

**PRICING IN
THE LONDON MARKET
PART 2
PRACTICAL PRICING
NON MARINE MARKET**

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1. Introduction

- 1.1 Following the paper produced for last year's GISG [5], in which we set out the basis for pricing, the Working Party decided to concentrate on the actual pricing issues. The original paper dealt with two main issues. The first part dealt with the actual pricing formula which may be used to determine a rate. The second part dealt with the control cycle and stochastic profit testing, and the interaction between the rating process and the profit performance of the business. It is proposed that the second part of the paper be expanded into an Institute paper for the 1987/88 session. The working party decided, for the 1996 GISG conference, to concentrate on developing the first part of the paper, namely the formal calculation of the rates.
- 1.2 The original paper dealt with pricing from the "formula" point of view. This forms an appropriate background to the process. The "formula" approach is well documented in the paper, and elsewhere in the various references. What tends to be lacking is the application of these formulated approaches to real life situations.
- 1.3 The problems with rating in the market is that many of the issues relate to the lack of ideal data, an uncertainty of the rating parameters from the underwriter, a changing market and so on. The use of a static formula approach does not necessarily answer these rating question in dynamic and changing conditions.

- 1.4 It was decided to divide the working party into specific three areas
1. Short Tail Non Marine
 2. Long Tail/ Casualty Non Marine
 3. Marine.
- 1.5 The Marine subgroup have not issued a report, and it is proposed to continue the work for next years session. There is lots of information available, but little in the right format. The Equitas project was, itself, a major strain on resources. There was insufficient time to undertake the analysis of the available information, and, accordingly, the Rating of Marine risks was deferred until the next GISG in 1987.
- 1.6 The Non Marine Casualty Group was led by Andrew Hitchcox. It is the intention to produce a detailed workshop type presentation on the day, with fuller documentation being presented . The main points that will be covered are given in section 2 of this report
- 1.7 The Non Marine Short Tail Group was led by Torbjorn Magnusson. A detailed paper setting out their work is given in section 3.
- 1.8 The rating issues in the Market are becoming more relevant as companies become increasingly aware that the involvement of actuaries in the process will give added value, and may provide a competitive edge.

2. Casualty Excess of Loss Pricing in the London Market

2.1 The presentation will deal with the following bullet points and issues:

- An overview of the process.
- The frequency/severity approach to experience rating.
- The dovetailing of this approach with exposure rating at the higher layers.
- The illustration of special features.
 - Contract terms and conditions:
 - Treatment of ALAE
 - Indexation of limits and deductibles
 - Items which depend on aggregate claims. e.g. swing-rates, aggregate deductibles and limits
 - The underlying business:
 - Underlying limits
 - Exposure changes
 - Underlying rate changes
 - Case reserve strength/adequacy
 - Sensitivity Assumptions:
 - Inflation
 - Frequency trends
 - Payment patterns.
- Communication with the underwriter
- Turning the loss cost into a rate.

3. TOPICS IN PROPERTY REINSURANCE PRICING

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3.1. Introduction

- 3.1.1 Property reinsurance is not an area where actuaries have been traditionally involved. Part of the reason for this is simply that property business, as a rule, is short tailed, and actuaries have historically been preoccupied with long-tailed business, where many of the problems are perceived to have been.
- 3.1.2 Nevertheless, there are almost as many important aspects of property business that require as solid a numerical and analytical background as in long-tailed business, and behind what seems to be straightforward experience rating techniques hide many difficult and interesting theoretical and practical issues. This has been made apparent by the big leap forward made by property catastrophe rating since the late eighties, where complex simulation techniques now are common market practice, whereas simple rules of thumb were still being used less than ten years ago. There is no reason to expect that this development will stop at catastrophe business, as there are as many reasons for sound mathematical modelling of noncatastrophe contracts as for catastrophe ones.

- 3.1.3 In this paper we do not propose to cover all the aspects of property pricing that may be used, for example, as a full manual for property reinsurance pricing, but have selected a few important practical issues which may not yet have been given enough attention in the marketplace.
- 3.1.4 The first of these issues is treated in a section on experience rating, specifically intended to illustrate the uncertainty in the traditional Poisson-Pareto parameter estimates widely used in market. The section should motivate actuaries and the reinsurance market to try and establish for better market statistics in property reinsurance.
- 3.1.5 The second issue is in the area of exposure rating, where we take a close look at what first loss curves (exposure rating curves, increased limits functions) actually are, and which sets of curves are available.
- 3.1.6 Finally, we discuss a few important techniques for dealing with aggregate conditions in excess of loss pricing, e.g. reinstatements and annual aggregate deductibles.
- 3.1.7 We have chosen not to include anything on catastrophe rating, as this area is extensively covered in many other contexts, and mathematical modelling techniques are getting so specialised that they benefit from a more detailed analysis.
- 3.1.8 A final introductory remark is that there is still considerable scope for practical, mathematical developments that could increase accuracy of technical pricing in the market. Textbooks still only give very simplistic guidance, generally ignoring most of the practical problems encountered. There is thus still much use for a good general mathematical and statistical

