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Model-independent Price Bounds for the Swiss Re Mortality Bond – 2003

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Quotation

"Nothing is certain in life except death and taxes."

— Benjamin Franklin

Agenda

- Introduction
- Historical Facts
- The Problem
- Available Methodologies
- Design of the Swiss Re Bond
- A Model-independent Approach
- Lower Bound for the Swiss Re Bond
- Upper Bound for the Swiss Re Bond
- Numerical Results
- Conclusions
- What Lies Ahead?
- Further Research

Motivation

- In the present day world, financial institutions face the risk of unexpected fluctuations in human mortality
- This Risk has two aspects
    - *Mortality Risk*: Actual rates of mortality are in excess of those expected
    - *Longevity Risk*: People outlive their expected lifetimes

Introduction(2)

- Life insurers interested in *mortality risk*
- Annuity providers, defined benefit plans & social insurance programs interested in *longevity risk*
- A quick note on *longevity risk*
  - Life Expectancy in developed world has been increasing by approximately 1.2 months every year
  - Global Life Expectancy has increased by 4.5 months per year
  - Substantial improvements in Longevity at older ages during 20th century
  - Difficulties in Longevity Risk Management in Pension Funds due to wrong estimation of mortality rate
- What are the implications?
  - Underestimation of expected lifetimes leads to aggregate deficit in pension reserves
  - In 2010 alone improved life expectancy added 5 billion pounds to corporate pension obligations in UK

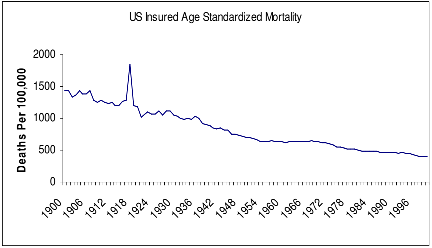
Introduction(3)

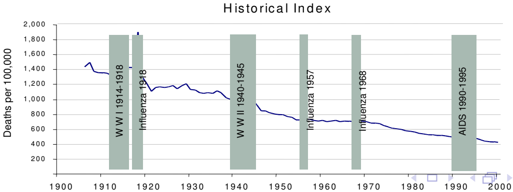
- A quick note on *mortality risk*
  - Life being shorter than expected is referred to as premature death or mortality risk
  - Factors that trigger mass premature deaths are CATASTROPHES!
- Catastrophes can be natural or man-made
- What is a catastrophe?
  - An event in which insured claims, total economic losses, or the number of casualties exceed a certain threshold
  - Lost or missing lives 20, injured 50, homeless 2000
- Number of catastrophes has risen sharply in the last four decades
- In the 1970's roughly 100 catastrophic events per year
- Number has more than tripled in the last decade
- In 2011, there were 325 such events: 175 natural and 150 man-made
- In 2014, 336 such events: 189 (highest ever) natural and 147 man-made
- In 2014, 12700 people lost their lives or went missing in the disasters

Introduction(4) : Possible Mortality Catastrophes

- Natural Disasters
- Terrorist Attacks
- Wars
- Influenza Epidemics
- Infectious diseases
- Meteorite Crashes

Historical Facts(1): Catastrophes lead to Mortality Spikes





Historical Facts(2)

Table 2: The change of death rates per 100,000 for each age group, from 1917 to 1919

Age groups	1917	1919	Ratio	Age groups	1917	1919	Ratio
All	1397.1	1810	1.296	35-44	900.8	1339.3	1.487
<1	10457.2	11167.2	1.068	45-54	1385.6	1524.1	1.100
1-4	1066.0	1573.5	1.476	55-64	2678.6	2648.1	0.989
5-14	256.0	412.8	1.613	65-74	5728.4	5505.0	0.961
15-24	468.9	1070.6	2.283	75-84	12386.2	11295.7	0.912
25-34	649.1	1643.5	2.532	>=85	24593.6	22213.5	0.903

- The 1918 influenza pandemic: Increase in mortality rate by 30% overall.
- Most affected age groups: 15-24 and 25-34
- For individuals aged 55 and over a little decrease in the death rate

Historical Facts(3)

- 13 or more influenza pandemics since 1500
- Four Influenza Pandemics in the 20th Century
  - Spanish Flu (1918)
    - most severe influenza pandemic ever experienced
    - more than 675,000 excess deaths b/w Sep 1918 & Apr 1919 in US
  - Asian Flu (1957)
  - Hong Kong Flu (1968)
  - Russian Flu (1977)
- H5N1 Avian Influenza in Hong Kong in 1997
- Swine Flu in 2009
- Could a flu happen again?
- Virologists and Epidemiologists say YES!
- Zika and Ebola: A taste of things to come?

The Problem - Extreme Mortality Risk

- Life insurance companies provide protection to their policyholders in the form of a payout made in the event of a policyholder's death, in exchange for a premium
- Extreme mortality events, such as a severe pandemic or a large terrorist attack, could result in a life insurance company needing to make sudden payouts to many policyholders
- This large payout would be exacerbated in that the investment portfolio would not yet have delivered sufficient returns – the payouts to policyholders are made sooner than expected
- Therefore it is crucial for life insurers, and life reinsurers, to manage their exposure to extreme mortality risks where insurance portfolio diversification by itself is insufficient

Available Methodologies for Hedging Extreme Mortality Risk

- *Natural Hedging*: compensating longevity risk by mortality risk
  - Drawback: Cost prohibitive
- *Mortality-linked Securities (MLS'S)* or *Catastrophe (CAT) Mortality (CATM) Bonds*: Cash flows linked to a mortality index such that the bonds get triggered by a catastrophic evolution of death rates of a certain population
  - Swiss Re Bond 2003 (VITA I): The first mortality bond
  - Swiss re Bond 2015 (VITA VI): The latest mortality bond

Valuation approaches on MLS's

- *Risk-adjusted process/ No-arbitrage Pricing:*
  - Estimate the distribution of future mortality rates in the real world probability measure
  - Transform the real-world distribution to its risk-neutral counterpart
  - Calculate the price of MLS by discounting the expected payoff under the risk-neutral probability measure at the risk-free rate
- *The Wang Transform:*
  - Employs a distortion operator that transforms the underlying distribution into a risk-adjusted distribution
  - MLS price is the expected value under the risk-adjusted probability discounted by risk-free rate
- *Instantaneous Sharpe Ratio:* Expected return on the MLS equals the risk-free rate plus the Sharp ratio times its standard deviation
- *The utility-based valuation:* Maximisation of the agent's expected utility subject to wealth constraints to obtain the MLS equilibrium

History of Mortality Linked Securities

- Tontines: 17th and 18th century in France
- Annuities in Geneva: Payoffs directly linked to the survival of Genevan "mademoiselles"
- Speculations came to an end during French Revolution
- Detailed overview in [Bauer(2008)]

Recent Developments(1)

