<u>REINSURANCE AND RETENTIONS</u>

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Section 7

Appendix 1

EXAMPLE APPLICATIONS OF STRAUB'S METHOD

A1.1 Introduction

Straub's method has been applied to the aviation, liability and property examples mentioned in the Introduction to Section 4. For simplicity we shall only consider the use of either Quota Share or Risk Excess reinsurance. These may not be the most appropriate forms of reinsurance for the class of business in the examples, but they serve to illustrate the use of Straub's method. In each example a discrete distribution was used for claim amounts (Exhibit 1) and a Poisson distribution for claim numbers.

The results are shown in Exhibit 2 pages 1-12. The graphs demonstrate the effect on the retention level of varying the capital at risk and the desired probability of exhausting that capital over an infinite period. The tables show the numeric results of using Straub's method. The graphs are not directly comparable with those of the other methods, which consider finite future time periods.

A summary of the results for a 60% solvency margin (that is, capital at risk of 60% of gross premiums) and probabilities at a one in one thousand level are shown in Table 2 below:-

Table 2 - Results of Straub's Method (Amounts in £000s)

	Aggregate Claims Coefficient of Variation	Quota Share Retention	Risk Excess Retention	Capital* at Risk for no R/I
Aviation	0.79	3%	405	1500%
Property	0.23	46%	75	130%
Liability	0.17	87%	1,875	68%

* Expressed as a percentage of premium.

The following general observations can be made from the results:-

1. The relationship between capital at risk and retention level is linear for a Quota Share, whereas it depends on the claim amount distribution for Risk Excess reinsurance. This is a direct result of the structure of Straub's formula.

2. The Quota Share graphs can be used to determine the point at which no reinsurance is required - that is, the level of capital at the point where the Quota Share retention is 100%. For a probability of one in one thousand this point is shown in the final column of Table 2.

3. For a given probability, the retention increases as the available capital at risk increases.

4. For a given capital at risk, the retention increases for companies which are less risk averse (that is, as the probability increases).

5. The rate of change of retention with respect to capital at risk is lower for a lower probability. In other words, the more risk averse a company is, the less will be the effect on its retention policy of an increase in available capital at risk (due to capital injections etc.)

6. The coefficient of variation (CV) of the aggregate claim amount distribution summarises the variability of this distribution. The above table indicates that the higher the CV, the greater the need for reinsurance and, hence, the lower the retention.

Some brief comments on each example based upon the stated capital and probability assumptions, are as follows:-

A1.2 Aviation Example (Exhibit 2 Pages 1 to 4)

1. There is a very high coefficient of variation, leading to very low retentions.

2. Annual expected gross claims are about £74 million.

3. Across a range of practical levels of capital at risk, the retention level changes very little and is very low.

4. These results indicate the highly volatile nature of this business. In practice, the use of coinsurance or pooled arrangements helps to spread the risk across the market.

A1.3 Liability Example (Exhibit 2 Pages 5 to 8)

1. In this example, annual expected claims are about £10m, with approximately 260 claims per annum.

2. Risk Excess reinsurance is likely to be used here (in conjunction with other forms of reinsurance).

3. The method suggests a retention of about £75,000 which seems reasonable.

4. Such a retention would lead to the reinsurer being involved in 10% of claims.

5. As the capital at risk approaches 100% of premium then there is a rapid increase in the retention and a reduced need for reinsurance.

A1.4 Property Example (Exhibit 2 Pages 9 to 12)

1. This example has the lowest coefficient of variation of the three examples and hence we might expect the retention to be higher. The graphs demonstrate that reinsurance is not needed when the capital at risk is greater than the 70% of premium.

2. The retention is quite high at 87% for a Quota Share and £1.9 million for a Risk Excess (above which there might only be three out of 13,000 claims!).

3. 87% could be considered as an average retention for a Surplus treaty, which is the commonly used form of reinsurance for this class. It is doubtful whether, in practice, an insurer would have a Surplus treaty which ceded such a small percentage of the business.