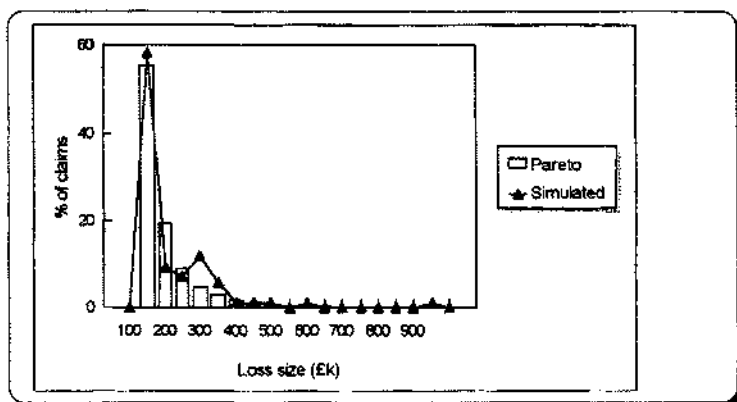
toolbox, that the practitioner-actuary can apply to new problems until solutions to these become more widely available.

3.2 Experience Rating

- 3.2.1 The definition of the expression "experience rating" varies between different contexts, but we are here concerned with trying to fit process parameters to a given claims material, and subsequently use those parameters for extrapolation purposes. This is a technique more frequently used than any other technique in property per risk rating, and is also still much in use in catastrophe rating. A standard reference for experience rating is Philbrick [7].
- 3.2.2 One way to look at the difference between exposure and experience rating is, that in exposure rating the claims distribution is given by market statistics in the form of first loss curves, and no weight is given to the experience of the individual cedant, whilst in experience rating we extrapolate using parameters fitted to the cedant's claims experience, ignoring (at least in the extreme case) how these parameters compare to market statistics.
- 3.2.3 There is a whole host of literature (see for example Sundt [6]) on how to mix the two approaches, often using what is called credibility theory, which can be very fruitful. This has unfortunately not been much used in the reinsurance market, and we believe that one of the missing links in the literature is an illustration of how important such a procedure is, and how uncertain the parameters from fitting parameters to a small become. We will therefore try to address this issue in this section, by simply going through a case study from a straightforward property risk x/l contract, and paying attention to some of the major sources of uncertainty and the corresponding

sensitivities.

- 3.2.4 So let our case cedant have a large commercial and industrial fire portfolio, with a known Pareto loss distribution characterised by a shape parameter $\alpha = 2$. What happens if we generate losses to this portfolio in excess of say £100,000 by way of simulations, and then estimate alpha from these losses? Using a standard maximum likelihood technique, this would exactly simulate the way many reinsurers find shape parameters for extrapolation purposes in experience rating of individual contracts!

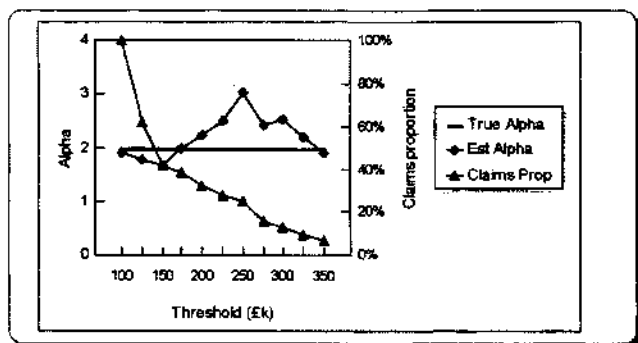


Graph 3.2.1. Comparison of a theoretical and a simulated distribution.

- 3.2.5 Graph 3.2.1 depicts a comparison of the theoretical, true Pareto distribution to the resulting distribution from one typical simulation. Obviously, the relative difference in the tail can be quite substantial, and in this simulation the proportion of losses exceeding £900k was 2.4%, whilst the theoretical proportion is 1.2%.
- 3.2.6 The next step is now to estimate the Pareto alpha from the simulated losses,

using straightforward maximum likelihood estimators, see for instance Hogg and Klugman [8]. To do this, you need to decide from which threshold you want to fit the data, and you will get different answers depending on the chosen threshold, something that is well-known but not widely appreciated in the market. Graph 3.2.2 shows estimated alphas as a function of this threshold compared to the true alpha = 2.0, and also the proportion of the claims above £100k used for the estimate.

- 3.2.7 As is shown in this graph 3.2.2, quite a small change in threshold, say from £200k to £250k which would be very realistic if you were rating a £300k x/s £300k layer, can give rise to a substantial change in alpha, from 2.2 to 3.0. If we were to use these two alphas to rate this same layer the difference in risk premium would be more than 40%. For a typical second layer above the first one of say £600k x/s £600k, the difference would be more than 60%, the figures getting worse the further away you get from the threshold.



Graph 3.2.2. Sensitivity of alpha estimates to claims threshold.

3.2.8 There are several questions arising from these simple graphs, both of a practical and theoretical nature, including

- ◆ What is the probability distribution of the alpha estimate given the threshold?
- ◆ How do you construct confidence intervals for the alphas?
- ◆ What is a suitable threshold, or how do you combine the estimates for different thresholds?