Blake & Burrows (2001)	derived the concept of longevity bond
Swiss Re. (2003)	issued the first mortality bond
European investment bank (2004)	issued the first longevity bond
Cowley & Cummins(2005)	show that securitization may increase a firm's value
Lin & Cox (2005)	study and price the mortality bonds and swaps
Cairns, Blake & Dowd(2006)	show how to price mortality-linked financial instruments such as the EIB bond
Blake et al. (2006)	Introduce five types of longevity bonds

Recent Developments(2)

Pessler (2000)	Criticism of Wang Transform
Chen and Cox (2009)	Modelling mortality with Jumps
Cox et al (2010)	Mortality Risk Modelling
Milidonis et al (2011)	A regime switching mortality model with two states
Shang et al (2011)	Recursive Approach to MLS
Deng et al (2012)	Double-exponential jump diffusion model for mortality jumps & cohort effects
Cox et al (2013)	Mortality portfolio Risk Management
Lin et al (2013)	Pricing mortality securities with correlated indexes
Huang et al (2014)	Price jumps of MLS in incomplete markets
Hunt & Blake (2015)	Analysing the Swiss Re Kortis Bond

Prime Focus(1)

- Catastrophe Mortality Bonds or CATM Bonds
- What are these?
  - Bonds designed to transfer the risk of extreme mortality from a sponsor to investors
  - Coupon & Principal payments depend on the non-occurence of a pre-defined catastrophic event
- Transaction involves three parties
  - The Ceding company or Sponsor
  - Special Purpose Vehicle (SPV) or issuer
  - Investors generally large institutional buyers
- Transaction begins with formation of a SPV

Prime Focus(2)

- SPV issues bonds to investors
- SPV invests the received capital in high quality securities such as government or corporate AAA bonds
- Generally held in a trust account
- Coupon Payment
  - Investment returns from trust account &
  - Risk premium from ceding company
- Embedded in the bonds is a call option
- This call option gets triggered by a defined catastrophic event
- Well defined *Attachment* and *Exhaustion Points*
- Principal is fully at risk
- Our choice: Swiss Re Bond 2003

Prime Focus(3)

Specifications	VITA I	VITA II	TARTAN
Sponsor	Swiss Re	Swiss Re	Scottish Re
Arranger	Swiss Re	Swiss Re	Goldman Sachs
Modelling Firm	Milliman	Milliman	Milliman
SPV domicile	Cayman Islands	Cayman Islands	Cayman Islands
Size	\$ 400M	\$ 362M	\$ 155M
No.of Tranches	1	3	2
Issue date	December 2003	April 2005	May 2006
Maturity	4 years	5 years	3 years
Index	US, UK, France, Italy, Switzerland	US, UK, Germany, Japan, Canada	US

Table 1: The Initial CAT Mortality Bonds

Prime Focus(4)

Specifications	OSIRIS	VITA III	NATHAN
Sponsor	AXA	Swiss Re	Munich Re
Arranger	Swiss Re	Swiss Re	Munich Re
Modelling Firm	Milliman	Milliman	Milliman
SPV domicile	Ireland	Cayman Islands	Cayman Islands
Size	€ 345M	$ 705M	$ 100M
No.of Tranches	3	2	1
Issue date	November 2006	January 2007	February 2008
Maturity	4 years	4 & 5 years	5 years
Index	France, Japan, US	US, UK, Germany, Japan, Canada	US, UK, Canada, Germany

Table 2: The Middle Stage CAT Mortality Bonds

Prime Focus(5)

Specifications	Vita IV	Vita IV	Vita V
Sponsor	Swiss Re	Swiss Re	Swiss Re
Arranger	Swiss Re	Swiss Re	Swiss Re
Modelling Firm	RMS	RMS	RMS
SPV domicile	Cayman Islands	Cayman Islands	Cayman Islands
Size	\$ 300M	\$ 180M	\$ 275M
No.of Tranches	4	2	2
Issue date	I: Nov'09; II: May'10 III & IV: Oct 2010	July 2011	July 2012
Maturity	4 & 5 years	5 years	5 years
Index	I:US, UK; II:US/UK III: US/Japan, IV: Germany/ Canada	IV:Canada/ Germany(Ger.), V:Canada/Ger./ UK/US	D-1:Australia, Canada E-1:Australia, Canada, US

Table 3: The Middle Stage CAT Mortality Bonds (Contd...)

Prime Focus(6)

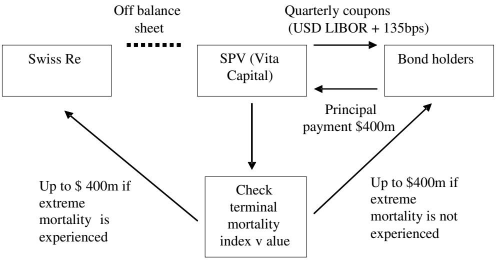
Specifications	Mythen Re	Atlas IX	VITA VI
Sponsor	Swiss Re	SCOR Re	Swiss Re
Arranger	Swiss Re	Aon, BNP Paribas, Natixis	Swiss Re
Modelling Firm	AIR/RMS	RMS	RMS
SPV domicile	Cayman Islands	Ireland	Cayman Islands
Size	\$ 200M	\$ 180M	\$ 100M
No.of Tranches	2	2	1
Issue date	November 2012	September 2013	December 2015
Maturity	4 & 5 years	5 years	5 years
Index	U.S. hurricane, UK mortality	US	Australia, Canada, UK

Table 4: The Latest CAT Mortality Bonds

Prime Focus(7)

- Why Swiss Re Bond...?
  - An Innovative Security...one of its kind
  - A kind of pioneer and path setter
  - Shifted the risk exposure from the balance sheet to the capital markets
- Attracted lot of attention and was fully subscribed (Euroweek, 19 December 2003)
- Investors included a large number of pension funds
- Established a Special Purpose Vehicle (SPV) called VITA I for the securitization
- A 3-year bond issued in December 2003 with maturity on Jan 1, 2007
- Principal s.t. mortality risk defined in terms of an index q_i in yr t_i
- Quarterly coupons of three-month US-dollar LIBOR + 135 basis points
- Strength: Extreme Transparency

The Bond Mechanism



The Mortality Index

- Mortality index constructed as a weighted average of mortality rates (deaths per 100,000) over age, sex (male 65%, female 35%) and nationality (US 70%, UK 15%, France 7.5%, Italy 5%, Switzerland 2.5%)

$$\text{Index} = \sum_j C_j \sum_i \left(G^m A_i q^m_{i,j,t} + G^f A_i q^f_{i,j,t} \right)$$

- $q^m_{i,j,t}$ and $q^f_{i,j,t}$ = mortality rates (deaths per 100,000) for males and females respectively in the age group i for country j
- C_j = weight attached to country j
- A_i = weight attributed to age group i (same for males and females)
- G^m and G^f = gender weights applied to males and females respectively
- q_0 = base index