4. In practice, Catastrophe Excess of Loss would also be used to cover against events such as windstorm.

Appendix 2

EXAMPLE APPLICATIONS OF HECKMAN AND MEYERS' METHOD

A2.1 Introduction

This appendix demonstrates the use of the approach as published by Heckman and Meyers (Reference 11). The Heckman and Meyers (H & M) method was applied to the same three data sets, namely, aviation, liability and property.

A2.2 Outline of Approach

The core of the approach is to use the H & M method to produce an aggregate claim distribution for given input frequency and severity distributions. In order to use this to provide information on varying retention levels, the algorithm must be used a number of times allowing for varying retention and reinsurance costs. The objective is to calculate the capital at risk for a given retention level and probability level. Capital at risk for a given probability level may be defined as follows:-

Capital at Risk	=	Net aggregate claims at given probability level, less net premium received.

Where:

Net Premium	=	Gross premium received less expenses less cost of reinsurance

The cost of reinsurance will depend on the retention level and market conditions. In this section we (unrealistically) assumed the cost of reinsurance is related to the risk premium with a constant percentage loading, regardless of the retention level. We have also assumed that the expenses are split in proportion to the risk premium independently of the retention level. This may also be unrealistic. In practice, one would aim to use realistic figures based on the current state of the reinsurance market. For all the examples in this paper we have:-

Table 3 - Cost of Reinsurance

		Percentage
Gross Premium	:	100
Risk Premium	:	70
Expenses	:	20
Profit Loading	:	10

For readers more familiar with the λ of Risk Theory, the above represents a λ equal to 1/7, (that is, approximately 14%).

For a particular retention, the first step is to calculate the reinsurance risk premium. The cost of reinsurance is then calculated as that risk premium loaded for profit and expenses. For example, say the net risk premium is 50% of the gross premium, we then have:-

Table 4 - Calculation of Net Risk Premium

	Percentage
Gross Premium	100
Less total expenses	20
Less reinsurance (net of expenses)	57
Net Premium	23

The next step is to adjust the gross claim severity distribution for the effect of the reinsurance retention. The frequency distribution does not require adjustment. The H & M algorithm is then run to produce a table of net aggregate claims at various probability levels. The amount of aggregate claims at the desired probability level is then read off and the net premium subtracted to give the capital at risk for that retention and probability.

The exercise is repeated a number of times to build up a picture of the capital at risk for varying retention levels. These may be represented graphically and interpreted to select an appropriate retention level. Exhibit 3 Page 1 shows an example graph.

For a given retention level, the capital at risk of the various probability levels may be determined from the graph. Alternately, for a given capital at risk the retention consistent with various probability levels may be read from the graph.

For a company as a whole, there are often many lines of business with differing retention levels. The H & M method is specified in their paper to handle multiple lines and so the corresponding capital at risk for an entire company can be easily derived for a given set of retention levels.

This general approach can also be used for other methods of calculating aggregate claims, for example analytical or recursive methods.

A2.3 Assumptions Made in Calculating Aggregate Claims

Claim count distribution : Poisson

This implicitly assumes that the variance of the number of claims is equal to the expected number of claims. A larger variance could have been assumed by use of the negative binomial distribution (that is by using a positive contagion parameter in the H & M algorithm)

Similarly, a smaller variance could have been assumed by use of the binomial distribution (negative contagion parameter).

Claim Severity distribution: Piecewise linear.

The distribution used is based on past claims experience. Past claims were sorted into ascending order and assumed to be equally spaced on the probability scale. The cumulative probability was then calculated and various claim sizes selected to represent the severity distribution. In the case of the liability claims, a log-normal distribution was fitted to the large claims and the actual largest two or three claims were replaced by their fitted values.

Parameter Uncertainty: None

The variation was assumed to come only from that implicit in the claim count and severity distributions. Additional variation could have been incorporated, for example to allow for uncertain future inflation by using a non-zero mixing parameter in the H & M algorithm.

A2.4 Aviation Example (Exhibit 3 Pages 2 to 7)

The frequency and severity distributions used are summarised in the table below. All figures in the example are in thousands. The underlying claim severity distribution is shown in Exhibit 1 Page 1.