3.2.9 We will not attempt to give detailed answers these questions in this paper, only point out that some of the answers are available in a more general form in textbooks on mathematical statistics, e.g. Arnold [9], or Hogg and Klugman's comprehensive work on loss distributions [8]. The distribution of the alpha estimators is given (in a different context) by Arnold [9] (chapter 7.2) as a gamma distribution with the number of claims as the parameter, which is of some practical interest, and can be used as a basis for confidence intervals.

3.2.10 In practice though, part of the answers to these questions have to be judgmental, taking a view on whether the actual claims are outliers or not. The graphs do imply though, that it is paramount not to ignore market information about what typical values alpha can take for various classes, and possibly to combine this information with the company specific claims patterns.

3.3 Exposure Rating

3.3.1 *The Basic Recipe*

The Basic Recipe for this technique was set out in detail in the 1995 GISG paper on Pricing in the London Market. This can be summarised as a three stage process:

- 1 Determine the "pure premium" for the risk(s) in question
 {e.g. by using the cedants original premium net of loadings}
- 2 Determine the portion of the pure premium for the whole risk that relates to the Excess Layer being costed.
 {e.g. by using Ludwig/Saltzman tables}
- 3 Convert the result of (2) to a Reinsurance Premium for the layer by adding appropriate loadings etc.

The 1995 Paper describes the situation where a whole portfolio of risks is to be protected. For the first part of this paper we discuss the situation where it is a single original risk that is to be protected - in effect a single facultative case - as this serves to clarify the issues. The portfolio situation is merely a case of repeating the operation several times for risks of different size.

3.3.2 *The Pure Premium*

The exposure rating technique is basically a means of splitting the overall pure premium between different layers of cover. For the result to have any

meaning it is necessary to have a meaningful pure premium in the first place. In practice most reinsurance proposals will give information about the "office" premium for the risk(s) and in some instances will give an indication of the original commission terms. A fairly common practice is to take the original gross ("office") rate less the original commission as an approximation to the pure premium. In some instances a further deduction is made so as to allow for the cedant's in house expenses. *The underlying assumption is that the original rate breaks even.* This needs to be tested and suitable adjustments made if thought not to be a reasonable assumption.

3.3.2.1 First Loss Curves

The phrase "First Loss Curve" is a term more usually used in the London Market than Ludwig or Salzmann Tables. The two expressions are, however, substantially the same thing which can be expressed mathematically as follows.

3.3.2.2 An algebraic aside

At the risk of oversimplifying:

Suppose a risk has a claim size distribution function $f(x)$ (and cumulative claim size distribution $F(x)$) where x lies in the range $(0,1)$. This means we are expressing the claim size as a proportion of the Sum Insured (here assumed to be the same as the PML) and consequently no claim can (in theory at least) have a size outside the range $(0,1)$. It is more convenient to work with the range expressed as $(0,1)$ rather than $(0\%, 100\%)$.

The mean claim size "m" is given by:

$$m = \int_0^1 x f(x) dx$$

and the pure premium is ϕm where ϕ is the expected claims frequency.

If we limit any claim to a maximum value of z (where z lies in the range $(0,1)$) then the mean "limited" claim becomes.

$$m_z = \int_0^z x f(x) dx + z \int_z^1 f(x) dx$$

and the "limited" pure premium is ϕm_z

The first loss curve can be written as

$$H(z) = (\phi m_z) / (\phi m) = m_z / m$$

Clearly $H(z)$ is an increasing function with the properties that $H(0) = 0$ and $H(1) = 1$.

In passing it can be noted that in the case where all losses are for 100% of the sum insured $H(z)$ reduces to a straight line. In all other cases the function $H(z)$ must lie above this line.

3.3.2.3 Market Curves

If a reasonable volume of data is available to determine the values of $f(x)$, or equivalently $F(x)$, then it is not a difficult exercise to produce a table of values for $H(z)$ - and an example of how this can be done is to be found at appendix A. Indeed, this is how the Ludwig/Salzmann tables were produced. It must be recognised that these were produced largely from data obtained from Household and small Commercial Property risks. It would be, at the least, a leap of faith to assume that these tables are necessarily appropriate for the kinds of business normally found in the London Market.

In this Market we are usually dealing with large, often non-standard, risks and the curves that are usually used do not always appear to have been

derived from claims data - indeed in many cases there is no way they could have been as there is an inadequate volume of data available.. Rather, they have been deduced from Underwriters expectations of the nature of the risk.

It may, perhaps, be helpful to think of *First Loss Tables* where these have been determined from relevant original data and *First Loss Curves* where these are working assumptions. Obviously the distinction is not black and white; a table may be a heavily smoothed interpretation of the original data and some of the curves in use have been derived at one time from some, albeit possibly imperfect, data but have had parameters greatly changed to reflect different conditions.

What curves are then used in the Market and where did they come from? Some examples follow.

