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PRACTICAL EXPERIENCES IN STOCHASTIC RESERVING

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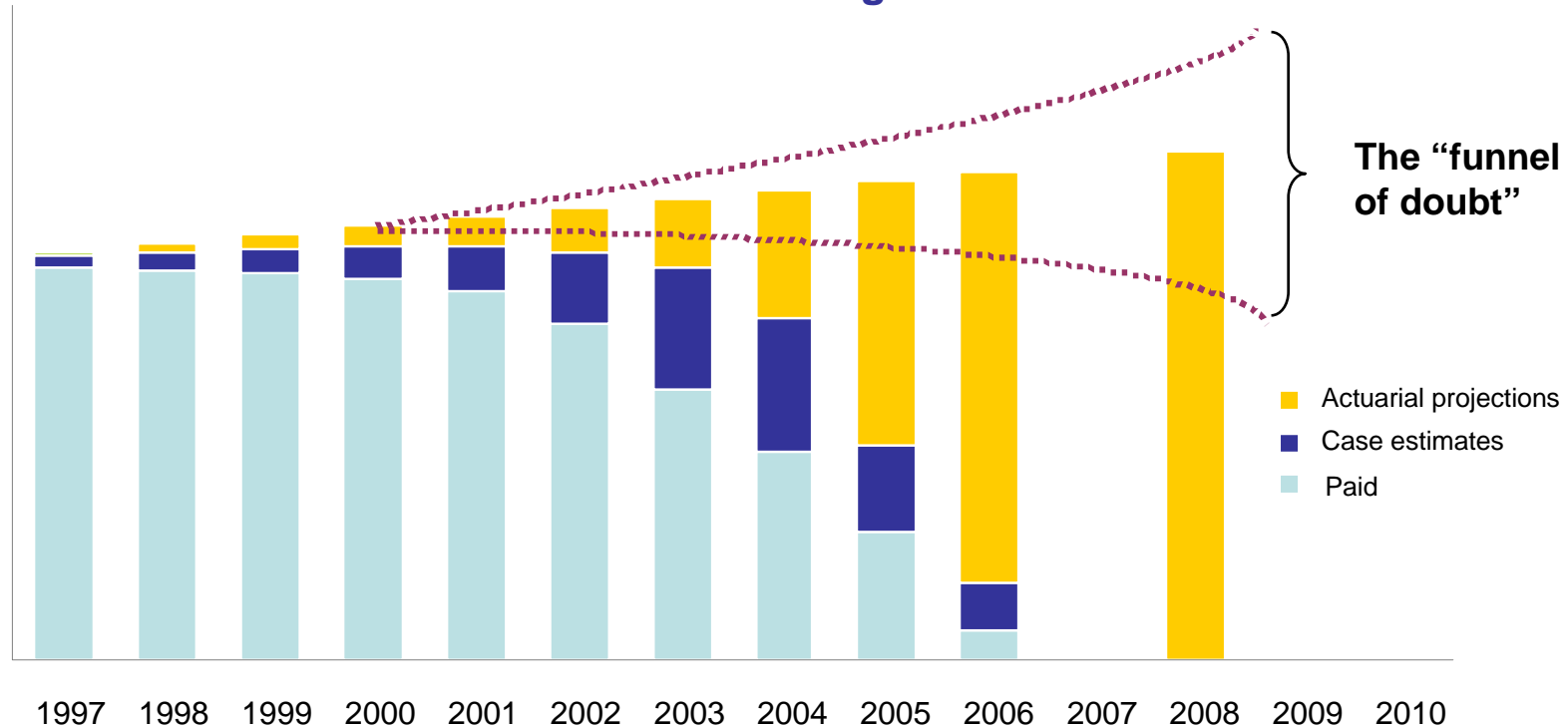
Towers Perrin Tillinghast

Structure

Stochastic Reserving:
The starting point
(Introduction)

Stochastic Reserving:
Opinions, Shortcomings
and Solutions

Stochastic Reserving:
How to get
the most out of it

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Assumptions used to make insurance decisions need to consider more than just one estimate

The Actuarial Reserving Problem

The "funnel of doubt"

Legend:
- Actuarial projections
- Case estimates
- Paid

X-axis: 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010

Variations in the past have to be observed and considered.