Table 5 - Aviation Example Frequency and Severity Distributions

Severity Mean	=	9175
Claim Frequency Distribution	=	Poisson
Mean Claims Per Year	=	8.000

Multiplying the means of the severity and claim count distributions gives expected aggregate claims of £73,398,000. Loading for expenses and profit produces a gross risk premium of £104,854,000. The gross data is initially used unadjusted as input into the H & M algorithm. The output produced from the calculation is contained in Exhibit 3 Page 2.

The column headed 'Entry Ratio' in the table refers to the ratio of claims on the aggregate distribution to the aggregate mean. The column headed 'Excess Pure Premium' refers to the stop loss risk premium. Some diagnostics from the numerical integration process are also included is the output.

From the columns of aggregate claim amounts and probabilities, the aggregate claims at 90%, 99% and 99.9% may be determined by interpolation.

Having calculated aggregate claims from the gross claims, the next step is to adjust the claim severity distribution for a retention level. The mean of the truncated distribution is easily calculated as the distribution remains piecewise linear; this is multiplied by the expected number of claims to obtain the net risk premium. The reinsurance risk premium is calculated as the difference between the gross and net risk premiums. This leads to figures for the capital at risk for the retention level under consideration. Repeating the process for a number of retention levels builds up the complete picture. Exhibit 3 Page 3 below summarises the results for this class of business. These results are plotted in the graphs in Exhibit 3 Page 4 to 7.

Checks for reasonableness

Beard, Pentikäinen and Pesonen (Reference 3) give a formula for a distribution free upper limit for the capital at risk (based on the normal power approximation):

$$U \leq y\sqrt{PM} - \lambda P + \frac{1}{6}\left(y^2 - 1 \right) M \qquad (1)$$

Where	U =	capital at risk
	P =	Net Risk Premium
	λ =	Profit loading
	M =	Retention
and	y =	normal variate for a given probability level

A further quick check on the level of aggregate claims may be constructed by assuming that all the claims are equal in size to the retention, and applying a poisson distribution to claim numbers. This gives:

$$\mathit{Aggregate\ Losses} \leq Mw \qquad (2)$$

w is the point where $\sum_{r=0}^{w} e^{-n} \frac{n^r}{r\,!}$ first exceeds the desired probability level. This check is only really helpful at small retention levels. Applying these checks to the results for a probability level of 99%, we have:

Table 6 - Reasonableness Checks on H & M Aviation Results

Retention (£000s)	**100**	**1,000**	**10,000**
H & M Capital at Risk	604	5,210	35,431
Compared with (1) above	608	5,585	41,696
H & M Aggregate claims	1,481	12,102	66,253
Compared with (2) above	1,500	15,000	150,000

This confirms the reasonableness of the results for the 99% probability level.

Interpretation of Results - Aviation

The results as presented show that very large amounts of capital would be needed if aviation were insured on a simple risk excess basis unless the retention were very small. Whilst this may be the case for consideration of the self insured deductible for a fleet operator, the actual aviation LMX market is based around some very complicated programmes involving numerous layers, co-insurance, aggregate deductibles, use of top and drops and so on. However, with some additional work, most of these features can be modelled by repeated application of the H & M method, and hence, the effectiveness of particular reinsurance programmes may be assessed.

A2.5 Liability Example (Exhibit 3 Pages 8 to 13)

Tables and graphs of results similar to the aviation example are set out in the exhibits as follows:-

Underlying claim severity distribution - Exhibit 1 Page 2

H&M aggregate claim distribution - Exhibit 3 Page 8

H&M results table - Exhibit 3 Page 9

Graphs of aggregate claim distribution vs retention - Exhibit 3 page 10

Graphs of capital at risk vs retention - Exhibit 3 Page 11

Graphs of capital at risk vs retention as a percentage of gross written premium - Exhibit 3 Page 12

Graphs of capital at risk vs retention as a percentage of net written premium - Exhibit 3 Page 13

Interpretation of Results - Liability

The tables and graphs indicate that relatively high retentions are possible without putting unreasonable amounts of capital at risk. This arises as a consequence of the high profit loading applied to the risk premium coupled with the assumption that there is no parameter uncertainty. It is interesting to note that the capital at risk at the 90% level becomes negative for a retention of 50,000. This means that at that retention and assumed cost of reinsurance, the premium loading is such that a profit can be expected for 9 out of 10 years.