Company A

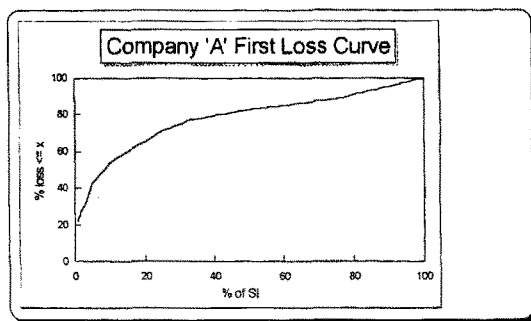
Graph 3.3.1 shows an impressive looking First Loss scale (and the corresponding First Loss table is shown in Appendix B) that it is believed has been widely used in the Market. However, closer inspection reveals it to be nothing more than a series of straight lines if allowance is made for roundings in the figures presented. (Note also that the gradients of the different sections are merely multiples of 0.115).

$y = 88.50\% + .460 * (x - 75\%)$	$100\% > x > 75\%$
$y = 82.75\% + .230 * (x - 50\%)$	$75\% > x > 50\%$
$y = 77.0\% + .345 * (x - 33.3\%)$	$50\% > x > 33.3\%$
$y = 71.25\% + .690 * (x - 25\%)$	$33.3\% > x > 25\%$
$y = 54.00\% + 1.15 * (x - 10\%)$	$25\% > x > 10\%$
$y = 42.50\% + 2.30 * (x - 5\%)$	$10\% > x > 5\%$

Below 5% the figures look particularly curious between 2% and 3% as the rate of increase is only half that in the surrounding intervals.

The slope of the "curve" at the top end gives cause for concern as it is fairly easy to show that a first loss scale cannot have an increasing slope!

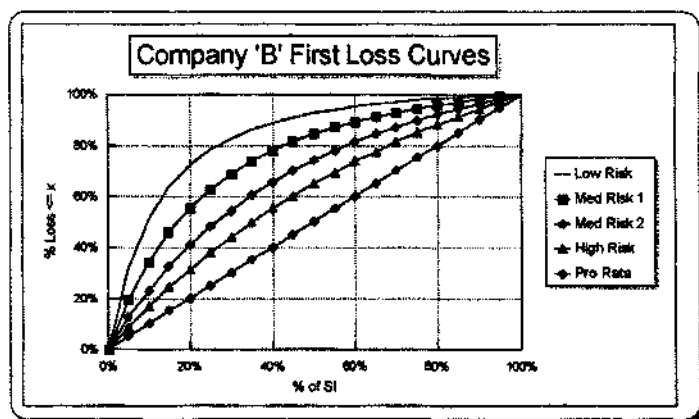
It is believed that this scale was based on data assembled in the mid 1970's - one underwriter suggests (tongue not altogether in cheek) on the strength of "half a dozen claims".



Graph 3.3.1

Company B

In graph 3.3.2, an example is given of a set of so called Pareto curves that were used by one company. This made no pretence as to having been explicitly derived from the data but were formulated in consultation with underwriters. The objective was to produce a set of curves which enabled the underwriting team to be able to produce consistent quotations. Although, in theory, the parameter "c" in the formula could be varied depending on the underwriters perception on the nature of the risk, in practice only the four standard curves were used; all risks were allocated to one of these. It was fashionable to use curves of this general shape in the mid 1980's. Later it became less so and curves of the "Company C" form became more popular.



Graph 3.3.2

The general form of the curve is:

$$H(z) = \lambda(1 - (\frac{z}{c\lambda})^2)$$

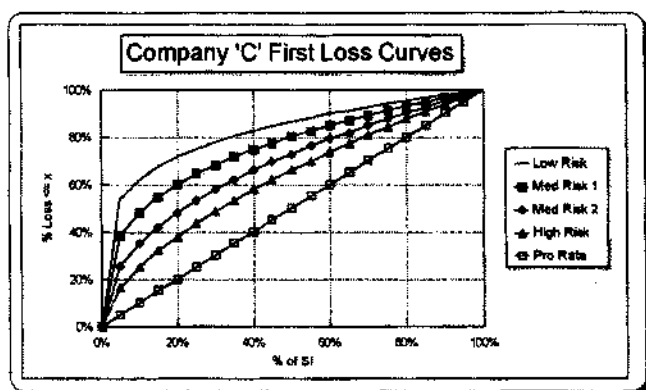
where λ is a scaling parameter to make $H(1)=1$.

Company C

Again four curves are shown, in graph 3.3.3, which represent differing risk types these all take the form:

$$H(z) = z^\alpha$$

The curves again appear to have been chosen because of their simple mathematical form rather than from any data considerations.