Change-over to stochastic reserving methods

Stochastic Reserving: The starting point

- Loss development is a stochastic cash flow of payments
- The observed paid losses are but one specific realisation of the underlying stochastic process
- The task of projecting reserves is to derive an assessment of the future development of this cash flow, given the data observed so far

- Deterministic methods provide a point estimate of claim liabilities
    - "Best estimate" is usually chosen judgementally
    - Applying multiple deterministic methods can give a"range of estimates"

- Stochastic reserving starts with solving questions like:
    - How far can the future loss development deviate from my expectation?
    - And how likely is that?

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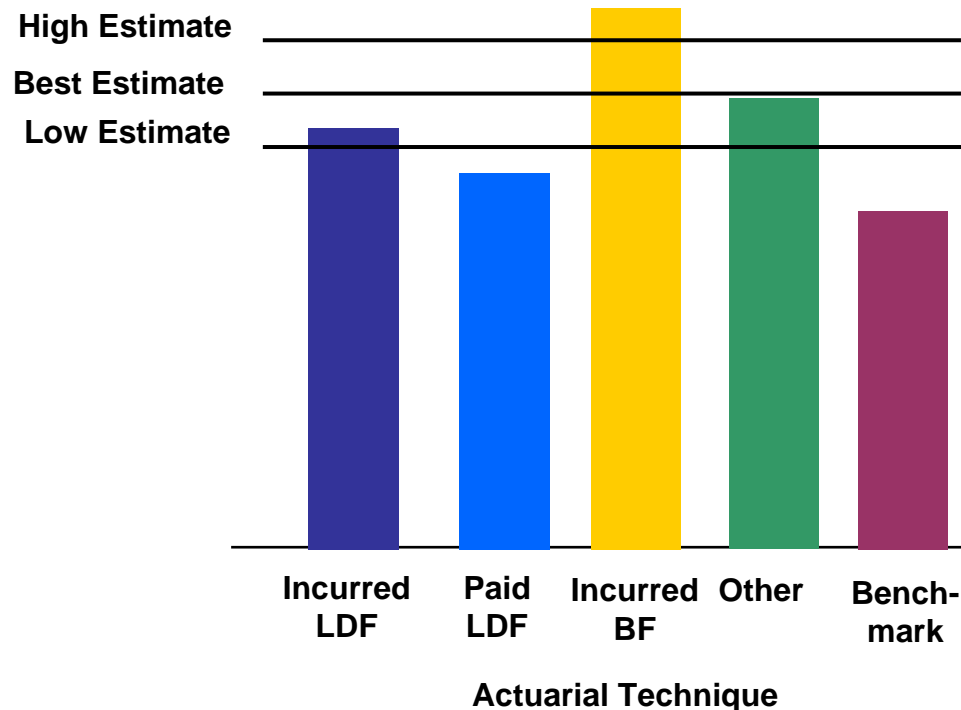
Some Assertions:

- Stochastic reserving does not add (much) value!
  - "We already use (deterministic) best practice reserving techniques."
  - "Stochastic methods are complicated to understand and to use.
    I do not have the time to read all those papers."
  - "Ranges can easier be estimated by stressing factor selections."

- Stochastic methods (often) produce implausible results!
  - "My data is not suitable for stochastic methods."
  - "The indicated range is too big / too narrow."
  - "Even the projected mean is considerably different
    from my best estimate."

- So why bother with stochastic reserving methods?!

