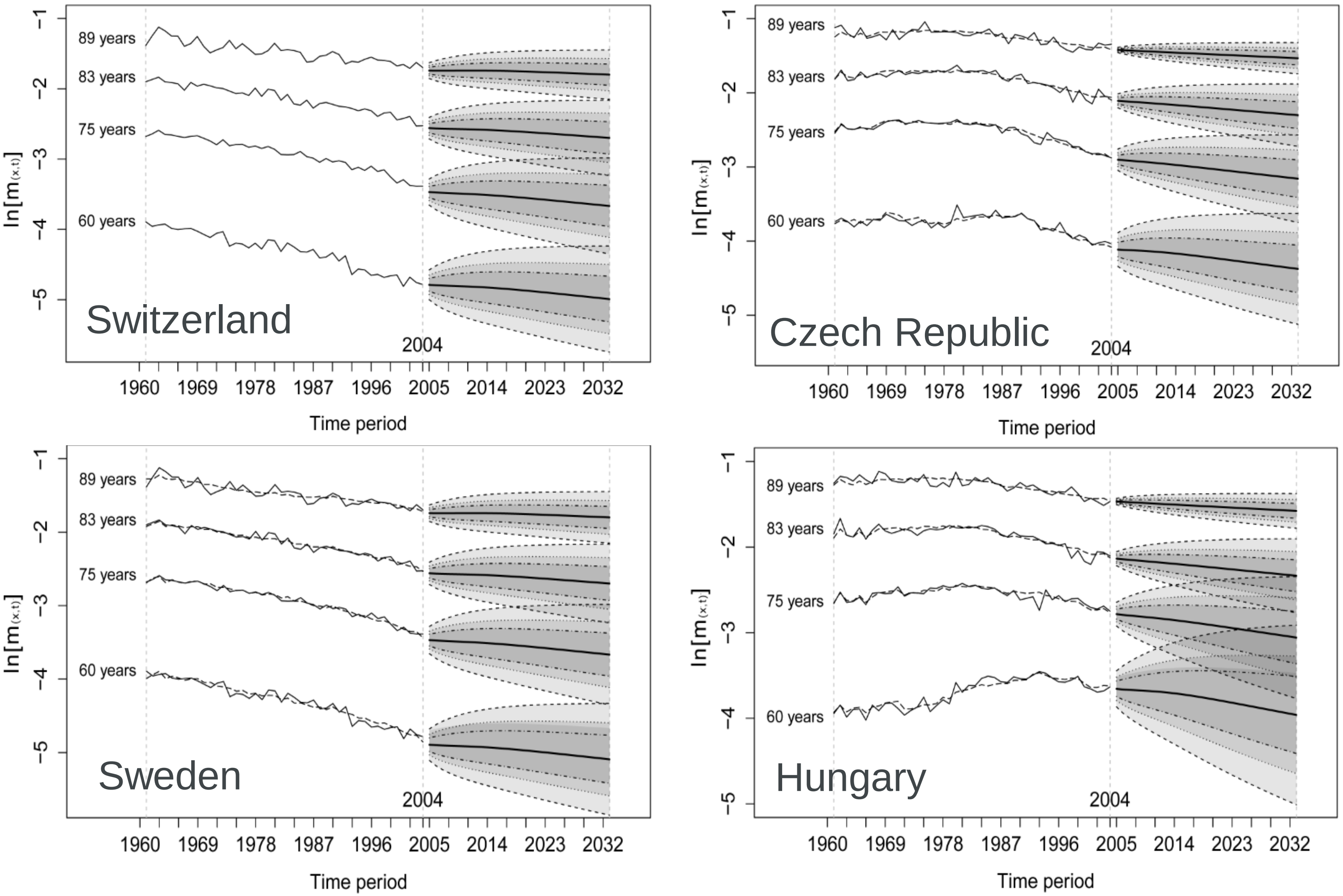


  - A time/country dependent $\kappa_{(t,i)}$ paramerer

Vector Autoregressive model

$$\kappa_{(t)} = \Phi\kappa_{(t-1)} + \Sigma Z_{(t)}$$

Li and Lee – mortality rates scenarios



Comparison of the fitted models

Property	Li and Lee	Variation I	Variation II	Kleinow
Number of parameters	*802*	*622*	*592*	*886*
Number of iterations	*Above 200*	*Below 20*	*Below 10*	*Below 30*
Identifiability problems	*Standard*	*Requires new constrain*	*Requires new constrain*	*Requires new constrain*
True identifiability constrains	*Yes*	*No*	*Yes*	*No*

Order of the models

Kleinow ⇐ Li and Lee ⇐ Variation I ⇐ Variation II

Smallest BIC *Biggest BIC*

BIC: -2*log-likelihood value + penalty term

Robustness test

Kleinow ⇐ Li and Lee ⇐ Variation I ⇐ Variation II

Forecasting through the models

- Suitable forecasting process for every model

Model	$K_{(t)}$	$\kappa_{(t,i)}$	$\kappa^{1}_{(t,i)}$	$\kappa^{2}_{(t,i)}$
Li and Lee	Random walk with drift	VAR(1)	✗	✗
Variation I	Random walk with drift	Multivariate random walk with drift	✗	✗
Variation II	Random walk with drift	Multivariate random walk with drift	✗	✗
Kleinow	✗	✗	Random walk with common drift	mvVAR(1)

- Narrower confidence interval in the forecasted mortality rates scenarios

Conclusion

- Comparison of multi population models
- Reduced parameter number
- Models that are estimated faster
- Generation of joint scenarios for future mortality rates