The Actuarial Profession
making financial sense of the future

Risk and Investment Conference 2010
Mark Greenwood and Simona Svoboda

LPI swaps
Pricing and Trading

13-15 June 2010

Agenda

1. What is LPI?
2. The inflation options market
3. The inflation volatility smile
4. Modelling the inflation volatility smile
5. SABR
6. A simple LPI model
7. Conclusion and discussion

Types of LPI

LPI, *Limited Price Indexation*, is the RPI link in pensions

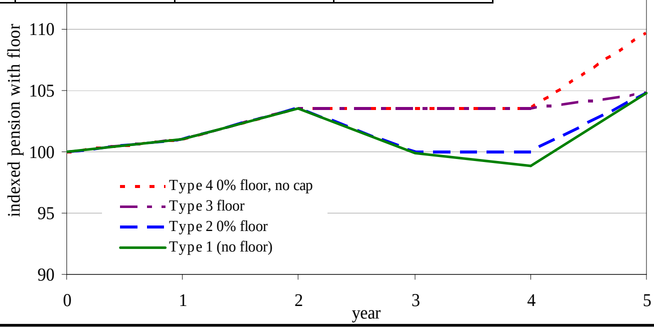
Following Wilkie(1988), we define various types of LPI indexation:

- Type 1: $LPI_t = RPI_t$
- Type 2: $LPI_t = \max[RPI_0*(1+floor\%)^t , RPI_t]$
- Type 3: $LPI_t = \max[RPI_0, RPI_1, \ldots, RPI_t]$
- Type 4: $LPI_t = LPI_{t-1} * \min[\max[1+floor\%, RPI_t / RPI_{t-1}], 1+cap\%]$
- Type 5: $LPI_t = LPI_{t-1} * [1+participation\% * (RPI_t/RPI_{t-1} - 1)]$

For 0% floors, Type 1 < Type 2 < Type 3 < Type 4

Illustration of types of LPI

	RPI_t	RPI_t	0% floor	Type 2 floor at 0%		Type 3 floor at 0%		Type 4 floor at 0%	
t	*(Type 1)*	*y/y%*	*$index_t$*	*pension*	*increase*	*pension*	*increase*	*pension*	*increase*
0	**100.00**		100.00	**100.00**		**100.00**		**100.00**	
1	**101.00**	**1.0%**	100.00	**101.00**	**1.0%**	**101.00**	**1.0%**	**101.00**	**1.0%**
2	**103.53**	**2.5%**	100.00	**103.53**	**2.5%**	**103.53**	**2.5%**	**103.53**	**2.5%**
3	**99.90**	**-3.5%**	100.00	**100.00**	**-3.4%**	**103.53**	**0.0%**	**103.53**	**0.0%**
4	**98.90**	**-1.0%**	100.00	**100.00**	**0.0%**	**103.53**	**0.0%**	**103.53**	**0.0%**
5	**104.84**	**6.0%**	100.00	**104.84**	**4.8%**	**104.84**	**1.3%**	**109.74**	**6.0%**



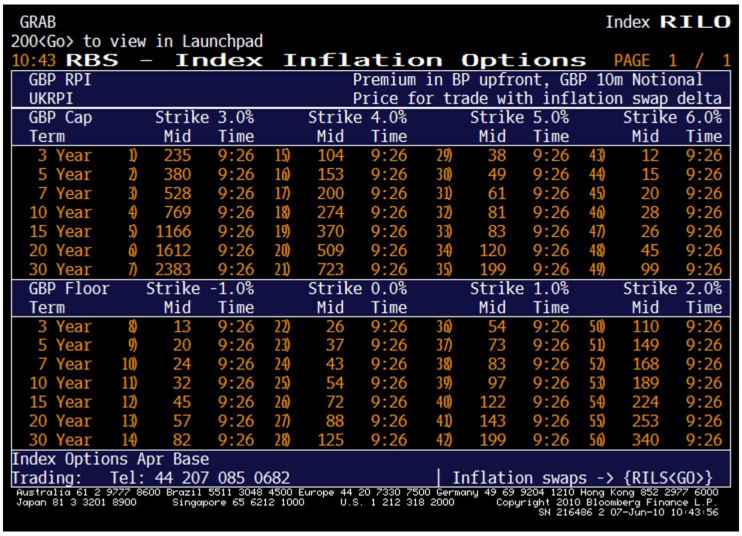
The UK inflation options market

- The building blocks of the RPI derivatives market are zero coupon inflation swaps. These are a hedge for type-1 LPI.
- Three forms of "vanilla" RPI inflation options trade:
  1. Year-on-year (**y/y**) **RPI caps and floors.** The cap has T caplets with payoffs $\max[\,0, (RPI_t/RPI_{t-1}-1) - K\%\,]$ at times t=1,2, …,T.
  2. **RPI index caps and floors** with a single payoff at maturity. The cap payoff is $\max[\,0, RPI_t/RPI_0 - (1+K\%)^T\,]$. Hedge for LPI type-2 liabilities.
  3. **LPI swaps** hedge LPI type-4 liabilities. The most common collar strikes traded are [0%,5%], [0%,3%] and [0%,∞] (i.e. no cap).

