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## PROJECTIONS, EMBEDDED VALUES AND BUSINESS PLANS: A SIMPLIFIED EXAMPLE

by
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## PROJECTIONS, EMBEDDED VALUES AND BUSINESS PLANS A SIMPLIFIED EXAMPLE.

1. This paper discusses the application of projection techniques to the financial control of a life insurance operation. It does so by considering a simplified example which the author believes can be reproduced readily by any person with a knowledge of life contingencies, access to a personal computer and reasonable mastery of a programming language.
2. The example consists of a life insurance operation in the form of a South African branch of an international insurance company. The operation commenced business on lst January 1980 and is still receiving annual subsidies from the parent fund, in the form of an annual transfer of the amount required to meet the operating deficit for the year. The purpose of this subsidy is to ensure that the Life Fund of the branch is sufficient, at each year end, to meet the actuarial liabilities of the branch, calculated on the South African statutory minimum basis.
3. Premium Rates.

From outset, the business of the branch has consisted exclusively of 20 -year With Profit Endowment Assurances, the premium rates for which were calculated by determining the rate at each entry age that would - on the basis of the expected experience - support an annual compound reversionary bonus of 30 per mille, and provide a return of exactly $12 \%$ per annum on the capital invested in meeting the first year valuation strain resulting from the issue of the policy. Specimen premium rates are shown in the following table:

Table 1
Specimen Premium Rates
for
20 year Endowment Assurances
\(\left.\begin{array}{cc}Entry \& Annual Premium <br>

Age for\end{array}\right\}\)| Sum Assured of |  |
| :---: | :---: |
|  | R1 000 |
| 20 | 47,71 |
| 25 | 47,87 |
| 30 | 48,32 |
| 35 | 49,30 |
| 40 | 51,12 |
| 45 | 54,09 |
| 50 | 58,52 |
| 55 | 65,01 |
| 60 | 71,06 |

The experience assumed, which allows for both withdrawals and escalating renewal costs, is summarised in Appendix 1 , and examples of the resulting strains and surpluses for individual policies with a Sum Assured of R100,000 are shown for specimen entry ages in Table 1 of Appendix 2.

No Commercial applicability is claimed for these rates. The use of a predetermined rate of return on the capital invested in new business strains is primarily to simplify, in the example, the relationship between the embedded value of the business in force and the total of the accumulated subsidies.

It is, however, interesting to note that the premium rates derived by the profit testing technique can be approximately reproduced, for most of the age range, by the classical formula:

$$
\begin{gathered}
P^{\prime}=\frac{\left.1000 \stackrel{A}{A}_{x}: \bar{n} \mid+4,75 \ddot{a}_{x}: \bar{n}\right]}{\left.\left[\ddot{a}_{x: n}\right]-1\right]}
\end{gathered}
$$

Where the unaccented ${ }^{\prime \prime}$ is calculated at $8 \%$ and the accented $A^{\prime}$ is calculated at a rate of [1.08/1.03-1] to allow for the $3 \%$ compound bonus, and

I is the loading for Initial Expenses (Commission and Management Expenses combined), as set out in Appendix $A$, and expressed as a proportion of the annual premium. i.e. 1.16 for age 55 or less and . 9575 for age 60.

Table 2
Comparison of Premium Rates
$\left.\begin{array}{rcc}\text { Entry } & \text { Annual Premium for Sum Assured of R1 } 000 \\ \text { Age } & \text { Calculated by using: } \\ \text { Profitability } & \text { Classical }\end{array}\right]$

The calculation of premium rates to meet a given profitability standard is not as straightforward as it might seem, since the rates so derived depend upon the assumed surrender values, and the appropriate surrender value basis must have some regard to the total of the premiums paid. In the example, the surrender basis has been shosen to ensure that policies that are surrendered provide a reasonable, but not excessive return on the first year raluation strain.

In order to simplify the example, it has been assumed that the age spread of the business received remains constant from year to year and that this spread can be represented by the following Sum Assured weights:

Table 3
Distribution of New Business
by age

| Entry <br> Age <br> Group | Representative <br> Age | Sum <br> Assured <br> Weight |
| :---: | :---: | :---: |
| $18-22$ |  |  |
| $23-27$ | 20 | 1 |
| $28-32$ | 25 | 3 |
| $33-37$ | 30 | 7 |
| $38-42$ | 35 | 9 |
| $43-47$ | 40 | 5 |
| $48-52$ | 45 | 3 |
| $53-57$ | 50 | 2 |
| $58-62$ | 55 | 1 |
|  | 60 | -2 |

Applying these weights to the projection programme that was constructed to profit-test the premium rates, one arrives at the projected finances for a tranche of new business consisting of R100 millions of Sums Assured received on lst July in any year. The resulting figures are set out in Table 2 of Appendix 2 .

It is this single injection table that establishes and underlies the finances of the operation and the production of it is the fundamental purpose of the projection calculations.

The projection programme essentially consists of a calculation loop that projects, policy by policy, the expected premiums, interest, expenses and claims for each consecutive calendar year and compares the resulting fund with the expected reserve, in order to arrive at the expected surplus for that year. The results for each policy are then combined with those for the other policies in the assumed policy mix, after multiplication of the results for each policy by the applicable sum assured weight.

To assist anyone wishing to reproduce this table, I have set out in Appendix 10 a note describing the calculation steps performed by the programme.

The projected Revenue Account in Appendix 2 shows the typical finances of a Reversionary Bonus series - a heavy first year strain that is only slowly recouped over the lifetime of the policy. Even if interest is ignored, it takes more than 10 years for the emerging surpluses to repay the first year strain. In current financial conditions, such a long repayment period would lead to unacceptable capital commitment and the motivating force behind much of modern product design is the radical reduction of the period required to recoup first year strains.

Further insight into the underlying finances of the operation can be obtained by studying the analysis of surplus figures shown in Table 3 of Appendix 2. An analysis of surplus is not an essential part of a profit-testing or projection programme, but its inclusion is recommended, because of the added insight it provides into the finances underlying each type of policy. In this particular case, it will be seen that the principal sources of surplus, the expected interest surplus and the expense margin in the premium rates - i.e. the difference between the projected office premiums (less the projected renewal costs) and the projected valuation net premiums - are, for many years, only marginally more than the cost of the bonus. The analysis also shows that a substantial part of the surplus is derived from surrenders and lapses, by the recoupment on terminating policies of the first year valuation strain.

The final column of the projected revenue account shows the embedded (asset) value of the policies in force at the end of each calendar year. This is the discounted value of the projected surpluses for all future years, at the rate of return required to be earned on the new business strains - (12\%).

Since the premium rates have been calculated to provide a return of precisely $12 \%$ per annum on the first year strain, the embedded value of each tranche of business at the end of the first calendar year is exactly equal to the valuation strain then incurred (the small difference of 20 cents being the effect of rounding the premiums to the nearest cent). In effect, the branch has, in writing business with a premium income of $R 5,138 \mathrm{million}$, incurred a first year valuation strain of $\mathrm{R} 3,292 \mathrm{million}$, but the management has the comfort of knowing that this apparent loss will, if the projection assumptions are realised, be recouped, over the lifetime of the policies concerned, with interest of $12 \%$ per annum.
5. The First Business Plan.

In commencing the operation, the $v i e w$ of the management was that it would be possible to build up the volume of new business to about $R 100$ millions of Sums Assured over
a period of about 6 years, after which an increase of about $8 \%$ per annum (representing the effect of inflation plus a modest increase in market share) could reasonably be expected. A Business plan was therefore presented on the basis of a projected new business flow in accordance with the following table.

Table 4
New Business Estimates
(Millions of Rands)

| Year | New <br> Sums <br> Assured | New <br> Premium <br> Income |
| :---: | :---: | :---: |
| 1981 | 50 | 2,569 |
| 1982 | 60 | 3,083 |
| 1983 | 70 | 3,597 |
| 1984 | 80 | 4,111 |
| 1985 | 90 | 4,624 |
| 1986 | 100 | 5,138 |

increasing at the rate of $8 \%$ per annum after 1986 .

This initial business plan was prepared by applying, to each of the 13 columns of the single injection projected revenue account, the following operator:

$$
\left.F_{t}=\sum_{j=1}^{j=t} f_{j} x n_{t-j+1}\right\}
$$

Where: $f_{t}$ is the selected item for year $t$ in the Single Injection Projected Revenue Account,
$F_{t} \quad$ is the corresponding item for year $1980+t$ in the Projected Revenue Account for the combined business, and
$n_{t}$ is the new business sum assured estimate for the year $1980+t$ (in units of R100 million)

The resulting revenue account is shown in Appendix 3
These figures show that, on the projection assumptions:
a) the operation was expected to require an annual subsidy for each of the first 10 years of about R1.75 million.
b) thereafter, the annual subsidy was expected to decline rapidly over the ensuing 4 years and, from the 15 th year, the operation was expected to become self-financing and to produce increasing annual surpluses.
c) by the end of the subsidy period (1994) the embedded value of the business in force was expected to be about R50 million, which is equal to the projected subsidies accumulated at $12 \%$ per annum to the end of 1994.

In presenting these figures some caution would have been necessary. The amount of the projected subsidies, and the number of years during which subsidies will be required, heavily depends upon the rate of growth of future business. By their nature, new business estimates, particularly for a new operation, are often unreliable. In this particular example, because of the long recoupment period, even a small increase in the rate of growth of the business will greatly increase the amount of finance required. Indeed, if, after the initial establishment period, the rate of growth is allowed to rise to more than about $12 \%$ per annum, annual subsidies may be required for an indefinite number of years.

This can be seen by examining the effect of altering the assumed rate of growth after 1986:

Table 5
Effect on Emerging Surplus of Increased Growth after 1986
(Rands Million)

| Year | Surplus for Year if Annual Growth Rate after 1986 is: |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 8\% | 10\% | 12\% | 15\% |
| 1987 | -1.75 | -1.82 | -1.89 | -1.99 |
| 1988 | -1.64 | -1.77 | -1.90 | -2.11 |
| 1989 | -1.50 | -1.70 | -1.90 | -2.22 |
| 1990 | -1.32 | -1.59 | -1.87 | -2.33 |
| 1991 | -1.10 | -1.45 | -1.82 | -2.44 |
| 1992 | -0.84 | -1.27 | -1.75 | -2.56 |
| 1993 | -0.54 | -1.06 | -1.65 | -2.68 |
| 1994 | -0.19 | -0.81 | -1.52 | -2.81 |
| 1995 | +0.20 | -0.52 | -1.37 | -2.94 |
| 1996 | +0.64 | -0.18 | 1.19 | -3.10 |
| 1997 | +1.12 | $+0.19$ | -0.97 | -3.26 |
| 1998 | +1.65 | $+0.61$ | -0.73 | -3.45 |
| 1999 | +2.23 | $+1.08$ | -0.45 | -3.66 |
| 2000 | +2.87 | $+1.60$ | -0. 0.13 | -3.91 |
| Subsidy period (years) | 14 | 16 | 20 | efini |

Total of Annual Subsidies

$$
19.25
$$

$$
22.54
$$

$$
29.51
$$

It is uncertainties of this magnitude that make it so difficult, with current levels of inflation, to justify the establishment of a life insurance business based upon the reversionary bonus system.

It must also be understood that the projection figures assume expenses in accordance with the assumptions made in calculating the premiums. In the early years of the operation, however, while the business is still small, expenses are likely to exceed those assumed in the actuarial calculations. The extent of this expense overrun should be establish by comparing the projected expenses with those in an expenditure budget based on expected staff recruitment and other expected costs. There should, of course, be a symbiosis between the expenditure assumed in the budget and the expenses assumed in the projection calculations, since the expenses assumed in the projection calculation must bear some relation to budgeted expenditure, and vice versa.
6. Experience for first 10 years.

For the purposes of the example it has been assumed that the experience of the operation for its first ten years has not differed from that assumed in the preparation of the initial business plan, except that, after a slow start, the growth of the business has been slightly faster than the growth that was anticipated in the preparation of the plan. The new business written each year being that shown in table 6.

For simplicity, it has been assumed that in all other respects - age spread of business, mortality, withdrawal, etc, the projection assumptions were exactly realised.

The subsidies required each year, ignoring any expense overruns were therefore as shown in Table 6:

Table 6
New Business Secured 1981 - 1990 and Resulting Subsidies.
(Rands Million)

| Year | New <br> Sums <br> Assured | Business <br> Annual <br> Premiums | Subsidy | Accumulated <br> Subsidy |
| :---: | :---: | :---: | :---: | :---: |
|  | (12\%) |  |  |  |

The financial effect of the additional business has therefore been to increase the annual subsidy for the first ten years to an average of more than $R 2$ million, with a total for the ten years of almost R 21 million (compared with the initial estimate of R19.25 million over a period of 14 years.