A2.6 <u>Property Example</u> (Exhibit 3 Pages 14 to 19)

Tables and graphs of results similar to the aviation and liability examples are set out in the exhibits as follow:-

Underlying claim severity distribution - Exhibit 1 Page 3

H&M aggregate claim distribution - Exhibit 3 Page 14

H&M results table - Exhibit 3 Page 15

Graphs of aggregate claim distribution vs retention - Exhibit 3 Page 16

Graphs of capital at risk vs retention - Exhibit 3 Page 17

Graphs of capital at risk vs retention as a percentage of gross written premium - Exhibit 3 Page 18

Graphs of capital at risk vs retention as a percentage of net written premium - Exhibit 3 Page 19

Interpretation of Results - Property

As was the case for the liability example, the tables and graphs indicate that relatively high retentions are possible without putting unreasonable amounts of capital at risk. As before, this arises as a consequence of the high profit loading applied to the risk premium coupled with the assumption that there is no parameter uncertainty. The unrealistic loadings applied to the reinsurance risk premiums also reduce the calculated figures for capital at risk.

In this example the capital at risk at the 90% level remains negative for all retentions shown in the results table, although the gross capital at risk is positive. This means that the premium loading is such that a profit can be expected for 9 out of 10 years for any retention of at least up to £1 million. At the 99.9% probability level, the results show positive capital at risk for retentions above £100,000. In a case like this, solvency aspects may not be as important in the analysis as the maximisation of expected profit subject to the cost and availability of reinsurance.

Appendix 3

AN EXAMPLE APPLICATION OF SIMULATION

A3.1 Introduction

This particular example is of a large insurer writing UK personal and commercial lines. The gross retention is acceptable to the company except for the aggregation exposure to weather events such as flood, windstorm and freeze. We shall consider the effect of weather catastrophes on the company. For this purpose, a catastrophe will be defined as any event giving rise to an insured claim in excess of £100 million to the market at 1990 values.

The results of the simulations lead us to the following conclusions for a hypothetical insurance company with a 10% share of the UK property market.

1. The company could reduce the variability of retained claims at no additional cost by purchasing higher layers of excess of loss reinsurance and retaining a greater coinsured share.

2. The company could raise the lower limit of the reinsurance programme. The outwards reinsurance premiums recouped from this could be used to purchase higher layers of reinsurance and reduce the variability of the claim retention.

3. The company could investigate other forms of reinsurance that will achieve the same level of variability at a reduced cost. One such reinsurance could be an annual aggregate stop loss on claims arising from catastrophe events.

4. The company's annual catastrophe excess of loss reinsurance premium is £22 million. The simulations indicate that the expected claim ratio to the reinsurer in the long term is 40%-60%. On this basis the annual long term cost to the company of smoothing their retentions using excess of loss reinsurance is £8.8 - £13.2 million.

5. If the company management are able to advise on their desired variability then the optimum reinsurance programme can be investigated.

A3.2 Methodology

The simulation divides into four parts:-

1. Determination of the model for the gross market claims distribution.

2. Estimation of the parameters for the gross market claims model.

3. Calculation of the effect of individual events on the company concerned.

4. Analysis of the retention strategy required to achieve the target net claims distribution.

A3.3 Model Identification and Parameter Estimation

It is possible to argue that a catastrophe occurrence is a Poisson process. In other words it satisfies:-

1. The probability of an event occurring in a time period t_1 to t_2 is proportional to $(t_2 - t_1)$.

2. The probability of two or more events occurring at the same time or an infinite number of events in a finite period is zero.