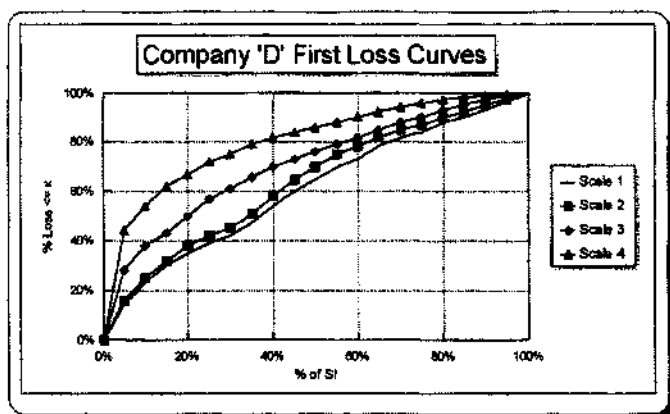


Graph 3.3.3

Company D

The data underpinning the curves in graph 3.3.4 is understood to have come from a study of all fire (but not business interruption?) losses in a small European country in the 1960's. It is believed that these curves are still much used in the Market today. The various scales relate to Sums Insured and, allowing for inflation and exchange rate changes could be described in today's values as:

Scale 1	Sums Insured over £800,000
Scale 2	Sums Insured between £400,000 and £800,000
Scale 3	Sums Insured between £160,000 and £400,000
Scale 4	Sums Insured between £80,000 and £160,000



Graph 3.3.4

Other Formulae and Conclusions

An extremely good fit to US Commercial Property First Loss Data has been obtained by use of a formula of the form:

$$H(z) = (B-1)*z - B*z^A$$

For Swedish householders risks, there exists a detailed study in Benckert and Jung [10], that could possibly be of interest of similar risks in northern Europe. This paper also discusses some of the statistical difficulties in deriving First Loss Curves.

Hybrid curves - weighted averages of some of the "simple" curves - are also used. Perhaps the most useful of these is the type which combines one of the Pareto or Power Curves with a straight line. This effectively allows for a "lump" of probability at a total loss.

As can be seen from the above, one must take great care to choose a relevant and suitable First Loss Curve to get an accurate risk premium. Using different curves produces as widely different answers as varying the Pareto shape parameter in experience rating. Just as in experience rating, the conclusion must be for the reinsurance industry to try to produce better market statistics, by class and country, if the uncertainty about excess of loss risk premiums is going to be reduced.

3.4 Special/Aggregate Property Conditions

3.4.1 Common themes

3.4.1.1 The heading "Special property conditions" of this section is something of a misnomer, as the treaty conditions to be discussed,

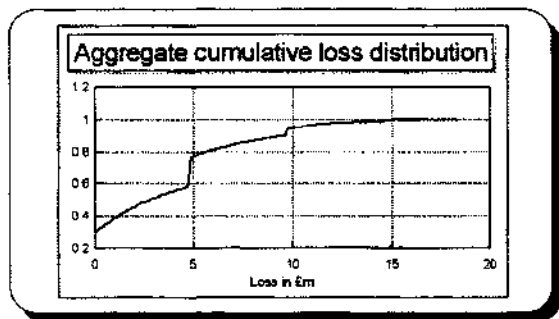
- reinstatements and annual aggregate limits (AAL's),
- aggregate annual deductibles (AAD's),
- no claims bonuses

are very common for property treaties. In fact, reinstatement conditions (other than unlimited free reinstatements) appear in almost all property treaties. Therefore, we will from hereon call the conditions aggregate conditions, as they all operate on the aggregate yearly claim.

3.4.1.2 These special conditions all have two things in common. Firstly, by introducing or modifying these conditions, the technical risk premium should normally be altered substantially. This fact is not always appreciated by the market, where these conditions are sometimes seen as secondary to the risk premium calculation before any of the aggregate conditions are taken into account.

3.4.1.3 Secondly, from an actuarial point of view, the effect of the aggregate conditions can only be seen from calculations on the yearly aggregate loss distribution. As is well-known from the actuarial literature, this distribution is relatively difficult to compute or approximate, and there are a large number of publications presenting classical methods of doing so, e.g. the Edgeworth or Esscher approximation or see, for example, Gerber [1].

3.4.1.4 A typical aggregate loss distribution is shown in the following graph, for a catastrophe treaty covering £5m x/s £5m. Please note the characteristic discontinuities at multiples of the cover limit. The graph was calculated by Panjer recursion, as described in a subsection of this paper.



3.4.1.5 It is very important for an actuary's credibility to gain a solid understanding of when the various conditions are used, and what the advantages and disadvantages of them are. We will therefore try to build in a number of practical examples in the text, giving the reader at least a starting point for this. Unfortunately, the conditions will vary from market to market and from line to line, which makes a full treatment of this to comprehensive for this text.

3.4.1.6 The subsections of the Aggregate Conditions section are classified according to which method is described, rather than according to the reinsurance condition, as all the described methods are applicable to all aggregate conditions. We will, however, look at specific examples to make clear exactly how this can be done in each section. The groups of methods described in the following are Total Loss Approximations, Panjer Recursion and Simulation Techniques.

3.4.1.7 In the section about simulation techniques, our example will be a multiyear treaty, as this is typically too difficult to treat by the other techniques, and a dedicated model often has to be built for every new case. These treaties frequently include combinations of no claims bonuses or profit commissions with changing the layering of a programme or renewal options, and solutions thus are tailor-made for each new client. Investment income also tends to play a more important role than is otherwise normal in property reinsurance.