Since the original estimate has been exceeded, and the annual subsidy is still about $R 2$ million, the management of the overseas company has called for the finances of the operation to be reviewed. In particular, they wish to know the extent of the finance likely to be required over the next 5 years and when the operation is likely to become self-financing.
7. Finances of Existing Business.

Step 1
The first step is to modify the basic projection programme to project the future surplus from the business that is already on the books. Although the alterations to the basic calculating loop are minimal (see Appendix 10), it is probably best to establish a separate programme, since the instruction that reads the details of the nine specimen policies used to project the finances for future business must be replaced by an instruction that reads grouped valuation data. Let us assume that the valuation data as at 31 st December 1990 is in accordance with the valuation summary set out in Appendix 4.

The valuation, which has already been made, has produced, on the prescribed minimum basis, a liability of R153, 12 million. In addition, it has been decided to declare, as at the valuation date, a $3 \%$ compound bonus, the additional liability for which will be R13, 02 million. The total liability, inclusive of the cost of the immediate bonus is therefore R166, 14 million . Since, at the valuation date, the life fund, inclusive of previous transfers from overseas, was only R164, 19 million, it was necessary, in accordance with the practice of the Parent Company, to make a further transfer of $R 1,95$ million, in order to ensure that the assets of the local fund were sufficient to meet to the local liabilities. This is the amount of the 1990 subsidy shown in Table 6.

Step 2
The second step is to use the modified projection programme to determine the expected future surplus from the policies already in force. Since there has been no change in financial conditions, and since the mortality and withdrawal experience for the first ten years supports the assumptions made in the initial business plan, the projection assumptions, as summarised in Appendix 1, will not be changed.

The projected revenue account that results from these calculations is shown in Appendix 5. From these figures it will be seen that, on the projection assumptions, the existing business is expected to provide over the next 5 years an average annual surplus of rather more than $R 4$ million.

It is the derivation of this information that is the most important result of the new projection calculations, because it provides the firm base upon which a new financial plan can be prepared.

By discounting the expected future surpluses at the rate of interest required to be earned on the first year valuation strains (12\%), it is possible to arrive at the 'embedded value' of the policies in force at the commencement of the projection period, i.e. lst January 1991, the figure for which is R35, 75 million. In the particular circumstances of the simplified example, this is exactly equal to the total of the subsidies made to the fund, accumulated at the required $12 \%$ per annum rate (see table 6).
8. The Impact of Future New Business

The third step is to use the original projection programme to calculate the strains and surpluses expected to arise from future new business. This merely requires an alteration to the base year and the insertion into the programme of the estimated future new business.

After discussion with the Marketing Department, it is decided to project the future new business on two bases:
a) A target of R190 million of sums assured for 1991 followed by an 8\% annual growth thereafter. This is the average rate of growth over the past 5 years.
b) The same target for 1991 followed by a 10\% annual growth thereafter. This is the rate of growth that the Marketing Department feel that they can achieve.

The projected revenue account resulting from assumption
(a) is shown in Appendix 6 and the projected revenue account resulting from assumption (b) is shown in Appendix 7
9. Alternative Business Plans.

The final step is to combine the projected revenue account for the existing business with each of the projected revenue accounts for future new business. This is most easily accomplisted by storing the results for the existing business into a sequential file on a hard or floppy disc and extending the programme for future new business to read the sequential file and produce the combined figures.

The combined projected revenue account, assuming an $8 \%$ per annum growth in new business after 1991, is shown in Appendix 8 and the combined projected revenue account, assuming a $10 \%$ per annum growth, is shown in Appendix 9.

From the figures so obtained, it is possible to secure a detailed picture of the projected finances of the operation from which the following information can be derived:

BUSINESS PLAN A (with 8\% growth)
On the projection assumptions and assuming a growth in new business after 1991 of 8 \% per annum:
a) the fund will grow, over the next five years from R166 million to about $R 470$ million. Over the same period, the premium income will grow from about R50 million in 1991 to about R82,5 million in 1995.
b) During the same period, the amount available each year for management expenses, excluding any expenditure to be financed from the allowance for commission related expenditure, will increase from R6, 59 million in 1991 to R10, 31 million in 1995.
c) The projected revenue account also provides for commission related expenditure - overriding commission and other direct procuration costs - of R2, 03 million in 1991, rising to $\mathrm{R} 2,76$ million in 1995.
d) The fund will continue to require subsidies, but the amount needed to be transferred is expected to diminish each year, from about R1, 7 million in 1991 to about R100 000 in 1995. The total of the subsidies for the five year period is expected to be about R5 million.
e) Over the same period, the embedded value of the policies in force is expected to increase from R35,75 million to almost R 70 million, this increase being derived as shown in the following table:

Table 7
Derivation of Increased Embedded Values.
(R's Million)

| Year | Embedded Value at Start | ```Interest at 12% on Embedded``` | $\begin{aligned} & \text { Subsidy } \\ & \text { for } \\ & \text { Year } \end{aligned}$ | Embedded Value at End |
| :---: | :---: | :---: | :---: | :---: |
|  | of Year | Value |  | of Year |
| 1991 | 35.75 | 4,29 | 1,72 | 41,76 |
| 1992 | 41.76 | 5,01 | 1,40 | 48, 17 |
| 1993 | 48,17 | 5,78 | 1,02 | 54,98 |
| 1994 | 54,98 | 6,60 | 0,60 | 62,17 |
| 1995 | 62,17 | 7,46 | 0,11 | 69,74 |
|  |  | 29,14 | 4,85 |  |

f) After 1995, the operation should become financially self-supporting and able to produce a surplis that grows rapidly from year to year.

BUSINESS PLAN B (10\% Growth)
However, if the new business after 1991 is allowed to grow at the rate of $10 \%$ per annum, the extra growth will increase the total subsidy required over the next 5 years from R4, 85 million to $R 6,10 \mathrm{million}$, and a further subsidy of R 220000 will be required in 1996. The effect of this additional new business on the growth, over the period 1991-95, of the fund, premium income, subsidy needs and embedded value of the business, is shown in Table 8:

Table 8
Comparison of Plan $A$ and Plan $B$.

| Projected Fund at end of 1995: | 469,63 | 472,15 |
| :---: | :---: | :---: |
| Projected Premium Income for 1995: | 82,47 | 84,53 |
| Total of additional subsidies | 4,85 | 6,32 |
| Embedded Value of policies in force | 69,74 | 71,16 |

* including R220 000 in 1996

The additional growth does not, therefore, greatly affect the finances of the operation over the next 5 years, but, if continued thereafter, it would materially reduce, for many years, the surpluses that could be expected to emerge from the operation. The comparison is as follows:

| Total Projected Surplus: |  |  |
| ---: | :---: | :---: |
| for years | Plan $A$ | Plan $B$ |
| $1996-2000$ | 8,92 | 3,94 |
| $2001-2005$ | 25,21 | 15,32 |
| $2006-2010$ | 39,11 | 23,99 |

In the light of these figures, it is difficult to escape the conclusion that a future growth of about $8 \%$ per annum is the right rate of expansion for this particular operation. Much will depend upon the size of the South African operation in relation to the overall finances of the parent fund, and the strategy that underlay the commencement of the business. If the feeling by the overseas management is that they would now like to see diminishing subsidies, and some cash return, as soon as possible, from the substantial investment that has been made, then it is clear that Plan $A$ must be the choice. If, however, the overseas management is not unhappy to find the extra R1, 5 million required to finance Plan $B$; is satisfied to invest part of its central reserves in South Africa to secure a $12 \%$ yield, and is in no hurry to see a cash return on its investment, then there is no reason why the local management should not try to secure the extra growth.

Practical Considerations.
The production of a business plan, for an established South African operation, in the form of a projected revenue account for the existing and future business combined, is not a task to be lightly undertaken.

An established office in South Africa is likely to have all kinds of policies on its books, ranging from conventional With and Without Profits business through many different kinds of linked policies to modern Universal plans. In addition, it is likely to have many different types of temporary assurances - level term, increasing term, decreasing term, convertible and renewable term - and all kinds of supplementary benefits. Many of these plans will be based on different financial principles and will require special routines to calculate the amounts payable under the policy.

The choice will have to be made between the construction of one programme to handle every class of policy or the construction of separate programmes for policies based upon different financial principles. This is essentially a matter of compromise. It is unlikely, for example, that the same projection programme would be used for both assurance and annuity business, or even for linked and non-linked policies, but most of the policies with easily ascertainable benefits - conventional With and Without Profits business and most temporary assurances - could easily be handled by the same programme. The ideal is to write programmes that can be understood by other members of the actuarial team and to avoid unduly long and complicated programmes.

If, as seems likely, the calculations are to be made class by class, a separate programme will be required to add together the results, since it would be pointless to attempt the addition of perhaps more than 400 items for each policy class by any other means.

Even the projection of the future business, which is far simpler, is not without its complications, since it is likel that some of the classes of business being sold will be found to be insufficiently profitable, or to be giving rise to unacceptable financial strains. The problem then is to decide whether these policies should be incorporated into the business plan as they stand, or whether, prior to their incorporation, the premium rates or policy terms should be altered to produce more satisfactory results.

These complications may seem too formidable to make the work worthwhile, but, once the programmes are written and the future new business restricted to policy classes that have been profit-tested, the work involved becomes routine. The calculations to be made for each valuation group are vastly more than those required for the valuation of the liability, but, if a really fast modern personal computer with a maths co-processor is used, the additional calculations take surprisingly little extra time. In the writer's experience, the work involved is far less than actuaries used to routinely perform, in pre-computer times, simply to produce the valuation and an approximate analysis of surplus.

The big job is to write the projection programmes and establish the system, and this is likely to take a small team ( 2 or 3 people) several months. But, thereafter, the active interplay between the decision making and accounting processes of the office and the financial planning is far more demanding than the work associated with the production of an annual valuation.
11. Embedded Values as a Management Tool

Given the limited actuarial resources available in South Africa, the question inevitably arises : Is all this extra work worthwhile? Could the management not derive all the information that it needs from the annual valuation. Does this peering into the unknowable future really serve any useful purpose? This is essentially a philosophical question. No one who has been responsible for the development of a new life insurance operation in the days when the calculation of embedded was physically impossible will seriously question the usefulness of being able to place a value, calculated on a consistent basis, on the policies in force. Without this measure, the management faces the problem of having to report each year an ever-increasing trading loss and of explaining, perhaps to a mystified Board, that the apparent losses are merely 'new business strains'. In these conditions, the financial reporting lacks the coherence that makes it comprehensible to the non-technical members of the Management or Board and easily becomes a meaningless ritual of reporting new business figures and of demonstrating that the current expenditure is within budgetary controls that, when set, may only have been a reflection of the ego of the management. It is not surprising if, in these circumstances, the production of new business, regardless of its quality or profitability, becomes the exclusive goal.

But a new sense of realism enters the reporting process once the embedded value of the policies in force is brought into the picture, since the apparent loss reflected in the accounts for the year can be compared with the increase in the embedded value of the business. This makes it possible to monitor the finances of the business against preset financial objectives, including the need to secure an adequate return on the capital invested in new business strains.

With an established business, the problem of reporting apparent losses does not arise, but, in the days when embedded value calculations were not available, the management of established offices had to face complicated problems and make difficult decisions on the basis of limited knowledge, with, perhaps, only an intuitive feel for the financial consequences of the decisions made. This may have been possible in the past, when policies were simpler and financial conditions more stable, but it is hardly practicable now that the policy range is so extensive and complicated and financial conditions so liable to change. Indeed, one wonders whether established offices would have fought so long and hard to continue to market reversionary bonuses policies in an inflationary environment, if they had had a clear understanding of the financial issues involved.

The principal advantage of calculating embedded values is that it makes it possible to disentangle the finances of the existing policies - which, in large measure, are the result of decisions already made - from the finances of the future new business - which will be the result of decisions yet to be made. It is the actuarial equivalent of the 'Christmas Carol' in which the Ghosts of Christmases Past, Present and Future help Scrooge to alter the course of his life.

It has the added advantage that it enables the finances of the business to be presented in a form that is consistent with the finances of other types of businesses.