3. The events in two disjoint time periods are independent.

If this is so, then the number of occurrences in a year has a Poisson distribution. Notice that for condition 2 to hold a catastrophe must be defined as all claims arising from one event. Counting two aeroplanes that crashed into each other as two events breaks condition 2. Further, the cyclical nature of weather conditions also undermines condition 1.

We commenced by examining the data concerning past losses above £40 million original cost in order to estimate parameters for the frequency and severity distributions. This is shown in Exhibit 4 Page 1. During the 11.5 years of experience there have been 12 claims in excess of £100 million at current costs or approximately one per year.

We decided to use a Pareto distribution to simulate the severity scaling all claims by £100 million. Thus a simulated value of 1.5 would correspond to a market claim of £150 million. The maximum likelihood estimator of the Pareto parameter based upon experience is 0.84. This gives a very skew distribution which has no mean.

This is probably a result of the fact that the sample of twelve claims includes two very large catastrophes which we expect to occur with much lower frequency than once every six years (unless weather patterns have changed significantly, which should be of more immediate concern to those responsible for gross pricing as well as those responsible for reinsurance pricing!). An adjustment to the severity distribution is required to reflect the finite amount of insured property that is at risk. We chose £10 billion as an upper limit to the severity distribution.

Table 7 shows what we consider to be a reasonable range of parameters to use in the simulations.

Table 7 - Simulation Parameters

Frequency	Severity
0.75	1.25
1.00	1.33
1.25	1.50

The combination of three frequency and three severity parameters gives nine possible distributions for the gross catastrophes. The three severity parameters 1.25, 1.33 and 1.5 indicate events such as the 1987 and 1990 storms as being one in thirty, forty or fifty occurrences respectively. That is one every so many events not years. The frequency of these measured in years will depend upon the number of events assumed per year. A low severity parameter has a high probability of yielding very large claims.

The actual simulation can be performed using the U(0,1) random variable function of the spreadsheet package. The practitioner should consider the randomness of the generator. Simple algorithms for the generation of the U(0,1) can be set up if required.

A3.4 The Company's Claims Distributions and Retention Policy

The estimation of a company's gross claim from that of the market has been assumed to follow a linear relationship with market share measured by premium volume. We believe that this is a reasonable approach due to the very high number of relatively homogeneous small units which compose the exposure of a large company. This assumption may not hold for smaller companies who could have very regionalised exposure. More complex methods can be used. A good example is the method described in Section 4.4 and used by some US insurers to estimate hurricane losses. Exhibit 4 Page 2 shows the mean and standard deviation of the aggregate gross annual cost of claims under the simulation for the company in our example on each of the nine bases.

For each set of parameters, a simulation of perhaps five thousand years' of claims should be performed. The higher the number of simulations, the greater the amount of information available concerning the extremes of the aggregate claims distribution. On the other hand, should events that occur once in ten thousand years have a material influence on the management of the operation?

The next stage is to set up a parameterised programme which calculates the net financial impact to the company for each year of simulated claims. The parameters determining the precise details of the reinsurance programme are required. The premiums paid plus reinstatements payable should be included in the costs of the reinsurance. For some purposes it may be best to use current market premium rates, for others an estimate of the mean long term rate chargeable may be better.

The mean of the resulting net claims distribution can be subtracted from that of the gross distribution to indicate the mean claims recovery. This in turn can be compared to the mean cost of the reinsurance including reinstatement premiums. This should demonstrate the cost of reinsurance to the company over the long term.

The aim of the reinsurance however is to reduce the variability of the retained claims distribution. One problem is to determine how to measure this variability. The standard deviation, 95% confidence limit or 99% confidence limit could be used. Again, a benefit of simulation is that any moment of the distribution can be estimated. The advantage of measures such as the standard deviation is that they look at the shape of the whole distribution. Two identical companies with the same capital and probability of losing that capital could have entirely different claims variability due to different reinsurance. As a result, they will experience very different profits. This demonstrates one problem of the probability of loss concepts: they look at only one point in the claims distribution.

It is worth investigating the effect that the truncation of the claim severity has on the measure of variability selected. Table 8 shows the results for a simulation of 5,000 years with a Poisson parameter of 1.25 and a Pareto parameter of 1.25.

