The Actuarial Profession
making financial sense of the future

Solvency II: Risk Margins and Technical Provisions

Peter England

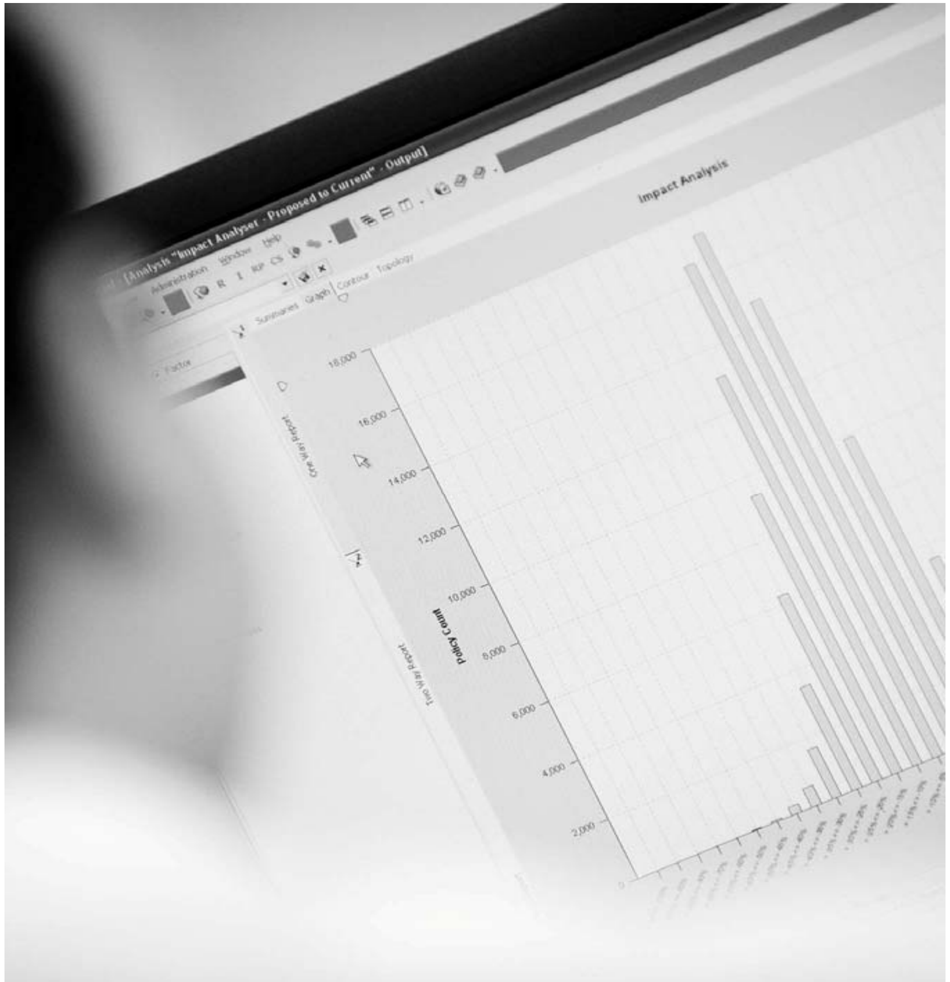
GIRO conference and exhibition

12-15 October 2010

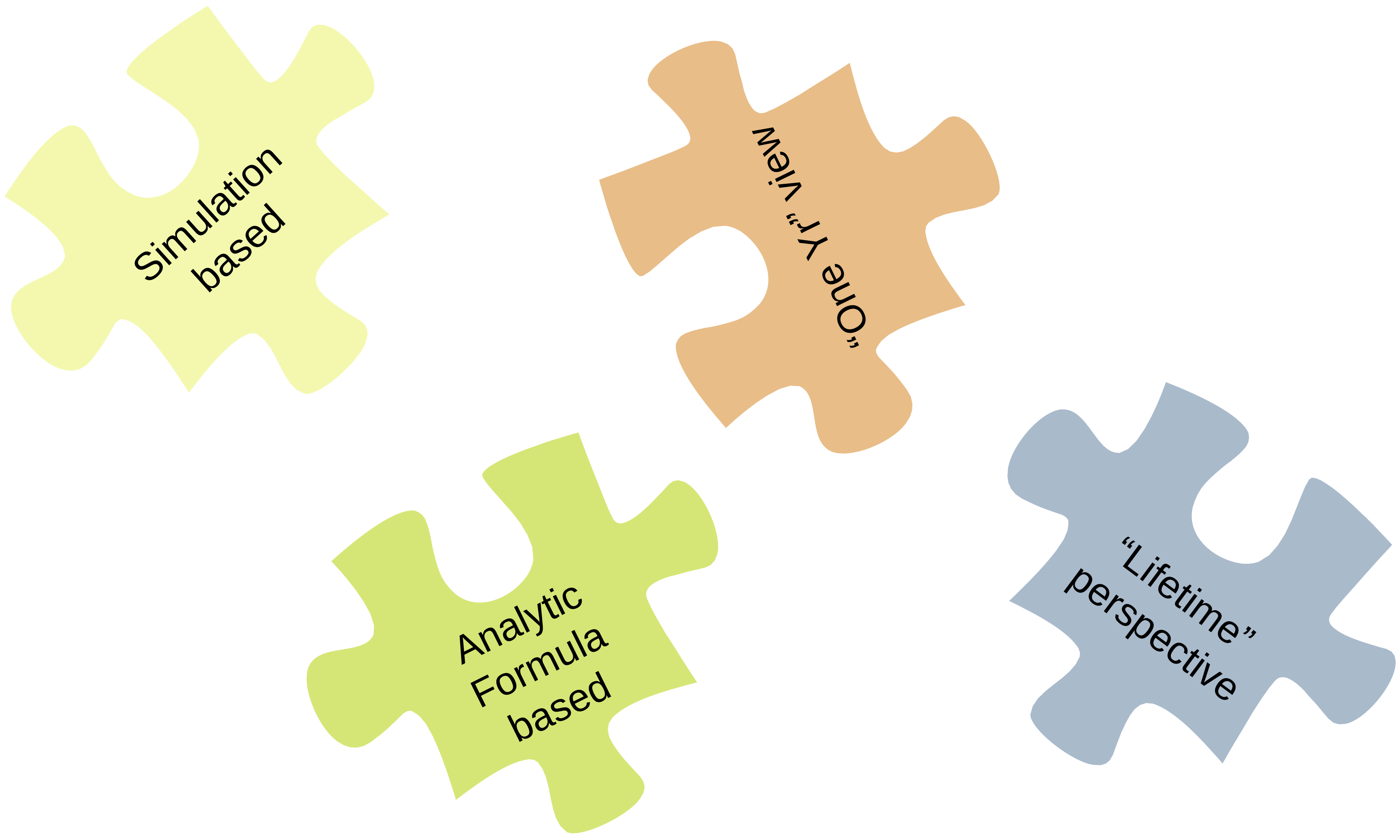
Agenda

- The traditional actuarial view of reserving risk vs Solvency II and the one-year view
  - We will look at the two perspectives, and try to reconcile them in some way

