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A PROPHET OF PROFITS

An introduction to the theory and applications of profit tests

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1. INTRODUCTION

Times change, and we change with them. (Anon.)

1.1. ONE of the most interesting developments of the last decade has been the advance in micro-computer technology, and the actuary has taken full advantage. The advent of the mainframe computer heralded the use of a whole range of techniques which had previously been impractical because of the lengthy calculations involved; it is, however, the convenience and flexibility of the micro-computer that has brought some of these techniques to life. As a result the micro-computer has become an indispensable tool for a wide variety of actuarial work. In no area of actuarial practice has the impact of the micro been more noticeable than in that of profit testing. It is profit tests and their applications that are the subject of this paper.

1.2. In the discussion on Smart's paper (*J.I.A.*, **104,** 125) it was suggested that, for all its general excellence, it did not make the subject of profit testing readily accessible to the student. One of my aims in writing this paper has been to present the subject in a more practical form. In an attempt to make the paper readily comprehensible many aspects have been simplified, which I consider to be necessary if the principles are to be expounded clearly and are not to be clouded by unnecessary detail. I have tried to avoid the use of formulae in the main text, putting the emphasis on describing the basic concepts and leaving interested readers to develop formulae and computer programs applicable to their own circumstances; however, an appendix is included to offer some limited guidance. Many of the parameters used, although chosen to be realistic, have been tailored to give simple answers and, as a result, they are not necessarily suitable for practical use.

1.3. Part I deals with the theoretical aspects of profit testing whilst Part II considers some of the many applications which follow from the theory, these have been chosen to illustrate the usefulness of the methods and help the reader to gain a deeper understanding of the subject. The applications cover two major areas:

(i) Contract design and premium rating.

(ii) Office modelling and its use in forecasting profits.

The paper considers methods applicable to all types of conventional and unit-linked business, but for illustrative purposes it concentrates on individual life business. A glossary is included for those unfamiliar with unit-linked terminology. Although most of the techniques described would be useful in the actuarial management of a mutual company, the paper has been written from the viewpoint of a proprietory company.

1.4. The content of the paper cannot be claimed as original thought but, rather, should be viewed as an attempt to summarize the current state of development with a few ideas of my own added for good measure. The views expressed in the paper are, however, my own and should not necessarily be regarded as the views of my office or colleagues.

This paper has been assisted in its development by helpful discussions with several colleagues representing a variety of viewpoints both inside and outside my office. In particular I should like to thank Peter Bairstow, Sidney Benjamin, Alasdair Brown, David Purchase, Mike Shelley and Michael Turner for all the constructive comments and advice that they have given me. I am also grateful to the typists who had to struggle through my many drafts. Lastly I am indebted to Steve Wood for his assistance, including checking the final version. Responsibility for the final content of the paper, and any errors that remain, is however mine alone.

Part I—The Theory of Profit Testing

Annual income twenty pounds, annual expenditure nineteen nineteen six, result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery.

(Dickens: *David Copperfield*)

2. THE PHILOSOPHY OF PROFIT TESTING

2.1. Ensuring a continuing flow of profits is one of the prime responsibilities placed upon the management of a life office. Profits are required to finance the growth and development of the office and, especially, to give the shareholders a reasonable return on their investment. Smart defines profit as "*the excess during any period of income over outgo, where outgo includes the necessary increase in valuation reserves*".

Traditionally this would be called surplus, a word that has a special meaning in actuarial practice. Taking an historical perspective, it is only quite recently that attempts have been made to project surplus quantitatively; traditionally surplus has been left to emerge from margins in the premium basis. It is, however, the explicit estimate of the surplus expected in each future period that is all important to the philosophy of profit testing. To distinguish the historical surplus from these explicit estimates I shall call the latter profits in this paper.

2.2. In this section consideration is given to the profit test in its most generalized form, concentrating on the basic principles rather than the detail.

2.3. Given a group of identical contracts to profit test the first step is to choose a time interval to use in studying the contracts. The expected profit arising over each of a series of these intervals is calculated, the profit arising being as defined above. The factors entering these calculations might be:

Profit in the period = Premiums
+ Investment income & capital gains
− Tax
− Expenses & commission
− Death claims
− Surrenders
− Maturities
− Increase in the reserves required

Thus a profit test merely evaluates, period by period, the profits emerging. The power of the profit test, however, lies in the way in which it considers these emerging profits from the shareholders' point of view.

2.4. The shareholders have invested risk capital in the company and will be requiring a return on their investment. The actual net rate of return they require is likely to reflect the market rate for risk investments and is called the *risk discount rate*. It is an important point that, because of the risks involved, the risk discount rate will probably be significantly higher than the net yields available on, relatively secure, gilt-edged stocks, but the exact differential will depend on market conditions. The name risk discount is chosen because the rate can be used to discount the emerging profits to the inception date of the contract, and thereby put a value on the shareholders' interest in the monies expected to emerge from the contract over its lifetime. By discounting, some consideration is given not only to the magnitude of the profits, but also to the timing of their emergence.

Using profit tests, therefore, it is possible not only to quantify the expected profit in each period, but also its discounted value to the shareholders.

2.5. There are two points arising from the above description which are of general interest:

(i) When determining the profit emerging in a given period the valuation reserves are usually calculated using the office's standard valuation basis, but the other items of income and outgo are found using an 'anticipated actual' experience basis which could be weaker than the valuation basis. The profit emerging in each period will, therefore, be affected by the interaction of the two bases.

(ii) Different people will have different views on an appropriate risk discount rate. In fact it does not have to be a constant rate; it is more likely to be a variable. Some may require a risk discount rate which is a function of the time until the profits are expected to emerge, others may prefer a rate

which depends upon the amount of initial capital needed. These ideas will be pursued further in Section 12. For practical purposes I shall use a fixed risk discount rate whilst profit testing but it should not be forgotten that this approximates to the average effect of a variable rate.

2.6. This section has considered the fundamental principles involved in profit testing; in the following sections the actual mechanics of the profit test are considered in more detail for a variety of types of contract.

3. PROFIT TESTING OF INDIVIDUAL UNIT-LINKED CONTRACTS

3.1. The aim of this section is to explain the principles involved in profit testing unit-linked contracts.

The nature of the profit

3.2. All unit-linked companies need risk capital to get them started. The profit structure of a unit-linked company is such that all the profits are due to the providers of that risk capital; in practice this is often the shareholders but could also be the long-term business fund of the parent company.

The nature of the contract

3.3. With any unit-linked contract there are a number of parameters which specify the contract and are independent of the demographic and economic parameters considered in the actuarial basis:

(i) The premium.
(ii) The premium frequency.
(iii) The age and sex of the policyholder.
(iv) The term.
(v) The allocation rates.
(vi) The use of capital and accumulation units.
(vii) The bid/offer spread.
(viii) The policy fee.
(ix) The management charge(s).
(x) The commission rates.
(xi) The guaranteed minimum death benefit.

The actuarial basis

3.4. There are further parameters, many outside the control of the actuary, for which assumptions have to be made when designing or assessing the contract. Each parameter has to be considered in relation to two bases:

(i) The standard valuation basis which is used to calculate the reserves required from period to period.

(ii) The anticipated actual experience basis which is used to calculate the expected items of income and outgo in each period.

The following items may be required for one or both bases depending on the valuation methods used.

Mortality and other decrements

(i) A mortality table.
(ii) A set of lapse rates.
(iii) A surrender value basis.

Interest

(i) The gross unit growth rate.
(ii) The risk discount rate.

Tax

(i) The relevant tax rate(s).

Expenses

(i) The initial expenses.
(ii) The renewal expenses.
(iii) The rate of expense inflation.

The choice of values for these parameters is considered in Part II.

The determination of the profits

3.5. In Section 2 profit was defined as the excess of income over outgo in a given period with allowance for any necessary increase in reserves.

The period—The most common periods are a month and a year. The year interval is simpler to use and gives rise to quicker calculations. It can, however, considerably distort the results because of the effect of discounting and lapses, particularly in the first few years of a monthly premium contract (see Section 12). An interval of a month gives greater precision and in most circumstances this extra precision is required.

The increase in the unit reserves—It is first necessary to calculate the unit reserves required by a survivor at the end of each period on the valuation basis. These are easily derived by:

(i) Calculating the premiums allocated to units in each period.
(ii) Accumulating the unit fund at the start of the period, plus the premiums allocated, at the appropriate growth rate to the end of the period.
(iii) Deducting the tax and management charges arising during the period.

If the contract uses actuarial funding then additional reserves will be required in accordance with the funding plan.

The change in reserves can be found by comparing the reserves required at the beginning and end of the period, making allowance for survivors in both cases.

Income—This is the premium received plus the gross interest (including an allowance for capital appreciation) in the period.

Outgo—This is the sum of expenses, commission, death payments, maturities, surrenders and tax liability arising during the period.

The company will have a tax liability on the excess of investment income over expenses and commission for life contracts, and on profits for pensions contracts.

The commission item should allow for any advances and clawback of commission if indemnity terms are offered.

Discounting

3.6. If the above calculations are made for each period then the net profits emerging in each period will be found. To value these emerging profits, a discount factor is applied to the profit for each period and the results are summed.

Assume that the profit emerges, on average, midway through the period. If $f(t)$ = time in years to the middle of period t and r is the risk discount rate per annum then:

$$\text{Present value of future profits (PVFP)} = \sum_t (\text{Profit in period } t) \times v_r^{f(t)}$$

The PVFP is the value of the profit expected to emerge over the lifetime of the contract. It is a measure of the worth of the contract to the suppliers of the risk capital and the standard profitability measure.

3.7. In § 1.2 I stated that there are a number of simplifications in this paper. Some of these are mentioned here both for completeness and because they highlight the sort of detailed considerations that are necessary when determining the items of income and outgo.

(i) Sterling reserves	I am assuming that these reserves may be ignored as a result of contract design.
(ii) Additional reserves	I am ignoring any additional reserves required to cover the solvency margins.
(iii) Maturity guarantees	I am assuming that there are none.
(iv) Switches	I am ignoring any profit or loss arising from the operation of a switching facility.
(v) Tax	I am assuming that all tax liabilities are paid immediately, whereas in practice there is likely to be some delay.

4. PROFIT TESTING OF INDIVIDUAL CONVENTIONAL BUSINESS

4.1. The aim of this section is similar to that of Section 3 but the structure of conventional business will be seen to be quite different.