Index Distribution

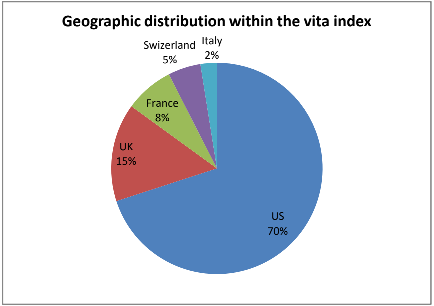


Table showing distribution by age within the VITA index

Age Group	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79
Weight	1%	5%	12.50%	20%	20%	16%	12%	7%	3%	2%	1%	0.50%

Design of the Swiss Re Bond(1)

Principal Loss Percentage

$$L_i = \begin{cases} 0 & \text{if } q_i \leq K_1 q_0 \\ \frac{(q_i - K_1 q_0)}{(K_2 - K_1) q_0} & \text{if } K_1 q_0 < q_i \leq K_2 q_0 \\ 1 & \text{if } q_i > K_2 q_0 \end{cases} \tag{1}$$

- For Swiss Re Bond $K_1 = 1.3$ $K_2 = 1.5$

Coupons

$$C_j = \begin{cases} \left(\frac{S + LI_j}{4}\right).C & \text{if } j = \frac{1}{4}, \frac{2}{4}, \ldots, \frac{11}{4}, \\ \left(\frac{S + LI_j}{4}.C + X\right) & \text{if } j = 3, \end{cases} \tag{2}$$

- S: Spread value, LI_j: LIBOR rates, C: Face Value, X: a random variable

Design of the Swiss Re Bond(2)

- Proportion of the principal returned to the bondholders on the maturity date:

$$X = C\left(1 - \sum_{i=1}^{3} L_i\right)^{+}, \tag{3}$$

- $C = \$400$ million
- Risk-neutral price of the random pay-off at time 0 with Q as the EMM

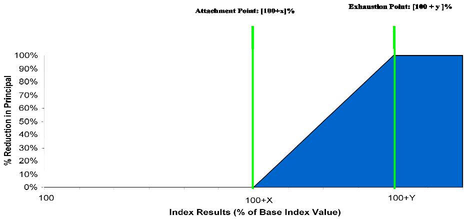
$$P = e^{-rT}\mathrm{E}_Q[X] \tag{4}$$

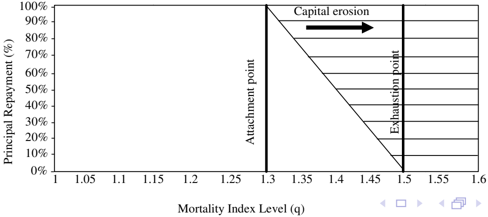
- r is nominal annual interest rate

Discounted Cashflow of Payments

$$DC(r) = \sum_{i=1}^{12} \frac{C_{\frac{i}{4}}}{\left(1 + \frac{r}{4}\right)^{i}} \tag{5}$$

Design of the Swiss Re Bond(3)





Our Approach for Bond Evaluation

- Adapt the payoff of the bond in terms of the payoff of an Asian put option
- Assume the existence of an Equivalent Martingale Measure (EMM)
- Present model-independent bounds
- Exploit comonotonic theory as illustrated in [Albrecher et al.(2008)Albrecher, Mayer, and Schoutens] for the pricing of Asian options
- Carry out Monte Carlo simulations to estimate the bond price under Black-Scholes Model
- Draw graphs of the bounds by varying the interest rate r and mortality rate q_0

Payoff as that of an Asian Put Option

Alternative form of writing Payoff

$$P = De^{-rT}\mathrm{E}\left[(q_0 - S)^+\right] \tag{6}$$

- $D = \frac{C}{q_0}$
- $S_i = 5\left(q_i - 1.3q_0\right)^+$
- $S = \sum_{i=1}^{3} S_i$

Call counterpart of the payoff

$$P_1 = De^{-rT}\mathrm{E}\left[(S - q_0)^+\right] \tag{7}$$

Put-call parity for the Swiss Re Bond

The relation

$$P_1 - P = De^{-rT}\left[5\sum_{i=1}^{3} e^{rt_i} C\left(1.3q_0, t_i\right) - q_0\right] \tag{8}$$

- Define

$$G = De^{-rT}\left[5\sum_{i=1}^{3} e^{rt_i} C\left(1.3q_0, t_i\right) - q_0\right] \tag{9}$$

- Bounding P_1 by bounds l_1 and u_1
- Corresponding bounds for the Swiss Re Mortality Bond:

$$(l_1 - G)^+ \leq P \leq (u_1 - G)^+ \tag{10}$$

Some Basic Concepts

Definition

Stop-loss Premium: The stop-loss premium with retention d of a random variable X is defined as $\mathbf{E}\left[(X-d)^{+}\right]$.

Definition

Stop-loss Order: Consider two random variables X and Y. Then X is said to precede Y in the stop-loss order sense, written as $X \leq_{sl} Y$, if and only if X has lower stop-loss premiums than Y:

$$\mathbf{E}\left[(X-d)^{+}\right] \leq \mathbf{E}\left[(Y-d)^{+}\right] \qquad -\infty < d < \infty \tag{11}$$

Definition

Convex Order: X is said to precede Y in terms of convex order, written as $X \leq_{cx} Y$, if and only if $X \leq_{sl} Y$ and $\mathbf{E}[X] = \mathbf{E}[Y]$.

Lower Bound for the Call Counterpart

Lower Bound using Jensen's Inequality

$$P_1 \geq De^{-rT}\mathbf{E}\left[\left(\sum_{i=1}^{n} 5\left(\mathbf{E}\left(q_i|\Lambda\right) - 1.3q_0\right)^+ - q_0\right)^+\right] \qquad (12)$$

- We define: $Z_i = 5\left(\mathbf{E}\left(q_i|\Lambda\right) - 1.3q_0\right)^+; i = 1, 2, ..., n$ & $Z = \sum_{i=1}^{n} Z_i$
- $S \geq_{sl} Z$ or $\mathrm{E}\left[(S - q_0)^+\right] \geq \mathrm{E}\left[(Z - q_0)^+\right]$
- The conditioning variable Λ is chosen in such a way that $\mathbf{E}\left[q_i|\Lambda\right]$ is either increasing or decreasing for every i
- This implies the vector: $\mathbf{Z^l} = (Z_1, \ldots, Z_n)$ is comonotonic & yields

Stop-loss lower bound for the call-counterpart

$$P_1 \geq De^{-rT}\sum_{i=1}^{n}\mathbf{E}\left[\left(5\left(\mathbf{E}\left(q_i|\Lambda\right) - 1.3q_0\right)^+ - F_{Z_i}^{-1}\left(F_Z\left(q_0\right)\right)\right)^+\right] \qquad (13)$$

The Trivial Lower Bound

- if the random variable Λ is independent of the mortality evolution $\{q_t\}_{t\geq 0}$ we get

The Trivial Lower Bound

$$P_1 \geq Ce^{-rT}\left(\sum_{i=1}^{n} 5\left(\exp\left(rt_i\right) - 1.3\right)^+ - 1\right)^+ =: \ \mathrm{lb}_0 \tag{14}$$

- Using

$$G = De^{-rT}\left[5\sum_{i=1}^{3} e^{rt_i} C\left(1.3q_0, t_i\right) - q_0\right] \tag{15}$$

