Probability: An Intermediate Text-Book. By M. T. L. BIZLEY, F.I.A., F.S.S., F.I.S.

[Pp. viii+230. Cambridge: Published for the Institute of Actuaries and the Faculty of Actuaries at the University Press, 1957. 20*s*.]

IN *Calculus and Probability*, Henry disposed of the latter subject in under twenty pages, and the chapters on probability in Freeman's *Mathematics for Actuarial Students* occupy about fifty pages. The actuary whose studies were based on either of those books may therefore suspect that the introduction, as the official textbook, of this volume, extending to more than 200 pages, will impose a serious extra burden on the newly enrolled student studying for Part I of the examinations, but this natural suspicion would be unlikely to survive a study of Bizley's book. The earlier textbooks demonstrated very clearly the solutions of various problems in probability, but they did not attempt a fully systematic treatment of the subject and, as a result, students have found their main difficulty to be that of deciding what method of approach would be likely to lead to a solution in a particular case: this state of affairs should now be remedied, since the great merit of the new textbook is that it provides a framework into which may be fitted almost all the probability problems that arise in practice (or, it might be more correct to say, in the examination room).

This systematic approach can best be brought out by giving a summary of the contents. The book opens, as it must, with a discussion of the meaning of probability. In essence, Bizley elects to employ a version of the classical or unitary definition—an almost inevitable choice for a book that is to discuss mainly problems dealing with coins, dice, packs of cards and the like—whilst setting out clearly the difficulties and limitations of such a definition. [An Appendix sets out briefly certain other definitions and theories of probability, but it is doubtful if the necessarily terse descriptions given will be comprehensible to the student new to the subject; this Appendix may, however, justify itself by encouraging the more enthusiastic student to further reading on some of the theories mentioned.] The author's definition of the probability of an event may be expressed as the ratio of the number of the favourable subclass to the number of the reference class: this necessitates his spending some time in defining 'classes' and 'subclasses' of 'elements'. These pages may not be easy for the student to comprehend at the outset of his course.

From this definition the author goes on to give illustrative examples which can be solved by the direct calculation of the number of the reference class and of the favourable subclass. Inevitably he assumes that the reader is familiar with the theory of permutations and combinations; in practice, students studying for Part I are very often far from having the necessary proficiency and the reviewer believes that this is the most common cause of failure in the examination. Some of these examples, and others elsewhere in the book, require a considerable amount of arithmetic: it would have been better had examples been chosen throughout, as they could have been, so that the arithmetic was simple; the student should work through the examples to ensure that he understands the principles, and heavy arithmetic is an unnecessary distraction.

The second chapter is entitled 'Rules of combination of probabilities' and in this the author derives the addition and multiplication rules and deals briefly with the binomial, multinomial and hypergeometric distributions. A feature of this chapter is the very careful discussion of the question of 'independence';

the student who masters this should avoid the errors that so often arise from the failure to recognize whether probabilities are dependent or independent. Next, in Chapter 3, the author develops and demonstrates the use of the theorems of Waring and Bayes. Waring's theorem, at any rate under that name, may be unfamiliar to some actuaries, but it is a most powerful tool in the solution of many otherwise difficult problems; it may be expressed in the form: 'the probability that just t of r events happen is $\sum_{i=t}^{r}(-1)^{i-t}\binom{i}{t}B_i$', where B_i is the sum of the $\binom{r}{i}$ probabilities that i specified events will happen, irrespective of whether any or all of the other $(r-i)$ events happen also. Bizley shows how this theorem can be used to solve such problems as the classical problem of derangements and the calculation of the probability that, when m objects are put into r cells, just t cells will be empty; the Z-formulae used in life contingencies also follow directly from Waring's theorem. Though the pages on this subject are perhaps the most difficult in the book, the power of the theorem is such that its inclusion is amply justified. Bayes's theorem is better known; it need only be said that great care has been taken to warn the reader of the dangers of its application in cases where the problem is so vaguely stated that no precise value can be given to the prior probabilities.