The UK inflation options market

- Participants
- Maturities
- Liquidity
- Equilibrium
- Execution costs
- Price transparency

Inflation options screens



GRAB Index RILO

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GBP RPI Premium in BP upfront, GBP 10m Notional

UKRPI Price for trade with inflation swap delta

GBP Cap		Strike 3.0%			Strike 4.0%			Strike 5.0%			Strike 6.0%	
Term		Mid	Time		Mid	Time		Mid	Time		Mid	Time
3 Year	1)	235	9:26	15)	104	9:26	29)	38	9:26	43)	12	9:26
5 Year	2)	380	9:26	16)	153	9:26	30)	49	9:26	44)	15	9:26
7 Year	3)	528	9:26	17)	200	9:26	31)	61	9:26	45)	20	9:26
10 Year	4)	769	9:26	18)	274	9:26	32)	81	9:26	46)	28	9:26
15 Year	5)	1166	9:26	19)	370	9:26	33)	83	9:26	47)	26	9:26
20 Year	6)	1612	9:26	20)	509	9:26	34)	120	9:26	48)	45	9:26
30 Year	7)	2383	9:26	21)	723	9:26	35)	199	9:26	49)	99	9:26

GBP Floor		Strike -1.0%			Strike 0.0%			Strike 1.0%			Strike 2.0%	
Term		Mid	Time		Mid	Time		Mid	Time		Mid	Time
3 Year	8)	13	9:26	22)	26	9:26	36)	54	9:26	50)	110	9:26
5 Year	9)	20	9:26	23)	37	9:26	37)	73	9:26	51)	149	9:26
7 Year	10)	24	9:26	24)	43	9:26	38)	83	9:26	52)	168	9:26
10 Year	11)	32	9:26	25)	54	9:26	39)	97	9:26	53)	189	9:26
15 Year	12)	45	9:26	26)	72	9:26	40)	122	9:26	54)	224	9:26
20 Year	13)	57	9:26	27)	88	9:26	41)	143	9:26	55)	253	9:26
30 Year	14)	82	9:26	28)	125	9:26	42)	199	9:26	56)	340	9:26

Index Options Apr Base

Trading: Tel: 44 207 085 0682 | Inflation swaps -> {RILS<GO>}

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
SN 216486 2 07-Jun-10 10:43:56

Source: Bloomberg 6 June 2010

The implied inflation volatility smile

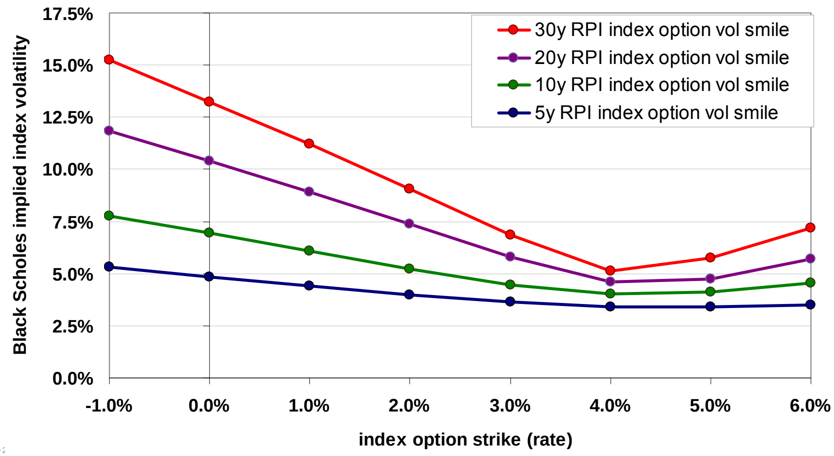
- Inflation options trade on price, not volatility
- Prices can be inverted to implied volatilities using the conventional options pricing formulae for each market
- For ***RPI index options***, it is natural to use the Black Scholes (lognormal) model to price since the index is always positive and is expected to grow exponentially:

$$C = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$P = Ke^{-r(T-t)} - S + C(S,t)$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Implied RPI index volatility smile

Type-2 LPI liabilities can be priced directly from the implied volatility given the strike and maturity



Source: Bloomberg composite 6 June 2010

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The inflation implied volatility smile: y/y options

- For ***RPI y/y options***, the y/y rate may be negative so the Black Scholes lognormal model is not appropriate
- The market convention is to assume the underlying y/y rate has a normal distribution and use the Bachelier(1900) model. The resulting vol σ is called the *normal vol* or *basis point vol*:

$$C = e^{-r(T-t)}[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}]$$

$$P = e^{-r(T-t)}[(K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}]$$

$$where\ d_1 = \frac{F-K}{\sigma\sqrt{T-t}}$$

See VBA code in the spreadsheet on the conference website.