In particular it shows:
a) the source from which new business strains are being financed and whether a satisfactory rate of return is expected to be earned on the capital used for this purpose.
b) whether, over and above this, the organisation is being adequately rewarded for the management and enterprise of the business.
c) how the experience under the existing business compares with the assumptions made in previous embedded value calculations: are the policies being administered economically? How does the current yield on investments compare with that previously assumed? Is the mortality experience satisfactory? This does not mean that these comparisons could not be made by other means; but the advantage of the embedded value calculation is that it shows the impact of any variances upon the profitability of the operation.

All this is exactly the kind of information that is needed for the active management of a modern life insurance business and its production has a profound effect on the philosophy of the management. New business is no longer seen as an end in itself. It simply part of the process of creating profit. The same applies to the administration of the business. It is not just a service to policyholders; it is part of the process of creating profit, since the economic management of the expense margins can have a profound effect on the profitability of the business.

In the circumstances of the simplified example, the questions that inevitably arise are:
a) Why does the new business not contribute to the growth in the asset values, except to the extent that it causes bigger transfers? Surely it ought to make some contribution of its own?
b) Instead of restricting the new business flow, would it not be better to design policies with less new business strains or with a shorter repayment period?
c) Is it really necessary for management expenses to increase by over $50 \%$ during the next five years?

It only needs the impact of questions like this to transform the operation from a complacent, service-motivated institution into a market-driven, profit-oriented business.

This applies equally to a mutual office, which, while it has no shareholders, faces the problem of preserving equity between different generations of policyholders. This essentially implies, in today's inflationary environment, that the investment of central reserves into new business strains should be made on terms that are consistent with the duty of the office to its existing policyholders.
12. Embedded Values and the Annual Accounts.

It is inevitable that, once embedded values are calculated, pressure will arise for their inclusion in the Annual Accounts and, at first sight, this would seem to be a good idea.

The customary accounts of a life insurance company are so artificial that it is almost impossible for an outside person to gauge the inherent profitability of the business or to ascertain the 'earnings' that underlie dividends, and it is argued that showing the embedded values as an asset in the balance sheet would improve the understandability of life insurance accounts and give a truer picture of the company's finances.

The inclusion of embedded values would also solve the problem that all new companies face of how to remunerate shareholders during the company's development stage, while the capital is being used to finance valuation strains. If, the embedded value of the policies could be shown as an asset in the balance sheet, transfers to the life fund from capital, made to finance valuation strains, could be offset by crediting the profit and loss account with the increase during the year in the embedded values. If the company was operating profitably, the profit and loss account would then show a positive balance from which a dividend could be financed.

It is clear that, if the company, instead of selling policies, had invested part of its capital in personal loans, no accounting problem would arise. The loans made could and would, under normal accountancy practice, be shown in the balance sheet as an asset (less any provision thought necessary for bad debts), and the interest charged on the loans would appear as income in the company's revenue account.

In theory, new business strains could also be thought of as loans to policyholders, repayable with interest over the lifetime of the policies, and embedded values as the unrecouped balances of these loans. Why then should embedded values be treated in the accounts differently from personal loans? Is there, perhaps, some different quality to embedded values that makes it impossible or unwise to treat them as assets in the accounts? Are they real assets or simply an actuarial concept?

They certainly have the appearance of a real asset, since, in theory, if a company wished to realise the embedded value of its policies, all that it would have to do is to stop writing new business, after which, provided that the expenses of administering the closed portfolio are not more than the renewal expenses assumed in the calculation of the
embedded values, and the other projection assumptions are realised, the surpluses that would then emerge would gradually turn the embedded values into cash. How does this differ from a moneylender gradually realising his loans?

Or the company could cut short this process by selling its portfolio of business to another life insurance company.

These arguments in favour of showing the embedded values as an asset in the balance sheet are attractive, but the counter arguments are also very strong:
(a) Looked at from the point of view of normal accountancy practice, embedded values look suspiciously like the discounted value of deferred profits and the normal accountancy rule is: First earn your profit and then bring it into account; in the meantime, deferred profits should be regarded as goodwill and eliminated from the balance sheet. Who knows what the future will be hold: Interest rates could fall; expenses could soar as a result of inflation; Aids could decimate the company's policyholders; a change of market or political conditions could greatly increase lapse and surrender rates. Any of these events would undermine the anticipated flow of surplus on which the embedded values are based.
(b) The value placed upon the embedded values has no historic basis. It is arbitrary, since the rate at which the future surplus is discounted is arbitrary. It is simply a rate that the actuary has decided to use. Why 12\%; Why not $20 \%$; or $50 \%$; or even $500 \%$ which would greatly reduce their value! Or, worse still, a company wishing to inflate its apparent value could discount the future surpluses at an arbitrarily low rate.
(c) Embedded values cannot be used to meet the actuarial liabilities; to do so would undermine the valuation basis. If, therefore, part of the dividends paid are financed from increases in embedded values, the capital available to finance new business strains will be correspondingly diminished. This will inevitably increase the capital requirements of the business.

In the example shown, the operation was started with a capital of R12 million. This was the amount that was estimated to be required to meet future strains, if no dividends were paid. If, however, the income from the investment of the capital had been paid out as dividends, R20,99 millions of capital would already have been consumed and the dividends
would have diminished each year, as the surviving capital shrunk. If a constant dividend had been required, equal to the first year's investment income, the capital needed to generate this dividend and to meet the expected strains shown in the business plan would have been about R 50 million .

The fact is that paying dividends out of embedded values could easily become a vicious spiral, in which more and more capital is required to replace the capital represented by embedded values, and the compulsion to raise more capital makes it necessary to declare higher and higher dividends financed from embedded values.

In short, turning embedded values into balance sheet assets will, at best, lead to confusion, and, at worst, create an ideal environment for the unscrupulous manipulation of life insurance accounts.

Given the strength of these arguments it is unlikely that that conservative actuarial opinion, will ever be happy with the idea of showing embedded values in the balance sheet of a life insurance company and, this means that they also cannot be used to justify the payment of a dividend.

If this is so, even quoting their value in a note to the accounts is dangerous, since it must inevitably give rise to the question: If your earnings and assets are so great, why is the dividend so low, or, worse still, why are you paying no dividend at all?.

The danger of showing earnings on an embedded value basis is that they could give rise to unrealistic shareholder expectations, since investors naturally feel that there should be some reasonable relationship between earnings and dividend distributions.

The fact is that there is a material difference of quality between the earnings of a trading company, represented by achieved profit on sales, and the 'earnings' of a life insurance company, represented by the discounted value of future surpluses, that are expected to emerge over many years and then to be wholly or partly absorbed in the financing of further valuation strains. Treating the latter as 'earnings in the hand' will cause more problems than it will solve.
13. Conclusion.

It seems unlikely, therefore, that the calculation of embedded values will solve the dividend problem of developing life insurance companies by enabling them to show their expenditure on first year strains as earnings in
their accounts, but this does not mean that embedded values serve no useful purpose.

Essentially, the calculation of embedded values is merely part of the information provided by the business plan. It is the business plan that, taken as a whole, provides the management of the operation with the information needed to control the finances of the business, namely:
a) the flow of surplus to be expected from existing policies,
b) the strains and surpluses that will result from the targeted new business, and the contributions to these from each segment of the business.
c) whether the new business is being written on a profitable basis.
d) a firm actuarial base for the expenditure budgets.

In these and other ways the information that the business plan provides is like a headlight illuminating the dark road along which the business is progressing.

In my opinion, the production each year of a business plan is an indispensable part of the financial management of any modern life insurance operation. Without the information that a business plan provides, the management is like a sailor crossing the ocean without navigational aids. In the past, actuaries had to manage, as best they could, with limited computational tools. But life insurance operations were then far simpler, since the product range was limited and financial conditions were stable for long periods of time. Today, an actuary faces far more complex problems and an ever-growing pressure for greater disclosure, but he (or she) now has access to the kind of computational power that could scarcely be conceived when I was an actuarial student, more than 40 years ago. It is the intelligent use of this new computational power that enables the modern actuary to use logic, instead of intuition, to solve these complex problems, and the whole process of profit-testing, surplus projection and embedded value calculation is the key that opens this door.

## APPENDIX 1.

## PROJECTION ASSUMPTIONS

All the projection calculations assume the following experience:

1. Mortality:

> SA 56-62 Ultimate with a selection factor for the first three policy years calculated as follows:
> $\begin{aligned} \text { Year 1:(SEL) } & =(105-x) / 100 \\ > & =.5\end{aligned}$
> Where $x$ is the age next birthday at entry
> Year 2: $(\operatorname{SEL})_{2}=\left(1+(\operatorname{SEL})_{1}\right) / 2$
> Year 3:(SEL $)_{3}=\left(2+(\operatorname{SEL})_{1}\right) / 3$
> For the convenience of readers who are not in South Africa, the mortality rates for the SA 56-62 Ultimate Table are shown in the attached table.
> Note : Policyholders were assumed to enter at age ( $x-1 / 2$ ) and attain exact age $x$ at the end of the calendar year.
2. Interest on Fund: 8\% per annum

Note: No attempt has been made to reproduce the financial conditions that have prevailed in South Africa over the period, since these would have reduced to absurdity the concept of developing a life insurance operation selling reversionary bonus policies. The example assumes a stable financial environment, with modest inflation and the kind of investment returns that might be acceptable in these circumstances.
3. Surrenders and Lapses:
a) Termination Rates.

The following termination rates were assumed:
Policy Termination Rate

Year
(\%)
15,0
10,0
5.0
2.5

Note: For purposes of simplicity, withdrawals were assumed to be spread evenly over the policy year. In practice, allowance would probably have to be made for the actual incidence of lapses during the first policy year.
b) Surrender Values

Surrender values were assumed to payable after the end of the second policy year and were calculated by multiplying the total of the proportionate PUP and declared bonus by an A factor based on the SA 56-62 Table with interest at $4 \%$ per annum. The A factor at the end of policy year $t$ is :

$$
A=A_{x+t: \bar{n}-t \mid}
$$

the factor for intermediate months being obtained by linear interpolation.

Note: See comment on surrender value basis in section 3 of paper

Commission and Commission Related Expenses
The following rates of commission were assumed:
First Policy Year: $3 \%$ policy term, except for policies entering after age 55, for which the commission term was limited to age 75 .

Thereafter Nil.

Allowance was also made for First Year Commission Related Costs of $35 \%$ of the First Year Commission.
5. Management Expenses:

First Policy Year: 35: of the Annual Premium.
Thereafter: 6.5\% of the Annual Premium, increasing by 5\% per annum after the second policy year, i.e. the renewal percentage for year $t$ is:

$$
.065 \times 1.05 \wedge(t-2)
$$

6. Premium Frequency:

All premiums were assumed to be payable annually, but the programme allows for a different premium mix to be assumed.
7. Bonus Declarations:

Bonus declarations are assumed to made at the end of each calendar year. At the end of the first policy year, half a year's bonus is declared, compensated by a final half year's bonus at the maturity date. The maturity value of each policy is therefore assumed to be:

Sum Assured $x(1.015)^{2} \times(1.03)^{19}$
8. Policy Reserves:

The policy reserve was calculated at the end of each calendar year using the minimum basis prescribed by the South African Life Insurance Act, namely a modified net premium reserve on the SA 56-62 Ultimate Mortality Table with interest at 4.5\% and an allowance for first year expenses of $1.5 \%$ of the sum Assured. (The alternative ( $x+1$ ) Sprague adjustment does not apply to 20-year Endowment Assurances).