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Excursion:
GIRO 2005 Reserve Uncertainty Survey

- "What did we learn?"
  - Using methods/models is easy
  - Understanding methods/models is more difficult
  - Understanding method/model output can be very difficult […]
  - No "correct" method/model apparent
  - Wide range of results from different methods/models
  - Range still wide even when same method/model used
  - Data issues will distort results
  - […]

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Take a step back: The deterministic approach

Indicated Unpaid Claim Liabilities as at 31 December 2006

High Estimate
Best Estimate
Low Estimate

Incurred LDF | Paid LDF | Incurred BF | Other | Bench-mark

Actuarial Technique

The General Deterministic Approach

- Usually starts with a chainladder projection of paid (and incurred) data

- (Almost) never ends there…

- Excellent knowledge of the underlying business is essential

- Applying actuarial expertise and judgement when selecting development patterns and projection methods is crucial for deriving sensible results

- Point Estimate usually picked judgementally

- Possible estimation of a range based on
  a) results of different projection methods or
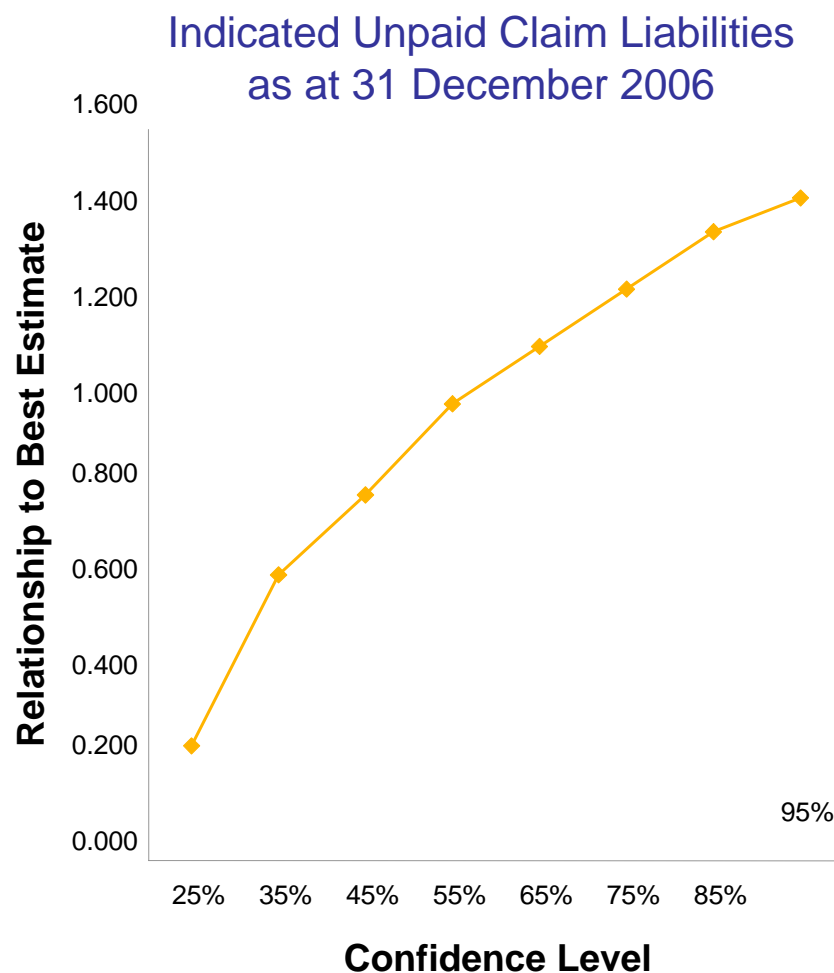 b) varying assumptions (e.g. loss ratios, tail)

Advantages

- Easy to understand and apply

- Established

Disadvantages

- Does not include confidence intervals

- No quantitative answer

- Simplistic and highly judgemental

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The Stochastic Approach:
Reserves are really distributions

Indicated Unpaid Claim Liabilities as at 31 December 2006

Chart: Relationship to Best Estimate (y-axis, 0.000 to 1.600) vs Confidence Level (x-axis, 25% to 85%, with 95% marked). Plotted points approximately: 25%→0.200, 35%→0.590, 45%→0.755, 55%→0.970, 65%→1.095, 75%→1.215, 85%→1.335, 95%→1.410.