The nature of the profits

4.2. The profit structure of individual conventional business is complex and has to be established before suitable profit tests can be developed. For unit-linked contracts the only interested party is the supplier of the initial risk capital. For conventional contracts there still exist suppliers of risk capital, but they are not the only party interested in the emerging profits. The position is complicated because of the existence of three other factors:

(i) The with profit policyholders.
(ii) The without profit policyholders.
(iii) The estate.

The financial conditions of the early 1980's are such that in general:

(i) With profit policies are unlikely to be self-supporting and will draw on the profits made from other parts of the portfolio to support the bonus rates declared.
(ii) Without profit policies make a profit and these increase the value of the estate.
(iii) The estate, which I shall define as the hidden reserves not required to support the business in force, is utilized to bring in extra income and capital gains which increase its value. It can do this directly by external investment or by financing additional new business, and also indirectly by allowing greater flexibility in overall investment policy.

The shareholders' interest in the business is often a variable percentage of the value of the bonuses declared and therefore reflects the fortunes of the with-profits policyholders within each class of with-profit business. The interaction of these items is illustrated in Figure 1.

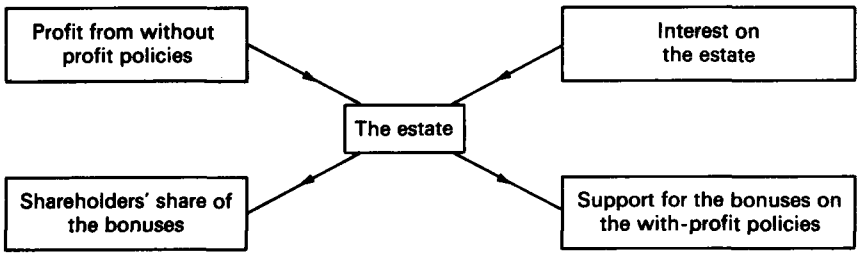

Figure 1. Profit structure of individual conventional business

It should be clear that it is the estate that should be considered when assessing emerging profits. It remains perfectly feasible to put a value on the expected transfers to the shareholders from a policy, since this depends on the value of bonuses declared, but it is also necessary to look at the effect of that policy on the estate. Further, since with-profit policies cause a loss and without profit policies a profit to the estate, one cannot sensibly look at policies in isolation unless one also looks at them collectively to see the overall effect. Mix of business is therefore going to be a key factor.

The nature of the contract

4.3. The parameters that specify the individual conventional contract are in general:

(i) The premium.
(ii) The premium frequency.
(iii) The age and sex of the policyholder.
(iv) The term.
(v) The commission rates.
(vi) The policy fee.
(vii) The basic sum assured.

The actuarial basis

4.4. Each parameter will have to be considered in relation to the anticipated actual basis and the valuation basis. The valuation would normally be a bonus reserve valuation but in certain circumstances a net premium valuation may be regarded as more appropriate.

Mortality & other decrements

(i) A mortality table.
(ii) A set of lapse rates.
(iii) A surrender value basis.

Interest

(i) The gross rate of interest.
(ii) The required rate of return on the estate.

Tax

(i) The relevant tax rate(s).

Expenses

(i) The initial expenses.
(ii) The renewal expenses.
(iii) The rate of expense inflation.

Bonus

(i) The rate of reversionary bonus.
(ii) The rate of terminal bonus.
(iii) The proprietors' share of bonuses declared.
(iv) The rate of discount used to convert reversionary bonuses into cash transfers to the proprietors' fund, which should be the published valuation rate of interest.
(v) The risk discount rate for future transfers to the proprietors' fund.

Determination of the profits and discounting

4.5. The principles are similar to those found in §§ 3.5 and 3.6 except that:

(i) An additional item of outgo is the transfer to the proprietors of their share of the value of bonuses declared.
(ii) The profit is the balance remaining to the estate; therefore the discount rate used is the required rate of return on the estate and the PVFP measures the effect of the contract on the estate.
(iii) The transfers to the proprietors can be discounted in a similar way to the profits, but using the shareholders' risk discount rate, to put a value on the contract from the shareholders' viewpoint.

5. PROFIT TESTING OF GROUP PENSION CONTRACTS

5.1. This paper concentrates on individual contracts but would not be complete without a brief look at group pensions contracts. To construct profit tests for premium rating group pensions contracts is difficult, but profit test ideas can be used to build a global view of the group pensions business. If group pensions business forms a large part of an office's portfolio then it is important that this is done, even if rather imprecisely, so that group pensions can be considered in an exercise such as that described in Sections 15 and 16.

5.2. It would be possible to consider a group contract as a collection of individual contracts, but this would be complicated and time consuming. One possible approach is to try to express the profits available for distribution to the shareholders in a period as a proportion of the funds under management in that period. With many group pensions contracts the employer and employees are effectively being given a yield on their contributions, so that it is not unreasonable to express the profit as a percentage of this yield and therefore a percentage of the funds under management. For managed funds and deposit administration contracts (with a cash bonus) the relationship is explicit and even for traditional group pensions policies with a reversionary bonus, the profits emerge in much the same way. To express profits as a proportion of funds under management is an exercise in itself, for which the previous few years' valuation results are a useful starting point. It should be appreciated that the results are

directly proportional to the percentage of funds assumed to be distributed as profits and care is necessary since the percentage will inevitably be small.

5.3. Once the profit, as a percentage of funds under management, has been determined then the funds in the future can be projected, by making suitable assumptions about interest rates, new business levels, expense inflation and withdrawals, enabling the profit expected to emerge in each future period to be found. These expected profits can be discounted using the risk discount rate to put a value on the business from the shareholders' viewpoint.

6. THE USE OF MODEL POLICIES

When we mean to build
We first survey the plot, then draw the model;
And when we see the figure of the house
Then we must rate the cost of the erection
Which if we find outweighs ability,
What do we then but draw anew the model
In fewer offices, or at last desist
To build at all?

(Shakespeare: *2 Henry IV*)

6.1. Apart from Section 5, this paper has so far considered profit testing a group of identical individual contracts. For the methods to be most effective it would be desirable if they could be applied to a block of business, or even a whole office. Whilst it would be possible to test every policy individually, as in a valuation, this would require a mainframe computer and so would be costly both in time and loss of flexibility. The profit test approach is to try and build a model of the block of business or office.

6.2. The art of building the model is to try to find a small set of policies which represent the business under consideration. Each of these policies is called a *model policy*, and a collection of these model policies a *model office*. If the number of model policies can be kept small then a micro computer is ideally suited for investigations using modelling techniques. The likely effect on profits of changes in bonus rates, marketing strategy and many other areas considered in Part II can be quickly assessed. The model policy is therefore an investigative tool of fundamental importance for actuarial or general managerial use.

6.3. The choice of model policies is as much an art as a science and will depend upon their required use. A careful survey of the data is a necessary first step. Then one is likely to find statistics such as average age and average premium size useful in the construction of model policies. As one becomes more experienced, greater sophistication can be used in finding ages and premiums which lead to average profit over the class or sub class.

When deciding on how many model policies to use there is always the dilemma that too few policies will result in a lack of precision but too many will increase the length of investigation. The number of policies required to represent a contract adequately will, to a large extent, depend on the sensitivity of the

contract to changes in parameters such as age, term and premium. I would suggest initially keeping the model as simple as possible, since it is always possible to add in more policies at a later stage if greater precision is required.

Part II—The Applications of Profit Tests

Example is always more efficacious than precept. (Samuel Johnson)

7. A SIMPLIFIED MODEL OFFICE

7.1. In Part II the paper moves from the general principles of profit testing to particular examples and applications. I have adopted a simplified approach, but there should be sufficient detail to enable the examples to be translated into real life situations.

7.2. The examples throughout Part II will be based on a well-established proprietory life office which is about to set up a unit-linked subsidiary, wholly owned by the shareholders. It has been assumed that the only business written is individual life business; the contracts written being:

(i) A conventional endowment assurance with profits (EAWP).
(ii) A conventional endowment assurance without profits (EANP).
(iii) A unit-linked single premium plan (SPP).
(iv) A unit-linked regular premium savings plan (RPP).

In addition a unit-linked protection plan is considered in Section 12.

7.3. For illustrative purposes it has been assumed that the future business of the life office and its subsidiary can be modelled by just four model policies which will be used in the examples. These are:

Model policy 1: Endowment assurance with profits (EAWP)

Term	10 years
Male age next birthday	55
Annualized premium	£350 payable monthly
Basic sum assured	£3,000
Commission: Initial	25% of the annualized premium on indemnity terms
Renewal	2·5%

Model policy 2: Endowment assurance without profits (EANP)

Term	25 years
Male age next birthday	40
Annualized premium	£400 payable monthly
Basic sum assured	£15,000
Commission: Initial	60% of the annualized premium on indemnity terms
Renewal	2·5%

Model policy 3: Unit-linked single premium plan (SPP)

Term	Whole life
Male age next birthday	50
Single premium	£5,000
Allocation rate	101%
Bid/offer spread	5%
Death benefit	1·1 times the bid value of units at death
Management charge	$\frac{3}{4}$% of the funds under management each year
Commission: Initial	$3\frac{1}{2}$% of the single premium
Renewal	Nil

Model policy 4: Unit-linked regular premium plan (RPP)

Term	10 years
Male age next birthday	40
Annualized premium	£500 payable monthly
Allocation rate	104%
Capital units	Allocated in year 1
Accumulation units	Allocated after year 1
Bid/offer spread	5%
Death benefit	$7\frac{1}{2}$ times the annualized premium or the bid value of units if greater
Management charge	$4\frac{1}{4}$% and $\frac{3}{4}$% of the unit funds each year for capital units and accumulation units respectively
Commission: Initial	25% of the annualized premium on indemnity terms
Renewal	2·5%
Policy fee	£12 p.a.

I do not propose to consider the existing business of the life office in the examples, but additional model policies to take account of the typical durations in force would be necessary in any modelling exercise.

7.4. Because the applications lend themselves more easily to unit-linked companies the examples will concentrate on the unit-linked subsidiary. Where greater insight can be obtained by illustrating the conventional business, in addition, this has been done.

8. PREMIUM RATING USING PROFIT TESTS

8.1. This can be considered in a series of steps.

(i) Decide on a valuation basis, an anticipated actual basis and a profit criterion for the contract under consideration.