## MORTALITY RATES

| x | q( x ) | x | $q(x)$ |
| :---: | :---: | :---: | :---: |
| 15 | 0.00142 | 60 | 0.01930 |
| 16 | 0.00142 | 61 | 0.02102 |
| 17 | 0.00143 | 62 | 0.02288 |
| 18 | 0.00143 | 63 | 0.02489 |
| 19 | 0.00143 | 64 | 0.02707 |
| 20 | 0.00143 | 65 | 0.02944 |
| 21 | 0.00144 | 66 | 0.03201 |
| 22 | 0.00144 | 67 | 0.03479 |
| 23 | 0.00145 | 68 | 0.03781 |
| 24 | 0.00146 | 69 | 0.04108 |
| 25 | 0.00147 | 70 | 0.04462 |
| 26 | 0.00148 | 71 | 0.04845 |
| 27 | 0.00150 | 72 | 0.05260 |
| 28 | 0.00153 | 73 | 0.05709 |
| 29 | 0.00156 | 74 | 0.06195 |
| 30 | 0.00159 | 75 | 0.06720 |
| 31 | 0.00164 | 76 | 0.07286 |
| 32 | 0.00170 | 77 | 0.07898 |
| 33 | 0.00177 | 78 | 0.08556 |
| 34 | 0.00186 | 79 | 0.09266 |
| 35 | 0.00197 | 80 | 0.10029 |
| 36 | 0.00210 | 81 | 0.10849 |
| 37 | 0.00225 | 82 | 0.11729 |
| 38 | 0.00244 | 83 | 0.12671 |
| 39 | 0.00266 | 84 | 0.13680 |
| 40 | 0.00291 | 85 | 0.14757 |
| 41 | 0.00320 | 86 | 0.15906 |
| 42 | 0.00353 | 87 | 0.17129 |
| 43 | 0.00391 | 88 | 0.18428 |
| 44 | 0.00433 | 89 | 0.19806 |
| 45 | 0.00479 | 90 | 0.21263 |
| 46 | 0.00531 | 91 | 0.22801 |
| 47 | 0.00587 | 92 | 0.24421 |
| 48 | 0.00649 | 93 | 0.26122 |
| 49 | 0.00716 | 94 | 0.27904 |
| 50 | 0.00788 | 95 | 0.29764 |
| 51 | 0.00867 | 96 | 0.31702 |
| 52 | 0.00951 | 97 | 0.33713 |
| 53 | 0.01043 | 98 | 0.35795 |
| 54 | 0.01142 | 99 | 0.37941 |
| 55 | 0.01249 |  |  |
| 56 | 0.01365 |  |  |
| 57 | 0.01490 |  |  |
| 58 | 0.01625 |  |  |
| 59 | 0.01772 |  |  |

SUMMARY OF PROFIT TESTS FOR INDIVIDUAL POLICIES
TYPE OF POLICY：WITH PROFIT ENDOWMENT ASSURANCES

|  |  | surplus for ： |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Year | Year | Year | Year | Year |
| 1 | 2 | 3 | 5 | 10 |

$$
\begin{array}{cc}
\text { Present } & \text { Illustr- } \\
\text { Value } & \text { ative } \\
\text { of } & \text { Maturity } \\
\text { Future } & \text { Value } \\
\text { Surplus } & (0 \%) \\
\left(\begin{array}{c}
12
\end{array}\right) &
\end{array}
$$

Initial
Annual
Premium
Entry
Age

으N오NㅇNNNNNN


のNぃMNートNm
$\dot{\sim}$



＊per 10000 basic annual premium

$$
\text { APPENDIX } 2-\text { TABLE } 1
$$




APPENDIX: 2 - TABLE 2
PROJECTED REVENUE ACCOUNT FOR SINGLE INJECTION OF R100 MILLIONS OF SUMS ASSURED

| TYPE OF POLICY: WITH PROFIT ENDOWMENT ASSURANCES |  |  |  |  |  |  | CURRENCY: RANDS THOUSANDS |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YEAR | PREM- <br> IUMS | INTEREST | COMMISSION |  | EXPENSES |  | DEATH CLAIMS | SURRENDERS | MATURITY PAYMENTS | YEAR END |  | SURPLUS | ASSET <br> VALUE | YEAR |
|  |  |  | INITIAL | RENEWAL | INITIAL | RENEWAL |  |  |  | S FUND | RESERVE |  |  |  |
| annualised premium: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 5138.3 | -32.0 | 4117.0 | 0.0 | 1798.4 | 0.0 | 95.1 | 0.0 | 0.0 | -904.3 | 2387.6 | -3291.9 | 3292.1 | 1 |
| 2 | 4357.8 | 343.5 | 0.0 | 0.0 | 0.0 | 283.3 | 215.5 | 0.0 | 0.0 | 6590.1 | 5942.6 | 647.6 | 3039.6 | 2 |
| 3 | 3909.5 | 606.7 | 0.0 | 0.0 | 0.0 | 266.8 | 253.9 | 160.1 | 0.0 | 9777.8 | 9249.3 | 528.5 | 2875.8 | 3 |
| 4 | 3700.1 | 852.0 | 0.0 | 0.0 | 0.0 | 265.2 | 296.5 | 302.9 | 0.0 | 12936.9 | 12712.1 | 224.8 | 2996.2 | 4 |
| 5 | 3590.8 | 1125.5 | 0.0 | 0.0 | 0.0 | 270.2 | 340.3 | 277.4 | 0.0 | 16540.5 | 16307.5 | 233.0 | 3122.7 | 5 |
| 6 | 3483.4 | 1405.0 | 0.0 | 0.0 | 0.0 | 275.2 | 368.1 | 352.7 | 0.0 | 20199.8 | 19916.8 | 283.0 | 3214.4 | 6 |
| 7 | 3377.8 | 1685.5 | 0.0 | 0.0 | 0.0 | 280.2 | 398.4 | 431.2 | 0.0 | 23870.2 | 23540.8 | 329.4 | 3270.7 | 7 |
| 8 | 3273.9 | 1967.0 | 0.0 | 0.0 | 0.0 | 285.2 | 431.4 | 512.9 | 0.0 | 27552.2 | 27179.4 | 372.7 | 3290.4 | 8 |
| 9 | 3171.6 | 2249.6 | 0.0 | 0.0 | 0.0 | 290.1 | 467.2 | 597.9 | 0.0 | 31245.4 | 30833.0 | 412.4 | 3272.9 | 9 |
| 10 | 3070.9 | 2533.1 | 0.0 | 0.0 | 0.0 | 294.9 | 505.9 | 686.1 | 0.0 | 34950.1 | 34501.1 | 449.0 | 3216.7 | 10 |
| 11 | 2971.6 | 2817.7 | 0.0 | 0.0 | 0.0 | 299.6 | 547.6 | 777.4 | 0.0 | 38665.8 | 38183.7 | 482.2 | 3120.5 | 11 |
| 12 | 2873.7 | 3103.3 | 0.0 | 0.0 | 0.0 | 304.3 | 592.0 | 871.9 | 0.0 | 42392.5 | 41880.6 | 511.9 | 2983.0 | 12 |
| 13 | 2777.2 | 3389.9 | 0.0 | 0.0 | 0.0 | 308.7 | 639.5 | 969.5 | 0.0 | 46129.9 | 45591.2 | 538.7 | 2802.3 | 13 |
| 14 | 2682.0 | 3677.3 | 0.0 | 0.0 | 0.0 | 313.1 | 690.0 | 1070.1 | 0.0 | 49877.5 | 49315.3 | 562.1 | 2576.4 | 14 |
| 15 | 2588.0 | 3965.8 | 0.0 | 0.0 | 0.0 | 317.2 | 743.4 | 1173.7 | 0.0 | 53634.8 | 53052.2 | 582.6 | 2303.0 | 15 |
| 16 | 2495.3 | 4255.0 | 0.0 | 0.0 | 0.0 | 321.1 | 799.7 | 1280.2 | 0.0 | 57401.5 | 56801.4 | 600.0 | 1979.3 | 16 |
| 17 | 2403.7 | 4545.2 | 0.0 | 0.0 | 0.0 | 324.8 | 858.7 | 1389.5 | 0.0 | 61177.4 | 60562.9 | 614.5 | 1602.3 | 17 |
| 18 | 2313.3 | 4836.1 | 0.0 | 0.0 | 0.0 | 328.2 | 920.3 | 1501.5 | 0.0 | 64962.4 | 64336.1 | 626.2 | 1168.4 | 18 |
| 19 | 2224.1 | 5127.9 | 0.0 | 0.0 | 0.0 | 331.4 | 984.3 | 1616.2 | 0.0 | 68756.3 | 68121.0 | 635.2 | 673.4 | 19 |
| 20 | 2136.1 | 5420.5 | 0.0 | 0.0 | 0.0 | 334.1 | 1050.7 | 1733.4 | 0.0 | 72.559 .3 | 71917.6 | 641.7 | 112.5 | 20 |
| 21 | 0.0 | 2797.6 | 0.0 | 0.0 | 0.0 | 0.0 | 550.5 | 921.2 | 73117.5 | 126.0 | 0.0 | 126.0 | 0.0 | 21 |

## table 3

ANALYSIS OF SURPLUS FOR SINGLE INJECTION OF R 100 MILLIONS OF SUMS ASSURED

| type of policy: with profit endowment assurances |  |  |  |  |  |  | CURRENCY: RANDS THOUSANDS |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YEAR | EXPENSE SURPLUS | INTEREST SURPLUS | SURRENDER sURPLUS | DEATH SURPLUS | NEW BUSINESS STRAIN | BONUS <br> STRAIN | maturity SURPLUS | TOTAL SURPLUS | year |
| FUND | 5138.3 | -32.0 | 0.0 | 95.1 | 5915.4 | 0.0 | 0.0 | -904.3 | 1 |
| 1 Reserve | 3525.1 | 41.5 | 193.6 | 161.1 | 1500.0 | 675.6 | 0.0 | 1036.3 |  |
| 1 Surplus | 1613.1 | -73.5 | 193.6 | 66.0 | 4415.4 | 675.6 | 0.0 | -3291.9 | 1 |
| FUND | 4074.5 | 343.5 | 0.0 | 215.5 | 0.0 | 0.0 | 0.0 | 6590.1 | 2 |
| 2 Reserve | 2989.6 | 159.0 | 506.3 | 298.2 | 0.0 | 1210.9 | 0.0 | 3520.8 |  |
| SURPLUS | 1084.9 | 184.6 | 506.3 | 82.7 | 0.0 | 1210.9 | 0.0 | 647.6 | 2 |
| FUND | 3642.7 | 606.7 | 160.1 | 253.9 | 0.0 | 0.0 | 0.0 | 9777.8 | 3 |
| 3 Reserve | 2682.0 | 307.1 | 549.9 | 297.3 | 0.0 | 1164.9 | 0.0 | 6919.6 |  |
| 3 SURPLUS | 960.7 | 299.6 | 389.7 | 43.3 | 0.0 | 1164.9 | 0.0 | 528.5 | 3 |
| FUND | 3435.0 | 852.0 | 302.9 | 296.5 | 0.0 | 0.0 | 0.0 | 12936.9 | 4 |
| 4 Reserve | 2538.2 | 456.1 | 398.1 | 315.7 | 0.0 | 1182.3 | 0.0 | 10347.6 |  |
| SURPLUS | 896.8 | 395.9 | 95.1 | 19.2 | 0.0 | 1182.3 | 0.0 | 224.8 | 4 |
| FUND | 3320.6 | 1125.5 | 277.4 | 340.3 | 0.0 | 0.0 | 0.0 | 16540.5 | 5 |
| 5 RESERVE | 2463.1 | 613.5 | 367.3 | 344.3 | 0.0 | 1230.5 | 0.0 | 13846.5 |  |
| 5 SURPLUS | 857.6 | 512:0 | 89.9 | 4.0 | 0.0 | 1230.5 | 0.0 | 233.0 | 5 |
| FUND | 3208.2 | 1405.0 | 352.7 | 368.1 | 0.0 | 0.0 | 0.0 | 20199.8 | 6 |
| 6 RESERVE | 2389.2 | 771.5 | 458.5 | 373.1 | 0.0 | 1280.3 | 0.0 | 17356.3 |  |
| 6 SURPLUS | 819.0 | 633.5 | 105.8 | 4.9 | 0.0 | 1280.3 | 0.0 | 283.0 | 6 |
| FUND | 3097.6 | 1685.5 | 431.2 | 398.4 | 0.0 | 0.0 | 0.0 | 23870.2 | 7 |
| 7 Reserve | 2316.6 | 930.1 | 550.1 | 404.3 | 0.0 | 1331.6 | 0.0 | 20877.6 |  |
| 7 SURPLUS | 781.0 | 755.4 | 118.9 | 5.8 | 0.0 | 1331.6 | 0.0 | 329.4 | 7 |

