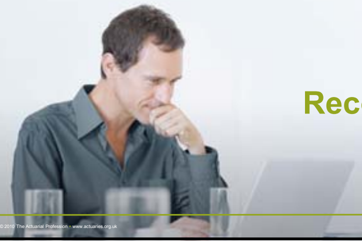
The Actuarial Profession
making financial sense of the future

Risk and investment conference 2010
Investing for success – balancing risk and reward



Reconciling Risk Measures

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Risk measure defined in terms of
- Time horizon
- Net assets definition
- Statistical measure
- Confidence level

Reconciling risk measures of interest to
- Management, for expressing risk appetite
- Modellers, for validating model output
- Analysts, for interpreting ERM claims

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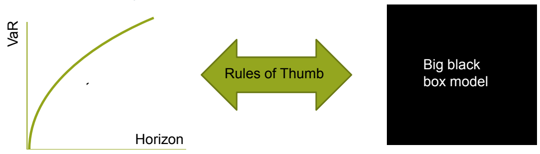
Rules of Thumb: Square Root of Time

Square Root of Time Rule
- Suppose VaR (value at risk) is €100 over one year.
  - Then 3-month VaR ≈ €50
  - And 2-year VaR ≈ €141

More complex conversions
- Capital required for runoff?
- Annual solvency tests?
- Tail Value at Risk?



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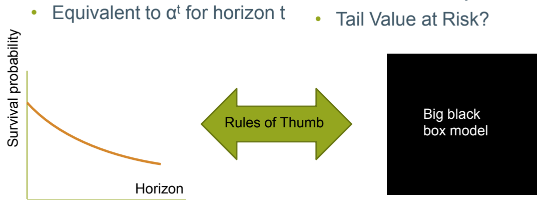
Rules of Thumb: Geometric Probability

Geometric Probability
- 1-year probability α of survival
- Equivalent to α^t for horizon t

More complex conversions
- Capital required for runoff?
- Annual solvency tests?
- Tail Value at Risk?



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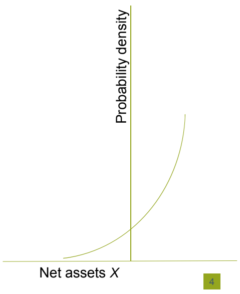
Comparing VaR to TVaR

At a given confidence level
- VaR (value at risk criterion)
  - eg Solvency 2: α = 99.5%
  - Prob{X>0} ≥ α
  - Quantile $q_{1-\alpha}(X) \ge 0$
  - 99.5% probability of success
- TVaR (tail value at risk)
  - eg Swiss Solvency Test: β= 99%
  - $\mathbf{E}(X|X \le q_{1-\beta}(X)) \ge 0$
  - Average of worst 1% still solvent



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VaR and TVaR confidence levels

Distribution	VaR success probability α	TVaR quantile β
Pareto *a* = 2	99.50%	98.00%
Pareto *a* = 4	99.50%	98.42%
Pareto *a* = 10	99.50%	98.57%
Exponential (Pareto limit for large *a*)	99.50%	98.64%
Normal	99.50%	98.70%
Swiss Solvency Test	#N/A	99.00%

Pareto distribution
$$F(x) = 1 - \left(\frac{b}{b+x}\right)^a$$
$$f(x) = \frac{ab^a}{(b+x)^{a+1}}$$

Conversion
$$1-\beta = \left(\frac{a}{1-a}\right)^a (1-\alpha)$$

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Back to Probability
Three Measures of Success

In this presentation α,β always refer to probabilities of success (ie 99.5%) and not probability of failure

Success Criterion	Symbol	Definition of Success
Value at Risk	$\alpha_{VaR}=\Phi(z_{VaR})$	Assets exceed technical provisions in one year.
Rip van Winkle	$\alpha_{RvW}=\Phi(z_{RvW})$	Assets, with investment returns, are sufficient to meet liabilities as they fall due.
Solvent Runoff	$\alpha_{SRO}=\Phi(z_{SRO})$	Assets, with investment returns, exceed technical provisions at all future dates.

Importance of Key Assumptions

In this presentation α,β always refer to probabilities of success (ie 99.5%) and not probability of failure

Success Criterion	Required assets depends on expected asset return?	Required assets depends on valuation discount rate?
Value at Risk	No	Yes
Rip van Winkle	Yes	No
Solvent Runoff	Yes	Yes

Value at Risk: The Lognormal Case

Lognormal assets against fixed liabilities

- Initial liabilities (=technical provisions) $Ce^{-\delta t}$
  - Represents a future cash flow C at time t discounted at δ
- Initial assets A_0
  - Annualised volatility σ = stdev of ln A_1
- Value-at-Risk for confidence level $\Phi(z)$ is $A_0\{1-e^{-\sigma z}\}$
  - Based on a downward shock of z standard deviations
  - Var capital adequacy: $A_0 \geq Ce^{-\delta t} + A_0\{1-e^{-\sigma z}\}$
  - Equivalent to $A_0 \geq Ce^{\sigma z-\delta t}$
  - Success probability $\Phi(z_{VaR}) = \Phi\left(\frac{\ln(A_0/C)+\delta t}{\sigma}\right)$