(ii) Profit test the contract using these assumptions over a range of ages and premium sizes.
(iii) Compare the results of the profit tests with the profit criterion.
(iv) Make any adjustments necessary to items such as policy fees, premium rates or surrender values until a policy design is found which satisfies the profit criterion to an acceptable degree.
(v) Test the rates found in (iv) for sensitivity to changes in the key parameters of the anticipated actual basis, e.g. interest rates, expenses.
(vi) Consider the market and make any adjustments necessary, including, possibly, a change in profit criterion, in which case repeat the exercise.

In the examples which follow in Sections 9–11, I shall assume that the above procedure has been adopted and merely state the basis and the results for the model policies, together with an example of sensitivity testing for the unit-linked contracts.

8.2. Profit tests are also useful when deciding on what the premium rates should be in non-standard circumstances, such as when commission is waived or a large premium is involved. A simplified version of the exercise described above, omitting the parts concerned with contract design and sensitivity testing and making any adjustments necessary to take account of the individual circumstances, is all that is required.

9. PREMIUM RATING OF UNIT-LINKED CONTRACTS

Although the unit-linked company is about to be set up, the basis used in premium rating will be the long-term anticipated actual basis. Any deviations from this basis in the short term will be considered in Section 13.

The profit criterion

9.1. The profit criterion is a matter for a management or board decision. Initial commission is a useful measure for a unit-linked profit criterion, but by no means the only one. If profit is related to initial commission this has the advantage that, whichever contracts are sold, there is a direct relationship between the amount of initial commission paid and the profits expected. The profit criteria used in the examples are:

(i) Single premium contract—100% of initial commission.
(ii) Regular premium savings contract—50% of initial commission.

The actuarial basis

9.2. The contracts are long-term insurance contracts and best estimates of the future experience should be used for the parameters of the anticipated actual basis. It has been assumed for simplicity that the valuation basis is the same as the anticipated actual basis.

Mortality—This should reflect the office's anticipated experience. Any

information regarding the office's existing business should be compared with a standard table, such as A67/70 select (possibly rated down 1 year). Experience in line with this table has been assumed.

Lapses—Lapses can have a financially significant effect on some unit-linked contracts. If the office has sufficient experience then a lapse investigation should be carried out to help in assessing the rates to be used.

For the SPP, a lapse rate of 10% p.a. has been assumed throughout (occurring uniformly over the year).

For the RPP, lapse rates of 10% in year 1, 7½% in year 2 and 5% p.a. thereafter have been assumed. It has also been assumed that in year 1 the lapses are higher in the earlier part of the year but that thereafter they occur uniformly over the year.

Interest—The growth in unit prices should reflect long-term expectations. The net risk discount rate should be higher than the net long-term growth rate to reflect the extra risks taken by the shareholders, especially the risk associated with setting up a subsidiary company operating in a new market. A unit growth rate of 10% p.a. (before tax and management charges) and a risk discount rate of 12% p.a. net have been assumed.

Tax—There should be two separate tax rates:

(i) An overall tax rate on investment income and capital gains.
(ii) An effective rate of tax relief on expenses and commission.

The two should be consistent and should be shown to be so. This test for consistency will be considered more fully in Section 13.

An investment income distribution will be assumed as follows (income being used loosely to include realized and unrealized capital gains):

(i) Unfranked investment income (liable to tax at 37½%)	40%
(ii) Franked investment income (liable to tax at 30%)	30%
(iii) Realized capital gains (liable to tax at 10%)	10%
(iv) Unrealized capital gains (liable to tax at 0%)	20%

This gives an overall rate of tax on income of

$$\cdot 375 \times \cdot 4 + \cdot 3 \times \cdot 3 + \cdot 1 \times \cdot 1 = 25\%.$$

The rate of tax on realized capital gains has been taken as 10% for illustrative purposes, taking account of the fact that index linking makes taxation of capital gains less likely.

It is assumed that in the long term, because of the high proportion of unfranked income (due to the existence of a large fixed interest fund), tax relief is available on expenses and commission at 37½%. It is also assumed that there is sufficient franked investment income for all the transfers to the proprietors to be made without incurring any further tax liability.

Thus the tax rates assumed are:

(i) An overall rate of 25% on income.
(ii) A rate of relief on expenses and commission of 37½%.

This tax basis is not intended to apply to one contract in isolation but assumes a block of existing business sufficient for the rates to be reasonable in aggregate. This is not the case for the new subsidiary in the short term and a global adjustment is considered in Section 13.

Expenses—A thorough investigation of expenses should be carried out. In the case of a new subsidiary company, where there is no prior experience, the starting point should be the office's current experience on conventional business, to which suitable adjustments could be made.

If it were possible, a split of expenses between marketing, administration and overheads for initial and renewal costs, would be useful in both modelling and controlling expenses.

When the initial expenses and renewal expenses have been determined, an allowance should be made for inflation which takes account of the delay between setting the premium rates and the time when the contracts are sold. Further allowance should be made for inflation on the renewal expenses once the business is written.

Initial expenses of £150 per policy and renewal expenses from year 2 onwards of £10 per policy have been assumed. It has been further assumed that the inflation rate is 8% p.a. (2% p.a. less than the gross growth rate), and this is applied to the renewal expenses.

Surrender basis—For the RPP a deduction is made on surrender, approximately equal to the difference between the full value of units and the actuarially funded value of the units.

For the SPP there is no deduction on surrender, the surrender value being the bid value of the units.

Results of the profit tests

9.3. The results for the SPP are shown in Table 1 and those for the RPP in Table 2. All the figures have been rounded to the nearer £ and only the first 10 years of the SPP are illustrated. Further details of the calculations can be found in

Table 1. *Results of profit testing the single premium plan contract*

Year	*Premium*	*Interest*	*Expenses*	*Tax*	*Deaths*	*Lapses*	*Δ Reserves*	*Profit*
1	5,000	455	325	−8	26	497	4,582	31
2	0	434	10	105	28	475	−208	24
3	0	414	9	100	30	453	−202	22
4	0	395	9	95	32	432	−196	21
5	0	376	9	91	34	412	−190	19
6	0	358	8	86	36	392	−184	18
7	0	341	8	82	39	373	−179	17
8	0	324	8	78	41	355	−174	15
9	0	307	7	74	43	336	−170	14
10	0	291	7	70	46	319	−165	13

Target profit based on the profit criterion of 100% of initial commission 175
Present Value of Future Profits (PVFP) 151

Table 2. *Results of profit testing the regular premium plan contract*

Year	*Premium*	*Interest*	*Expenses*	*Tax*	*Deaths*	*Lapses*	*Δ Reserves*	*Profit*
1	469	16	266	−96	4	1	304	−2
2	434	51	20	5	4	37	412	6
3	405	91	20	15	6	45	404	6
4	384	129	19	25	6	65	388	9
5	364	165	19	34	6	85	373	11
6	345	201	19	43	7	103	359	14
7	327	235	19	52	8	122	345	15
8	310	267	18	60	9	140	331	17
9	293	299	18	68	12	157	317	19
10	277	329	18	75	15	3,709	−3,233	20

Target profit based on profit criterion of 50% of initial commission 63
Present Value of Future Profits (PVFP) 60

the Appendix. The figures under each column heading can be explained as follows:

Year—Although the period used in profit testing was a month, the results have been presented as yearly items of income, outgo and changes in reserves and are expressed per initial entrant.

Premium—This is the total premium expected during the year. Note that for the RPP where the premiums are due monthly, exits during the year make the premium receivable less than 12 monthly premiums.

Interest—This is the total gross investment income and capital gains expected during the year.

Expenses—These are the total expenses, including commission, incurred during the year. The commission is on indemnity terms, therefore all of the initial commission is paid in the first month. This only affects monthly premium contracts when the initial commission would otherwise be paid in monthly instalments over the first year. For lapses during the first year on monthly premium contracts, there is a clawback of any commission that would not have been due if the initial commission had been paid in monthly instalments.

Tax—This is the total tax liability incurred during the year. A negative tax item can arise if $\cdot 25\,I < \cdot 375\,E$ and it is assumed that there is sufficient investment income generated elsewhere to enable the expenses to be relieved immediately. This is considered further in Section 13.

Deaths—This is the total benefit paid out during the year in respect of death claims.

Lapses—This is the total amount paid out during the year to those who exit through lapse and maturity. Note that for the RPP in year 1, the amount is very small even though there is a high lapse rate. This is because of a large surrender deduction on lapses during the first year.

Δ Reserves—This is the change over the year in the reserves needed. Note that for the SPP the change is negative after year 1 due to the lapse rate exceeding the net unit growth rate.

Profit—This is the value of the total profit emerging during the year. The profit emerging is calculated monthly and then discounted to the start of the year. I have assumed that the profit emerges at the end of the month. Because the other items of income and outgo have been shown differently one would not expect income–outgo–Δ reserves to exactly equal profit even though the differences are small.

The profits emerging are net profits, and could be transferred to the shareholders without incurring any further tax liability, provided that there is sufficient franked investment income.

PVFP—This is the present value of future profits for the contract. Because profit has been discounted to the start of each year the PVFP is:

$$\sum_t (\text{Profit in year } t) \times v_r^{t-1}$$

where r is the risk discount rate.
The PVFP is compared with the profit criterion.

Comments on the results

9.4. Table 1 shows that the PVFP for the single premium plan are £151 compared to the profit criterion of 100% of initial commission, which equals £175. The test in Table 2 for the regular premium plan gives the PVFP as £60, whereas the original profit criterion was 50% of initial commission, which is £63. In both cases the PVFP is sufficiently close to the profit criterion. The SPP is only just outside a 10% band around the target profit; were it much further away from target then some of the courses of action set out in § 8.1 should be considered. It is interesting to note that both contracts appear relatively strain free. This will be followed up in more detail in section 12.

10. SENSITIVITY TESTING

10.1. Because the profit tests are being carried out on a micro-computer, it is an easy exercise to perform the tests on different bases. This is an important part of the premium rating exercise since it identifies those parameters for which the contract is most sensitive to change.

10.2. First consider the effects of changing the premium size and age at entry. This would normally be done as part of the premium rating exercise, where rates

Results of profit tests and profit criterion for some changes to policies

	Regular premium plan		*Single premium plan*	
Change to policy	*Actual*	*Profit criterion*	*Actual*	*Profit criterion*
No change—Standard contract	60	63	151	175
Double premium	194	125	433	350
Half premium	−8	31	12	88
15 years older	49	63	60	175
15 years younger	61	63	180	175

are found for a range of ages and premium sizes, but is included here since attention has been focused on only four model policies.
What can we deduce from these results?

