The Actuarial Profession
making financial sense of the future

Projection of mortality rates as illustrated by Weibull distribution

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Building mortality models

Possible forms of mortality models:

- Projection of past trends without adjustment
- Projection of past trends with adjustment
  - Adjust for changes in risk factors
  - Application to alternative populations
- Projections from medical cohort data by disease condition, allowing for effects of treatment and disease interaction

Development of actuarial mortality models

- Gompertz - Bc^x
- Makeham – $A + Bc^x$
- Kannisto – $Bc^x / (1 + Bc^x)$
  - Other variants by Beard, Perks
- Each model approximates actual mortality experience either in **specified calendar year** or over lifetime of individual

Weibull distribution

- Published in 1939 by Waloddi Weibull
- Probability density function

$$f(x;\lambda,\kappa) = \frac{\kappa}{\lambda}\left(\frac{x}{\lambda}\right)^{\kappa-1} e^{-(x/\lambda)^{\kappa}}$$

- Cumulative distribution function

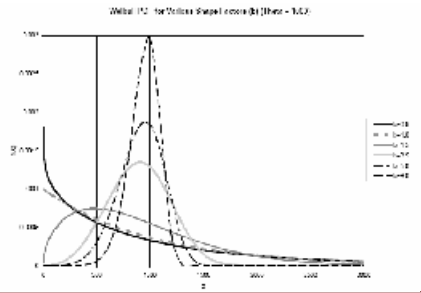
$$F(x;\lambda,\kappa) = 1 - e^{-(x/\lambda)^{\kappa}}$$

  - λ is scale parameter, and κ is shape parameter
  - 3 parameter version available

Uses of Weibull distribution

- Weibull distribution normally used as a reliability distribution to model:
  - Material strength
  - Times-to-failure of electronic and mechanical components
- More recent application to model various phases of mortality experience

Value of “slope” or “shape” parameter



Value of “slope” or “shape” parameter

- Effect of value of slope parameter:
  - <1 – Infant mortality or burn-in
  - 1 – Random failures
  - 1-4 – Wear out failure
  - 3.5 – Similar to Normal distribution
  - >4 – Old age or rapid wear out

Weibull distribution in mortality models

- Oldest-old mortality in China – Demographic Research (2003) – Vaupel & Yi
  - Comparison of Weibull to Kannisto
- Application of mortality models to Japan – SOA’s Living to 100 (2005) – Ozeki
  - Use of Mixed Weibull distributions at different ages
  - Use in construction of Japanese life tables

Weibull distribution in mortality models

- Complementarity between survival and mortality - Weon
  - Weon model derived from Weibull distribution using age-dependent shape parameter
- Analysis of trends in the age-specific shape of mortality curves for populations in the US and Japan – SOA’s Living to 100 (2005) – Dugan et al
- Analysis of trends in mortality near or during retirement for four European countries – ICA (2006) – Humble et al

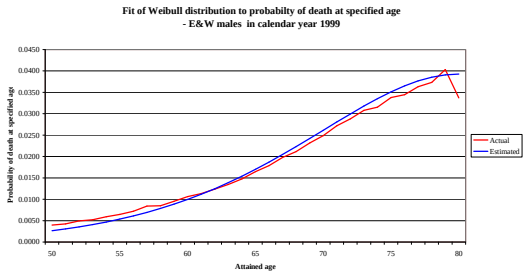
Source of mortality experience and choice of measure of mortality

- Population mortality experience by individual age and for both sexes taken from Human Mortality Database over the period 1960 to 1999 for:
  - England & Wales
  - USA
  - Japan
  - France
  - Italy
- Probability of death from starting age – chose age 50 - for each subsequent age from vertical life table

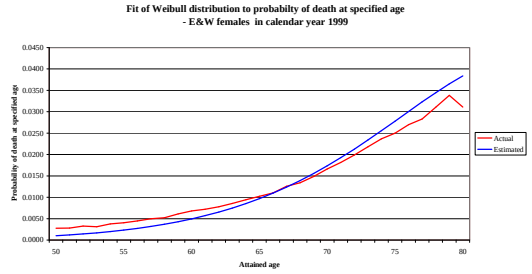
Method for fitting Weibull mortality models

- Method of percentiles
  - Fit Weibull cumulative probability distribution to actual experience at selected percentiles
  - Chose 50th and 95th percentiles to highlight old age mortality
- Comparison of parameter values from Weibull mortality models in different calendar years

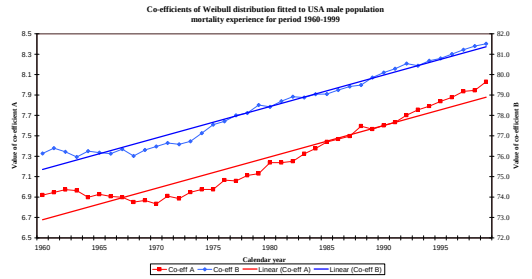
E&W males – appropriateness of fit



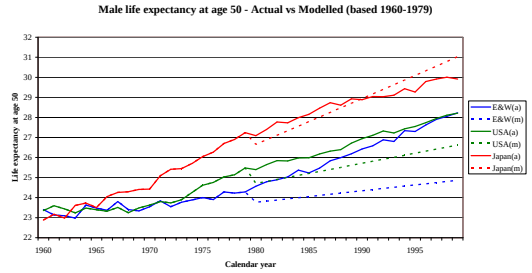
E&W females – appropriateness of fit



Specimen parameter values for Weibull mortality models in different calendar years