- Corresponding bound for the Swiss Re Mortality Bond:

$$P \geq \left(\ \mathrm{lb}_0 - G\right)^+ =: \ \mathrm{LB}_0 \tag{16}$$

The Lower Bound LB_1

- We choose $\Lambda = q_1$ in (13)
- Use the martingale argument for the discounted mortality process

$$\mathbf{E}\left[q_i|q_1\right] = \mathbf{E}\left[e^{rt_i}e^{-rt_i}q_i|q_1\right] = e^{r(t_i-t_1)}q_1 .$$

The Lower Bound LB_1

$$P_1 \geq 5D\sum_{i=1}^{n} e^{-r(T-t_i)}C\left(q_0\left(\frac{1.3}{e^{r(t_i-t_1)}} + \left(x - \frac{1.3}{e^{r(t_i-t_1)}}\right)^+\right), t_1\right) =: \mathrm{lb}_1 \tag{17}$$

- where x is the solution of $\sum_{i=1}^{n}\left(e^{r(t_j-t_1)}x - 1.3\right)^+ = 0.2$
- $C(K, t_1)$ is the price of a European call on the mortality index with strike K, maturity t_1 and current mortality index q_0

The Lower Bound $\mathrm{LB}_t^{(1)}$

- Further improvement using additional assumptions
- The following inequality holds for every random variable Y and every constant c

$$\Rightarrow \mathbf{E}\left[a^+\right] \geq \mathbf{E}\left[a\mathbb{1}_{\{Y \geq c\}}\right] \tag{18}$$

- Utilizing the above inequality twice
- and further assume: q_i and $\mathbb{1}_{\{q_t \geq c\}}$ are non-negatively correlated for $t > t_i$

The Lower Bound $\mathrm{LB}_t^{(1)}$

$$P_1 \geq 5De^{-rT} \max_{0 \leq t \leq T} C\left(\tilde{c}_t, t\right) \sum_{i=j}^{n} e^{rt_i} =: \mathrm{lb}_t^{(1)} \tag{19}$$

- where $j = \min\{i \,:\, t_i \geq t\}$ and

$$\tilde{c}_t = q_0 \left(\frac{(0.2 + 1.3n) - \sum_{i=1}^{j-1} e^{rt_i}}{\sum_{i=j}^{n} e^{r(t_i - t)}} \right) \tag{20}$$

A Model-independent Lower Bound(1)

- Additional assumption that holds good for stationary exponential Lèvy models

$$\sum_{i=1}^{n} q_i \geq_{sl} \left(\sum_{i=1}^{j-1} q_0^{(1-t_i/t)} q_t^{t_i/t} + \sum_{i=j}^{n} e^{r(t_i-t)} q_t \right) \tag{21}$$

- for $0 \leq t \leq T$ and $j = \min\{i \,:\, t_i \geq t\}$
- We then use the following two results

Proposition

Let $(X,\ Y) \sim BVN\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, where BVN stands for bivariate normal distribution. The conditional distribution function of X, given the event $Y = y$, is given as

$$F_{X|Y=y}(x) = \Phi\left(\frac{x - \left(\mu_X + \rho\frac{\sigma_X}{\sigma_Y}(y - \mu_Y)\right)}{\sigma_X\sqrt{1-\rho^2}}\right) \tag{22}$$

A Model-independent Lower Bound(2)

Proposition

Let $W = (W_t)\,, t \geq 0$ be a standard Brownian motion. Then the conditional expectation of W_{t_i} given W_t is given as

$$\boldsymbol{E}\left[W_{t_i} | W_t\right] = \frac{t_i}{t} W_t \quad \textit{for any } t_i < t$$

- The above proposition then leads to the following proposition

Proposition

The additional assumption (21) holds for stationary exponential Lèvy models with mortality evolution $q_t = q_0 \exp(U_t)$, where $(U_t)_{t \geq 0}$ is a Lèvy process

A Model-independent Lower Bound(3)

- We use this result to achieve the lower bound for the Asian-type call option

$$\sum_{i=1}^{n} 5\left(\mathbf{E}\left(q_i|q_t\right) - 1.3q_0\right)^+ = \sum_{i=1}^{j-1} 5q_0\left(\left(\frac{q_t}{q_0}\right)^{t_i/t} - 1.3\right)^+ + \sum_{i=j}^{n} 5q_0\left(\frac{q_t}{q_0}e^{r(t_i-t)} - 1.3\right)^+ =: S^{l_2}. \tag{23}$$

- S^{l_2} is the same as Z with Λ being replaced by q_t
- So we have $S \geq_{sl} S^{l_2}$

A Model-independent Lower Bound(4)

- Define $\mathbf{Y} = (Y_1, \ldots, Y_n)$ with

$$Y_i = \begin{cases} 5q_0 \left(\left(\frac{q_t}{q_0} \right)^{t_i/t} - 1.3 \right)^+ & i < j \\ 5q_0 \left(\left(\frac{q_t}{q_0} \right) e^{r(t_i - t)} - 1.3 \right)^+ & i \geq j \end{cases}$$

- $i = 1, 2, ..., n$
- $\mathbf{Y}$ is comonotonic:-components are strictly increasing functions of q_t
- By the comonotonic theory

$$\mathbf{E}\left[\left(S^{l_2} - q_0 \right)^+ \right] = \sum_{i=1}^{n} \mathbf{E}\left[\left(Y_i - F_{Y_i}^{-1}\left(F_{S^{l_2}}\left(q_0 \right) \right) \right)^+ \right] \qquad (24)$$

- where $F_{S^{l_2}}(q_0)$ is the distribution function of S^{l_2} evaluated at q_0

- such that for an arbitrary t, we have:

$$
\begin{aligned}
F_{S^{l_2}}(q_0) &= \mathbf{P}\left[S^{l_2} \leq q_0\right] \\
&= \mathbf{P}\left(\sum_{i=1}^{j-1}\left(\left(\frac{q_t}{q_0}\right)^{t_i/t} - 1.3\right)^+ \right. \\
&\qquad \left. + \sum_{i=j}^{n}\left(\left(\frac{q_t}{q_0}\right)e^{r(t_i-t)} - 1.3\right)^+ \leq 0.2\right) \qquad (25)
\end{aligned}
$$

- Substitute x for q_t/q_0 in (25)
- where x solves

$$
\sum_{i=1}^{j-1}\left(x^{t_i/t} - 1.3\right)^+ + \sum_{i=j}^{n}\left(xe^{r(t_i-t)} - 1.3\right)^+ = 0.2 \qquad (26)
$$

- Then $S^{l_2} \leq q_0$ if and only if $q_t \leq xq_0$

A Model-independent Lower Bound(6)

- This yields

$$F_{S^{l_2}}(q_0) = F_{q_t}(xq_0) = \begin{cases} F_{Y_i}\left(5q_0\left(x^{t_i/t} - 1.3\right)^+\right) & i < j \\ F_{Y_i}\left(5q_0\left(xe^{r(t_i-t)} - 1.3\right)^+\right) & i \geq j \end{cases}$$