- Risk margins in internal models
  - Practical considerations



The Reserve Risk Puzzle



Reserve Risk: The traditional actuarial view
Looking over the lifetime of the liabilities

- The traditional actuarial view of reserve risk looks at the uncertainty in the outstanding liabilities over their lifetime
  - We have to start talking statistics
  - Given a statistical model, we can derive analytic formulae for the standard deviation of the forecasts
  - Given a statistical model, we can also generate distributions of outstanding liabilities, and their associated cash-flows, using simulation techniques (eg bootstrap or MCMC techniques)
  - We can do this in a way that reconciles the analytic and simulation approaches

Simulation vs analytic approaches to reserve risk

"We can do this the easy way, or we can do it the hard way"

DIRECTIVE OF THE EUROPEAN PARLIAMENT
Article 101

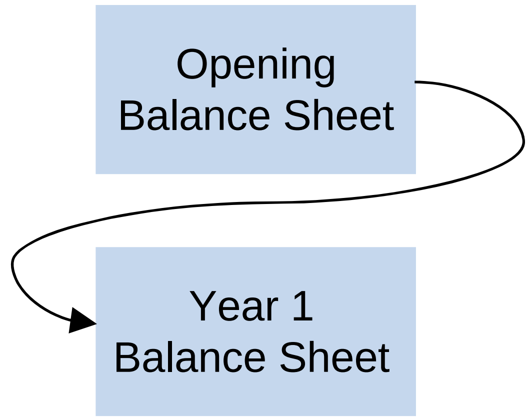
“The Solvency Capital Requirement shall be calibrated so as to ensure that all quantifiable risks to which an insurance or reinsurance undertaking is exposed are taken into account. With respect to existing business, it shall cover unexpected losses.

It shall *correspond* to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.”

So it seems straightforward to estimate the SCR using a simulation-based model: simply create a simulated distribution of the basic own funds over 1 year, then calculate the VaR @ 99.5%.

A Projected Balance Sheet View

- When projecting Balance Sheets for solvency, we have an opening balance sheet with expected outstanding liabilities
- We then project one year forwards, simulating the payments that emerge in the year
- We then require a closing balance sheet, with (simulated) expected outstanding liabilities conditional on the payments in the year



Solvency II

- For Solvency II, a 1 year perspective is taken, requiring a distribution of the *expected* value of the liabilities after 1 year, for the 1 year ahead balance sheet in internal capital models
- If the standard formula is used, a 1 year-ahead "reserve risk" standard deviation % is required. This could be:
  - The standard parameter for the line-of-business
  - An undertaking specific parameter
- The 1 year-ahead "reserve risk" standard deviation is the SD of the distribution of profit/loss on reserves after 1 year
  - Note: this is a different definition of risk from the traditional actuarial view

The one-year run-off result (undiscounted)

(The view of profit or loss on reserves after one year)

- For a particular origin year, let:
- The opening reserve estimate be R_0
- The reserve estimate after one year be R_1
- The payments in the year be C_1
- The run-off result (claims development result) be CDR_1
- Then

$$CDR_1 = R_0 - C_1 - R_1 = U_0 - U_1$$

- Where the opening estimate of ultimate claims and the estimate of the ultimate after one year are U_0, U_1

The One-year Run-off Result

(the view of profit or loss on reserves after one year)

- Merz & Wuthrich (2008) derived analytic formulae for the standard deviation of the claims development result after one year assuming:
  - The opening reserves were set using the pure chain ladder model (no tail)
  - Claims develop in the year according to the assumptions underlying Mack's model
  - Reserves are set after one year using the pure chain ladder model (no tail)
  - The mathematics is quite challenging. This is the HARD way

- The M&W method is gaining popularity, but has limitations. What if:
  - We need a tail factor to extrapolate into the future?
  - Mack's model is not used – other assumptions are used instead?
  - We want another risk measure, not just a standard deviation (*eg* VaR @ 99.5%)?
  - We want a distribution of the CDR?

Merz & Wuthrich (2008)

Data Triangle

Accident Year	12m	24m	36m	48m	60m	72m	84m	96m	108m
0	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633
1	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	
2	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825		
3	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422			
4	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812				
5	2,290,664	3,338,197	3,550,332	3,641,036					
6	2,148,216	3,219,775	3,428,335						
7	2,143,728	3,158,581							
8	2,144,738								

Merz & Wuthrich (2008)

Prediction errors

	Analytic	
	Prediction Errors	
Accident Year	1 Year Ahead CDR	Mack Ultimate
0	0	0
1	567	567
2	1,488	1,566
3	3,923	4,157
4	9,723	10,536
5	28,443	30,319
6	20,954	35,967
7	28,119	45,090
8	53,320	69,552
Total	81,080	108,401

Expressed as a percentage of the opening reserves, this forms a basis of the reserve risk parameter under Solvency II (QIS 5 Technical Specification)

The one-year run-off result in a simulation model

The EASY way

- For a particular origin year, let:
- The opening reserve estimate be R_0
- The expected reserve estimate after one year be $R_1^{(i)}$
- The payments in the year be $C_1^{(i)}$
- The run-off result (claims development result) be $CDR_1^{(i)}$
- Then