APPENDIX: 2 - TABLE 3 - CONTINUED
ANALYSIS OF SURPLUS FOR SINGLE INJECTION OF R 100 mILLIONS OF SUMS ASSURED - CONTINUED
TYPE OF POLICY: WITH PROFIT ENDOWMENT ASSURANCES

| TYPE OF POLICY: WIth profit endowment assurances |  |  |  |  |  |  | CURRENCY: RANDS THOUSANDS |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YEAR | EXPENSE SURPLUS | INTEREST SURPLUS | SURRENDER SURPLUS | DEATH SURPLUS | NEW BUSINESS STRAIN | BONUS <br> STRAIN | MATURITY SURPLUS | TOTAL SURPLUS | Year |
| FUND | 2988.7 | 1967.0 | 512.9 | 431.4 | 0.0 | 0.0 | 0.0 | 27552.2 | 8 |
| 8 RESERVE | 2245.2 | 1089.5 | 642.0 | 438.5 | 0.0 | 1384.5 | 0.0 | 24410.5 |  |
| 8 SURPLUS | 743.5 | 877.5 | 129.1 | 7.1 | 0.0 | 1384.5 | 0.0 | 372.7 | 8 |
| FUND | 2881.5 | 2249.6 | 597.9 | 467.2 | 0.0 | 0.0 | 0.0 | 31245.4 | 9 |
| 9 RESERVE | 2174.8 | 1249.5 | 734.3 | 475.4 | 0.0 | 1438.9 | 0.0 | 27955.3 |  |
| 9 SURPLUS | 706.7 | 1000.0 | 136.4 | 8.2 | 0.0 | 1438.9 | 0.0 | 412.4 | 9 |
| FUND | 2776.0 | 2533.1 | 686.1 | 505.9 | 0.0 | 0.0 | 0.0 | 34950.1 | 10 |
| 10 RESERVE | 2105.6 | 1410.3 | 827.0 | 515.7 | 0.0 | 1494.9 | 0.0 | 31511.4 |  |
| 10 SURPLUS | 670.4 | 1122.8 | 140.9 | 9.7 | 0.0 | 1494.9 | 0.0 | 449.0 | 10 |
| FUND | 2672.0 | 2817.7 | 777.4 | 547.6 | 0.0 | 0.0 | 0.0 | 38665.8 | 11 |
| 11 RESERVE | 2037.3 | 1571.9 | 920.0 | 559.0 | 0.0 | 1552.4 | 0.0 | 35079.0 |  |
| 11 SURPLUS | 634.6 | 1245.9 | 142.6 | 11.4 | 0.0 | 1552.4 | 0.0 | 482.2 | 11 |
| FUND <br> 12 PESERVE | $2569.5$ | $3103.3$ | $871.9$ | $592.0$ | $0.0$ | $\begin{array}{r} 0.0 \\ 1611.4 \end{array}$ | 0.0 0.0 | $42392.5$ | 12 |
| 12 RESERVE | 1970.0 | 1734.1 | 1013.4 | 605.2 | 0.0 | $1611.4$ | 0.0 | $38657.8$ |  |
| 12 SURPLUS | 599.4 | 1369.2 | 141.5 | 13.2 | 0.0 | 1611.4 | 0.0 | 511.9 | 12 |
| FUND | 2468.5 | 3389.9 | 969.5 | 639.5 | 0.0 | 0.0 | 0.0 | 46129.9 | 13 |
| 13 RESERVE | 1903.7 | 1897.1 | 1107.2 | 654.9 | 0.0 | 1671.9 | 0.0 | 42247.4 |  |
| 13 SURPLUS | 564.8 | 1492.8 | 137.7 | 15.4 | 0.0 | 1671.9 | 0.0 | 538.7 | 13 |
| FUND | 2368.9 | 3677.3 | 1070.1 | $690.0$ | $0.0$ | $0.0$ | 0.0 | $49877.5$ | 14 |
| 14 RESERVE | 1838.2 | 2060.8 | 1201.3 | $707.5$ | $0.0$ | $1733.9$ | 0.0 | $45847.5$ |  |
| 14 SURPLUS | 530.7 | 1616.5 | 131.2 | 17.6 | 0.0 | 1733.9 | 0.0 | 562.1 | 14 |

aGnNIdNOD - $\varepsilon$ gTgyd - $\tau$ :XIGNGddy
ANALYSIS OF SURPLUS FOR SINGLE INJECTION OF R 100 MILLIONS OF SUMS ASSURED - CONTINUED
TYPE OF POLICY: VITH PROFIT ENDOWMENT ASSURANCES

| type of policy: Vith profit endowment assurances |  |  |  |  |  |  | CURRENCY: RANDS THOUSANDS |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | EXPENSE SURPLUS | INTEREST SURPLUS | SURRENDER SURPLUS | DEATH SURPLUS | NEW BUSINESS STRAIN | Bonus STRAIN | MATURITY SURPLUS | total SURPLUS | YEAR |
| FUND | 2270.8 | 3965.8 | 1173.7 | 743.4 | 0.0 | 0.0 | 0.0 | 53634.8 | 15 |
| 15 RESERVE | 1773.6 | 2225.2 | 1295.7 | 763.7 | 0.0 | 1797.4 | 0.0 | 49457.4 |  |
| 15 SURPLUS | 497.2 | 1740.5 | 122.0 | 20.3 | 0.0 | 1797.4 | 0.0 | 582.6 | 15 |
| FUND | 2174.2 | 4255.0 | 1280.2 | 799.7 | 0.0 | 0.0 | 0.0 | 57401.5 | 16 |
| 16 RESERVE | 1709.8 | 2390.4 | 1390.5 | 822.9 | 0.0 | 1862.4 | 0.0 | 53076.7 |  |
| 16 SURPLUS | 464.3 | 1864.6 | 110.3 | 23.2 | 0.0 | 1862.4 | 0.0 | 600.0 | 16 |
| FUND | 2078.9 | 4545.2 | 1389.5 | 858.7 | 0.0 | 0.0 | 0.0 | 61177.4 | 17 |
| 17 RESERVE | 1646.9 | 2556.3 | 1485.5 | 885.0 | 0.0 | 1928.8 | 0.0 | 56705.3 |  |
| 17 SURPLUS | 432.0 | 1988.9 | 96.1 | 26.4 | 0.0 | 1928.8 | 0.0 | 614.5 | 17 |
| FUND | 1985.1 | 4836.1 | 1501.5 | 920.3 | 0.0 | 0.0 | 0.0 | 64962.4 | 18 |
| 18 RESERVE | 1584.8 | 2722.9 | 1580.9 | 950.3 | 0.0 | 1996.7 | 0.0 | 60342.7 |  |
| 18 SURPLUS | 400.3 | 2113.2 | 79.4 | 30.0 | 0.0 | 1996.7 | 0.0 | 626.2 | 18 |
| FUND | 1892.8 | 5127.9 | 1616.2 | 984.3 | 0.0 | 0.0 | 0.0 | 68756.3 | 19 |
| 19 RESERVE | 1523.4 | 2890.3 | 1676.5 | 1018.3 | 0.0 | 2066.1 | 0.0 | 63988.9 |  |
| 19 SURPLUS | 369.3 | 2237.6 | 60.4 | 34.0 | 0.0 | 2066.1 | 0.0 | 635.2 | 19 |
| FUND | 1801.9 | 5420.5 | 1733.4 | 1050.7 | 0.0 | 0.0 | 0.0 | 72559.3 | 20 |
| 20 RESERVE | 1462.9 | 3058.4 | 1772.5 | 1089.1 | 0.0 | 2136.9 | 0.0 | 67643.9 |  |
| 20 SURPLUS | 339.0 | 2362.1 | 39.1 | 38.4 | 0.0 | 2136.9 | 0.0 | 641.7 | 20 |

APPENDIX 3 - TABLE 1
initial business plan

| (CALENDAR YEAR BASIS) |  |  |  |  |  |  |  |  |  |  | CURRENCY: RANDS's MILLIONS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YES $R$ | B/E | $\begin{aligned} & \text { PREM- } \\ & \text { IUMS } \end{aligned}$ | INTEREST | COMMISSION |  | EXPENSES |  | DEATH <br> CLAIMS | SURRENDERS | MATURITY <br> PAYMENTS | FUND RESERVE |  | SURPLUS | $\begin{aligned} & \text { ASSET } \\ & \text { VALUE } \end{aligned}$ | Year |
|  |  |  |  | INITIAL | RENEWAL | INITIAL | RENEWAL |  |  |  |  |  |  |  |  |
| 1981 | 0.00 | 2.57 | -0.02 | 2.06 | 0.00 | 0.90 | 0.00 | 0.05 | 0.00 | 0.00 | -0.45 | 1.19 | -1.65 | 1.65 | 1981 |
| 1982 | 1.19 | 5.26 | 0.15 | 2.47 | 0.00 | 1.08 | 0.14 | 0.16 | 0.00 | 0.00 | 2.75 | 4.40 | -1.65 | 3.50 | 1982 |
| 1983 | 4.40 | 8.17 | 0.49 | 2.88 | 0.00 | 1.26 | 0.30 | 0.32 | 0.08 | 0.00 | 8.21 | 9.86 | -1.65 | 5.57 | 1983 |
| 1984 | 9.86 | 11.36 | 1.00 | 3.29 | 0.00 | 1.44 | 0.49 | 0.53 | 0.25 | 0.00 | 16.22 | 17.98 | -1.75 | 7.98 | 1984 |
| 1985 | 17.98 | 14.86 | 1.74 | 3.71 | 0.00 | 1.62 | 0.71 | 0.78 | 0.43 | 0.00 | 27.34 | 29.16 | -1.82 | 10.77 | 1985 |
| 1986 | 29.16 | 18.67 | 2.74 | 4.12 | 0.00 | 1.80 | 0.95 | 1.09 | 0.68 | 0.00 | 41.93 | 43.78 | -1.85 | 13.91 | 1986 |
| 1987 | 43.78 | 22.68 | 4.01 | 4.45 | 0.00 | 1.94 | 1.23 | 1.44 | 1.01 | 0.00 | 60.40 | 62.15 | -1.75 | 17.33 | 1987 |
| 1988 | 62.15 | 26.91 | 5.59 | 4.80 | 0.00 | 2.10 | 1.53 | 1.85 | 1.42 | 0.00 | 82.95 | 84.60 | -1.64 | 21.05 | 1988 |
| 1989 | 84.60 | 31.41 | 7.49 | 5.19 | 0.00 | 2.27 | 1.86 | 2.31 | 1.92 | 0.00 | 109.95 | 111.45 | -1.50 | 25.08 | 1989 |
| 1990 | 111.45 | 36.19 | 9.75 | 5.60 | 0.00 | 2.45 | 2.22 | 2.84 | 2.51 | 0.00 | 141.77 | 143.09 | -1.32 | 29.41 | 1990 |
| 1991 | 143.09 | 41.29 | 12.39 | 6.05 | 0.00 | 2.64 | 2.61 | 3.44 | 3.22 | 0.00 | 178.81 | 179.91 | -1.10 | 34.03 | 1991 |
| 1992 | 179.91 | 46.72 | 15.45 | 6.53 | 0.00 | 2.85 | 3.03 | 4.12 | 4.06 | 0.00 | 221.48 | 222.32 | -0.84 | 38.96 | 1992 |
| 1993 | 222.32 | 52.51 | 18.97 | 7.06 | 0.00 | 3.08 | 3.50 | 4.88 | 5.03 | 0.00 | 270.25 | 270.79 | -0.54 | 44.17 | 1993 |
| 1994 | 270.79 | 58.70 | 22.96 | 7.62 | 0.00 | 3.33 | 4.00 | 5.75 | 6.14 | 0.00 | 325.62 | 325.81 | -0.19 | 49.66 | 1994 |
| 1995 | 325.81 | 65.31 | 27.49 | 8.23 | 0.00 | 3.59 | 4.54 | 6.71 | 7.42 | 0.00 | 388.11 | 387.91 | 0.20 | 55.41 | 1995 |
| 1996 | 387.91 | 72.39 | 32.59 | 8.89 | 0.00 | 3.88 | 5.14 | 7.78 | 8.88 | 0.00 | 458.31 | 457.67 | 0.64 | 61.43 | 1996 |
| 1997 | 457.67 | 79.97 | 38.30 | 9.60 | 0.00 | 4.19 | 5.78 | 9.00 | 10.53 | 0.00 | 536.84 | 535.72 | 1.12 | 67.68 | 1997 |
| 1998 | 535.72 | 88.08 | 44.67 | 10.37 | 0.00 | 4.53 | 6.47 | 10.35 | 12.39 | 0.00 | 624.37 | 622.72 | 1.65 | 74.15 | 1998 |
| 1999 | 622.72 | 96.78 | 51.77 | 11.20 | 0.00 | 4.89 | 7.23 | 11.85 | 14.48 | 0.00 | 721.63 | 719.40 | 2.23 | 80.81 | 1999 |
| 2000 | 719.40 | 106.12 | 59.65 | 12.09 | 0.00 | 5.28 | 8.05 | 13.52 | 16.81 | 0.00 | 829.41 | 826.54 | 2.87 | 87.64 | 2000 |