The author then proceeds to a chapter on repeated trials and expectations. This is excellently done, particularly the section on expectations, where the explanations of the various ways in which expected values may be calculated are very clear: there is a tendency for students to think that the number of trials required for a success can only be obtained by calculating $\sum_{r=1}^{\infty} r\phi_r$, where ϕ_r is the chance that the first success occurs at the rth trial; this is often far from being the simplest approach and there can now be no excuse for ignorance of other methods.

So far, direct methods of calculating probabilities and expectations have been explained. The author now turns to indirect methods, which are so often necessary when the direct approach either fails altogether or becomes too cumbersome. He shows how difference equations (perhaps better known to some as recurring series) may be used in the solution of various problems in probability, and goes on to explain how similar methods can also be used in obtaining expected values. It is the reviewer's experience that students seldom realize that these indirect methods are applicable to expectations as well as to probabilities, and these pages should be of special value on this account. It is to be feared, however, that the knowledge of difference equations that the author assumes is not possessed by all, or even the majority of, Part I students. These indirect methods are also used extensively in Chapter 6 which is entitled, with commendable brevity, 'Runs' and which discusses the probability of the occurrence of sequences of various types in a number of repeated trials, and the allied problem of the expected number of trials required to obtain a particular sequence. The sequences considered are much more varied than the usual runs of exactly or at least n successes or failures, and this adds much to the interest of this chapter. Both here and in the previous chapter several of the methods used do not appear to have been given in other books on this subject.

The final chapter deals with continuous variables; the author, like Freeman, has found it impossible to avoid the separation of the discrete and the continuous case, but a valuable link has been provided here by a most interesting section in

which it is shown how certain 'continuous' problems may be solved by regarding them as limiting cases of analogous 'discrete' problems. In this chapter geometrical methods are dismissed too briefly; though, as the author indicates, they are simply devices for evaluating the corresponding integrals, they are so frequently of service that more attention might well have been paid to them, both in drawing attention to their value and in warning of the dangers of their use in unsuitable cases. Another point that is insufficiently stressed is the need for extreme care in the choice of limits when the method of integration is employed; it will indeed be a red-letter day when the examiners find it unnecessary to comment on students' shortcomings in this matter.

Finally, some impression of the book as a whole should be given. This addition to the series of textbooks published under the authority of the Institute and Faculty has fully maintained, to say the least, the standard set by those already published, and that is a very high standard indeed. Two things in particular stand out. First, a probability textbook depends largely for its success on its worked examples; those given by Bizley are always appropriate to the context, instructive and fully and clearly worked out. Secondly, it is a real pleasure to read this book if only to appreciate the clarity, simplicity and elegance with which the author has expressed himself. Those who have heard Bizley's speeches at Institute meetings will expect something of unusual quality from him; they will not be disappointed.

L. V. M.

Introduction to Statistical Analysis. By WILFRID J. DIXON and FRANK J. MASSEY, Jr.

[2nd. Edn. Pp. xiii+488. New York, Toronto and London: McGraw-Hill Book Co. Inc., 1957. 45*s*.]

SUBJECT to certain qualifications which will appear below, this book should prove an excellent text for use by teachers of elementary (and immediately post-elementary) courses in statistical method, and also for those students whose circumstances force them to commence the study of statistics on their own. The main points of excellence of the book are the liberal supply of class exercises, problems and discussion questions, the detailed treatment of each statistical concept, so helpful to the beginner and so often ignored by the expert author, forgetful of his own period of training, and the unusually varied selection of tables provided. The book would have been of even greater value had outline solutions been given for at least some of the problems and exercises; perhaps this will be made good in a third edition.

The level of mathematics required from the reader is quite low, though a considerable facility in relatively simple algebraic manipulation would greatly reduce the time taken to master the contents. Indeed, the procedures of integration and differentiation appear nowhere in the book, though this necessitates giving a number of results without proof, e.g. in the estimation, by least squares, of the coefficients of regression equations.

The development of the subject matter follows well-established lines from descriptive statistics through definitions of universe (or population) to the Normal distribution, confidence intervals, tests of significance, analysis of variance and covariance and regression analysis. Unusual features of the book are the late introduction of discontinuous distributions (Poisson, binomial, etc.) and of the formal study of probability. The latter is not considered till the final