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PROJECTING THE SPREAD OF AIDS INTO THE GENERAL POPULATION—APPLICATION TO LIFE ASSURANCE

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ABSTRACT

The following paper sets out the actuarial basis for the Cologne Re model projecting the spread of AIDS. We project new infections, new AIDS cases, and deaths for the sexually active population and demonstrate the effect on the German life assurance industry.

1. INTRODUCTION

In 1987 we felt the necessity of being able to produce our own calculations projecting the spread of AIDS and its effects on the life assurance industry. Looking at the models available at the time, we found that the more complex ones had not been published in all detail. Furthermore, the models did not comply with all of the following requirements:

We wanted to be able to study the effects of selection. Therefore, we needed a model for the incidence of HIV infection rather than of new AIDS cases.

We wanted to 'guesstimate' how the disease would be likely to spread from the high risk groups (homosexuals, bisexuals, drug-abusers) to the rest of the population.

Finally, we wanted the model to be simple enough to be handled by actuaries working practically and not only by those able to dedicate large amounts of time to research.

Bearing these prerequisites in mind, we decided to follow the ideas of J. Weyer *et al.* and work with a multiple group model. At that time the model had not been published and it was not possible to use the work done by his group immediately. For the time being, Weyer's model has been described at least in part in *Die Zeitschrift für AIDS-Forschung*[3].

Our model, however, is much simpler and can be used in a straightforward manner by actuaries in insurance companies.

It should be pointed out that our main intent was not to set up an additional model projecting the spread of AIDS into the population. We wanted to have a simple tool available to check the possible effects of this disease on the insurance industry, and to study the effects of selection.

2. CURRENT SITUATION IN THE FEDERAL REPUBLIC OF GERMANY—ROUTES OF INFECTION

The model presented is of a general nature and applicable not only to the situation in West Germany. The projections of the model set out in Section 5, however, are based on West German data. We feel that the spread of the disease in other European countries is close enough to the German situation, so that the results of our calculations will be of interest there as well.

The current situation regarding the spread of the disease in the Federal Republic of Germany can be seen in Table 1.

Table 1. *AIDS cases reported by risk groups*

Risk	Last 12 months (31.12.87–31.12.88) Male	Female	Total since 1.1.1982 Male	Female	Proportion of total (%)
1. Homosexuals and bisexual men	770	—	2003	—	72·1
2. Drug addicts	92	43	180	98	10·0
2a. Homosexual drug addicts	8	—	27	—	1·0
3. Haemophiliacs	48	—	138	—	5·0
4. Blood transfusions etc.	22	14	44	30	2·7
5. Heterosexual contacts with group 1–4	17	11	56	28	3·0
6. Pre-natal infection	6	4	16	8	·9
7. Unknown	58	17	125	26	5·4
Grand Total	1021	89	2589	190	100·0

Since the introduction of voluntary reporting, the following trend has been observed:

Year	AIDS cases	Cumulative AIDS cases
1983	36	36
1984	84	120
1985	243	363
1986	449	812
1987	857	1,669
1988	1,110	2,779

We assume that in the Federal Republic of Germany as well as in other countries, not all AIDS cases are reported. We reckon that the numbers set out above represent at the most 80% of all AIDS cases. Nevertheless we have used these published figures as a basis for future projections.

Like J. Weyer, we simulated infection with the HIV virus for eight interacting groups. Within the model sexual contacts and needle contacts among drug addicts have been considered the only relevant routes of infection. The resulting network of infections is given in Figure 1.

In this diagram straight lines indicate infections from one group to another. Circles around the group names indicate infections within the group. We have