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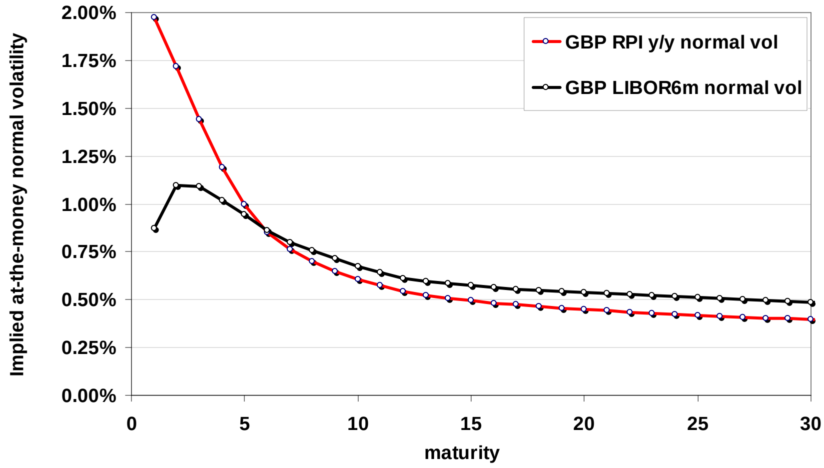
The RPI y/y inflation options market

RPI y/y atm inflation vol > GBP LIBOR atm caplet floorlet vol out to 5 years



Source: RBS, 6 June 2010

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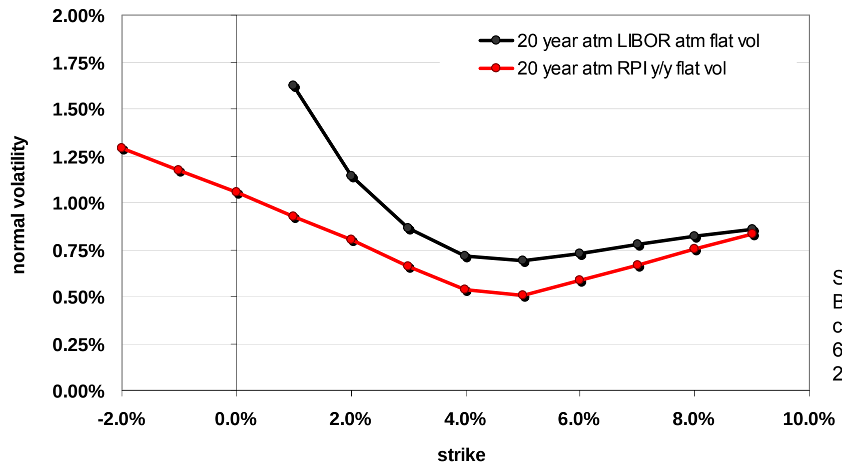
The RPI y/y inflation options market

RPI y/y inflation vol smile compared with LIBOR cap floor vol smile



Source: Bloomberg composite 6 June 2010

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Modelling the implied volatility smile

- Historically, the implied volatility smile first appeared in equity option markets
- Techniques developed for equities have subsequently been used in interest rate and inflation markets
  - Alternate stochastic process
  - Local volatility models
  - Stochastic volatility models
  - Lévy processes

Volatility smiles - alternate stochastic process

- The presence of a smile may interpreted as a deviation from lognormality in the dynamics of the underlying
- Popular processes include the CEV (constant elasticity of variance) process

$$dS = \mu dt + \sigma S^{\beta} dW \quad \text{where} \quad 0 \leq \beta \leq 1$$

- And the shifted-lognormal (displaced-diffusion) process

$$d(S + \alpha) = \mu dt + (S + \alpha)\sigma dW$$

- Both processes allow a monotonically downward sloping skew, however alternate techniques are needed to introduce curvature.

Volatility smiles – local volatility models

- Volatility is a function of the underlying itself, hence

$$dS = \mu S dt + \sigma(t, S) dW$$

- First introduced by Dupire(1994) and Derman and Kani(1994).

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Stochastic volatility models

- The underlying and its volatility are driven by correlated Brownian motions
- A well known stochastic volatility model is due to Heston(1993)

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^1$$
$$dv_t = \kappa[\theta - v_t]dt + \sigma\sqrt{v_t} dW_t^2 \quad \text{where} \quad dW_t^1 dW_t^2 = \rho dt$$

- Semi-analytical option prices may be found via Fourier transform techniques as

$$C(S, v, t) = SP_1 - KP(t, T)P_2$$

where $P(t,T)$ is the T-maturity discount bond and P_1 and P_2 are the associated probabilities evaluated via numerical integration.

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Lévy processes

- General class of processes with independent and stationary increments
- This includes familiar processes such as Brownian motion and Poisson process which introduces random jumps into the dynamics
- All other Lévy processes are generalisations of a Brownian motion and a possibly infinite number of Poisson processes.

Lévy processes

Lévy processes that have become popular in finance include:

– *Jump-diffusion* – dynamics are driven by a diffusion and a finite number of Poisson processes,
– *Variance-gamma* – may be interpreted as a Brownian motion evaluated at a time given by a gamma process, see Madan et. al. (1990,1991,1998),
– *Normal-Inverse-Gaussian* – may be interpreted as a Brownian motion evaluated at a time given by an Inverse-Gaussian process, see Barndorff-Nielsen (1997,1998).