$$CDR_1^{(i)} = R_0 - C_1^{(i)} - R_1^{(i)} = U_0 - U_1^{(i)}$$

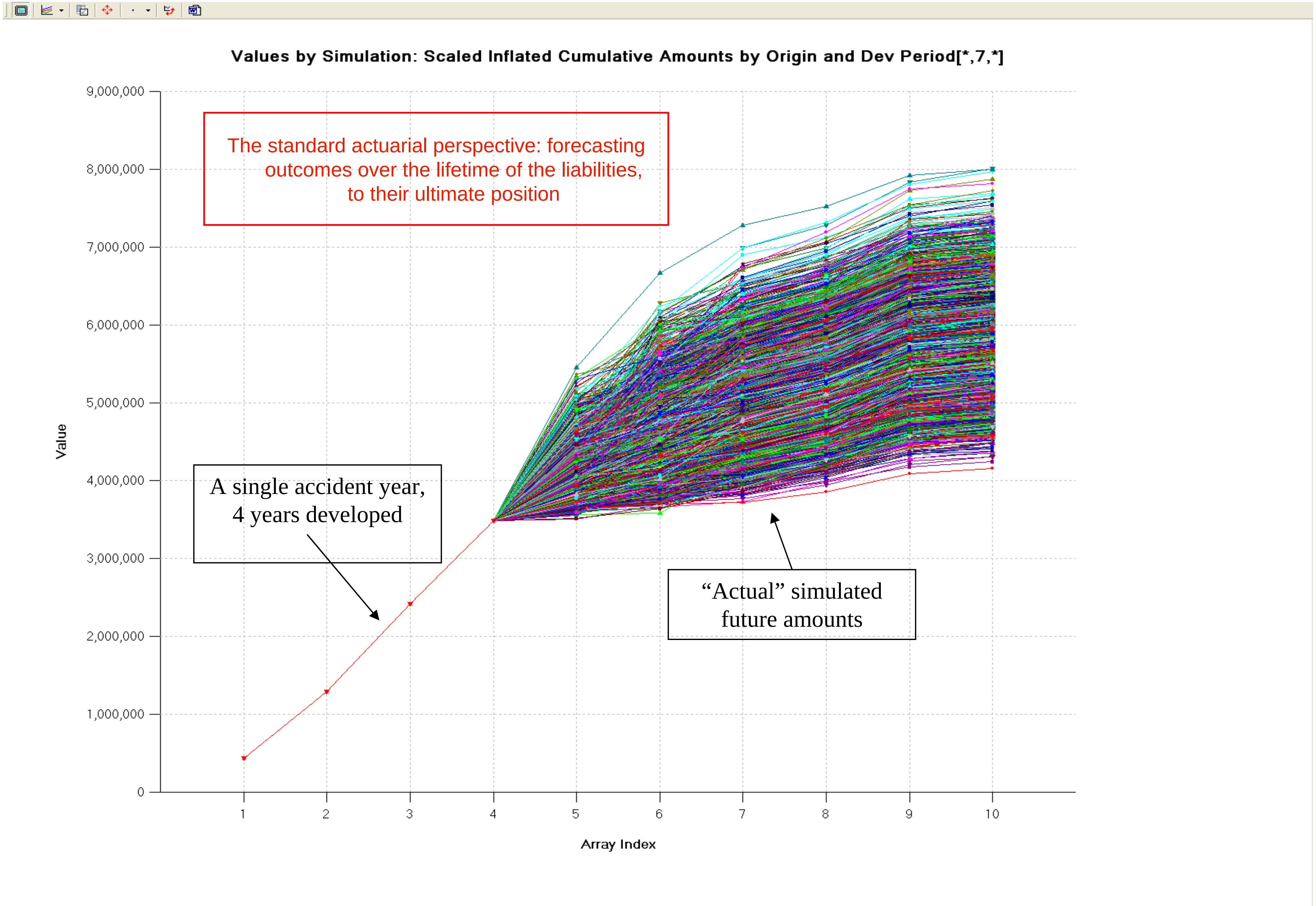
- Where the opening estimate of ultimate claims and the expected ultimate after one year are $U_0, U_1^{(i)}$
- for each simulation i

The one-year run-off result in a simulation model

The EASY way

1. Given the opening reserve triangle, simulate all future claim payments to ultimate using bootstrap (or Bayesian MCMC) techniques.
2. Now forget that we have already simulated what the future holds.
3. Move one year ahead. Augment the opening reserve triangle by one diagonal, that is, by the simulated payments from step 1 in the next calendar year only. An actuary only sees what emerges in the year.
4. For each simulation, estimate the outstanding liabilities, conditional only on what has emerged to date. (The future is still "unknown").
5. A reserving methodology is required for each simulation – an "actuary-in-the-box" is required*. We call this re-reserving.
6. For a one-year model, this will underestimate the true volatility at the end of that year (even if the mean across all simulations is correct).

** The term "actuary-in-the-box" was coined by Esbjörn Ohlsson*


Values by Simulation: Scaled Inflated Cumulative Amounts by Origin and Dev Period[*,7,*]
9,000,000
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
0
Value
1
2
3
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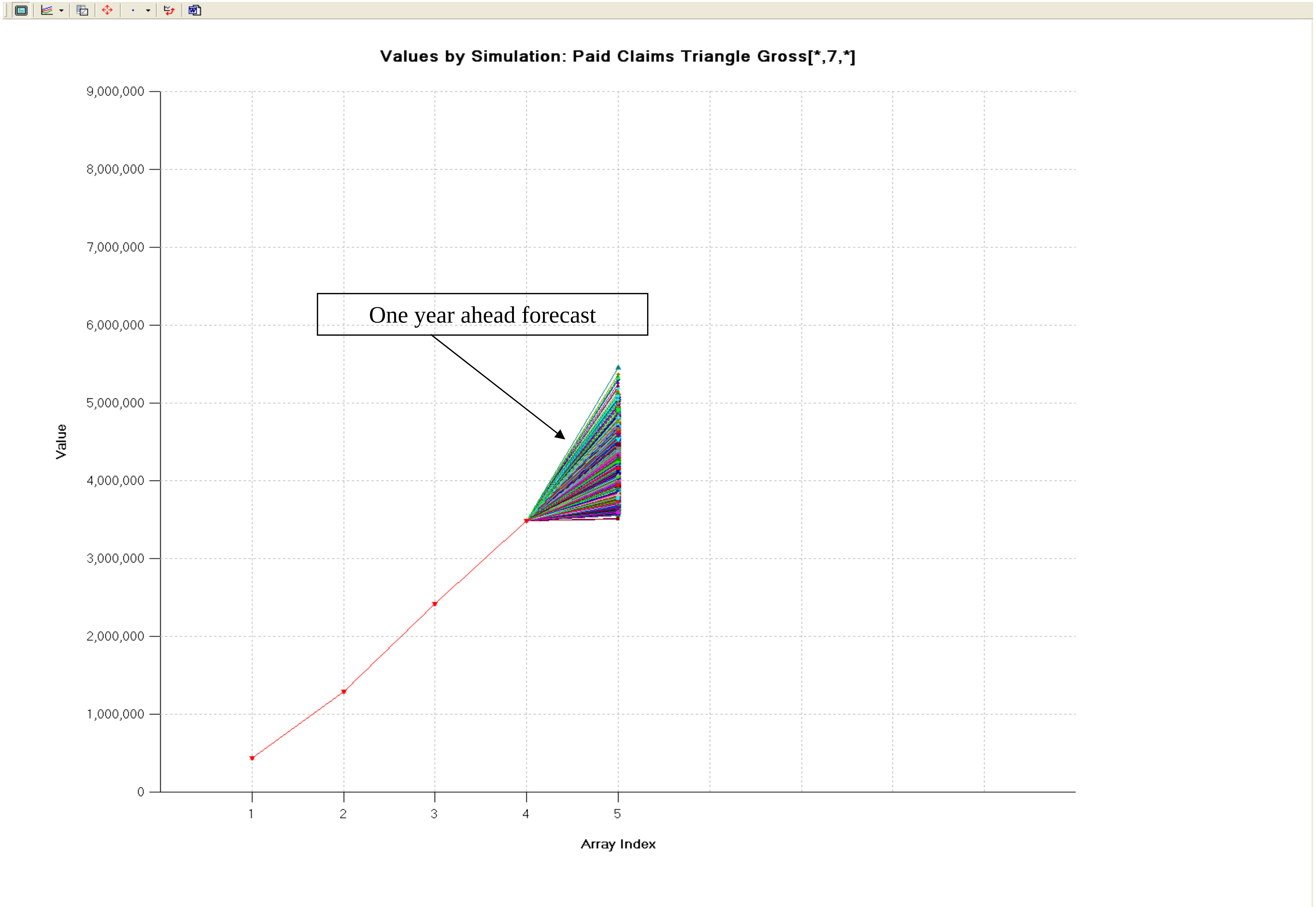
10
Array Index
The standard actuarial perspective: forecasting outcomes over the lifetime of the liabilities, to their ultimate position
A single accident year, 4 years developed
“Actual” simulated future amounts



