JIA 120 (1993) 211-214

J.I.A. **120**, I, 211–214

PRACTICAL APPLICATIONS OF RISK THEORY

SEMINAR, 29–30 SEPTEMBER 1992

A seminar on 'The Practical Applications of Risk Theory' was held at Staple Inn on 29–30 September 1992, organised jointly by the Institute and the Department of Actuarial Mathematics and Statistics at Heriot-Watt University. The aim of the seminar was to combine introductory talks on several aspects of risk theory with detailed presentations of case studies by practitioners.

The first three sessions dealt with risk models.

Ms Mary Hardy gave a short survey of the classical collective risk model, compound Poisson distributions, and some simple approximations to such distributions.

Professor Axel Reich from Cologne then presented a case study based on a portfolio of industrial fire policies in Germany. The portfolio suffered a comparatively high loss ratio, which analysis showed to be due to a small number of large claims. The problem was to devise a rating structure which would optimise the gross portfolio, and to determine what reinsurance would optimise the net portfolio. The risk profile of the business was categorised by the probable maximum loss (PML) into 21 groups. The ratio (premium ÷ PML) indicated quite different risks in different groups. Using data from 1987 to 1991, the claims degree (claims ÷ PML) in each group was modelled by a Beta distribution; modelling the claims degree was easier than modeling the claim amounts, because the claims degree is normalised. Overall, claims degree decreased with increasing risk. The mean claim frequency in each risk group was modelled as an exponential function of PML, and the claim number distribution in each risk group was then assumed to follow a Poisson distribution. The aggregate claims distribution was then estimated by Panjer's recursion formula.

The expected loss ratios varied considerably according to the risk group, but having modelled the claims distribution explicitly, the standard deviation of the loss was also available, and the gross portfolio had a very high probability of loss. The model results suggested that the rating structure could be improved, and in practice it would also be used to find the optimum reinsurance arrangement. The practical advantages of modelling the claims distribution included the analysis of the effects of segmentation of the portfolio into different risk groups, and the contribution of each risk group to the overall loss ratio and the total fluctuation potential.

The next three sessions were on credibility theory.

Professor Howard Waters outlined the origins of the subject in American credibility theory, and its development along Bayesian lines, culminating in the work of Bühlmann and Straub.

Dr Alois Gisler from Winterthur then presented a case study based on the rating structure used in Switzerland for employees' Group Health Insurance, one of a number of lines for which Swiss companies use an industry-wide rating structure. The problem was to devise an experience rating system for adjusting the premiums charged to different employers, while maintaining overall the premium volume indicated by the standard tariff. Initial analysis by the working party of actuaries indicated that the conditions for credibility-type estimators were satisfied.

The basic Bühlmann–Straub model gave unacceptable results, because of the effects of large claims on small policies. Thus, a claim for sick pay in respect of an employee who subsequently died might result in a large increase in the premium, despite the cessation of the particular risk. To deal with this problem, the working party turned to credibility estimators in which the largest claims were truncated. The smaller policies caused further problems in estimating the variance components of the model, since they tended to show no claims at all or else comparatively large claims; these problems were resolved by excluding the smaller policies from the fitting process. The fitted model was not put into use as it stood, but served as the basis for a simpler rating structure based upon three bands of policy sizes and associated truncation points for claims. Certain anomalies could still arise, and to limit these a pragmatic set of rules was adopted. During this stage, the results from the model indicated where simplifications could be made without losing important features.

Dr Gisler referred to the mathematical theory which lay behind the model,[(1)] and discussed the need to test any model against real data and to work towards a solution capable of implementation. He concluded that the use of sophisticated models and mathematics is not at all in conflict with what is needed in practice.

The first three sessions on the second day were on ruin theory and solvency.

Dr David Dickson introduced the topic by outlining some basic models of the claims process and ruin in finite and infinite time. If the claims process possessed a particular compound Poisson distribution, explicit results were available. The model could also be adapted to allow for reinsurance.

Then Professor Ragnar Norberg from Copenhagen described the solvency reporting system being set up by the Norwegian Banking, Insurance and Securities Commission. This stemmed from work done by a working party which reported in 1984,[(2)] and will be described by Norberg.[(3)] Solvency is to be evaluated on a break-up basis. The supervisors will collect data from each company on each line of business, with details of reinsurance, and use these to model the distribution of the outcome of the run-off. The model will then yield estimates of the ruin probability, on a company by company basis. For this type or work a statistical model may be preferred, as it is relatively objective and not easily manipulated, although within each company the estimation of future claims development may use other methods.

The basis of the system is provided by modelling the occurrence and development of claims under a policy as a marked Poisson process; that is, a non-homogeneous Poisson process describes the timing of claims and the subsequent claim development is a random process associated with the time of the claim. At the time of the investigation, the insurance portfolio can be decomposed into claims which have been settled, reported but not settled, incurred but not reported, or which are covered but not incurred. The basic problem is then to predict the outstanding amounts of these separate claims processes, possibly allowing for unobservable risk characteristics. To do this, the intensities of the timing of the jumps in the claim development process need to be estimated, by regressing the claims data supplied onto the known risk characteristics, and by using credibility methods to deal with unknown risk characteristics. Similar methods may be used to estimate the claims size distributions.

Professor Norberg described a pilot study which had been undertaken, resulting in a solvency reserve similar to the current, more *ad hoc* reserving requirements, and some of the implications of the new reporting system for insurance companies' record keeping.

In the final sessions, Dr Henrik Ramlau-Hansen from Copenhagen described a model for windstorm claims in Denmark.[(4)] The need for this study arose because it is common practice in Denmark to combine cover for fire, glass and windstorm damage, but the characteristics of windstorm claims are quite different from fire and glass claims. The aggregate risk is catastrophic in nature, since the individual risks are not independent.

The study was based on single-family houses, classified by number of floors and floor area, and larger dwellings, classified by floor area alone. There was a clear association between windstorm damage and the number of floors. During the period covered by the data (1977–81) there had been 13 windstorms, according to the definition used, causing widely differing amounts of damage. Upon analysing the claims data, the following features were identified: (*a*) the number of windstorms varied from year to year, (*b*) the proportion of policies giving rise to claims varied from storm to storm, (*c*) the mean claim amounts varied from storm to storm, (*d*) there was no clear evidence of correlation between the proportion of policies giving rise to claims and the mean claim amounts, (*e*) damage depended on floor space, and houses with two floors suffered more damage than houses with one floor, (*f*) claims in a given storm and for a given type and size of building were approximately lognormally distributed, and (*g*) damage to houses and damage to dwellings were correlated.

The model which was used allowed for all of these features; in particular the number of storms each year was modelled by a Poisson distribution, and the strength, or capacity to cause damage, of a given windstorm was modelled as a (possibly correlated) pair of random variables, representing the proportion of policies giving rise to claims and the mean claim amounts. The fitted model was applied to calculate solvency reserves and to study the effect of reinsurance. The solvency reserves were high, and because of the dependence of the individual risks did not decrease with volume.

All of the case studies were followed by useful discussions, some comparing the methods used by different practitioners. The participants found the seminar most interesting and useful, in bringing out the links between theory and practice which are often not covered in such depth in the actuarial literature.

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