General Approach - Stochastic

- Estimate probability distribution
- Based on statistical methods
- Approach varies and depends on the actual data

Advantages

- Produces estimates of confidence intervals
- More complete description of loss generating process
- Feeds other analyses (ERM, DFA, QIS)
- Can approximately separate parameter and process risk

Disadvantages

- Involves relatively complex statistical analyses
- There are many different approaches, but the variety is not yet developed well enough as to have a general agreement among actuaries on which approach is best practice
- Some exposures not amenable to this approach (e.g. APH)

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Conclusion

- Stochastic reserving techniques are an emerging technology

- There are some challenges

- Big opportunities for more refined information about claim liabilities make the extra effort worthwhile

- Naïve application of stochastic models seldom provides good results (as with deterministic models)

- Testing the model assumptions against the data is essential!

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Testing Model Assumptions

- The most common stochastic methods are based on the basic chainladder assumptions:
    - Mack's model
    - Over-dispersed Poisson ($\rightarrow$ GLM, Bootstrapping)

- These models have only limited allowance for actuarial judgement

- Hence it is not possible to judgementally overrule parameter calculations as is done in the deterministic Loss Development Method

- **Testing and verifying the underlying model assumptions is crucial!**

The Reserving Problem

Best Model?

Ideally, minimize parameter and model uncertainty

Loss triangle data

Cumulative Payments or Incurred Claims										
1995	87.7	118.0	122.2	124.7	127.2	129.7	131.0	133.2	133.9	134.6	134.9
1996	89.2	120.4	124.7	127.6	129.9	131.6	134.1	135.7	136.7	137.6
1997	81.4	109.2	113.6	116.3	119.0	121.8	123.7	124.8	125.8
1998	83.9	112.4	116.3	119.3	121.3	123.5	124.9	125.9
1999	92.6	124.4	129.7	133.2	135.5	137.0	138.2
2000	95.3	132.1	139.4	143.3	146.9	149.9
2001	115.9	162.6	172.6	178.7	182.1
2002	120.6	162.1	170.1	175.2
2003	107.6	140.4	146.9
2004	96.7	124.8
2005	77.6

Model
→ Assumptions

Distribution of Outstanding Claims
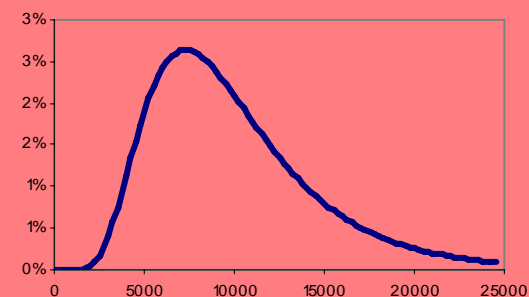
Process Uncertainty

Additional Information
- Trends
- Exposure data
- Pricing information

Model Uncertainty?

Parameter Uncertainty

Some Sample data (German Motor Liability)

A Y/DY	1	2	3	4	5	6	7	8	9	10
1996	87	117	120	121	122	123	123	123	123	123
1997	88	118	121	123	125	125	126	126	126	
1998	80	106	109	111	112	113	114	114		
1999	83	111	114	116	117	118	118			
2000	92	123	128	130	131	132				
2001	95	130	136	138	140					
2002	115	160	166	170						
2003	120	159	165							
2004	106	138								
2005	96									