(a) Relationship with premium size

There is a linear relationship with premium size.

The results based on half the standard premium are unacceptable. Three simple solutions are:

(i) Charge an increased policy fee for small premiums.
(ii) Reduce the allocation rate for small premiums.
(iii) Introduce a minimum premium.

The results based on double the standard premium are high compared with the profit criterion and may, therefore, not be marketable. Possible solutions are:

(i) Increase the allocation rates for large premiums.
(ii) Reduce the policy fee for RPP's with large premiums.

(b) Relationship with age

The RPP contract is relatively stable with respect to age although there may be a problem with high ages. A possible solution would be to reduce the death benefit at high ages but care must be taken over the qualification rules.

For the SPP contract there is a high degree of sensitivity to changes in age at entry. This could be reduced by making the death benefit decrease for higher ages at death or by having a death benefit with an increase in unit values of less than 10%.

10.3. The table shows the effect of changing some of the important parameters of the anticipated actual basis. This is the more usual context for sensitivity.

Results of profit tests for some changes to anticipated actual basis

Change to anticipated basis	*Regular premium plan*	*Single premium plan*
No change—Standard contract	60	151
No lapses	59	304
Growth 5% gross, inflation 4%, risk discount 7%	78	162
Initial expenses £200 tax relief 35%	18	110
Renewal expenses £15 p.a.	41	131

For the RPP the surrender deductions have the effect of eliminating a lapse risk. For the SPP, where there is no surrender deduction, profits improve very significantly when the lapse rates are reduced. Thus for the SPP it is important to monitor the lapse rates carefully.

These contracts are relatively unaffected by changes in the magnitude of the interest rates provided a reasonable relationship between growth, inflation and risk discount is experienced.

One of the major risks to profit, for unit-linked contracts, lies in the levels of expenses being experienced. Not only do higher initial expenses mean less initial profit, they can also delay the relief of expenses and commission against tax. This is why the two changes have been combined in the example above; a higher level of initial expense and, as a result, not all the expenses being relieved at $37\frac{1}{2}\%$. Initial expense levels are particularly important to a new company in its early stages. The assumed increase in renewal expenses, although undesirable, has not had so large an impact on profits as increasing initial expenses. It can, however, put pressure on the actuary to raise the management charge, this would affect marketability and may not be possible if the management charge has a fixed limit. Clearly both initial and renewal expenses should be carefully monitored and poor experience on either or both can be potentially very dangerous.

10.4. Sensitivity testing has clear implications for the financial management of a unit-linked office. It detects the parameters for which the contracts are vulnerable to change. Once detected it may be possible to re-design the contract to reduce its sensitivity. Clearly the experience of any items for which this is not possible should be carefully monitored. In the examples in Sections 13 and 15, I shall assume that the model policies described earlier adequately represent the expected experience of the unit-linked subsidiary.

11. PREMIUM RATING OF CONVENTIONAL CONTRACTS

The profit criterion

11.1. As was explained in Part I, profit for conventional contracts is difficult to define if the contracts are considered in isolation. It is possible to set a profit criterion for each class of policy, but as with profit policies are likely to need support from the without-profit policies the mix of business becomes important.

I propose to use a profit criterion which requires that, for the expected mix of business, the conventional business taken as a whole at least breaks even so far as transfers to and from the estate are concerned.

The idea of a profit criterion for conventional business is interesting and controversial. It depends upon the view that is taken on what the estate is and how it should be used, which is a subject worthy of a paper of its own. The above profit criterion assumes that the estate grows in line with the required rate of return on its underlying assets. Some would argue that it is reasonable that the policyholders of each future generation should help to maintain, or even increase, the value of the estate that has been built up by past generations of policyholders. Others would argue that the estate should be no more than a minimal contingency reserve, and that any income over and above that required to maintain the reserve should be paid out, as it arises, in the form of special bonuses. These are two extremes, but whatever view is taken it will have an effect

not only on the profit criterion but also on the valuations described in Sections 15 and 16.

The actuarial basis

11.2. The contracts are long-term insurance contracts and best estimates of the future experience should be used for the anticipated actual basis.

Mortality—This should reflect the office's anticipated experience. Any information regarding the office's existing business should be compared with a standard table such as A67/70 select. Experience in line with this table has been assumed.

Lapses—A lapse investigation of the office's experience is useful in determining the rates to use. It is assumed that the business experiences lower lapse rates than the new unit-linked subsidiary. The rates of lapse used start at 5% p.a. and reduce over the term.

Surrenders—The office's current non-guaranteed surrender basis should be used unless there is good reason for expecting it to change. A basis which approximates to 95% of the net premium reserve using 5% net interest has been used, with appropriate adjustments at early durations to cover expenses.

Interest—A gross rate of interest, reflecting long-term expectations, of 10% p.a. has been used.

The net risk discount rate should be higher than the expected net rate of return, otherwise the shareholders could invest directly in a similar portfolio. A rate of 10% p.a. net will reflect the risks they are taking and give them a reasonable return. This is lower than for the unit-linked subsidiary, reflecting the lower levels of risk attaching to the conventional business of a well-established office and the absence of a requirement for new capital.

The required rate of return on the estate will depend upon the view taken of the estate as discussed in § 11.1. For illustrative purposes I shall take the view that the estate should be utilized to yield a higher return than the rest of the funds and accordingly use a required rate of return of 10% p.a. net. The implication of this is that the estate is expected to grow faster than the assumed inflation rate of 8% p.a.

Tax—The rate of tax on investment income has been taken as 25% and the rate of relief on expenses 37½%. These are consistent with the assumptions made for the unit-linked contracts and an explanation of their derivation will be found in §9.2. It would be possible to argue that the income distribution for the conventional business would differ from the income distribution for unit-linked contracts since a more conservative investment approach may be adopted for the former.

Expenses—A thorough investigation of current expense levels should be carried out and allowance for future expense inflation should be made. The results could well lead to different assumptions for conventional and unit-linked contracts. The same assumptions have, however, been made, i.e. initial expenses of £150 per policy and renewal expenses of £10 per policy inflating at 8% p.a.

Table 3. *Results of profit testing the conventional endowment assurance without profits (EANP)*

Year	*Premium*	*Interest*	*Expenses*	*Tax*	*Deaths*	*Lapses*	*Props*	*Δ Reserves*	*Profit*
1	388	13	396	−145	20	1	0	0	129
2	367	16	15	−2	21	16	0	0	333
3	338	12	18	−4	21	42	0	62	210
4	311	17	18	−3	23	62	0	192	36
5	285	34	17	2	23	73	0	246	−42
6	265	58	17	8	25	75	0	227	−29
7	248	79	16	13	26	80	0	210	−18
8	233	99	16	19	28	84	0	199	−14
9	220	118	16	23	29	79	0	193	−2
10	209	137	16	29	31	80	0	189	1
11	198	155	16	33	34	79	0	185	6
12	191	174	16	37	37	70	0	192	13
13	183	192	16	42	39	66	0	194	18
14	177	212	17	47	43	65	0	196	21
15	171	231	17	51	46	56	0	205	27
16	167	251	18	57	50	54	0	210	29
17	161	271	18	61	54	57	0	209	33
18	157	292	19	65	59	61	0	209	36
19	152	312	19	70	63	65	0	207	40
20	148	332	21	76	68	69	0	205	41
21	142	352	21	80	73	73	0	203	44
22	138	372	21	85	79	77	0	201	47
23	132	390	22	89	85	81	0	197	48
24	128	410	23	94	91	85	0	194	51
25	123	424	23	97	95	4,601	0	−4,325	56

Present Value of Future Profits (PVFP) 625 to the estate.

Other assumptions made are as follows:

(i) Bonus rates: Reversionary bonus—4·5% p.a. compound.
Terminal bonus—10% of the Reversionary bonuses.
(ii) Shareholders' share of bonus: 10% of total bonuses.
(iii) Published net premium valuation rate: $3\frac{1}{2}\%$ p.a. net (used only to evaluate the proprietors' share).

Reserves—To calculate the reserves the profit test uses a bonus reserve method with interest at $6\frac{1}{4}\%$ p.a. net. The mortality, expense and bonus assumptions are the same as for the anticipated actual basis. No account is taken of lapses.

Results of the profit test

11.3. The results for the EANP are shown in Table 3 and those for the EAWP in Table 4. All figures have been rounded to the nearer £. Many of the column headings are the same as in the unit-linked context. The following differences should be noted:

Year—The period used was a year; the greater precision obtainable by using a month was not thought necessary for well-established conventional business

Table 4. *Results of profit testing the conventional endowment assurance with profits (EAWP)*

Year	*Premium*	*Interest*	*Expenses*	*Tax*	*Deaths*	*Lapses*	*Props*	*Δ Reserves*	*Profit*
1	343	55	244	−79	24	3	11	683	−488
2	329	82	16	15	27	16	11	298	28
3	312	109	19	21	31	32	11	274	33
4	294	136	19	27	34	44	12	257	37
5	278	160	18	33	37	45	12	252	37
6	264	184	18	39	41	47	12	254	37
7	253	210	18	45	46	52	13	249	40
8	241	233	18	51	50	55	13	246	41
9	231	256	19	57	56	55	13	250	41
10	223	277	19	62	63	3,048	26	−2,763	45

Present Value of Future Profits (PVFP) −264 to the estate
Present Value of Future Profits (PVFP) 82 to the shareholders

particularly since the lapse rates were not so high. All the results are expressed per initial entrant.

Interest—This is the total gross interest received during the year based on the average reserves.

Props—This is the transfer to the proprietors in respect of the bonuses declared in the year. The larger transfer in year 10 with the EAWP is due to the terminal bonus. There is, of course, no transfer from the EANP.

Δ *Reserves*—Note: for the EANP the required reserves are zero in the first 2 years. This is because negative reserves have been eliminated.

Profit—This is the total profit to the estate during the year. It should be the balancing item in Income − Outgo − Δ Reserves.

Note the large loss in year 1 for the EAWP which results from setting up the required reserves and also meeting the initial expenses. The EANP, however, makes a substantial profit in the early years. It does suffer losses in years 5–9 caused by a loss of surrenders due to the reserves on $6\frac{1}{4}\%$ interest being lower than the surrender values on the stated basis. For a large, well-established office it should not be necessary to set up additional reserves to cover these losses, especially as the surrender values are not guaranteed.

PVFP—This is the present value of future profits to the estate. The rate used for discounting is the required rate of return on the estate. The profits are assumed to emerge mid-way through the year when discounting.