Table 8 - Gross Market Catastrophe Claims

	No Truncation	£10 Billion	£5 Billion
Average Annual Cost	549	448	433
SD of Average Annual Cost	2,771	920	753

Clearly, if conclusions are being drawn on the basis of the value of standard deviations it is important to investigate whether the conclusions are the same whatever the truncation point.

We are now ready commence investigation of the retention of the company. As we have touched on earlier, the retention philosophy must come from a consideration of the objectives of the company and may well incorporate shareholder utility curves. These discussions are outside the scope of this section. Here, we shall demonstrate some of the ways in which we can use this work to improve retention decisions.

Our starting point is to assume that the company in question has a catastrophe reinsurance programme covering claims arising from one event for £170 million excess of £30 million. The cover has been 95% placed at an initial cost of £22 million and has unlimited reinstatements paid 100% for time irrespective of the unelapsed exposure and pro-rata to the size of the recovery.

Exhibit 4 Page 3 shows the mean gross and net claims costs for this company for each combination of simulation parameters. The standard deviations are also shown. As expected the reinsurance programme results in a lower coefficient of variation for the net claims distribution than for the gross. Even under the most severe claim assumptions the expected reinsurance recovery net of reinstatements is £13 million against the original premium of £22 million. Can the reinsurance programme be improved without increasing the cost? We can investigate what happens when the height of the layers purchased is changed, both above £30 million and above £200 million. The cost is kept the same by increasing the amount of coinsurance, after all, who said "Placing 100% of the layer is the most efficient thing to do."!?

The graphs in Exhibit 4 Pages 4 to 6 show that with a fixed lower limit the standard deviation of the net claims reduces as the upper limit is raised! Further, raising the lower limit also reduces the standard deviation as is shown in Exhibit 4 Page 7. Perhaps the result of this is that companies should be encouraged to take higher layers of cover with more coinsurance? This will provide a reduction in the standard deviation of the retained claims at no additional cost.

We have concentrated, thus far, on one type of reinsurance. The variability that we are trying to control is the standard deviation of the retained catastrophe claims in one year. So why are we considering a reinsurance programme focusing on each event? What about an aggregate stop loss contract that caps the aggregate claims from all catastrophe events in the year? In order to perform a full analysis of this, the company would have to obtain quotes for this insurance.

The simulation allows us to investigate the levels of variability that would result from such contracts. These variabilities are shown in Exhibit 4 Page 8 for a stop loss of £100 million xs £50 million. The results look very promising. This is not wholly surprising since this reinsurance protects against frequency as well as severity of catastrophe.

We have not really discussed which of the nine sets of parameters we consider to be the most appropriate. The main reason for this is that our conclusions have been non-parametric. The results have held for all nine combinations. Exhibit 4 Pages 9 and 10 shows a hundred year simulation of catastrophes under each of these nine combinations. We hope that you will agree, based on your experience of UK weather claims, that they cover a reasonable range from the optimistic to the pessimistic.

Finally, a word of caution: we have used the standard deviation as a measure of variability. Exhibit 4 Page 11 compares the actual 95% and 99% confidence limits for the simulated net claims with the same limits estimated using the normal approximation. There are very considerable differences which demonstrate the skewness of these distributions and the care required when interpreting simulation results.

On the same note, examination of simulation results in Exhibit 4 Page 2 shows that the most severe set of claim assumptions, Pareto 1.25 and Poisson 1.25, do not have the highest standard deviation. The Pareto 1.33 and Poisson 1.25 standard deviation is higher. This could either be a genuine result, a random variation in the simulation or an effect of capping the claim severity distribution. If the same sample of U(0,1) variables are used for both sets of simulations then the Pareto 1.25 and Poisson 1.25 has the highest standard deviation. This is shown in Table 9 below:-

Table 9 - Comparison of Simulations (£ millions)

Simulation Parameters	Simulation Mean	Simulation Standard Deviation
* Pareto 1.33 Poisson 1.25	421	940
* Pareto 1.25 Poisson 1.25	448	920
+ Pareto 1.25 Poisson 1.25	469	1,064

* As shown in Exhibit 4 Page 2.