SABR (Stochastic Alpha Beta Rho) model

- This stochastic volatility model is the market standard for fitting an implied volatility smile,

$$dF_t = \alpha_t F_t^{\beta} dW_t^1, \qquad F_0 = f$$
$$d\alpha_t = \nu \alpha_t dW_t^2, \qquad \alpha_0 = \alpha$$

  where $dW_t^1 dW_t^2 = \rho dt$ and ν, β are constants such that $0 \le \beta \le 1$ and $\nu \ge 0$.
- It is not a true stochastic volatility model since a separate set of parameters ν, β and ρ are associated with each maturity.
- The great advantage is that the equivalent Black-Scholes (lognormal) implied volatility may be approximated analytically by $\sigma_B(K,f)$ defined as follows:

SABR (Stochastic Alpha Beta Rho) model

$$\sigma_B(K,f) = \frac{\alpha}{(fK)^{\omega/2}\left\{1 + \frac{\omega^2}{24}\log^2 f/K + \frac{\omega^4}{1920}\log^2 f/K + \cdots\right\}} \cdot \left(\frac{z}{\chi(z)}\right)$$
$$\cdot \left\{1 + \left[\frac{\omega^2}{24}\frac{\alpha^2}{(fK)^{\omega}} + \frac{1}{4}\frac{\rho\beta\nu\alpha}{(fK)^{\omega/2}} + \frac{2-3\rho^2}{24}\nu^2\right]T + \cdots\right\}$$

where $\omega = 1 - \beta$ and

$$z = \frac{\nu}{\alpha}(fK)^{\omega/2}\log f/K \qquad \chi(z) = \log\left\{\frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1-\rho}\right\}$$

SABR Normal model

- For options on the RPI y/y rate, we use a normal model of the underlying and the normal option pricing formula of Bachelier. Hence:

$$dF_t = \alpha_t dW_t^1, \qquad F_0 = f$$
$$d\alpha_t = \nu\alpha_t dW_t^2, \qquad \alpha_0 = \alpha$$

  where $dW_t^1 dW_t^2 = \rho dt$ and ν is constant such that $\nu \geq 0$.
- The equivalent normal implied volatility may be approximated analytically as

$$\sigma_N(K,f) = \alpha \cdot \left(\frac{z}{\chi(z)}\right) \cdot \left\{1 + \frac{2-3\rho^2}{24}\nu^2 T + \cdots\right\}$$

  where

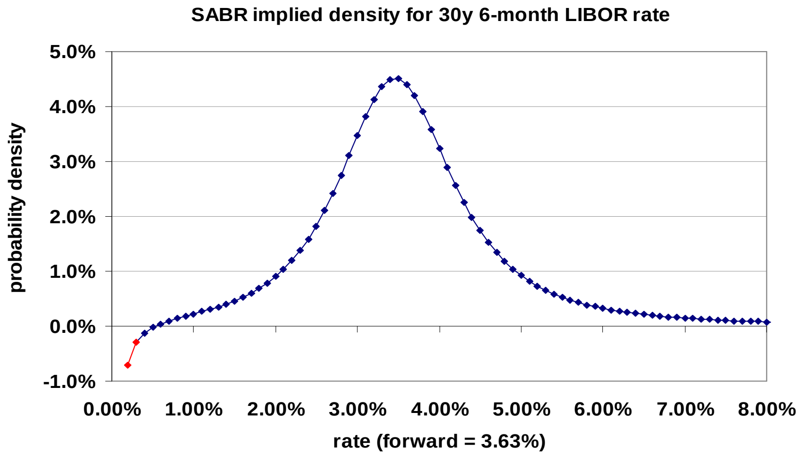
$$z = \frac{\nu}{\alpha}(f-K), \qquad \chi(z) = \log\left\{\frac{\sqrt{1-2\rho z + z^2} + z - \rho}{1-\rho}\right\}$$

Weaknesses of the SABR model

- These analytic approximations are derived via singular perturbation techniques relying on a 'small volatility' expansion, hence assuming both volatility α and volatility-of-volatility ν are small.
- For extreme parameter values and strikes far away-from-the-money, these approximations break down.
- This is well known by the market and each market participant has their own set of proprietary 'fixes'.
- The breakdown of these approximations is most clearly visible if one examines the probability density function derived from call spreads using implied volatilities, which becomes negative at extreme parameter values and away-from-the-money.

