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Coherent forecasting of mortality for multiple populations using functional data models

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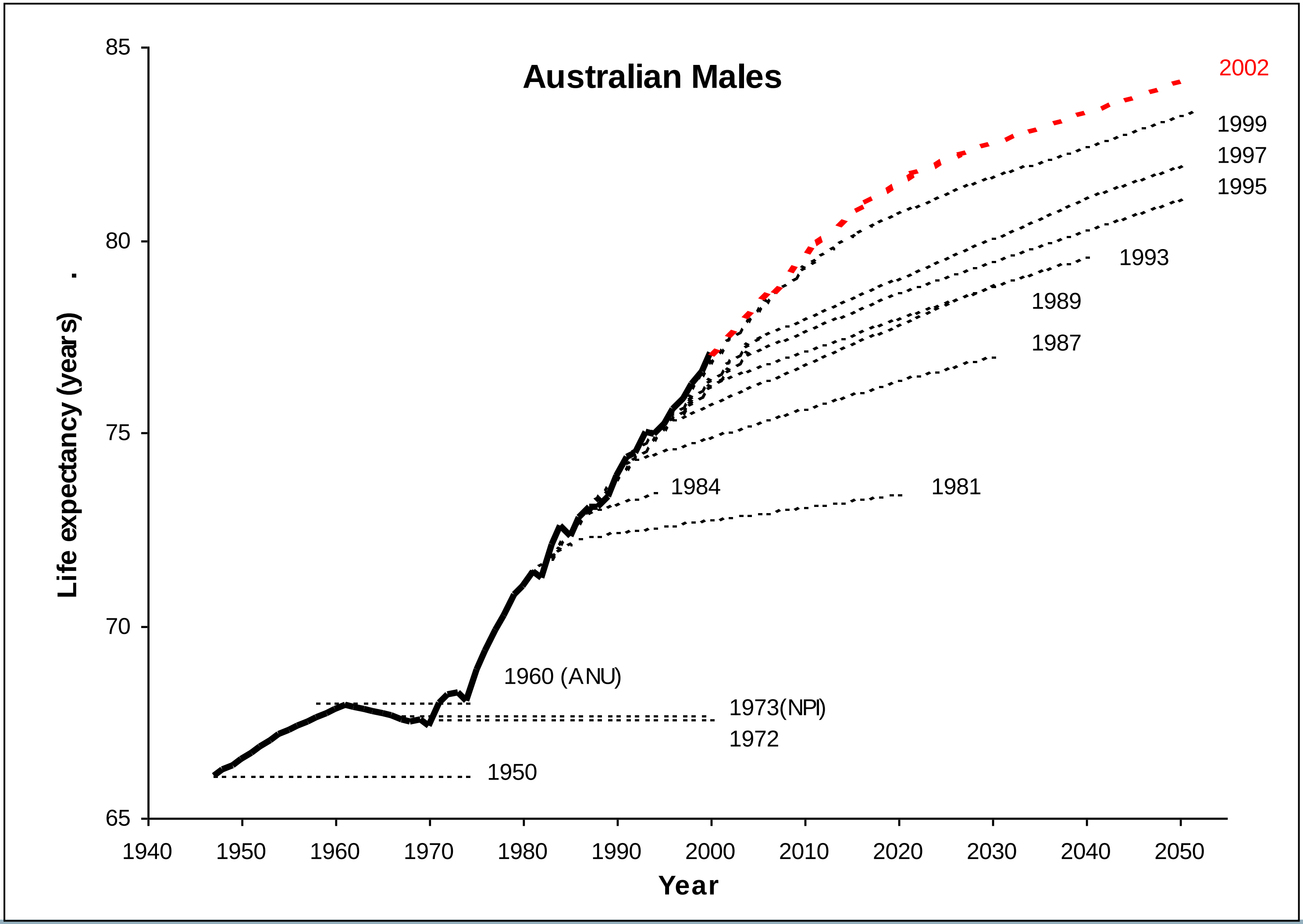
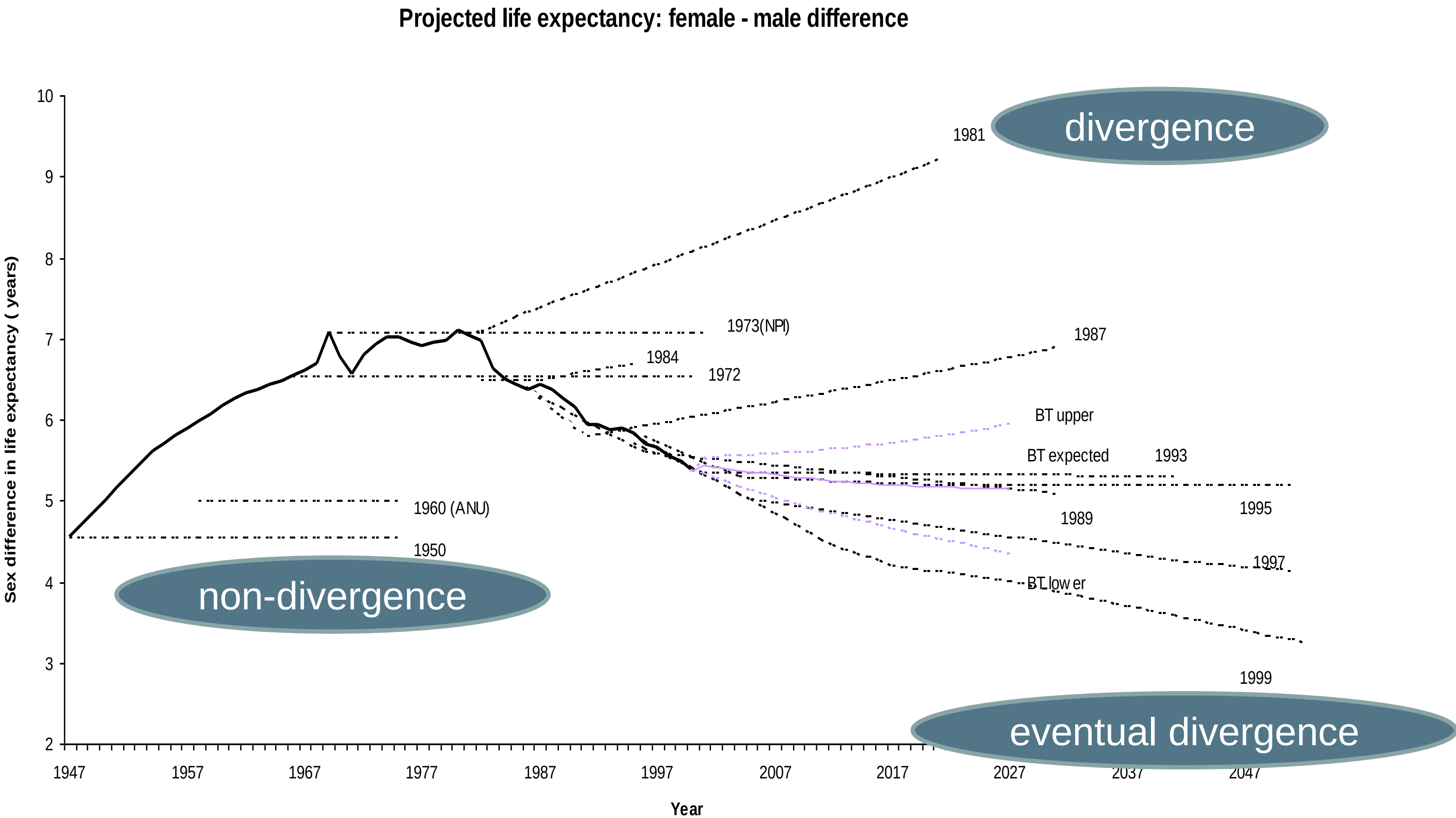
Historical experience in mortality ‘forecasting’



Better methods are needed….

The divergence problem



The sex gap has been largely ignored in ‘forecasting’

This presentation is about …

- Better forecasting methods …
- … that are capable of taking the sex gap into account
- … in other words, taking **other** mortality into account
- I consider how we might improve forecast accuracy by focussing on that other mortality

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Better forecasting methods

LEE-CARTER METHOD (1992)

Principal components in forecasting

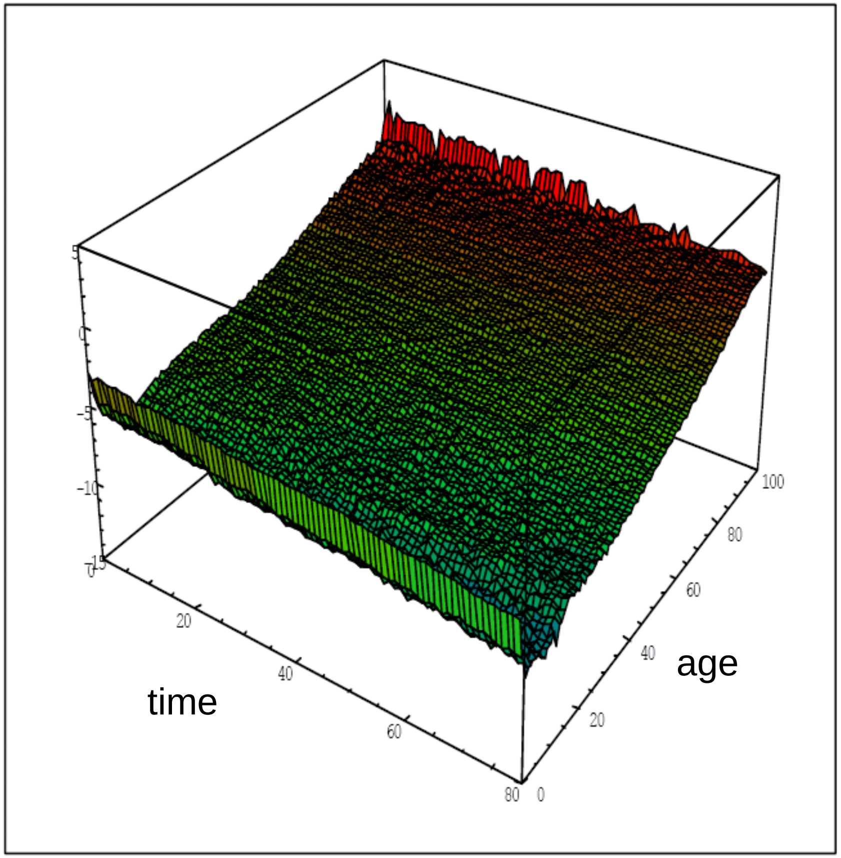
Decompose age x time matrix of death rates into its 2 dimensions

- Age effect
- Time effect

Singular value decomposition

Interpretable

Time effect useful for forecasting



Lee-Carter Model (one Principal Component)

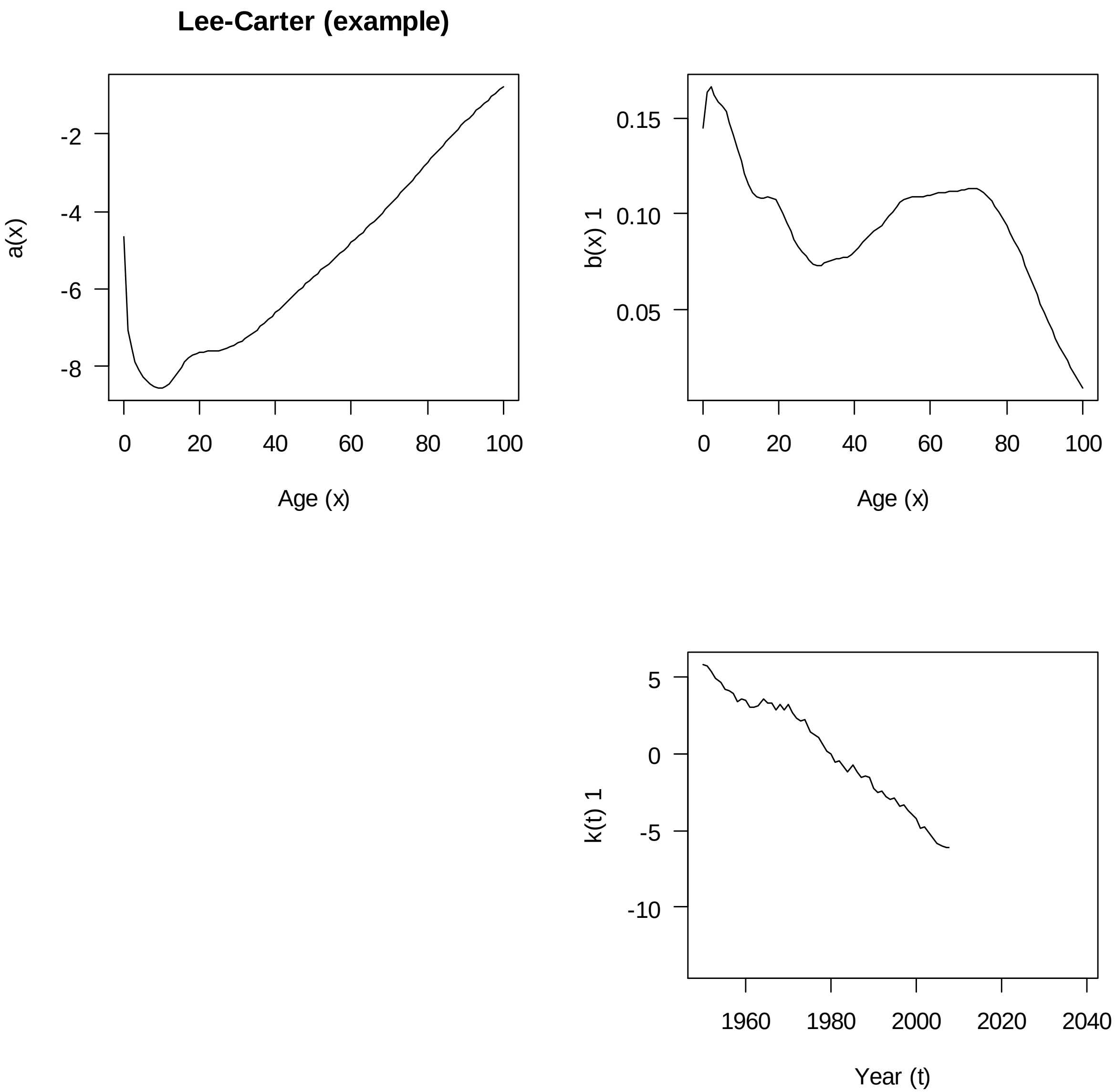
$$\ln[m(x,t)] = a(x) + \mathbf{b(x)k(t)} + e(x,t)$$

m(x,t)	central death rate at age x in year t
a(x)	mean ln[m(x,t)] over time
k(t)	index of the level of mortality — Time effect
b(x)	relative speed of change at each age — Age effect
e(x,t)	residual at age x and time t, Normal(0,σ^2)

High percentage (>90%) of variation explained by this model

i.e. by the first Principal Component (PC)

This is a model of age and time – the changing **age structure** is modelled



Mortality (mean adjusted) is decomposed into:

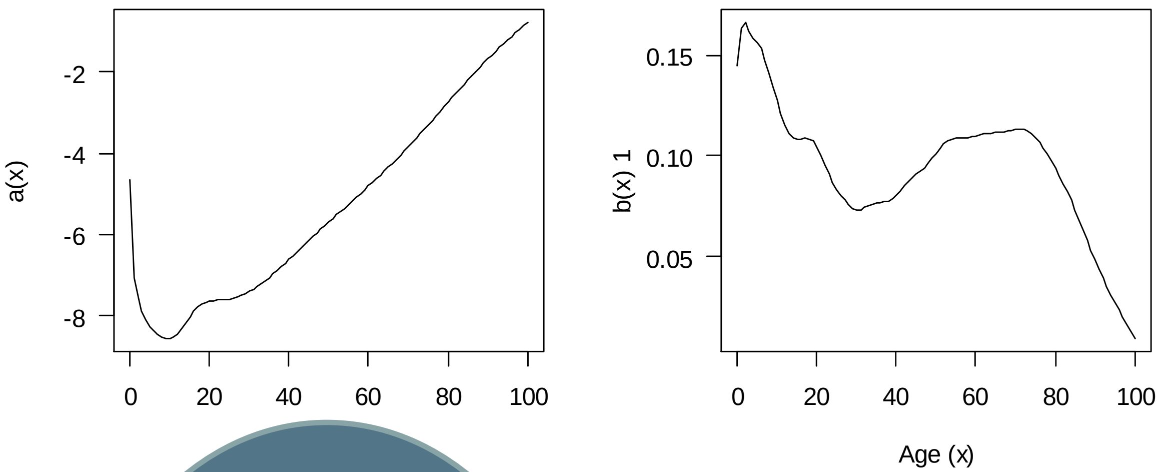
Age pattern of mortality change (assumed fixed)

and

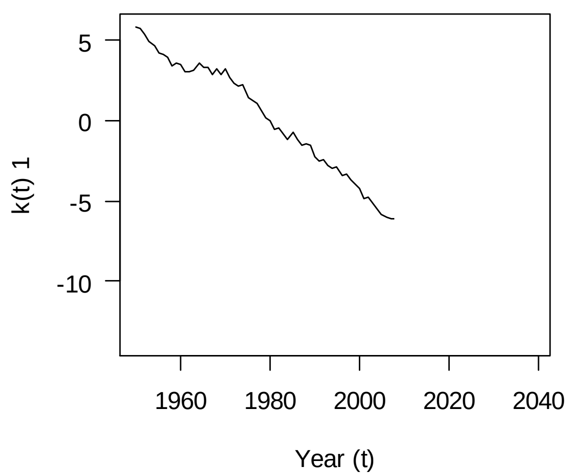
Time pattern of mortality change

This is a model of age and time – the changing **age structure** is modelled

Lee-Carter (example)



Since 1950, decline fastest at ages 0-19 and 50-80



Mortality (mean adjusted) is decomposed into:

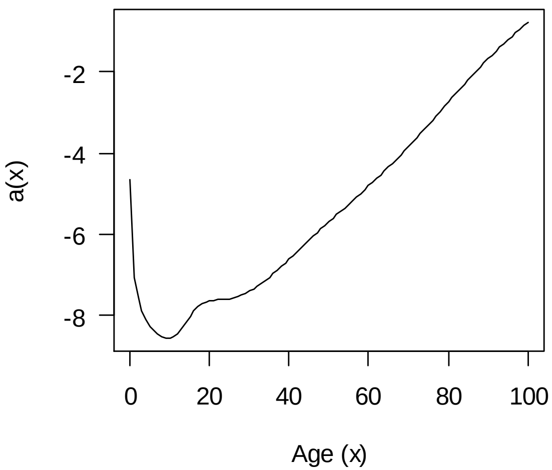
Age pattern of mortality change (assumed fixed)

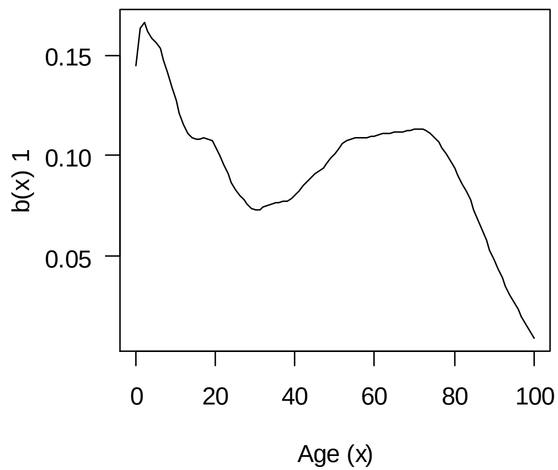
and

Time pattern of mortality change: ~ LINEAR
Roughly constant rate of mortality improvement

This is a model of age and time – the changing **age structure** is modelled

Lee-Carter (example)





Mortality (mean adjusted) is decomposed into:

Age pattern of mortality change (assumed fixed)

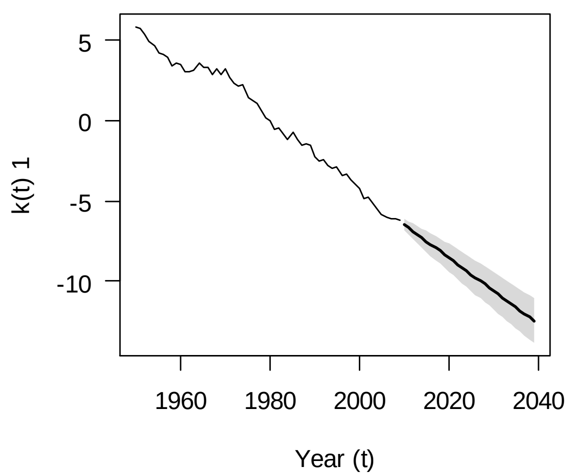
and

Time series model of k(t)

Random walk with drift

Linear decline

Uncertainty from RWD and e(x,t)

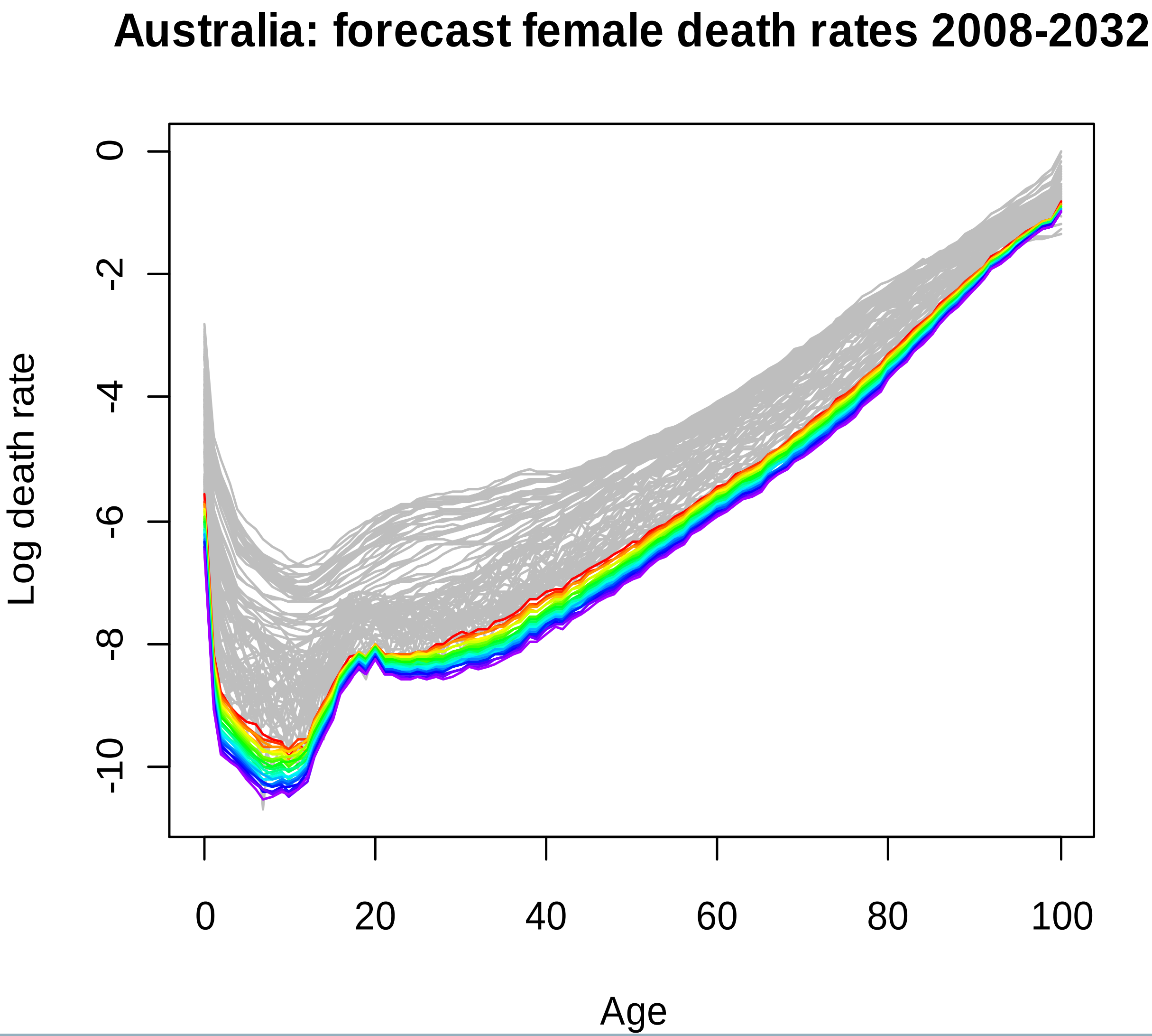


Time pattern of mortality change (with linear time series forecast)

Forecast m(x,t) uses **future values of time parameter k(t)**

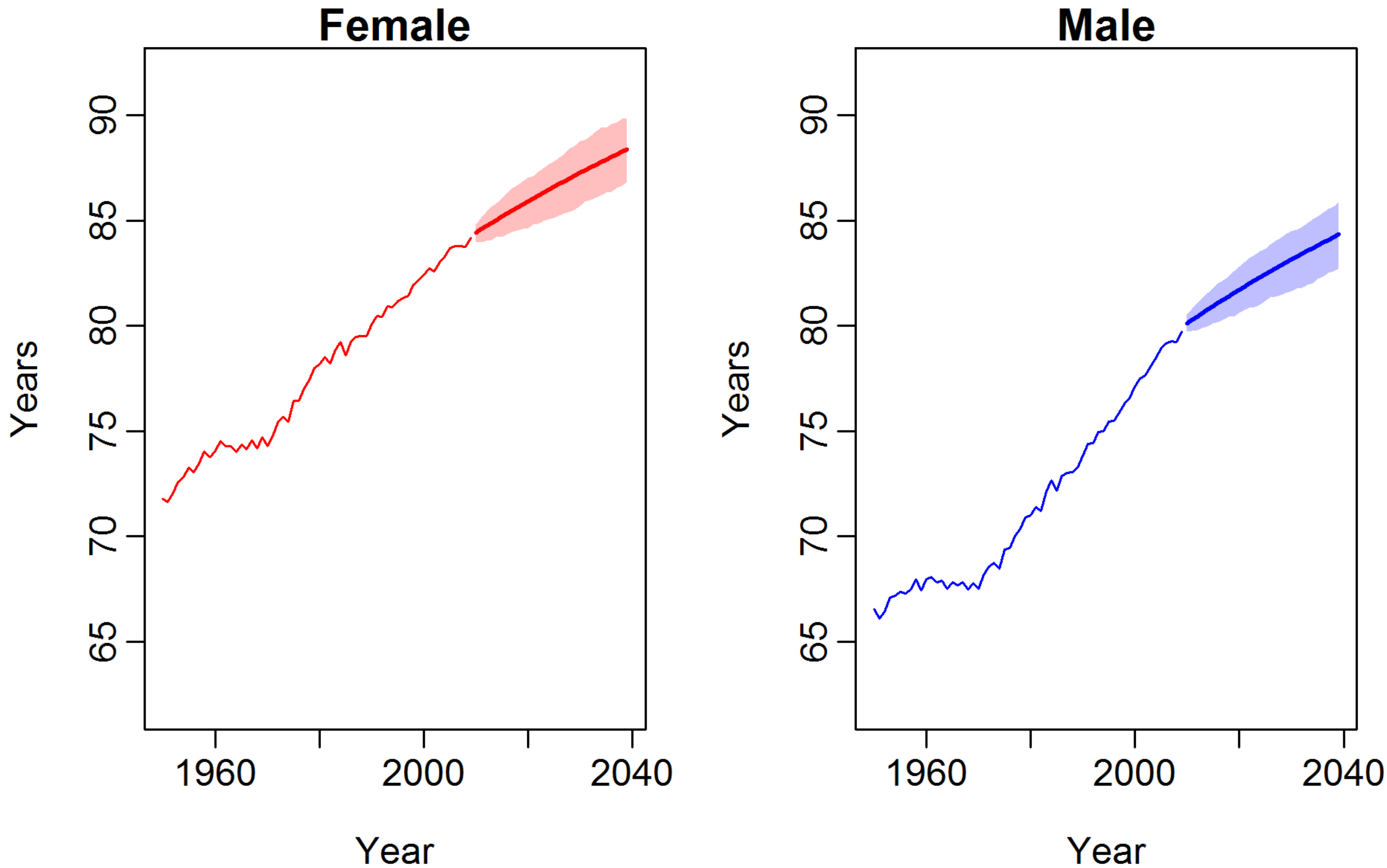
This is a model of age and time – the changing **age structure** is modelled