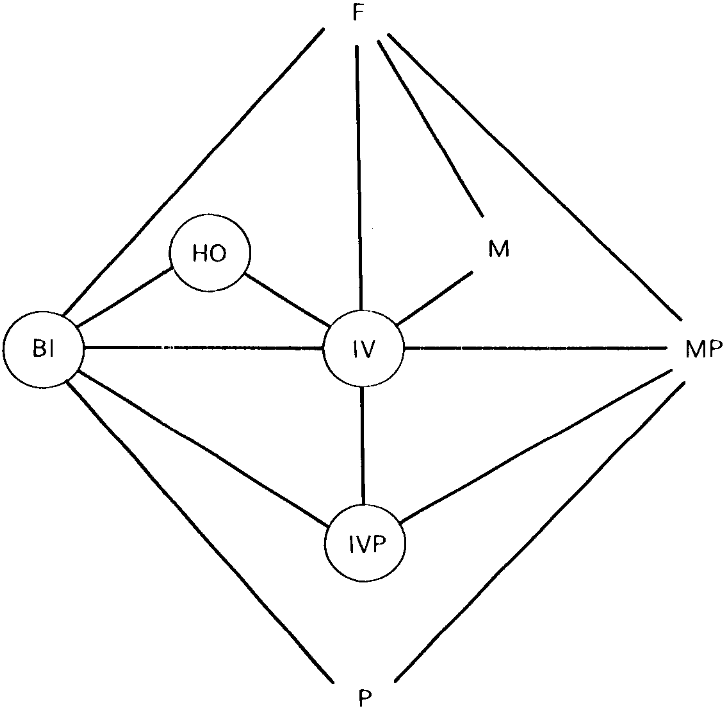

Figure 1. Diagram illustrating the routes of infection with AIDS.

considered the distribution of the sexually active population in the Federal Republic of Germany (per 1.1.1989) (Table 2).

For reasons of simplicity we have assumed that the sexually active population is the group of 18- to 60-year-olds. Sexual activity outside the group is compensated by inactivity within the group. The distribution of males and

Table 2. *Distribution of the sexually active population in the Federal Republic*

Heterosexual females (not drug addicts or prostitutes)	17,477,000	(F)
Prostitutes, not drug addicts	51,000	(P)
Intravenous drug-addicted prostitutes	20,000	(IVP)
Other intravenous drug addicts (inclusive of homosexual drug addicts)	60,000	(IV)
Heterosexual males with no prostitute contacts	15,798,000	(M)
Heterosexual males with prostitute contacts	1,866,000	(MP)
Bisexual males	464,000	(BI)
Homosexual males	464,000	(HO)
Total	36,200,000	

females is based on the 1985 figures of the Federal Statistics Office. It was assumed that 2·5% of the male population are homosexuals and another 2·5% bisexuals. The exact numbers of prostitutes and drug addicts are not known. The estimates given in Table 2 seem to be in line with experts' reports which appeared in *Die Zeitschrift für AIDS-Forschung* and other publications.

3. THE MODEL

3.1 *Input Parameters*

We set out the parameters used in the calculations. The first three parameters refer to the development of the population without AIDS. They are (m indicates the year of projection):

Q	average mortality of the active population;
$\mathrm{NEW}_m(j)$	young generations entering the sexually active population in group j;
OUT_m	proportion leaving the sexually active population (old generations).

These parameters can be estimated using figures from the statistical year books.

The next three parameters refer to AIDS itself:

$Q\mathrm{AIDS}$	mortality of an AIDS victim;
$I\mathrm{AIDS}(k)$	probability of a person in his/her k-th year of infection getting AIDS;
$\lambda(j,s)$	number of persons from group j a virus-carrier belonging to group s would infect if all his/her contact partners were not infected.

Whereas the population-related parameters are easy to estimate using known statistical data, the last three parameters relate directly to AIDS. We shall come to the method of estimation later.

3.2 *Definitions*

We have split the eight interacting groups of persons into non-infected persons, infected persons, AIDS victims, etc. The number of persons within these sub-groups are labelled as follows (m indicates the year of projection):

$LA_m(i,j)$	non-infected persons in group j at the beginning of day i;
$LI_m(i,j)$	infected persons in group j at the beginning of day i.

Since the transition from the stage of infection to the stage of sickness with AIDS is dependent on the number of years that a person has already been infected, we have split $LI_m(i,j)$ into

$LI^k_m(i,j)$	infected persons in group j at the beginning of day i in their kth-year of infection.

Furthermore, we have:

$LAIDS_m(i,j)$	persons sick with AIDS in group j at the beginning of day i;
$NI_m(i,j)$	newly-infected persons, who became infected on day i, in group j;
$NAIDSm(i,j)$	new AIDS victims within group j, falling sick on day i.