SABR model: negative probabilities

SABR model implied density for F=3.63%, α=1.25%, β=50%, ρ=15%, ν=22%



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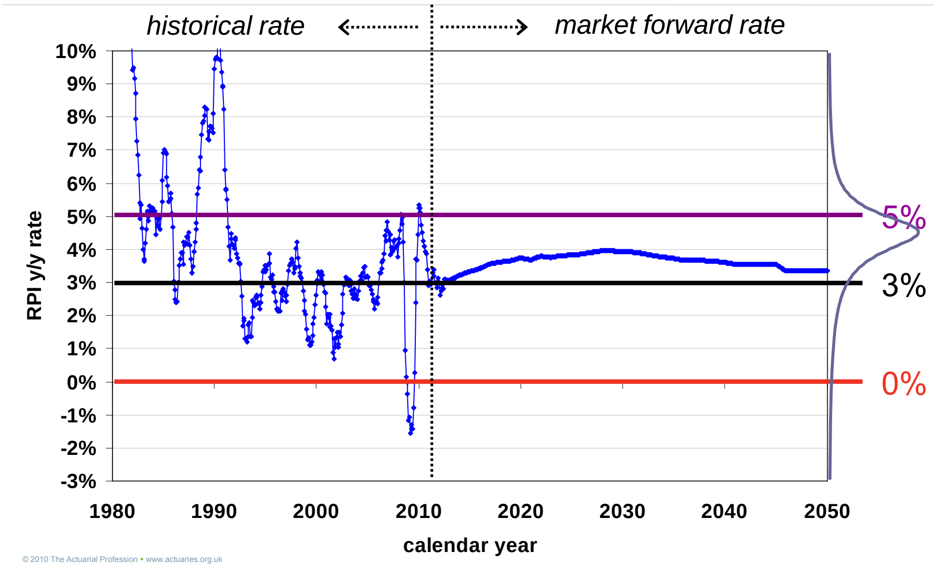
Simple LPI model: RPI y/y history and forward rates



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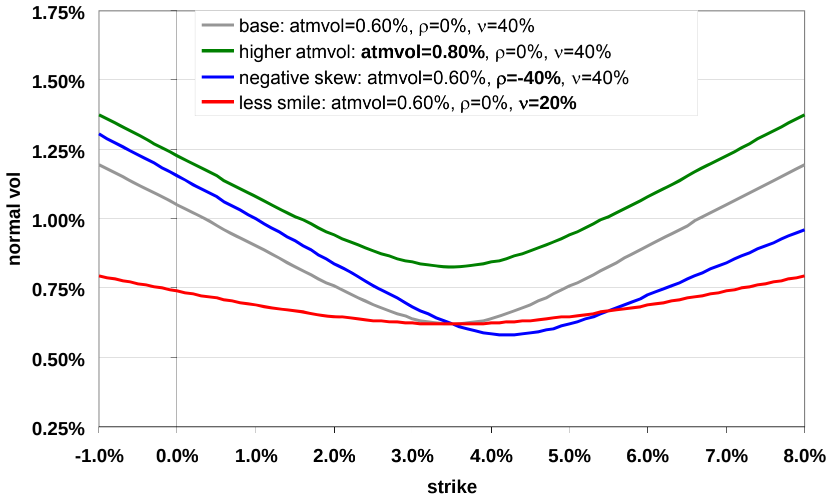
SABR normal volatility smiles: effect of parameters



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Simple LPI model

- Type 4: $LPI_t = LPI_{t-1}$* min[max[1+floor%, RPI_t /RPI_{t-1}], 1+cap%]
- A 30y LPI swap has 60 RPI y/y options embedded in the swap:

$$LPI_{30} = LPI_0 * (RPI_1 / RPI_0 - 1 + floor_1 - cap_1)$$
$$* (RPI_2 / RPI_1 - 1 + floor_2 - cap_2)$$
$$: \quad : \quad :$$
$$* (RPI_{30} / RPI_{29} - 1 + floor_{30} - cap_{30})$$

- Most LPI swap trades use 3 strikes: [0,5], [0,3] and [0,∞]. SABR RPI y/y normal model has 3 parameters: α, ρ and ν *(in our example spreadsheet we reparameterise as: atm vol, ρ and ν)*
- In practice a unique fit can be usually identified

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m2 mark greenwood, 08/06/2010