Values by Simulation: Paid Claims Triangle Gross[*,7,*]
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8,000,000
7,000,000
6,000,000
5,000,000
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3,000,000
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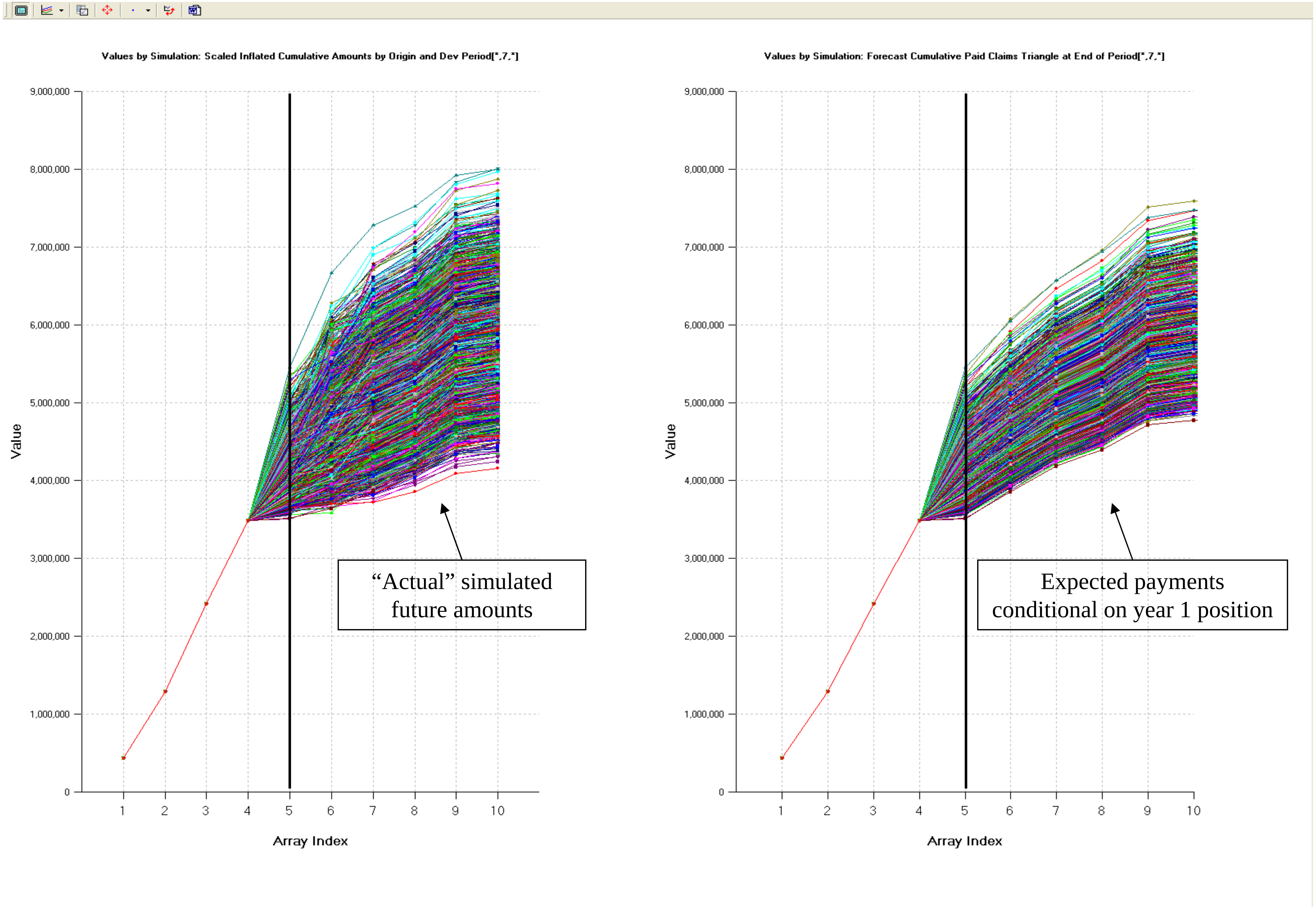
0
Value
One year ahead forecast
1
2
3
4
5
Array Index



Values by Simulation: Scaled Inflated Cumulative Amounts by Origin and Dev Period[*,7,*]
9,000,000
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4,000,000
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Value
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Array Index
"Actual" simulated future amounts
Values by Simulation: Forecast Cumulative Paid Claims Triangle at End of Period[*,7,*]
9,000,000
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
3,000,000
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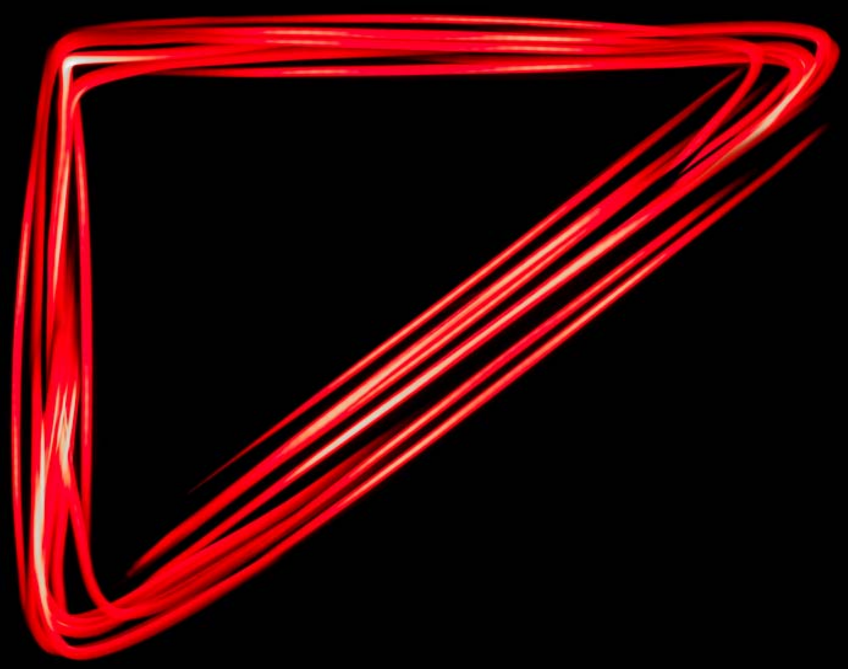
Array Index
Expected payments conditional on year 1 position