\+ Calculated using the U(0,1) variables from the simulation of Pareto 1.33 and Poisson 1.25 in Exhibit 4 Page 2.

It would appear that the results arose from random variations in the simulation.

APPENDIX 4

EXAMPLE APPLICATIONS OF THE RECURSIVE METHOD

A4.1 Introduction

We have applied the recursive method to the aviation and liability data sets in order to estimate the aggregate claims distributions. The property data set is so large that we would not recommend the use of the recursive method. There are two reasons for this: first, the normal approximation should be reasonably robust when used with such a high number of claims; second, if the number of claims assumed for the future is very high then the computation of the aggregate claims distribution using the recursive formula becomes arduous.

A4.2 Methodology

The data sets are rescaled. The rescaled data points are then rounded to the nearest integer. This results in an approximation for the severity distribution. Essentially, the continuous severity distribution is substituted by a mass function on the first few dozen integers. We input the empirical severity distributions as implied by the data. An alternative approach would be to fit one of the classical distributions to the data before scaling and grouping the severities for use in the recursive formula.

The choice of scaling factor represents a trade-off. If the scaling factor chosen is too small, then the number of mass points for the proxy distribution is large, and the application of the recursive formula becomes more difficult. However, if the scaling factor is too large the recursive formula may be more easily applied, but the proxy distribution may not reflect all the characteristics of the parent distribution from which it is derived.

Fortunately, this process is quite robust in that the accuracy gained at having three hundred mass points rather than forty, say, is outweighed by the added computational complexity when applying the recursive formula. The scaled data sets are shown in Exhibit 5 Pages 1 and 2.

We assumed a Poisson distribution for claim frequency taking the number of claims as assumed in Appendices 2 and 3 as the estimate of the mean of the distribution.

A4.3 Aviation Example

Exhibit 5 Page 3 shows graphs of various classical points on the aggregate claims distribution against the per risk claim retention. These graphs are directly comparable to those produced by the H & M method as shown in Exhibit 3 Page 4.

A4.4 Liability Example

Exhibit 5 Page 4 shows graphs of various classical points on the aggregate claims distribution against the per risk claim retention. These graphs are directly comparable to those produced by the H & M method as shown in Exhibit 3 Page 10.

A4.5 Property Example

For the reasons outlined above, we used the normal approximation on this data set. Exhibit 5 Page 5 shows graphs of various classical points on the aggregate claims distribution against the per risk claim retention. These graphs can be compared to those produced by the H & M method as shown in Exhibit 3 Page 16 in order to assess the reasonableness of normal approximation.

Section 8

EXHIBITS

Exhibit 3 - Exhibits for Appendix 2

Exhibit 4 - Exhibits for Appendix 3

Page 4 - Graphs of retention vs net aggregate claims for the liability example.

Page 5 - Graphs of the normal approximation confidence intervals for the property example.

Reinsurance and Retentions Working Party
Sample Data Distribution Used in Examples
Aviation LMX
Amounts in £000s

Claim Amount	Probability Point
22	2.439%
35	4.878%
235	7.317%
236	9.756%
244	12.195%
280	14.634%
332	17.073%
332	19.512%
338	21.951%
360	24.390%
598	26.829%
666	29.268%
693	31.707%
723	34.146%
750	36.585%
766	39.024%
795	41.463%
997	43.902%
1,006	46.341%
1,035	48.780%
1,080	51.220%
1,615	53.659%
2,507	56.098%
2,635	58.537%
2,635	60.976%
3,622	63.415%
3,832	65.854%
4,042	68.293%
4,551	70.732%
4,868	73.171%
5,800	75.610%
6,247	78.049%
8,865	80.488%
15,714	82.927%
20,160	85.366%
24,670	87.805%
25,587	90.244%
49,912	92.683%
52,211	95.122%
83,445	97.561%

Severity Mean = 9175

Claim Frequency Distribution = Poisson

Mean Claims Per Year = 8.000