The Lower Bound $\mathrm{lb}_t^{(2)}$

$$\begin{aligned} P_1 \quad &\geq \quad 5De^{-rT}\left(\sum_{i=1}^{j-1} q_0^{1-t_i/t}\mathbf{E}\left[\left(q_t^{t_i/t} - q_0^{t_i/t}\left(1.3 + \left(x^{t_i/t} - 1.3\right)^+\right)\right)^+\right]\right. \\ &\qquad \left. + \sum_{i=j}^{n} e^{rt_i} C\left(q_0\left(\frac{1.3}{e^{r(t_i-t)}} + \left(x - \frac{1.3}{e^{r(t_i-t)}}\right)^+\right), t\right)\right) \\ &=: \quad \mathrm{lb}_t^{(2)} \qquad (27) \end{aligned}$$

A Model-independent Lower Bound(7)

- $\mathrm{lb}_t^{(2)}$ is a lower bound for all t and can be maximized w.r.t. t to yield the optimal lower bound:

$$P_1 \geq \max_{0 \leq t \leq T} \mathrm{lb}_t^{(2)} \tag{28}$$

- As before, we have on using the put-call parity

$$P \geq \left(\mathrm{lb}_t^{(2)} - G\right)^+ =: \mathrm{LB}_t^{(2)} \tag{29}$$

A Lower Bound under Black-Scholes Model(1)

- Assume that the mortality evolution process $\{q_t\}_{t\geq 0}$ follows the Black-Scholes model written as $q_t = e^{U_t}$
- where

$$U_t = \log_e(q_0) + \left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t^* \tag{30}$$

and $\{W_t^*\}_{t\geq 0}$ denotes a standard Brownian motion

-

$$U_t \sim N\left(\log_e q_0 + \left(r - \frac{\sigma^2}{2}\right)t,\ \sigma^2 t\right) \tag{31}$$

Proposition

If $(X,\ Y) \sim BVN\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, the conditional distribution of the lognormal random variable e^X, given the event $e^Y = y$ is

$$F_{e^X|e^Y=y}(x) = \Phi\left(\frac{\log_e x - \left(\mu_X + \rho\frac{\sigma_X}{\sigma_Y}(\log_e y - \mu_Y)\right)}{\sigma_X\sqrt{1-\rho^2}}\right) \tag{32}$$

A Lower Bound under Black-Scholes Model(2)

- Given the time points t_i, t for each i
- let ρ be the correlation between U_{t_i} and U_t
- Then, $(U_{t_i}, U_t) \sim \text{BVN}\left(\mu_{U_{t_i}}, \mu_{U_t}, \sigma^2_{U_{t_i}}, \sigma^2_{U_t}, \rho\right)$
- where $\mu_{U_{t_i}}, \mu_{U_t}, \sigma^2_{U_{t_i}}$ and $\sigma^2_{U_t}$ are given by (46)
- Now $q_t = e^{U_t}$
- The distribution function of q_i conditional on the event $q_t = s_t$ is given as

$$F_{q_i|q_t=s_t}(x) = \Phi\left(a\left(x\right)\right)$$

- where $a(x)$ is given by

$$a\left(x\right) = \frac{\log_e x - \left(\log\left(q_0\left(\frac{s_t}{q_0}\right)^{\rho\sqrt{\frac{t_i}{t}}}\right) + \left(r - \frac{\sigma^2}{2}\right)\left(t_i - \rho\sqrt{t_i t}\right)\right)}{\sigma\sqrt{t_i\left(1-\rho^2\right)}}. \tag{33}$$

A Lower Bound under Black-Scholes Model(3)

- For the mortality evolution process $\{q_t\}_{t\geq 0}$ defined as $q_t = e^{U_t}$

$$\mathbf{E}\left(q_i | q_t\right) = \begin{cases} q_0 \left(\frac{q_t}{q_0}\right)^{\frac{t_i}{t}} e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} & t_i < t, \\ q_t e^{r(t_i - t)} & t_i \geq t. \end{cases} \tag{34}$$

- Use this result to achieve the lower bound for the Asian-type call option
-
- Define $\mathbf{Y} = (Y_1, \ldots, Y_n)$
- where

$$Y_i = \begin{cases} 5q_0\left(\left(\frac{q_t}{q_0}\right)^{t_i/t} e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} - 1.3\right)^+ & i < j \\ 5q_0\left(\left(\frac{q_t}{q_0}\right) e^{r(t_i-t)} - 1.3\right)^+ & i \geq j \end{cases}$$

- $i = 1, 2, ..., n$
- $\mathbf{Y}$ is comonotonic

A Lower Bound under Black-Scholes Model(4)

- Define $S^{l_3} = \sum_{i=1}^{n} Y_i$
- By the comonotonic theory

$$\mathbf{E}\left[\left(S^{l_3} - q_0\right)^+\right] = \sum_{i=1}^{n} \mathbf{E}\left[\left(Y_i - F_{Y_i}^{-1}\left(F_{S^{l_3}}\left(q_0\right)\right)\right)^+\right] \tag{35}$$

- where $F_{S^{l_3}}(q_0)$ is the distribution function of S^{l_3} evaluated at q_0
- such that for an arbitrary t, we have:

$$\begin{aligned} F_{S^{l_3}}\left(q_0\right) &= \mathbf{P}\left[S^{l_3} \leq q_0\right] \\ &= \mathbf{P}\left(\sum_{i=1}^{j-1}\left(\left(\frac{q_t}{q_0}\right)^{t_i/t} e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} - 1.3\right)^+ \right. \\ &\qquad \left. + \sum_{i=j}^{n}\left(\left(\frac{q_t}{q_0}\right) e^{r(t_i-t)} - 1.3\right)^+ \leq 0.2\right) \end{aligned} \tag{36}$$

- Substitute x for q_t/q_0 in (36)
- where x solves

$$\sum_{i=1}^{j-1}\left(x^{t_i/t}e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} - 1.3\right)^+ + \sum_{i=j}^{n}\left(xe^{r(t_i-t)} - 1.3\right)^+ = 0.2 \quad (37)$$

- Then $S^{l_3} \leq q_0$ if and only if $q_t \leq xq_0$
- This yields

$$F_{S^{l_3}}(q_0) = F_{q_t}(xq_0) = \begin{cases} F_{Y_i}\left(5q_0\left(x^{t_i/t}e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} - 1.3\right)^+\right) & i < j, \\ F_{Y_i}\left(5q_0\left(xe^{r(t_i-t)} - 1.3\right)^+\right) & i \geq j \end{cases}$$