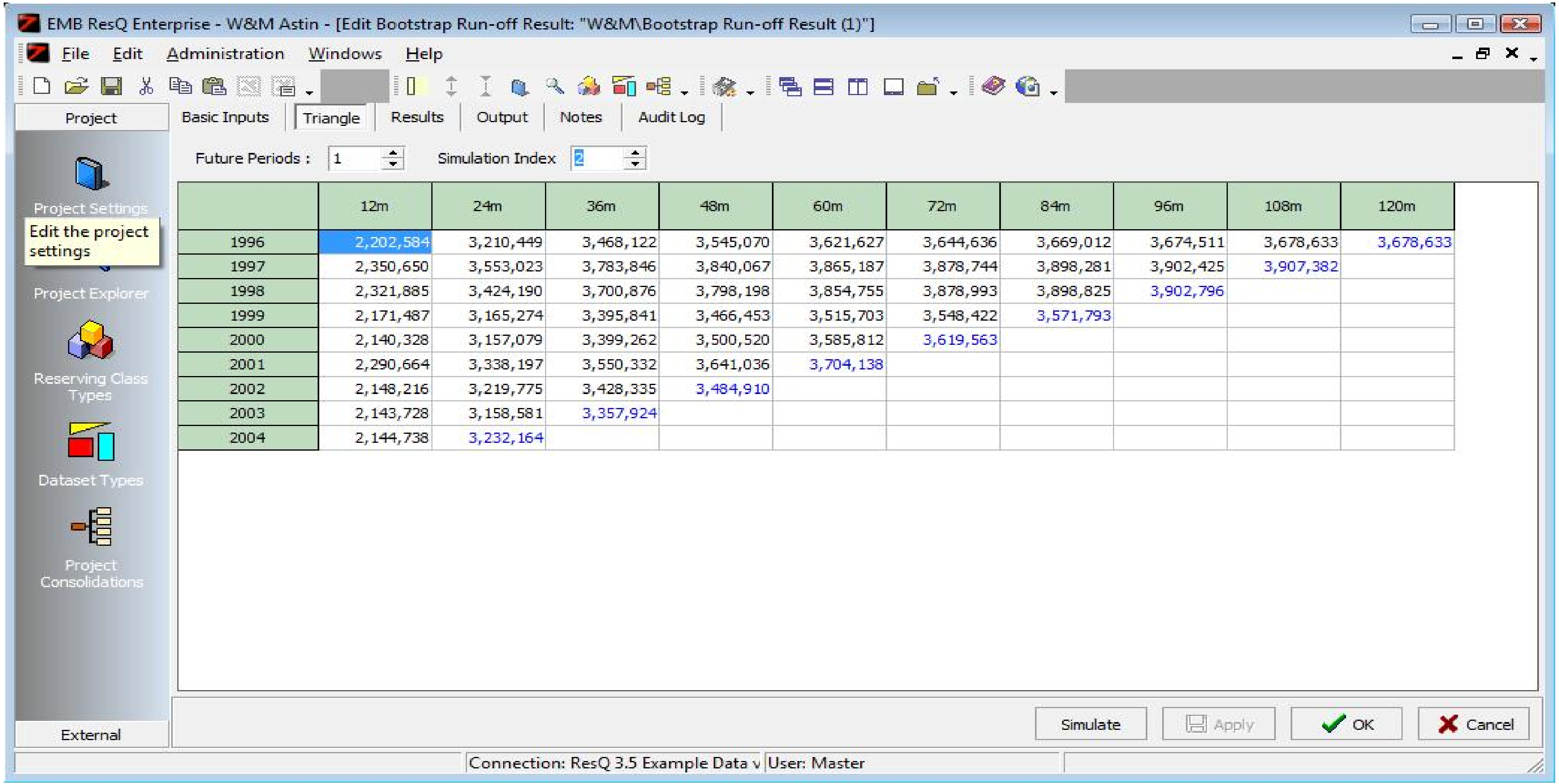
Example



1 Year ahead – Simulation 1

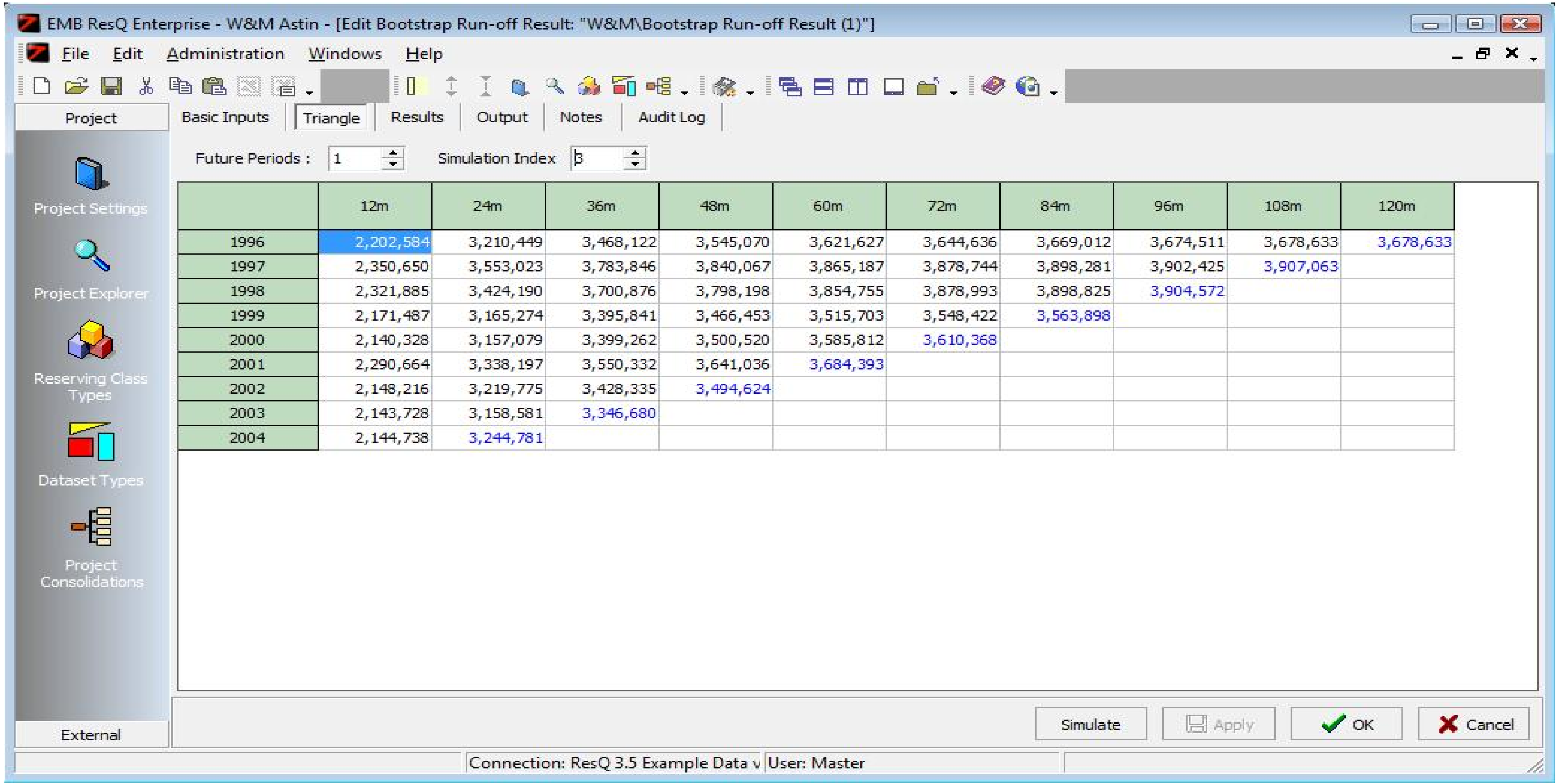
	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,232	
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,901,701		
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,577,173			
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,599,948				
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,737,909					
2002	2,148,216	3,219,775	3,428,335	3,496,277						
2003	2,143,728	3,158,581	3,394,672							
2004	2,144,738	3,221,989								