Again we break down this group into different years of infection and define:

$NAIDS_m^k(i,j)$	new AIDS victims in group j, falling ill on day i, who are in their k-th year of infection.

Furthermore, we define

$DA_m(i,j)$	deaths among the non-infected persons in group j on day i;
$DI_m^k(i,j)$	non-AIDS deaths among the infected persons in group j on day i who are in their k-th year of infection;
$DAIDS_m(i,j)$	deaths among the AIDS victims in group j on day i.

3.3 *Formulae*

In the model we simulate the outbreak of the disease on a daily basis, i.e. Q, New, Out, $\lambda(j,s)$ etc. are values per day. We assume that AIDS victims do not continue to infect other people. The formulae for the spread of the infection read as follows:

$$NI_m(i,j) = \min\left\{\frac{LA_m(i,j) \times \sum_{s=1}^{8} \lambda(j,s) \times LI_m(i,s)}{LA_m(i,j) + LI_m(i,j)};\ LA_m(i,j) \times (1-Q)\right\}$$

$$DA_m(i,j) = (LA_m(i,j) - \cdot 5 \times NI_m(i,j)) \times Q$$

$$LA_m(i+1,j) = \mathrm{NEW}_m(j) + (1-\mathrm{OUT}_m) \times (LA_m(i,j) - DA_m(i,j) - NI_m(i,j))$$

$$\left.\begin{aligned} DI_m^k(i,j) &= (LI_m^k(i,j) - \cdot 5 \times NAIDS_m^k(i,j)) \times Q \\ NAIDS_m^k(i,j) &= (LI_m^k(i,j) - \cdot 5 \times DI_m^k(i,j) \times IAIDS\ (k) \end{aligned}\right\} \text{simple equation}$$

$$LI^k(i+1,j) = (1-\mathrm{OUT}_m) \times (LI^k(i,j) - DI_m^k(i,j) - NAIDS_m^k(i,j))$$

It must be noted that here multiplication with $(1-\mathrm{OUT}_m)$ means that infected persons leaving the group of sexually active people are no longer taken into account in the model. This leads to the number of AIDS cases being estimated too low later on in the population model. This incorrectness does not apply to the calculations for life assurance, as will be seen later.

$$DAIDS_m(i,j) = LAIDS_m(i,j) \times QAIDS + \cdot 5 \times NAIDS_m \times QAIDS$$

$$LAIDS_m(i+1,j) = LAIDS_m(i,j) + NAIDS_k(i,j) - DAIDS_m(i,j)$$

4. ESTIMATION OF THE INPUT PARAMETERS

In Section 3 we gave a description of our model for the spread of AIDS into the general population. The model does not involve any advanced mathematical tools and can easily be programmed by actuaries to enable them to perform calculations of their own.

The main problem in the application of the model is the estimation of the parameters. In the following we shall give a brief description of the techniques we have used.

We have taken active life mortality $Q = 2{\cdot}864‰$ from the Statistical Year Book. $NEW_m(j)$ are based on the figures within the age groups below 18 which were then distributed to the various risk groups according to their share in the total population as set out in Section 2.

The factors OUT_m were also estimated taking into account the numbers in the generations reaching the age of 61 in projection year m. Possible differences in the AIDS mortality between one group and the other have been ignored but could easily be incorporated. The same applies to introducing the possibility of a behavioural change in infected persons. These are, however, included implicitly in scenario 1 presented later where we make assumptions regarding a future reduction of infectiousness.

As far as IAIDS is concerned, we assume that in the first year after infection ·5% of the infected developed AIDS; in the second year 1%; in the third year 1·75%; in the fourth year 6·2%; in the fifth year 11·5%; in the sixth year 13%, increasing to 16% in the 26th year. This value then stays constant in future years.

For QAIDS, we assumed a mortality of 50% per annum. We kept that percentage unchanged, although there is some indication that, thanks to developments in medicine, AIDS mortality will be reduced.

The most difficult set of parameters to estimate is the set of $\lambda(j,s)$, i.e. the number of persons a virus-carrier in group s would infect from group j if all his/her contact partners were non-infected.

This set of parameters consists of 64 elements, and it is obviously impossible to estimate these parameters directly from observing the development of the numbers of AIDS cases.