- As a result we have:

$$
\begin{aligned}
P_1 \quad \geq \quad & 5De^{-rT}\left(\sum_{i=1}^{j-1} q_0^{1-t_i/t}\mathbf{E}\left(\left(q_t^{t_i/t}e^{\frac{\sigma^2 t_i}{2t}(t-t_i)}\right.\right.\right. \\
& \left.\left.\left. - q_0^{t_i/t}\left(1.3+\left(x^{t_i/t}e^{\frac{\sigma^2 t_i}{2t}(t-t_i)}-1.3\right)^{+}\right)\right)^{+}\right) \\
& + \sum_{i=j}^{n} e^{rt_i}C\left(q_0\left(\frac{1.3}{e^{r(t_i-t)}}+\left(x-\frac{1.3}{e^{r(t_i-t)}}\right)^{+}\right),t\right)\Bigg)
\end{aligned}
$$

A Lower Bound under Black-Scholes Model(7)

- Denote the term within the first summation as E_1 and its value is given below.

$$\mathbf{E}_1 = 5q_0 \left(e^{rt_i} \Phi\left(d_{1ai}\right) - \left(1.3 + \left(x^{t_i/t} e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} - 1.3 \right)^+ \right) \Phi\left(d_{2ai}\right) \right) \tag{38}$$

- where d_{2ai} and d_{1ai} are given respectively as

$$d_{2ai} = \frac{-\log_e\left(\frac{da_i}{q_0}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{39}$$

$$d_{1ai} = d_{2ai} + \sigma\frac{t_i}{\sqrt{t}} \tag{40}$$

- and da_i is given as

$$da_i = q_0 \left(\frac{1.3}{e^{\frac{\sigma^2 t_i}{2t}(t-t_i)}} + \left(x^{t_i/t} - \frac{1.3}{e^{\frac{\sigma^2 t_i}{2t}(t-t_i)}} \right)^+ \right)^{t/t_i} \tag{41}$$

A Lower Bound under Black-Scholes Model(8)

- As a result we have

The Lower Bound $\mathrm{lb}_t^{(3)}$

$$
\begin{aligned}
P_1 \geq\ & 5De^{-rT}\left(\sum_{i=1}^{j-1} q_0 \left(e^{rt_i}\Phi\left(d_{1ai}\right) - \left(1.3 + \left(x^{t_i/t}e^{\frac{\sigma^2 t_i}{2t}(t-t_i)} - 1.3\right)^+\right)\right.\right. \\
& \left.\left. + \sum_{i=j}^{n} e^{rt_i} C\left(q_0\left(\frac{1.3}{e^{r(t_i-t)}} + \left(x - \frac{1.3}{e^{r(t_i-t)}}\right)^+\right), t\right)\right)\right) \\
=:\ & \mathrm{lb}_t^{(3)}
\end{aligned}
$$

- The bound $\mathrm{lb}_t^{(3)}$ can undergo treatment similar to $\mathrm{lb}_t^{(2)}$ in sense of maximization with respect to t yielding

$$P_1 \geq \max_{0\leq t\leq T} \mathrm{lb}_t^{(3)} \tag{43}$$

An Upper Bound for the Swiss Re Bond(1)

Proposition

The payoff of the call option is a convex function[a] *of the strike price, i.e.,* $\boldsymbol{E}\left[(X-x)^{+}\right]$ *is convex in x.*

[a] A function $f : I \to \mathbb{R}$, where I is an interval in $\mathbb{R}$, is convex if and only if $f\left(ax+(1-a)\,y\right) \leq af\left(x\right)+(1-a)\,f\left(y\right) \quad \forall a \in [0,1]$ and any pair of elements $x, y \in I$.

- Define a vector $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ such that $\lambda_i \in \mathbb{R}$ and $\sum_{i=1}^{n} \lambda_i = 1$
- With the help of $\boldsymbol{\lambda}$ we can write the payoff of the Asian-type call option as

$$P_1 = Ce^{-rT}\mathrm{E}\left[\left(\sum_{i=1}^{n}\left(5\left(\frac{q_i}{q_0}-1.3\right)^{+}-\lambda_i\right)\right)^{+}\right]. \qquad (44)$$

- The above result for the call option implies

$$P_1 \leq 5De^{-rT}\sum_{i=1}^{n} e^{rt_i}C\left(q_0\left(1.3+\frac{\lambda_i}{5}\right), t_i\right) \qquad (45)$$

An Upper Bound for the Swiss Re Bond(2)

- Employing the Lagrangian with ϕ as the Lagrange's multiplier, we have

$$L(\boldsymbol{\lambda}, \phi) = \frac{5}{q_0} \sum_{i=1}^{n} e^{rt_i} C\left(q_0\left(1.3 + \frac{\lambda_i}{5}\right), t_i\right) + \phi\left(\sum_{i=1}^{n} \lambda_i - 1\right) \quad (46)$$

The Upper Bound ub_1

$$P_1 \le 5De^{-rT} \sum_{i=1}^{n} e^{rt_i} C\left(F_{q_i}^{-1}(x), t_i\right) =: \mathrm{ub}_1 \quad (47)$$

- where $x \in (0,1)$ solves $\sum_{i=1}^{n} F_{q_i}^{-1}(x) = \frac{q_0}{5}(1 + 6.5n)$
- Put-Call parity yields: $P \le (\mathrm{ub}_1 - G)^+ =: \mathrm{UB}_1$

Numerical Results(1)

- Assume that the mortality evolution process $\{q_t\}_{t\geq 0}$ obeys the Black-Scholes model, specified by the following stochastic differential equation (SDE)
$$dq_t = rq_t dt + \sigma q_t dW_t.$$
- In order to simulate a path, we will consider the price of the asset on a finite set of $n = 3$ evenly spaced dates $t_1, ..., t_n$.

The Brownian Simulation

$$q_{t_j} = q_{t_{j-1}} \exp\left[\left(r - \frac{1}{2}\sigma^2\right)\delta t + \sigma\sqrt{\delta t}\, U_j\right] \quad U_j \sim N(0,1)\,, \quad j = 1, 2, \ldots, n \tag{48}$$

Parameter choices in accordance with [Lin and Cox(2008)]

$$q_0 = 0.008453,\ r = 0.0,\ T = 3,\ t_0 = 0,\ n = 3,\ \sigma = 0.0388$$

Numerical Results(2)

Table 5: Table showing the various lower bounds, upper bound and the Monte Carlo estimate for the B-S Model for varying values of r

r	LB0	LB1	LBt_(1)	LBt_(2)	LBt_(3)	UB	MC
0.035	0.899130889131400	0.899130889153152	0.899130889163207	0.899131563852078	0.899131577418890	0.899131637780299	0.899130939228525
0.03	0.913324024542464	0.913324024546338	0.913324024548259	0.913324251738880	0.913324256505855	0.913324320930395	0.913324120543246
0.025	0.927447505802074	0.927447505802722	0.927447505803066	0.927447578831809	0.927447580428344	0.927447619324390	0.927447582073642
0.02	0.941626342686440	0.941626342686542	0.941626342686600	0.941626365090140	0.941626365599735	0.941626384748977	0.941626356704134
0.015	0.955935721003105	0.955935721003120	0.955935721003129	0.955935727561107	0.955935727716106	0.955935736078305	0.955935715488521
0.01	0.970419124545862	0.970419124545864	0.970419124545865	0.970419126377220	0.970419126422140	0.970419129771609	0.970419112046475
0.005	0.985101139986133	0.985101139986134	0.985101139986134	0.985101140473942	0.985101140486345	0.985101141738075	0.985101142704466
0	0.999995778015617	0.999995778015617	0.999995778015617	0.999995778139535	0.999995778142797	0.999995778583618	0.999995730678518