Cumulative Payments,
Figures in mn EUR
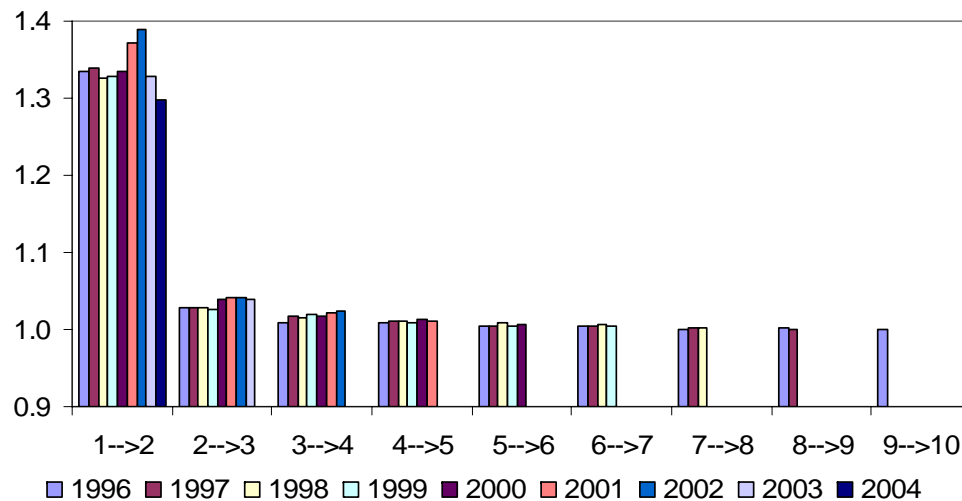
Plotted Link Ratios
Can we identify any
trends in this chart?

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A first glance at the data: Check for Linearity in the Functional Form of Development Year Trends

- **Graphical judgment:**
  Plot cumulative (or incremental) claims for year *d+1* versus cumulative claims for year *d* for all accident years.

y = 0.3783x - 3.7287

(Chart: Incremental Claims (d+1) vs. Cumulative Claims (d))
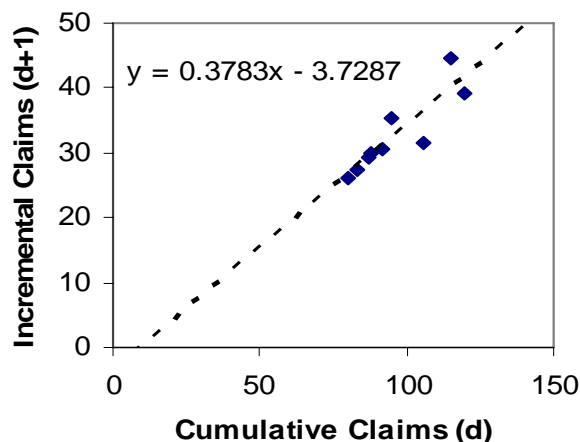
In this example (for *d=1)*, the linearity assumption seems to fit well!
The estimated intercept is not significantly different from zero. This is a necessary requirement for the Chain-Ladder models.

- **Statistical measure:**
  R^2 value = Fraction of the variance that can be explained by the linear relationship.
  In the above example: R^2 = 79%.

Other common tests use a (model-specific!) definition of residuals

$$R_{s,d+1} = (C_{s,d+1} - f_{d+1} C_{s,d}) / \left(\alpha_{s,d+1} \sqrt{C_{s,d}} \right)$$

"Standardised Residuals"

*s:* accident year
d: development year

f: Chain Ladder development factor
α: volatility coefficient
  (*= std. dev. of* $C_{s,d+1}$ *divided by* $\sqrt{C_{s,d}}$)

Remarks:

- Also called "Pearson Residual".

- This definition of residual is used in all models that are based on chainladder assumptions, in particular:

  - Mack's Chain Ladder

  - Over-dispersed Poisson, etc → GLM

  - ODP-Bootstrapping (without volatility coefficient)

Testing Model Assumptions: Independence of Residuals

- The standardised residuals should be independent.

- A necessary condition is that the residuals for two consecutive development years are independent or uncorrelated.