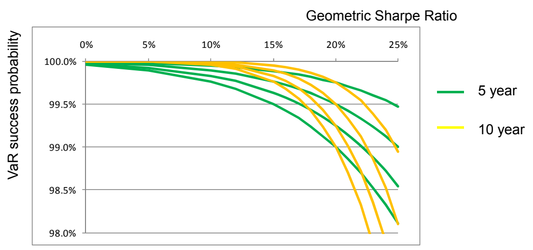
Success Probability: The Rip van Winkle Case

Geometric Brownian assets against fixed liabilities

- Initial liabilities $Ce^{-\delta t}$
  - Represents a future cash flow C at time t discounted at δ
- Now replace stress test with explicit projection
  - Assets at time t are $A_0\exp\{\mu t+\sigma B_t\}$; B_t = Brownian motion
  - Note explicit use of (geometric) expected return μ
- Probability of sufficiency

$$\alpha_{RvW} = \Phi(z_{RvW}) = \Phi\left(\frac{\ln(A_0/c)+\mu t}{\sigma\sqrt{t}}\right)$$

Comparing Value at Risk to Rip van Winkle



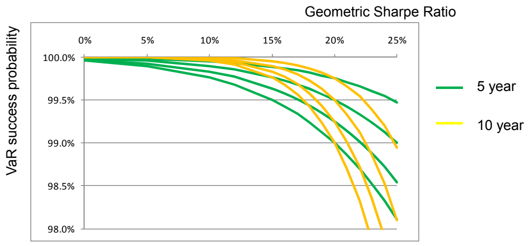
Curves are contours of equal Rip van Winkle probability, letting required assets vary.
More optimism in the return assumption has the effect of using lower 1-year probability.
The effect is more marked for longer cash flow horizons

Measuring the Strength of a Basis

We define the Sharpe Ratio as the expected return on risky assets, measured in excess of the valuation discount rate, then divided by the asset volatility.

Sharpe Ratio	Vary expected return. Fix discount rate.	Vary discount rate. Fix expected return
Low Sharpe ratio	Prudent basis; Low expected returns.	Optimistic basis: High discount rate
High Sharpe ratio	Optimistic basis; High expected returns	Prudent basis: Low discount rate

Effect of Weakening the Valuation Basis



Fixing the expected return, high Sharpe ratios mean more prudent valuation bases.
Required assets is a total of technical provisions and required capital
Prudence in the technical provisions can substitute prudence in VaR.

Here's the Maths:

$$\alpha_{VaR} = \Phi(z_{VaR}) = \Phi\left(\frac{\ln(A_0/c) + \delta t}{\sigma}\right)$$

$$\alpha_{RvW} = \Phi(z_{RvW}) = \Phi\left(\frac{\ln(A_0/c) + \mu t}{\sigma\sqrt{t}}\right)$$

Sharpe ratio

$$z_{RvW} = \frac{z_{VaR}}{\sqrt{t}} + \frac{\mu - \delta}{\sigma}\sqrt{t}$$

Square root of time rule
For small *t*

Solvent Runoff Probability

Solvent Runoff allows for Intermediate Solvency Tests

- Probability formula:

$$\Phi(z_{SRO}) = \begin{cases} \Phi\left(\frac{\ln(A_0/c)+\mu t}{\sigma\sqrt{t}}\right) - \exp\left(\frac{2(\delta-\mu)[\delta t + \ln(A_0/c)]}{\sigma^2}\right)\Phi\left(\frac{-\ln(A_0/c)+\mu t - 2\delta t}{\sigma\sqrt{t}}\right) & A_0 \geq Ce^{-\delta t} \\ 0 & A_0 \leq Ce^{-\delta t} \end{cases}$$

- The first term is the Rip van Winkle probability
- Second term is a deduction for solvency failure before maturity
- Example calculations (5 year term, 20% Sharpe ratio)

α_{VaR}	α_{RvW}	α_{SRO}
99.50%	94.51%	85.93%

Relating Solvent Runoff to Rip van Winkle: The Formulas

We can express Solvent Runoff probability in terms of Rip van Winkle probabilities and Sharpe Ratios