Forecast death rates

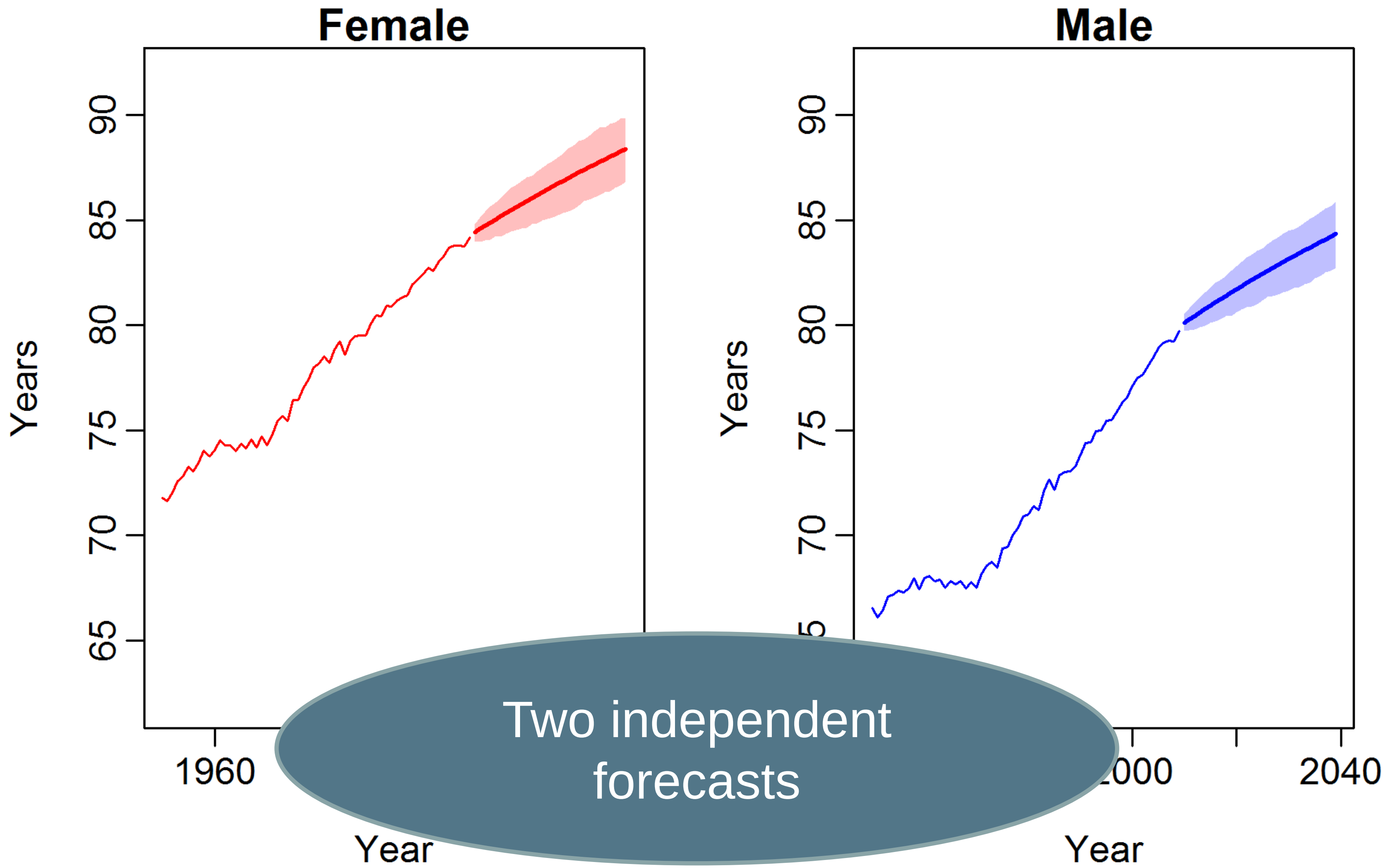


Limitations: fixed age pattern of change and increasing jaggedness over age

LC: Point forecasts with prediction intervals

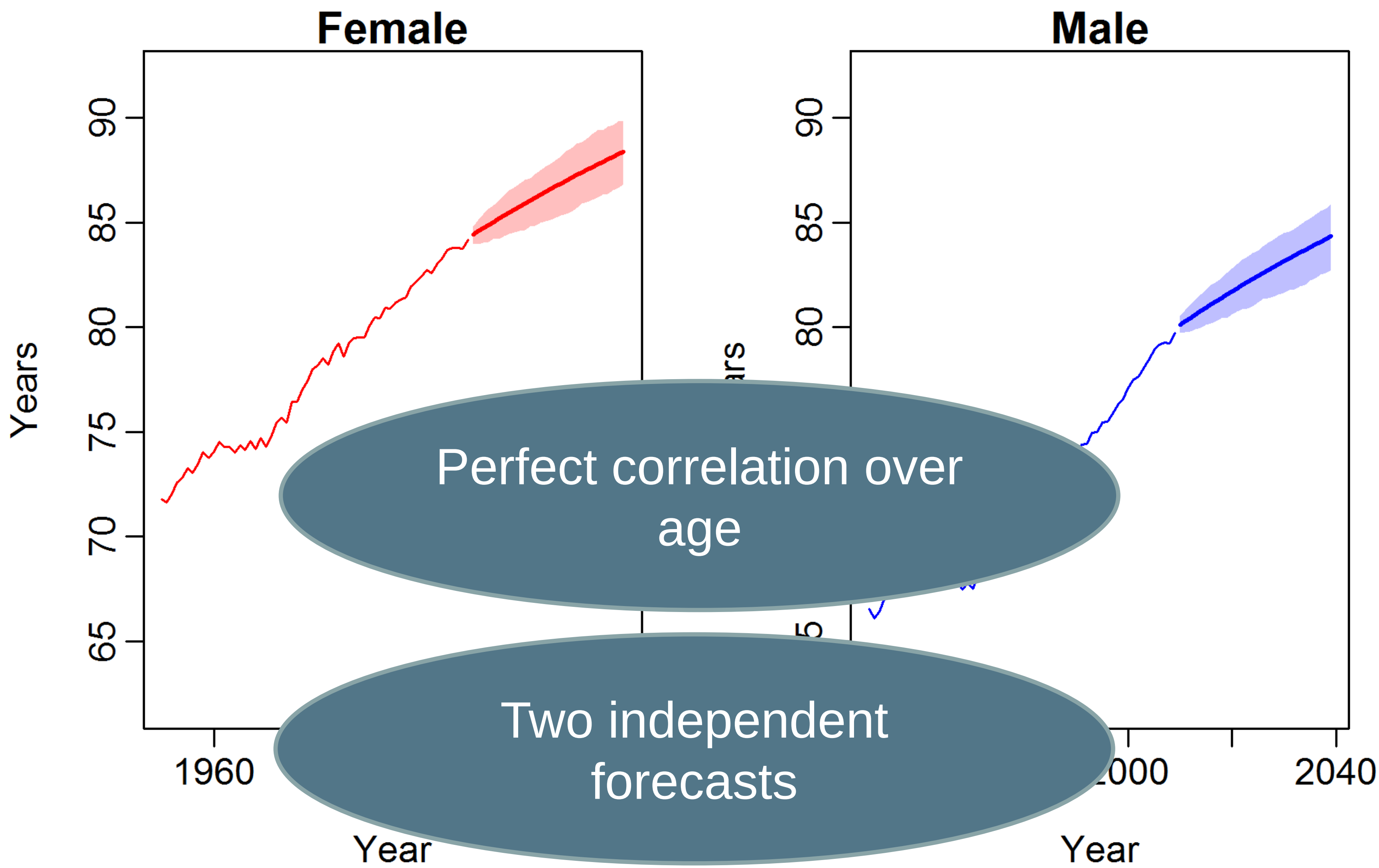


LC: Point forecasts with prediction intervals

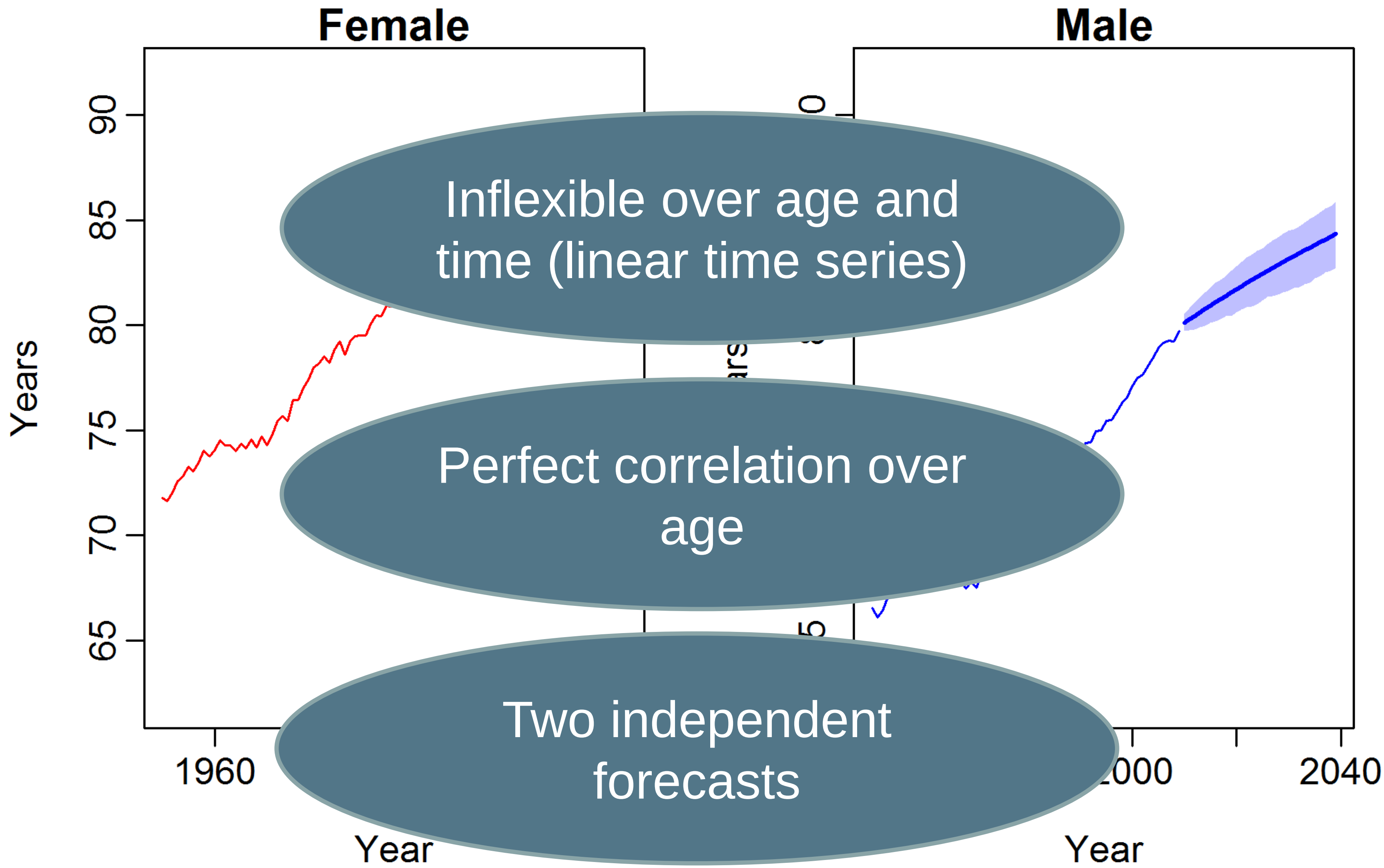


Two independent forecasts

LC: Point forecasts with prediction intervals



LC: Point forecasts with prediction intervals



Inflexible over age and time (linear time series)

Perfect correlation over age

Two independent forecasts

An improved method:

FUNCTIONAL FORECASTING OF MORTALITY

Functional forecasting - overview

- Greater sophistication but essentially LC
- Uses **functional** principal components
  - Achieve continuous functions by **smoothing** using splines
- Models and forecasts **multiple** (≤ 6) functional PCs
  - Allows some **flexibility** in age pattern of change
- Uses **non-linear** time series models as appropriate
- Adopts improved estimation (Poisson deaths, MLE, age-varying variance) (e.g., Booth, Maindonald, Smith 2002; Brouhns, Denuit, Vermunt 2002)

Hyndman & Ullah (2007); Hyndman & Booth (2008)

In terms of the Lee-Carter Model ……

$\ln[m(x,t)]$ **– random error** =

$$a(x) + b_1(x)k_1(t) + \mathbf{b_2(x)k_2(t) + \ldots\ldots} + e(x,t)$$

where:

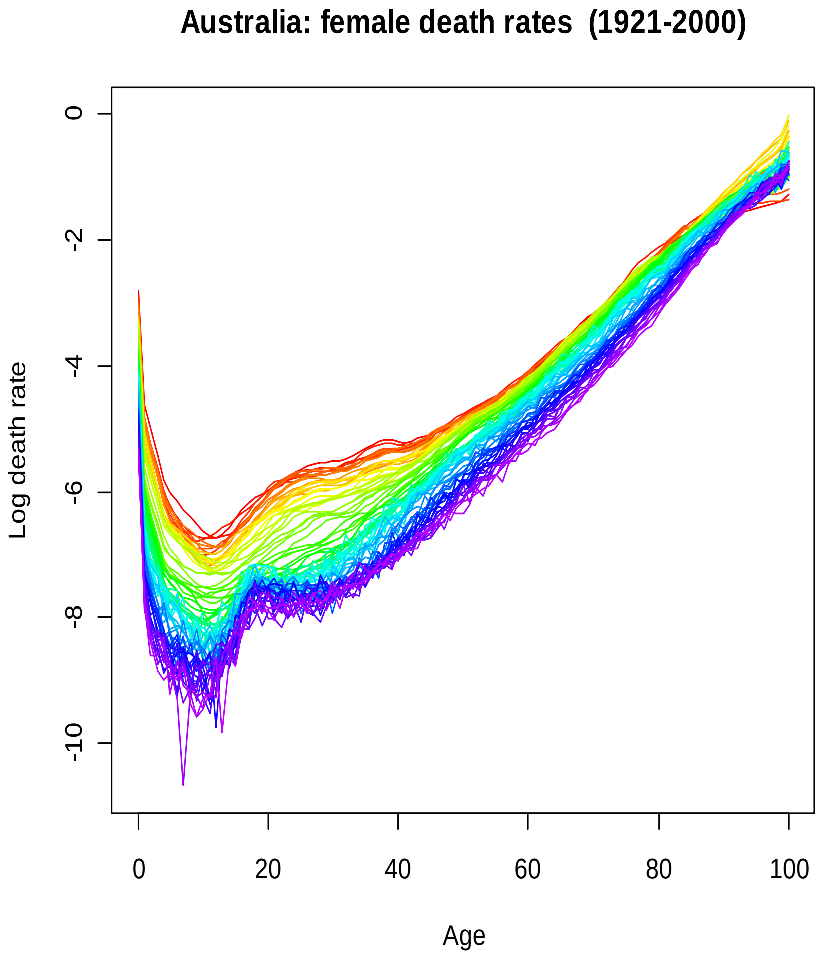
$m(x,t)$	central death rate at age x in year t
$a(x)$	mean death rate by age
$k_1(t)$	coefficient for 1st PC
$b_1(x)$	age pattern for 1st PC **(smooth)**
$\mathbf{k_2(t)}$	coefficient for 2nd PC
$\mathbf{b_2(x)}$	age pattern for 2nd PC **(smooth)**

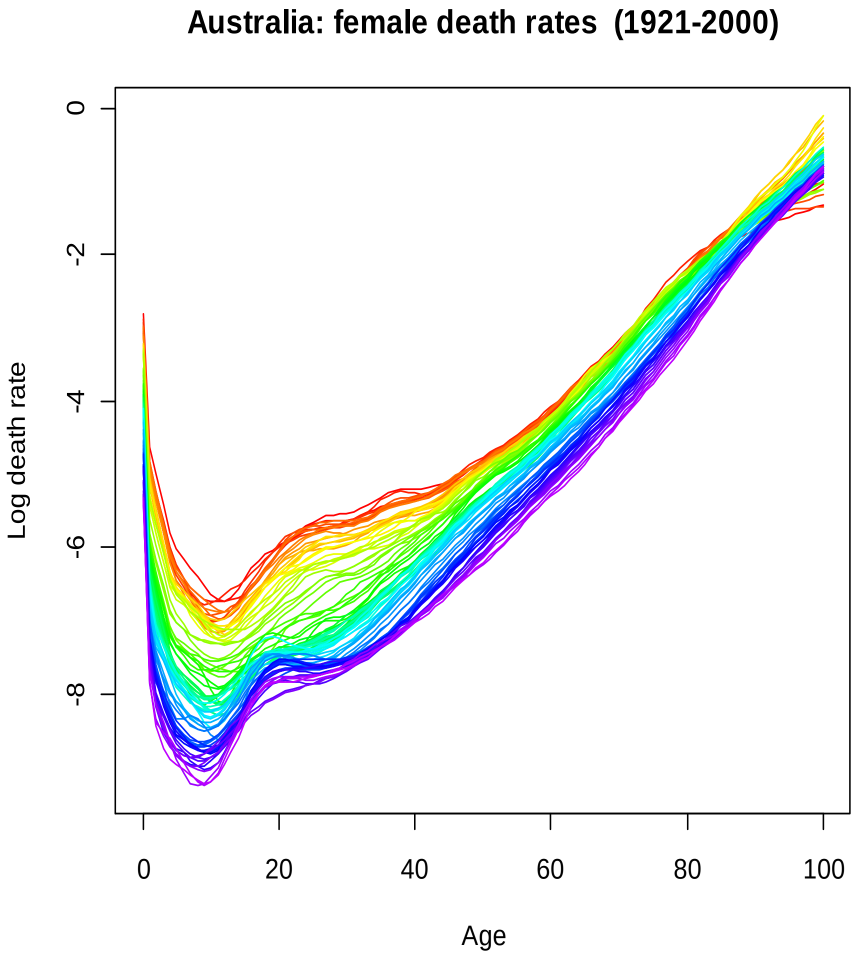
….

$e(x,t)$	**new (smaller)** residual at age x and time t, mean=0, var=σ_x^2

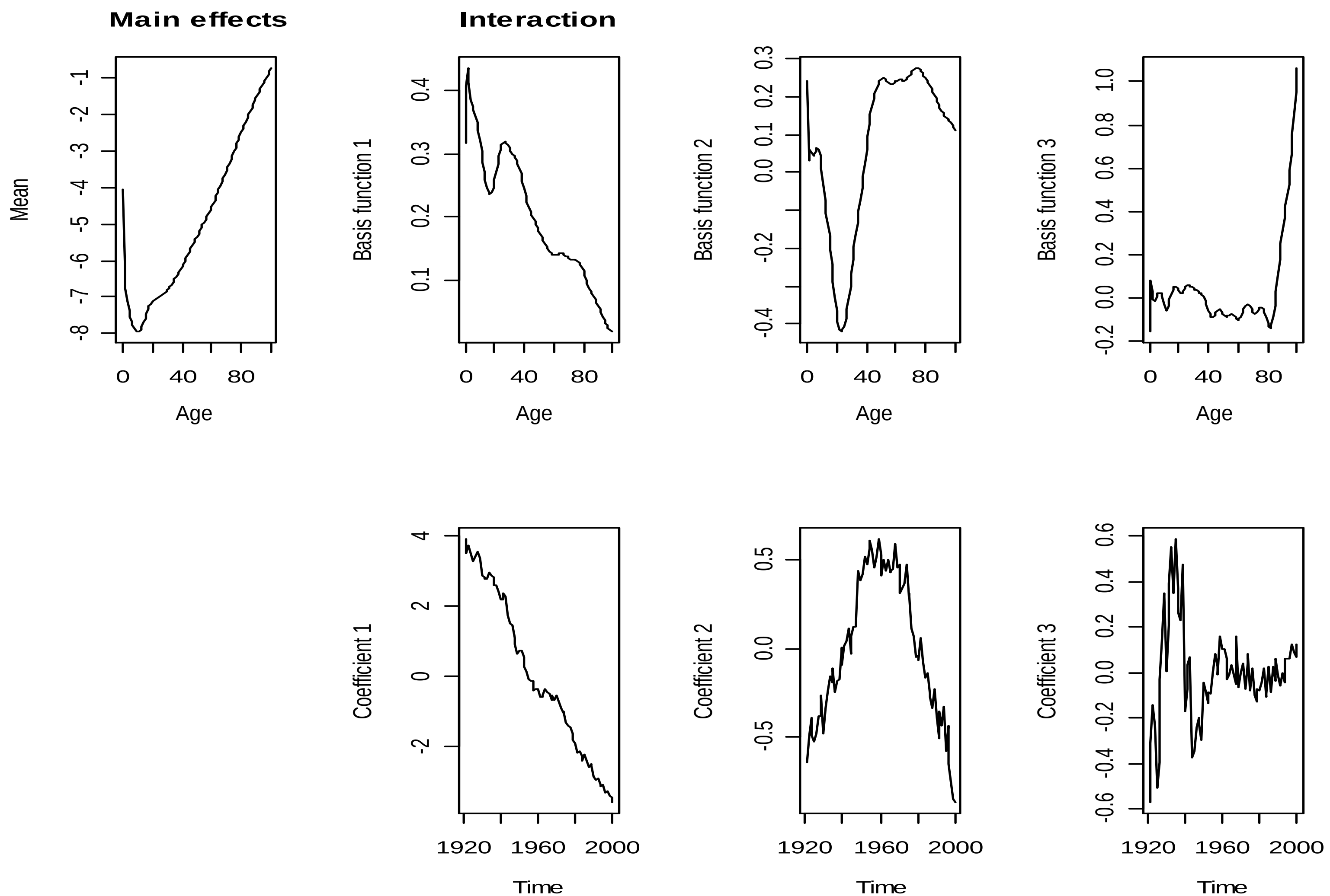
Lee-Carter is a special case of the functional model

Raw and smoothed death rates



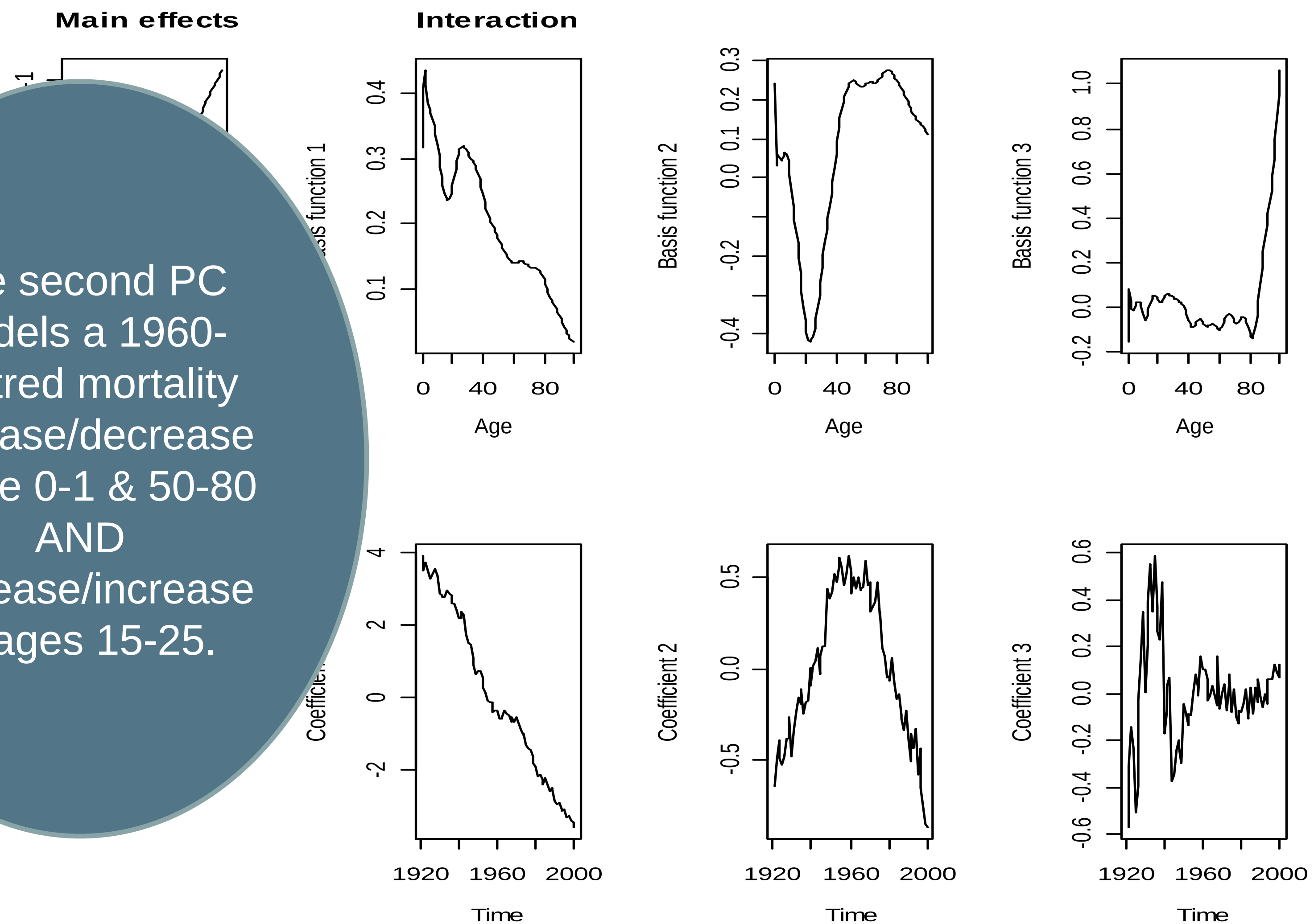


First 3 PCs, Australia, females, 1921-2000



The PCs are orthogonal

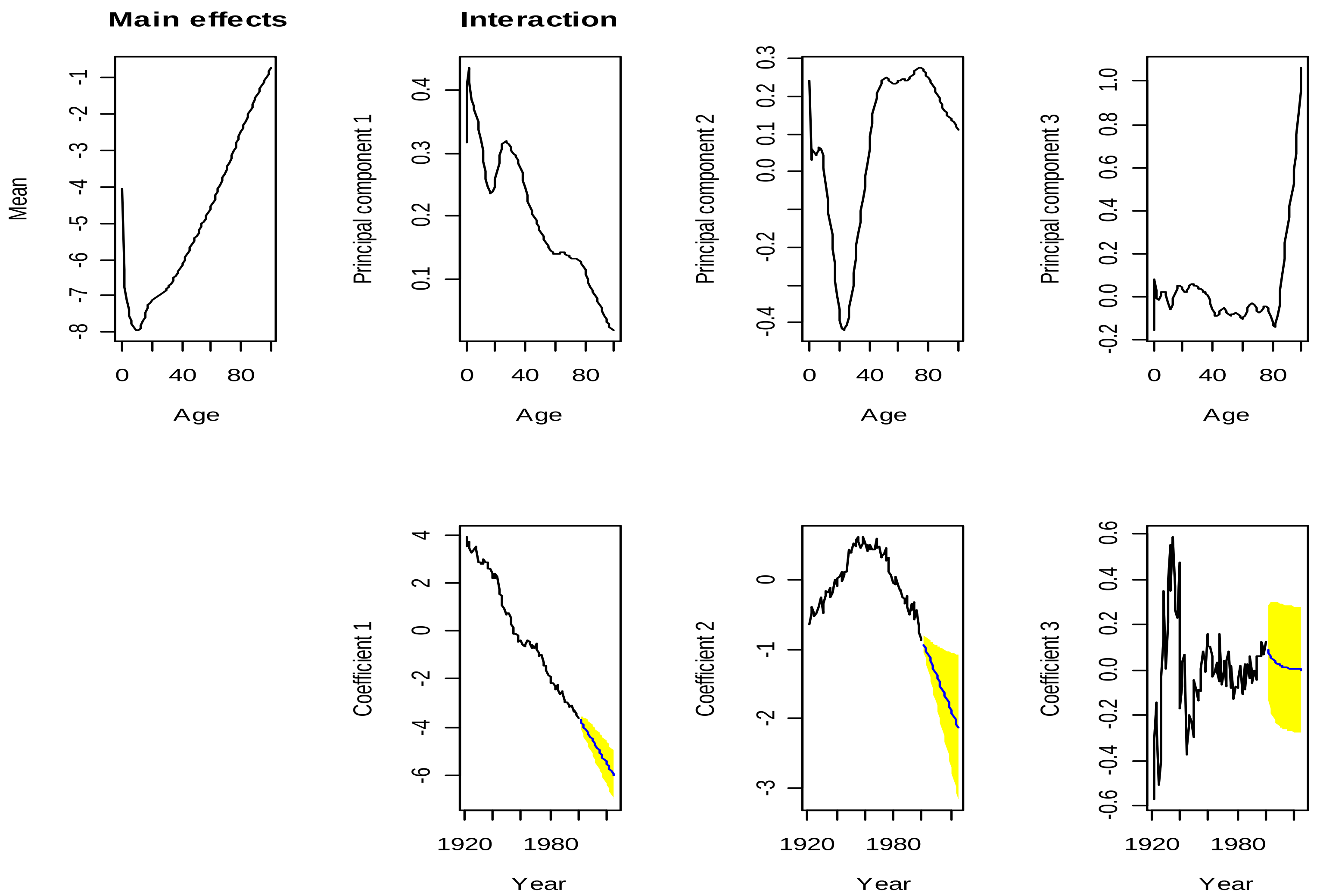
First 3 PCs, Australia, females, 1921-2000



The second PC models a 1960-centred mortality increase/decrease at age 0-1 & 50-80 AND decrease/increase at ages 15-25.

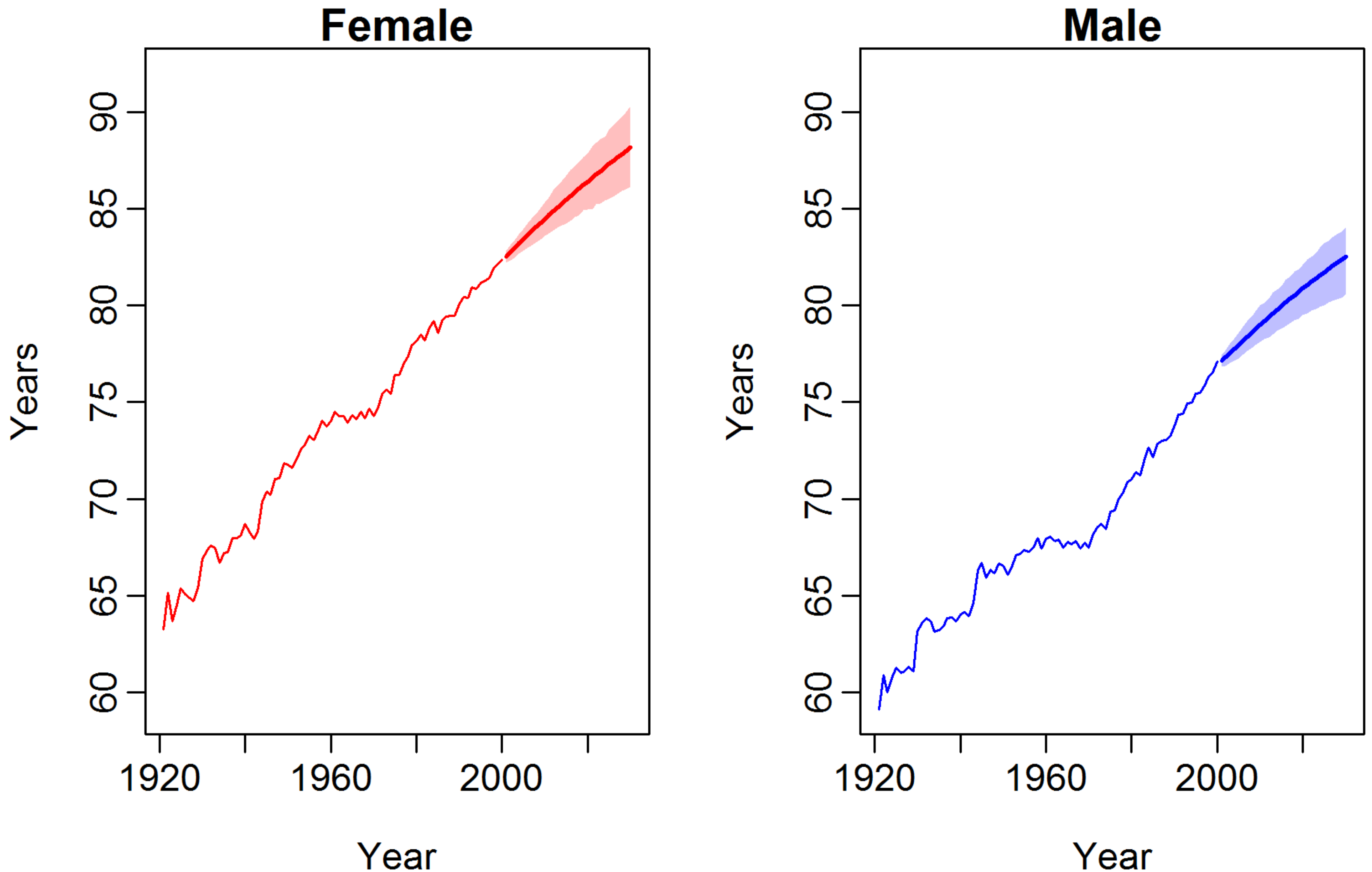
The PCs are orthogonal

Forecast coefficients using ARIMA models



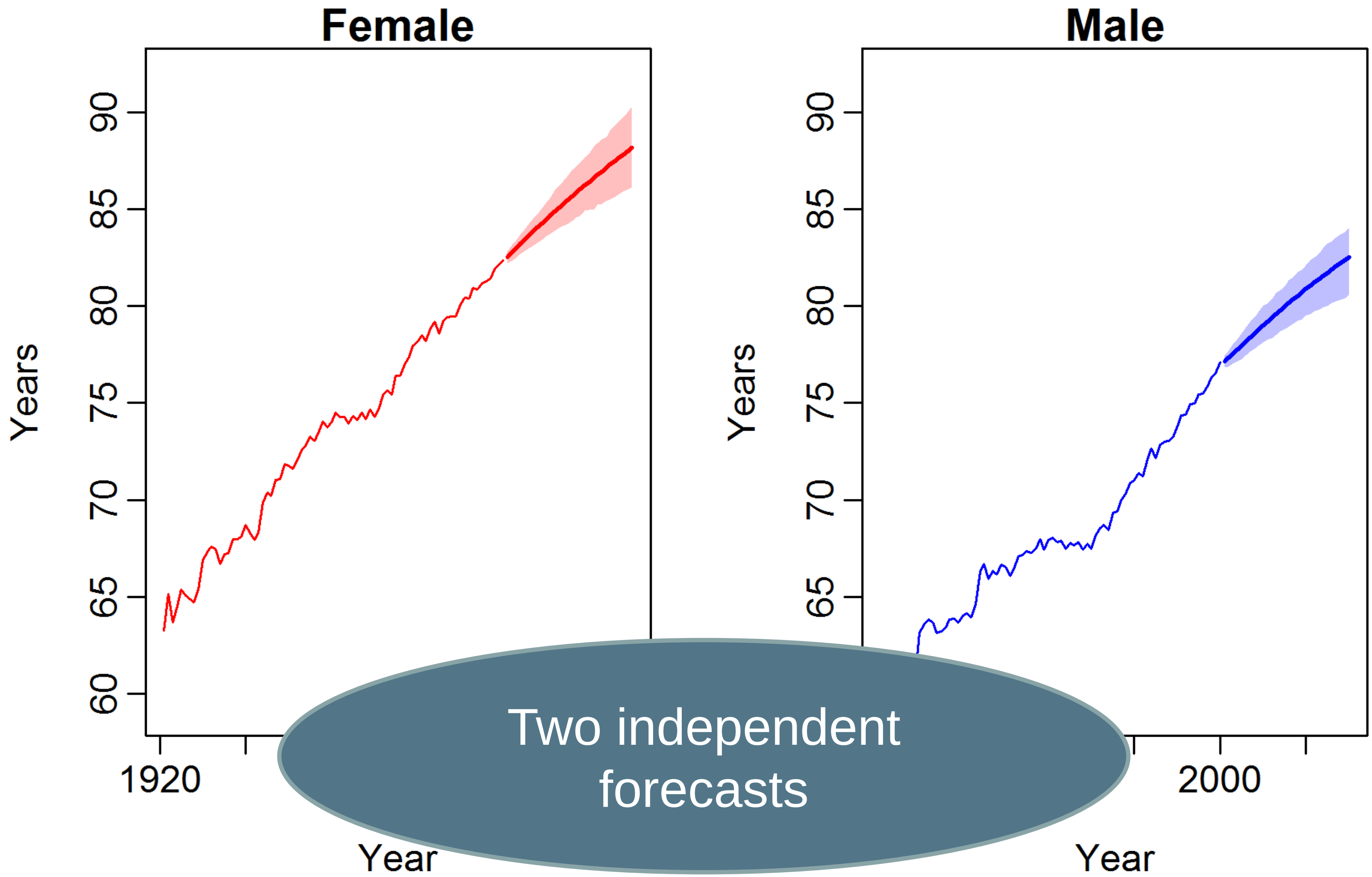
The PCs are orthogonal

Functional forecasts & prediction intervals



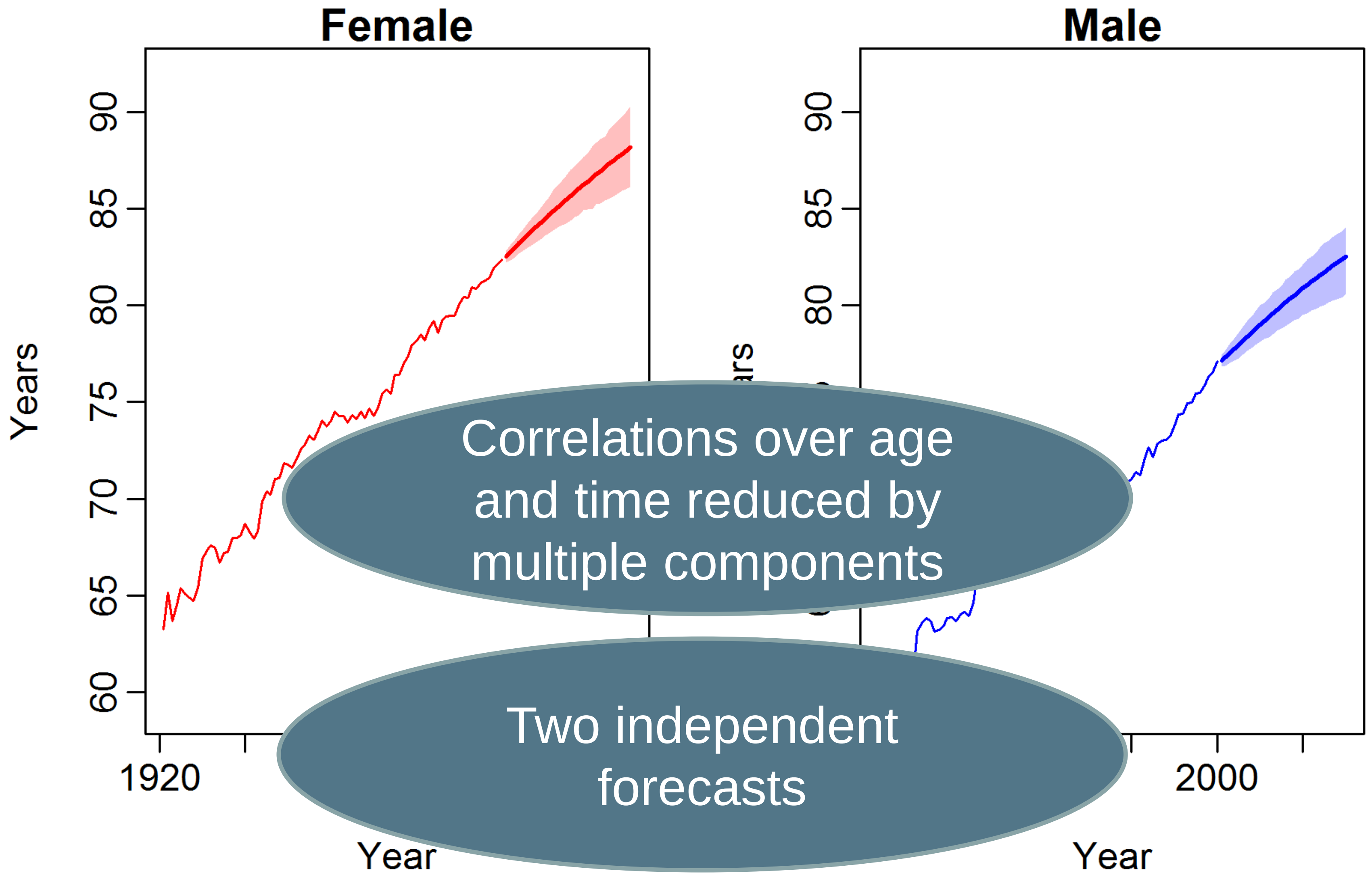
Prediction intervals for e(0) derived from variance of m(x,t) using simulation