Weyer carries out this estimation by splitting each of the parameters into so-called micro-parameters. He represents each of the $\lambda(j,s)$ as a product of the frequency of intercourse/needle-sharing, the time during which a couple exists, and the infectiousness of each contact.

With regard to the first two of these micro-parameters, detailed information is needed about the sexual behaviour of the population. Of course, such information is generally not available. Using incomplete information, however, one determines sexual behaviour by weighing up different statements from different studies on this topic, thus arriving at relatively accurate results. Remarks on this technique can be found in the paper by E. G. Knox[2]. As for the last of the micro-parameters, infectiousness per contact, some estimates are available.

We employ a much simpler technique for our first projections, as presented in Lörper (1988)[1], as well as for those presented later, by reducing the parameters $\lambda(j,s)$ to 4 basic parameters:

λ_1 number of persons a heterosexual would infect if all his/her partners were non-infected;

λ_2 number of persons a non-drug-abusing prostitute would infect if all her contact partners were non-infected (only professional contacts; private contacts are not included here);

λ_3 number of persons a drug addict would infect by way of needle-sharing, if all his/her partners were non-infected;

λ_4 number of persons a homosexual would infect if all his partners were non-infected.

We then draw up equations for $\lambda(j,s)$ using simple assumptions regarding the behaviour of the various groups, e.g.

Bisexuals infect at a rate of $\cdot 5 \times \lambda_1$ through male/female contact and $\cdot 5 \times \lambda_4$ through male/male contact.

Drug addicts have half their sexual contacts (private, not as prostitutes) within the group of drug addicts and the other half outside the group.

Drug-addicted prostitutes are less likely to protect themselves and their partners adequately. Therefore the number of persons they infect in professional contact is assumed to be twice as high as λ_2, etc.

As far as the estimation of λ_1 is concerned, we have used the technique of splitting them up into micro-parameters. We have assumed that in one partnership an average of 100 sexual contacts take place per year, and furthermore that a partnership lasts an average 5 years. An infection probability of approximately 92% per partnership results. This result is based on the assumption that with heterosexual contact the infection probability per contact is on average 5‰. The latter figure results from studies on partners of haemophiliacs in the United States of America. It follows that $\lambda_1 = \cdot 1825$ persons per year.

We had assumed λ_2 to be significantly higher than λ_1. This, however, would have led to a much higher number of AIDS cases among heterosexuals in the past. We have therefore set λ_2 at $\cdot 1825$ persons per year.

As far as λ_3 and λ_4 are concerned, these can be estimated from the past development of the disease.

In our previous publication[1] we arrived at $\lambda_3 = \lambda_4 = \cdot 73$ persons per year.

The above assumptions were based on the data available up to September 1987. In 1988, however, we noted that the increase in the number of new AIDS cases was significantly lower than anticipated, which means that the number of people infected at the beginning of 1988 must have been significantly lower than the 100,000 which we had used in our model calculation. This led us to believe that during the years 1983 to 1987 people had already changed their behaviour, thus giving rise to a smaller number of new infections. To express this in more detail, we have assumed λ_1-λ_3 would have reduced linearly to 75% of their initial values during the years 1983–1988 whereas λ_4 would have reduced to 50%. On the basis of these assumptions we arrive at Table 3 in respect of $\lambda(j,s)$ for the beginning of 1989.

Table 3. λ(j,s) *per year (as per 1.1.1989)*

j/s	1	2	3	4	5	6	7	8
1	·00000	·00000	·00000	·04127	·13680	·13632	·06838	·00000
2	·00000	·00000	·00000	·00000	·00000	·00740	·00702	·00000
3	·00000	·00000	·13688	·14719	·00000	·00557	·00549	·00000
4	·00014	·00000	·45189	·47639	·00008	·00008	·00056	·00105
5	·12073	·00000	·00000	·06043	·00000	·00000	·00000	·00000
6	·01424	·38969	·57904	·00713	·00000	·00000	·00000	·00000
7	·00177	·00969	·01440	·00496	·00000	·00000	·06066	·12132
8	·00000	·00000	·00000	·00815	·00000	·00000	·12132	·24263

We do not know for sure how the infection parameters $\lambda(j,s)$ will develop in the future. There is good reason to believe that in the next few years at least they will not drop any further, whereas it can be hoped that later on, with the number of AIDS victims on the increase, the behaviour of the population will show a more marked change. Furthermore, there is hope that some medical devices (vaccines, etc.) to prevent the further spread of AIDS will have been developed.