- In this particular example:
    - Graphically, a slight positive correlation may be identified (significant?).
    - R^2-Value: very low (~ 20%).
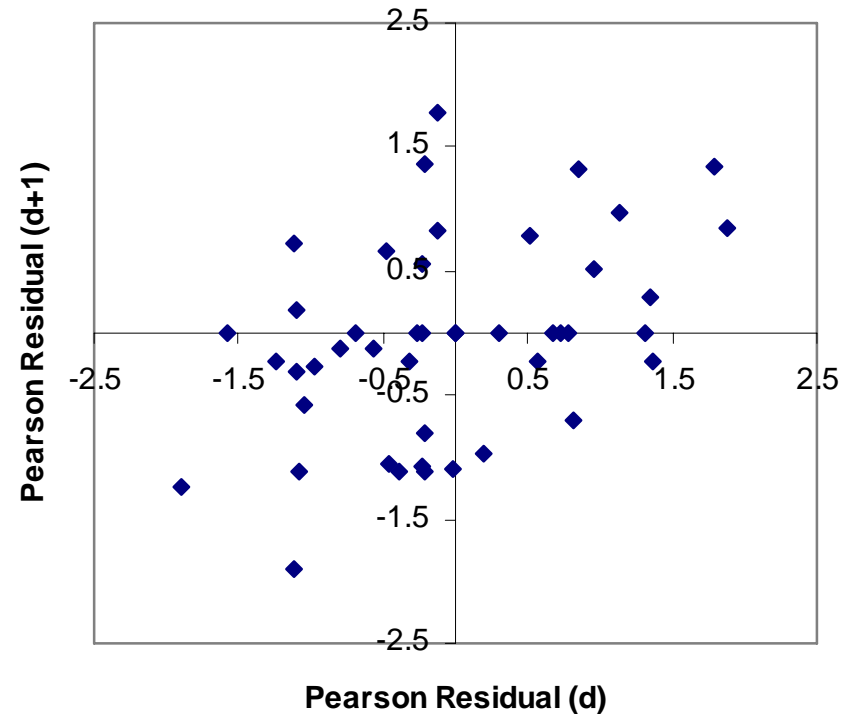
**Remark:**

Note that independence of the residuals is also a necessary condition so that bootstrap for Chain-Ladder may be applicable.

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Testing Model Assumptions:
Distribution of Residuals

- Consider the cumulative distribution function for the residuals:
    - ***What is the underlying distribution?***
    - ***Outliers?***

In our example:

- Plot shows comparison of
    - Empirical distribution (♦)
    - Fit with Normal distribution (–)

- The "periodic" deviations from the Normal curve are likely to be caused by calendar year trends.

- No outliers
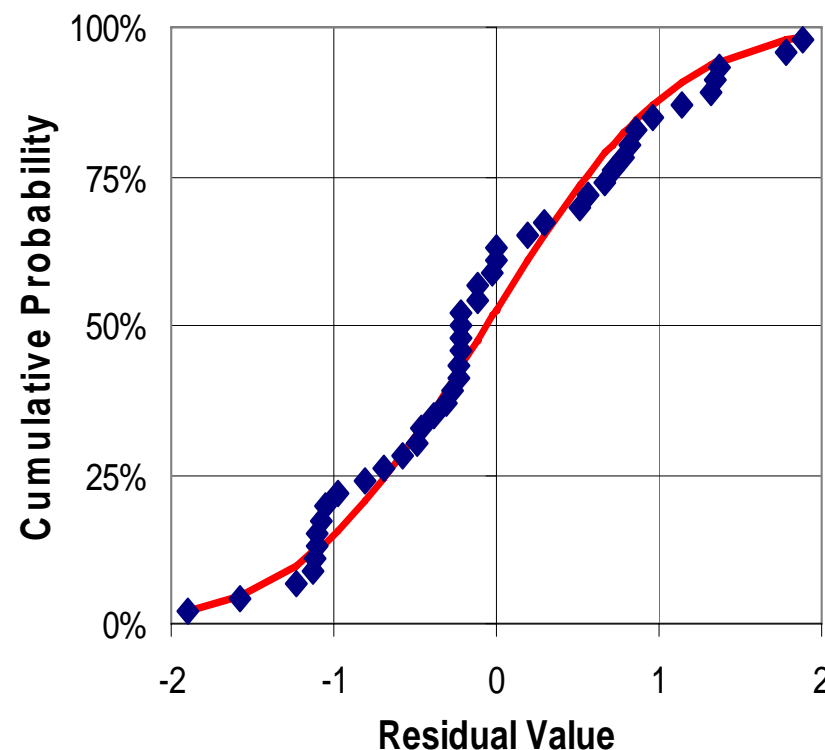
Average   = -0.05
Median   = -0.22
Stdev   = 0.91