Simple LPI model

1. FIND SABR PARAMETERS

2. TO GIVE BEST FIT

solveSABRparms

FITTING A SIMPLE SABR LPI MODEL

04-Jun-10	valuation date
01-Apr-10	RPI zc swap base
01-Apr-10	last published RPI

maturity	pay date	index date	RPI zc%	y/y fwd	SABR atmvol	SABR rho	SABR sigma	0% LPI[0,5] zc%	0% LPI[0,3] zc%	0% LPI[0,inf zc%	goodness of fit	LPI[0,5] market	LPI[0,inf market	LPI[0,3] market
0	04-Jun-10	01-Apr-10			1.98%	-26.0%	48.4%							
1	04-Jun-11	01-Apr-11	**2.990%**	**2.990%**	1.976%	-26.0%	48.4%	-0.049%	-0.694%	0.112%				
2	04-Jun-12	01-Apr-12	3.061%	3.131%	1.717%	-29.0%	47.7%	-0.063%	-0.767%	0.172%				
3	04-Jun-13	01-Apr-13	3.059%	3.055%	1.459%	-32.0%	46.9%	-0.047%	-0.768%	0.215%				
4	04-Jun-14	01-Apr-14	3.097%	3.212%	1.200%	-35.1%	46.2%	-0.039%	-0.781%	0.234%				
5	**04-Jun-15**	**01-Apr-15**	**3.143%**	**3.329%**	0.941%	-38.1%	45.4%	**-0.032%**	**-0.790%**	**0.236%**	0.1	**-0.031%**	**0.236%**	**-0.790%**
6	04-Jun-16	01-Apr-16	3.188%	3.414%	0.865%	-37.1%	44.2%	-0.029%	-0.805%	0.240%				
7	04-Jun-17	01-Apr-17	3.241%	3.558%	0.789%	-36.1%	42.9%	-0.031%	-0.828%	0.243%				
8	04-Jun-18	01-Apr-18	3.288%	3.619%	0.713%	-35.1%	41.7%	-0.033%	-0.849%	0.244%				
9	04-Jun-19	01-Apr-19	3.329%	3.653%	0.638%	-34.1%	40.4%	-0.035%	-0.866%	0.242%				
10	**04-Jun-20**	**01-Apr-20**	**3.370%**	**3.743%**	0.562%	-33.1%	39.2%	**-0.037%**	**-0.884%**	**0.239%**	0.1	**-0.037%**	**0.239%**	**-0.883%**
11	04-Jun-21	01-Apr-21	3.399%	3.683%	0.549%	-35.5%	38.7%	-0.036%	-0.893%	0.239%				
12	04-Jun-22	01-Apr-22	3.432%	3.796%	0.536%	-37.9%	38.1%	-0.036%	-0.909%	0.240%				
13	04-Jun-23	01-Apr-23	3.457%	3.756%	0.522%	-40.3%	37.6%	-0.033%	-0.919%	0.243%				
14	04-Jun-24	01-Apr-24	3.482%	3.816%	0.509%	-42.7%	37.1%	-0.031%	-0.932%	0.247%				
15	**04-Jun-25**	**01-Apr-25**	**3.505%**	**3.825%**	0.496%	-45.1%	36.5%	**-0.027%**	**-0.942%**	**0.252%**	0.0	**-0.027%**	**0.252%**	**-0.942%**
16	04-Jun-26	01-Apr-26	3.527%	3.859%	0.487%	-47.0%	36.2%	-0.024%	-0.953%	0.257%				
17	04-Jun-27	01-Apr-27	3.550%	3.911%	0.478%	-48.9%	35.8%	-0.020%	-0.965%	0.263%				
18	04-Jun-28	01-Apr-28	3.572%	3.957%	0.469%	-50.9%	35.5%	-0.017%	-0.978%	0.269%				
19	04-Jun-29	01-Apr-29	3.592%	3.956%	0.460%	-52.8%	35.1%	-0.013%	-0.989%	0.275%				
20	**04-Jun-30**	**01-Apr-30**	**3.609%**	**3.923%**	0.450%	-54.8%	34.7%	**-0.008%**	**-0.997%**	**0.282%**	0.0	**-0.008%**	**0.281%**	**-0.998%**
21	04-Jun-31	01-Apr-31	3.624%	3.919%	0.445%	-54.8%	34.5%	-0.003%	-1.004%	0.288%				
22	04-Jun-32	01-Apr-32	3.635%	3.875%	0.439%	-54.8%	34.3%	0.003%	-1.009%	0.295%				
23	04-Jun-33	01-Apr-33	3.643%	3.814%	0.433%	-54.9%	34.1%	0.009%	-1.010%	0.302%				
24	04-Jun-34	01-Apr-34	3.648%	3.767%	0.428%	-54.9%	33.8%	0.016%	-1.010%	0.310%				
25	**04-Jun-35**	**01-Apr-35**	**3.651%**	**3.726%**	0.422%	-54.9%	33.6%	**0.023%**	**-1.007%**	**0.317%**	0.1	**0.023%**	**0.317%**	**-1.007%**
26	04-Jun-36	01-Apr-36	3.652%	3.674%	0.417%	-54.9%	32.9%	0.030%	-1.004%	0.324%				
27	04-Jun-37	01-Apr-37	3.652%	3.663%	0.413%	-54.9%	32.2%	0.037%	-1.000%	0.331%				
28	04-Jun-38	01-Apr-38	3.653%	3.663%	0.408%	-54.8%	31.5%	0.043%	-0.996%	0.337%				
29	04-Jun-39	01-Apr-39	3.653%	3.644%	0.403%	-54.8%	30.8%	0.048%	-0.993%	0.343%				
30	**04-Jun-40**	**01-Apr-40**	**3.651%**	**3.608%**	0.399%	-54.8%	30.0%	**0.054%**	**-0.989%**	**0.348%**	0.1	**0.054%**	**0.348%**	**-0.989%**