Table 6: Table showing the various lower bounds, upper bound and the Monte Carlo estimate for B-S Model for varying values of q0 when r=0.0

q0	LB0	LB1	LBt(1)	LBt(2)	LBt(3)	UB	MC
0.007	0.999999999999517	0.999999999999517	0.999999999999517	0.999999999999517	0.999999999999517	0.999999999999517	1.000000000000000
0.008	0.999999915251651	0.999999915251651	0.999999915251651	0.999999915252160	0.999999915252175	0.999999915253115	0.999999935586330
0.008453	0.999995778015617	0.999995778015617	0.999995778015617	0.999995778139535	0.999995778142797	0.999995778583618	0.999995730678518
0.009	0.999821987943444	0.999821987949893	0.999821987949893	0.999822025862818	0.999822025862818	0.999822875816246	0.999816103328680
0.01	0.978292691034648	0.978310383929407	0.978310383929037	0.978503560221413	0.978503560221499	0.986262918346612	0.978738658827918
0.011	0.572750782003669	0.610962124257773	0.610962123857399	0.610962123857399	0.610962123857400	0.877336305501968	0.652440509314875
0.012	0.000000000000000	0.040209774144029	0.040209770810356	0.040209770810359	0.040209770810359	0.395672911251278	0.094615386163640
0.013	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.083466184427206	0.001662471990070
0.014	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.008942985848261	0.000003376858132

Numerical Results(3)

Figure1: Rel. Diff. of LBt(2), LBt(3) and UB1 w.r.t. MC estimate under Black-Scholes model

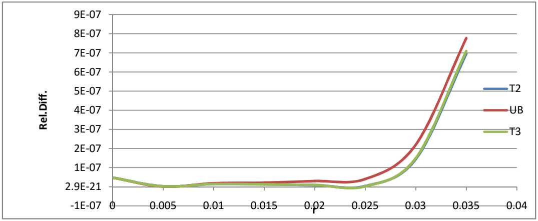
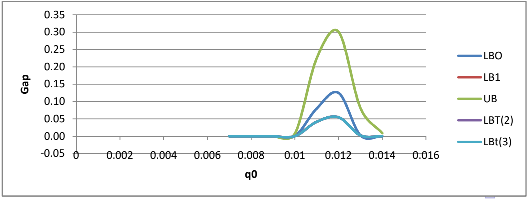


Figure2: Comparison of different bounds under B-S model in terms of difference from MC estimate for r=0



Numerical Results(4)

Figure3: Price Bounds under Black-Scholes model for the parameter choice of Lin and Cox(2008) Model

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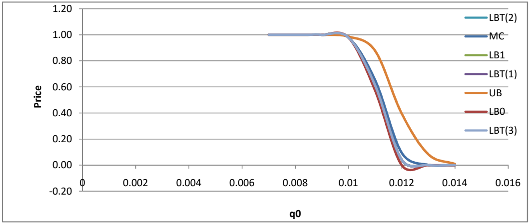


Numerical Results(5)

- Assume that the mortality rate 'q' obeys the four-parameter transformed Normal (S_u) distribution ([Johnson(1949)] and [Johnson et al.(1994)Johnson, Kotz, and Balakrishnan]) which is defined as follows

$$sinh^{-1}\left(\frac{q-\alpha}{\beta}\right) = x \sim N\left(\mu, \sigma^2\right), \tag{49}$$

- α, β, μ and σ are parameters ($\beta, \sigma > 0$) and $sinh^{-1}$ is the inverse hyperbolic sine function
- Let $q_0 = 0.008453$.

Parameter choices in accordance with [Tsai and Tzeng(2013)]

$\alpha = [0.008399, 0.008169, 0.007905]$, $\beta = [0.000298, 0.000613, 0.000904]$,

$\mu = [0.70780, 0.58728, 0.58743]$ and $\sigma = [0.67281, 0.50654, 0.42218]$.

Numerical Results(6)

Table 7: Table showing the various lower bounds, upper bound and the Monte Carlo estimate for the Su distn. for varying values of r

r	LB0	LB1	LBt1	LBt2	UB1	MC
0.035	0.883255461690070	0.884321427701533	0.884321427701545	0.885548150428771	0.886806565750194	0.884689900254432
0.03	0.903403981322902	0.904010021303490	0.904010021303483	0.904693957669362	0.905481788284534	0.904223406591320
0.025	0.921607066867317	0.921935518850858	0.921935518850855	0.922291170234705	0.922759498340311	0.922030679117868
0.02	0.938407830148741	0.938576980453810	0.938576980453809	0.938747560828014	0.939010425491579	0.938598989786277
0.015	0.954287129640998	0.954369722665066	0.954369722665065	0.954444088119093	0.954582647473048	0.954415686472720
0.01	0.969639544072264	0.969677756802278	0.969677756802278	0.969706604342752	0.969774875755017	0.969683647401862
0.005	0.984762743262391	0.984779521693468	0.984779521693468	0.984789115794819	0.984820459036106	0.984784143645972
0	0.999861354235404	0.999868375732131	0.999868375732131	0.999870879263060	0.999884274666239	0.999871208429012

Note: LBt2 obtained by Numerical Integration in MATLAB

- Assume that the mortality index $\{q_t\}_{t\geq 0}$ follows log gamma distribution, which is defined as

$$log\left(\frac{q-\mu}{\sigma}\right) = x \sim Gamma\left(p, a\right), \tag{50}$$

- μ, σ, p and a are parameters (> 0) and *log* is the natural logarithm.

Parameter choices in accordance with [Cheng et al.(2014)Cheng, Tzeng, Hsieh, and Tsai]

$$q_0 = 0.0088,\ p = [61.6326, 64.2902, 71.8574],\ a = [0.0103, 0.0098, 0.0080],$$

$$\mu = [-5.2452, -5.4600, -5.7238]\ \&\ \sigma = [7.4\times10^{-5}, 9.5\times10^{-5}, 9.4\times10^{-5}].$$

Numerical Results(8)

Table 8: Table showing the various lower bounds, upper bound and the Monte Carlo estimate for the TGD for varying values of r

r	LB0	LB1	LBt1	LBt2	UB1	MC
0.035	0.848032774815386	0.848424044789595	0.848490721686796	0.855969730838120	0.866104360048102	0.854167495146694
0.03	0.873577023530120	0.873813448730075	0.873845296962823	0.879110918002518	0.887240130128182	0.878026709161428
0.025	0.897102805167311	0.897242672828637	0.897255685548042	0.900881660116024	0.907283088296566	0.900486935407607
0.02	0.918896959516680	0.918977921696450	0.918981602796468	0.921421185492669	0.926366403382851	0.921030195923945
0.015	0.939240965473512	0.939286791778834	0.939286791778834	0.940888331577441	0.944633306794068	0.941092453291025
0.01	0.958403723325991	0.958429070673721	0.958429070673721	0.959452704642603	0.962230654369936	0.959485386731500
0.005	0.976635430514097	0.976649121750369	0.976649121750369	0.977286229664468	0.979302971604630	0.977322136744823
0	0.994162849651329	0.994170066410978	0.994170066410978	0.994555652671267	0.995987334249625	0.994698510160850