Comments on the results

11.4. The EAWP causes a loss to the estate and the EANP a profit, a result which undoubtedly occurs in practice.

Comparing the £264 loss from the EAWP (Table 4) with the £625 profit from the EANP (Table 3) it can be seen that a mix of business by contracts of not more than 2·37 EAWP's to each EANP is required if no overall loss to the estate is to result.

The value of the EAWP to the shareholders can be found by discounting the props transfers at the shareholder's risk discount rate, which gives £82 (Table 4). It is therefore in the shareholders' interest that as much EAWP business is sold as possible, but there is a constraint on the company because of the loss to the estate resulting from writing each contract.

Suppose the mix of business were four EAWP's to one EANP, then for each EANP sold the shareholders' interest in the five policies is $4 \times 82 = £328$. The overall effect on the estate, however, is $625 - 4 \times 264$, an expected loss of £431. This would improve the shareholders' immediate return but the company could not continue writing business with this mix for any length of time without failing to achieve the required rate of return on the estate or even exhausting the estate. This illustrates the conflicts of interest that could arise and the way in which profit tests can be used to investigate them. Once decisions have been taken to resolve the various interests, profit tests can be used again to determine an optimum business strategy.

12. PATTERN OF EMERGING PROFITS

12.1. In calculating the present value of future profits, a profit test calculates the profits expected to emerge period by period. A study of these emerging profits can be very enlightening and give a greater understanding of the contracts. This can help both in designing contracts and in choosing a desirable mix of business.

12.2. The profits emerging month by month are shown in the table on page 24 for three unit-linked contracts. The contracts have been chosen to give a reasonable cross-section of the different unit-linked contract designs and are:

(i) Single premium plan (SPP) with a premium of £5,000.
(ii) Regular premium savings plan (RPP) with an annualized premium of £500 and having an allocation to capital units for one year.
(iii) Protection plan—a unit-linked whole life plan with an annualized premium of £400. This gives high cover at low cost and uses a front-end loading which gives no allocation to units until month 20.

The profits in the table on page 24 have been expressed in the money values of month 1, i.e. by discounting back to commencement at the risk discount rate.

The variation in profit per month is greatest during the first 24 months. I will, therefore, consider this period in detail when investigating and comparing contracts.

The SPP has a relatively uniform emergence of profit over the period, small in amount but with no initial loss.

The RPP has a substantial initial loss in month 1, followed by steady profits in months 2–12 whilst capital units are being allocated. At the end of the period of allocation to capital units the profit emergence is steady but at a much reduced level.

The protection plan has a very large initial loss, almost as large as the initial

Monthly emergence of profit for three contracts

Month	*Single premium plan*	*Regular premium plan*	*Protection plan*
1	2·2	−148·0	−388·0
2–12	2·6	13·3	27·1
13–21	1·8	0·4	22·0
22–24	1·7	0·4	2·0
25–36	1·4	0·4	1·0
37–48	1·2	0·5	0·9
49–60	1·0	0·6	0·9
61–72	0·9	0·6	0·8
73–84	0·7	0·6	0·7
85–96	0·6	0·7	0·7
07–108	0·5	0·6	0·6
109–120	0·4	0·6	0·5

annualized premium. Sizeable profits then emerge steadily during the period whilst no allocation to units is being made. At the end of this period the profits continue at the same level for 2 months. One might expect the profits to fall immediately the allocation to units commences; this is not so because the policy charges the cost of the death benefit on a monthly basis by deduction from the units. In the first 19 months, whilst no allocation is being made, the cost of the death cover is calculated and deducted as soon as an allocation is made. It is this recovery which is contributing to profits in months 20 and 21. After month 21 the profits fall substantially but continue to emerge steadily.

12.3. As an aside, the profit emergence in year 1 of a contract such as the RPP or the protection plan highlights the need to discount profits month by month. Consider a policy with an undiscounted profit emergence of $-z$ in month 1 and y in each of months 2–12. If profits are discounted monthly the value of profits in year 1 is

$$-z\, v_r^{1/24} + y[v_r^{3/24} + v_r^{5/24} + \ldots + v_r^{23/24}] \simeq -z + 11y\, v_r^{\frac{1}{2}}$$

If profits are discounted annually and assumed to emerge on average mid way through the year then the value of profits in year 1 is

$$(-z + 11y)\, v_r^{\frac{1}{2}} \text{ an overstatement of profits of approximately } \frac{zr}{2},$$

i.e. half a year's interest at the risk discount rate on the *initial* loss, not the year's profit/loss.

There is a further problem associated with a monthly premium contract. If there are high year 1 lapse rates which do not occur uniformly over the year but are heavily weighted towards the early months, a feature not uncommon with monthly premium protection plan type policies, then using the period of a year can make the contract look more profitable than it really is. Early lapses give rise to a loss because initial expenses will have been incurred but insufficient

premiums received from which to recover them. The period of a year will assume that lapses occur on average midway through the year but the period of a month can take account of the heavy lapses in the early months, when losses are greater.

If, therefore, the contract suffers a large month 1 loss and/or has a high year 1 lapse rate, weighted towards the early months, then the period of a month is essential if accuracy is to be obtained.

12.4. To formalize the results seen in § 12.2 five statistics are defined which will be useful when investigating emerging profits. Let $PV(t)$ = the present value of future profits emerging in the first t months. The statistics are:

(a) *Present value of future profits* (PVFP)

We define the present values of future profits as before:

$$PVFP = PV(\infty).$$

(b) *The break-even month*—B

A contract which causes an initial loss to the company is going to hinder cash flow. It is important to know how long it takes for a contract to generate a profit under realistic assumptions. The break-even month B is:

$$B = \min (t : PV(t) > 0).$$

If B is large then the company will find problems in trying to expand without incurring losses.

(c) *The profit to loss ratio*—L

An initial loss may be acceptable if profits thereafter emerge sufficiently quickly. An indication of the rate at which profits are emerging compared with capital injection required is useful. We define the profit to loss ratio for month t as:

$$L(t) = -\frac{PV(t)}{PV(1)}$$

For the comparison of different contracts a value for t of 24 is probably reasonable. Then:

$$L = L(24).$$

(d) *The profit ratio*—F

This is an alternative measure of how fast the profits are emerging. We define the profit ratio for month t as:

$$F(t) = \frac{PV(t)}{PV(\infty)}$$

Again 24 is probably a good choice for t. Then:

$$F = F(24).$$

(e) *Expense relief delay*—E

A contract may generate profits quickly but generate little income in the process, thus delaying the tax relief on expenses.

$$E = \min (n: \sum \text{Investment income} > \sum \text{Expenses})$$

where n is measured in years; i.e. E is the first year in which all expenses could be relieved against tax were the contract sold in isolation.

Should any of these statistics show undesirable features then this may affect the choice of the risk discount rate (see § 2.5).

12.5. The table shows the values of these statistics (and some of their components) for the three contracts.

Plan statistic	*Single premium plan*	*Protection plan*	*Regular premium plan*
Premium	£5,000	£400 p.a.	£500 p.a.
Initial loss	none	£388	£148
Present value of first 24 months' profits—PV(24)	£52	£114	£3
Present value of future profits—PVFP	£151	£232	£60
Break-even month—B	month 0	month 17	month 17
Profit to loss ratio—L	no loss	·29	·02
Profit ratio—F	·34	·49	·05
Expense relief delay—E	1 year	11 years	5 years

The results in the table can be summarized.

Single premium plan—This contract is ideally suited to avoiding initial losses (B = month 0). The PVFP of £151 is in line with the profit criterion. Profits are not slow to emerge (F = ·34) and form a steady stream throughout the lifetime of the contract. The excess of income over expenses in year 1 means that such contracts are useful in reducing the unrelieved expenses accumulated by other, less income productive, contracts.

Protection plan—This contract takes a long time to get over the large loss in month 1 (B = month 17). Profits, however, emerge quickly once the initial loss has been recovered (L = ·29 and F = ·49). Thus at the cost of a substantial initial loss, a large proportion of the total profits can emerge within 2 years. There is one major drawback from this type of contract, and that is the low income generating capacity (E = 11 years). Thus when considering the mix of business there must be sufficient single premium or mature regular premium business to enable expenses to be relieved against tax, otherwise the problem of unrelieved expenses will arise. A new company in particular should think very carefully before writing large quantities of this type of business.

Regular premium plan—This contract takes as long to recover the initial loss as the protection policy (B = month 17) and profits are slow to emerge (L = ·02 and F = ·05). There is not such an acute expense problem as with the protection policy (E = 5 years) but the right mix of business is going to be important if this contract is sold.

12.6. These examples show that consideration of the PVFP, although necessary, is not always sufficient when designing contracts. They illustrate the

desirability of getting a mix of business that is not going to lead to problems in the future from unrelieved expenses or cash flow difficulties from too rapid growth. They should help in making decisions on a suitable mix of business and in contract design. The effect of a particular mix of business can be tested using a revenue account projection, this is the subject of the next section.

13. REVENUE ACCOUNTS

13.1. The profit test calculates the expected items of income, outgo and changes in reserves for each period during the term of the contract. These items are effectively a revenue account for the individual contract. If a set of new business assumptions is introduced then it is possible to use these, together with the profit test items, to produce a revenue account for a class of business, and by summing over classes, the whole office, e.g. expenses in period t would be

$$\sum_{\substack{\text{all durations} \\ s}} (\text{Expenses at duration } s) \times \left[\begin{array}{c} \text{Expected number of} \\ \text{contracts in force of} \\ \text{duration } s \text{ during period } t \end{array} \right]$$

There is a problem in dealing with inflation since once expense inflation is introduced the expenses at duration s are not constant for all periods. There is a simple solution to this problem. If both average premiums and expenses are assumed to increase in line with inflation then, for a typical policy to be written in t years time, all items of income and outgo in any period of this policy can be expressed as the corresponding items for a policy written immediately, increased by t years' inflation. Putting this another way, one model policy written in t year's time is equivalent to $(1+i)^t$ policies written immediately, i being the inflation rate. This relationship can be utilized to produce revenue accounts which allow for inflation.

Once the basic revenue account is produced some adjustments may be necessary and these are considered in § 13.8.

13.2. Revenue accounts are useful tools in the planning and development of a life office. Their applications cover a wide range of investigations including:

(i) To show the financial progress of the company under a set of assumptions.
(ii) To enable estimates of the profits emerging in the short term to be made.
(iii) To allow the effect of a change in the mix of business to be seen.
(iv) To allow the effect of the introduction of a new contract to be investigated.
(v) To allow the effect of a change in the income distribution or taxation laws to be investigated.