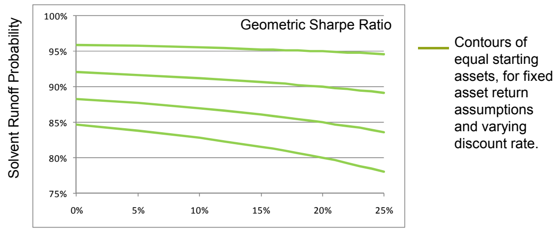
- Probability formula:

$$\Phi(z_{SRO}) = \begin{cases} \Phi(z_{RvW}) - \exp\left(-2\lambda\sqrt{t}\left(z_{RvW} - \lambda\sqrt{t}\right)\right)\Phi\left(-z_{RvW} + 2\lambda\sqrt{t}\right) & z_{RvW} \geq \lambda\sqrt{t} \\ 0 & z_{RvW} \leq \lambda\sqrt{t} \end{cases}$$

$$\lambda = \frac{\mu - \delta}{\sigma}$$

Trade-off between Basis Strength and Solvent Runoff Survival Probability

Solvent Runoff Probabilities still depend on the choice of basis, but not as strongly as is the case for one-year Value at Risk



What about Jumps?

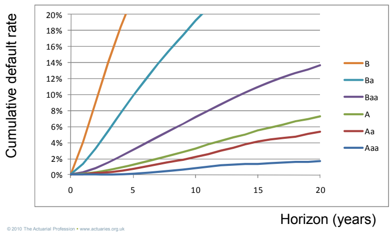
- Brownian motion models assume continuous paths
- Many insurance and market risks produce jumps, fat tails and other aspects of non-normality
- At the extreme of many small jumps, over moderate time horizons the central limit theorem kicks in
- At the other extreme, ruinous catastrophe losses occur as a Poisson process and the geometric probability rule applies:
  - 1-year probability α of survival
  - Equivalent to α^t for horizon t

Variation by Term: Moody's Default Study (2010)

EXHIBIT 33 Average Cumulative Issuer-Weighted Global Default Rates, 1920-2009

RATING	0	1	2	3	4	5	6	7	8	9	10
Aaa	0.000%	0.000%	0.008%	0.026%	0.081%	0.162%	0.248%	0.354%	0.507%	0.666%	0.850%
Aa	0.000%	0.068%	0.202%	0.309%	0.468%	0.721%	1.015%	1.321%	1.618%	1.901%	2.222%
A	0.000%	0.092%	0.281%	0.574%	0.909%	1.257%	1.627%	2.026%	2.425%	2.858%	3.303%
Baa	0.000%	0.285%	0.843%	1.549%	2.317%	3.136%	3.953%	4.735%	5.530%	6.370%	7.213%
Ba	0.000%	1.356%	3.287%	5.466%	7.741%	9.901%	11.939%	13.815%	15.668%	17.416%	19.223%
B	0.000%	4.029%	9.046%	14.049%	18.502%	22.420%	25.918%	29.143%	31.870%	34.273%	36.372%
Caa-C	0.000%	14.276%	24.028%	31.365%	36.888%	41.183%	44.346%	46.672%	48.804%	50.863%	52.795%

RATING	11	12	13	14	15	16	17	18	19	20
Aaa	1.014%	1.143%	1.282%	1.326%	1.364%	1.438%	1.515%	1.575%	1.653%	1.703%
Aa	2.592%	2.996%	3.402%	3.814%	4.131%	4.367%	4.564%	4.799%	5.110%	5.383%
A	3.779%	4.245%	4.661%	5.056%	5.509%	5.925%	6.282%	6.640%	6.973%	7.297%
Baa	8.004%	8.801%	9.585%	10.301%	10.933%	11.591%	12.177%	12.683%	13.159%	13.684%
Ba	20.846%	22.426%	23.987%	25.373%	26.645%	27.817%	29.008%	30.131%	31.127%	32.095%
B	38.317%	40.169%	41.828%	43.356%	44.745%	46.090%	47.291%	48.284%	48.992%	49.451%
Caa-C	54.736%	56.666%	58.494%	60.413%	62.361%	64.214%	65.844%	67.305%	68.674%	70.022%



Multiple Cash Flows:
Effective time horizon is shorter than you think

	Discount at 5% pa		Discount at 6% pa	
	Exact	Normal	Exact	Normal
Annuity liability	1 pa, decreasing at 10% pa		1 pa, decreasing at 10% pa	
Expenses, taxes, dividends, new business	none		none	
Liability value	6.67		6.25	
Initial assets	7.29		7.29	
Volatility σ	5%		5%	
α_{VaR}	96.4%		99.9%	
Geometric mean return	6%		6%	
α_{RvW}	95.3%		95.3%	
Implied horizon *t*		1.6		3.4
α_{SRO}	80.74%	89.39%	90.81%	90.59%

Conclusions

Stated survival probabilities have to be seen in the context of the underlying assumption strength

- Expected asset return
- Liability discount rate

Simple but approximate rules exist for converting between different sets of assumptions

- Effective time horizon and Sharpe ratios are key inputs

These can help management to understand directional relationships between formulations of risk appetite

- Not a substitute for heavy models (which are also wrong!)