Functional forecasts & prediction intervals



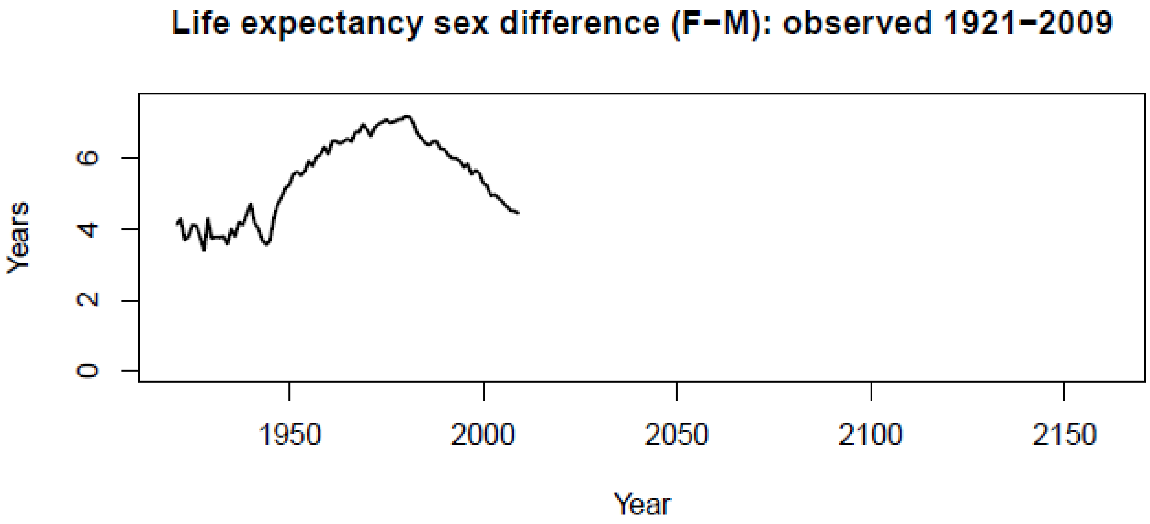
Prediction intervals for e(0) derived from variance of m(x,t) using simulation

Functional forecasts & prediction intervals



Prediction intervals for e(0) derived from variance of m(x,t) using simulation

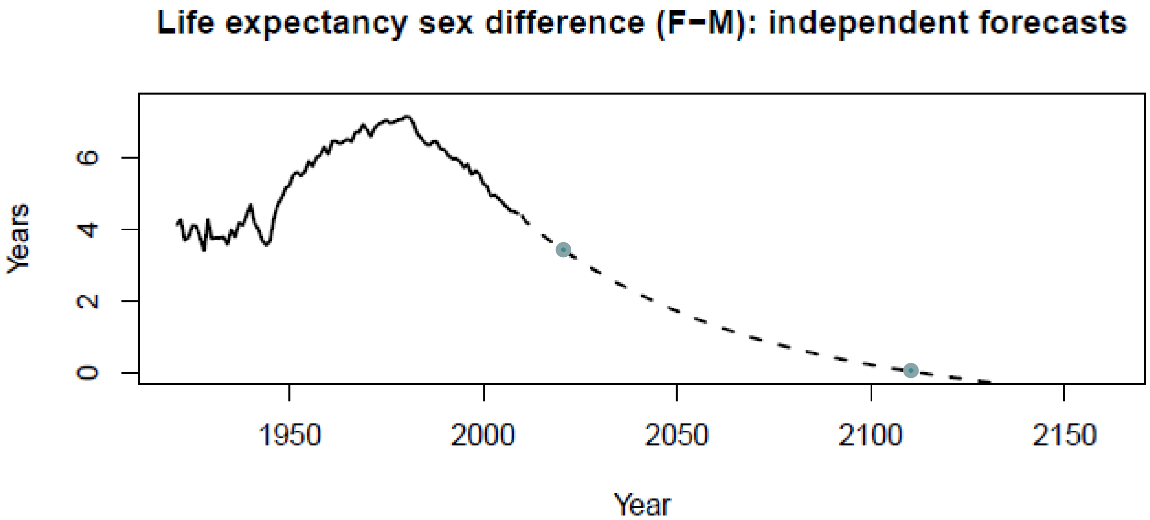
Two independent forecasts



AUSTRALIA

Sex difference in e(0):

1980 7.2 years (peak)
2009 4.5 years



Based on **independent** forecasts, the mean sex gap will reach the recent minimum of 3.4 years by 2021 and be **negative by 2112**

Coherent forecasts prevent such divergence

Taking account of the ‘other’:

COHERENT MORTALITY FORECASTING

Product-Ratio (Coherent) Method

Hyndman, Booth and Yasmeen (2013) *Demography*

- ‘Other’ mortality can be incorporated by modelling and forecasting:
  - a joint mortality function: geometric mean (√**product**)
  - a mortality difference function: square root of ratio (√**ratio**)
- In the two population case:
  - Joint function is √**product**
  - Difference function is √**ratio**
- For n populations:
  - Joint function is $\sqrt[n]{\text{product}}$
  - Multiple (n-1) difference functions are √**ratio**

Simple model for sex-coherent forecasting

F = female mortality rate

M = male mortality rate

Geometric mean rate = √**product** = √(FM)

Square root of sex ratio = √**ratio** = √(M/F)

Using **forecasts** of these functions:

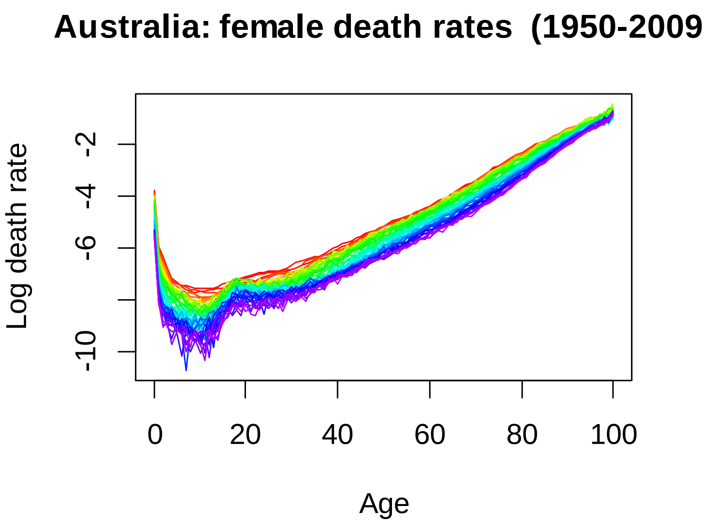
√(FM) x √(M/F) = √(FMM/F) = M forecast

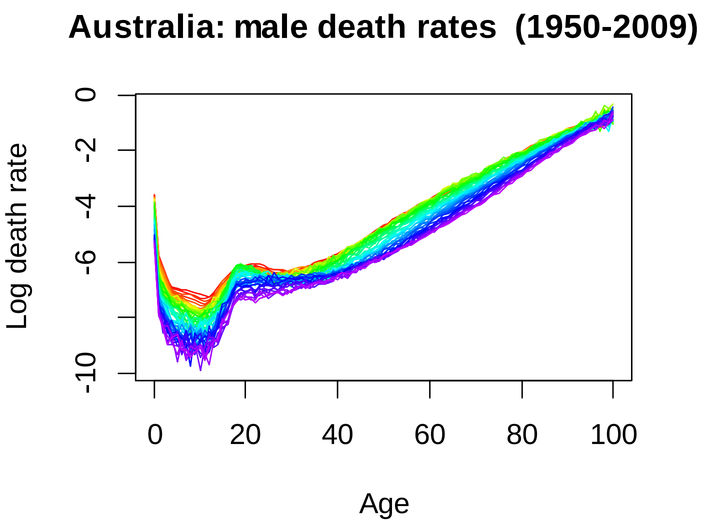
√(FM) / √(M/F) = √(FMF/M) = F forecast

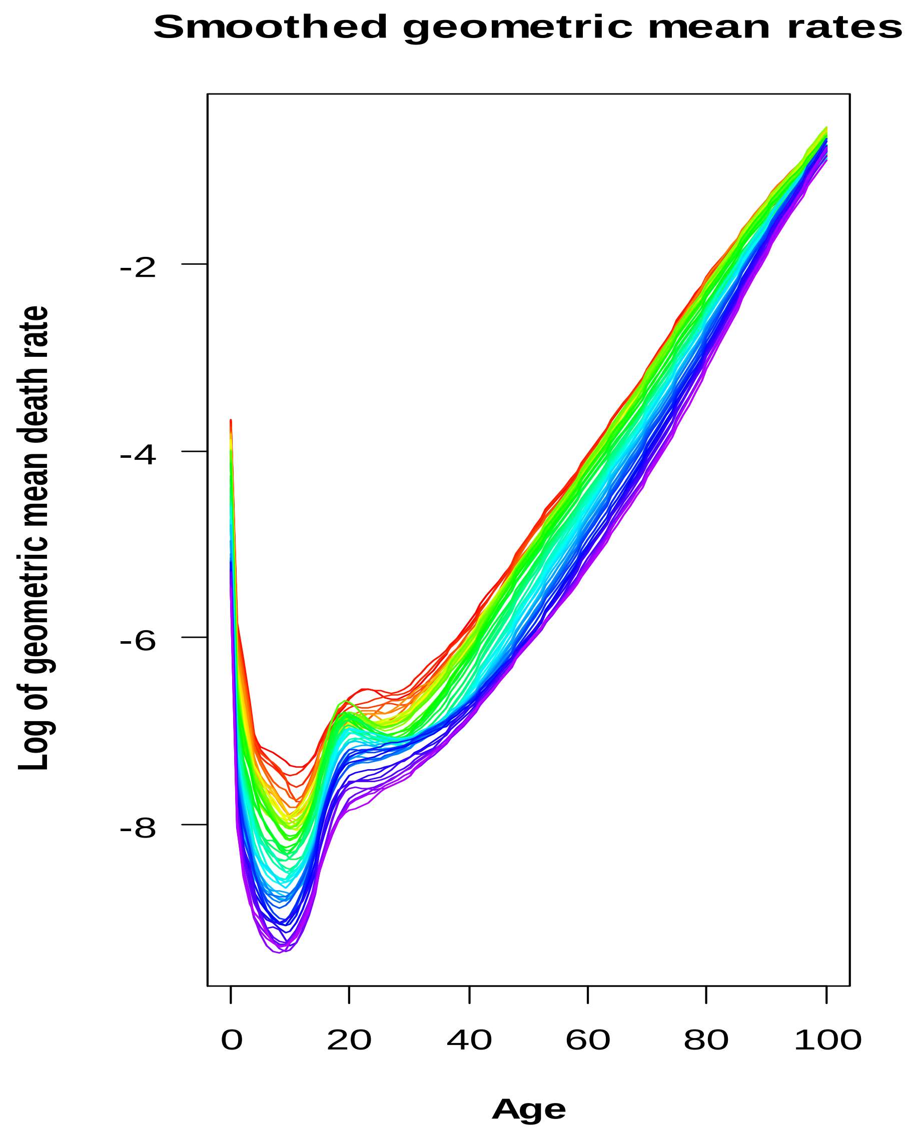
Product-Ratio (Coherent) Method

Hyndman, Booth and Yasmeen (2013) *Demography*

- Method makes use of the fact that the product and ratio of two variables are generally uncorrelated
  - hence the two forecasts ($\sqrt{}$**product** and $\sqrt{}$**ratio**) can be multiplied or divided without having to take covariance into account
- Both functions are forecast using the functional principal components method


Australia: female death rates (1950-2009
Log death rate
-2
-4
-6
-10
0
20
40
60
80
100
Age

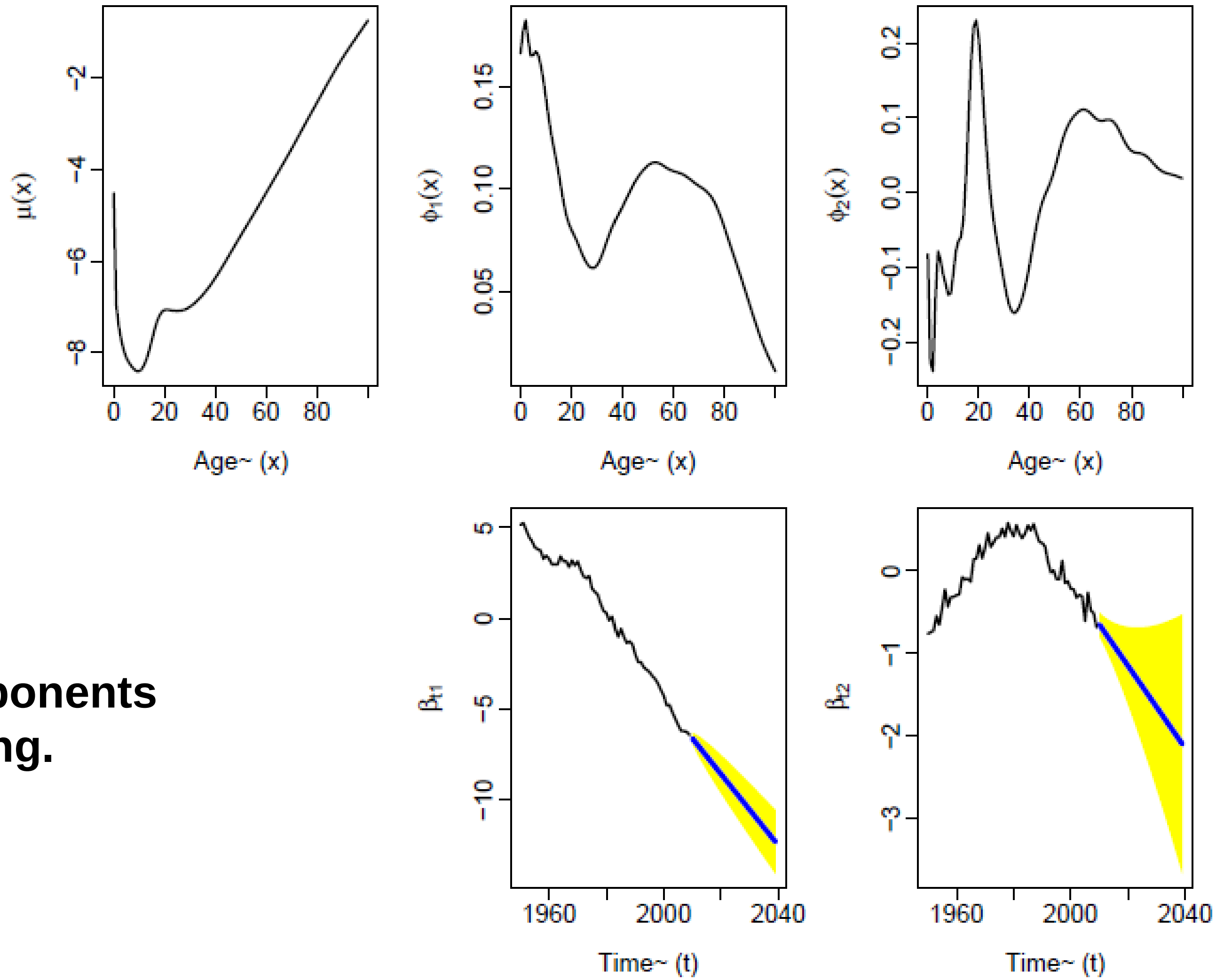

Australia: male death rates (1950-2009)
Log death rate
0
-2
-4
-6
-10
0
20
40
60
80
100
Age


Smoothed geometric mean rates
Log of geometric mean death rate
-2
-4
-6
-8
0
20
40
60
80
100
Age

Product function: components & forecast

Australia 1950-2009

Geometric mean mortality



Method:
Functional principal components with time series forecasting. Identical to that used in independent forecasting

Mean age pattern & first 2 components (explaining 97% & 1% of variance)

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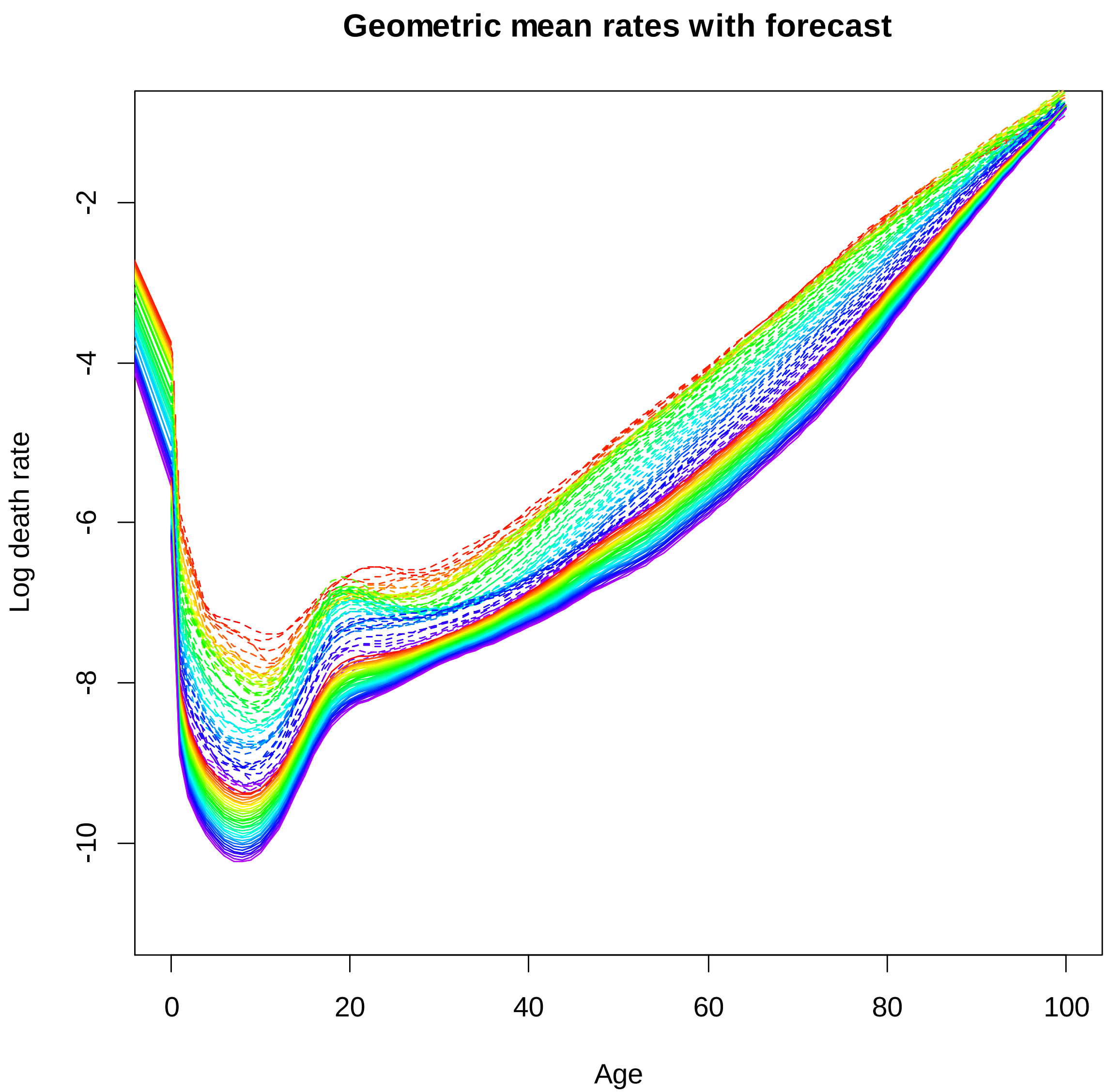
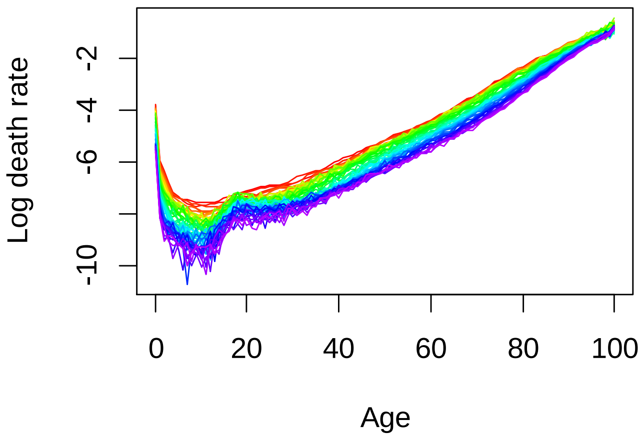
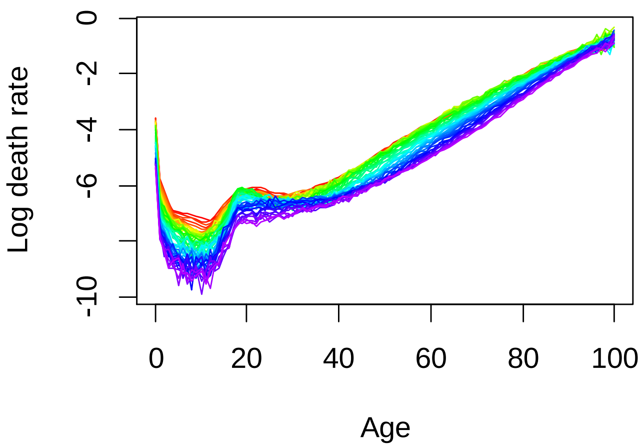


Dotted lines: Observation 1950-2009 Solid lines: Forecast 2010-2039

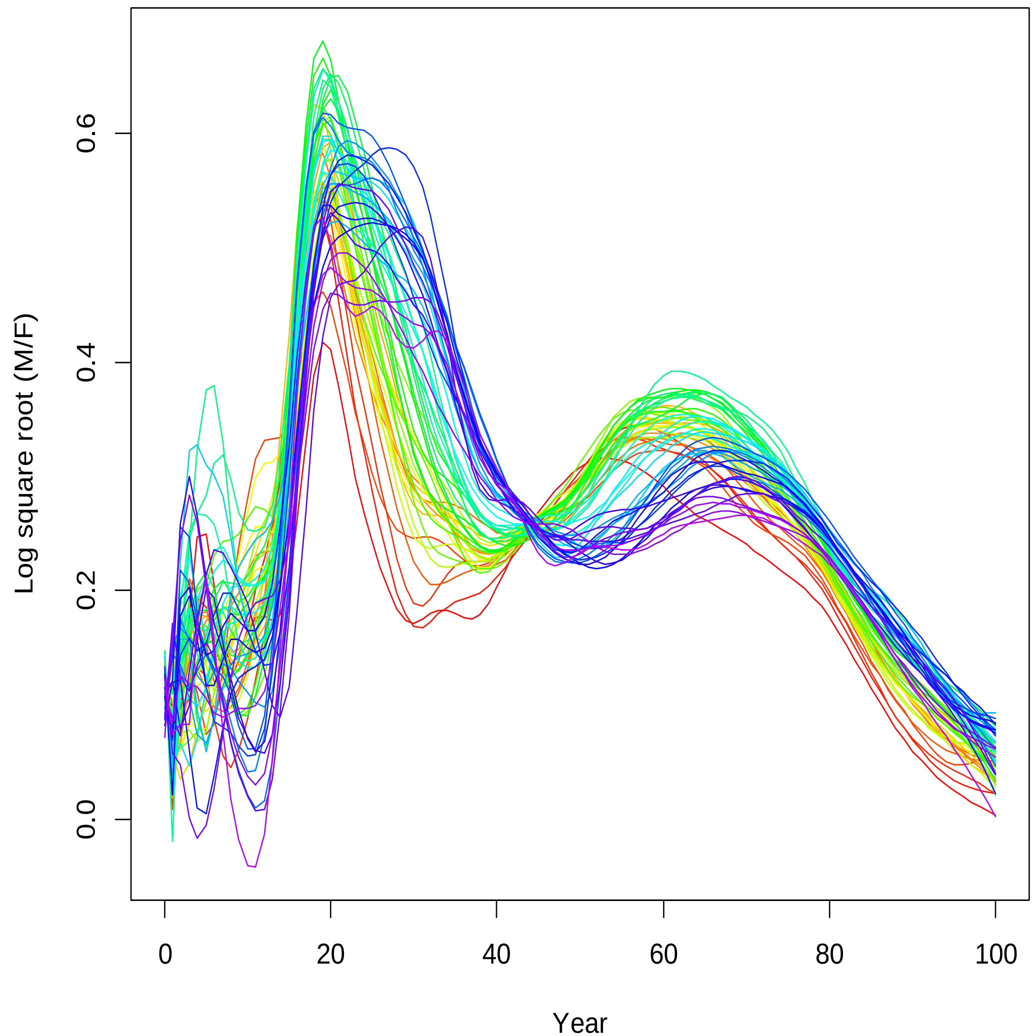
Australia: female death rates (1950-2009



Australia: male death rates (1950-2009)



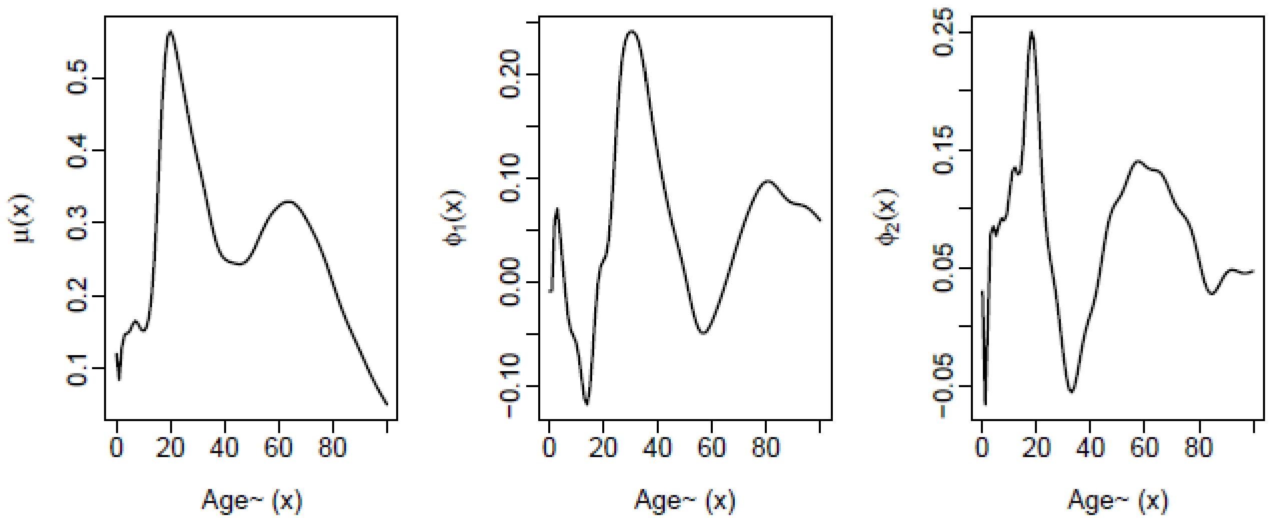
Log square root of mortality sex ratio, 1950-2009



Ratio function: components & forecast

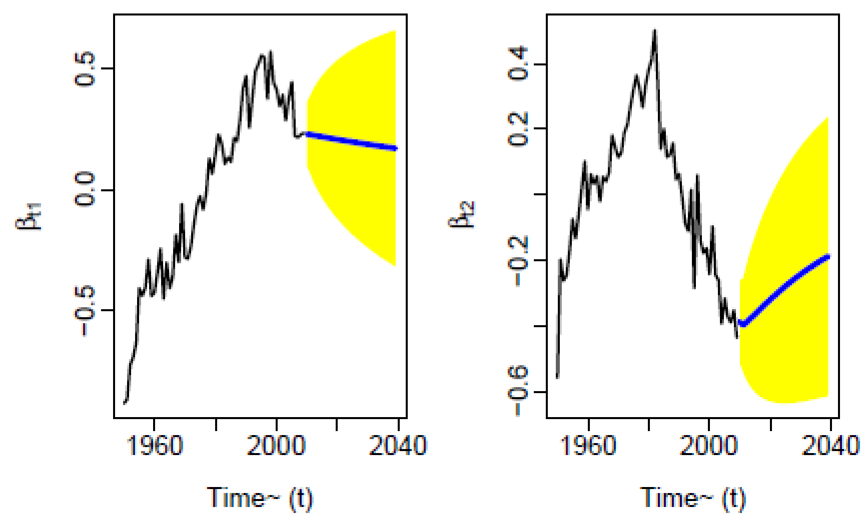
Australia 1950-2009

Log square root of mortality sex ratio)

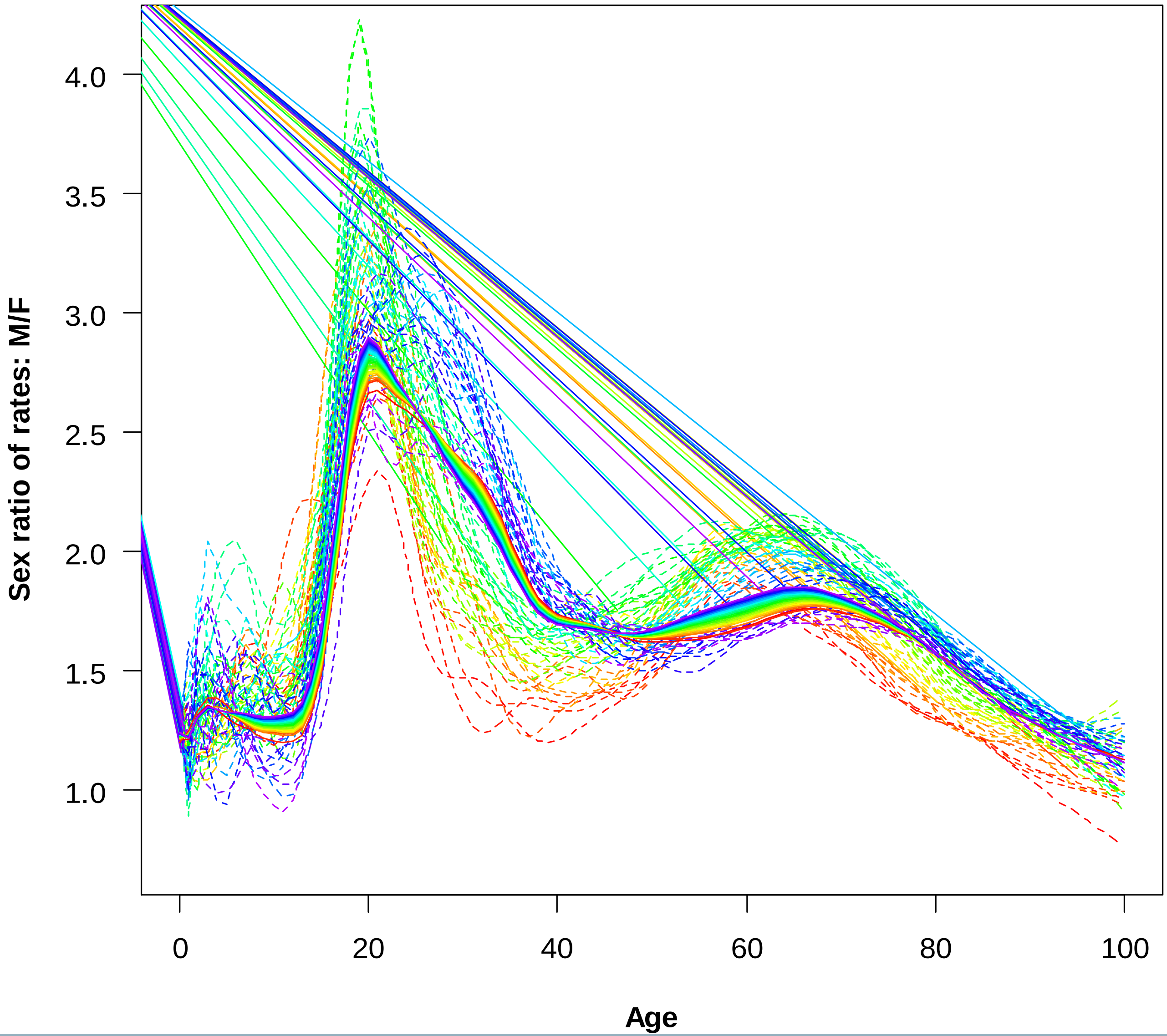


Method:
Functional forecasting method (as used in independent forecasting) EXCEPT that time effects are forecast with the constraint that each eventually reaches zero (stationarity).