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| YEAR | B/F | PRMS | INTETAEST | $\begin{gathered} \text { COMMN } \\ \text { INITIAL } \end{gathered}$ | COMMN RENEWAL | $\begin{gathered} \text { EXP } \\ \text { INITIAL } \end{gathered}$ | EXP RENEWAL | DEATH COST | $\begin{gathered} \text { SV } \\ \text { PAID } \end{gathered}$ | MATURITY PAYMENTS | FUND | RSERVE | SURPLUS | ASSET <br> VALUE | YEAR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1991 | 166.14 | 41.34 | 14.52 | 0.00 | 0.00 | 0.00 | 3.17 | 3.94 | 3.75 | 0.00 | 211.15 | 206.61 | 4.54 | 35.51 | 1991 |
| 1992 | 206.61 | 39.38 | 17.63 | 0.00 | 0.00 | 0.00 | 3.17 | 4.36 | 4.76 | 0.00 | 251.33 | 247.20 | 4.13 | 35.64 | 1992 |
| 1993 | 247.20 | 38.01 | 20.77 | 0.00 | 0.00 | 0.00 | 3.22 | 4.79 | 5.65 | 0.00 | 292.32 | 288.39 | 3.94 | 35.98 | 1993 |
| 1994 | 288.39 | 36.83 | 23.97 | C. 00 | 0.00 | 0.00 | 3.27 | 5.22 | 6.42 | 0.00 | 334.28 | 329.94 | 4.34 | 35.96 | 1994 |
| 1995 | 329.94 | 35.67 | 27.20 | 0.00 | 0.00 | 0.00 | 3.33 | 5.64 | 7.41 | 0.00 | 376.44 | 371.67 | 4.77 | 35.50 | 1995 |
| 1996 | 371.67 | 34.53 | 30.44 | 0.00 | 0.00 | 0.00 | 3.38 | 6.10 | 8.43 | 0.00 | 418.72 | 413.55 | 5.17 | 34.59 | 1996 |
| 1997 | 413.55 | 33.40 | 33.69 | 0.00 | 0.00 | 0.00 | 3.44 | 6.59 | 9.48 | 0.00 | 461.13 | 455.60 | 5.53 | 33.21 | 1997 |
| 1998 | 455.60 | 32.29 | 36.95 | 0.00 | 0.00 | 0.00 | 3.49 | 7.11 | 10.58 | 0.00 | 503.66 | 497.80 | 5.86 | 31.34 | 1998 |
| 1999 | 497.80 | 31.20 | 40.22 | 0.00 | 0.00 | 0.00 | 3.54 | 7.67 | 11.70 | 0.00 | 546.31 | 540.16 | 6.15 | 28.95 | 1999 |
| 2000 | 540.16 | 30.12 | 43.50 | 0.00 | 0.00 | 0.00 | 3.58 | 8.26 | 12.86 | 0.00 | 589.07 | 582.66 | 6.40 | 26.02 | 2000 |
| 2001 | 582.66 | 28.44 | 45.91 | 0.00 | 0.00 | 0.00 | 3.53 | 8.71 | 13.78 | 21.94 | 609.07 | 602.60 | 6.47 | 22.67 | 2001 |
| 2002 | 602.60 | 25.98 | 46.25 | 0.00 | 0.00 | 0.00 | 3.33 | 8.78 | 14.04 | 51.18 | 597.49 | 591.24 | 6.25 | 19.14 | 2002 |
| 2003 | 591.24 | 23.18 | 44.68 | 0.00 | 0.00 | 0.00 | 3.07 | 8.49 | 13.69 | 65.81 | 568.05 | 562.20 | 5.85 | 15.58 | 2003 |
| 2004 | 562.20 | 20.27 | 42.01 | 0.00 | 0.00 | 0.00 | 2.76 | 8.00 | 12.98 | 73.12 | 527.62 | 522.28 | 5.34 | 12.11 | 2004 |
| 2005 | 522.28 | 17.25 | 38.48 | 0.00 | 0.00 | 0.00 | 2.42 | 7.35 | 11.99 | 80.43 | 475.81 | 471.07 | 4.74 | 8.82 | 2005 |
| 2006 | 471.07 | 14.13 | 34.06 | 0.00 | 0.00 | 0.00 | 2.04 | 6.53 | 10.70 | 87.74 | 412.24 | 408.19 | 4.05 | 5.84 | 2006 |
| 2007 | 408.19 | 10.81 | 28.58 | 0.00 | 0.00 | 0.00 | 1.61 | 5.50 | 9.06 | 98.71 | 332.70 | 329.46 | 3.24 | 3.29 | 2007 |
| 2008 | 329.46 | 7.31 | 21.84 | 0.00 | 0.00 | 0.00 | 1.11 | 4.23 | 6.98 | 109.68 | 236.61 | 234.28 | 2.33 | 1.36 | 2008 |
| 2009 | 234.28 | 3.74 | 13.96 | 0.00 | 0.00 | 0.00 | 0.58 | 2.72 | 4.51 | 116.99 | 127.18 | 125.86 | 1.32 | 0.20 | 2009 |
| 2010 | 125.86 | 0.00 | 4.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.96 | 1.61 | 127.96 | 0.22 | 0.00 | 0.22 | 0.00 | 2010 |



○


APPENDIX 6 －TABLE 1
REVISED BUSINESS PLAN ASSUMING NEW BUSINESS GROWTH AFTER 1991 of $8.0 \%$ PER ANNUM

| TURITY | YEAR END |  | SURPLUS | ASSET | YEAR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| YMENTS | FUND | RESERVE |  | Value |  |
| 0.00 | －1．72 | 4.54 | －6． 25 | 6.26 | 1991 |
| 0.00 | 10.67 | 16.19 | －5．52 | 12.53 | 1992 |
| 0.00 | 30.10 | 35.06 | －4．96 | 19.00 | 1993 |
| 0.00 | 57.08 | 62.02 | －4．93 | 26.21 | 1994 |
| 0.00 | 93.08 | 97.96 | －4．88 | 34.24 | 1995 |
| 0.00 | 138.90 | 143.64 | －4．74 | 43.09 | 1996 |


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APPENDIX 7 - TABLE
REVISED BUSINESS PLAN ASSUMING NEW BUSINESS GROWTH AFTER 1991 OF 10.0 PER ANNUM
PROJECTED REVENUE ACCOUNT FOR EXPECTED FUTURE BUSINESS
CURRENCY: RANDS's MILLIONS

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| ENDERS PAYMENTS FUND RESERVE |  |  |

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APPENDIX 8 - TABLE
revised business plan assuming new business growth after 1991 of 8.0\% per annum
projected revenue account for existing and future business combined

| (CALENDAR YEAR BASIS) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| YEAR | B/E | PREM- <br> IUMS | INTEREST | COMMIS INITIAL | SSION RENEWAL | $\begin{aligned} & \text { EXPEN } \\ & \text { INITIAL } \end{aligned}$ | ES RENEWAL | DEATH clatms | SURRENDERS | MATURITY <br> PAYMENTS | $\begin{aligned} & \text { YEA } \\ & \text { FUND } \end{aligned}$ | AR END RESERVE | SURPLUS | ASSET <br> VALUE | YEAR |
| 1991 | 166.14 | 51.11 | 14.46 | 7.82 | 0.00 | 3.42 | 3.17 | 4.12 | 3.75 | 0.00 | 209.43 | 211.15 | -1.72 | 41.76 | 1991 |
| 1992 | 211.15 | 58.21 | 18.22 | 8.45 | 0.00 | 3.69 | 3.71 | 4.97 | 4.76 | 0.00 | 261.99 | 263.39 | -1.40 | 48.17 | 1992 |
| 1993 | 263.39 | 65.77 | 22.55 | 9.12 | 0.00 | 3.99 | 4.31 | 5.93 | 5.95 | 0.00 | 322.42 | 323.44 | -1.02 | 54.98 | 1993 |
| 1994 | 323.44 | 73.84 | 27.52 | 9.85 | 0.00 | 4.30 | 4.95 | 7.01 | 7.33 | 0.00 | 391.36 | 391.96 | -0.60 | 62.17 | 1994 |
| 1995 | 391.96 | 82.47 | 33.17 | 10.64 | 0.00 | 4.65 | 5.66 | 8.22 | 8.91 | 0.00 | 469.52 | 469.63 | -0.11 | 69.74 | 1995 |
| 1996 | 469.63 | 91.68 | 39.56 | 11.49 | 0.00 | 5.02 | 6.42 | 9.59 | 10.72 | 0.00 | 557.63 | 557.19 | 0.43 | 77.68 | 1996 |
| 1997 | 557.19 | 101.55 | 46.74 | 12.41 | 0.00 | 5.42 | 7.25 | 11.11 | 12.78 | 0.00 | 656.50 | 655.46 | 1.04 | 85.96 | 1997 |
| 1998 | 655.46 | 112.11 | 54.78 | 13.41 | 0.00 | 5.86 | 8.14 | 12.82 | 15.11 | 0.00 | 767.01 | 765.29 | 1.72 | 94.55 | 1998 |
| 1999 | 765.29 | 123.43 | 63.75 | 14.48 | 0.00 | 6.32 | 9.12 | 14.72 | 17.74 | 0.00 | 890.09 | 887.63 | 2.46 | 103.44 | 1999 |
| 2000 | 887.63 | 135.56 | 73.73 | 15.64 | 0.00 | 6.83 | 10.17 | 16.83 | 20.68 | 0.00 | 1026.76 | 1023.48 | 3.27 | 112.58 | 2000 |
| 2001 | 1023.48 | 147.96 | 83.91 | 16.89 | 0.00 | 7.38 | 11.21 | 19.01 | 23.70 | 21.94 | 1155.24 | 1151.24 | 4.00 | 122.09 | 2001 |
| 2002 | 1151.24 | 160.52 | 93.18 | 18.24 | 0.00 | 7.97 | 12.21 | 21.02 | 26.41 | 51.18 | 1267.91 | 1263.34 | 4.56 | 132.18 | 2002 |
| 2003 | 1263.34 | 173.76 | 101.81 | 19.70 | 0.00 | 8.60 | 13.24 | 22.93 | 28.89 | 65.81 | 1379.74 | 1374.69 | 5.05 | 142.99 | 2003 |
| 2004 | 1374.69 | 187.99 | 110.70 | 21.27 | 0.00 | 9.29 | 14.35 | 24.91 | 31.44 | 73.12 | 1499.01 | 1493.47 | 5.54 | 154.61 | 2004 |
| 2005 | 1493.47 | 203.31 | 120.20 | 22.98 | 0.00 | 10.04 | 15.54 | 27.02 | 34.15 | 80.43 | 1626.82 | 1620.76 | 6.06 | 167.10 | 2005 |
| 2006 | 1620.76 | 219.81 | 130.40 | 24.81 | 0.00 | 10.84 | 16.81 | 29.30 | 37.07 | 87.74 | 1764.40 | 1757.78 | 6:62 | 180.53 | 2006 |
| 2007 | 1757.78 | 237.52 | 141.26 | 26.80 | 0.00 | 11.71 | 18.18 | 31.72 | 40.17 | 98.71 | 1909.27 | 1902.08 | 7.19 | 195.01 | 2007 |
| 2008 | 1902.08 | 256.55 | 152.73 | 28.94 | 0.00 | 12.64 | 19.63 | 34.29 | 43.44 | 109.68 | 2062.72 | 2054.95 | 7.78 | 210.63 | 2008 |
| 2009 | 2054.95 | 277.14 | 165.06 | 31.26 | 0.00 | 13.65 | 21.22 | 37.06 | 46.95 | 116.99 | 2230.02 | 2221.61 | 8.42 | 227.49 | 2009 |
| 2010 | 2221.61 | 299.34 | 178.38 | 33.76 | 0.00 | 14.75 | 22.92 | 40.05 | 50.75 | 127.96 | 2409.15 | 2400.05 | 9.10 | 245.69 | 2010 |