Investigation period affecting comparisons between actual and projected mortality



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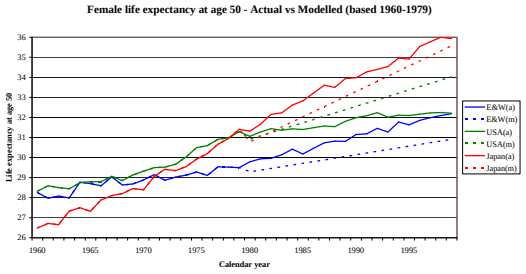
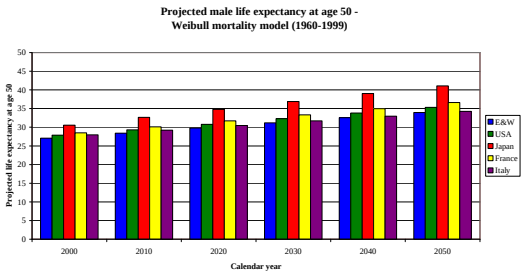
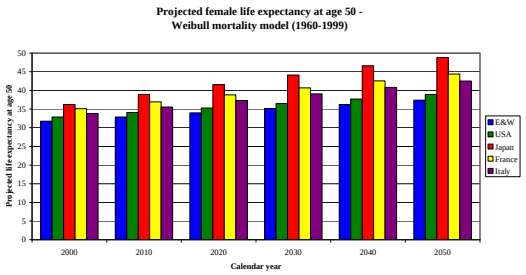


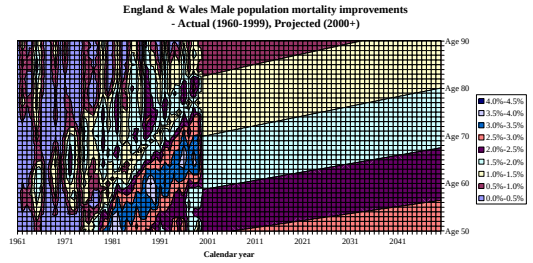
Comparisons of male life expectancy from Weibull mortality model



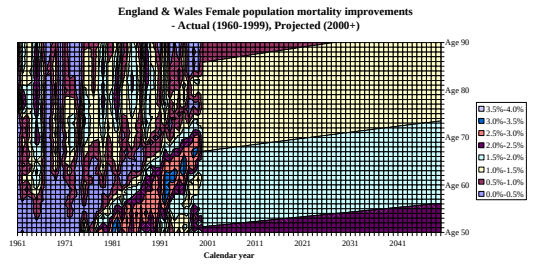
Comparisons of female life expectancy from Weibull mortality model



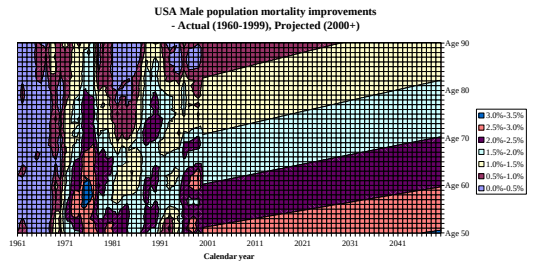
E&W males - mortality improvements from Weibull mortality model



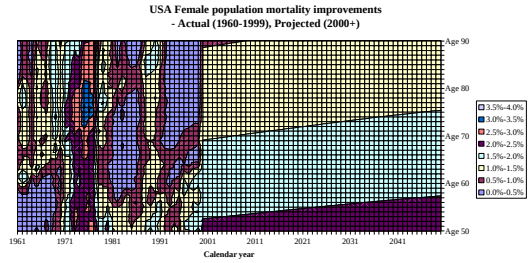
E&W females - mortality improvements from Weibull mortality model



USA males - mortality improvements from Weibull mortality model



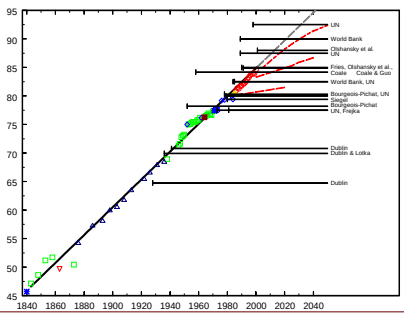
USA females - mortality improvements from Weibull mortality model



Further areas for consideration over application of Weibull mortality model

- Differing views over scope for further improvements in life expectancy
- Application to insured population experience
- Quantifiable differences between future expectation and past trends for prevalence of known risk factors e.g. smoking
- Adjustment where appropriate for publicly available disease specific models, with recent attention on cardiovascular mortality

Broken limits to life expectancy



Science 2002: Vaupel & Oeppen.