In scenario 1 of our projections we have assumed that, from 1995 onwards, the parameters will drop linearly to 0 over the next 10 years.

In scenario 2 we have, for illustrative purposes only, assumed that these parameters will remain constant. For both scenarios we use the following numbers of infected lives in the various groups.

F	2,200
P	150
IVP	2,250
IV	6,850
M	1,600
MP	2,750
BI	7,400
HO	14,800
	38,000

The results of the calculations are summarized in Figures 2–7 and Tables 4–6. In these we have set out separately the numbers for the 'classical' risk group, i.e. homosexuals, bisexuals and drug addicts and for males and females not belonging to this risk group.

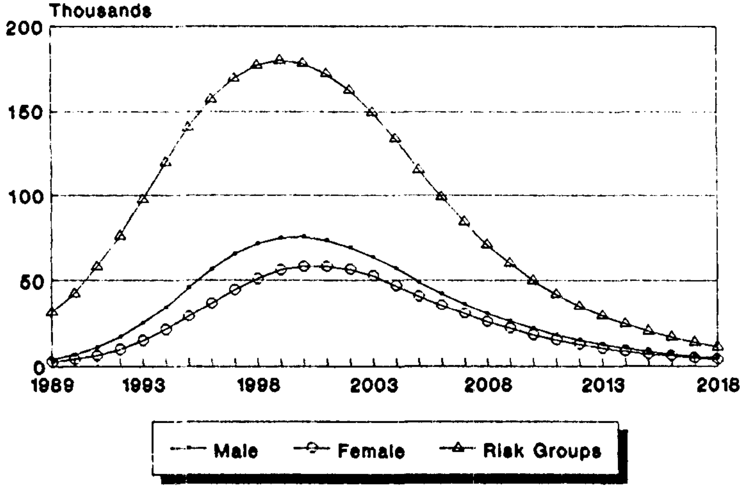

Figure 2. Scenario 1—infected lives.

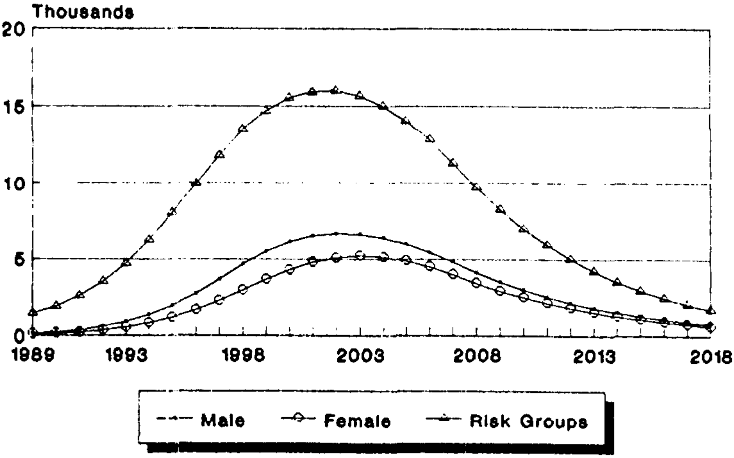

Figure 3. Scenario 1—new AIDS cases in the active population.

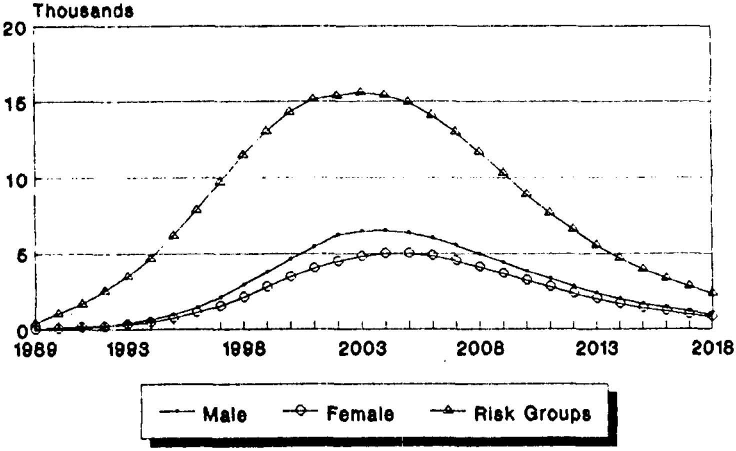

Figure 4. Scenario 1—AIDS deaths in the active population.