1 Year ahead – Simulation 2



	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,382	
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,902,796		
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,571,793			
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,619,563				
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,704,138					
2002	2,148,216	3,219,775	3,428,335	3,484,910						
2003	2,143,728	3,158,581	3,357,924							
2004	2,144,738	3,232,164								

1 Year ahead – Simulation 3



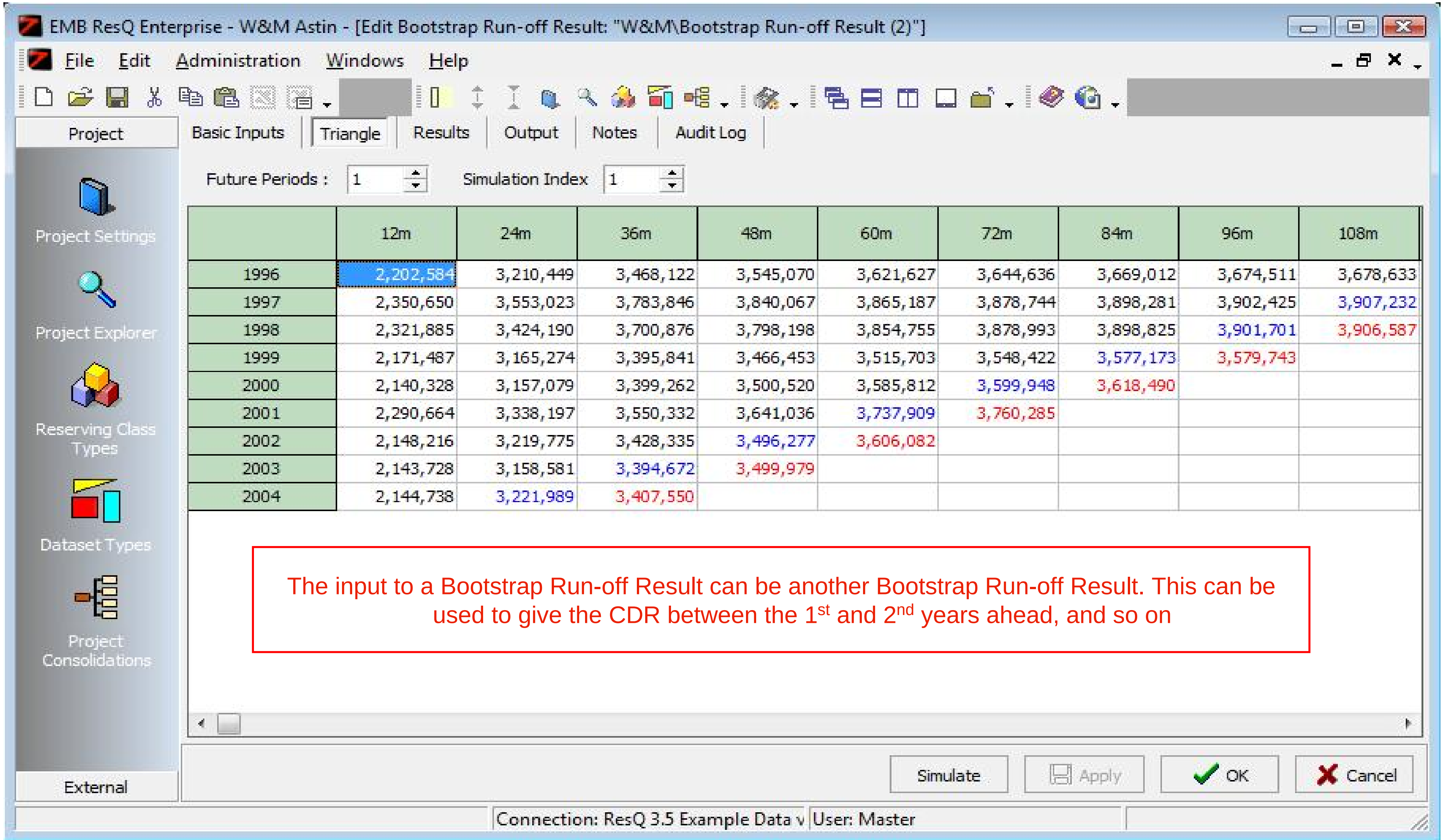
	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,063	
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,904,572		
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,563,898			
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,610,368				
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,684,393					
2002	2,148,216	3,219,775	3,428,335	3,494,624						
2003	2,143,728	3,158,581	3,346,680							
2004	2,144,738	3,244,781								

Merz & Wuthrich (2008)

Analytic vs Simulated: Summary

	Analytic		Simulated	
	Prediction Errors		Prediction Errors	
Accident Year	**1 Year Ahead CDR**	**Mack Ultimate**	**1 Year Ahead CDR**	**Mack Ultimate**
0	0	0	0	0
1	567	567	568	568
2	1,488	1,566	1,486	1,564
3	3,923	4,157	3,916	4,147
4	9,723	10,536	9,745	10,569
5	28,443	30,319	28,428	30,296
6	20,954	35,967	20,986	35,951
7	28,119	45,090	28,110	44,996
8	53,320	69,552	53,406	69,713
Total	81,080	108,401	81,226	108,992

Cascading Bootstrap Run-off Results



	12m	24m	36m	48m	60m	72m	84m	96m	108m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,232
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,901,701	3,906,587
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,577,173	3,579,743	
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,599,948	3,618,490		
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,737,909	3,760,285			
2002	2,148,216	3,219,775	3,428,335	3,496,277	3,606,082				
2003	2,143,728	3,158,581	3,394,672	3,499,979					
2004	2,144,738	3,221,989	3,407,550						

The input to a Bootstrap Run-off Result can be another Bootstrap Run-off Result. This can be used to give the CDR between the 1st and 2nd years ahead, and so on

Multiple 1 yr ahead CDRs

An interesting result

- Creating cascading CDRs over all years gives the following results:

Accident Year	Number of years ahead								Sqrt(Sum of Squares)	Mack Ultimate
	1 Yr	2 Yrs	3 Yrs	4 Yrs	5 Yrs	6 Yrs	7 Yrs	8 Yrs		
1	0	0	0	0	0	0	0	0	-	0
2	568	0	0	0	0	0	0	0	568	568
3	1,486	487	0	0	0	0	0	0	1,564	1,564
4	3,916	1,306	431	0	0	0	0	0	4,151	4,147
5	9,745	3,837	1,277	425	0	0	0	0	10,560	10,569
6	28,428	9,679	3,824	1,272	425	0	0	0	30,303	30,296
7	20,986	27,438	9,343	3,693	1,226	409	0	0	35,998	35,951
8	28,110	20,404	26,922	9,162	3,613	1,208	402	0	45,055	44,996
9	53,406	27,798	20,236	26,687	9,111	3,600	1,203	402	69,600	69,713
Total	81,226	52,344	38,513	29,010	10,120	3,879	1,285	402	108,543	108,992

- The sum of the variances of the repeated 1 yr ahead CDRs (over all years) equals the variance over the lifetime of the liabilities
  - Under Mack’s assumptions/chain ladder, this can be proved
- Therefore we expect the risk under the 1 year view to be lower than the standard “ultimo” perspective

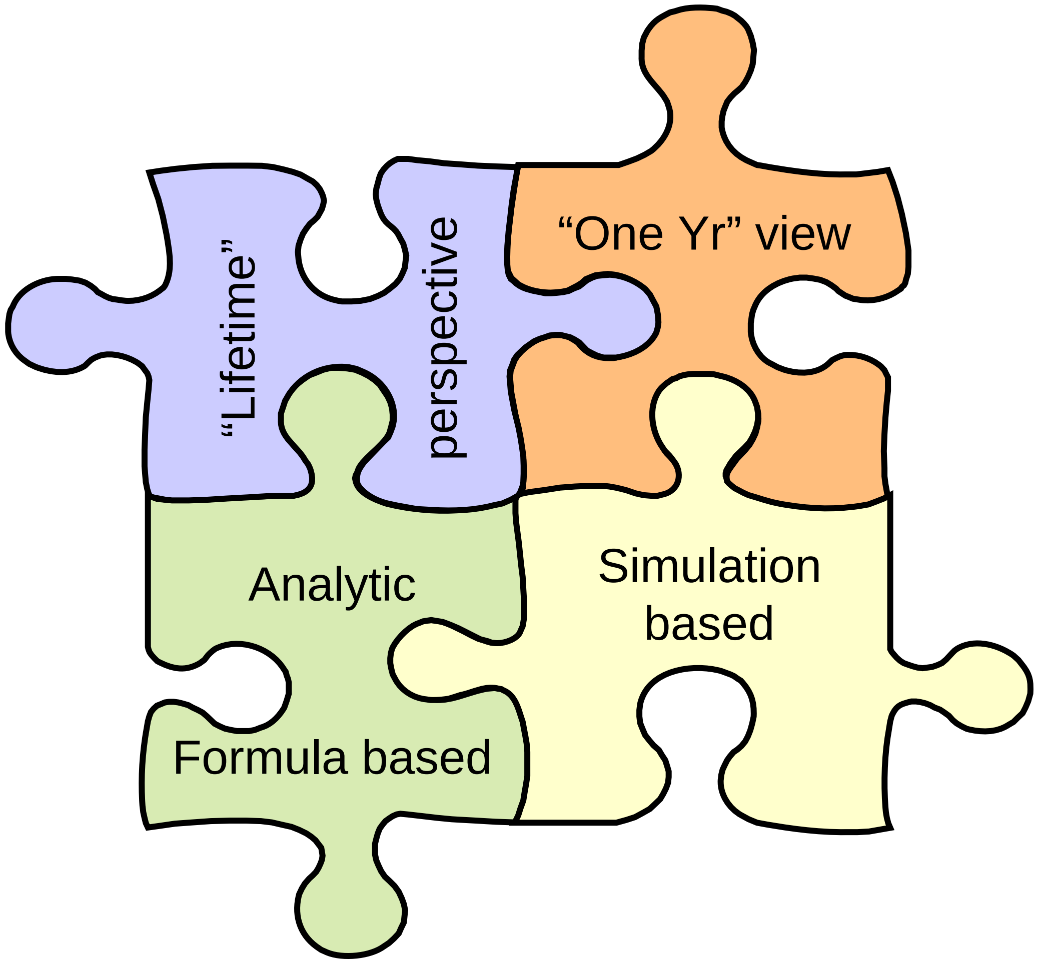
Re-reserving in Simulation-based Capital Models

- The advantage of investigating the claims development result (using re-reserving) in a simulation environment is that the procedure can be generalised:
  - Not just the chain ladder model
  - Not just Mack’s assumptions
  - Can include curve fitting and extrapolation for tail estimation
  - Can incorporate a Bornhuetter-Ferguson step
  - Can be extended beyond the 1 year horizon to look at multi-year forecasts
  - Provides a *distribution* of the CDR, not just a standard deviation

- But it is not without its difficulties, so we need simpler alternatives
  - Simply allow the “ultimo” variability to emerge steadily over time (but there is the problem of calibration)

The Reserve Risk Puzzle

Harmony has been restored



Risk Margins in Internal Models

DIRECTIVE OF THE EUROPEAN PARLIAMENT
Article 77

Article 77

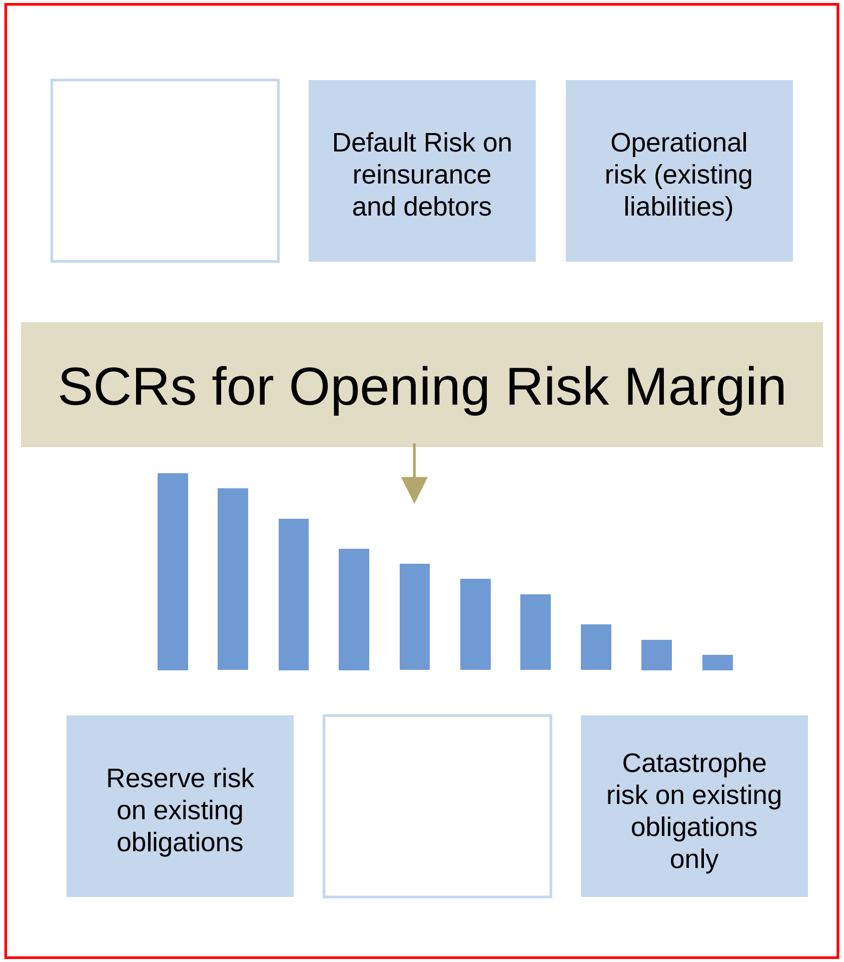
“The risk margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount insurance undertakings would be expected to require in order to take over and meet the insurance obligations…”