(vi) To enable a value to be placed on the existing business.
(vii) To provide a bench mark for an analysis of profit (see Section 14).

13.3. To illustrate some of these applications, consider the example of the unit-linked subsidiary about to be launched. For simplicity the period investigated will be restricted to 5 years.

New business assumptions

13.4. Assume that in each future year 1,000 policies of each contract are sold and that the new premiums increase each year in line with the inflation assumption of 8%.

This gives a new annual premium income (£000's) projections for the next 5 years of:

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*
Single premium plan	5,000	5,400	5,832	6,299	6,802
Regular premium plan	500	540	583	630	680

In practice, with a new company, one might expect initial growth rates well in excess of inflationary growth, but in the examples lower rates have been used so that the main points of interest occur within 5 years of launching.

13.5. It is now possible, using these forecasts of new business and the profit tests shown earlier in this paper, to produce 5-year revenue accounts. (Because the company only commences its operations in year 1 there is no business in force.)

5-year revenue account assuming no growth in real terms and 8% inflation

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*
Income (£000's)					
Unit fund b/f	0	4,850	10,242	16,274	22,979
Premiums	5,259	6,128	7,044	7,994	9,003
Investment income	254	753	1,310	1,930	2,618
Outgo (£000's)					
Commission	293	321	358	396	436
Expenses	295	338	384	433	486
Deaths	16	49	87	131	181
Surrenders	266	791	1,359	1,966	2,621
Tax	−157	−59	49	172	309
Profit	**−50**	**49**	**85**	**121**	**165**
Unit fund c/f	4,850	10,242	16,274	22,979	30,402

13.6. After a loss in year 1 (£50,000), which would need financing, the revenue account shows a steady improvement in profitability over the period.

13.7. Using a risk discount rate and assuming that the profits emerge, on

average, half-way through the year, a value can be placed on this stream of emerging profits. For the 5-year period, using a 12% risk discount rate, the value is

$$1{,}000\,(-50\,v^{\frac{1}{2}}+49\,v^{1\frac{1}{2}}+85\,v^{2\frac{1}{2}}+121\,v^{3\frac{1}{2}}+165\,v^{4\frac{1}{2}})=£239{,}000.$$

If the revenue account was extended beyond year 5 then a steady growth in profits would continue, the growth rate decreasing monotonically to 8% p.a.

Global adjustments

13.8. The figures above assume that the experience in each of the 5 years is in line with the profit-test assumptions. This is unlikely to be the case in practice, particularly since the company is just commencing its operations, and a number of adjustments may be needed. The two items most likely to need further consideration are looked at in some detail.

Expenses—In the initial stages of the operation the development costs and lack of scope for economies of scale mean that the expense loadings in the premium rates are unlikely to be adequate.

Assume, therefore, that the expenses incurred in each of the first 3 years are in fact £400,000. The expenses assumed in the projected revenue account are £295,000, £338,000 and £384,000. Thus the adjusted levels of expenses will reduce the profits in the early years.

Taxation—The taxation assumptions in the profit tests were based on an income distribution of:

40% taxable at 37½%
30% taxable at 30%
10% taxable at 10%
20% taxable at 0%

Expenses can only be relieved against tax when there is sufficient income liable to tax available. This should be tested and fed back into the revenue account.

Determination of actual tax position of the fund

Year	Total income (£000's)	Income (£000's) available for relief at 37½%	30%	10%	Expenses & commission (£000's)	Unrelieved expenses & commission c/f (£000's)	Tax (£000's)	Rate of tax relief %
1	254	102	76	25	693	490	Nil	9
2	753	301	226	75	721	609	Nil	26
3	1,310	524	393	131	758	319	Nil	43
4	1,930	772	579	193	829	Nil	80	49
5	2,618	1,047	785	262	922	Nil	309	37½

The tax row in the revenue account should be replaced by the tax figures in the table. The effect of the unrelieved expenses and commission is to defer the

emergence of profits. There is also a loss of profit, in that not all the expenses and commission are relievable against income liable to tax at 37½% during the first 4 years. It should be clear, however, that after year 4 all the expenses will be relievable against income liable to tax at 37½%, which is the assumption made in the profit tests.

13.9. With these adjustments the revenue account becomes:

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*
Income (£000's)					
Unit fund b/f	0	4,850	10,242	16,274	22,979
Premiums	5,259	6,128	7,044	7,994	9,003
Investment income	254	753	1,310	1,930	2,618
Outgo (£000's)					
Commission	293	321	358	396	436
Expenses	400	400	400	433	486
Deaths	16	49	87	131	181
Surrenders	266	791	1,359	1,966	2,621
Tax	0	0	0	80	309
Profit	**−312**	**−72**	**118**	**213**	**165**
Unit fund c/f	4,850	10,242	16,274	22,979	30,402

There is a subtle point which deserves mention. The tax liability of a life fund is based on 'I−E' or 'profits', whichever is the greater. For a new company it is often the case that the 'profits' basis applies at some stage during the early years. In this example the 'profits' basis should apply in year 3, when a profit is made but no tax paid. I shall overlook this point in the rest of the section, except to mention that by careful tax planning it should be possible to withhold the profits until the 'I−E' basis applies, but this would delay the emergence of profit.

13.10. The effect of these adjustments is to change, quite dramatically, the profit emergence.

Year	*Profits (£000's) before adjustments*	*Profits (£000's) after adjustments*
1	−50	−312
2	49	−72
3	85	118
4	121	213
5	165	165

The discounted value of the first 5 year's profits is £239,000 before adjustments and −£24,000 after adjustments.

After 4 years the profits emerging year by year would be the same, whether or not these adjustments are made.

The effect of the increased expenses is to reduce the profits of the first 3 years.

The effect of the unrelieved expenses and commission, which lasts for 4 years, is to both defer and reduce the emergence of profits; defer because not all

expenses can be relieved when they are incurred, reduce because not all expenses are relievable at 37½% until year 5. Most of the unrelieved expenses are brought through in years 3 and 4, hence the higher levels of profits in these years.

13.11. The financial progress of the office over a 5-year period has been illustrated by using profit tests to construct a revenue account. It becomes clear that, if the experience is in line with the assumptions made, financing will be required for the first 2 years of operation, and it will take about 5 years for this to be recouped out of profits (the discounted value of the first n years' profits/losses, after adjustment, is positive if $n > 5$). In practice further reserves may be required as a result of the solvency requirements in the 1981 Insurance Companies Regulations, but these are outside the scope of this paper.

Change of assumptions

13.12. This example has illustrated how a revenue account, constructed from profit tests, can be used to study the short-term emergence of profit. However, these techniques enable further investigations, which consider the effect on profits if certain parameters are changed, to be carried out with ease.

To illustrate the flexibility of these methods the effects of two changes in assumptions are now investigated; these are:

(i) A revised income distribution of:

24% income liable to tax at 37½%
50% income liable to tax at 30%
10% income liable to tax at 10%
16% income liable to tax at 0%

This gives the same overall tax rate, 25%, as before but there is a reduction in the proportion of unfranked investment income. This might arise if there were a switch away from a gilt fund to an equity fund. It should be pointed out that in practice the income distribution is not easy to control.

(ii) A growth in new business of 10% p.a. in real terms on top of the 8% p.a. inflationary growth.

A revised income distribution

13.13. The tax adjustment calculations are repeated using the revised income distribution assumptions.

Considering each year in turn:

	Income (£000's) for relief at			*Expenses & commission*	*Unrelieved expenses*	*Tax*	*Rate of tax relief*
Year	*37½%*	*30%*	*10%*	*(£000's)*	*c/f (£000's)*	*(£000's)*	*%*
1	61	127	25	693	480	Nil	9
2	181	377	75	721	568	Nil	26
3	314	655	131	758	226	Nil	43
4	463	965	193	829	Nil	131	42
5	628	1,309	262	922	Nil	331	35

This changes the profit in years 4 and 5 to £162,000 and £143,000 respectively, compared to the previous values of £213,000 and £165,000. The effect is to reduce the value of the first 5 years' business even further, to −£71,000 compared to the previous figure of −£24,000.

Thus, although the overall rate of tax is still the same, because the composition of the income has changed, so that more of the expenses and commission are relieved against income liable to tax at 30% instead of 37½%, there is an increase in losses over the 5 year period of £47,000.

There is one further point; by year 5 all unrelieved expenses and commission have been brought through but the rate of relief for expenses in year 5 is 35%, not the 37½% assumed in the profit tests. It would take about 3 more years before 24% of investment income exceeded expenses and commission, thereby making the 37½% profit test assumption correct. There would, therefore, be a further reduction of profits in the intervening period, although the magnitude of the reduction would be small. This highlights how important it is to investigate the income distribution when deciding upon the rate of tax relief on expenses and commission to be used for profit testing.

Growth in new business of 10% p.a. in real terms

13.14. A 5-year revenue account can be produced as before. The expense adjustment can be made by assuming the same level of overrun during the first 3 years. Expenses are still assumed to follow the assumptions in § 9.2. The validity of this assumption is debatable, since one might expect economies of scale from the extra business, but it will serve for the purposes of this illustration.

The taxation adjustment can be made as before and the resulting revenue account, ignoring tax on profit, is:

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*
Income (£000's)					
Unit fund b/f	0	4,850	10,727	17,861	26,482
Premiums	5,259	6,654	8,267	10,156	12,358
Investment income	254	778	1,414	2,184	3,112
Outgo (£000's)					
Commission	293	350	424	510	612
Expenses	400	429	468	552	670
Deaths	16	50	94	148	214
Surrenders	266	817	1,478	2,232	3,131
Tax	0	0	0	33	301
Profit	**−312**	**−91**	**83**	**244**	**181**
Unit fund c/f	4,850	10,727	17,861	26,482	36,843

These results can be compared with those assuming no growth shown in the table on page 33.

The discounted value of the first 5 years' profits is −£24,000 with nil growth and −£36,000 with 10% real growth.

Year	*Profits (£000's) assuming* Nil real growth	10% real growth
1	−312	−312
2	−72	−91
3	118	83
4	213	244
5	165	181

It is not surprising to see that in the case of 10% p.a. real growth, although ultimately higher levels of profits will emerge, the extra strain caused by the growth reduces profits in years 2 and 3. By year 4 the extra profits begin to emerge, and from year 4 onwards the profits are larger than with nil real growth.