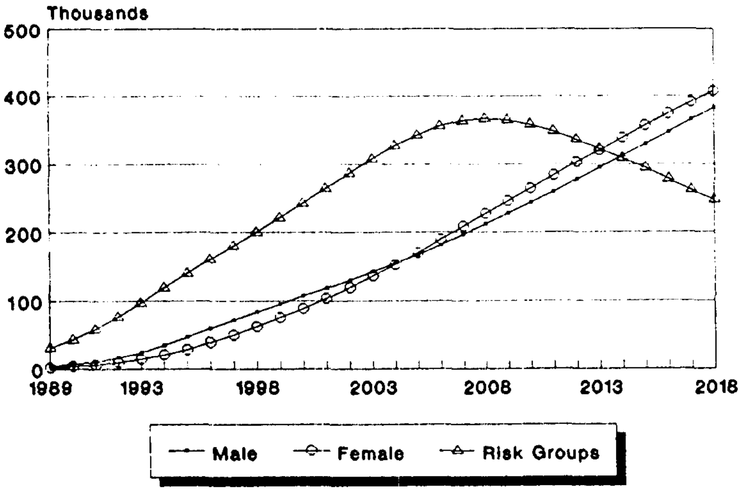

Figure 5. Scenario 2—infected lives.

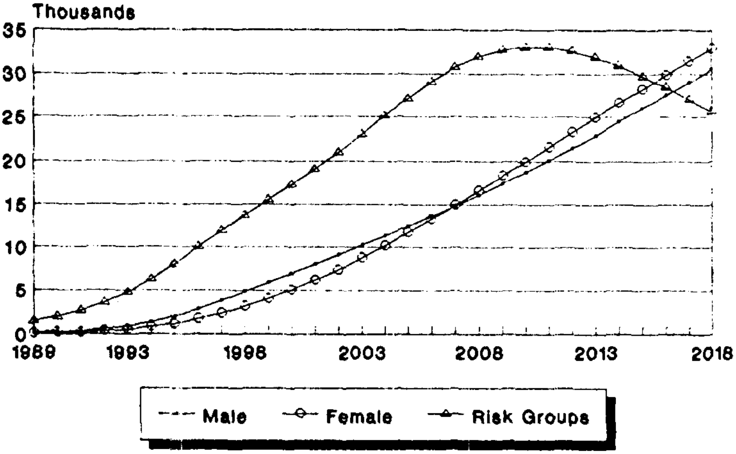

Figure 6. Scenario 2—new AIDs cases in the active population.

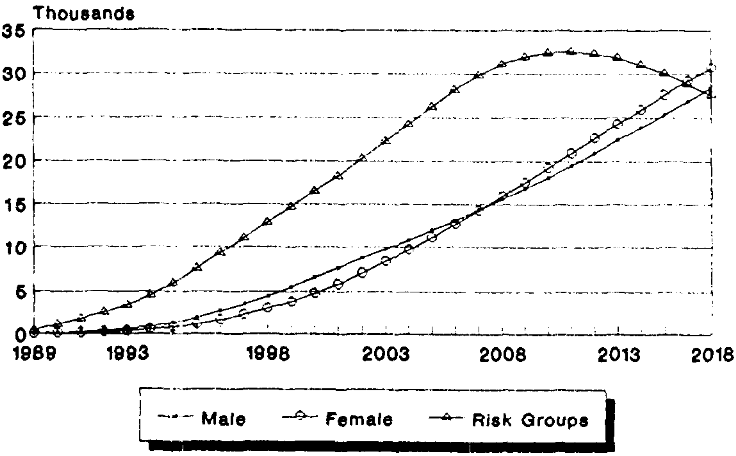

Figure 7. Scenario 2—AIDS deaths in the active population.