REVISED BUSINESS PLAN ASSUMING NEW BUSINESS GROWTH AFTER 1991 OF $10.0 \%$ PER ANNUM

| PROJECTED REVENUE ACCOUNT FOR EXISTING AND FUTURE BUSINESS COMBINEDNDAR YEAR BASIS) CURRENCY: RANDS'S MILLIONS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YEAR | B/F | $\begin{aligned} & \text { PREM- } \\ & \text { IUMS } \end{aligned}$ | INTEREST | COMMISSION |  | EXPENSES |  | DEATH <br> CLAIMS | SURRENDERS | MATURITY PAYMENTS | Y YEAR END |  | SURPLUS | ASSET value | YEAR |
|  |  |  |  | INITIAL | RENEWAL | INITIAL | EnEwal |  |  |  | FUND | RESERVE |  |  |  |
| 1991 | 166.14 | 51.11 | 14.46 | 7.82 | 0.00 | 3.42 | 3.17 | 4.12 | 3.75 | 0.00 | 209.43 | 211.15 | -1.72 | 41.76 | 1991 |
| 1992 | 211.15 | 58.40 | 18.21 | 8.60 | 0.00 | 3.76 | 3.71 | 4.97 | 4.76 | 0.00 | 261.96 | 263.48 | -1.52 | 48.30 | 1992 |
| 1993 | 263.48 | 66.36 | 22.57 | 9.47 | 0.00 | 4.13 | 4.32 | 5.94 | 5.95 | 0.00 | 322.60 | 323.87 | -1.27 | 55.36 | 1993 |
| 1994 | 323.87 | 75.05 | 27.57 | 10.41 | 0.00 | 4.55 | 4.99 | 7.05 | 7.33 | 0.00 | 392.16 | 393.13 | -0.97 | 62.98 | 1994 |
| 1995 | 393.13 | 84.53 | 33.29 | 11.45 | 0.00 | 5.00 | 5.73 | 8.30 | 8.94 | 0.00 | 471.53 | 472.15 | -0.62 | 71.16 | 1995 |
| 1996 | 472.15 | 94.89 | 39.81 | 12.60 | 0.00 | 5.50 | 6.54 | 9.73 | 10.78 | 0.00 | 561.71 | 561.93 | -0.22 | 79.92 | 1996 |
| 1997 | 561.93 | 106.22 | 47.20 | 13.86 | 0.00 | 6.05 | 7.44 | 11.34 | 12.89 | 0.00 | 663.77 | 663.54 | 0.23 | 89.28 | 1997 |
| 1998 | 663.54 | 118.61 | 55.55 | 15.24 | 0.00 | 6.66 | 8.44 | 13.15 | 15.30 | 0.00 | 778.91 | 778.18 | 0.73 | 99.27 | 1998 |
| 1999 | 778.18 | 132.17 | 64.95 | 16.77 | 0.00 | 7.32 | 9.53 | 15.20 | 18.03 | 0.00 | 908.44 | 907.15 | 1.29 | 109.89 | 1999 |
| 2000 | 907.15 | 147.03 | 75.52 | 18.44 | 0.00 | 8.06 | 10.74 | 17.50 | 21.13 | 0.00 | 1053.83 | 1051.91 | 1.91 | 121.16 | 2000 |
| 2001 | 1051.91 | 162.68 | 86.49 | 20.29 | 0.00 | 8.86 | 11.97 | 19.92 | 24.35 | 21.94 | 1193.77 | 1191.32 | 2.44 | 133.26 | 2001 |
| 2002 | 1191.32 | 179.11 | 96.78 | 22.32 | 0.00 | 9.75 | 13.20 | 22.23 | 27.32 | 51.18 | 1321.21 | 1318.41 | 2.80 | 146.45 | 2002 |
| 2003 | 1318.41 | 196.90 | 106.71 | 24.55 | 0.00 | 10.72 | 14.51 | 24.51 | 30.14 | 65.81 | 1451.79 | 1448.71 | 3.07 | 160.95 | 2003 |
| 2004 | 1448.71 | 216.46 | 117.22 | 27.00 | 0.00 | 11.80 | 15.94 | 26.93 | 33.11 | 73.12 | 1594.49 | 1591.14 | 3.35 | 176.92 | 2004 |
| 2005 | 1591.14 | 237.97 | 128.75 | 29.71 | 0.00 | 12.98 | 17.52 | 29.58 | 36.37 | 80.43 | 1751.28 | 1747.62 | 3.66 | 194.49 | 2005 |
| 2006 | 1747.62 | 261.66 | 141.44 | 32.68 | 0.00 | 14.27 | 19.26 | 32.51 | 39.95 | 87.74 | 1924.32 | 1920.32 | 4.00 | 213.82 | 2006 |
| 2007 | 1920.32 | 287.67 | 155.33 | 35.94 | 0.00 | 15.70 | 21.16 | 35.71 | 43.87 | 98.71 | 2112.22 | 2107.86 | 4.36 | 235.12 | 2007 |
| 2008 | 2107.86 | 316.25 | 170.46 | 39.54 | 0.00 | 17.27 | 23.25 | 39.21 | 48.13 | 109.68 | 2317.51 | 2312.76 | 4.75 | 258.59 | 2008 |
| 2009 | 2312.76 | 347.80 | 187.19 | 43.49 | 0.00 | 19.00 | 25.56 | 43.06 | 52.83 | 116.99 | 2546.81 | 2541.62 | 5.19 | 284.44 | 2009 |
| 2010 | 2541.62 | 382.52 | 205.74 | 47.84 | 0.00 | 20.90 | 28.11 | 47.34 | 58.07 | 127.96 | 2799.68 | 2793.98 | 5.69 | 312.88 | 2010 |

## APPFNDTX 10

## PROGRAMME NOTES.

These notes are provided to assist anyone seeking to write a projection programme and wishing to reproduce the results shown in the paper. Some of the assumptions used in the projection calculations have been made in order to simplify the example and would not necessarily be used in practice, depending upon the degree of refinement that was felt appropriate.

Apart from its set-up and printing routines, a projection programme essentially consists of two nested loops:
a) A loop for each policy in the assumed policy mix or , in the case of a projection of existing business. for each valuation group
b) Within the policy loop a loop for each policy year with an inner loop to calculate the fund at the end of each month.

The successive programming steps are described in these notes.
STAGE 1. - SET UP THE CONSTANTS THAT WILL BE REQUIRED BY THE LOOPS:
(i) The Commutation Factors on the Valuation Basis
(ii) The rate of interest assumed to be earned by the fund (I) and its corresponding monthly rate:
$(I M)=\left\{(1+I)^{\wedge}(1 / 12)-1\right\}$
Note: $\sim$ denotes exponentiation
(iii) The experience mortality table, if this is different from the mortality table used to calculate reserves.
(iv) The withdrawal rates - (qw).
(v) The premium frequency factors and their derivatives, namely:
(1) the factors to determine the proportion of the annual premium paid each month during the policy year:

$$
\text { pfact }(m) \quad(\text { for } m=1 \text { to } 12)
$$

(2) and the resulting deferred instalment factor (DI).

Note: in the example, all premiums are assumed to be payable annually and hence:

```
pfact(m)=1 if m=1
pfact(m)=0 if m>1
and (DI) = 0
```

(v) The factors needed to calculate the commission and expenses resulting from each premium payment.

STAGE 2 - THE POLICY LOOP.
(vi) Read policy details, namely:
(a) Entry age ( $x$ )
(b) policy term (n)
(c) premium term (m)
(d) Sum Assured (SA)
(e) Annual or Annualised Premium (AP)
(f) Weight to be given to policy.
(vii) Calculate Net Premium ( $\Pi$ )
(viii) Calculate, for each policy year, the A factors required to determine surrender values.

In the example, surrender values are calcuated by multiplying the total of the proportionate PUP and the accrued bonus by an $A$ factor, the factor for the end of policy year $t$ being:

$$
A^{\prime}=\bar{A}_{x+t: \overline{n-t}}
$$

Note: factors for the intervening months are obtained by linear interpolation

It is convenient to calculate the year-end $A^{\prime}$ factors in advance of the policy year loop, using the relationship:

$$
\begin{aligned}
A_{n}= & 1, \text { and } \\
A_{t}^{\prime}= & v^{1 / 2} \times q+v \times(1-q) \times A_{t+1}^{\prime} \\
& \text { if } t<n
\end{aligned}
$$

The $v$ factors are at $4 \%$ and the $q$ is $q_{x+t}$ from the SA 56-62 Ultimate table.
(ix) Calculate Initial Reserve ( $V_{0}$ )
i.e. the reserve at entry before payment of the first premium - this reserve is required for the analysis of surplus, in order to determine the new business strain.
(x) Define the radix of the double decrement table:

$$
(1 p)_{0}=1
$$

(xi) Zeroise all counters and variables that need to be set afresh for each policy.

STAGE 3 THE POLICY YEAR LOOP
For each of the policy years ( $t=1$ to $n$ ):
(xii) calculate the expected mortality rate $\left(q^{d}\right)_{t}=.5 x\left(q_{y}+q_{y-1}\right) *(S E L)_{t}$

Where:

$$
\left.\begin{array}{rl}
y= & x+t-1, \\
q_{y} & \text { is the SA } 56-58 \text { ULT mortality } \\
& \text { rate for age } y \text {, and }
\end{array}\right\}
$$

(xiii) calculate the next line of the double decrement table:
$d_{t}=(1 p)_{t-1} \times(q d)_{t} \times\left\{1-.5 \times(q w)_{t}\right\}$
$w_{t}=(1 p)_{t-1} \times(q w)_{t} \times\left\{1-.5 \times(q d)_{t}\right\}$
$(l p)_{t}=\left(l_{p}\right)_{t-1}-d_{t}-w_{t}$
(xiv) Calculate the commission and expenses to be paid during the policy year, assuming the policy remains in force.
(xv) Calculate the bonus declared during the policy year, if the policy is in force on the declaration date, and add this to the existing bonus:

New Bonus $=\left\{(S A)+(B)_{t-1}\right\} * .03 * k$
Where: (B) $t_{t-1}$ is the bonus in force at the end of year $(t-1)$, and

$$
\begin{aligned}
\text { k is } & =.5 \text { if } t=1 \\
& =1 \text { if } t>1
\end{aligned}
$$

(xvi) Calculate the policy reserve at the midpoint of the policy year (i.e. the end of the calendar year) and multiply it by the double decrement survival factor for the same point of time. using the formula:

$$
\begin{gathered}
v_{t}=\left\{(S A)+B_{t}\right\} \bar{A}_{y: \overline{n-t+.5}}-\prod \bar{a}_{y: m-t\rceil} \\
-\frac{T}{} \quad \text { (DI) }
\end{gathered}
$$

Where:
$V_{t}$ is the policy reserve at the end of
calendar year $t$, and
(DI) is the deferred instalment factor for the predefined premium frequency mix.

Hence the reserve multiplied by the survival
factor is given by the formula:
$(W)_{t}=V_{t} x(l c a l)_{t}$
Where (lcal) ${ }_{t}=\left(1 p_{t}+\left(1 p_{t-1}\right) / 2\right.$
Note: to simplify the analysis of surplus, the premium factor is calculated by using the formula:

$$
\bar{a}_{y: \overline{m-t}}=\frac{\langle N H\rangle}{-\cdots+1-(N H\rangle} \frac{x+m}{D_{y}}
$$

Where (DH) and (NH) are special commutation
factors defined by the relationships:
$(\mathrm{DH})_{x}=.5 *\left\{1_{x}+1_{x-1}\right\} x V^{(x-.5)}$ and
$(N H)_{x}=\sum_{t=0}(\mathrm{DH})_{x+t}$
Note: $l_{x}$ is the SA 56-58 Ultimate survival
factor: not the double decrement factor (lp)
(xvii) Calculate and accumulate the projected revenue and expenditure for each month of the policy year and derive from the monthly receipts and payments the projected revenue and expenditure for each calendar year, using the formula:

$$
\begin{aligned}
& F_{m}=(F B)_{m}+P_{m}+I_{m}-C_{m}^{i}-C_{m}^{r}-E_{m}^{i}-E_{m}^{r}-D_{m}-S_{m} \\
& \text { Where, for each of the months } m=1 \text { to } 12: \\
& F_{m} \text { is the expected amount of the fund at the end } \\
& \text { of month m. } \\
& (F B)_{m} \text { is the expected amount at the fund at the } \\
& \text { beginning of month } m .
\end{aligned}
$$

Note: (FB) $=$ the fund brought forward from the previous month, except for month 7, which is the beginning of a new calendar year, for which (FB) must be set to equal the policy reserve $-\left(W_{t}\right)$.
$P_{m}$ is the expected premiums for month m

$$
\begin{gathered}
=(A P) \times(p f a c t)_{m} \times 1_{m} \\
\text { Where } l_{m}=(m-1) \times(1 p)_{t}+(13-m) \times(1 p)_{t-1}
\end{gathered}
$$

$$
C_{m}^{i} \text { is the expected amount of any initial }
$$

Note: if $t>1, C_{m}^{i}=0$
$C_{m}^{r}$ is the expected amount of any renewal
commission for month m
Note: in the example $C_{m}^{r}=0$ for all policy years
$E_{m}^{i}$ is the expected amount of any initial expenses for month m

Note: $E_{m}^{i}=0$ unless $t=1$
$E_{m}^{r}$ is the expected amount of any renewal expenses
for month $m$

These four expense and commission factors are, in the example, all calculated by multiplying the expected premiums for the month by the applicable expense or commission factor.
$D_{m}$ is the expected death claims for month m
$=$ Death Value for Month $x d_{t} / 12$
Where the Death Value for the month is:

$$
\begin{aligned}
& (S A)+B_{t-1} \text { if } m<=6 \text {, and } \\
& (S A)+B_{t} \text { if } m>6
\end{aligned}
$$

Note: This assumes that no interim bonus is paid
$S_{m}$ is the expected surrender payments for month $m$ $=$ Surrender Value for month $m \times w_{t} / 12$ Where the Surrender Value for the month is: $(S V)_{m}=\left\{(S A) \times \frac{(t-1)+p p}{\text { premium term }}+(E B)\right\} \times A$, $\mathrm{pp}=$ Proportion of annual premium paid by end of month m . In the example pp always equals 1
$\mathrm{EB}=\mathrm{B}_{\mathrm{t}-1}$ if $\mathrm{m}<=6$
$=B_{t}$ if $m>6$, and
A. is obtained by interpolating between the previously calculated year-end factors.