13.15. If an office has existing business it is necessary to model the business in force as well as the future new business.

The methods of this section can be applied to the conventional business. There is, however, little extra to be gained by illustrating a revenue account for conventional business in addition.

Revenue accounts constructed by these methods are useful tools for investigating the short-term financial progress of a life office.

14. ANALYSIS OF PROFIT

14.1. In constructing the premium rates certain assumptions are made. An important task in the actuarial management, especially of a new unit-linked office, is to check that the experience is in line with the assumptions made. Probably the most important assumptions that need checking are the expense assumptions, but lapses, investment income distribution, mortality and unit growth rates also need investigating. If the experience is found to differ from the original assumptions then the actuary will have to consider what action, if any, is necessary. It is enlightening for the actuary if these differences can be translated into the effect that they are having on profits, and this is what an analysis of profit exercise attempts to do.

14.2. A suggested approach to the analysis of profit, since producing a revenue account on different assumptions is quick and easy if the methods in Section 13 are used, is:

(i) Run the revenue account model for the period under investigation, say 1 year, using the actual business written during the year. This should produce the profits which would be expected during the year, were the experience in line with the assumptions used to calculate the premium rates.

(ii) Compare the profit forecast by the revenue account with the actual profit that has emerged.

(iii) Try to explain any difference in profit found in (ii) by any deviations from the assumptions, for expenses, lapses, mortality or other parameters, that

have been experienced. Where applicable the revenue account model can be re-run using the revised parameters until the forecast profit agrees sufficiently closely with that actually experienced.

Having performed this exercise it should become clear which parameters have contributed most to the deviation from expected profit.

15. VALUATION OF FUTURE NEW BUSINESS

15.1. In Section 13 a method for valuing the emerging short-term profits was considered. This idea can be extended to the longer term.

Let $V(t)$ be the value of the profits expected to emerge during the next t years. To value all future new business it is necessary to calculate

$$\lim_{t\to\infty} V(t)$$

Although it is possible to produce a revenue account for a period of T years such that

$$\left|V(T) - \lim_{t\to\infty} V(t)\right| < \varepsilon$$

where ε is sufficiently small. This method is not very practicable and a much easier approach exists.

Let i be the inflation rate p.a., r be the risk discount rate p.a., $N_c(t)$ be the number of new contracts of class c in year t and $P_c(t)$ be the PVFP in year t to the shareholders of a contract of class c written in year t. Then the value of future new business can be expressed as

$$\sum_c \sum_t \frac{N_c(t)\, P_c(t)}{(1+r)^{t-\frac{1}{2}}}$$

assuming that the profits emerge on average midway through the year.

A key assumption that can be made is that the average premium, expenses and profits all increase in line with inflation.

$$\text{i.e.} \quad P_c(t) = P_c\,(1+i)^{t-1}$$

where $P_c = P_c(1)$ is the PVFP for contract c in year 1.

If, further, it is assumed that new premium income increases in line with inflation then the number of new contracts is constant in each future year,

$$\text{i.e.} \quad N_c(t) = N_c \text{ for all } t$$

where N is the number of contracts of class c written in year 1, and the value of future new business is

$$\sum_c \sum_{t=1}^{\infty} \frac{N_c\, P_c\,(1+i)^{t-1}}{(1+r)^{t-\frac{1}{2}}} = \frac{(1+r)^{-\frac{1}{2}}}{1-\left(\frac{1+i}{1+r}\right)} \sum_c N_c\, P_c$$

This is the value of the business written in year 1 multiplied by a perpetuity factor, which is easily evaluated.

15.2. An example should make this clearer, so consider again the unit-linked subsidiary. The assumptions are that the inflation rate, $i = \cdot 08$, the risk discount rate, $r = \cdot 12$, the number of single premium contracts in year 1, $N_s = 1{,}000$, the number of regular premium contracts in year 1, $N_r = 1{,}000$, the PVFP for the single premium contract, $P_s = £151$ and the PVFP for the regular premium contract, $P_r = £60$. The value of profits from 1 year's new business is $N_s P_s + N_r P_r$, i.e. $1{,}000 \times £151 + 1{,}000 \times £60 = £211{,}000$. On average, this emerges half way through the year, so divide by $(1 \cdot 12)^{\frac{1}{2}}$ to give £199,376 which can be taken as £200,000.

Algebraically, the value of 1 year's new business is

$$\sum_c N_c P_c (1+r)^{-\frac{1}{2}}$$

If we assume that the average premium increases in line with inflation and that the number of contracts in each future year remains at 1,000 then the factor which can be applied to 1 year's value to convert it into a perpetuity is

$$\left[1 + \frac{(1+i)}{(1+r)} + \frac{(1+i)^2}{(1+r)^2} + \dots\right] = \frac{1}{1 - \dfrac{(1+i)}{(1+r)}} = \frac{1}{1 - \dfrac{(1 \cdot 08)}{(1 \cdot 12)}} = 28$$

Thus the value of all future new business is £200,000 × 28, i.e. £5·6m.

This value assumes that the profit-test assumptions are followed throughout. In Section 13 it was seen that the value of profits emerging in the first 5 years, using the profit-test assumptions, was £239,000 but that this was reduced to −£24,000 after allowing for additional initial expenses and the delay in obtaining tax relief on expenses. The value of future new business should, therefore, be reduced by £263,000 to take account of these deviations from the profit-test assumptions. The value becomes £5·3m after adjustment.

There are two more points worth making. Firstly, the value of the future new business is very sensitive to the risk discount rate and its relationship with the assumed growth rate for new business (here the inflation rate). If the risk discount rate were 11% p.a. with inflation still at 8% p.a. then the perpetuity factor would be 37, making the value of future new business £7·4m before adjustment. Thus a reduction of 1% p.a. in the risk discount rate has increased the value of future new business by 32%!

Secondly, since the company is just commencing operations there is likely to be a large degree of uncertainty over its potential. The shareholders may require a higher rate of return to reflect the extra risks that they are taking. A more accurate model, having regard to the uncertainty over future new business levels, might be to use a 12% risk discount rate once the business has actually been written but to use a higher rate, say 15%, up to the point in time when the business

is expected to be written. This is easily calculated by changing r in the above formula; there is no need to recalculate the PVFP's. The effect on the value of the company is significant, the perpetuity factor becomes 16·43 and the value of future new business £3·3m before adjustment (£3·0m after adjustment). This is probably a more realistic value for the future new business than either of the two derived above.

Value of future conventional business

15.3. This is less straightforward but more interesting than the unit-linked valuation because of the sophisticated profit structure. Two items need consideration, first in isolation and secondly together:

(i) The value of future profits transferred to shareholders.
(ii) The effect on the estate of future business.

Suppose the expected conventional new business is 4,000 with-profits contracts per year with an A.P.I. of £350 and 2,000 without profits contracts per year with an annual premium income of £400. This gives an annual premium income in the first year of $4{,}000 \times 350 + 2{,}000 \times 400 = £2{\cdot}2\text{m}$.

With the assumption that the number of contracts in each future year remains constant but the annual premium income grows in line with inflation, the above two items can be investigated.

The value to the shareholders

15.4. The shareholders only have a direct interest in the with-profits business. The value to the shareholders of one with-profits contract was seen to be £82 (§11.3). Thus the value to the shareholders of 1 year's new business is $£82 \times 4{,}000 = £328{,}000$.

The value of all future years' new business, using the formula in §15.1 and assuming that 10% is the required rate of return on the estate, is

$$£328{,}000\left[1 + \frac{(1{\cdot}08)}{(1{\cdot}1)} + \frac{(1{\cdot}08)^2}{(1{\cdot}1)^2} + \ldots\right] \div (1{\cdot}1)^{\frac{1}{2}} = £17{\cdot}2\text{m}.$$

The extra uncertainty introduced in §15.2 when discounting the profit from future years' unit-linked business is not considered to be necessary here, since the business is well established and the assumption of no growth in real terms is considered to be realistic.

The effect on the estate

15.5. In Section 4 the effect of a contract on the estate was seen to be important. Using the results in Section 11 the effect of 1 year's new business on the estate is: with profits; $4{,}000 \times -£264 = -£1{\cdot}06\text{m}$ and without profits; $2{,}000 \times £625 = +£1{\cdot}25\text{m}$. The result is a small profit, £0·19m, to the estate from

the business, which is acceptably close to the profit criterion (§ 11.1). The value to the estate of all future years' new business is therefore

$$£190{,}000\left[1+\frac{(1{\cdot}08)}{(1{\cdot}1)}+\frac{(1{\cdot}08)^2}{(1{\cdot}1)^2}+\ldots\right]\div(1{\cdot}1)^{\frac{1}{2}}=£10{\cdot}0\text{m}.$$

Thus, overall, writing the future new business is expected to generate an addition to the estate worth £10m in present values. If this additional source of profit to the estate is ultimately distributed by means of higher bonuses, then the shareholders have an interest in this extra estate. Their interest, based on their 10% share of the value of bonuses, is £1·0m.

It is worth pointing out that the extra profit to the estate from writing the new business is a result of the mix of new business (2 with-profits contracts to 1 without-profits contract). Were the new business in the ratio 2·37:1 (§ 11.3) then there would be no overall effect on the value of the estate. It is not unreasonable, therefore, to assume that this addition to the estate is distributed by way of increased bonuses.

15.6. Thus the combined interest of the shareholders in future new business is £18·2m; £17·2m directly and £1·0m from the effect on the estate.

16. VALUATION OF A COMPANY

16.1. The value of a company, to its shareholders, is the value of future dividends discounted at the required risk discount rate. Using the profit test methods which have been developed throughout Part II it is possible to estimate this value as the sum of three items:

(i) The value of future new business to the shareholders.
(ii) The value of existing business to the shareholders.
(iii) The net asset value.

16.2. The value of future new business to the shareholders has been considered in Section 15. It should be remembered that for conventional business it is also necessary to consider the effect of writing new business on the estate.

16.3. The value of existing business to the shareholders can be calculated by similar methods to those used in Section 15. To do this it is necessary to model the business in force by choosing suitable model policies and then carrying out profit tests on these policies to find the PVFP expected to emerge over the unexpired terms. In practice it may be possible to use the same model policies for existing business as those used for future business, which would reduce the work involved. Another possible method for valuing the existing business is to carry out a valuation using the profit test parameters. Whichever method is used for conventional business, the value of the existing estate should be determined and consideration given to the arguments put forward in § 11.1, since this will affect the shareholders' interest in the estate.