5. RESULTS FOR THE LIFE ASSURANCE INDUSTRY

We have used the new infections from scenario 1 of our population model to establish the discounted values of additional AIDS claims in respect of new business in 1989.

We carried out our calculations by deriving an infection probability per risk group per policy year and we used our parameters *I*AIDS (k) to calculate the actual excess payments resulting from AIDS. All AIDS deaths in an insured portfolio were taken into account, not only the AIDS deaths among the sexually active assureds of a portfolio.

Tables 4–6 show the results for term assurances with durations of 10 and 20 years as well as for endowment assurances with a 20-year duration. In each case, age at entry is 35. The mortality used is the West German mortality 1986 M/F. Interest used in premium calculation is 3·5%, interest used in the discounting is 7%.

Our model so far is age-independent. We have chosen $x = 35$ because we believe that this is the average age at entry in the policies of a typical portfolio. In Lörper (1988)[1] we have also performed calculations with varying ages at entry.

In Tables 4–6 the AIDS payments are set out for the various groups. We have also calculated the following totals:

Total 1 gives the average for the total population.

Total 2 gives the average for an insured population, i.e. a portfolio not including drug addicts and with a 33% proportion of females.

Total 3 gives the average for an insured population, but under the additional assumption that no homosexuals or bisexuals are included.

Tables 4–6 show that the main contribution to future AIDS claims will come from the high risk groups. The differences between the totals in the Non-infected and the Total columns indicate that testing 100% of the population would cut

Table 4. *Discounted value of AIDS claims in ‰ (term assurance with duration 10—production year 1989)*

Group	Non-infected	Infected	Total	Discounted mortality claims excluding AIDS
F	·12	368·23	·16	15·15
P	1·81	375·58	2·91	15·15
IVP	80·14	361·38	111·78	15·15
IV	82·60	356·93	113·91	18·87
M	·07	362·36	·11	20·70
MP	1·22	354·34	1·74	20·70
BI	5·24	399·05	11·52	20·70
HO	10·06	403·81	22·62	20·70
Total 1	*·51*	*376·26*	*·91*	*18·01*
Total 2	·41	386·23	·81	18·87
Total 3	·17	361·06	·25	18·87

Table 5. *Discounted value of AIDS claims in ‰ (term assurance with duration 20—production year 1989)*

Group	Non-infected	Infected	Total	Discounted mortality claims excluding AIDS
F	1·01	475·02	1·07	30·20
P	14·76	480·12	16·13	30·20
IVP	282·78	470·37	303·88	30·20
IV	278·63	458·46	299·16	39·78
M	·62	458·28	·67	44·39
MP	7·92	452·40	8·57	44·39
BI	34·02	484·06	41·20	44·39
HO	62·67	487·54	76·23	44·39
Total 1	2·95	473·13	3·44	37·51
Total 2	2·81	479·25	3·29	39·71
Total 3	1·28	461·42	1·38	39·71

Table 6. *Discounted value of AIDS claims in ‰ (endowment assurance with duration 20—production year 1989)*

Group	Non-infected	Infected	Total	Discounted mortality claims excluding AIDS
F	·33	321·31	·37	16·22
P	4·81	327·61	5·76	16·22
IVP	124·22	315·61	145·75	16·22
IV	125·16	310·48	146·32	20·68
M	·21	315·05	·24	22·85
MP	2·87	307·59	3·32	22·85
BI	11·91	347·20	17·25	22·85
HO	22·24	351·63	32·74	22·85
Total 1	1·07	327·73	1·42	19·64
Total 2	·98	336·40	1·32	20·66
Total 3	·44	314·09	·50	20·66

out between 20 and 40% of future AIDS claims. It becomes obvious that the most efficient way of reducing future AIDS claims is to reduce the high-risk proportion of the portfolio. Nevertheless, in the long run persons not considered to be at high risk can also have an impact on future AIDS claims.

It seems that the results presented here are much less worrying than the results presented in Lörper (1988)[1]. It can only be hoped that the behavioural changes on which we have based our assumptions will actually materialize.

6. ACKNOWLEDGEMENT

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