And

| $I_{m}=$ | $\left(P_{m}-C_{m}^{i}-C_{m}^{r}-E_{m}^{i}-E_{m}^{r}\right) x(M I)$ |
| ---: | :--- |
| Where (MI) is the monthly accrual rate: |  |
|  | $(1+i) \sim(1 / 12)-1$ |$\quad$| at the fund accumulation rate |
| ---: | :--- |

(xviii) Derive the surplus for the calendar year:
(Surplus) ${ }_{t}=$ Fund at end of month $6-(W)_{t}$
Note: The technique used assumes that at the end of each calendar year the fund will be made equal to the liability at that time, by the transfer into the fund of any deficit, or the release from the fund of any surplus.
(xix) The final calendar half-year
a) Calculate the Maturity Value (MV) $=(S A)+B_{n}+(I B)$, where $(I B)=$ the interim bonus paid at the maturity date, namely:

$$
\left\{(S A)+B_{n}\right\} * .015
$$

b) Calculate the maturity payment (M)

$$
=(M V) \times(l p)_{r}
$$

c) Total the revenue receipts and payments for final six months and the amount of the fund at the maturity date
d) $\quad$ Set $V_{n+1}$ to $\left\{(S A)+B_{n}+\right.$ interim bonus $\}$
e) Calculate the surplus for calendar year ( $n+1$ )
$=$ Fund at maturity date less the maturity payment
(xx) Analysis of Surplus

The surplus for the calendar year can be analysed by comparing the expected net premiums, interest, death strain, and the reserves expected to be released by surrenders and lapses, with the corresponding items in the projected revenue account, the expected values being obtained by using the following formulae:

Expected Net Premiums (E) ${ }_{t}=$
[Net Premium + deferred instalments b/f - deferred instalments c/f] $x(l p)_{t-1}$

$$
\text { Expected Interest (I) } \quad=I_{1}+I_{2}-I_{3}
$$

## Where:

$$
\begin{aligned}
I_{1} & =\left[V_{t-1}+\begin{array}{rl}
\text { deferred instalment } b / f] \times(l b) \\
x & i
\end{array}\right) \\
I_{2} & =\text { Net Premium } \times(1 b) \times\left[(1+i)^{1 / 2}-1\right] \\
I_{3} & =\left[(S A)+B_{t-1}-5 \times \text { Net Premium }\right] \times(1 b) \times Q \\
i & =\text { the valuation rate of interest } \\
(1 b) & =(1 p)_{t-1} \text { and } \\
Q & =\left(l_{y-1}-l_{y}\right) / l_{y} \text { (SA } 56-6 i 2 \text { ULT Table) }
\end{aligned}
$$

Expected Surrender Release ( S$)_{t}=S_{1}+S_{2}$

## Where:

$$
\begin{aligned}
& s_{1}=.5 \times(w)_{t-1} \times{ }^{w}{ }_{t-1} \quad \text { and } \\
& s_{2}=.5 \times(w)_{t} \times{ }^{w}{ }_{t}
\end{aligned}
$$

Expected Death Strain (D) ${ }_{t}=D_{1}+D_{2}+D_{3}$
Where:

$$
\begin{aligned}
& D_{1}=\left\{\left\{(S A)+B_{t-1}-.5 \times N_{1 e t} \text { Premium }\right\}-\right. \\
& \left.\left\{V_{t-1}+\text { deferred instalment } b / f\right\}\right] \times Q \times(1 b) \\
& D_{2}=.5 \times W_{t-1} \times{ }^{d_{t-1}} \text { and } \\
& D_{3}=.5 \times W_{t} \times d_{t} \\
& \text { Expected New Business Strain }(N)_{t}^{\prime}
\end{aligned}
$$

$$
\begin{aligned}
& =-v_{0} \text { if } t=1 \\
& =0 \quad \text { if } t>1
\end{aligned}
$$

Expected Bonus Strain (B) ${ }_{t}$

$$
\left.=\left(B_{t}-B_{t-1}\right) x(1 b) \quad x \bar{A}_{y: n}-\overline{t+5}\right)
$$

Expected Maturity Release (M) ${ }_{t}^{\prime}$

$$
\begin{aligned}
& =0 \text { if } t<n+1 \\
& =v_{n+1} \times(1 p)_{n} \text { if } t=n+1
\end{aligned}
$$

Total Surplus $=E_{t}^{\prime}+I_{t}^{\prime}-S_{t}^{\prime}-D_{t}^{\prime}-N_{t}^{\prime}-B_{t}^{\prime}-M_{t}^{\prime}$

Note: The above formulae can be deduced from the equation of equilibrium linking the reserve at the beginning of the calendar year with the reserve at the end of the year, which in its simplest form is:

$$
\begin{aligned}
& v_{t}=\left(v_{t-1}+\text { deferred instalment } b / f\right) \quad x-\frac{D_{x+t-1}}{D_{x+t}} \\
& + \text { Net Premium } x \underset{D_{x+t}}{(D H)}+\ldots+t \text { New Bonus } x \bar{A}_{y: \overline{n-t+.5}} \\
& \text { - deferred instalment } c / f \\
& -\left\{(S A)+B_{t-1}\right\} x-\frac{\bar{C}_{x+t-1}}{D_{x+t}}
\end{aligned}
$$

Notes:

To simplify the analysis, the deaths and withdrawals in each policy year are assumed to occur at the midpoint of the year, one-half being allocated to the calendar year terminating at that date and the other half to the calendar year commencing at that date. The survival factor by which the the components of the above equation of equilibrium must be multiplied is then (lp)

The above formulae need modification for the first and final calendar years, since the analysis for these years relates only to a period of six months:

First year: replace $Q$ by $Q / 2$

$$
\text { replace } i \text { by }\left[(1+i)^{1 / 2}-1\right]
$$

Final year: $\quad Q=\left(1_{x+n-1}-l_{x+n}\right) /\left(1_{x+n-1}+l_{x+n}\right)$

$$
\begin{aligned}
& \text { replace }\left[(1+i)^{1 / 2}-1\right] \text { by } 0 \\
& \text { replace } i \text { by }\left[(1+i)^{1 / 2}-1\right]
\end{aligned}
$$

These replacement factors are obtained by appropriately modifying the equation of equilibrium shown above:

First year:

$$
v_{1}=V_{0} \frac{(D H)}{D_{x}}+\cdots \operatorname{Net} \text { Premium } \underset{D_{x}}{(D H)}
$$

+ New Bonus $\times \bar{A}_{x: \overline{n-.5}}-(S A) \times \underset{D_{x}}{.5 \times \bar{C}_{x-1}}$
- deferred instalment $c / f$


## Final year:

$$
\begin{aligned}
v_{n+1} & =\left(v_{n}+\text { deferred instalment } c / f\right) \\
& x_{(D H)_{x+n}}^{D_{x+n-1}} \\
& + \text { Final Bonus }-\left\{(S A)+B_{n}\right\} \times \bar{C}_{x+n-1}
\end{aligned}
$$

(xxi) Accumulate the results for the policy with results of the other policies in the assumed policy mix by applying to each of the revenue account and analysis of surplus components the operator:

$$
R V(t, k)=R V(t, k)+f(t, k) \quad x \quad \begin{aligned}
& \text { policy weight } \\
& \text { total of policy weights }
\end{aligned}
$$

Where $f(t, k)$ is the $k$ th item from the list made up of the totals for calendar year $t$ namely:
(1) the Revenue Account items:

$$
\begin{aligned}
& V_{t-1}, P_{t}, I_{t}, C_{t}^{i}, C_{t}^{r}, E_{t}^{i}, E_{t}^{r}, D_{t}, S_{t}, \\
& M_{t}, F_{t}, V_{t}, \text { and }(\text { Surplus })_{t}
\end{aligned}
$$

(2) the Analysis Component items:
$E_{t}^{\prime}, I_{t}^{\prime}, N_{t}^{\prime}, D_{t}^{\prime}, S_{t}^{\prime}, B_{t}^{\prime}$ and $M_{t}^{\prime}$
(xxii) calculate the embedded value of the single injection by discounting the projected surpluses at the required discount rate, using the relation:

$$
\begin{aligned}
& (E V)_{n+1}=0 \\
& (E V)_{t}=\left\{(E V)_{t+1}+(S S I)_{t+1}\right\} / 1.12, \text { if } t<n
\end{aligned}
$$

Where: (SSI) $t_{t}=$ the $R V(t, k)$ value representing the surplus for the assumed policy mix

## STAGE 5 PRINTING OF RESULTS

If the results for individual policies are required:
Print the projected Revenue Account for Individual policies during the Policy Year Loop, with a control to print the detailed calculations should these also be required.

If print of Analysis of Surplus is required, store the Analysis figures during the Policy Year Loop and print analysis and completion of all the Policy year loops

After completion of all the policy loops, print the Single Injection Revenue Account and the Single Injection Analysis.

$$
-0-0-0
$$

## MODIFICATIONS TO PROJECT EXISTING POLICIES

Section (vi) : Read details of Valuation Group
(a) Year of Entry - (YOE)
(b) Entry Age next birthday - (x)
(c) Total Sums Assured - (SA)
(d) Bonus attaching at Valuation Date - (EB)
(excluding bonus to be declared)
(e) Total of Annualised Premiums - (AP)
(f) Total of (e) payable after Valuation Date - (TDI)

Section (ix): Calculate Initial Reserve:

For existing business this is the reserve at the beginning of the first projection year, exclusive of the immediate bonus, hence, if $t$ is the curtate duration, (valuation year - YOE), the reserve is:

$$
v_{t}=\{(S A)+E B\} \bar{A}_{x+t: \overline{n-t}-.5}-\prod^{1} \bar{a}_{x+t}: \overline{n-t-1}
$$

- (TDI)

The amount of the immediate - (IMB) is:

$$
=\{(S A)+(E B)\} \times .03 \times k
$$

Where $k=1$ if $t>0$

$$
=.5 \text { if } t=0
$$

And the cost of the immediate bonus is:
(IMB) $\mathrm{x} \overline{\mathrm{A}}$

$$
x+t: \overline{n-t-.5}
$$

Note: These values are required only for balancing against the valuation, as the inital reseve, inclusive of the cost of the immediate bonus is calculate by the first policy year loop.

```
    Section (x): Define Radix of double decrement table
    Since the valuation date is the end of calendar t
    year: (lcal)
```



```
        Where: (aqd)
        (aqw)}\mp@subsup{\mathbf{t}}{}{(aw)
        and y = x + t - 1
    Note if t < 3 qd is select
STAGE 3 - THE POLICY YEAR LOOP
    loop for t = dur to n
where: dur = the curtate duration + l
Note: The first loop is for the policy year in which
        the valuation date falls. The reserve for this
        year is the reserve on the valuation date,
        inclusive of the immediate bonus.
```

Stage 4 - THE INNER LOOP
If $t=$ dur commence the monthly calculations with month 7 .
The opening fund will then be set equal to the reserve
on the valuation date, inclusive of the immediate bonus
and no monthly values will be calculated or accumulated
for the first six months of the policy year, since these
precede the valuation date.
Section (xx) Analysis of Surplus
Bypass the analysis routines if $t=$ dur, since the
analysis for the first calandar year can only be made
during the loop for $t=$ dur +1
Section (xxi) Accumulation of results

The Operator is:
$R V(t, k)=R V(t, k)+f(t, k)$
since there are no policy weights.


[^0]:    YEAR OF ENTRY 1981