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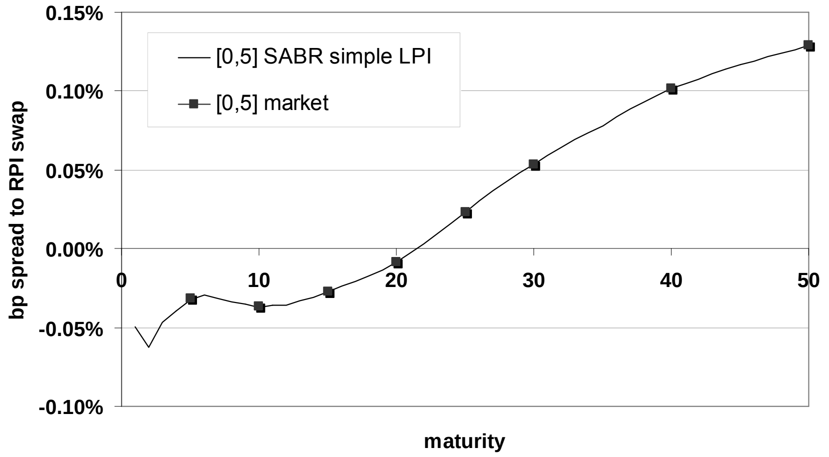
Simple LPI model: LPI[0,5] fit good



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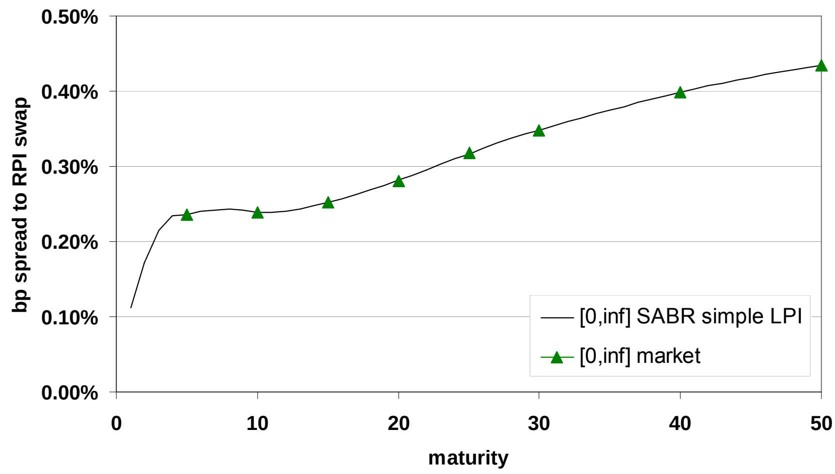
m4 mark greenwood, 08/06/2010

Simple LPI model: LPI[0,∞] fit good



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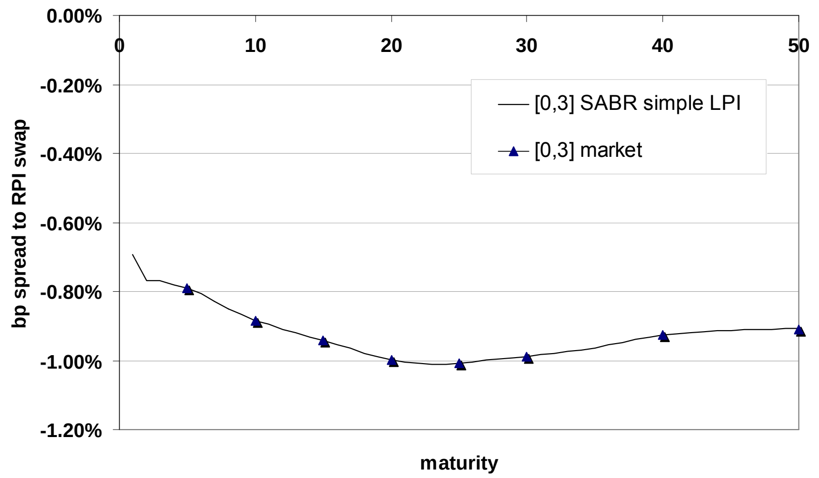
Simple LPI model: LPI[0,3] fit good



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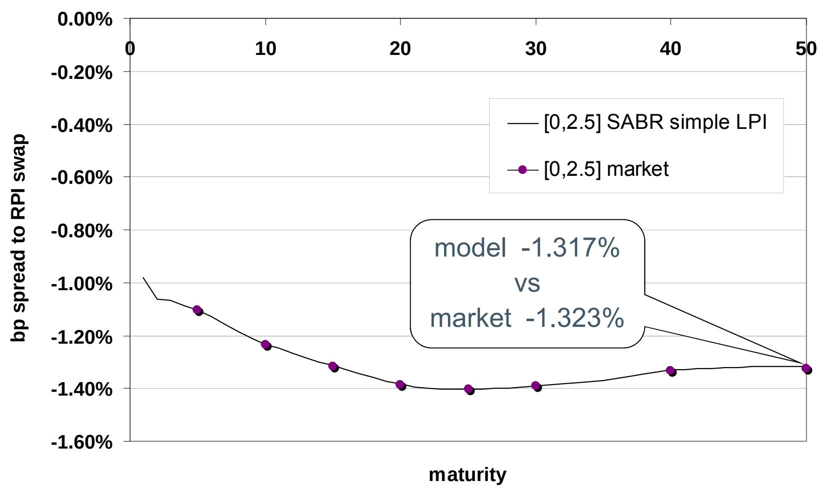
Slide 30