Table 9: Table showing the various lower bounds, upper bound and the Monte Carlo estimate for the TGD for varying values of q0 when r=0.0

q0	LB0	LB1	LBt1	LBt2	UB1	MC
0.008	0.999766066714250	0.999766066846378	0.999766071151593	0.999772840361840	0.999779562416927	0.999793281501976
0.0088	0.994162849651329	0.994170066410978	0.994170066410978	0.994555652671267	0.995987334249625	0.994686720834666
0.009	0.989104987070782	0.989146149900171	0.989146149900171	0.989952105692831	0.993383346707654	0.990012775482666
0.01	0.876692543049394	0.888049181229988	0.888049181229988	0.896376305638172	0.958189590378894	0.891609413787780
0.011	0.410971060715423	0.596089667856852	0.596089667856850	0.596089667856850	0.837207974723077	0.568675584083477
0.012	0.000000000000000	0.271045973759684	0.271045973759678	0.271045973759680	0.613838720959082	0.207081909248152
0.013	0.000000000000000	0.082740708460284	0.082740708460275	0.082740708460278	0.381822437530697	0.045779872978350
0.014	0.000000000000000	0.012702023135424	0.012702023135415	0.012702023135418	0.212229375394606	0.006694089213835
0.015	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.110420349200491	0.000883157235603
0.016	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.055539272590864	0.000084710725625
0.017	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.027576845294053	0.000004497045497
0.018	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000	0.013697961782757	0.000000019842250

Numerical Results(9)

Figure 4: Rel. Diff. of Lower Bounds and UB1 w.r.t. MC estimate under Transformed Gamma Distribution

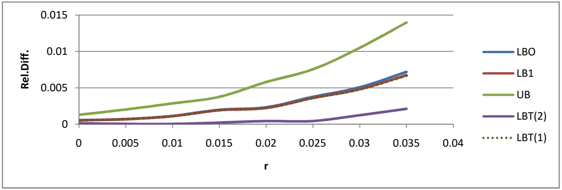
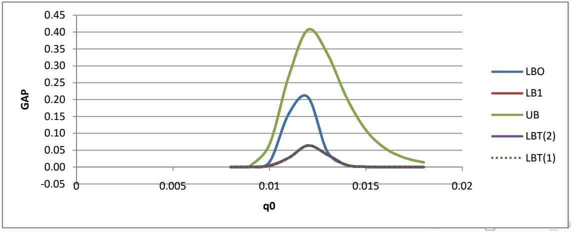
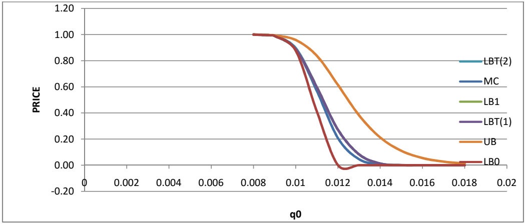


Figure 5: Comparison of different bounds under Transformed Gamma distn in terms of difference from MC estimate for r=0



Numerical Results(10)

Figure 6: Price Bounds under Transformed Gamma Distn. for the parameter choice of Lin and Cox(2008) Model



Conclusions

- Swiss Re thrives from Life Insurance Business
- It achieved Mortality Risk Transfer
- Main purpose of Swiss Re:- Protection against extreme mortality events
- Profitability negatively correlated to mortality rates
- Needed counter parties to offload mortality risk
- No dependence on retrocessionaire
- Methodology: Catastrophic bond with loss measurement based on a parametric index
- Investors in the bond took opposite position
- Received an enhanced return if an extreme mortality event doesn't occur

What Lies Ahead...?

- Mortality risk transfer expected to become more of a concern for life insurers and reinsurers
- Under Solvency II access to fully collateralized ILS capacity beneficial on a capital efficiency basis
- More such transactions predicted in the future
- ILS investors pleased to see VITA VI
- A new extreme (or excess) mortality catastrophe bond deal
- Keen to access the diversification it can offer
- The fact that it is Swiss Re again welcomed
- The giant has transferred over $ 2.2 billion of mortality risk to the capital market
- A lot of variations being tried
- Swiss Re has experimented with
  - Longevity Trend Bond - KORTIS (2010)
  - Multiple Peril Bond - MYTHEN RE (2012)
- A more transparent and liquid Longevity and mortality market is emerging (since the formation of LLMA (2010))

Further Research

- This research is a crucial breakthrough in the pricing of catastrophic mortality bonds
- Model-independent bounds give freedom of choice for selecting mortality models
- Only one earlier publication by [Huang et al.(2014)Huang, Tsai, Yang, and Cheng] in direction of price bounds for the Swiss Re bond
- These authors propose gain-loss bonds that suffer from model risk
- The present scenario poised for further research
- Deriving even more tighter upper bound
- Using these bounds for the *Longevity Trend Bond* - KORTIS
- The success of our research hinges upon the trading of vanilla options written on the mortality index

"If there will be one day such a severe world-wide pandemic that one of the bonds I bought will be triggered, there will be more important things to look after than an investment portfolio."

— ANONYMOUS CATM INVESTOR

Thanks!

Bibliography I

H. Albrecher, P. A. Mayer, and W. Schoutens.
General Lower Bounds for Arithmetic Asian Option Prices.
Applied Mathematical Finance, 15(2):123–149, 2008.

D. Bauer.
Stochastic Mortality Modelling and Securitizaton of Mortality Risk.
ifa-Verlag, Ulm, Germany, 2008.

H. W. Cheng, C.-F. Tzeng, M.-H. Hsieh, and J. T. Tsai.
Pricing of Mortality-Linked Securities with Transformed Gamma Distribution.
Academia Economic Papers, 42(2):271–303, 2014.

Y.L. Huang, J.T. Tsai, S.S. Yang, and H.W. Cheng.
Price Bounds of Mortality-linked Security in Incomplete Insurance Market.
Insurance: Mathematics and Economics, 55:30–39, 2014.

Bibliography II

N.L. Johnson.
Systems of Frequency Curves Generated by Methods of Translation.
Biometrika, 36:149–176, 1949.

N.L. Johnson, S. Kotz, and N. Balakrishnan.
Continuous Univariate Distributions, Vol. 1.
Wiley, New York, 1994.

Y. Lin and S.H. Cox.
Securitization of Mortality Risks.
Insurance: Mathematics and Economics, 42:628–637, 2008.

J.T. Tsai and L.Y. Tzeng.
Pricing of Mortality-linked Contigent Claims: an Equilibrium Approach.
Astin Bulletin, 43(2):97–121, 2013.