3.4.2 *Total Loss Only Approximations*

3.4.2.1 Using simple formulae has certain big advantages over more advanced techniques, including

- speed,
- facilitating sensitivity testing, and
- better acceptance from underwriters due to less of a “black box” feeling.

3.4.2.2 As stated in the introduction, aggregate conditions apply to the aggregate loss distribution, which is difficult to treat analytically with formulae. However, there is one special case where it is feasible to carry out the calculations, and that is when it can be assumed that all losses are going to

be total losses.

3.4.2.3 The assumption of total losses arises most naturally in catastrophe reinsurance. The average loss to a typical catastrophe layer is often more than 70% of the cover limit.

3.4.2.4 The most important aggregate conditions in catastrophe business are reinstatements and annual aggregate deductibles. The standard reinstatement condition is one reinstatement at 100%. Let us look at how to calculate the effect of this condition, assuming total losses only (TLO).

3.4.2.5 Let the risk premium with unlimited free reinstatements be P , let us assume Poisson distributed loss frequency with intensity q , and cover limit L . The risk premium (with unlimited free reinstatements) must equal the expected loss by definition, so $P = qL$. Let furthermore the risk premium with 1 reinstatement at 100% be Q . The equation expected premium equals expected claims now reads

$$Qe^{-q} + 2Q(1 - e^{-q}) = Lqe^{-q} + 2L(1 - e^{-q} - qe^{-q})$$

which after simplification gives

$$Q = L(2 - 2e^{-q} - qe^{-q}) / (2 - 2e^{-q})$$

3.4.2.6 Interestingly enough, developing this formulae in a Taylor series gives Q approximately equal to $q(1 - q)$ for small q 's, which is the rule of thumb frequently used in the London Market for many years. For q 's bigger than say 0.05, the full expression should be used, as the approximation becomes poor.

3.4.2.7 Similar calculations can obviously be carried out for other reinstatement conditions, or AAD's/AAL's. However, the calculations do get more

cumbersome with the complexity of the aggregate condition considered and the value of having a formula instead of doing simulations to convince underwriting staff of the correctness of the answer decreases.

- 3.4.2.8 In addition to the TLO assumption, we have in the above made an assumption about Poisson distributed losses. There is nothing to stop us from doing similar calculations for example with negative binomial distributions, although the calculations get quite a lot more involved with this distribution. As there are reasons to believe catastrophe losses may not follow a Poisson distribution (windstorms often coming in groups, earthquakes building up energy over time) some care should be taken in the choice of distribution. Obviously, there is no reason not to carry out the TLO based calculations with a Poisson assumption as a first approximation if that is the distribution used already in the other risk premium calculations.

3.4.3 Panjer Recursion

- 3.4.3.1 If one cannot assume total losses only, one does not necessarily have to use simulations. During the last 15 years, a technique known as Panjer recursion (or as Heckmann-Myers algorithm in the US) has gained ground. This is a fast, recursive algorithm that can approximate the aggregate distribution, in most practically interesting cases, to any degree of accuracy.
- 3.4.3.2 As it is an algorithm, and not a formula, one does not achieve the advantage of easily being able to demonstrate the calculation to the underwriting staff. Nevertheless, the algorithm is very fast, and can be programmed into rating systems or spreadsheets, which can be made available directly to the underwriters, something which is difficult with simulation techniques.

3.4.3.3 We will here discuss practical aspects of Panjer recursion, as a full derivation of the formulae falls beyond the scope of this paper. A good recent theoretical reference for the basic technique is Dickson [2], which does not apply the technique to reinsurance conditions. Other useful references are Beard et al [3] and Straub [4]. There are in fact not many papers on the application of Panjer recursion to aggregate conditions, so we will work out an example below in some detail.

3.4.3.4 When can Panjer recursion be used to calculate the aggregate claims distribution? There are basically two prerequisites. The first one is that the claims frequency distribution must be possible to express in a recursive way, as

$$p_k = (a + b/k)p_{k-1}$$

where p_k is the probability of k claims and a and b are parameters. This condition is satisfied for many useful distributions including the Poisson, negative binomial and logarithmic distributions. The parameters a and b are particularly simple for the Poisson process, a equals zero and b the intensity parameter of the distribution.

3.4.3.5 The second prerequisite for being able to use Panjer recursion is even less of a restriction. The formulae work on a discrete severity distribution, not a continuous one, so therefore one has to be able to discretize the distribution used. Naturally, approximating for instance a Pareto distribution with a discrete distribution in a large number of equidistant sample points would come very close to using the Pareto distribution itself. One of the attractive features of Panjer recursion, though, is that all practical evidence indicates that the number of sample points can be small, say between 5 and 20, and the accuracy will still be very good.

3.4.3.6 Now for the actual formulae. Let q_i be the value of the discrete severity distribution in sample point x_i , the i th sample point. Let furthermore $f(x_i)$ be the aggregate probability-density at x_i , f_0 the probability of no loss, and s the highest index for a sample point. Then, Panjer's core recursion formula says that

$$f(x_j) = \sum_{i=1}^{\min(j,s)} (a + ib/x_i) q_i f(x_{j-i})$$

This is obviously not difficult to programme in any programming language, or for example a Visual Basic macro for a spreadsheet. The extension to calculate the cumulative distribution function from the density is straightforward, keeping in mind that this one will also by necessity be discretized.