In our example, all stochastic methods provide reasonable results

	Mack B.E. Reserve	Mack S.E.	GLM B.E. Reserve	GLM S.E.	Bootstrap B.E. Reserve	Bootstrap S.E.	Bayesian B.E. Reserve	Bayesian S.E.
1996	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1997	0.1	0.0	0.1	0.1	0.1	0.2	0.1	0.1
1998	0.2	0.1	0.2	0.2	0.2	0.2	0.2	0.2
1999	0.5	0.2	0.5	0.3	0.4	0.3	0.5	0.3
2000	1.2	0.3	1.2	0.4	1.2	0.4	1.2	0.4
2001	2.1	0.4	2.1	0.6	2.0	0.6	2.1	0.6
2002	4.3	0.5	4.3	0.8	4.2	0.8	4.3	0.8
2003	7.3	1.0	7.3	1.1	7.1	1.1	7.3	1.1
2004	11.1	1.4	11.1	1.3	10.9	1.2	11.1	1.3
2005	43.0	3.3	43.0	2.9	44.0	3.0	43.2	2.9
Total	69.8	4.1	69.8	4.1	70.1	4.4	70.1	4.1

- **GLM:** Over-Dispersed Poisson (ODP) parameterization used.
- **Bayesian:** Based on ODP.
  Prior information: Gamma distributed ultimate loss ratio with Average = 75%; Stdev = 8%

Note: All calculations without tail extrapolation.

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But even in our data we could have observed
some trends in the residuals:

AY/DY	1	2	3	4	5	6	7	8
1996	-0,2	-1,1	-1,9	-1,2	-0,2	-1,1	-1,1	0,7
1997	0,0	-1,1	-0,3	-0,2	-0,8	-0,1	0,8	-0,7
1998	-0,5	-1,1	-0,6	-0,1	1,8	1,4	0,3	
1999	-0,4	-1,1	0,2	-1,0	-0,3	0,0		
2000	-0,2	0,6	-0,2	1,4	-0,2			
2001	1,1	1,0	0,5	0,8				
2002	1,9	0,9	1,3					
2003	-0,5	0,7						
2004	-1,6							

Accident Year Trend

Calendar Year Trend
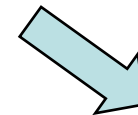
What to do if things are not that easy – How to deal with significant trends in the data?

- Option 1: Apply judgement (e.g. Practical method)
- Option 2: De-trend data before model input:
    - Introduce trends such that residuals appear independent of accident year, calendar year or development year.
    - Trends introduced should be explainable.
    - Estimate chainladder factors and volatility parameters (scale-independent!) based on de-trended data.
    - It may be difficult to re-introduce the trends on results!
- Option 3: Incorporate trends into the model:
    - Use GLM's since they offer a much richer parameter space.
    - This is not complicated!

Conclusion

- The available stochastic methods are based on different approaches and can lead to different results.

- A solid understanding of the models is essential.

- As models are more complex, they require the actuary to be more rigorous in assessing how well a model fits the given data.

- The actuary is responsible for selecting the appropriate approach, depending on the specific situation.

- Not only mathematical and statistical arguments count. Experience, expert knowledge and additional, non-actuarial knowledge is also necessary.

Thank you!

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