This is equivalent to convergence to the historic mean. Convergence is slow due to chosen ARFIMA time series model.

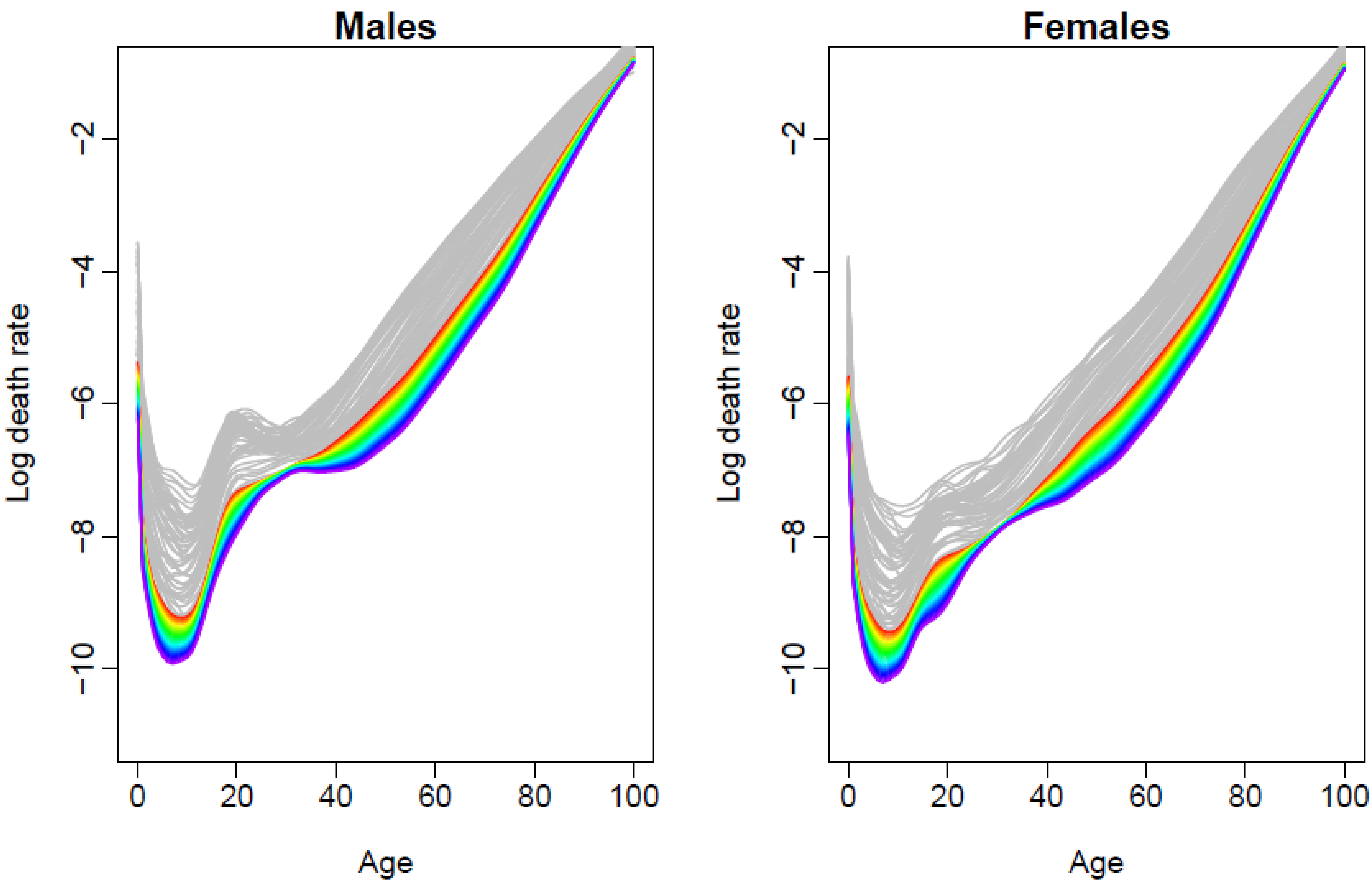


Mortality sex ratio with forecast



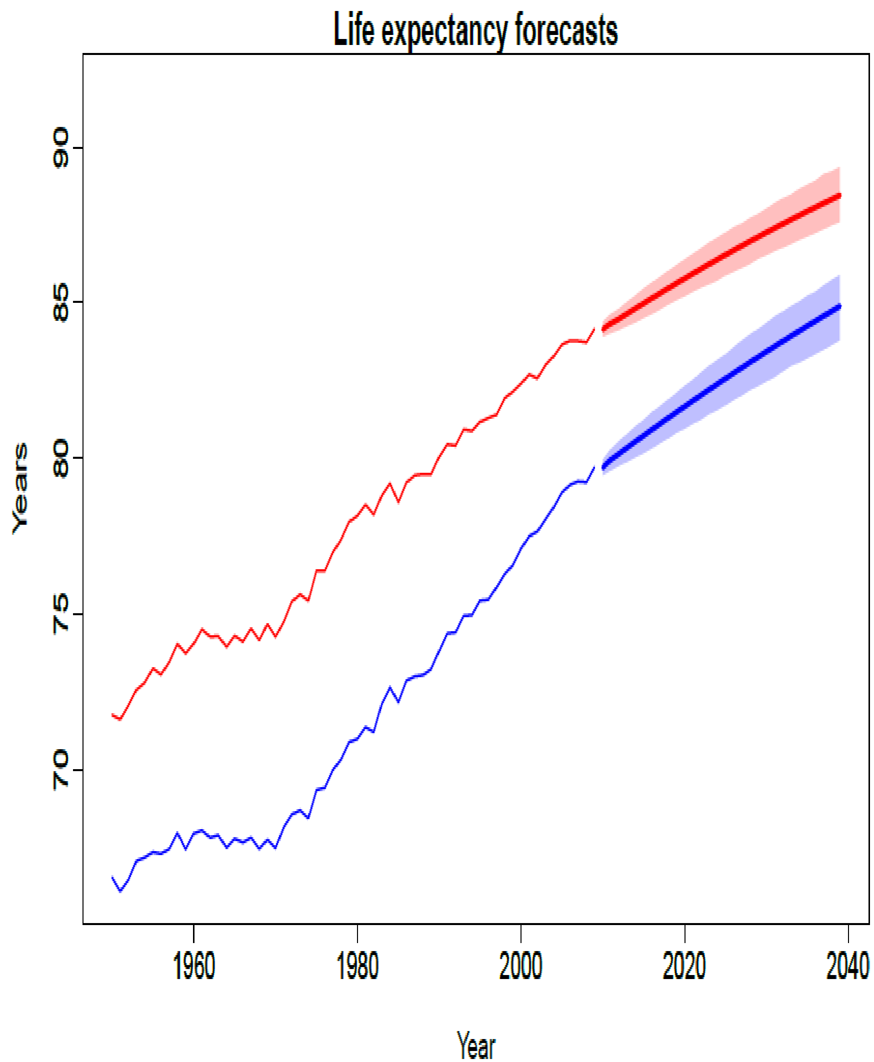
Dotted lines: Observation 1950-2009 Solid lines: Forecast 2010-2039

Forecast rates, 2010-2039



Data: Australia 1950-2009

Coherent forecasts: 2010-2039



Data: Australia 1950-2009

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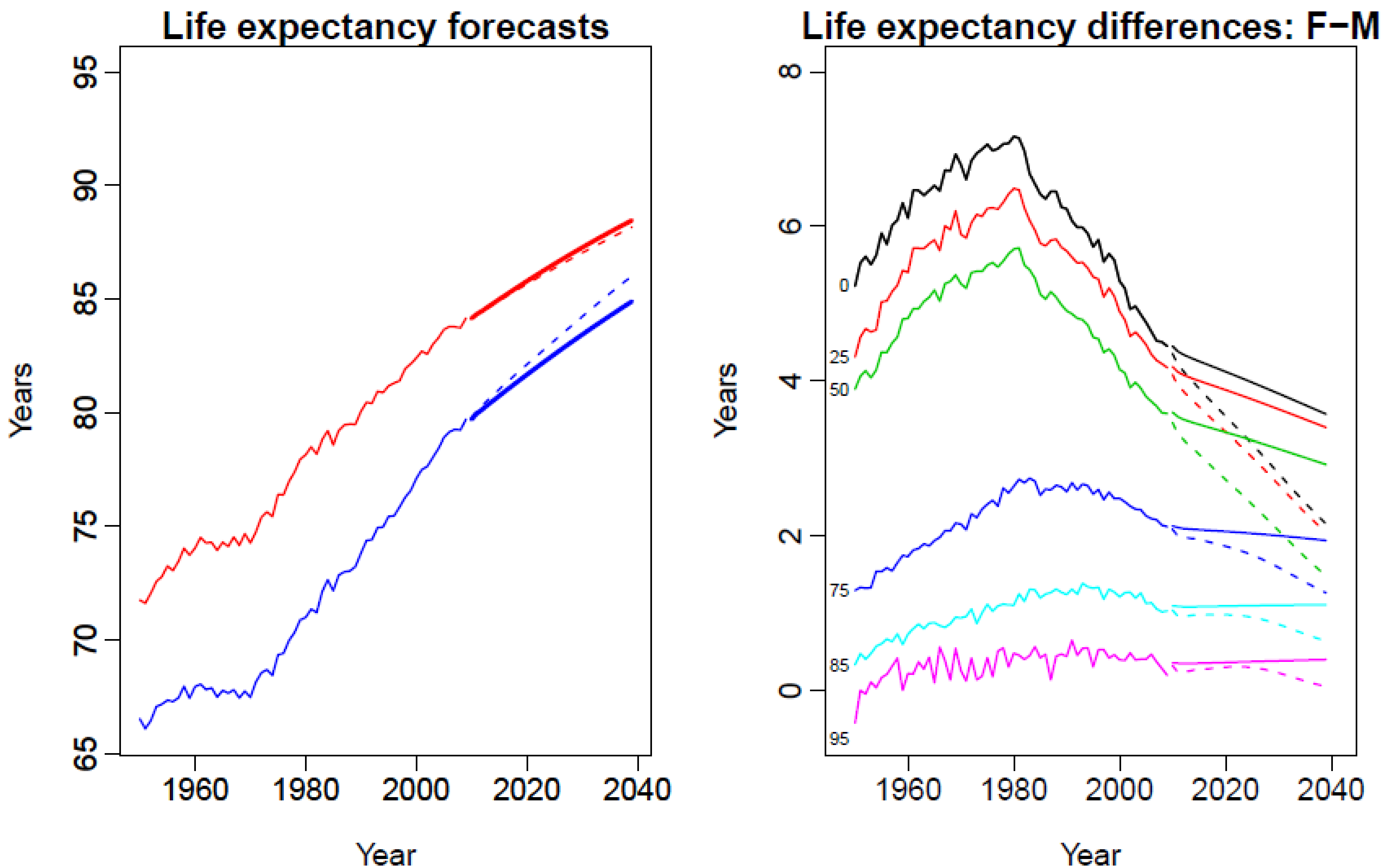
Comparing coherent with independent forecasts:

ADVANTAGES OF COHERENT FORECASTING

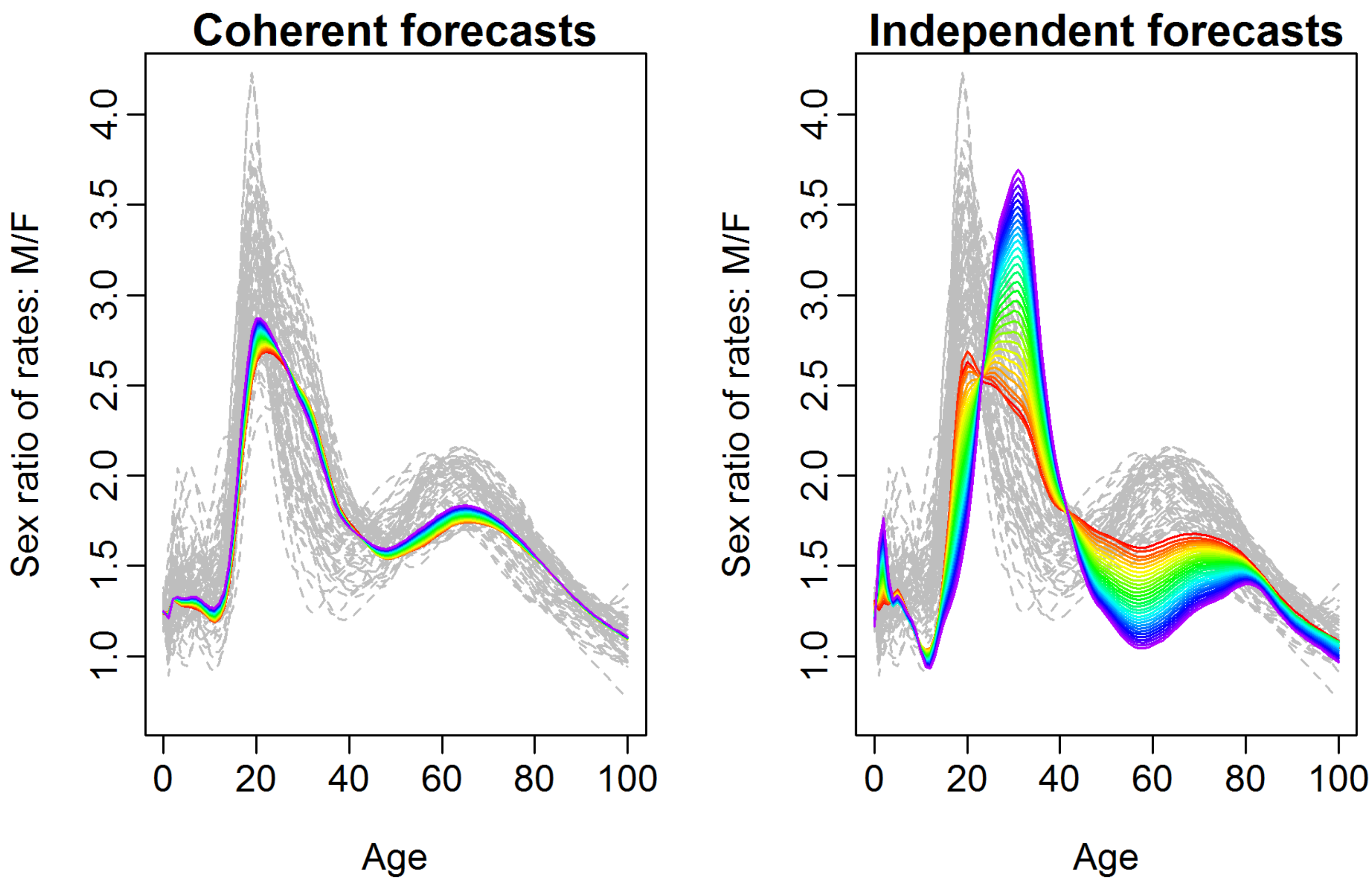
Advantage: Coherence



Data: Australia 1950-2009

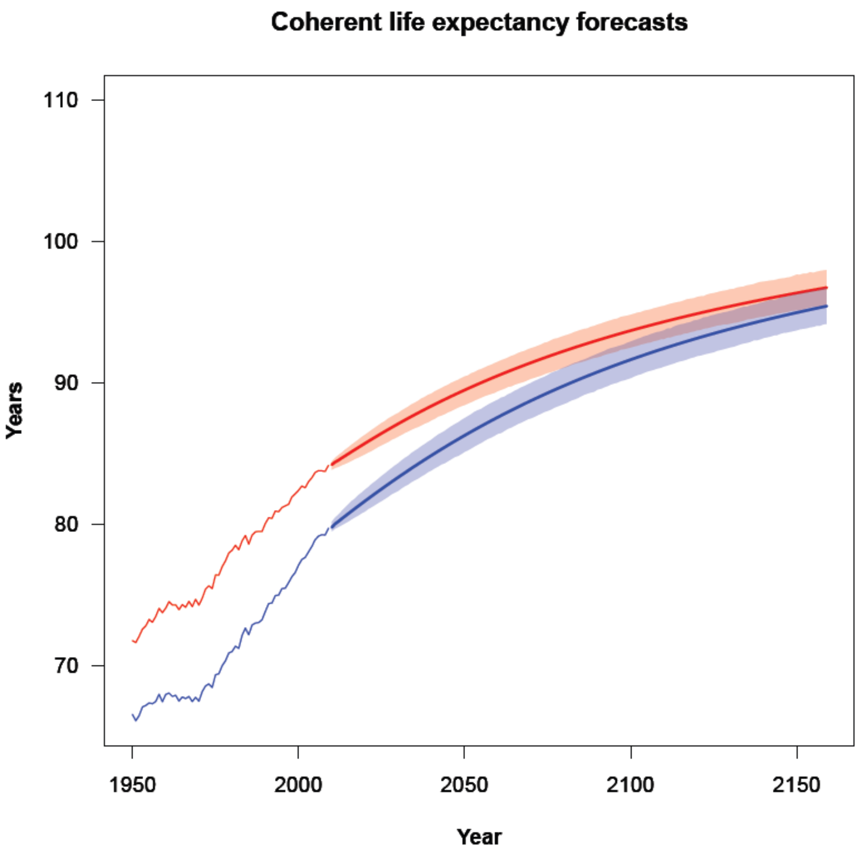
Advantage: Coherence

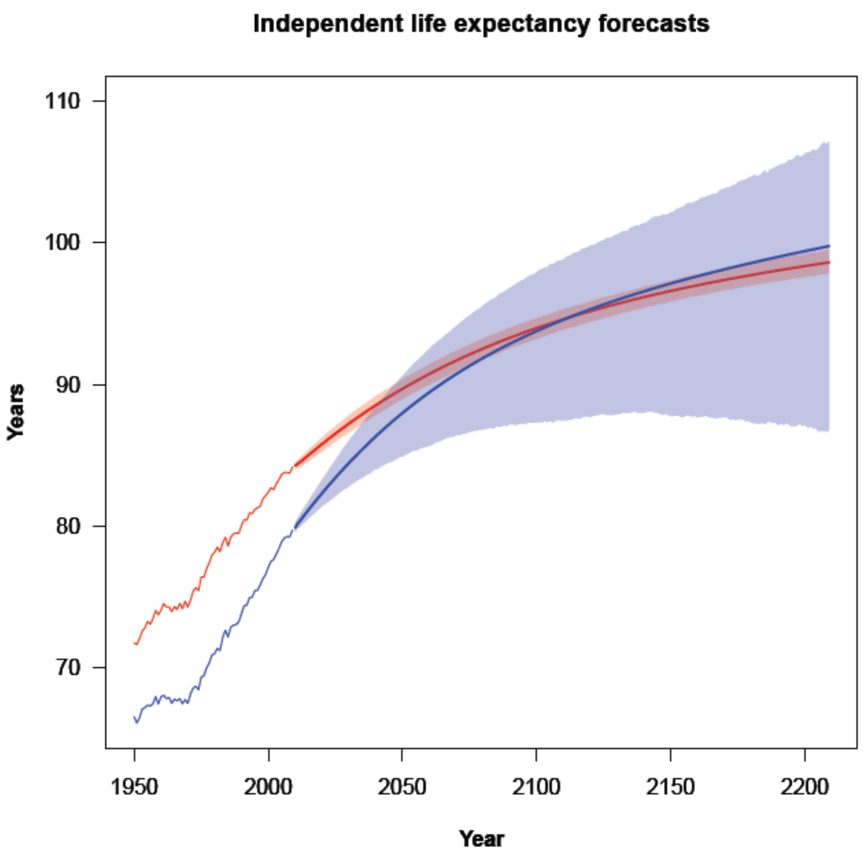
Sex ratios are more stable and exceed 1



Data: Australia 1950-2009

Advantage: Uncertainty is reduced & uncertainty of uncertainty is reduced





More research is needed to examine this effect in different situations

Advantage: Forecast accuracy is improved

Reduced error AND reduced error variability

Accuracy mean

Mean Square Forecast Error:

Coherent = 0.355

Independent = 0.367

Accuracy variation

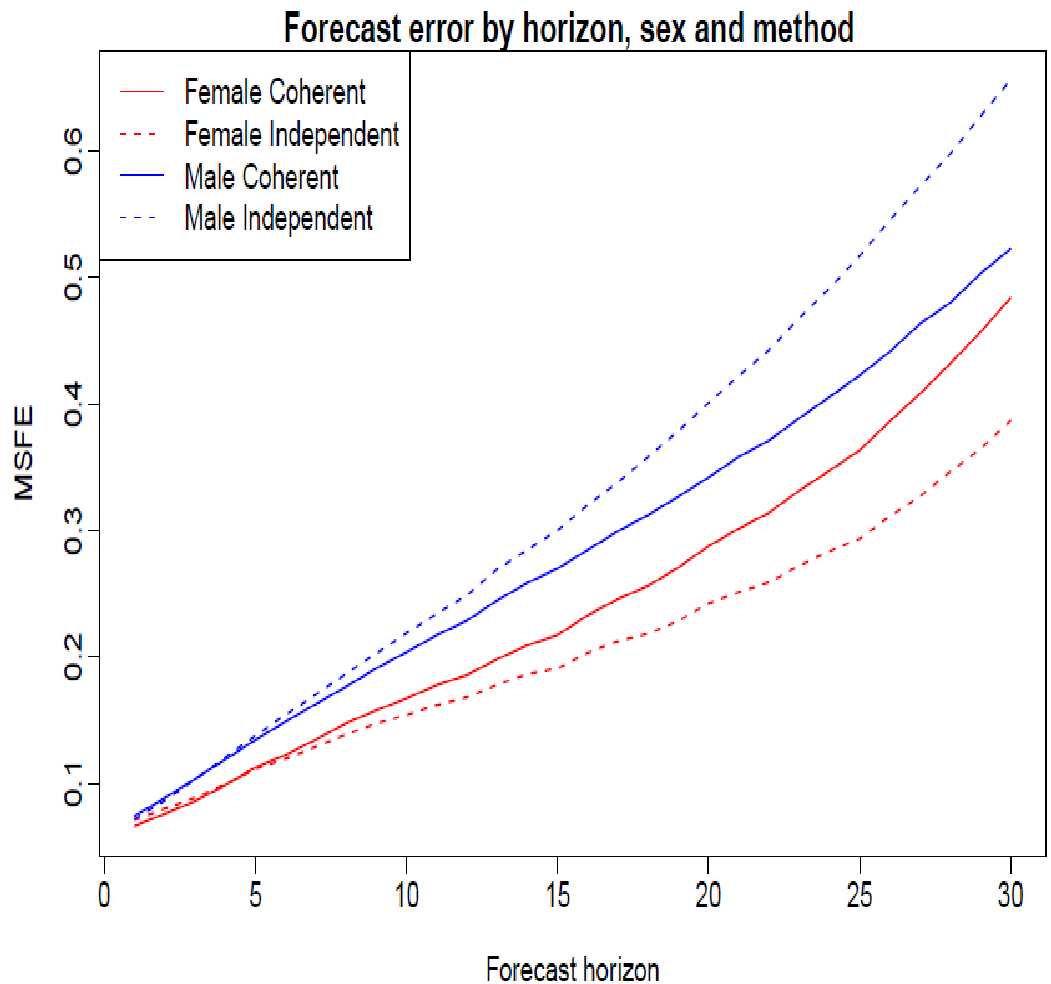
MSFE is 'homogenised'

Coherent diffs = 0.038

Independent diffs = 0.154

Greater confidence in forecasts as a group

Reduced uncertainty in group differences



Based on at least 20 years Australian data starting in 1950 and averaged

Advantage: Forecast accuracy is improved

Reduced error AND reduced error variability

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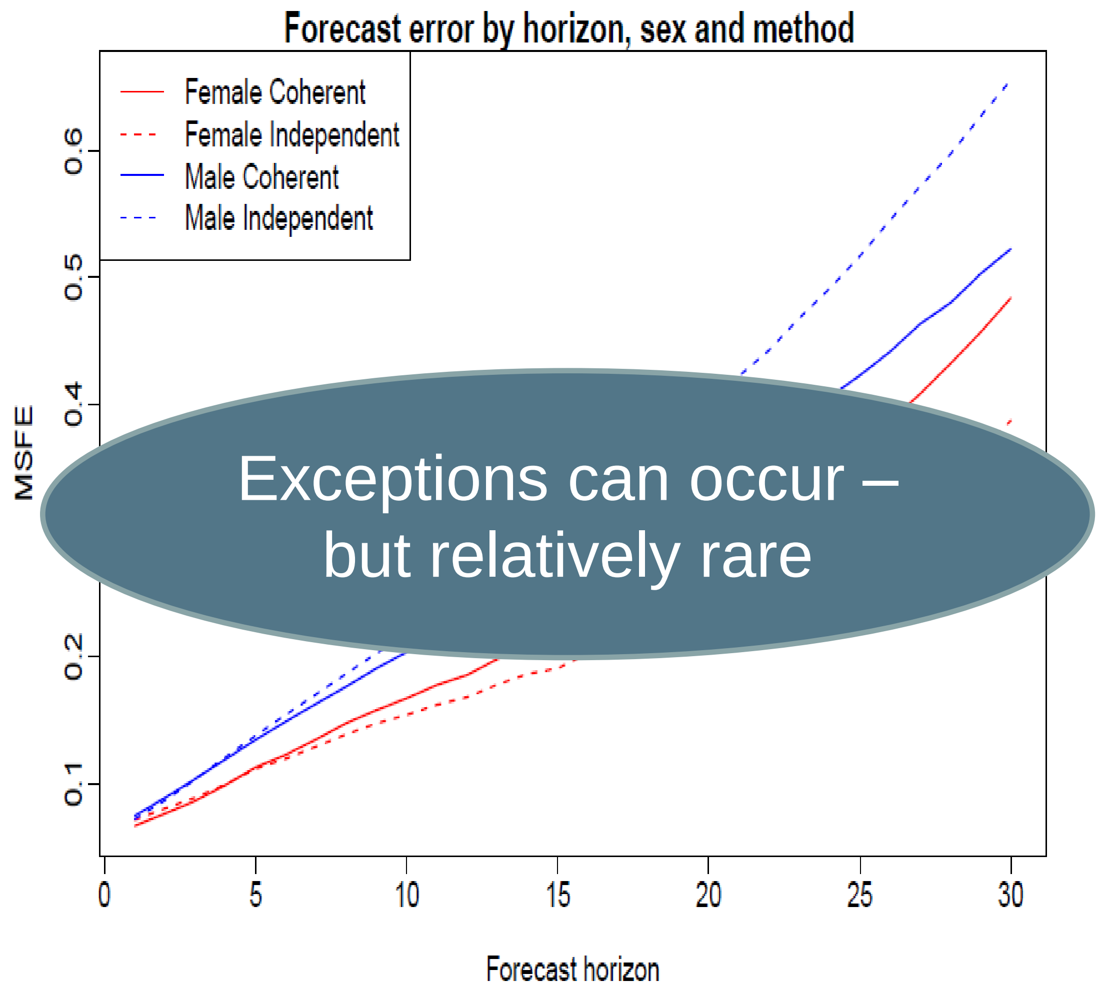


Exceptions can occur – but relatively rare

Based on at least 20 years Australian data starting in 1950 and averaged

Advantage: Forecast bias is improved

Reduced bias AND reduced bias variability

Bias mean

Mean Error:

Coherent = -0.262

Independent = -0.268

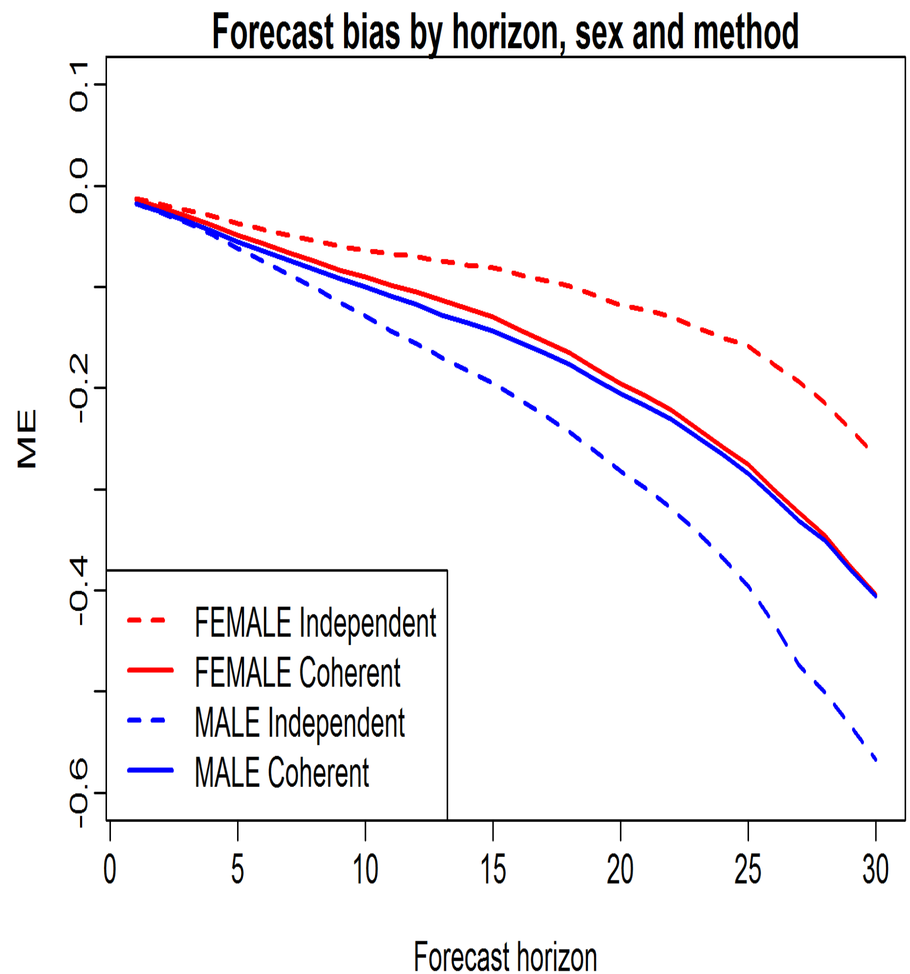
Bias variation

ME is ‘homogenised’

Coherent diffs = 0.006
Independent diffs = 0.168

Greater confidence in forecasts as a group

Reduced uncertainty in group differences



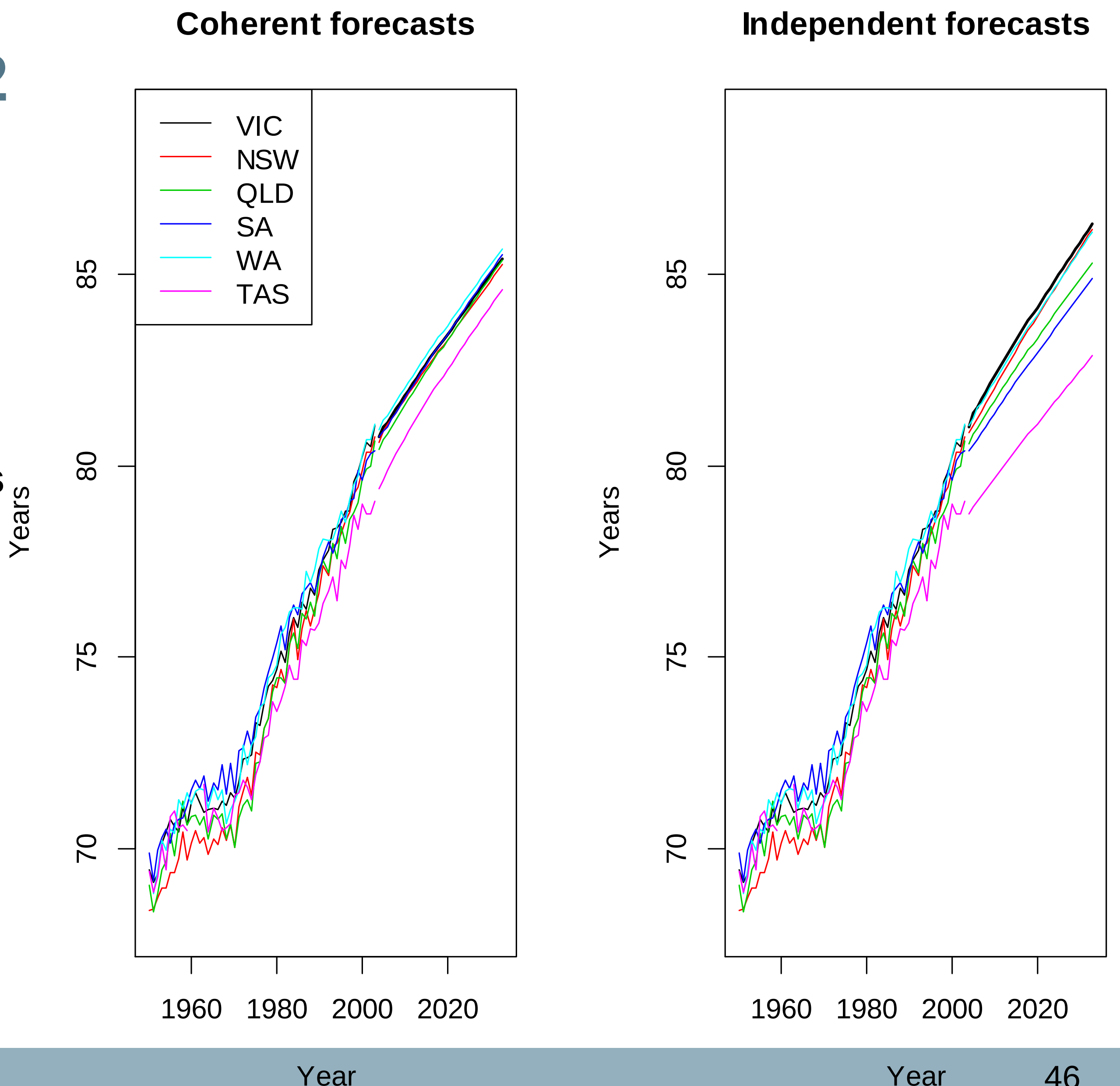
Extension to >2 populations: state-coherent forecasting

Application to six Australian states shows similar advantages

Divergence is prevented

Accuracy is improved: Mean and variation

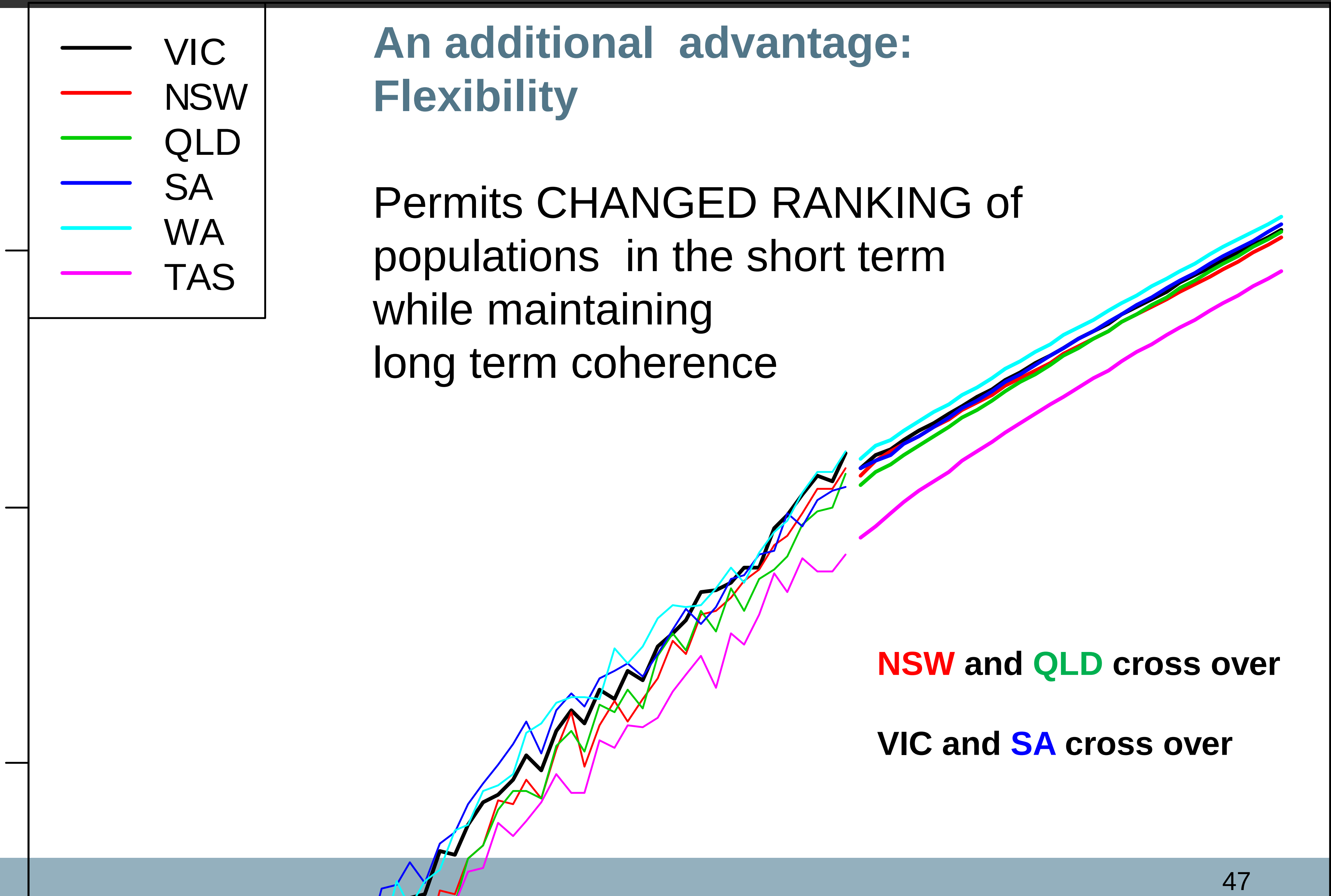
Bias is reduced: Mean and variation



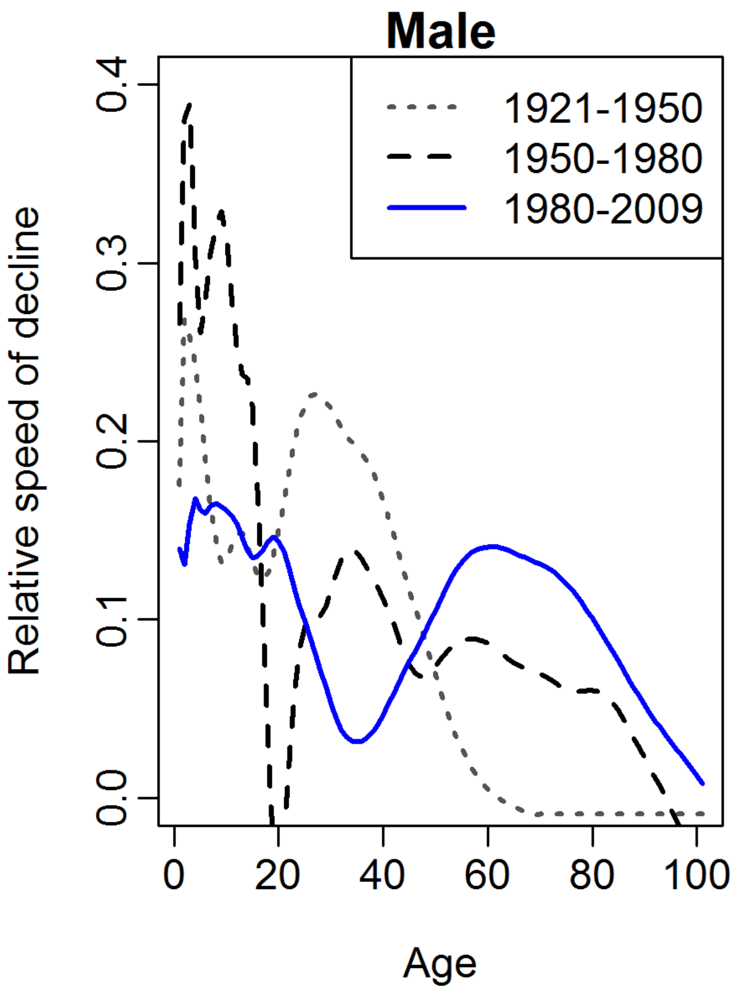
Coherent forecasts

An additional advantage: Flexibility

Permits CHANGED RANKING of populations in the short term while maintaining long term coherence



Address fundamental problem of PC model



- Principal components model assumes a **fixed age effect**
- But the age effect actually changes by fitting period
- Multiple PC approach allows some flexibility
- Ratio function adds further flexibility
- Correlations between ages further reduced
- Partial solution

Data: Australia 1950-2009

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Can we do better?

IMPROVING THE FORECAST

Questions

- Is sex-coherent forecasting or state-coherent forecasting more accurate for sex-state mortality? What can we learn from this?

- How can forecasting methods be further improved? How can we better use other information to improve forecasting?

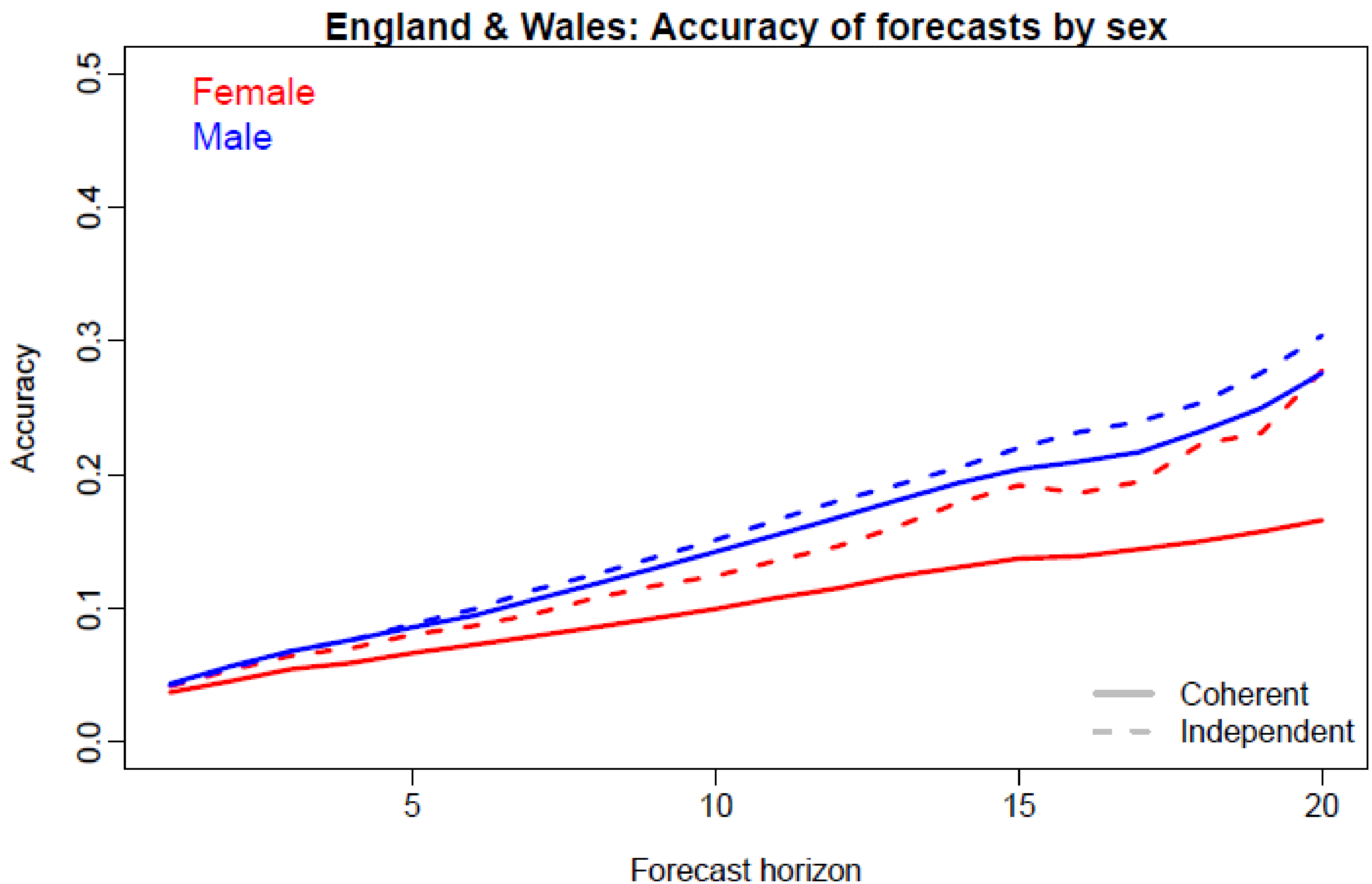
A Case Study: UK

- Human Mortality Database
- 1950-2009
- Male and female populations
- Constituent populations
  - England & Wales
  - Scotland
  - Northern Ireland

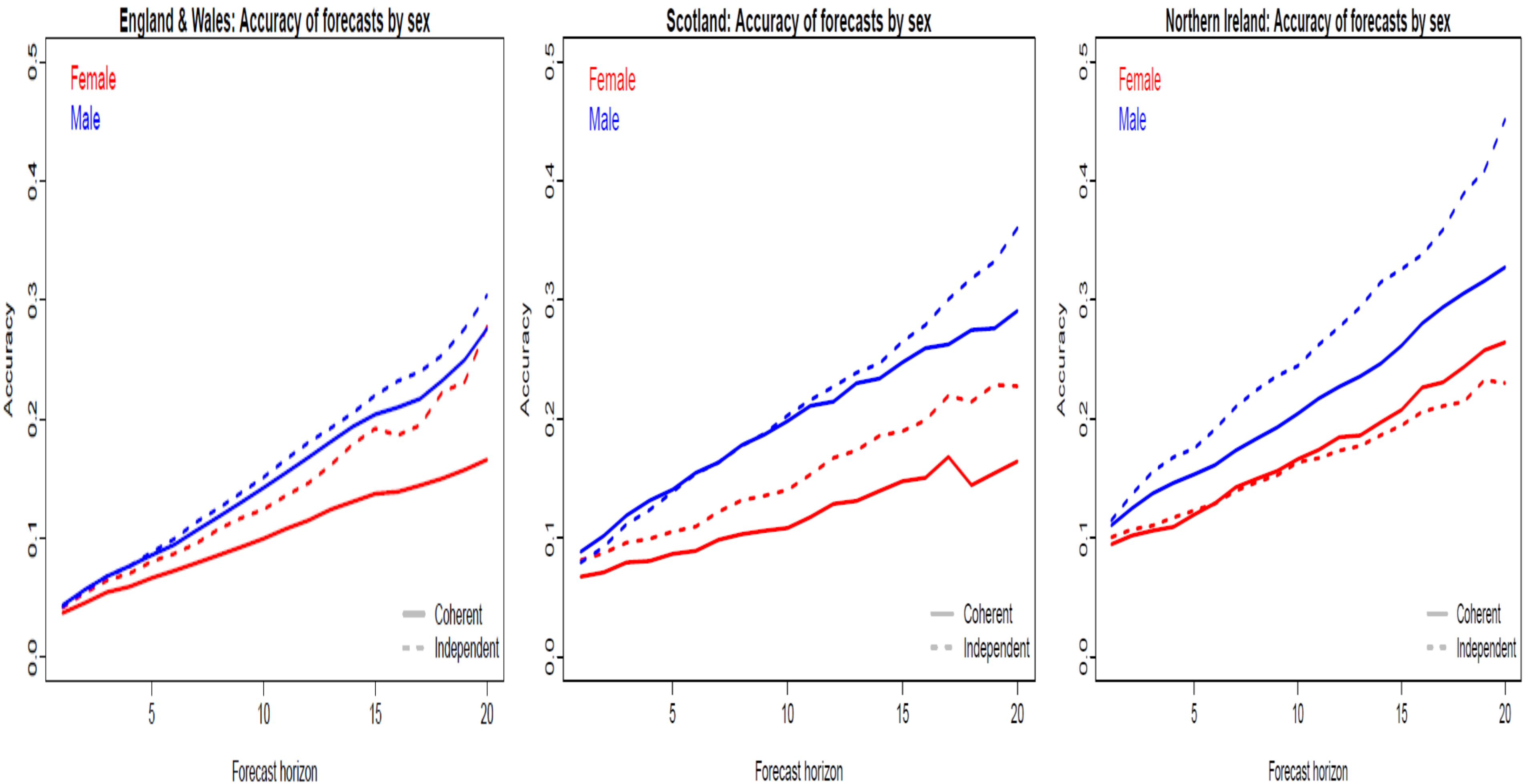
Establishing the first improvement

COHERENT VS INDEPENDENT

Accuracy of sex-coherent forecasts, E&W

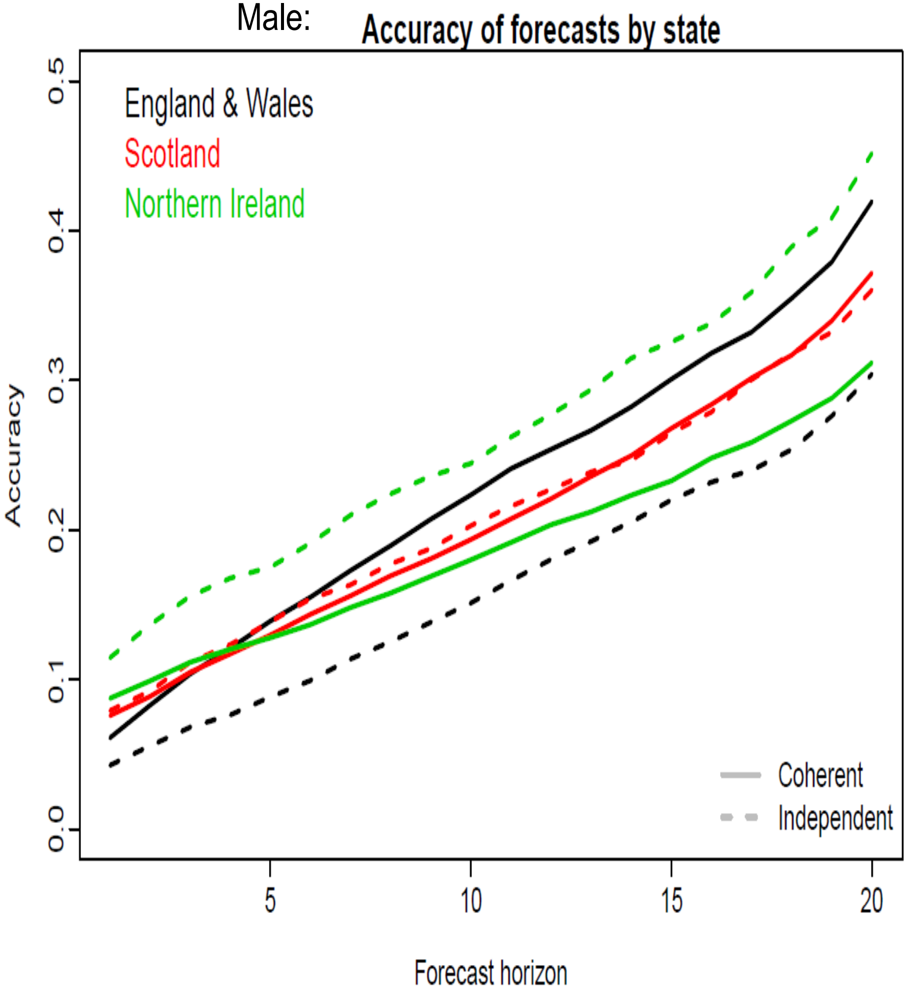


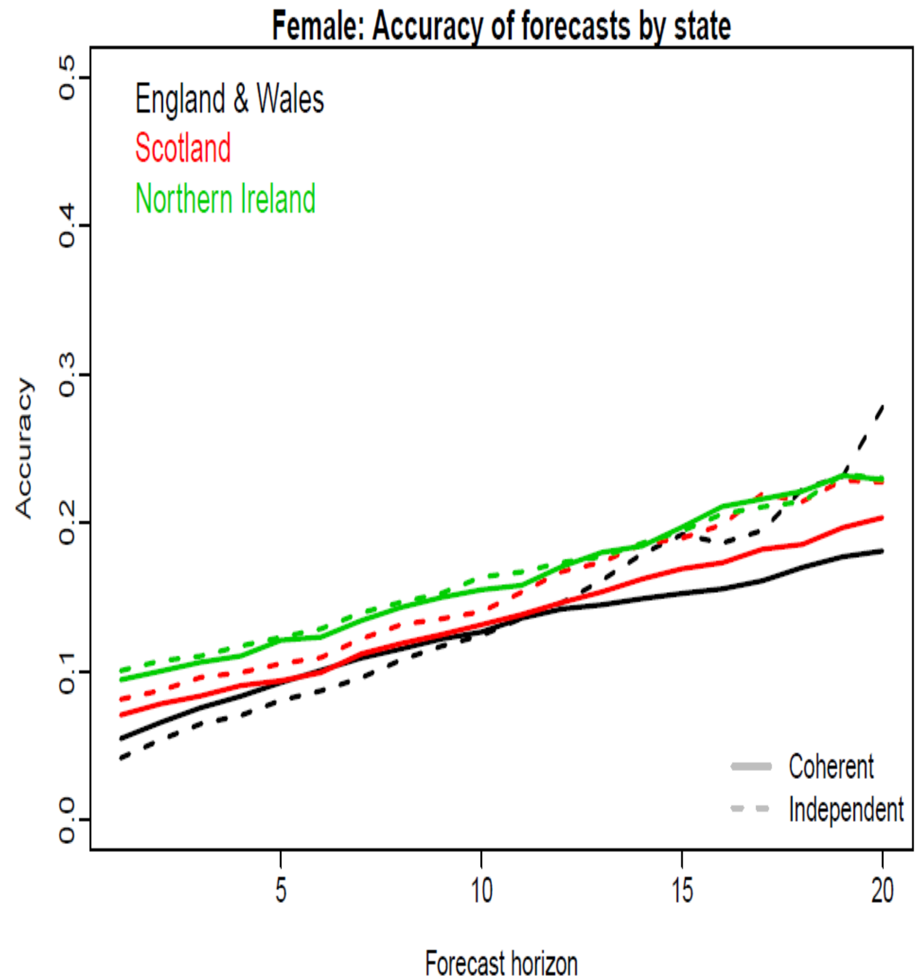
Accuracy of sex-coherent forecasts, three UK populations



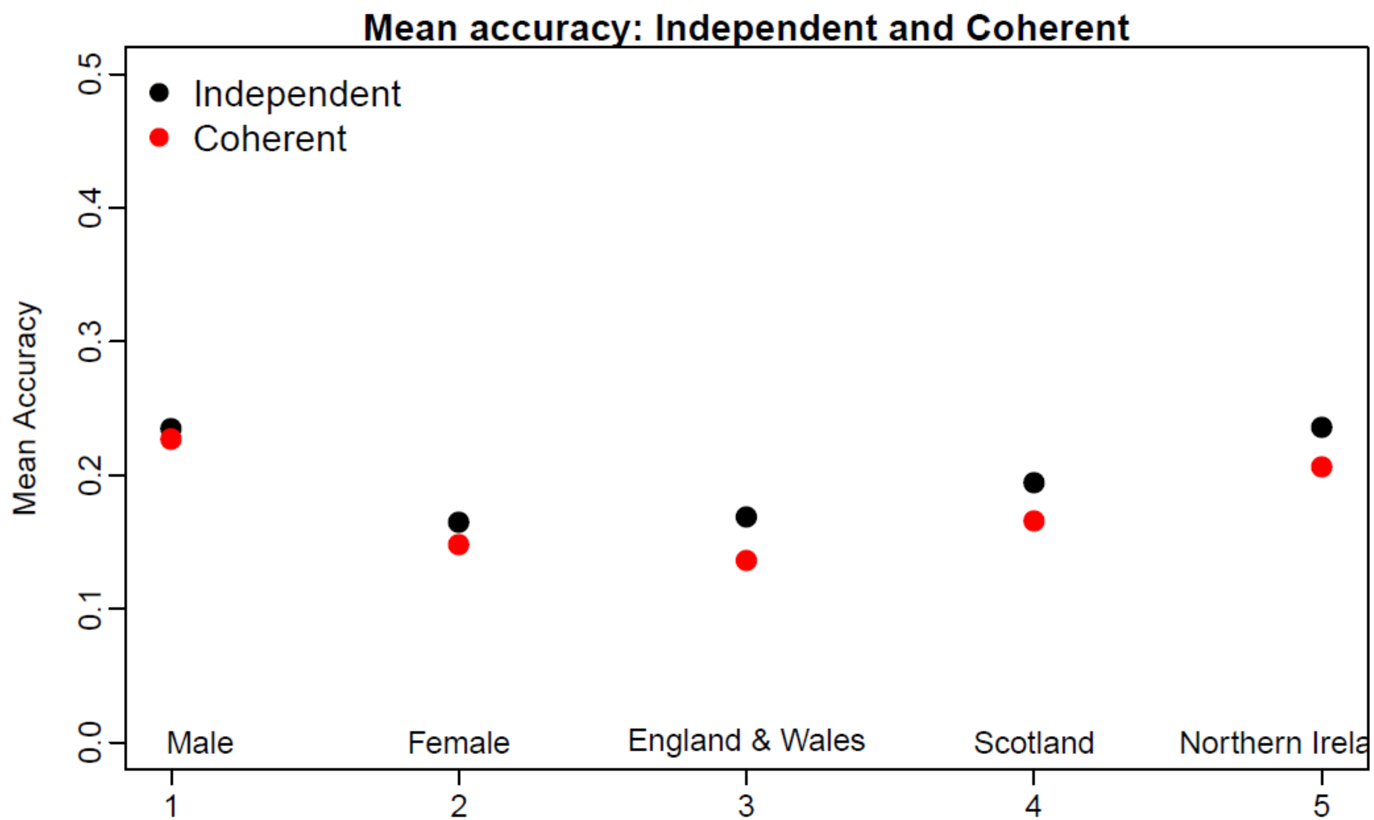
Accuracy of state-coherent forecasts, UK males and females

Male:





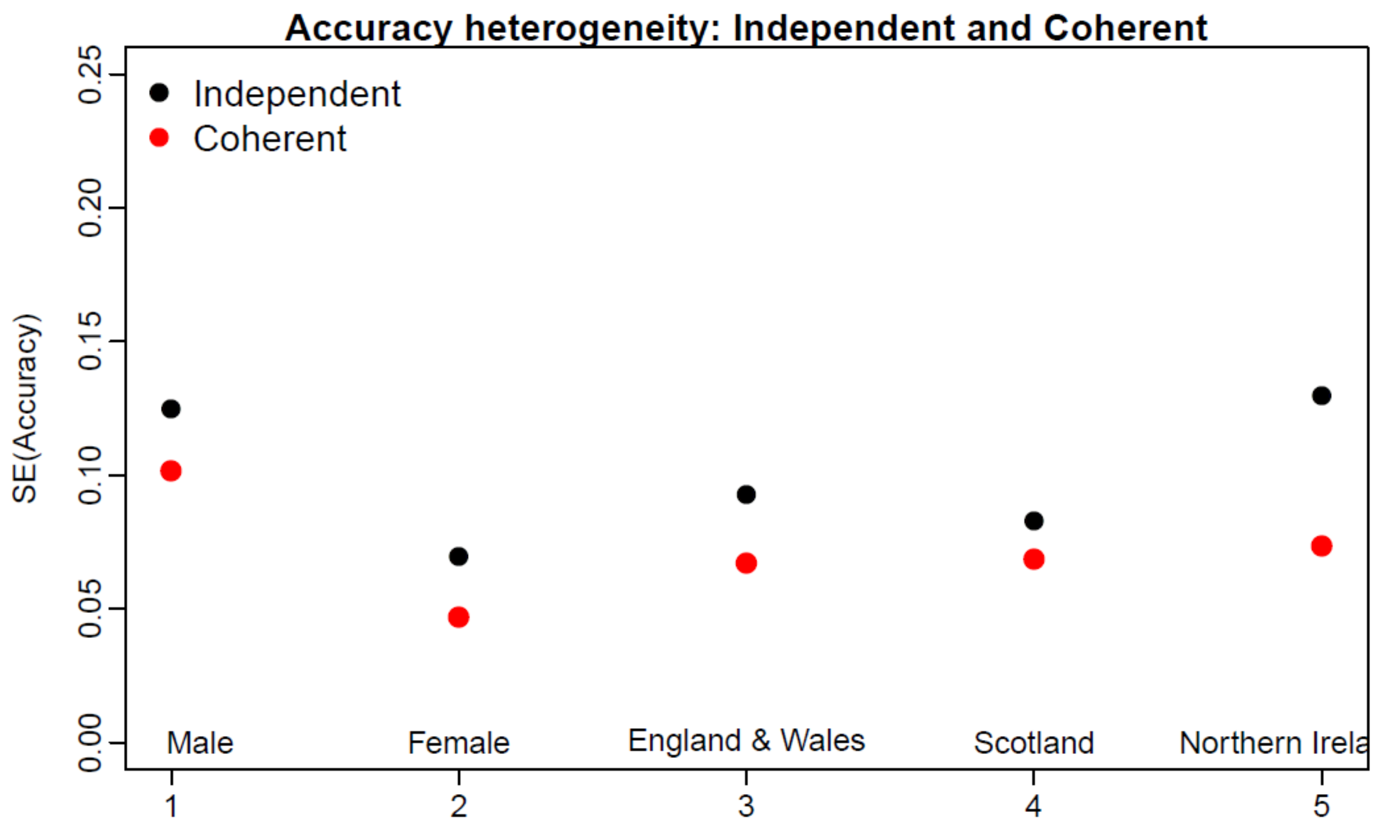
Coherent more accurate than Independent



State-coherent: averaged over states

Sex-coherent: averaged over sexes

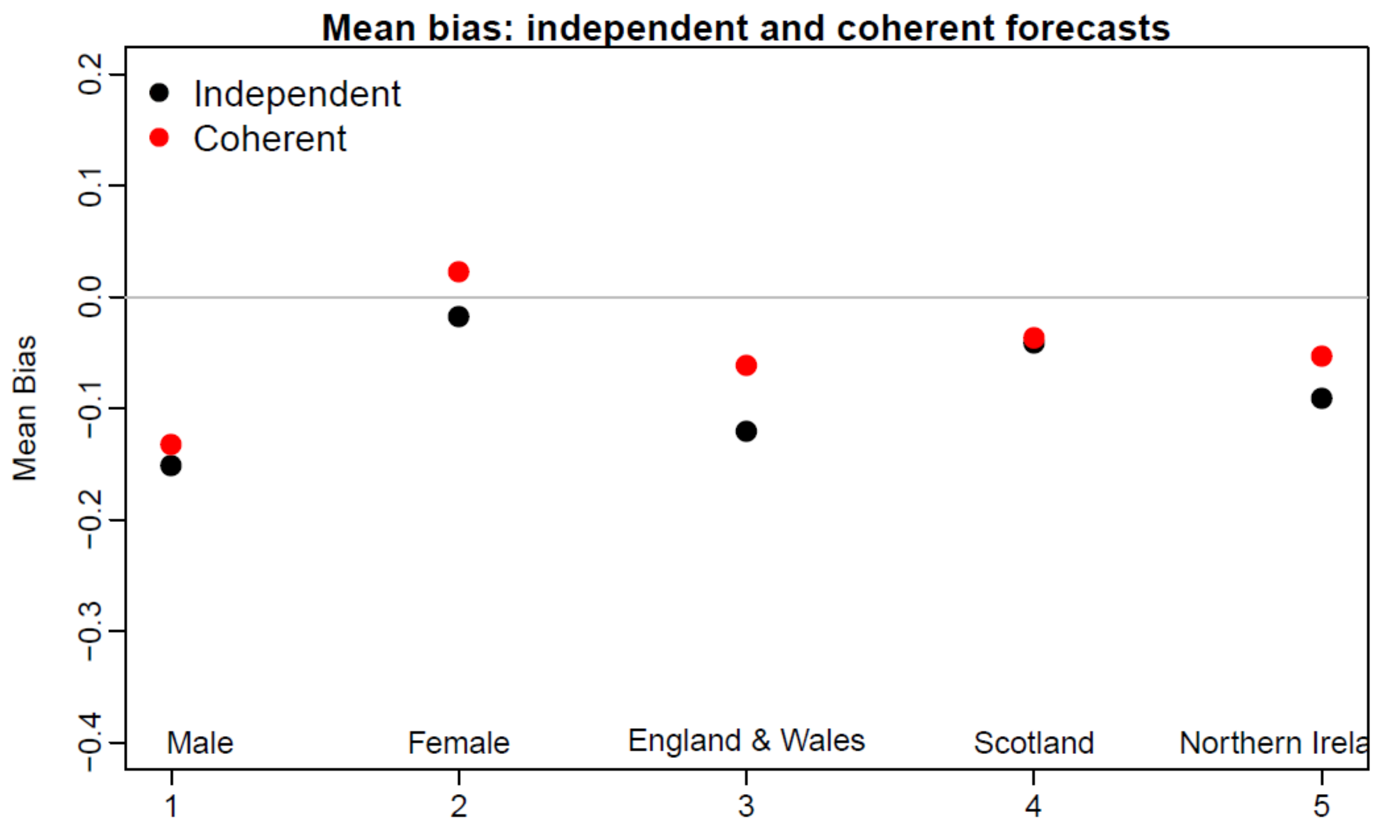
Coherent accuracy less heterogeneous



State-coherent: averaged over states

Sex-coherent: averaged over sexes

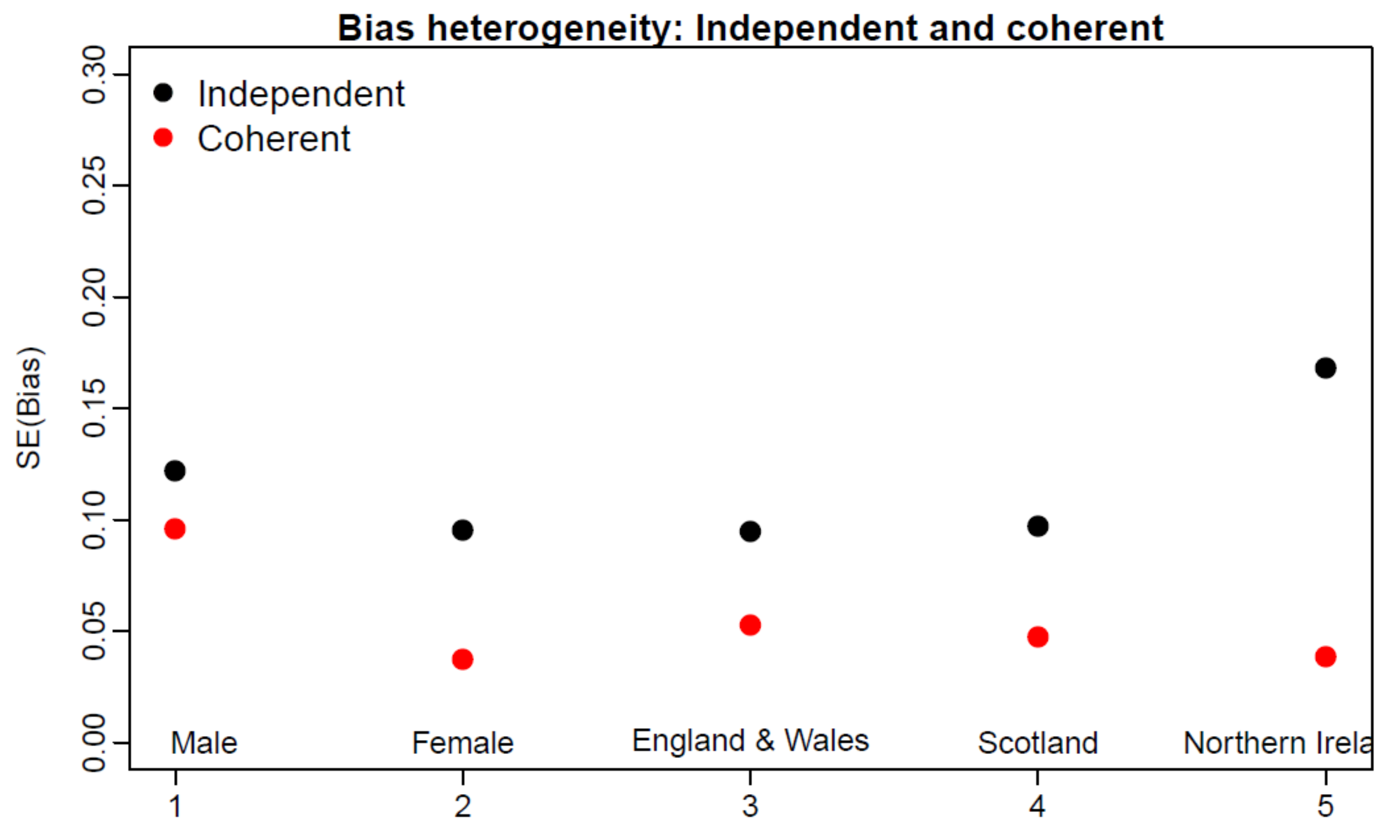
Coherent less biased than Independent



State-coherent: averaged over states

Sex-coherent: averaged over sexes

Coherent bias less heterogeneous



State-coherent: averaged over states

Sex-coherent: averaged over sexes

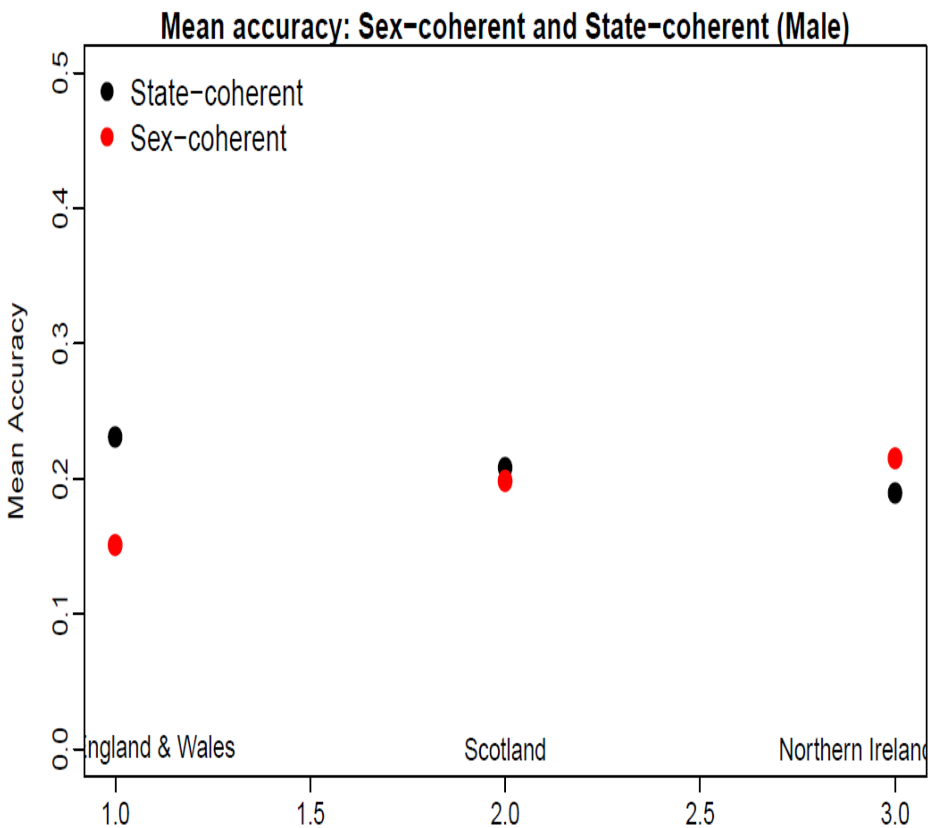
Which is more accurate?

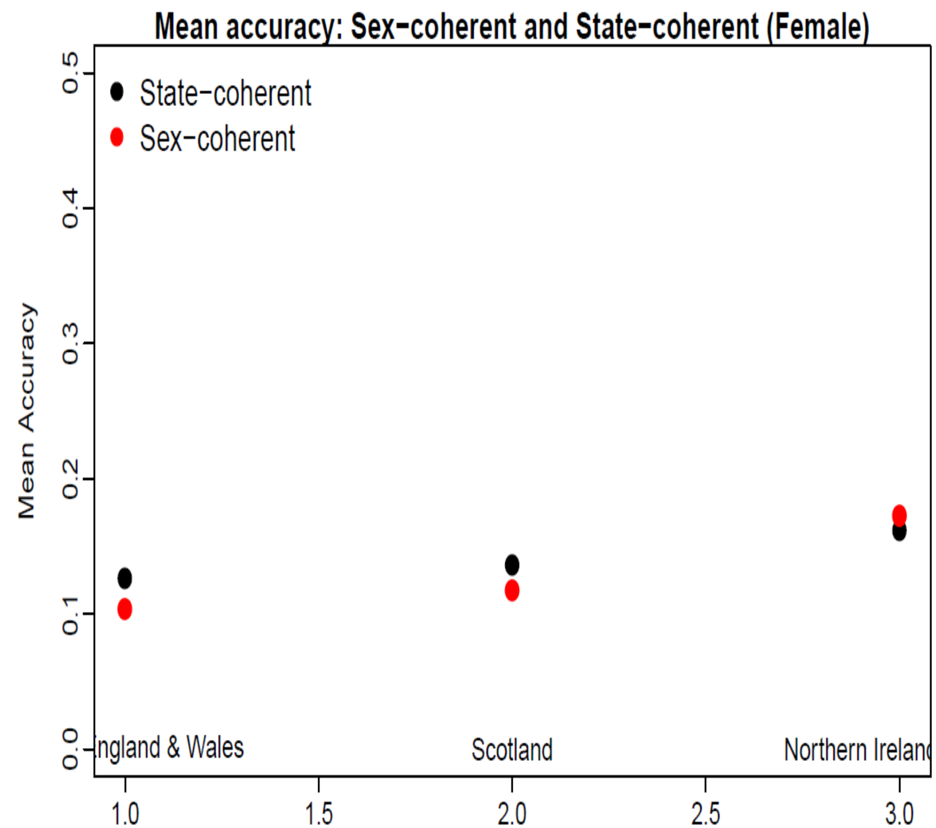
SEX OR STATE COHERENCE?

Comparison of two coherent forecasts for each sex-state population (six)

- Sex-coherent forecast:
  - Male and female forecasts each taking other sex into account [per state: 3 states x 2 sexes = 6]
- State-coherent forecast:
  - Several individual states, each taking others into account [per sex: 2 sexes x 3 states = 6]
- Example: Female population of E&W:
  - Sex-coherent forecast taking males in E&W into account
  - State-coherent forecast taking females in other states into account

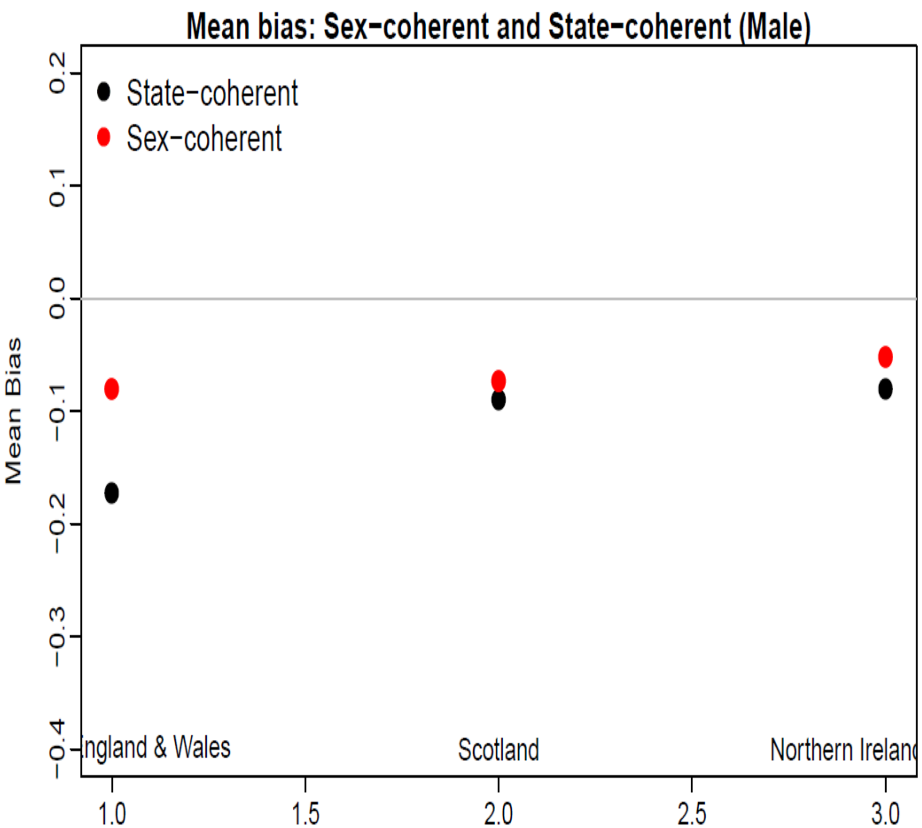
ACCURACY: Comparison of sex-coherent and state-coherent forecasts for each sex-state

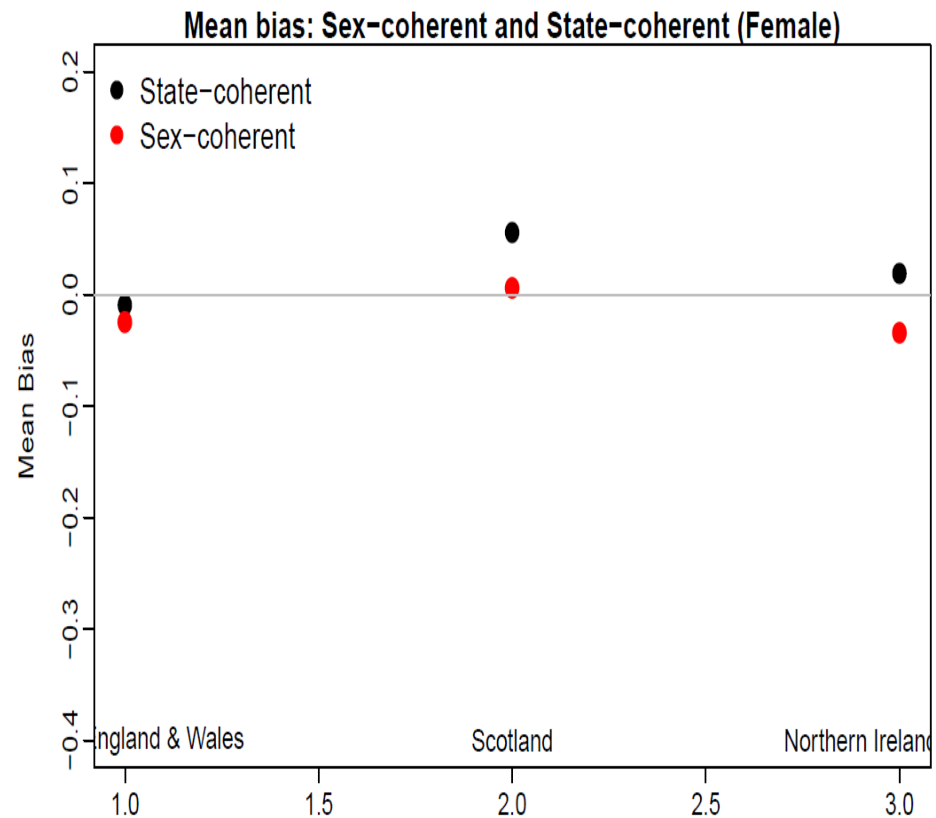




Sex-coherent more accurate overall

BIAS: Comparison of sex-coherent and state-coherent forecasts for each sex-state





Sex-coherence less biased overall

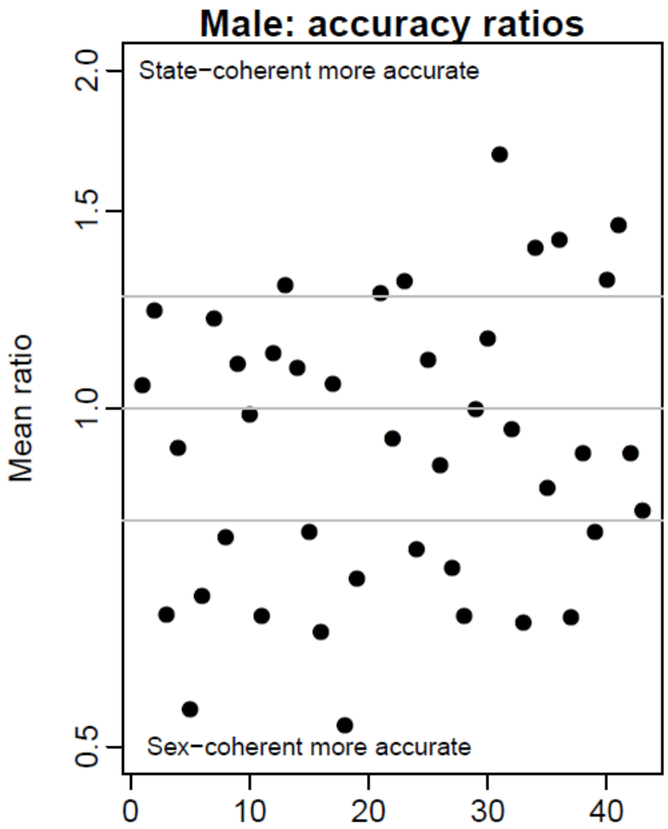
What (if anything) can we deduce?

- Sex-coherent forecasts appear to be more accurate and less biased….
- ….at least for the UK

BUT clearly we need more evidence…..

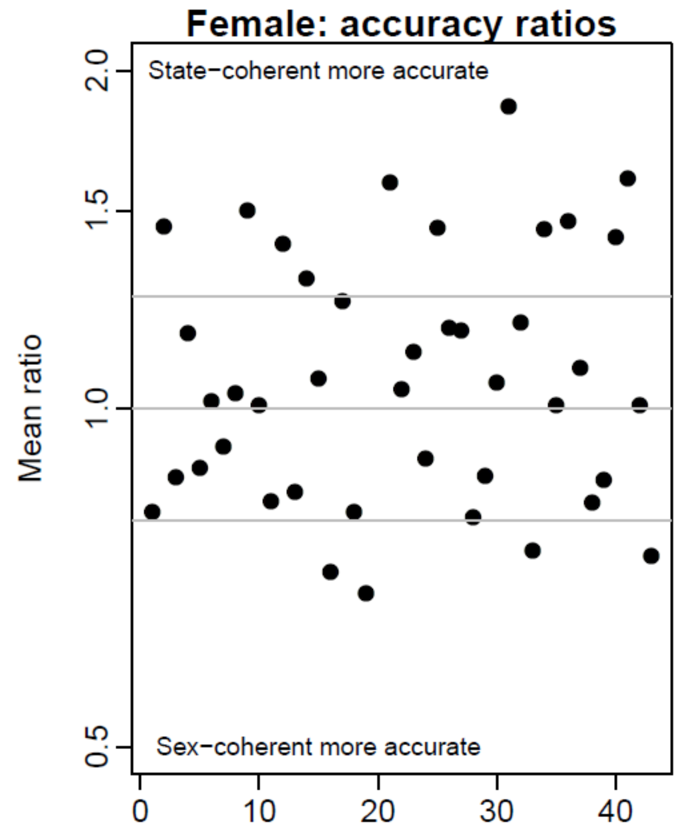
Accuracy: Sex-coherent vs State-coherent

No.
8
9
10
15



Sex:State = 25:17

Sex-coherent



No.
11
14
13
4

Sex:State = 17:25

State-coherent

State-coherent based on regions of Europe: sub-regions; English-speaking

Is this a good result……?

Male – better accuracy:

- Sex-coherent
- Taking female mortality into account

Female – better accuracy:

- State-coherent
- Taking other female mortality into account

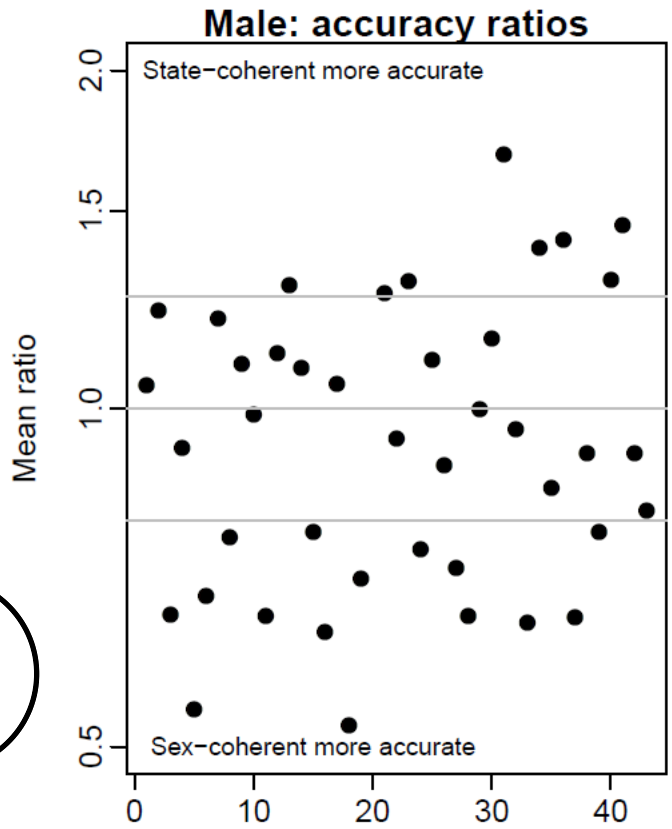
For both sexes, female mortality as 'other' tends to give a better forecast

Why does female mortality as ‘other’ tend to give a better forecast?

- Forecasts are in the future
- Future mortality is expected to be lower than current mortality
- It makes sense that the ‘other’ has lower mortality
- Female mortality is lower than male in same state
- May explain why male mortality is more accurate if female mortality is used as ‘other’ (sex-coherent)

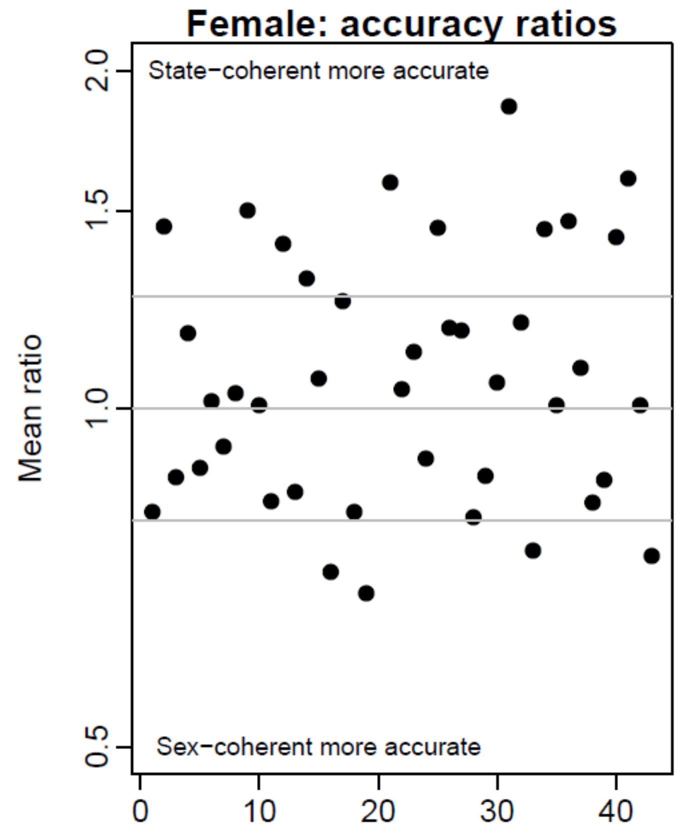
Accuracy: Sex-coherent vs State-coherent

No.
8
9
10
15



Sex:State = 25:17

Sex-coherent



Sex:State = 17:25

State-coherent

No.
11
14
13
4

State-coherent based on regions of Europe: sub-regions; English-speaking

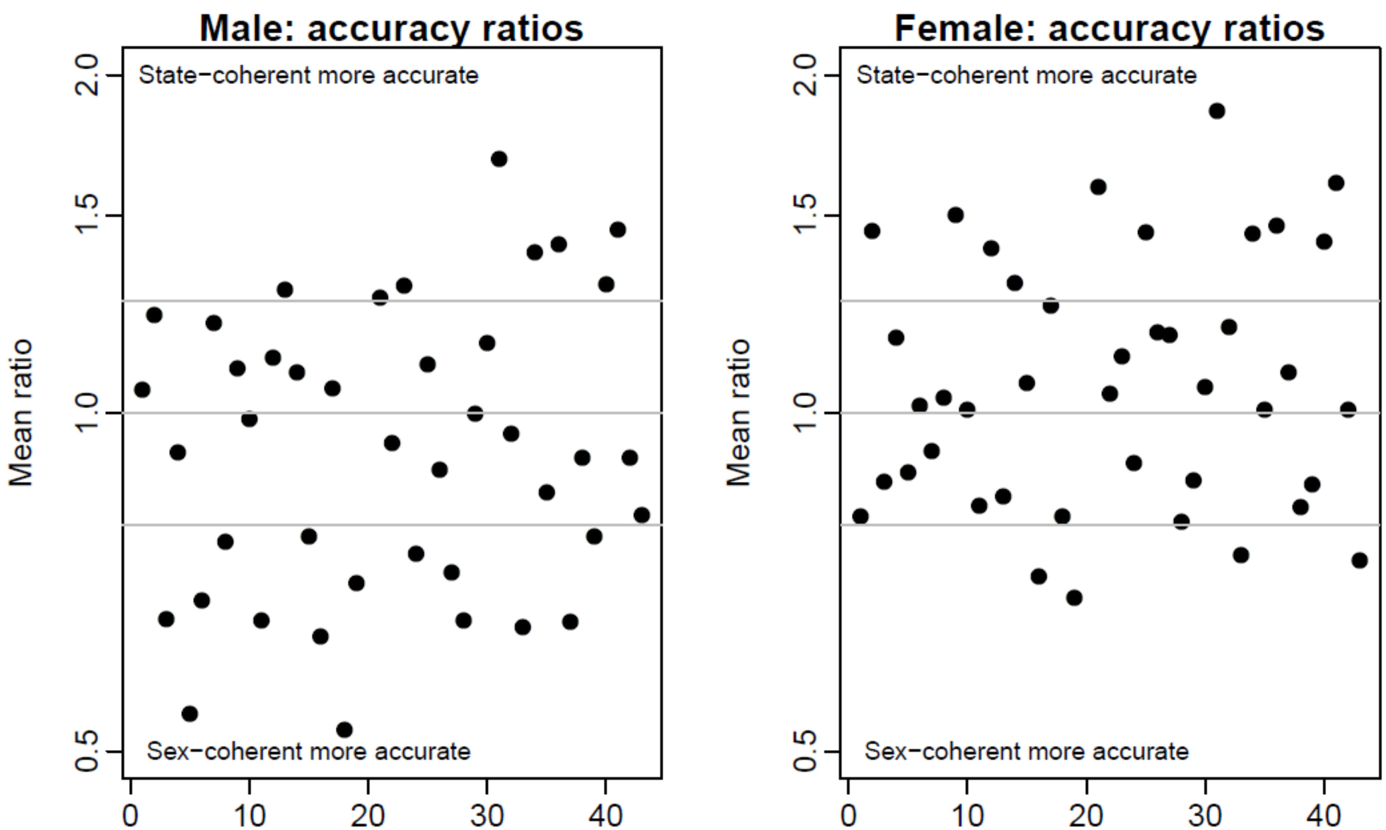
Why does female mortality as 'other' tend to give a better forecast?

- Forecasts are in the future
- Future mortality is expected to be lower than current mortality
- It makes sense that the 'other' has lower mortality
- Female mortality is lower than male in same state
- May explain why male mortality is more accurate if female mortality is used as 'other' (sex-coherent)
- Not inconsistent with more accurate state-coherent forecast for female mortality

Not female, but lower….?