3.4.3.7 As there are so relatively few papers applying Panjer recursion to reinsurance, we will here go through the specialisation to the Poisson-Pareto case, and look at how to apply the result to calculate the effect of an AAD. Let us denote the Poisson intensity v , and use the continuous Pareto density cux^{u-1} , where c is a suitable constant, and u is the shape parameter. We will use R , L and U for the retention, cover limit and upper point. A simple discretization for $i = 0, 1, \dots, s$ and $x_i = R, R+Li/s, \dots, U$, gives $q_i = cux_i^{u-1}L/s$ for $i = 0, 1, \dots, s-1$ and $q_s = 1 - \sum_{i=0}^{s-1} q_i$, with obvious possibilities to improve accuracy using a less simplistic discretization technique. Panjer's recursion equation, now reads

$$f(x_j) = (L/s)vcu \sum_{i=1}^{\min(j,s)} \left(\frac{i}{(R+Li/(s+1))} \right) x_i^{u-1} f(x_{j-i}) + (s/U)vq_s f(x_{j-s})\delta_{js}$$

In order to estimate the effect of an AAD to an x/l treaty, the only thing one has to do is to integrate (which reduces to a sum and can be done in a

spreadsheet, as the distribution is discrete) this aggregate distribution with the condition that the corresponding loss is reduced by the AAD (but always stays non-negative).

3.4.3.8 Typical reinstatement conditions for property risk x/l treaties often have a few cheap, or free, reinstatements followed by increasingly expensive ones, usually depending on if it is a bottom, medium or top layer. Any combination of reinstatements and AAD's can easily be treated by the Panjer recursion formula, and AAL's are nothing else than a special case of reinstatements, mathematically speaking. Thus, the Panjer technique can cope with most common situations in property business. No claims bonuses can also often be incorporated, however they are less common, and often require special treatment with simulation techniques because of combinations with other multiperiod conditions.

3.4.3.9 The Panjer technique has shown very powerful and versatile in a number of companies, as well as easy to implement, and will very likely gain even more ground over the next few years, in the London market and elsewhere. As said in a previous paragraph, the results are relatively insensitive to the number of discretization points, and there are few other parameters to select. The only other practically important issue is probably the decision of how long a tail for $f(x_j)$, i.e. the number of discretization points one wants to calculate for the aggregate distribution. If one has say $s = 20$, and the number of losses in the tail of the distribution is more than 10, this means that the sum in the recursion formula will have to be iterated more than 200 times, which may slow it down a bit, depending on the computer solution chosen. Few property x/l treaties have more than a couple of expected losses per year, so a case with 10 losses is certainly an extreme one.

3.4.4 Simulation Techniques

3.4.4.1 In [5], attention was drawn to the versatility of simulation techniques and the widespread use of these in the reinsurance industry by actuaries. It is not the purpose of this paper to teach actuaries how to perform simulation studies, suffice it here to say that the technique is

- ♦ extremely flexible - and there is probably no reinsurance treaty that cannot be modelled in a simulation,
- ♦ very illustrative - as the loss scenarios are shown explicitly in detail to the analyst, but
- ♦ sometimes slow - both the modelling and the executions can be a bit cumbersome, especially when one wants to vary many parameters and conditions.

In this short section, we will just illustrate what can be done by simulation techniques by way of an example, an aggregate risk excess of loss treaty with some unusual features from the European market.

3.4.4.2 The example treaty (which has been doctored slightly for the purpose of this paper) covers commercial and industrial property losses from a relatively big insurance company. The treaty loss is calculated by adding up the 10 biggest losses from ground up, in one year, and applying a layer of £3m x/s £3m to the aggregate loss. The cedant has the right, but not the obligation, to renew the treaty for up to three years, and will receive a profit commission of 20% of total premiums minus total claims and brokerage after three years if he does so. The cedant also has a risk x/l programme inuring to the benefit of this treaty, which in practice means no individual loss can exceed £1m.

3.4.4.3 Obviously, neither a TLO approach nor distribution calculus according to Panjer would be feasible to carry out in this case. With a simulation software

however, the modelling just becomes a question of following the steps outlined in the treaty wording, and trying to find statistical data for the loss distributions necessary.

3.4.4.4 In our example treaty, let us assume that our from ground up losses are Pareto distributed, and that we know a suitable shape parameter and the frequency of losses at a certain observation point. These are by no means a trivial assumptions, as we will have to take into account the change in portfolio size and inflation in history to estimate them. With analytical probability theory, it would be relatively cumbersome to find a closed expression for the distribution of the ten largest losses, but simulations avoid this complication. We simply simulate the outcome for a large number (say 50) of losses, and select the biggest ten in each set of outcomes. This can be done fairly conveniently in most modern simulation packages/spreadsheets. There is also the complication of choosing a small enough threshold to always get at least ten non-zero losses in the simulation, but since our interest in is a layer in excess of £3m in the aggregate, we would not have to consider very small losses.