16.4. The net asset value is the current value of the shareholders' fund. The assets of the fund are locked in and their value can only be taken as the current balance in the shareholders' fund if it is assumed to be utilized, by direct investments or by financing expansion, so as to earn a return equal to the required risk discount rate. If this is not the case then the current balance will need adjustment.

16.5. Consider the unit-linked subsidiary. Assume that the company requires £1m of capital to cover the solvency margins and the expected losses of £384,000 in the first 2 years. The £1m is the current balance in the shareholders' fund and in this situation there is a restriction on the way it can be used. Assume that, because of this restriction the capital grows only at the gross interest rate of 10% p.a. instead of the shareholders' required rate of return of 12% p.a. net.

The value at 12% of a stream of income of 12% p.a. net is £1·0m. The value at 12% of a stream of income of 10% p.a. gross, 7½% p.a. net, is approximately £·6m. Thus, because the capital is locked in, its present value is only £·6m and this is the net asset value.

The components of the value of the unit-linked company can now be estimated

The value of future business to the shareholders (§ 15.2)	= £3·0m
The value of existing business to the shareholders	= £0·0m
The net asset value	= £0·6m

This is a total value of £3·6m in return for an initial capital requirement of £1·0m and therefore looks an attractive investment!

16.6. It should be appreciated that the values obtained by the above methods are sensitive to changes in parameters, particularly risk discount rates, but if a range of values is calculated based on small changes in the major parameters it should be possible to have a reasonable degree of confidence that the range of values is realistic.

16.7. This method of valuing a company is clearly of considerable use as a management tool. It enables many of the applications set out in § 13.2 to be extended to the long term and, in particular, the trend of values from year to year is a helpful guide to the financial progress of the office.

17. CONCLUSION

For we know in part, and we prophesy in part. (I Corinthians 13: 9)

In writing this paper, my aim has been to treat the subject of profit testing in a practical way. By the use of examples I have tried to show the many and varied uses to which the basic profit tests can be put. These techniques are now in frequent use throughout the insurance industry; they are, and should continue to be, important tools for use in the actuarial management of a life office.

The methods are particularly useful for projecting the profits of a whole office into the future, based on a set of assumptions. By varying these assumptions it is

possible to measure the sensitivity of these future profits to changing conditions, enabling limits within which the profits are likely to fall to be determined; indeed, it could be said that here is a *prophet of profits*.

APPENDIX 1

Example of the development of a profit test program

In my university days I was never very impressed when a lecturer stated an important theorem but left the proof as an exercise for the student. I am aware that, by concentrating on the principles and applications of profit tests, this paper has given little guidance on how to develop a profit test program. In an attempt to bridge the gap between understanding the theory and putting it into practice this Appendix sets out the calculations involved in producing the results for year 1 of the SPP profit test, §9.3. Throughout this Appendix any monetary values are in £'s.

Period—The period used is a month.

Decrement table—The exits are by death and lapse, therefore, a double decrement approach on a monthly basis is required. Define

$(al)_0$,	the number of entrants age 50 n.b.d.
$(al)_t$,	the number of the $(al)_0$ entrants surviving to the end of month t
$(aq)^d_t$,	the number of the $(al)_0$ entrants who exit through death in month t
$(aq)^w_t$,	the number of the $(al)_0$ entrants who exit through lapse or surrender in month t.

Let $(al)_0$ be 100,000 then the expected deaths in year 1 using A67/70 are $100{,}000 \times q_{50} = 480$. Ultimate rates have been used because it has been assumed that with the small death strain at risk no selection is necessary.

The expected lapses in year 1 are 10,000 (10% lapse rate).

It is assumed that the dependent rates are equal to the independent rates, which is acceptable as the expected deaths are much smaller than the expected lapses. It is further assumed that both decrements operate uniformly over the year so

$$(aq)^d_t = 40 \qquad (aq)^w_t = 833$$

The double decrement table for year 1 is, therefore,

Month t	$(al)_{t-1}$	$(aq)^d_t$	$(ap)^w_t$	$(al)_t$	*Month* t	$(al)_{t-1}$	$(aq)^d_t$	$(ap)^w_t$	$(al)_t$
1	100,000	40	833	99,127	7	94,762	40	833	93,889
2	99,127	40	833	98,254	8	93,889	40	833	93,016
3	98,254	40	833	97,381	9	93,016	40	833	92,143
4	97,381	40	833	96,508	10	92,143	40	833	91,270
5	96,508	40	833	95,635	11	91,270	40	833	90,397
6	95,635	40	833	94,762	12	90,397	40	833	89,524

Reserves—The initial unit reserves are premium × bid/offer spread × allocation rate, i.e. $5000 \times \cdot 95 \times 1 \cdot 01 = 4797 \cdot 5$. The additional reserves to cover the solvency requirements and the excess of death payments over unit values have been ignored throughout.

If V_t are the reserves required by a survivor at the end of month t then $V_0 = 4797 \cdot 5$ and

$$V_t = V_{t-1}\,(1+g)^{1/12}\left(1-\frac{c}{12}\right)$$

where g is the net growth rate, 7·5% p.a., and c is the annual management charge, ·75%.

Then unit reserves are

Month, t	0	1	2	3	4	5	
Reserves, V_t	4,797·5	4,823·5	4,849·6	4,875·9	4,902·3	4,928·8	
Month, t	6	7	8	9	10	11	12
Reserves, V_t	4,955·5	4,982·4	5,009·4	5,036·5	5,063·8	5,091·2	5,118·8

Premiums—The premium is a single premium of 5,000 received in month 1.

Interest—If I_t is the gross income (including capital gains) in month t, then

$$\begin{aligned} I_t = \quad & (\text{the reserves for a survivor to the end of month } t-1) \times \\ & (\text{the grossed up net interest rate for one month}) \times \\ & (\text{the proportion surviving to the end of month } t-1) \\ = \quad & V_{t-1}\frac{[(1+g)^{1/12}-1]}{1-T_i}\frac{(al)_{t-1}}{(al)_0} \end{aligned}$$

where T_i is the tax rate on income, 25%.

Expenses—The expenses in month 1 are the initial expenses of 150. In months 2–12 a possible assumption is a uniform expense at 1/12 of the annual renewal expense. However, the actual assumption made is that all the renewal expenses are incurred on the policy anniversary and there are, therefore, no expenses in these months.

Commission—The commission in month 1 is 3½% of 5000, i.e. £175.

Tax—The tax liability in month t is

$$T_i\,I_t - T_e\,(E_t + C_t)$$

where T_e is the rate of tax relief on expenses and commission, E_t is the expenses in month t, C_t is the commission in month t and T_i is the average rate of tax on income.

Deaths—The exits by death and lapse in the period are assumed to take place at the end of the period, which is acceptable when the period used is a month. The payment on death in month t is

$$1 \cdot 1\; V_t \frac{(aq)_t^d}{(al)_0}$$

The factor 1·1 includes allowance for the excess of the death benefit over the unit value.

Lapses—The payment on lapse in month t is:

$$V_t \frac{(aq)^w_t}{(al)_0}$$

Change in reserves—The increase in reserves required in month t is

$$V_t \frac{(al)_t}{(al)_0} - V_{t-1} \frac{(al)_{t-1}}{(al)_0}$$

Monthly cash flows—The results of the calculations outlined above are shown in Table 5, profit being the balancing item. The *Total* figures correspond with those for year 1 in §9.3, except for profit. The profit in §9.3 was derived by discounting each month's emerging profit to the start of the year at the risk discount rate. If the monthly profits are discounted then the value of the profits emerging in year 1 is found to be 30·6. The profit emergence in the table should be steady in months 2–12 and the fluctuations are due to rounding throughout to one decimal place.

Table 5. *Monthly cash flows for year 1 of the unit-linked single premium plan contract*

Month	*Premium*	*Interest*	*Expenses*	*Commission*	*Tax*	*Deaths*	*Lapses*	*Δ Reserves*	*Profit*
1	5,000	38·7	150	175	−112·2	2·1	40·2	4,781·4	2·2
2	0	38·5	0	0	9·6	2·1	40·4	−16·5	2·9
3	0	38·4	0	0	9·6	2·1	40·6	−16·7	2·8
4	0	38·3	0	0	9·6	2·2	40·8	−17·1	2·8
5	0	38·1	0	0	9·5	2·2	41·1	−17·5	2·8
6	0	38·0	0	0	9·5	2·2	41·3	−17·7	2·7
7	0	37·8	0	0	9·5	2·2	41·5	−18·0	2·6
8	0	37·7	0	0	9·4	2·2	41·7	−18·4	2·8
9	0	37·6	0	0	9·4	2·2	42·0	−18·8	2·8
10	0	37·4	0	0	9·4	2·2	42·2	−19·1	2·7
11	0	37·3	0	0	9·3	2·2	42·4	−19·4	2·8
12	0	37·1	0	0	9·3	2·3	42·6	−19·7	2·6
Total	5,000	454·9	150	175	−8·1	26·2	496·8	4,582·5	32·5

APPENDIX 2

Glossary of terms

This glossary has been designed to aid those unfamiliar with unit-linked business.

Allocation rate—When a premium is paid a percentage (the allocation rate) of the premium is invested in units and the remainder is used to cover the non-unit liabilities.

Offer price—The price at which the office allocates units.

Bid price—The price at which the office redeems units.
Bid/offer spread—(Offer price–Bid price) ÷ Offer price.
Management charge—A charge deducted from units, which is expressed as a percentage of the value of units per annum.
Accumulation unit—A unit subject to a 'small' management charge, in the examples $\frac{3}{4}\%$ p.a., designed to cover the renewal expenses.
Capital unit—A unit subject to a 'large' management charge, in the examples $4\frac{1}{4}\%$ p.a., designed to cover the renewal expenses and also to recover the initial expenses over the term of the contract.

Two methods for early recovery of the initial expenses:

(i) *Actuarial funding*—A method mostly used with capital units; credit is taken for the income expected to be generated from all future management charges. To facilitate this the unit values are reduced by an appropriate actuarial factor. Over the term of the policy, as the management charges are received, the actuarial factors are increased in line with a *funding plan* so that by the end of the *funding period* the full value of units is restored. Account must be taken of the funding plan when the surrender values are constructed. This method was used for the RPP contracts.

(ii) *Front-end loading*—A reduced or nil allocation rate is used for the first few months. This method was used for the protection plan in Section 12.