Accuracy: Sex-coherent vs State-coherent

No.
8
9
10
15



No.
11
14
13
4

Sex:State = 25:17
Sex-coherent

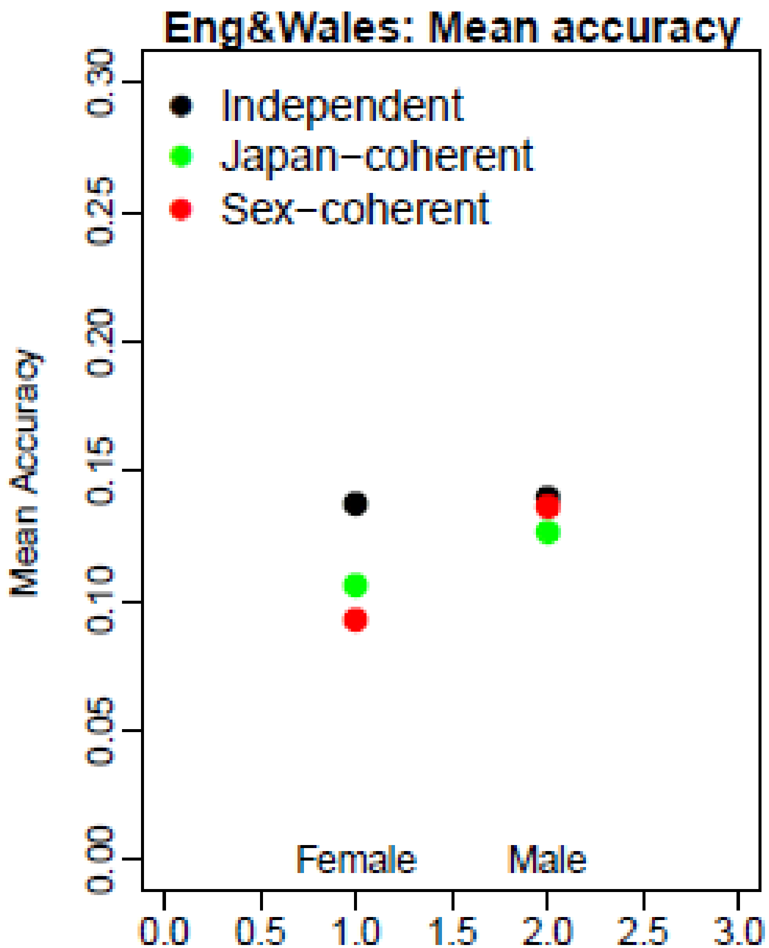
Sex:State = 17:25
State-coherent

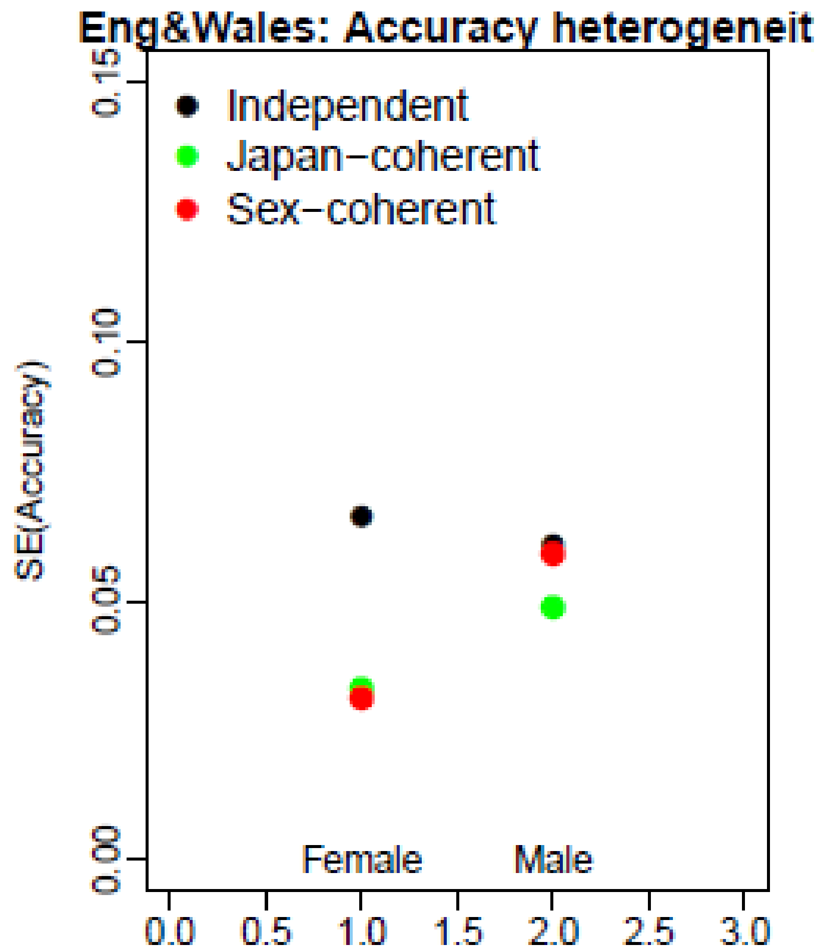
State-coherent based on regions of Europe: sub-regions: English-speaking

Low mortality as ‘other’

JAPAN AS STANDARD

Japan as Standard for England &Wales: accuracy





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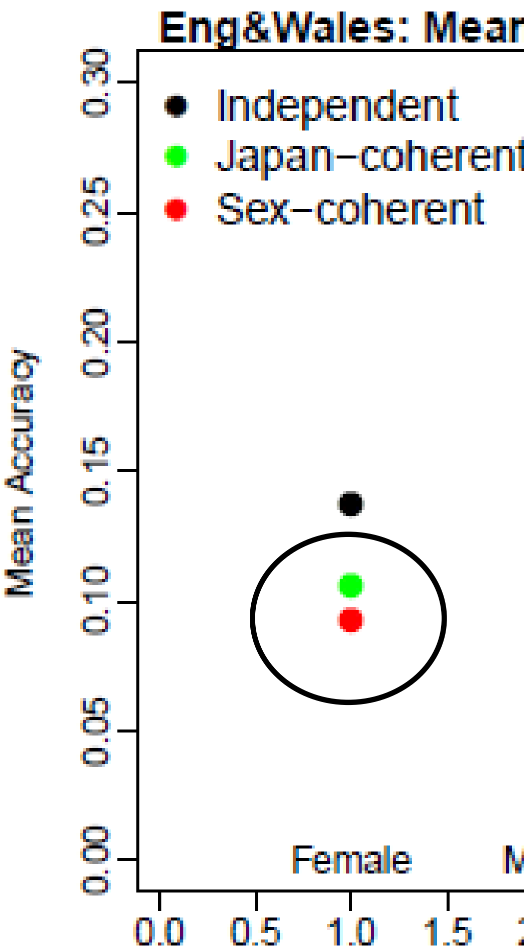
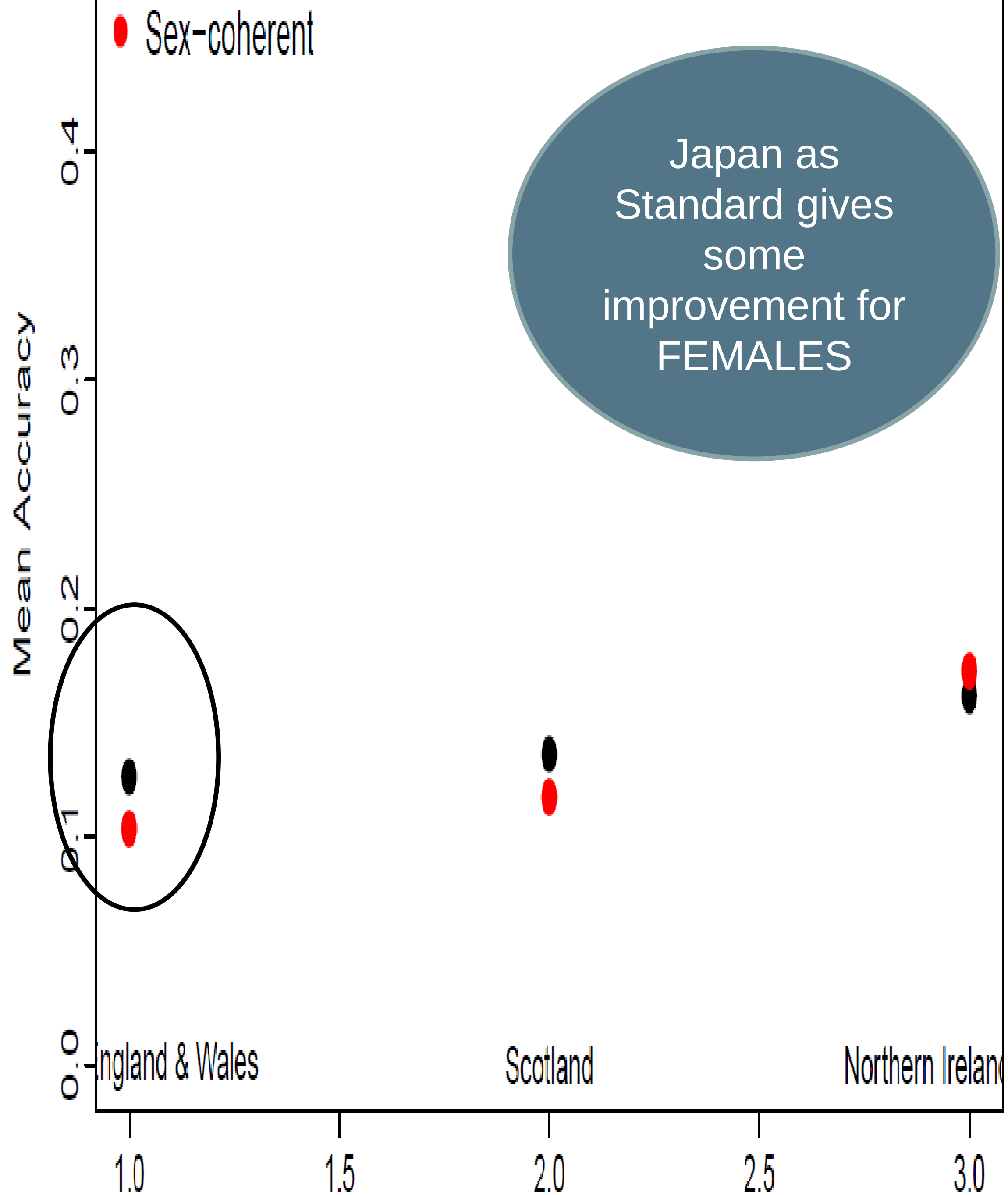
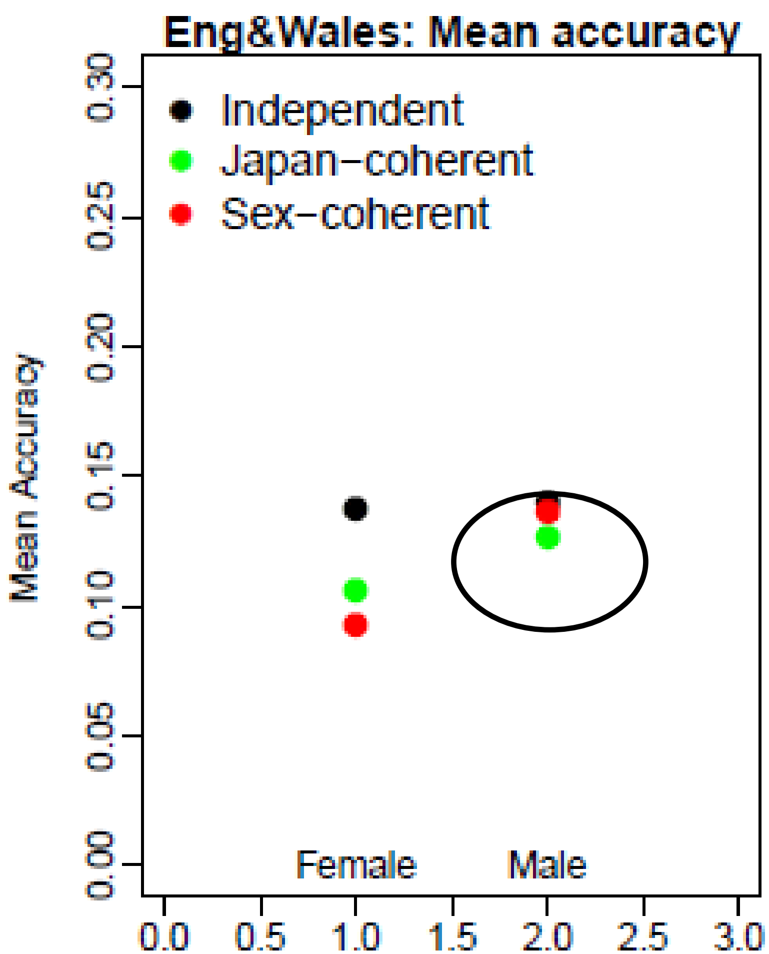
Japan as Standar y

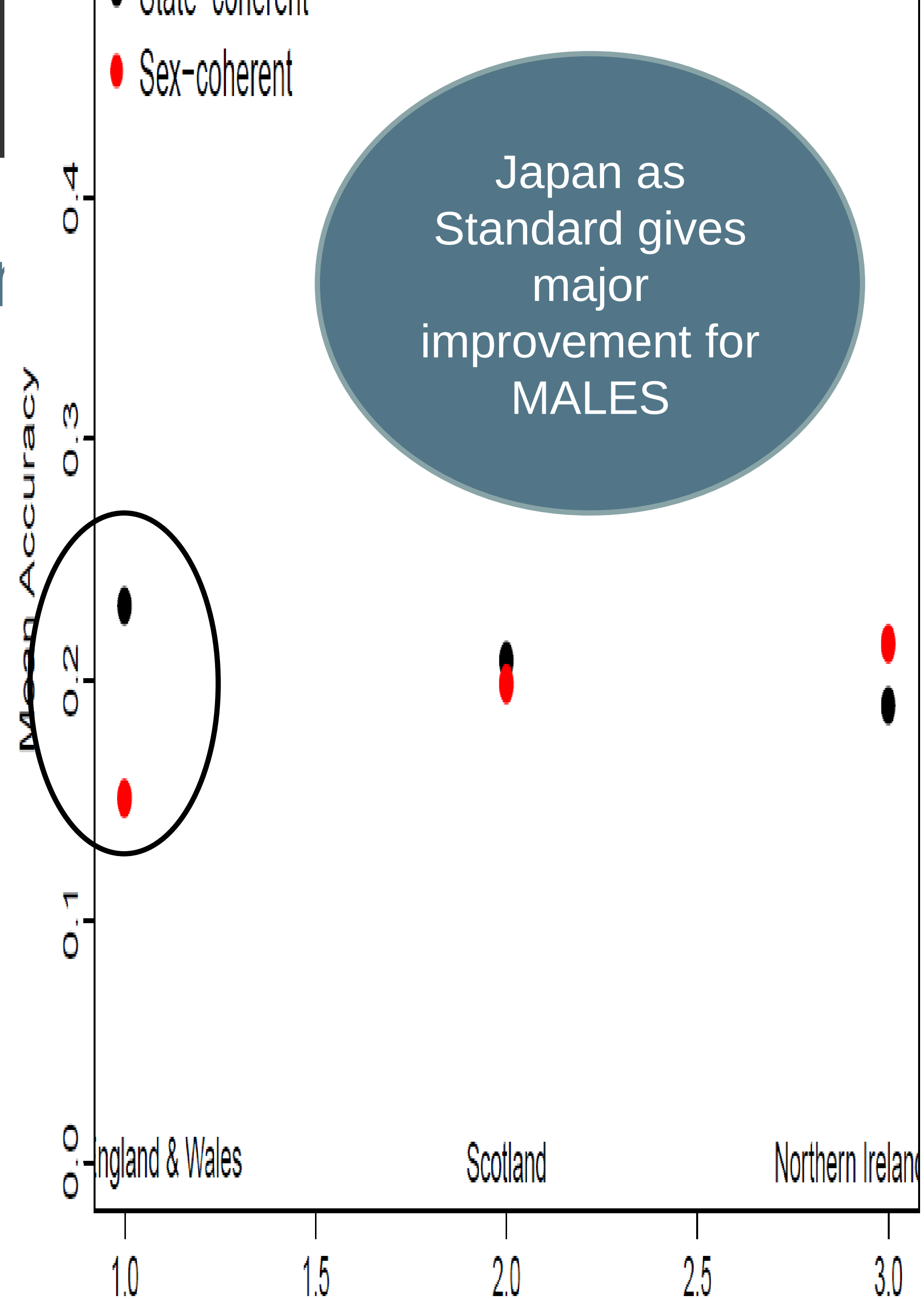




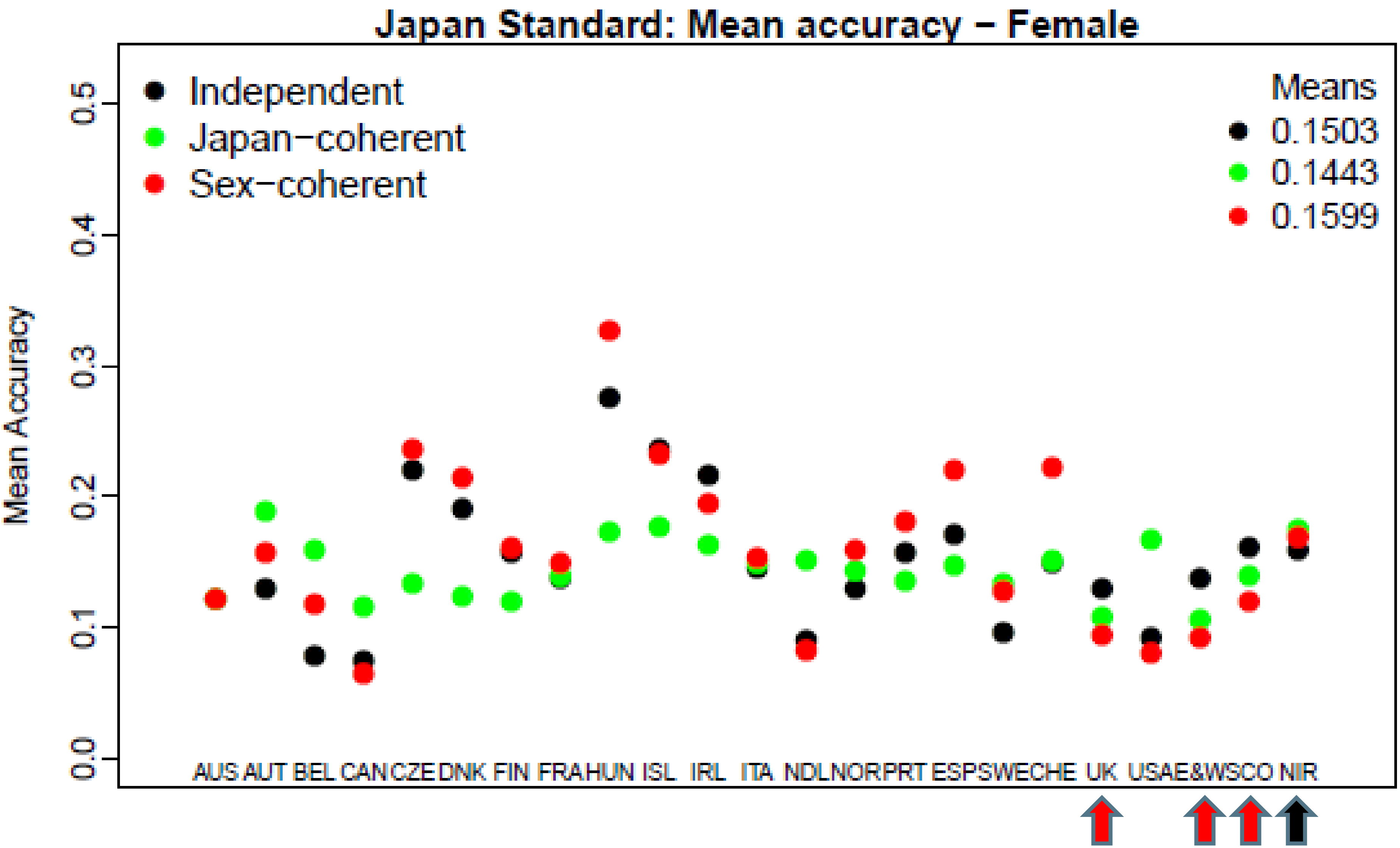
Japan as Standard gives some improvement for FEMALES

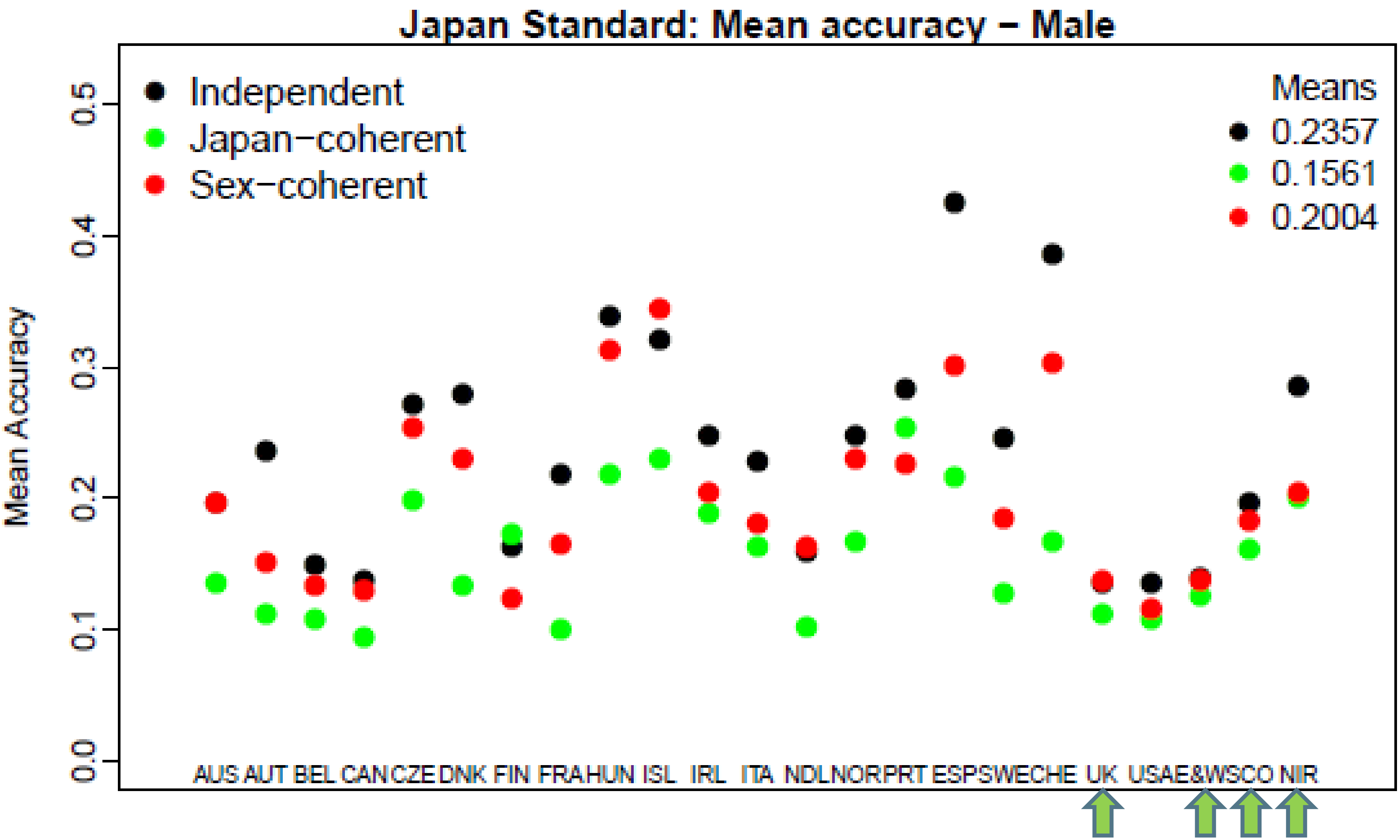
Japan as Standard for En

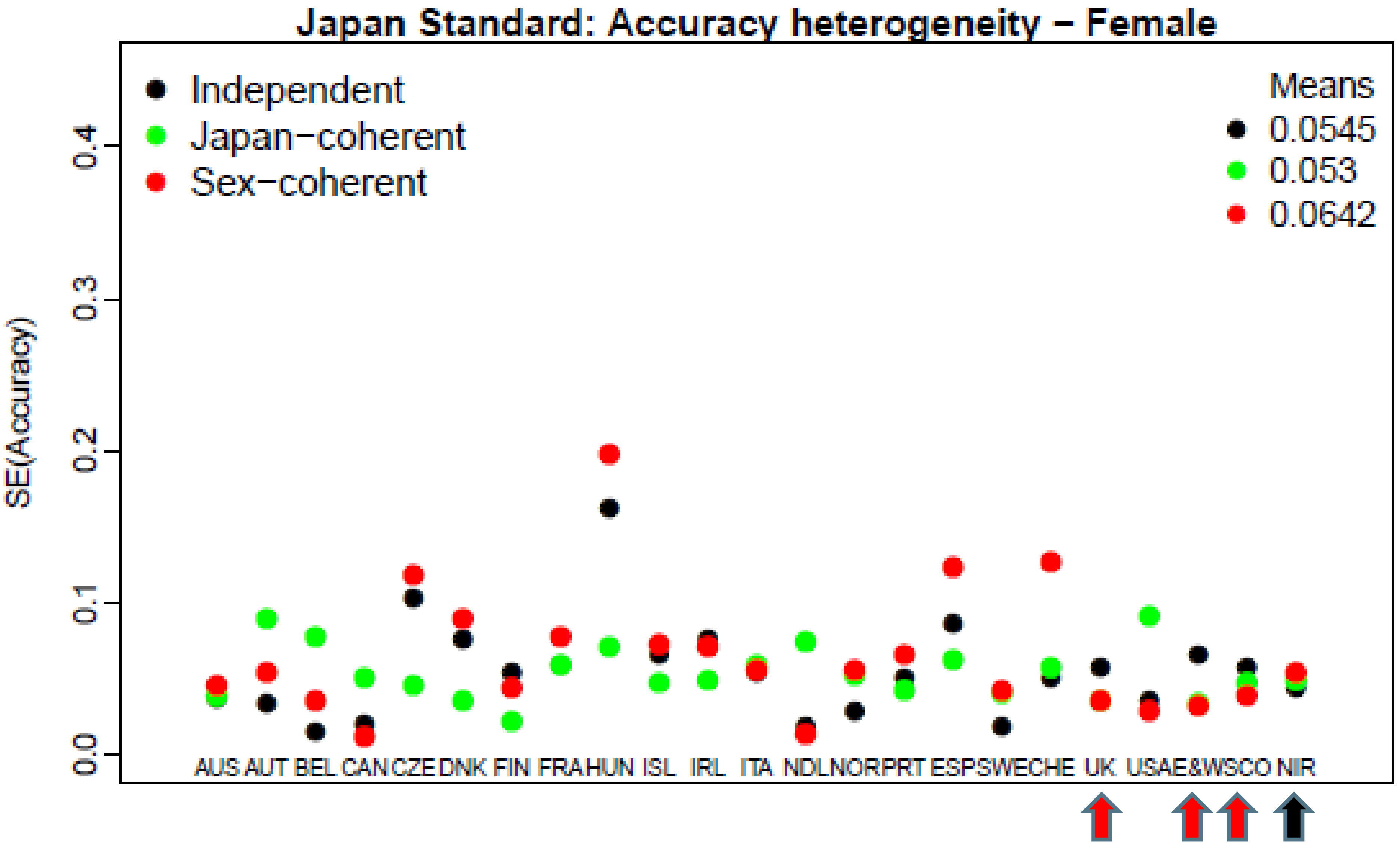




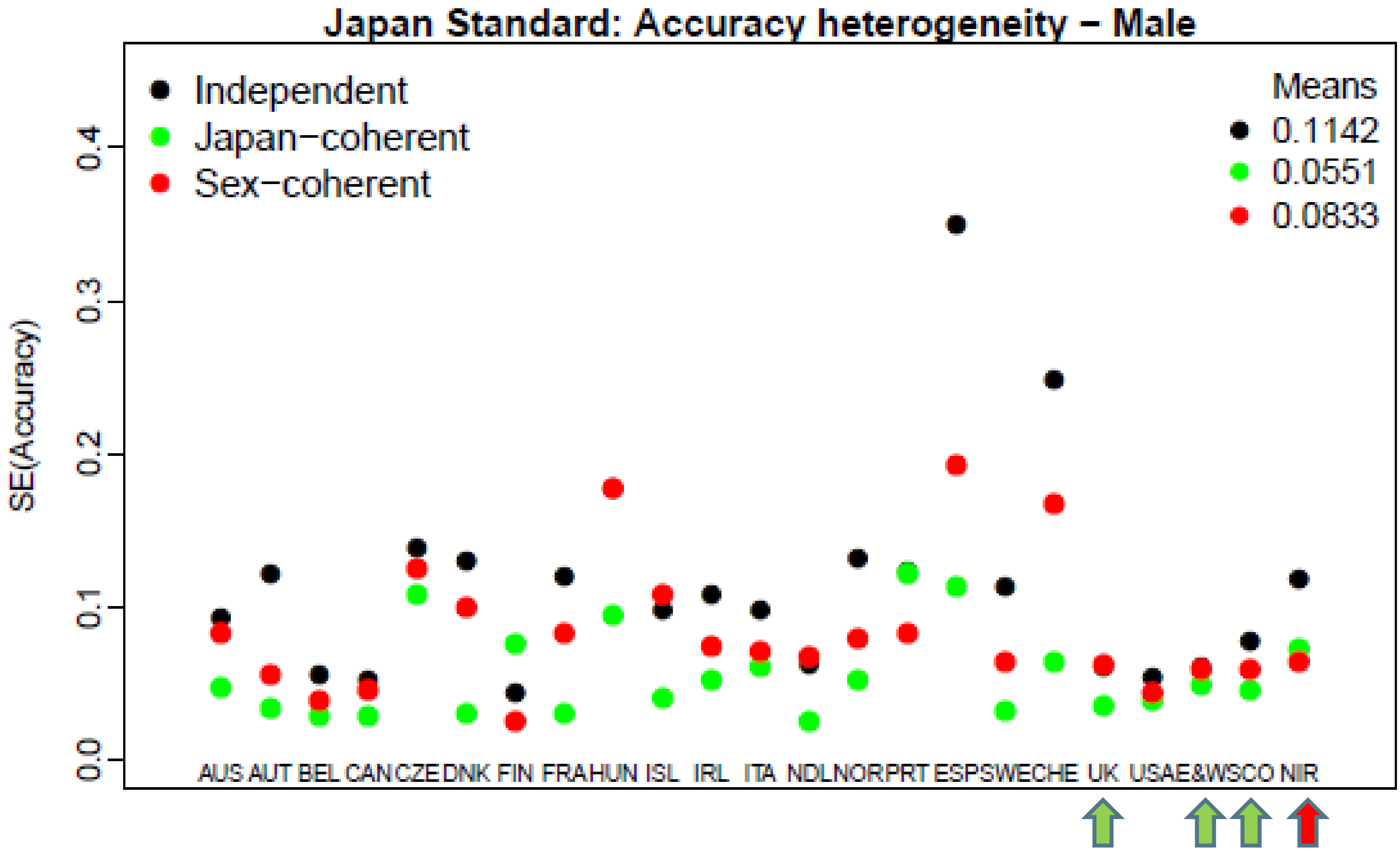
Japan as Standard gives major improvement for MALES


Japan Standard: Mean accuracy – Female
Independent
Japan–coherent
Sex–coherent
Means
0.1503
0.1443
0.1599
Mean Accuracy
0.0
0.1
0.2
0.3
0.4
0.5
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR

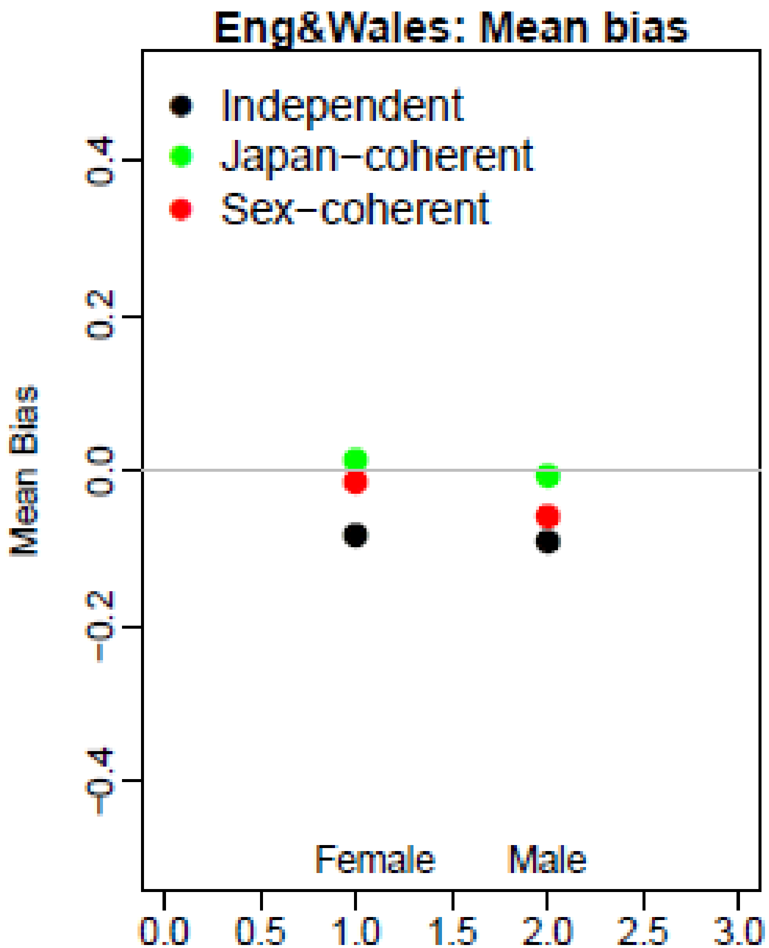

Japan Standard: Mean accuracy – Male
Independent
Japan−coherent
Sex−coherent
Means
0.2357
0.1561
0.2004
Mean Accuracy
0.0
0.1
0.2
0.3
0.4
0.5
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR

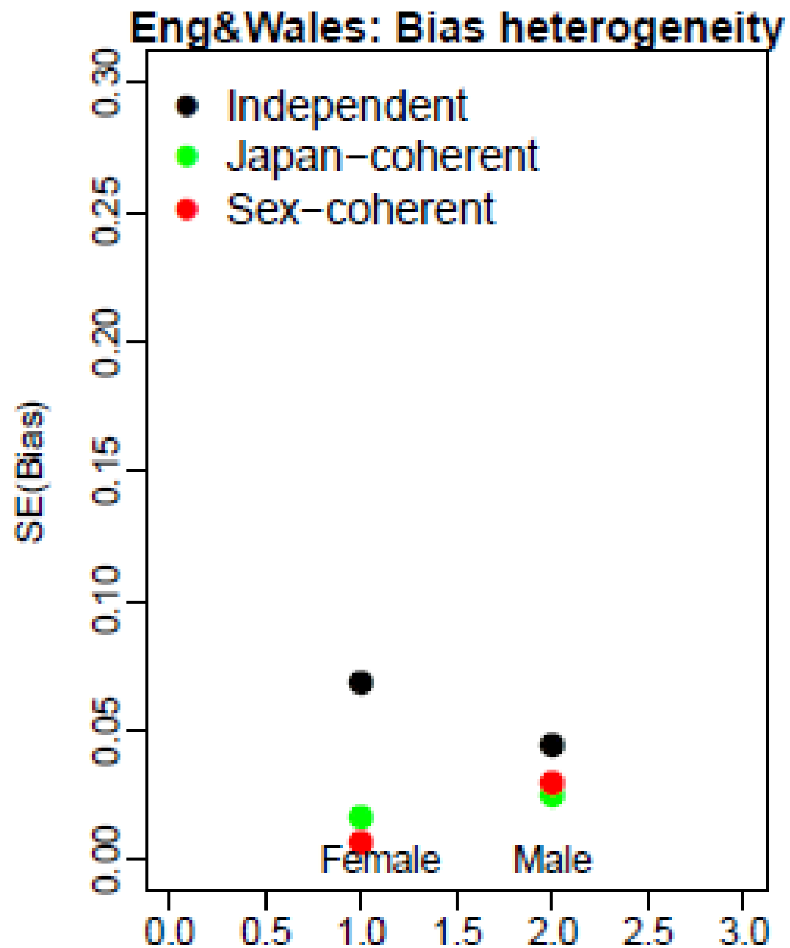

Japan Standard: Accuracy heterogeneity − Female
Independent
Japan−coherent
Sex−coherent
Means
0.0545
0.053
0.0642
SE(Accuracy)
0.0
0.1
0.2
0.3
0.4
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR

Australian National University

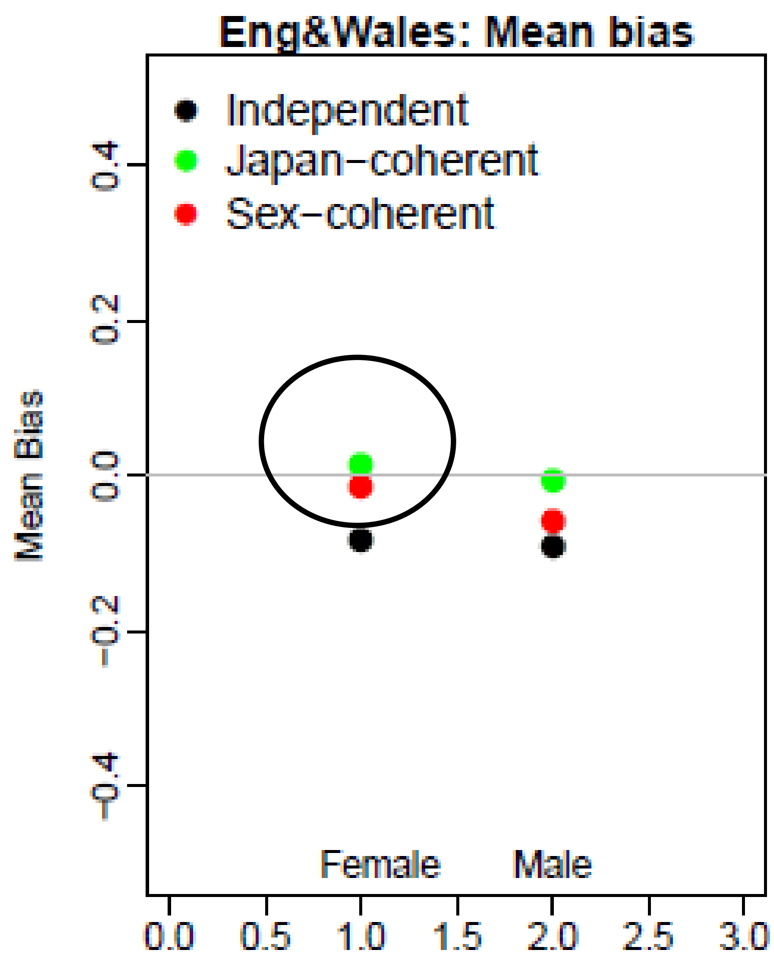


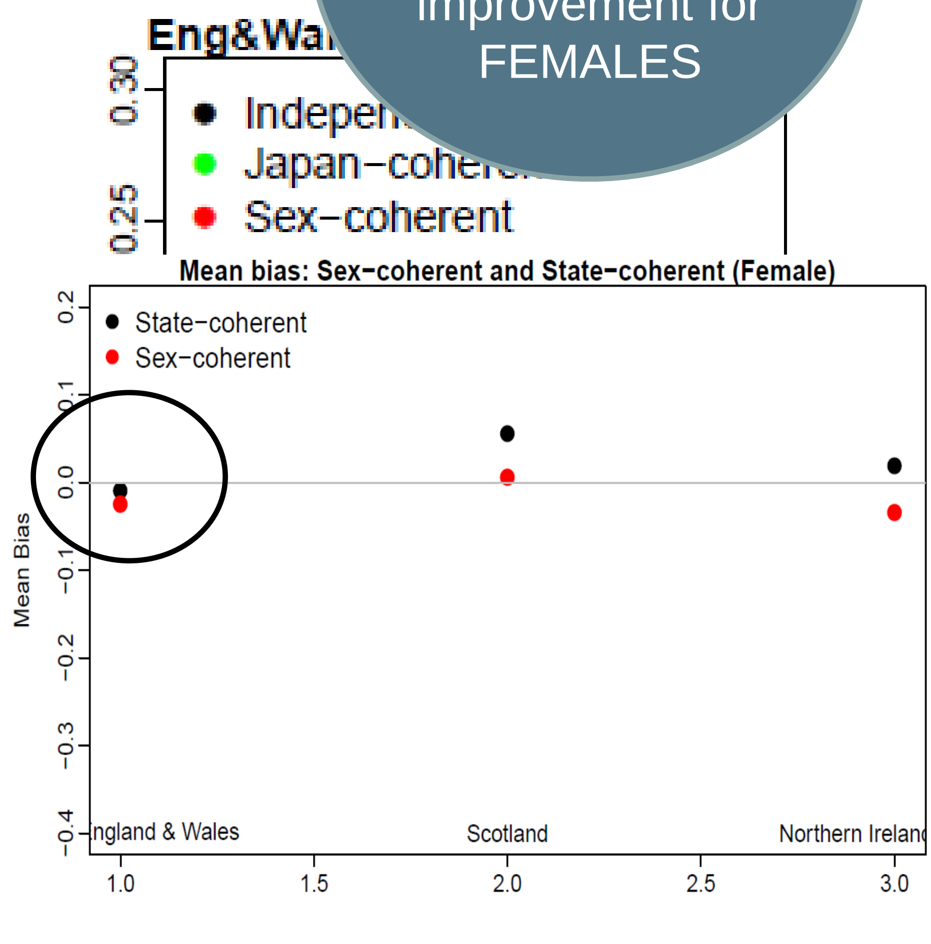
Japan as Standard for England & Wales: bias





Japan as Standard for England &





Japan as Standard gives no improvement for FEMALES

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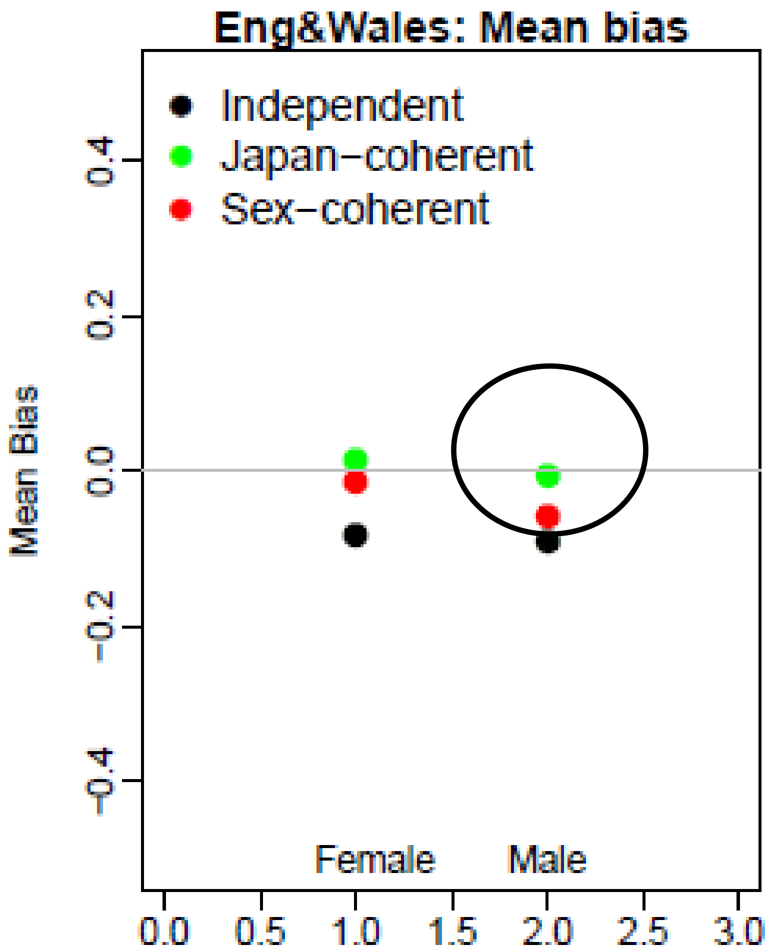
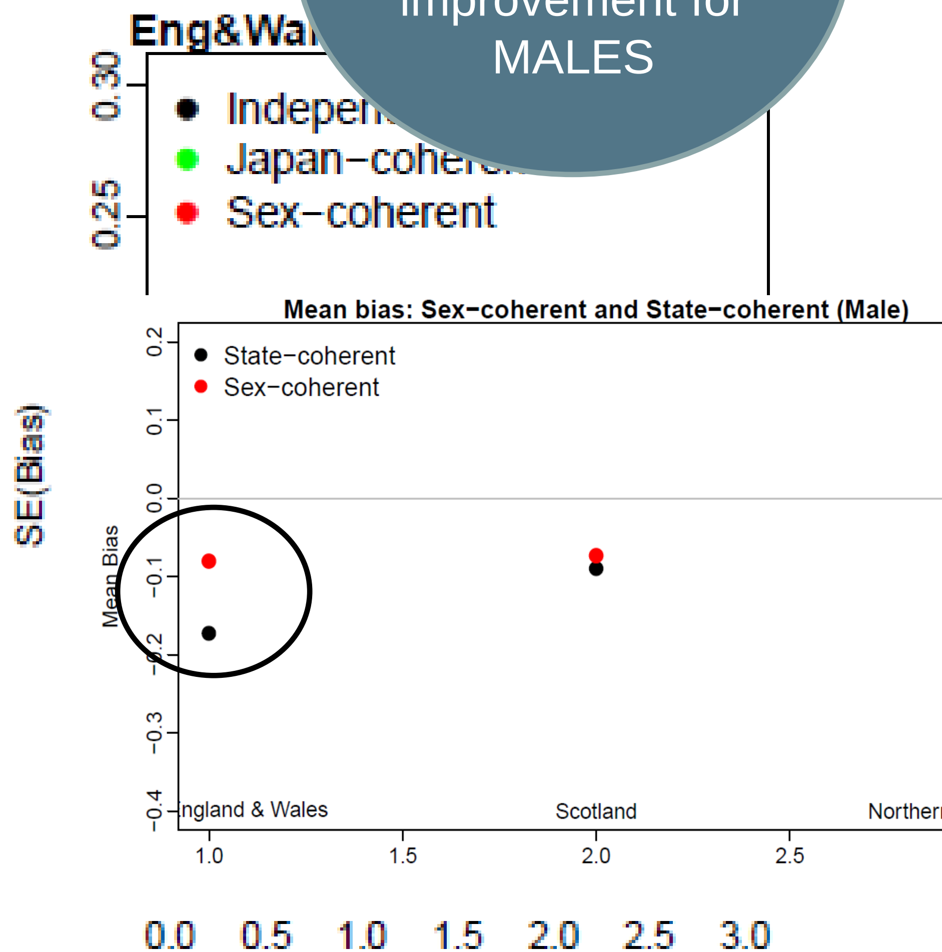
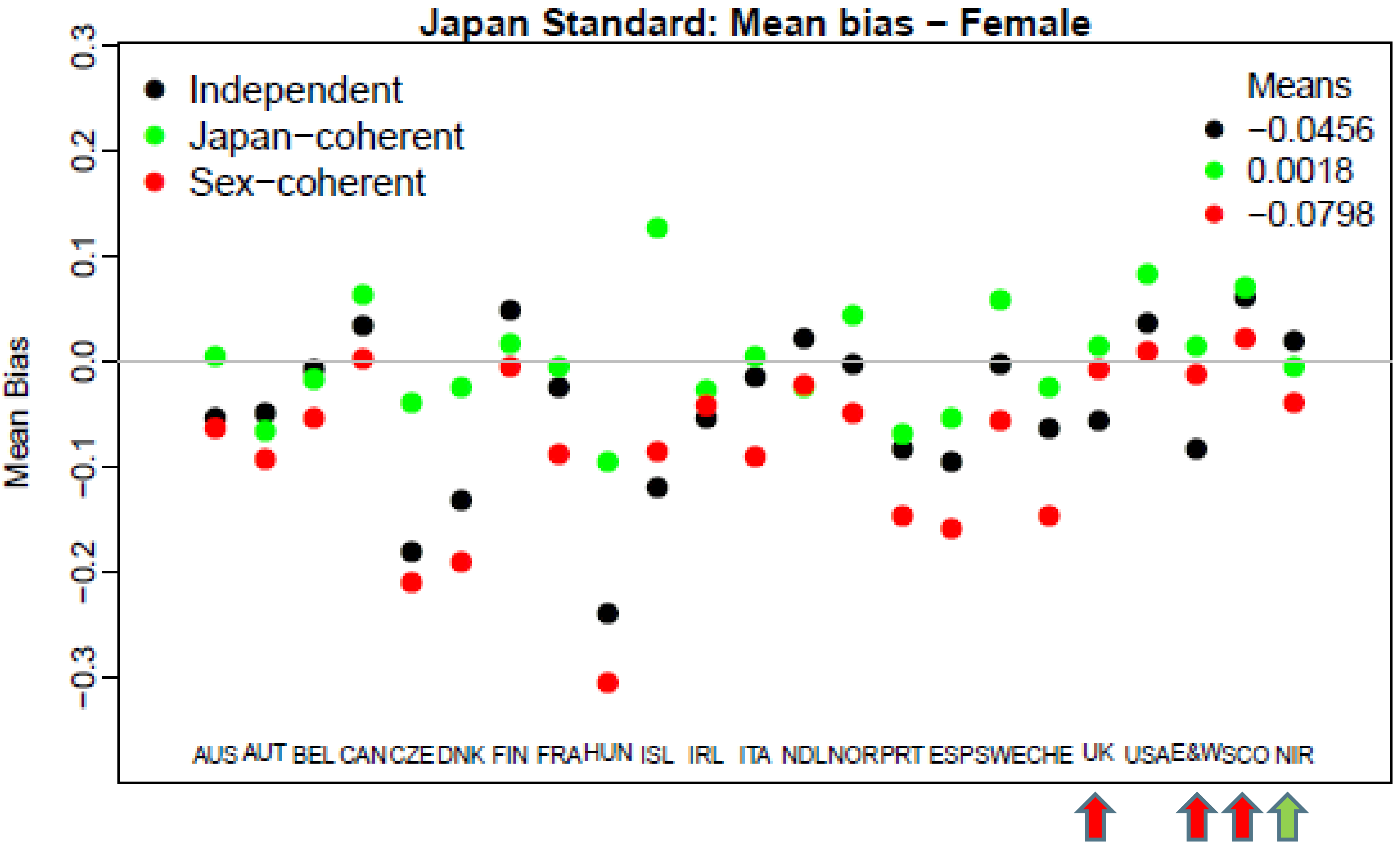
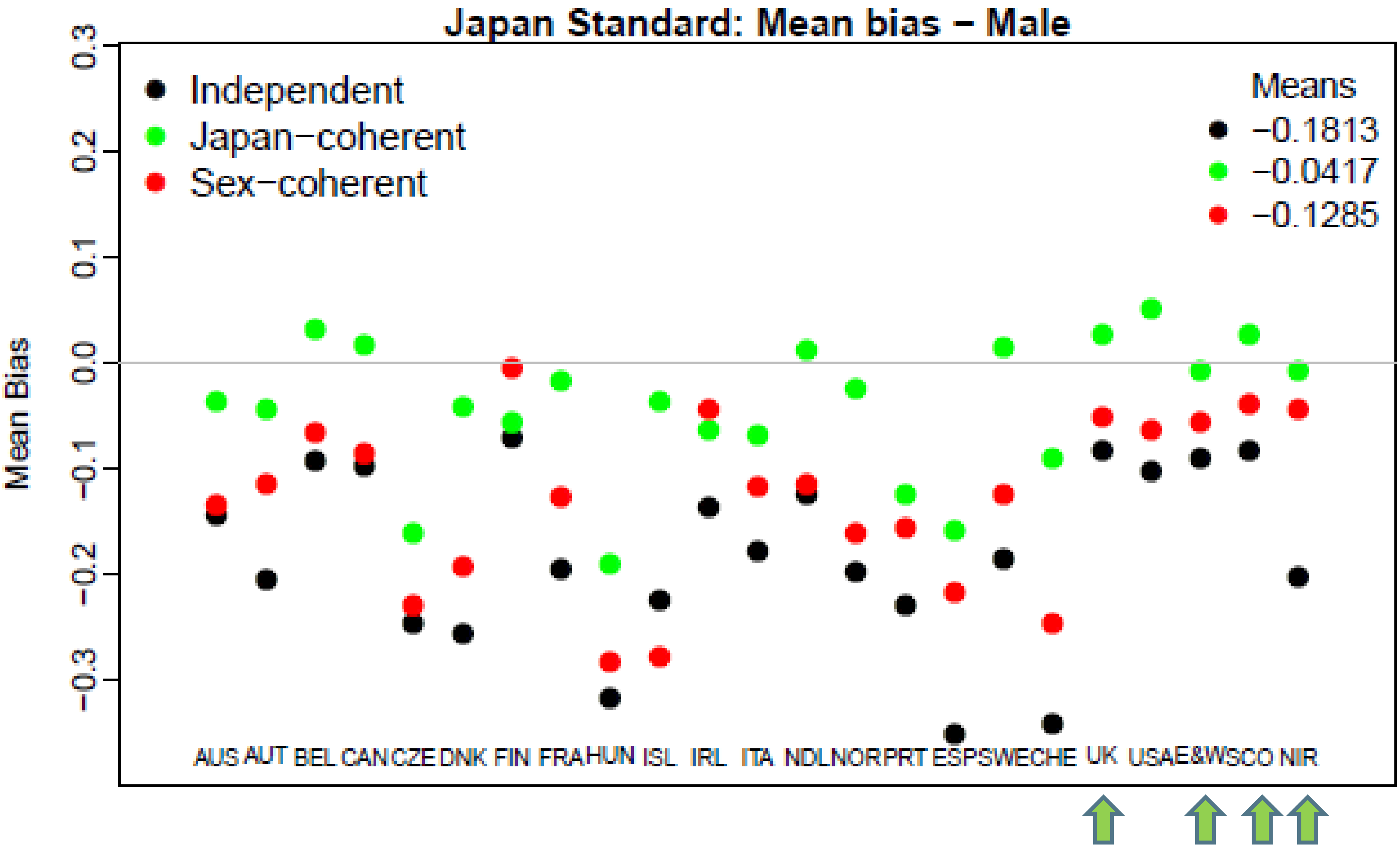
Japan as Standard for England &

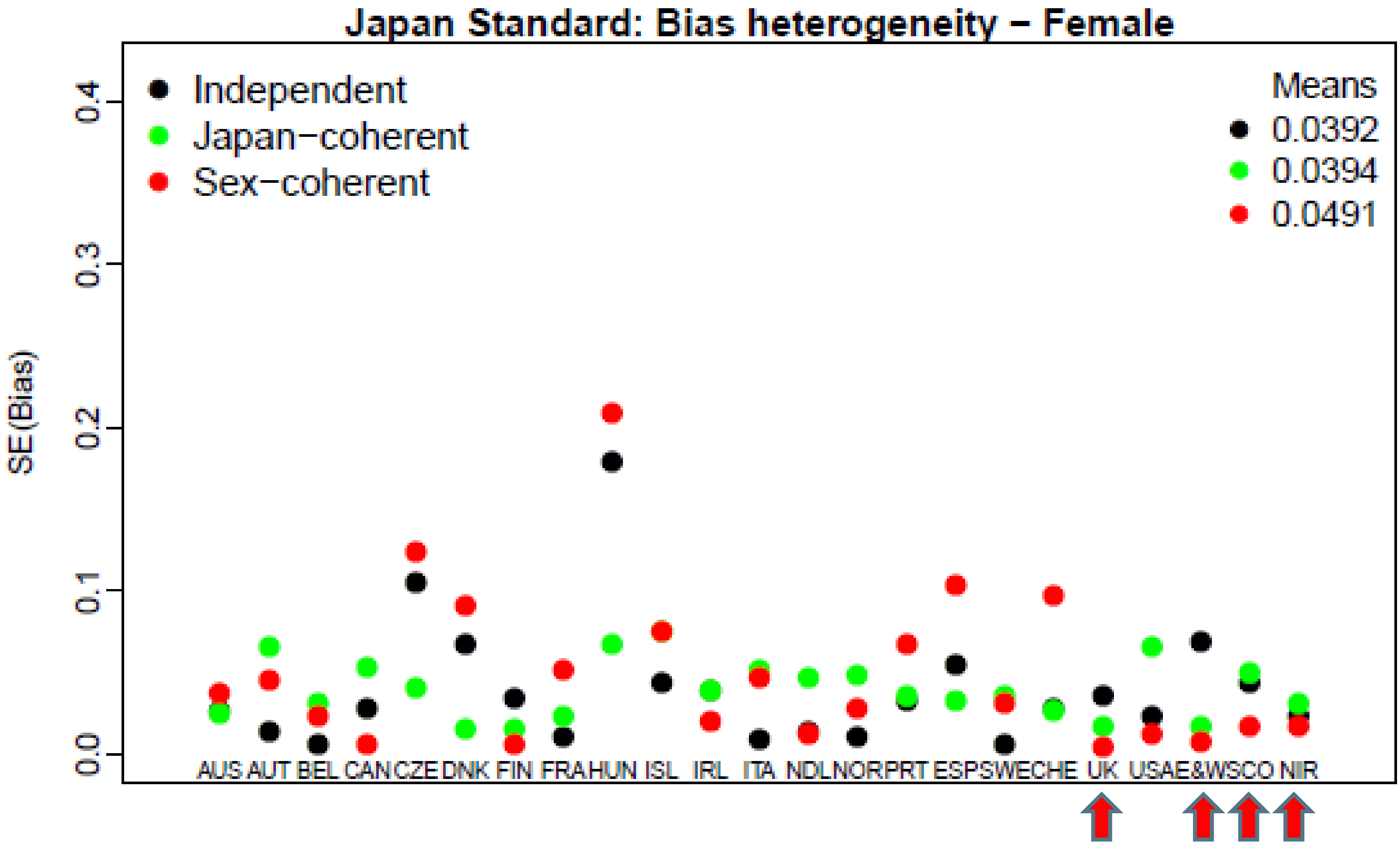
Japan as Standard gives major improvement for MALES

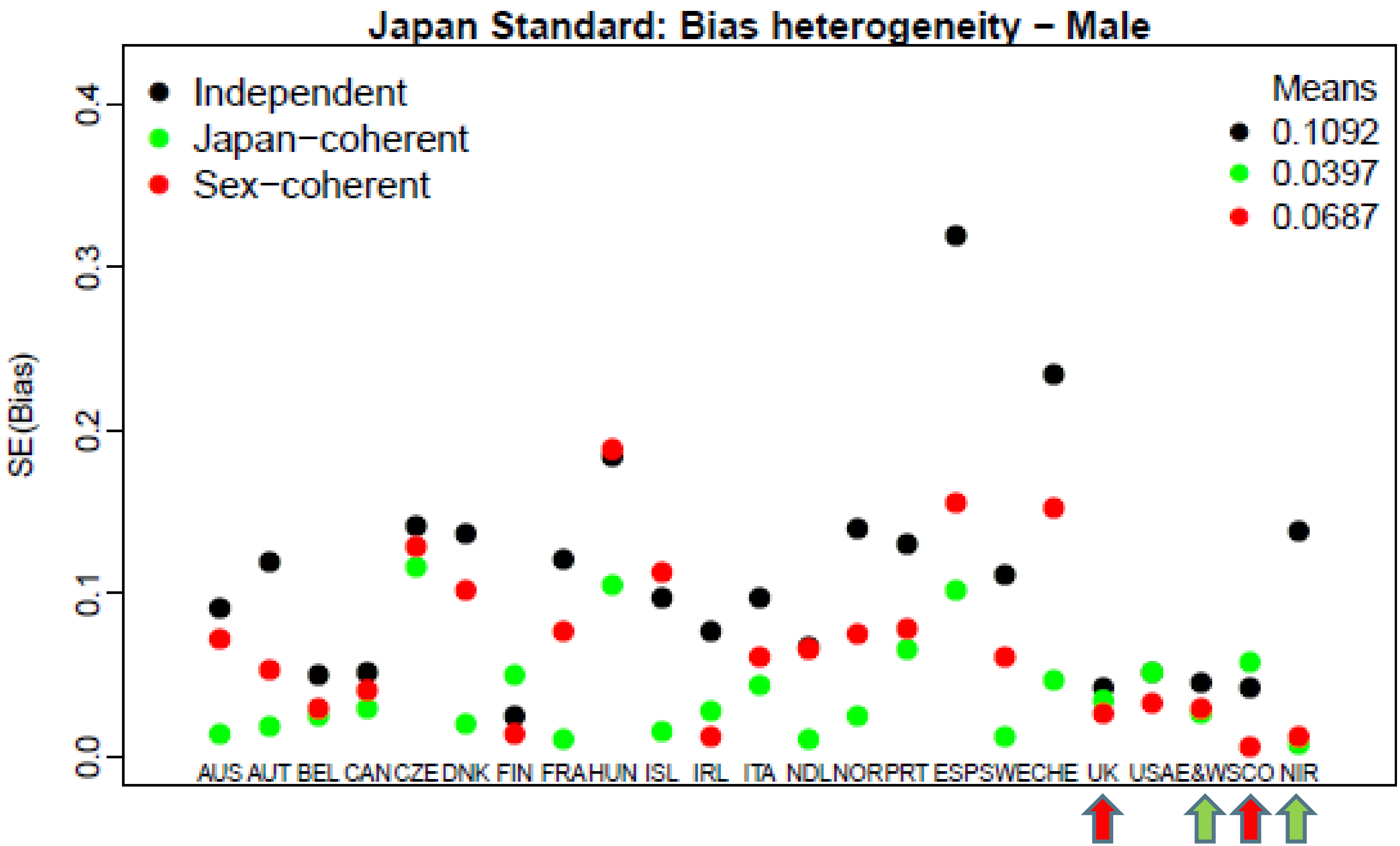





Japan Standard: Mean bias – Female
Mean Bias
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
Independent
Japan−coherent
Sex−coherent
Means
−0.0456
0.0018
−0.0798
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR


Japan Standard: Mean bias – Male
Mean Bias
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
Independent
Japan−coherent
Sex−coherent
Means
−0.1813
−0.0417
−0.1285
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR


Japan Standard: Bias heterogeneity − Female
Independent
Japan−coherent
Sex−coherent
Means
0.0392
0.0394
0.0491
SE (Bias)
0.0
0.1
0.2
0.3
0.4
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR


Japan Standard: Bias heterogeneity − Male
Independent
Japan−coherent
Sex−coherent
Means
0.1092
0.0397
0.0687
SE(Bias)
0.0
0.1
0.2
0.3
0.4
AUS AUT BEL CAN CZE DNK FIN FRA HUN ISL IRL ITA NDL NOR PRT ESP SWE CHE UK USA E&W SCO NIR

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CONCLUSION

Low mortality standard

- Overall improvements in forecast performance
  - Improved accuracy (esp for male mortality)
  - Reduced accuracy heterogeneity (esp for male mortality)
  - Less biased (both sexes)
  - Reduced bias heterogeneity (esp for male mortality)
  - Reduced heterogeneity of heterogeneity
- In real world forecasting, these are valuable advantages

Forecasting full circle?

- Targets and standards are not new in mortality modelling and forecasting
  - Revival: use of observation is advantageous
- Avoid ‘forecasting the past’
  - Take account of moving b(x) by forecasting ratio of age pattern to that of a more advanced population
- Choice of standard is important

Answering the initial questions

- Is sex-coherent forecasting or state-coherent forecasting more accurate for sex-state mortality? Wrong question! Pointed to female mortality as instrumental.
- What can we learn from this? Lower mortality appears to be key.
- How can forecasting methods be further improved? Use low mortality as standard.
- How can we better use other information to improve forecasting? Further research esp on the optimal standard.

References and Acknowledgments

- Hyndman, R. J. and H. Booth (2008). "Stochastic population forecasts using functional data models for mortality, fertility and migration." *International Journal of Forecasting* **24**(3): 323-342
- Hyndman, R. J. and S. Ullah (2007). "Robust forecasting of mortality and fertility rates: a functional data approach." *Computational Statistics and Data Analysis* **51**: 4942-4956.
- Hyndman, R., H. Booth and F. Yasmeen (2013) Coherent mortality forecasting: the product-ratio method with functional time series models. *Demography,* 50(1), pp.261-283
- Software: Hyndman et al. 'demography' package @ The Comprehensive R Archive Network @ http://cran.r-project.org/
- Human Mortality Database @ http://www.mortality.org/
- Australian Demographic Data Bank (ADDB) http://robjhyndman.com/software/addb/

ARE YOU INTERESTED IN LEARNING TO USE THESE FORECASTING METHODS?

heather.booth@anu.edu.au