3.4.4.5 Having added up the biggest 10 losses, we just proceed to set them against the aggregate layer, working out the expected result in a number of simulations.

3.4.4.6 In order to account for the profit commission clause, we will have to make assumptions about circumstances under which the cedant would renew the treaty. One could, for instance, imagine a modelling rule saying that the cedant would renew the treaty unless the result in the first year was too good for the reinsurer, or even that the cedant would do this with a certain probability. With the renewal model worked out, it is simply a matter of

duplicating the one-year model, running it for three years in each execution of the simulation, and finally working out the (expected) impact of the profit commission clause from a large number of simulations.

- 3.4.4.7 The above model will now, at least in spreadsheet-based simulation packages, have become somewhat slow to execute. Despite this, the illustrations produced and the fruitful discussions that can be triggered by showing the model both internally to underwriters and externally to cedants can be invaluable to a better understanding of the cedant's needs and an agreement of a fair price.

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Appendices

Appendix A. Accumulated Loss Cost Distribution by % of Insured Value

x% of Insured Value	#	Losses <=x \$000	Losses >x \$000	1st x% in Losses >x \$000	1st x% cost (3) + (5) \$000	1st x% cost Distribution %
1	114,519	60,964	264,493	36,498	97,462	29.9%
2	134,568	90,542	234,915	42,051	132,593	40.7%
3	143,967	111,091	214,366	42,068	153,159	47.1%
4	149,339	126,695	198,762	40,271	166,966	51.3%
5	152,655	138,424	187,033	38,513	176,937	54.4%
6	154,742	147,163	178,294	37,435	184,598	56.7%
7	156,118	153,887	171,570	36,923	190,810	58.6%
8	157,165	159,621	165,836	36,446	196,067	60.2%
9	157,909	164,195	161,262	36,418	200,613	61.6%
10	158,541	168,476	156,981	36,175	204,651	62.9%
15	160,132	182,589	142,868	37,515	220,104	67.6%
20	160,808	190,571	134,886	41,014	231,585	71.2%
30	161,415	200,732	124,725	49,198	249,930	76.8%
40	161,780	209,715	115,742	55,331	265,046	81.4%
50	162,022	217,337	108,120	60,804	278,141	85.5%
60	162,225	225,270	100,187	64,367	289,637	89.0%
70	162,365	231,490	93,967	68,456	299,946	92.2%
80	162,499	238,725	86,732	70,594	309,319	95.0%
90	162,624	247,005	78,452	70,741	317,746	97.6%
100	163,923	325,457	0	0	325,457	100.0%

Appendix B. Company 'A' First Loss Table

x	% loss < x	% loss < x	B	% loss < x	B
1.00	22.4	10.00	54.0	56	84.1
1.20	23.5	12.00	56.3	58	84.6
1.40	24.7	14.00	58.6	60	85.0
1.60	25.8	16.00	60.9	62	85.5
1.80	27.0	18.00	63.2	64	86.0
2.00	28.1	20.00	65.5	66	86.4
2.10	28.4	21.00	66.6	67	86.7
2.20	28.7	22.00	67.8	68	86.9
2.3	29.0	23.0	68.9	69	87.1
2.40	29.3	24.00	70.1	70	87.3
2.50	29.6	25.00	71.2	71	87.6
2.60	29.8	26.00	72.0	72	87.8
2.70	30.1	27.00	72.7	73	88.0
2.80	30.4	28.00	73.4	74	88.3
2.90	30.7	29.00	74.1	75	88.5
3.00	31.0	30.00	74.8	76	89.0
3.10	31.6	31.00	75.6	77	89.4
3.20	32.1	32.00	76.3	78	89.9
3.30	32.7	33 1/3	77.0	79	90.3
3.40	33.3	34	77.3	80	90.8
3.50	33.9	35	77.6	81	91.3
3.60	34.4	36	78.0	82	91.7
3.70	35.0	37	78.4	83	92.2

X	% loss < x	% loss < x	B	% loss < $\frac{B}{x}$	B
3.80	35.6	38	78.8	84	92.6
3.90	36.2	39	79.2	85	93.1
4.00	36.7	40	79.5	86	93.6
4.10	37.3	41	79.9	87	94.0
4.20	37.9	42	80.2	88	94.5
4.30	38.5	43	80.4	89	94.9
4.40	39.0	44	80.8	90	95.4
4.50	39.6	45	81.1	91	95.9
4.60	40.2	46	81.5	92	96.3
4.70	40.8	47	81.8	93	96.8
4.80	41.3	48	82.1	94	97.2
4.90	41.9	49	82.4	95	97.7
5.00	42.5	50	82.7	96	98.2
6.00	44.8	51	83.0	97	98.6
7.00	47.1	52	83.2	98	99.1
7.50	48.2	53	83.4	99	99.5
8.00	49.4	54	83.7	100	100.0
9.00	51.7	55	83.9		