m5 mark greenwood, 08/06/2010

Simple LPI model: LPI[0,2.5] fit good



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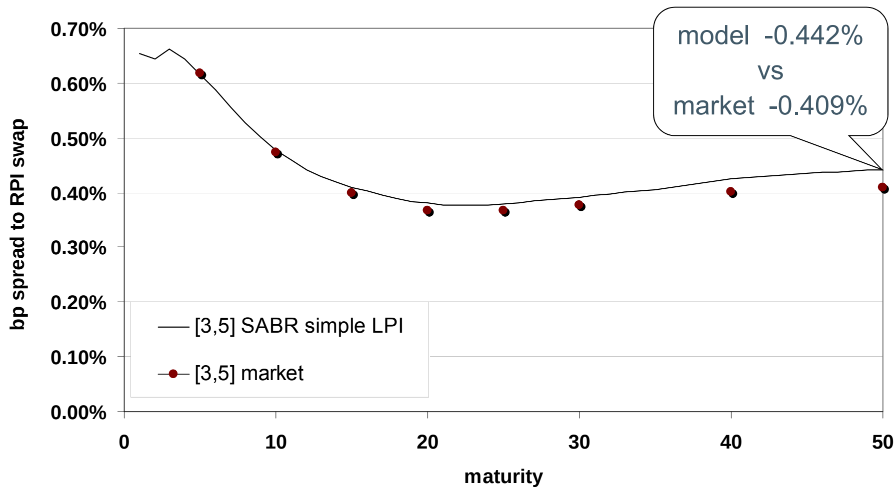
Simple LPI model: LPI[3,5] fit poor in 50y



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Simple LPI model: pros and cons

- + recovers RPI and main LPI swap rates and allows alternative strikes, maturities and RPI reference dates to be priced quickly
- + risk to the model parameters (*the greeks*) is quick and simple

Simple LPI model: the greeks

DELTA AND SABR VEGA LADDERS
per £1m notional LPI swap

	DELTA notional('£m')	DELTA 1bp	DELTA notional('£m')	VEGA atmvol 1bp	VEGA ρ 1%	VEGA ν 1%
	RPI zc 30y	LPI[0,5] 30y	LPI[0,5] 30y	LPI[0,5] 30y	LPI[0,5] 30y	LPI[0,5] 30y
5y	498	-11	-0.02	63	251	-251
10y	944	-12	-0.01	207	214	-309
15y	1339	45	0.03	170	230	-467
20y	1756	-40	-0.02	237	243	-625
25y	2186	-72	-0.03	48	270	-667
30y	2620	2094	0.80	-56	169	-432
40y	3436	0	0.00	0	0	0
50y	4204	0	0.00	0	0	0
TOTAL		2003		669	1376	-2752

RPI zc swap notionals (in £'m) to delta hedge £1million LPI[0,5] liability

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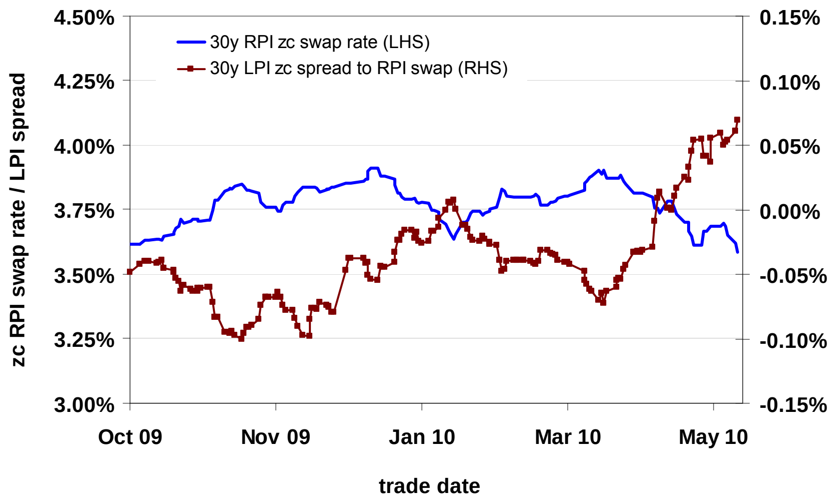
Simple LPI model: pros and cons

+ recovers RPI and main LPI swap rates and allows alternative strikes, maturities and RPI reference dates to be priced quickly
+ risk to the model parameters (*the greeks*) is quick and simple
+ effects on LPI swap rates and greeks of RPI swap scenarios or curve moves are readily explored
- is not a true model, recovers RPI zc swap rates but does not recover market prices of y/y and index options
- does not price other types of LPI swaps or other inflation derivatives

RPI vs LPI[0,5] swap market rates



Source: RBS

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Conclusion and discussion

- The implied RPI volatility smile is an important feature of the inflation options market. The skew towards expensive floors/cheaper caps is extreme as a result of lack of natural supply of floors
- LPI models proposed in literature have had far more general applicability, but have not emphasised the effect of the smile
- The simple type-4 LPI model presented may assist with "interpolating" values for LPI liabilities, calculation delta and vega risks and understanding dealer execution charges for these greeks.

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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