“… the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance obligations over the lifetime thereof.”

So we need an SCR for each future year as the reserves run-off

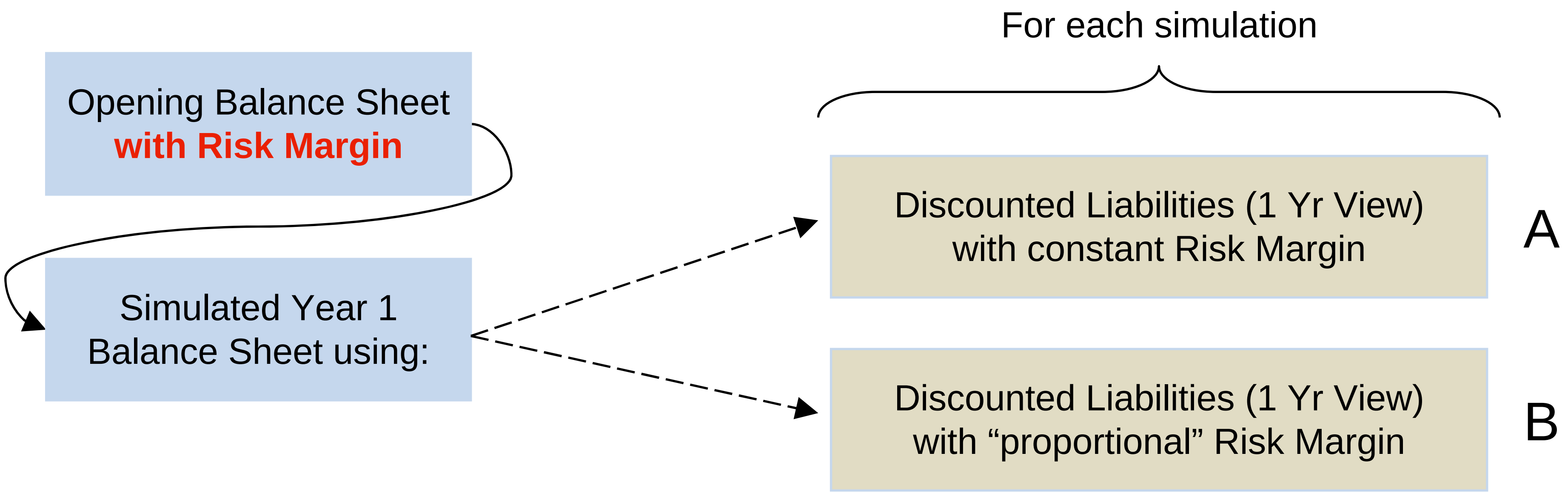
Solvency Capital Requirements

Non-Life Companies



Overall SCR

GIRO 2010: Simulated Year 1 balance sheet options?



Whichever method is used, we still need a risk margin for the opening balance sheet.

The "opening" SCR for the risk margin calculation could be calculated using the standard formula (maybe) or a modified version of the internal model itself.

Use a simple approach for the future SCRs for the risk margin calculation.

The Opening Risk Margin in Internal Models
Using the Standard Formula

- This has the advantage of appearing to be simple
  - There is no need to justify the assumptions in the standard formula
  - The risk margin method would be standardised across companies
- Calculate the opening SCR by entering reserve and premium volumes in respect of the (expected) technical provisions (legal obligations basis only)
  - Market risk not required (usually)
- Calculate future SCRs:
  - In proportion to the emergence of the (expected) reserves in each future year, or
  - By repeatedly calculating the SCR using the standard formula, but adjusting reserve and premium volumes in each future year
    - The capital requirement percentages can be calculated, relative to the opening SCR

The Opening Risk Margin in Internal Models

Using the Internal Model

- The internal model basis itself could* be used
  - Assume opening assets = 0**
  - For premium volumes, use “legal obligations” basis only (no new business in the forthcoming year)
  - Remember to modify assumptions about cat exposures, reinsurance and expenses
- VaR @ 99.5% will give the TOTAL capital required, for the SCR calculation
- Calculate future SCRs:
  - In proportion to the emergence of the (expected) reserves in each future year, or
  - Using the proportions implied by the recursive standard formula method

* It is possible that the internal model basis *should* be used, but given the concept of proportionality, using the standard formula may be sufficient

** Other assumptions could be used

The Important Question

- When calculating risk margins, it is impossible to satisfy the Solvency II requirements without simulation on simulation, which is impracticable
- Simplifications must be made
  - When calculating the opening SCR for the risk margin calculations
  - When calculating future SCRs
- Simplifications must be made for risk margins for each simulation on the 1 year ahead balance sheet
  - Assume a constant risk margin?
  - Use a simple ratio method?
- What we don't know is: "What methods will be approved?"
- The question can only be answered by the regulators

What We Asked the FSA

1. Will it be acceptable to have opening and 1 year ahead balance sheets excluding risk margins, and use the change in the balance sheet on that basis to estimate the overall SCR (after adding the opening risk margin back in)? If that is not acceptable, what simplifications will be approved for calculating risk margins for each simulation in the 1 year ahead balance sheet?
2. If the proposal in (1) is acceptable, will it also be acceptable to use the standard formula for estimating the opening risk margin, even with an internal model?
3. If the standard formula basis is not acceptable for estimating the opening risk margin when using an internal model, what methods will be approved for estimating the initial “SCR” for the risk margin calculation from the internal model, and what simplifications will be approved for estimating the future “SCRs” for the risk margin calculation?

What the FSA has said so far…*

- “At present there is no definitive answer”
- “We don’t want to give an answer that turns out to be wrong”
  - QIS 5 is not final: it is only a test
- “Do something sensible and explain why it's sensible”
- “Worry more about the technical provisions; the risk margin will usually be a lot smaller”
  - “Proportionality” should be borne in mind

** Thanks to the FSA for clarifying the current position*

Questions or comments?

“You’ve heard it from the ORSA’s mouth”